

19CS10071

Q2)  $a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$  First  $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} y \\ 0 \\ 2 \end{bmatrix} \quad y = \sqrt{1+2^2} = \sqrt{5}$

$$\begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} \sqrt{5} \\ 0 \\ 2 \end{bmatrix}$$

$$c - 2s = \sqrt{5} \quad \Rightarrow s = -2c$$

$$s + 2c = 0 \quad \therefore c + 4c = \sqrt{5} \Rightarrow c = \frac{1}{\sqrt{5}} \\ s = \frac{-2}{\sqrt{5}}$$

$$\therefore Q_1^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now  $\begin{bmatrix} \sqrt{5} \\ 0 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} y' \\ 0 \\ 0 \end{bmatrix} \quad y' = \sqrt{5+4} = 3$

$$\begin{bmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c \end{bmatrix} \begin{bmatrix} \sqrt{5} \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore c\sqrt{5} - 2s = 3$$

$$s\sqrt{5} + 2c = 0 \Rightarrow s = \frac{-2}{\sqrt{5}} c$$

$$\therefore c \left( \sqrt{5} + \frac{4}{\sqrt{5}} \right) = 3 \Rightarrow c \left( \frac{9}{\sqrt{5}} \right) = 3 \\ \Rightarrow c = \frac{\sqrt{5}}{3}$$

$$\therefore s = -\frac{2}{3}$$

$$\therefore Q_2^T = \begin{bmatrix} \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$Q = Q_2^T Q_1^T = \begin{bmatrix} \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} + 0 - 0 & \frac{2}{3} + 0 + 0 & 0 + 0 + \frac{2}{3} \\ 0 - \frac{2}{\sqrt{5}} + 0 & 0 + \frac{1}{\sqrt{5}} + 0 & 0 + 0 + 0 \\ -\frac{2}{3\sqrt{5}} + 0 + 0 & -\frac{4}{3\sqrt{5}} + 0 + 0 & 0 + 0 + \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{2}{3\sqrt{5}} & -\frac{4}{3\sqrt{5}} & \frac{\sqrt{5}}{3} \end{bmatrix}$$

b)  $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$   $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 - 2 \times 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 2 - 2 \times 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad l_{21} = -\frac{a_{21}}{a_{11}} = -\frac{2}{1} = -2$$

$$L_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$l_{31} = -\frac{a_{31}}{a_{11}} = -\frac{2}{1} = -2$$

$$L_{21} L_{31} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$