19CS10071

Q6) A matrix B that satisfies

$$BA = I$$, for matrix A is said to be the left inverse of A.

ii) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}_{5\times 1}$ It is one nonzero column so columns are linearly independent. Hence A is left invertible

Let $b = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$ be left inverse

$\therefore \ bA = I \Rightarrow A^T b^T = I$

Now if we have one solution of b, like

$$b^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ i.e } b = [1 \ 0 \ 0 \ 0 \ 0]$$

Consider $A^T x = 0$, then

$$A^T b^T = A^T (b^T + \alpha x) = I$$

So in general we have solution as $b + \alpha x^T$

$$A^T x = 0 \Rightarrow [1 \ 0 \ 0 \ 1 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix} = 0$$

$\therefore \ x_1 + x_4 = 0$

$\therefore$ Sol^n : $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ -x_1 \\ x_5 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Dim of Solution Space = 4

(ii) $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix}$ $\quad a_1 \quad a_2$

$a_2 \neq k a_1$, So a_1 and a_2 are L.I.

[For two vectors a_1, a_2, they are Linearly dependent iff $k_1 a_1 = k_2 a_2$, for some k_1, k_2]

$\therefore$ A has L.I columns, A is left invertible

Let left inverse be $B = \begin{bmatrix} — b_1 — \\ — b_2 — \end{bmatrix}$

$\therefore BA = \begin{bmatrix} — b_1 — \\ — b_2 — \end{bmatrix} \begin{bmatrix} | & | \\ a_1 & a_2 \\ | & | \end{bmatrix} = \begin{bmatrix} b_1 a_1 & b_1 a_2 \\ b_2 a_1 & b_2 a_2 \end{bmatrix}$

$a_1 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$

$\therefore b_1 a_1 = 1 \quad b_1 a_2 = 0$

$b_2 a_1 = 0 \quad b_2 a_2 = 1$

$\therefore A^T b_1^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ — ① $\quad A^T b_2^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ — ②

$A^T = \begin{bmatrix} 2 & 0 & 3 \\ 0 & -2 & 3 \end{bmatrix}$ $\Rightarrow$ Rank = 2 (Echelon Form)

For ①

Particular Sol^n $b_{1,p}^T = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}$

General Sol^n: $A^T x = 0 \Rightarrow \begin{bmatrix} 2 & 0 & 3 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\Rightarrow \begin{array}{l} 2x_1 + 3x_3 = 0 \\ -2x_2 + 3x_3 = 0 \end{array}$ $\therefore x = \begin{bmatrix} -\frac{3}{2} \\ 3/2 \\ 1 \end{bmatrix} x_3$

In general $b_1^T = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -3/2 \\ 3/2 \\ 1 \end{bmatrix}$

$\Rightarrow b_1 = [\frac{1}{2} \ 0 \ 0] + \alpha_1 [-3/2 \ \frac{3}{2} \ 1]$

similarly for ②,

$$b_{2p}^{T} = \begin{bmatrix} 0 \\ -1/2 \\ 0 \end{bmatrix} \qquad x(\text{Same}) = \begin{bmatrix} -3/2 \\ 3/2 \\ 1 \end{bmatrix} x_3$$

$$\therefore b_2 = \begin{bmatrix} 0 & -1/2 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} -3/2 & 3/2 & 1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} - b_1 - \\ - b_2 - \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & -1/2 & 0 \end{bmatrix} + \alpha_1 \begin{bmatrix} -3/2 & 3/2 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 & 0 & 0 \\ -3/2 & 3/2 & 1 \end{bmatrix}$$

is a left inverse of A for any α_1, α_2.