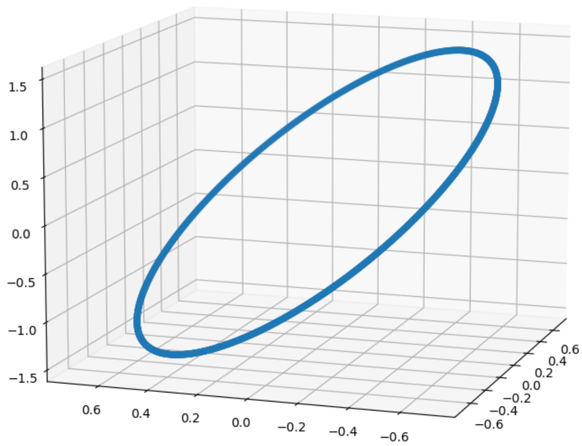
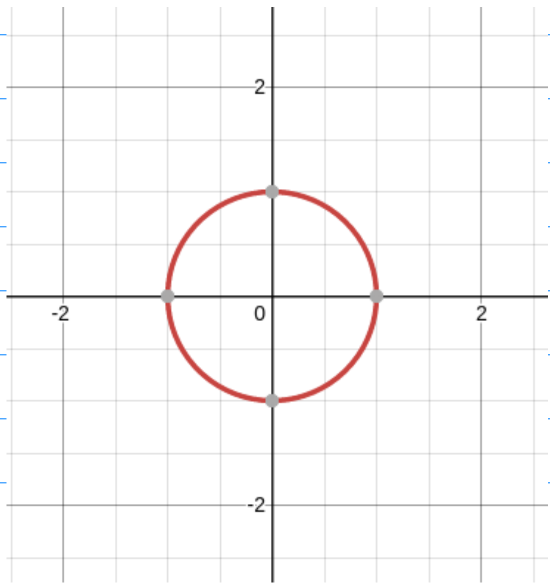
19CS10071

Q3) a) $\begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \\ -1 & 1 \end{pmatrix}$ Consider $(\cos\theta, \sin\theta)$

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}}\cos\theta \\ -\frac{1}{\sqrt{2}}\sin\theta \\ -\cos\theta + \sin\theta \end{bmatrix}$$

Plotting the parametric eq^n in 3D in geogebra we get an ellipse

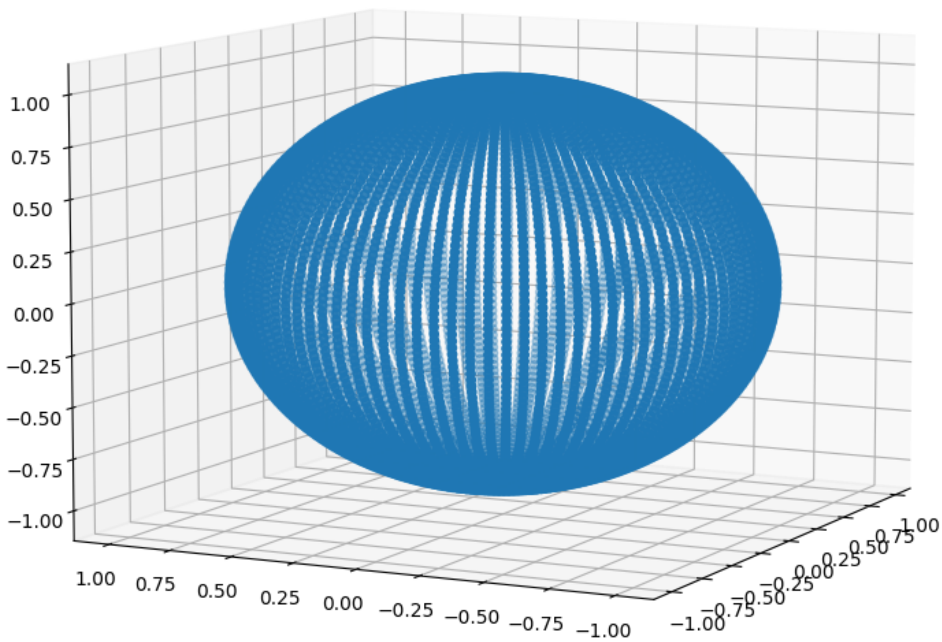


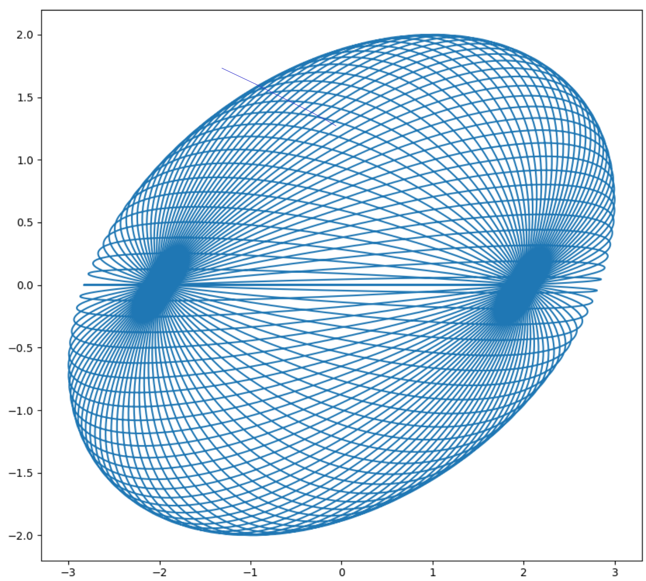


A

$$k(A) = 2.23606797749979$$

(ii) $\begin{pmatrix} -2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}$



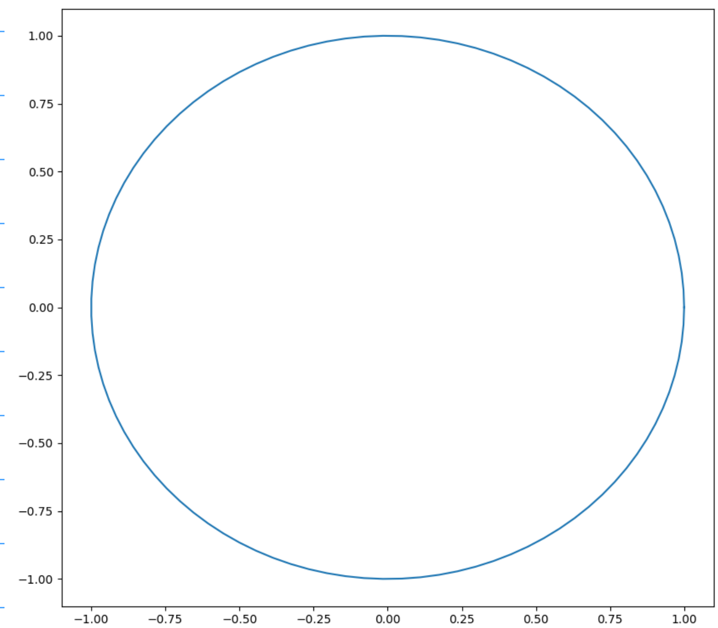


A

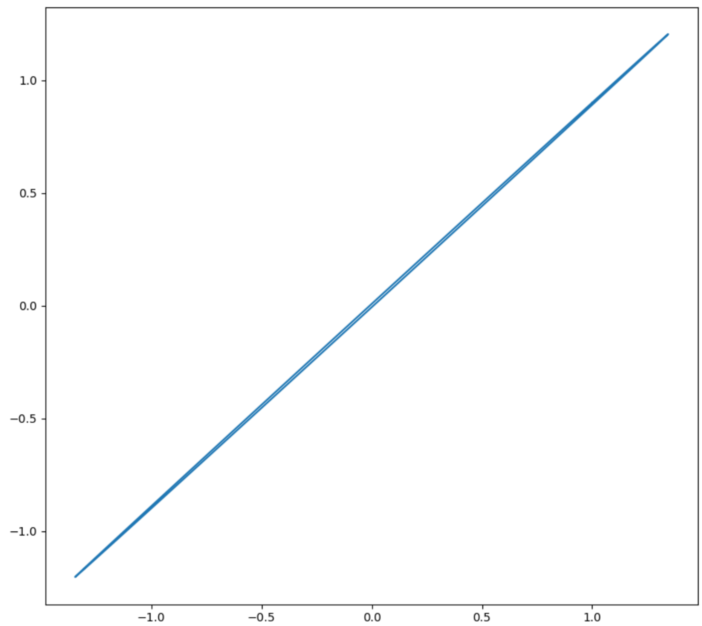
On taking more points we would see that the image will span the entire elliptical area

$\kappa(A) = 1.715010090561728$

c) $\begin{pmatrix} 1 & 0.9 \\ 0.9 & 0.8 \end{pmatrix}$



A



Highly eccentric ellipse

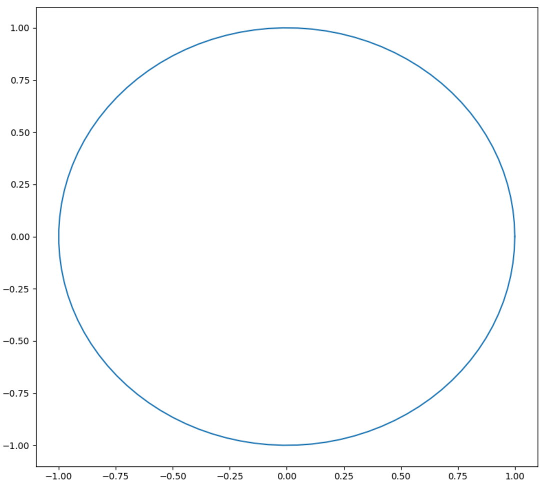
$k(A) =$ 325.99693248647924

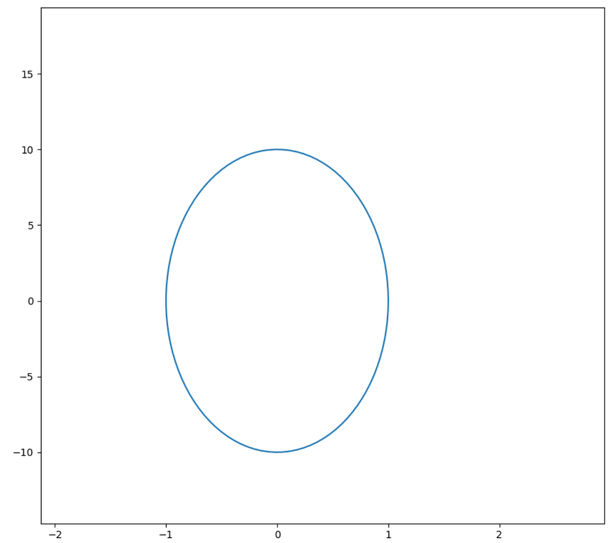
$\det(A) =$ -0.010000000000000004

$\therefore \; |A| \neq 0 \quad A$ is invertible

We see it is an almost singular matrix and has a high condition number

(iv) $\begin{pmatrix} 1 & 0 \\ 0 & -10 \end{pmatrix}$



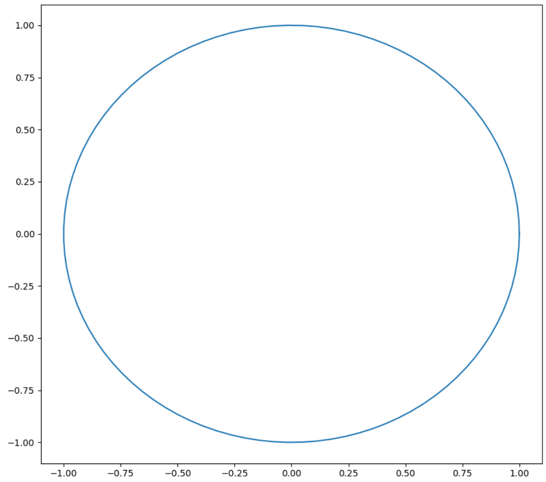


A

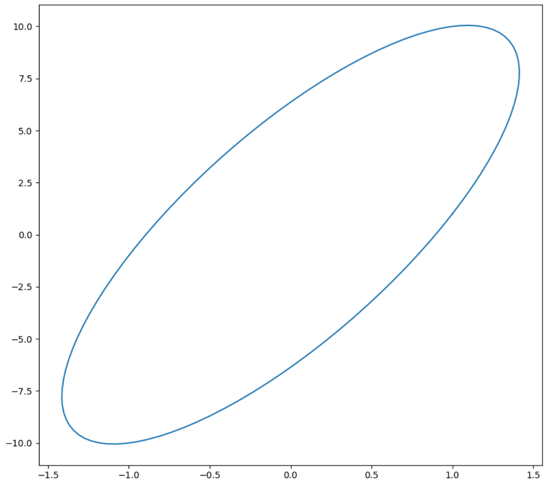
$k(A) =$ 10.0 .

$|A| = -10 \qquad |A| \neq 0$ hence invertible

(e) (i) $\begin{pmatrix} 1 & 1 \\ 1 & 10 \end{pmatrix}$ $\epsilon = 10$



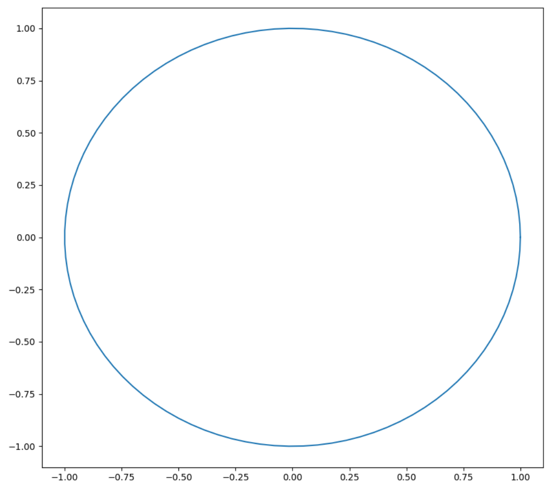
$\xrightarrow{A}$



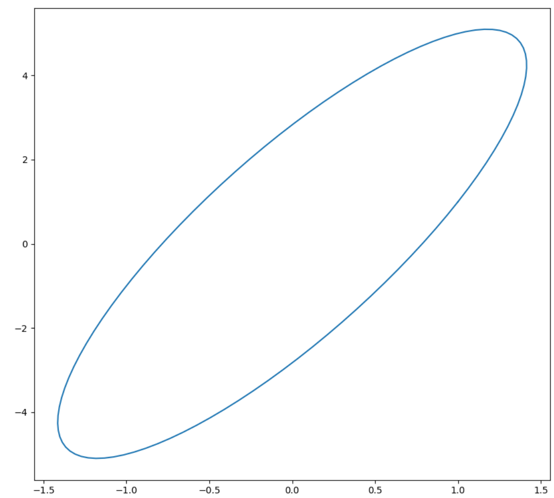
$k(A) =$ 11.35638827945676

$det(A) = 9$ $|A| \neq 0$ hence invertible

(ii) $\begin{pmatrix} 1 & 1 \\ 1 & 5 \end{pmatrix}$ $\epsilon = 5$



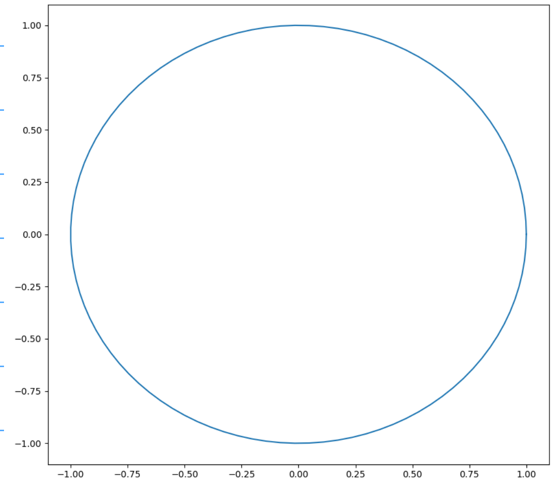
$\xrightarrow{A}$



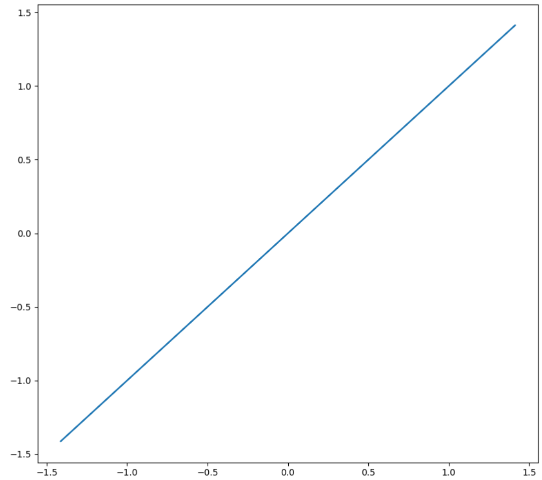
$k(A) =$ 6.854101966249685

$|A| = 4 \neq 0$ so ~~inte~~ invertible

(iii) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\epsilon = 1$



$\xrightarrow{A}$



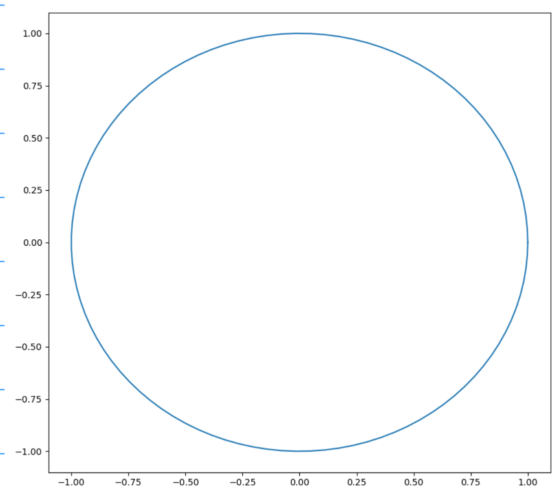
$|A| = 0$ NOT invertible

$k(A) =$ undefined (minimag$(A) = 0$)

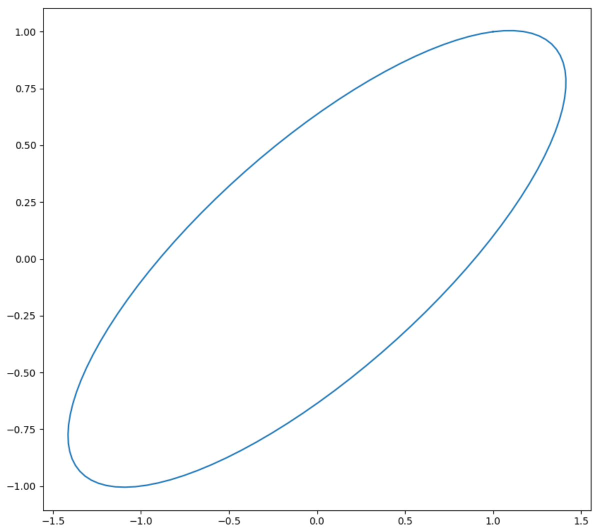
(iv) $\epsilon = 10^{-1}$

$A = \begin{pmatrix} 1 & 1 \\ 1 & 0.1 \end{pmatrix}$ $k(A) =$ 3.0124935233004138

$|A| = 0.9$ (Hence Invertible)



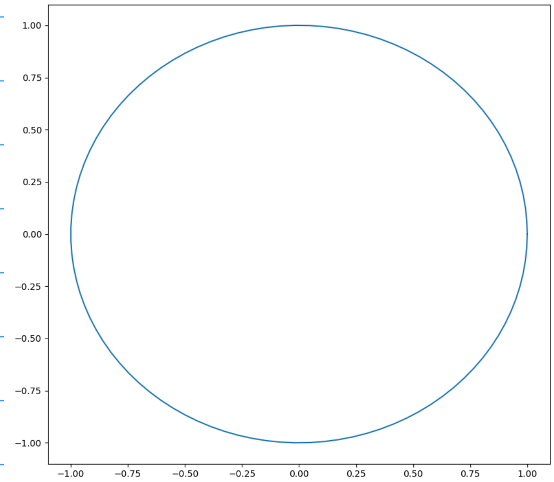
$\xrightarrow{A}$



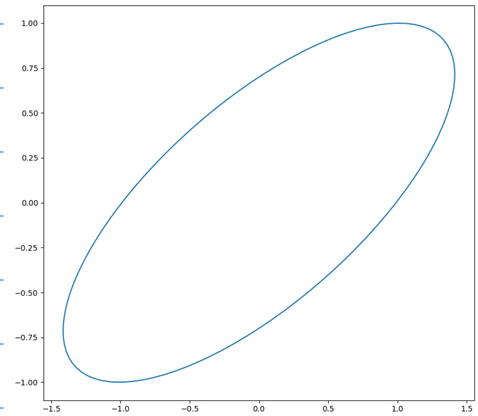
(v) $\epsilon = 10^{-2}$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0.01 \end{pmatrix} \qquad k(A) = 2.653550456325284\text{7}$$

$$|A| = -0.99$$



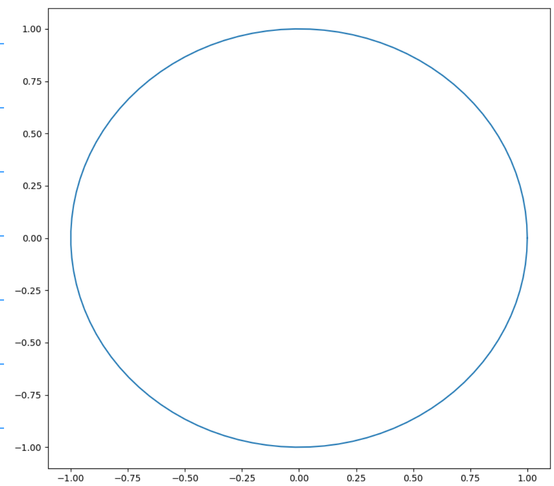
$\xrightarrow{A}$



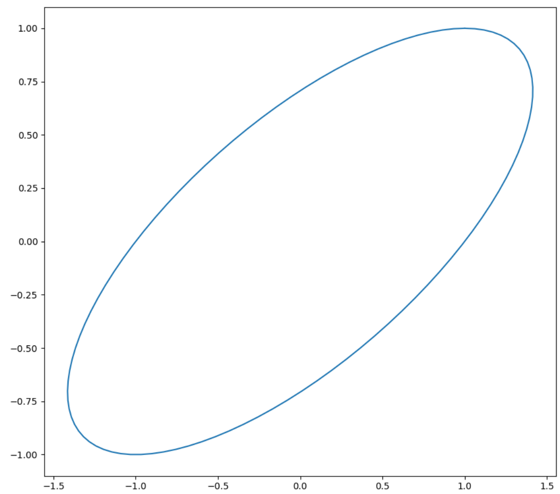
(vi) $\epsilon = 10^{-4}$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0.0001 \end{pmatrix} \qquad k(A) = 2.618385273654826$$

$|A| = -0.9999$ (Hence Invertible)

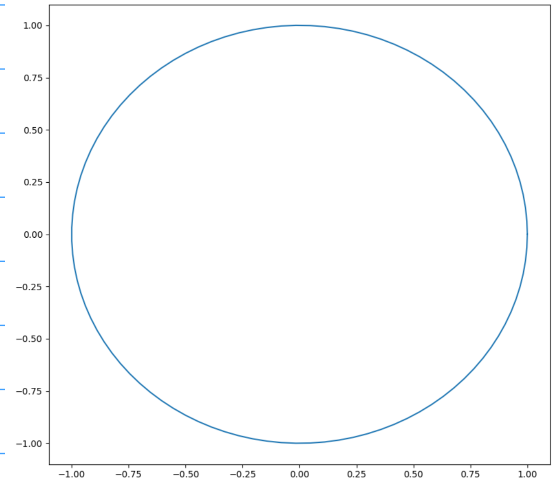


$\xrightarrow{A}$

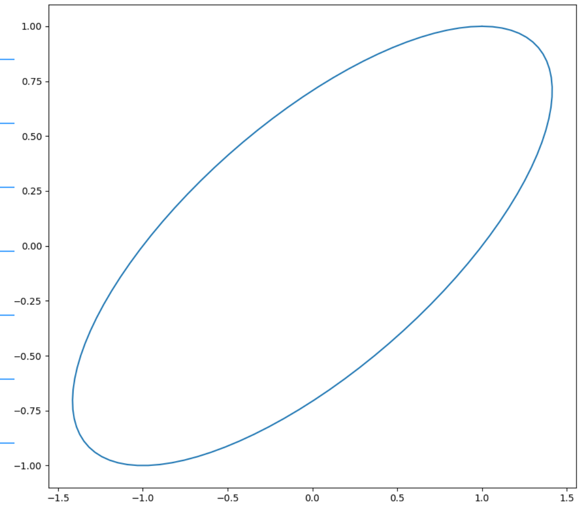


(vii) $\epsilon = 0$ $\qquad k(A) =$ 2.6180339887498953

$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad |A| = -1$ (Hence Invertible)



$\xrightarrow{A}$



Observation on det & $k(A)$

→ As a matrix → singular i.e $|A| \to 0$, $k(A) \to \infty$

Code Used to plot (Parts were edited while running the different parts

```
import matplotlib.pyplot as plt
import numpy as np
plt.rcParams["figure.figsize"] = [10,10]
            .
fig = plt.figure()

r = 1
#u,v = np.mgrid[0:2 * np.pi:100j,0:np.pi:100j]
u = np.mgrid[0:2*np.pi:100j]
#print(u.shape)
#X = np.array([np.cos(u)*np.sin(v) ,np.sin(u)*np.sin(v),np.cos(v) ])
X = np.array([np.cos(u),np.sin(u)]).reshape((2,100))
#X = X.reshape((3,1,10000))

#ax = plt.axes(projection='3d')
#ax.scatter3D(X[0], X[1], X[2])
#print(X)
plt.plot(X[0],X[1])
plt.show()
X = X.reshape((2,1,100))
#x = X[0,0]
#y = X[1,0]

#axes.plot(x, y)
#plt.show()
A = np.array([[1,1],[1,0]]).reshape((2,2))#input matrix
#Print condition number
print(np.linalg.cond(A))

#print determinant
print('Determinant : ',np.linalg.det(A))
Y = np.array([np.dot(A,X[:,:,i]) for i in range(100)]).T.reshape(2,100)#do transformation



x = Y[0]
y = Y[1]
#z = Y[2]

print(x.shape)

#ax = plt.axes(projection='3d')
#ax.scatter3D(x, y, z)
plt.plot(x,y)
plt.show()
```