19CS10071

Q7) $A \in \mathbb{R}^{n \times n}$ be a invertible matrix

$\therefore\ l_{ij} = \frac{a'_{ij}}{a'_{jj}}$ $[a'_{ij}$ current A$]$ $L_{ij}^{-1} = I + i\begin{array}{c} j \\ \begin{bmatrix} 0 & 0 & \cdots \\ 0 & & \\ \vdots & -l_{ij} & \end{bmatrix} \end{array}$

We find $L_{21}^{-1}, L_{31}^{-1} \cdots L_{n1}^{-1}$, the $L_{32}^{-1}\ L_{42}^{-1} \cdots L_{n2}^{-1} \cdots\cdots$

$$Ax = b$$

$$\Rightarrow L_{21}^{-1}Ax = L_{21}^{-1}b$$

$$\Rightarrow \cdots L_{n1}^{-1}L_{n-1\,1}^{-1} \cdots L_{21}^{-1}Ax = L_{n1}^{-1} \cdots L_{21}^{-1}b$$

$$\Rightarrow \underbrace{L_{n\,n-1}^{-1} \cdots\cdots L_{21}^{-1}A}_{U}x = \underbrace{L_{n\,n-1}^{-1} \cdots L_{21}^{-1}}_{L^{-1}}b$$

$\Rightarrow\ Ux = L^{-1}b$ [From here we use back substitution to find x]

To find L, We can simply replace $-l_{ij}$ By l_{ij} in L^{-1},

For example Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}$

$l_{21} = \frac{2}{1} = 2$ $\therefore\ L_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$L_{21}^{-1}A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 4 & 1 & 1 \end{bmatrix}$

$l_{31} = \frac{4}{1} = 4;\ L_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$ $L_{31}^{-1}L_{21}^{-1}A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & -3 \end{bmatrix}$

$$\ell_{32} = \frac{1}{1} = 1 \qquad L_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$L_{32}^{-1} L_{31}^{-1} L_{21}^{-1} A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore L_{32}^{-1} L_{31}^{-1} L_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

And $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$

Where $A = LU$