

Q6

$$f(u,v) = \theta_1 + \theta_2 u + \theta_3 v + \theta_4 uv = \begin{bmatrix} 1 & u & v & uv \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_M & y_M & x_M y_M \end{bmatrix}_{MN \times 4}$$

$$A_{ij} = \begin{cases} 1 & ; j=1 \\ x_{(i-1) \bmod M + 1} & ; j=2 \\ y_{\lfloor \frac{i-1}{M} \rfloor + 1} & ; j=3 \\ x_{(i-1) \% M + 1} y_{\lfloor \frac{i-1}{M} \rfloor + 1} & ; j=4 \end{cases}$$

Shape of A:  $MN \times 4$ 

$$b_{MN \times 1} = \begin{bmatrix} p_{1,1} \\ p_{1,2} \\ \vdots \\ p_{MN} \end{bmatrix} \quad b_i = p_{(i-1) \% M + 1, \lfloor \frac{i-1}{M} \rfloor + 1}$$

Shape of b:  $MN \times 1$ 

$$Q = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}_{4 \times 1} \quad \text{Shape of } Q: 4 \times 1$$

b) For unique solution to  $AQ = b$ 

- (i) Columns should be L.I.
- (ii)  $b \in \text{Col-Span}(A)$

We have 4 columns in A, So for LI of columns, # rows  $\geq 4$

$\Rightarrow M \geq 4$  Values of M, N can be (1,4), (2,2), (4,1) for minimal M, N.

So the above condition is necessary for unique solution to  $AQ = b$