

Q8) $\hat{z}_{t+1} = \theta_1 z_t + \theta_2 z_{t-1} + \dots + \theta_M z_{t-M+1}$

$$\therefore A = \begin{bmatrix} z_M & z_{M-1} & \dots & z_1 \\ z_{M+1} & z_M & \dots & z_2 \\ z_{M+2} & \dots & \dots & \dots \\ \vdots & \dots & \dots & \vdots \\ z_{100} & \dots & \dots & z_{101-M} \end{bmatrix}_{101-M \times M}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_M \end{bmatrix}_{M \times 1}$$

$$b = \begin{bmatrix} z_{M+1} \\ \vdots \\ z_{101} \end{bmatrix}_{101-M \times 1}$$

$A\theta = b$: LS problem.

Observations:

1. Diagonally elements in A are same

$$\begin{bmatrix} z_M & z_{M-1} & \dots & z_1 \\ & z_M & \dots & z_2 \\ & & \ddots & \vdots \\ & & & z_M & \dots & z_n \end{bmatrix}$$

2. Thus only 100 values are required to specify entire A , i.e. z_1, z_2, \dots, z_{100}

Relation of Rank(A) with M .

$$\text{Rank} \leq \min(M, 101-M) \left[\begin{array}{l} \because \text{Row Rank} = \text{Col Rank} \\ \& \text{Row Rank} \leq 101-M \\ \& \text{Col Rank} \leq M \end{array} \right]$$