19CS10071

Q5)

i) $f_1 \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1} \rightarrow \begin{pmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ \vdots \\ \frac{x_{n-1}+x_n}{2} \\ x_n \end{pmatrix}_{(n+1)\times 1}$

Let transformation be denoted by f,

$$\therefore f(x+y) = f\begin{pmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{pmatrix} = \begin{pmatrix} x_1+y_1 \\ \frac{x_1+y_1+x_2+y_2}{2} \\ \vdots \\ \frac{x_{n-1}+y_{n-1}+x_n+y_n}{2} \\ x_n+y_n \end{pmatrix}$$

$$= \begin{pmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ \vdots \\ \frac{x_{n-1}+x_n}{2} \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ \frac{y_1+y_2}{2} \\ \vdots \\ \frac{y_{n-1}+y_n}{2} \\ y_n \end{pmatrix} = f(x)+f(y)$$

for α scalar,

$$f(\alpha x) = \begin{pmatrix} \alpha x_1 \\ \frac{\alpha x_1+\alpha x_2}{2} \\ \vdots \\ \frac{\alpha x_{n-1}+\alpha x_n}{2} \\ \alpha x_n \end{pmatrix} = \alpha \begin{pmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ \vdots \\ \frac{x_{n-1}+x_n}{2} \\ x_n \end{pmatrix}$$

$$= \alpha f(x)$$

Hence f is a linear Transformation

(ii) $f_2 \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \rightarrow \sum_{i=1}^{n} |x_i|$

Consider $\alpha = -1$, and all $x_i \geq 0$

$$f_2(\alpha x) = f_2 \begin{pmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{pmatrix} = \sum_{i=1}^{n} |-x_i|$$

$$= \sum_{i=1}^{n} |x_i|$$

But $\alpha f_2(x_1) = (-1) \sum_{i=1}^{n} |(x_i)| = -\sum_{i=1}^{n} x_i$

$[x_i \geq 0]$

$\therefore f_2(\alpha x) \neq \alpha f(x)$ in this case

Since we get a contradiction f_2 is not linear

(iii)

$$f_3 : \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \rightarrow \begin{bmatrix} \max(x_1, 0) \\ \vdots \\ \max(x_n, 0) \end{bmatrix}$$

Here Consider $\alpha = -1$ & all $x_i > 0$

$$\therefore f_3(\alpha x) = f_3\left(\begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Since $-x_i < 0$ $\forall i$ in $1 \ldots n$

$$\text{But } \alpha f_3(x) = -\begin{bmatrix} \max(x_1, 0) \\ \vdots \\ \max(x_n, 0) \end{bmatrix} = -\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$\therefore \alpha f_3(x) \neq f_3(\alpha x)$

Hence f_3 is not a linear transformation