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Q4) $L = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mid x_1 + x_2 + x_3 = 0 \quad \in \mathbb{R}^3$

To prove L is subspace of $\mathbb{R}^3$

$\because$ $\mathbb{R}^3$ is already a vector space, if we show the (i) Closure under Addition
(ii) Closure under Scalar Multiplication
(iii) Existese of Additive Identity
(iv) Existence of Additive Inverse

The L will be a subspace of $\mathbb{R}^3$

(i) Addition Closure

Let $u, v \in L$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \mid u_1 + u_2 + u_3 = 0$$

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \mid v_1 + v_2 + v_3 = 0$$

$$\therefore w = u + v = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{pmatrix}$$

Now $(u_1 + v_1) + (u_2 + v_2) + (u_3 + v_3) = (u_1 + u_2 + u_3) + (v_1 + v_2 + v_3)$
$= 0$

$\Rightarrow w \in L$ hence closed under Addition

(ii) Scalar Multiplication Closure

Let $u \in L$ $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

α be a scalar

$\therefore w = \alpha u = \begin{pmatrix} \alpha u_1 \\ \alpha u_2 \\ \alpha u_3 \end{pmatrix}$, Now,

$\alpha u_1 + \alpha u_2 + \alpha u_3 = \alpha(u_1 + u_2 + u_3) = 0$

$\therefore w = \alpha u \in L$, hence closed under scalar multiplication

(iii) Additive Identity

$\underline{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in L$, $\because 0+0+0=0$

(iv) Let $u \in L$, $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \Big| u_1 + u_2 + u_3 = 0$

$\therefore v = \begin{pmatrix} -u_1 \\ -u_2 \\ -u_3 \end{pmatrix}$ is additive inverse of u since $u + v = \underline{0}$

Now, $-u_1 - u_2 - u_3 = -(u_1 + u_2 + u_3) = 0$

$\Rightarrow v \in L$

Hence Additive Inverse exists $\forall u \in L$,

So proved that L is a subspace of $\mathbb{R}^3$

b) $\underset{1}{x_1} + \underset{1}{x_2} + \underset{1}{x_3} = 0$ defines a plane (say Δ) with normal along, $n = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

i.e normal of reflectionplane $(u) = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$
(normalized)

$$= \begin{pmatrix}\frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\end{pmatrix}$$

$$\therefore Q = I - 2uu^T$$

$$uu^T = \begin{pmatrix}\frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix} = \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix}$$

$$\therefore Q = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}\frac{2}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3}\\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3}\\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3}\end{bmatrix} \quad \text{(Ans)}$$