

5) $Ax=b$

$$\text{Condition number of } A = \|A\|_2 \|A^{-1}\|_2 = \frac{\max \text{mag}(A)}{\min \text{mag}(A)}$$

For A to be orthogonal,

(i) $\max \text{mag}(A) = \min \text{mag}(A) = 1$ // For Orthogonal A

$$\text{Condition number of } A = 1,$$

$$\text{Hence } \frac{\| \delta x \|}{\| x \|} \leq 1 \cdot \frac{\| \delta b \|}{\| b \|}$$

So the solution is stable w.r.t perturbations.

Also thus orthogonal matrices effectively do some rotation/reflection or combination of both and does not do any scaling

(ii) Benefits in Computation

$$\text{for orthogonal } A, A^{-1} = A^T$$

Hence $x = A^T b$ is a solution to the equation.

Thus inverse computation is very simple