Q5) Associativity of matrix multiplication:

Consider $A_{p\times q}$, $B_{q\times r}$, $C_{r\times t}$

$$(A_{p\times q}\ B_{q\times r})\ C_{r\times t} = D_{p\times t}\ ;\ A(BC) = D'$$

$$d_{ij} = \sum_{k=1}^{r}\left(\sum_{l=1}^{q} a_{il}\ b_{lk}\right)\times c_{kj}$$

$$= \sum_{k=1}^{r}\sum_{l=1}^{q} a_{il}\ b_{lk} c_{kj}$$

$$= \sum_{k=1}^{r}\sum_{l=1}^{q} a_{il}\ (b_{lk}\ c_{kj})$$

$$= \sum_{l=1}^{q} a_{il}\left(\sum_{k=1}^{r} b_{lk}\cdot c_{kj}\right)$$

$$\Rightarrow d_{ij} = d'_{ij}$$

$$\Rightarrow D = D'$$

$$\Rightarrow (AB)\,C = A(BC)$$

$\therefore$ Matrix multiplication is associative

Not Commutative, Consider $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ $B = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$$\therefore AB = \begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$$

$$BA = \begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix} \quad \therefore AB \neq BA$$

So matrix multiplication is not commutative.

$A_{p \times q} \quad B_{q \times r} \quad C_{r \times t} \quad D_{p \times t}$

$A\,B\,C = D$

If we do $(AB)C$, Number of scalar multiplications

$= pqr + prt$

For $A(BC) \rightarrow pqt + qrt$

For $(AB)C$ to be more effective,

$$pqr + prt < pqt + qrt$$

$$\Rightarrow pr(q+t) < qt(p+r)$$

$$\Rightarrow \boxed{\frac{1}{q} + \frac{1}{t} < \frac{1}{p} + \frac{1}{r}}$$