The LP Model - Formulation and Solution

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2023-09-27

library(lpSolveAPI)

Problem Statement: A renowned chocolatier, Francesco Schröeder, makes three kinds of chocolate confectionery: artisanal truffles, handcrafted chocolate nuggets, and premium gourmet chocolate bars. He uses the highest quality of cacao butter, dairy cream, and honey as the main ingredients. Francesco makes his chocolates each morning, and they are usually sold out by the early afternoon. For a pound of artisanal truffles, Francesco uses 1 cup of cacao butter, 1 cup of honey, and 1/2 cup of cream. The handcrafted nuggets are milk chocolate and take 1/2 cup of cacao, 2/3 cup of honey, and 2/3 cup of cream for each pound. Each pound of the chocolate bars uses 1 cup of cacao butter, 1/2 cup of honey, and 1/2 cup of cream. One pound of truffles, nuggets, and chocolate bars can be purchased for $35, $25, and $20, respectively. A local store places a daily order of 10 pounds of chocolate nuggets, which means that Francesco needs to make at least 10 pounds of the chocolate nuggets each day. Before sunrise each day, Francesco receives a delivery of 50 cups of cacao butter, 50 cups of honey, and 30 cups of dairy cream. 1) Formulate and solve the LP model that maximizes revenue given the constraints. How much of each chocolate product should Francesco make each morning? What is the maximum daily revenue that he can make? 2) Report the shadow price and the range of feasibility of each binding constraint. 3) If the local store increases the daily order to 25 pounds of chocolate nuggets, how much of each product should Francesco make? We will solve this problem with two approaches: First by directly encoding the variables and coefficients and secondly, by using a .lp file \*1.Formulate and solve the LP model that maximizes revenue given the constraints. How much of each chocolate product should Francesco make each morning? What is the maximum daily revenue that he can make? We define for Decision Variables: Let P pounds of Artisanal Truffles, and Q pounds of handcrafted Chocolate nuggets, R pounds of premium gourmet Chocolate bars.

Objective Maximization = 35P + 25Q + 20R The following constraints Cacao butter: 1x1 + 1/2x2 + 1x3 <= 50; Honey: 1x1 + 2/3x2 + 1/2x3 <= 50; Cream: 1/2x1 + 2/3x2 + 1/2x3 <= 30; Chocolate nuggets: x2>= 10; x1,x3>=0 (Non negativity)

# Create lp object with 0 constraints and 3 decision variables  
lprec <- make.lp(0, 3)  
# Now create the objective function.  
set.objfn(lprec, c(35, 25, 20))  
# As the default is a minimization problem, so we change that to maximization  
lp.control(lprec,sense='max')

## $anti.degen  
## [1] "fixedvars" "stalling"   
##   
## $basis.crash  
## [1] "none"  
##   
## $bb.depthlimit  
## [1] -50  
##   
## $bb.floorfirst  
## [1] "automatic"  
##   
## $bb.rule  
## [1] "pseudononint" "greedy" "dynamic" "rcostfixing"   
##   
## $break.at.first  
## [1] FALSE  
##   
## $break.at.value  
## [1] 1e+30  
##   
## $epsilon  
## epsb epsd epsel epsint epsperturb epspivot   
## 1e-10 1e-09 1e-12 1e-07 1e-05 2e-07   
##   
## $improve  
## [1] "dualfeas" "thetagap"  
##   
## $infinite  
## [1] 1e+30  
##   
## $maxpivot  
## [1] 250  
##   
## $mip.gap  
## absolute relative   
## 1e-11 1e-11   
##   
## $negrange  
## [1] -1e+06  
##   
## $obj.in.basis  
## [1] TRUE  
##   
## $pivoting  
## [1] "devex" "adaptive"  
##   
## $presolve  
## [1] "none"  
##   
## $scalelimit  
## [1] 5  
##   
## $scaling  
## [1] "geometric" "equilibrate" "integers"   
##   
## $sense  
## [1] "maximize"  
##   
## $simplextype  
## [1] "dual" "primal"  
##   
## $timeout  
## [1] 0  
##   
## $verbose  
## [1] "neutral"

Now Adding the all the 4 constraint values in the model

# Adding four constraints  
add.constraint(lprec, c(1, 1/2, 1), "<=", 50)  
add.constraint(lprec, c(1, 2/3, 1/2), "<=", 50)  
add.constraint(lprec, c(1/2, 2/3, 1/2), "<=", 30)  
add.constraint(lprec, c(0, 1, 0), ">=", 10)  
  
# Set bounds for variables.  
set.bounds(lprec, lower = c(0, 0, 0), columns = c(1, 2, 3))  
  
# To identify the variables and constraints, we can set variable names and name the constraints  
RowNames <- c("CacaoButter", "Honey", "DiaryCream", "NUggetsOrder")  
ColNames <- c("AritisanTruffel", "ChocalateNuggets", "ChocalateBars")  
  
dimnames(lprec) <- list(RowNames, ColNames)  
lprec #Printing the model

## Model name:   
## AritisanTruffel ChocalateNuggets ChocalateBars   
## Maximize 35 25 20   
## CacaoButter 1 0.5 1 <= 50  
## Honey 1 0.666666666667 0.5 <= 50  
## DiaryCream 0.5 0.666666666667 0.5 <= 30  
## NUggetsOrder 0 1 0 >= 10  
## Kind Std Std Std   
## Type Real Real Real   
## Upper Inf Inf Inf   
## Lower 0 0 0

To save the model

write.lp(lprec, filename = "lpmodel.lp", type = "lp")  
solve(lprec) #Solving the above problem

## [1] 0

get.objective(lprec)

## [1] 1780

varV <- get.variables(lprec)

Using the LP problem, we created a text file using write.lp statement. Using the read.lp statement, we can take a look at the lpmodel.lp file.

x <- read.lp("lpmodel.lp")  
x

## Model name:   
## AritisanTruffel ChocalateNuggets ChocalateBars   
## Maximize 35 25 20   
## CacaoButter 1 0.5 1 <= 50  
## Honey 1 0.666666666667 0.5 <= 50  
## DiaryCream 0.5 0.666666666667 0.5 <= 30  
## NUggetsOrder 0 1 0 >= 10  
## Kind Std Std Std   
## Type Real Real Real   
## Upper Inf Inf Inf   
## Lower 0 0 0

solve(x)

## [1] 0

get.objective(x)

## [1] 1780

get.variables(x)

## [1] 40 12 4

get.constraints(x)

## [1] 50 50 30 12

According to the solution, the revenue is 1780. The first variable value is 40, and the second variable value is 12 and the third variable value is 4. To get shadow price and reduced cost

get.sensitivity.rhs(lprec)

## $duals  
## [1] 2 30 6 0 0 0 0  
##   
## $dualsfrom  
## [1] 4.750000e+01 3.000000e+01 2.916667e+01 -1.000000e+30 -1.000000e+30  
## [6] -1.000000e+30 -1.000000e+30  
##   
## $dualstill  
## [1] 5.166667e+01 5.200000e+01 5.000000e+01 1.000000e+30 1.000000e+30  
## [6] 1.000000e+30 1.000000e+30

get.sensitivity.obj(lprec)

## $objfrom  
## [1] 20.00 22.50 18.75  
##   
## $objtill  
## [1] 38.00000 26.66667 35.00000

3.If the local store increases the daily order to 25 pounds of chocolate nuggets, how much of each product should Francesco make?

lprec1 <- make.lp(0, 3)   
set.objfn(lprec1, c(35, 25, 20))   
lp.control(lprec1,sense='max')

## $anti.degen  
## [1] "fixedvars" "stalling"   
##   
## $basis.crash  
## [1] "none"  
##   
## $bb.depthlimit  
## [1] -50  
##   
## $bb.floorfirst  
## [1] "automatic"  
##   
## $bb.rule  
## [1] "pseudononint" "greedy" "dynamic" "rcostfixing"   
##   
## $break.at.first  
## [1] FALSE  
##   
## $break.at.value  
## [1] 1e+30  
##   
## $epsilon  
## epsb epsd epsel epsint epsperturb epspivot   
## 1e-10 1e-09 1e-12 1e-07 1e-05 2e-07   
##   
## $improve  
## [1] "dualfeas" "thetagap"  
##   
## $infinite  
## [1] 1e+30  
##   
## $maxpivot  
## [1] 250  
##   
## $mip.gap  
## absolute relative   
## 1e-11 1e-11   
##   
## $negrange  
## [1] -1e+06  
##   
## $obj.in.basis  
## [1] TRUE  
##   
## $pivoting  
## [1] "devex" "adaptive"  
##   
## $presolve  
## [1] "none"  
##   
## $scalelimit  
## [1] 5  
##   
## $scaling  
## [1] "geometric" "equilibrate" "integers"   
##   
## $sense  
## [1] "maximize"  
##   
## $simplextype  
## [1] "dual" "primal"  
##   
## $timeout  
## [1] 0  
##   
## $verbose  
## [1] "neutral"

using the new constraints now,

add.constraint(lprec1, c(1, 1/2, 1), "<=", 50)  
add.constraint(lprec1, c(1, 2/3, 1/2), "<=", 50)  
add.constraint(lprec1, c(1/2, 2/3, 1/2), "<=", 30)  
add.constraint(lprec1, c(0, 1, 0), ">=", 25)

set.bounds(lprec1, lower = c(0, 0, 0), columns = c(1, 2, 3))

RowNames <- c("CacaoButter", "Honey", "DiaryCream", "NUggetsOrder")  
ColNames <- c("AritisanTruffel", "ChocalateNuggets", "ChocalateBars")  
  
dimnames(lprec1) <- list(RowNames, ColNames)  
lprec1

## Model name:   
## AritisanTruffel ChocalateNuggets ChocalateBars   
## Maximize 35 25 20   
## CacaoButter 1 0.5 1 <= 50  
## Honey 1 0.666666666667 0.5 <= 50  
## DiaryCream 0.5 0.666666666667 0.5 <= 30  
## NUggetsOrder 0 1 0 >= 25  
## Kind Std Std Std   
## Type Real Real Real   
## Upper Inf Inf Inf   
## Lower 0 0 0

write.lp(lprec1, filename = "chocalte.lp", type = "lp")  
  
solve(lprec1)

## [1] 0

get.objective(lprec1)

## [1] 1558.333

x <- read.lp("lpmodel.lp")  
x

## Model name:   
## AritisanTruffel ChocalateNuggets ChocalateBars   
## Maximize 35 25 20   
## CacaoButter 1 0.5 1 <= 50  
## Honey 1 0.666666666667 0.5 <= 50  
## DiaryCream 0.5 0.666666666667 0.5 <= 30  
## NUggetsOrder 0 1 0 >= 10  
## Kind Std Std Std   
## Type Real Real Real   
## Upper Inf Inf Inf   
## Lower 0 0 0

solve(lprec1)

## [1] 0

get.objective(lprec1)

## [1] 1558.333

get.variables(lprec1)

## [1] 26.66667 25.00000 0.00000

get.constraints(lprec1)

## [1] 39.16667 43.33333 30.00000 25.00000