19CS10071

Q3) $A_{3\times 2}$ with full column rank

$\eta^T_{3\times 1}$ (non zero) $\mid$ $\eta^T A = 0$

To prove for any $b \mid n^T b = 0$, $Ax = b$ has unique sol^n

$\because$ A has full col^n rank, if $b \in$ ColSpace(A),

$A = \begin{bmatrix} | & | \\ a_1 & a_2 \\ | & | \end{bmatrix}$; then $b = x_1 a_1 + x_2 a_2$

$\because a_1, a_2$ are L.I, in this case x_1, x_2 are unique

Obsv 1: $\therefore$ If $b \in$ Col Space(A) $\Rightarrow$ Unique Solution —— (1)

So if we prove $b \in$ Col Space(A), we are done

$\because n^T A = 0$ & $n^T b = 0$ $\qquad n^T_{1\times 3} A_{3\times 2} = (0 \;\; 0)$

$\Rightarrow A^T \eta = 0 \quad \Rightarrow b^T \eta = 0$

Let us assume b not in ColSpace of A

$\therefore a_1, a_2, b$ are linearly independent

Let $C = (A|b)_{3\times 3}$

$\therefore$ C has rank 3

Consider $D = C^T$ $\therefore$ D also has rank 3,

Now $n^T A = 0$ & $n^T b = 0 \Rightarrow n^T (A|b) = (0\,0\,0)_{1\times 3}$

$\Rightarrow n^T C = 0_{1\times 3} \Rightarrow C^T \eta = 0_{3\times 1}$

$\Rightarrow \underset{3\times3}{D}\,\underset{3\times1}{\eta} = 0_{3\times1}$

But D has rank 3 so 3 linearly independent columns,

$\Rightarrow \eta = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

But η is given as nonzero

So we get contradiction

$\Rightarrow$ Our assertion b is not in colspace(A) is wrong

Hence $b \in$ ColSpace(A) [Solution exists]

By Obsv. 1, (showed earlier)

$Ax = b$ has unique solution

(Hence Proved