Q2) a) $\operatorname{avg}(x_{n\times 1}) = \frac{1}{n} 1_n^T x$

$\therefore \operatorname{avg}(\alpha x + \beta 1_n) = \frac{1}{n} 1_n^T [\alpha x + \beta 1_n]$

$= \frac{1}{n} \alpha 1_n^T x + \frac{1}{n} \beta 1_n^T 1_n$

$= \alpha \operatorname{avg}(x) + \frac{\beta}{n} \times n$

$= \alpha \operatorname{avg}(x) + \beta$ [Hence Proved]

(ii) $\operatorname{std}(x) = \frac{\|x - \operatorname{avg}(x)\|}{\sqrt{n}}$

$= \frac{\left\| x - \frac{(1_n^T x) 1_n}{n} \right\|}{\sqrt{n}}$

$\operatorname{std}(\alpha x + \beta 1_n) = \frac{\left\| \alpha x + \beta 1_n - \frac{(1_n^T(\alpha x + \beta 1_n)) 1_n}{n} \right\|}{\sqrt{n}}$

$= \frac{\left\| \alpha x + \beta 1_n - \frac{\alpha (1_n^T x) 1_n + \beta (1_n^T 1_n) 1_n}{n} \right\|}{\sqrt{n}}$

$= \frac{\left\| \alpha\left(x - \frac{(1_n^T x) 1_n}{n}\right) + \beta 1_n - \frac{\beta \cancel{n} 1_n}{\cancel{n}} \right\|}{\sqrt{n}}$

$= \frac{\left\| \alpha\left(x - \frac{(1_n^T x) 1_n}{n}\right) \right\|}{\sqrt{n}} = |\alpha| \operatorname{std}(x)$

[Using $\|\alpha A\| = |\alpha| \|A\|$]