UNIVERSITY OF TORONTO

Engineering Science

CHE260: Thermodynamics and Heat Transfer

Practical Session: PRA 0101 (Wednesday 3 - 6 p.m.)

Instructor: Professor Sanjeev Chandra and Arthur Chan

Lab Report No. 2

Investigating the First Law of Thermodynamics within a Pressurized Air-Tank System

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Date Performed: October 25th, 2023

Date Submitted: November 5th, 2023

Abstract

In this lab, the first law of thermodynamics was used to determine the specific heat capacity of air at constant volume (for temperatures of 40°C and 60°C). This quantity is essential to understanding how the internal energy of a volume of air changes with respect to temperature. The work done by the tank's propeller was also determined, and found to be negligible relative to the amount of heat transferred into the system (specifically, propeller work was only 0.00388% of the energy added to the system).

1 Introduction

The first law of thermodynamics, $Q + W = \Delta E$, is a fundamental principle in the field of thermodynamics. It is often referred to as the law of energy conservation and it states that the total energy of a closed system remains constant over time. This law is crucial because it provides a foundational understanding of how energy behaves in various physical and chemical processes. The purpose of this experiment is to better understand this first law. This will be done by first looking at the heat transfer across a system boundary and then by looking at the work done onto a given system.

2 Experimental Method

The aim of this experiment is three-fold: first to determine the mass in the left tank of the system, second to determine the heat lost in the left tank and specific heat capacity and third to compute the work done by the propeller experimentally.

2.1 Materials

The apparatus utilized in this practical include: a Pressurized air tank system (attached to pressure valve and gauges, refer to figure 1), a manometer, safety goggles and LabVIEW (data recording software connected to pressurized air tank system)

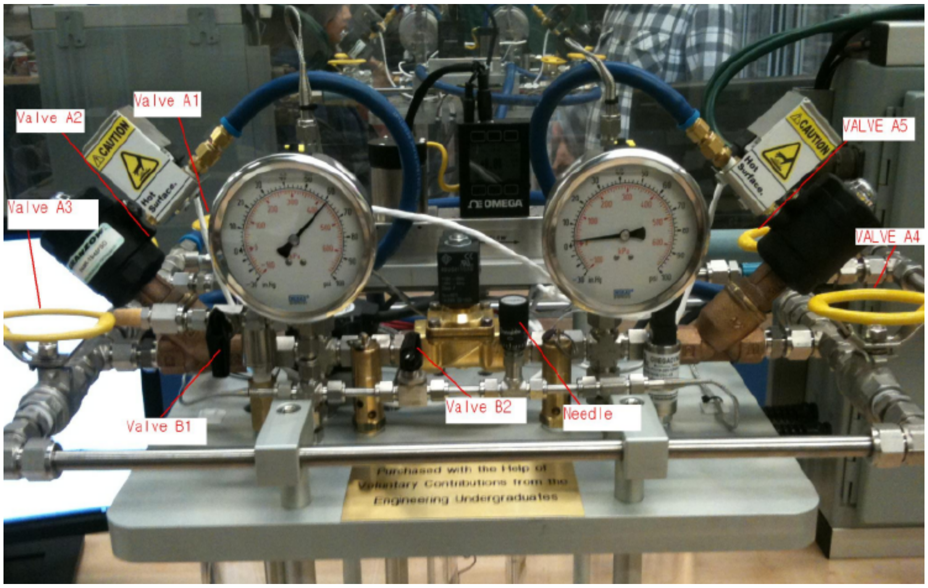

Figure 1: Pressurized Air-Tank System

2.2 Process

Part 1: The ambient pressure of the room was recorded using a manometer. The left tank was then pressurized to 40 psig. This was done by first opening valve A2 and then opening the left solenoid using LabVIEW. The mass flow rate was set to 50 g/min to increase the pressure of the tank. When the pressure in the left tank reached 40 psig, the left solenoid was closed using LabVIEW and Valve A2 was closed. The pressure and temperature of the left tank were then left to stabilize.

Part 2: The heaters in the left tank were turned on using LabVIEW and the target temperature was set to 40°C. Once the left tank reached 40°C, the heaters were left on for 5 minutes. The heaters were then turned off using LabVIEW and the left tank was cooled. This was done by first opening the left solenoid using LabVIEW and opening the bar valve. Once the tank reached the target temperature, the left solenoid was closed using LabVIEW and the bar valve was closed. Valve B2 was then opened to evacuate the left tank and Valve B2 was closed when the pressure stabilized.

Parts 1 and 2 were repeated with tank pressure and temperatures of 70 psig at 40°C, 40 psig at 60°C, and 70 psig at 60°C. All results were recorded using the LabVIEW software.

Part 3: The third section of the lab was to numerically determine the work done by the propellers by using the power consumption and fan similarity equations. It can be assumed that the system under adiabatic conditions (i.e the work done is equivalent to the change in internal energy).

3 Data Collection and Analysis

3.1 Part 1: Mass of the Left Tank

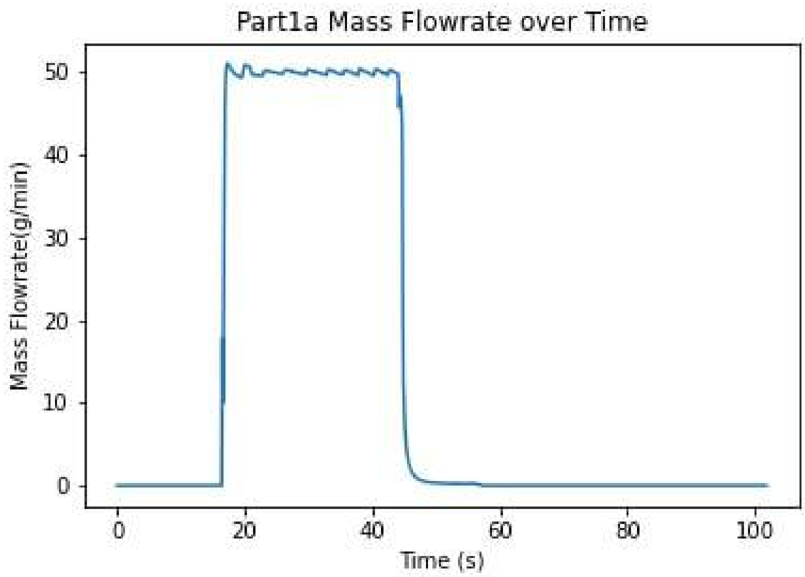

Figure 2: Mass flow rate over time from Trial A

The mass in the left tank can be determined by first finding the mass added to the tank during the process. This can be done by taking the integral under the mass flow rate graph (Figure 2). Using this method,

$$m_{added} = \int_{t_1}^{t_2} \dot{m} dt \tag{1}$$

The mass in the left tank can then be determined by rearranging the ideal gas law to get,

$$m_{Left} = m_{added} \left[1 + \frac{1}{\frac{P_2 T_1}{P_1 T_2} - 1} \right] \tag{2}$$

The integral of the mass flow rate was approximated by taking the midpoint Riemann sum of the data using Excel. This process was repeated for all four trials to get,

$m_{Left,a} = 34.6 \pm 0.067g$, $m_{Left,b} = 51.6 \pm 0.099g$, $m_{Left,c} = 32.1 \pm 0.062g$, $m_{Left,d} = 49.6 \pm 0.096g$

3.2 Part 2: Heat Transferred

The heat loss from the tank can be calculated by taking the total heat transferred into the tank by the heaters to keep the temperature at the target temperature, and dividing by the total time for which the target temperature was maintained. The software automatically calculates the total cumulative heat energy transferred into the system by the heaters at each time. Thus, the total heat transferred to maintain the target temperature is calculated by subtracting the final value for heater energy (kJ) from the value for heater energy given at the moment the tank reaches the target temperature. This is then divided by the time between the moment the target temperature is reached and the end of the experiment (s). The resulting average heat loss values are given in the row titled "Average Q' (W)" in Table 3.2.

The heat loss through the acrylic walls can also be calculated using the heat conduction equation for cylindrical walls given by Equation 3 below:

$$\dot{Q} = 2k\pi l \frac{\Delta T}{\ln\left(\frac{r_2}{r_1}\right)} \tag{3}$$

Where:

- $k = 0.185 \pm 0.015$ W/mK is the conductive heat transfer coefficient
- $l = 0.2858 \pm 2.54 \times 10^{-5}$ m is the length of the cylindrical tank.
- ΔT [K] is the temperature across the wall (i.e. difference between Target Temperature and Ambient Temperature, which was 26.5°C)
- $r_1 = 3.625 \pm 0.10125$ inches is the inner radius of the cylinder.
- $r_2 = 4 \pm 0.0225$ inches is the outer radius of the cylinder

Note that the radii of the cylinder do not need to be converted to meters since the Equation 3 only considers the ratio between the two.

Part	a	b	c	d
Target Pressure (psig)	40.0	70.0	40.0	70.0
Initial Temp (Deg C)	29.10	30.50	35.50	37.60
Target Temp (Deg C)	40.00	40.00	60.00	60.00
Time to Target Temp (s)	91.40	61.30	107.10	98.30
Initial Heater Energy (kJ)	40.20	34.30	95.60	85.70
End Time (s)	391.70	363.50	405.30	402.70
Final Heater Energy (kJ)	57.70	57.90	146.30	147.60
Δt (s)	300.30	302.20	298.20	304.40
Heat to Maintain Temp (kJ)	17.50	23.60	50.70	61.90
Average Q' (W)	58.28	78.09	170.02	203.35
Q' Through Walls (W)	45.55 ± 3.9	45.55 ± 3.9	113.03 ± 9.7	113.03 ± 9.7
Q' Through Plates (W)	12.72 ± 3.9	32.54 ± 3.9	56.99 ± 9.7	90.32 ± 9.7
mass (g)	34.6 ± 0.067	51.6 ± 0.099	32.1 ± 0.062	49.6 ± 0.096
c_v air J/(g°C)	106.59 ± 0.31	69.97 ± 0.21	121.56 ± 0.36	77.13 ± 0.23

Table 1: Data from Part 2, along with calculated heat loss and specific heat capacity.

The heat loss through the top and bottom plates can be calculated by subtracting the heat losses through the walls from the total heat losses given in the row "Average Q' (W)" for each part in Table 3.2. The results are shown in the row titled "Q' Through Plates (W)". Finally, the specific heat capacity of air (c_v) can be calculated. This is determined from the following formula:

$$Q = mc_v\Delta T$$

Which can be rearranged to:

$$c_v = \frac{Q}{m\Delta T} \tag{4}$$

where the Q values in this case are from the row titled Initial Heater Energy (i.e., the heat transferred into the tank to raise the temperature to the target temperature); m is the mass of air in the tank calculated in Part 1, and ΔT is the difference between the Target Temperature and the Initial Temperature.

3.3 Part 3: Work Done by the Propeller

The work done by the propeller in the left tank can be determined by considering the fan blade performance of the system and utilizing the pump and fan similarity equations (i.e. the affinity laws). By considering the volume capacity of a fan:

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 \tag{5}$$

where Q is the volumetric flow rate, n is the fan shaft speed and D is the impeller diameter, subscript 1 refers to the manufacturers test conditions, and subscript 2 refer to the laboratory operating conditions. The manufacturer's testing conditions are as follows: $Q_1 = 25$ cfm, $D_1 = 2.5$ inches and $n_1 = 4200$ rpm.

During the laboratory, the mass flow rate of air in the fan was set to 50 g/min. Because equation 5 requires volumetric flow rate, the density of air needs to be found using the Ideal Gas Law. Since the lab was conducted under the ambient temperature $T = 299.15$ K and ambient pressure $P = 101.05$ kPa conditions.

$$\rho = \frac{m}{V} = \frac{P}{RT} = \frac{101.05 \text{ kPa}}{(0.2870 \text{ kJ/kgK}) \times (299.65 \text{ K})} = 1.175\ kg/m^3 \tag{6}$$

Converting the density from kilogram per cubic meter to gram per cubic feet, $\rho = 33.27$ g/ft^3. The volumetric flow rate under operating conditions Q_2 can be determined using the mass flow rate $\dot{m}$ and density ρ from equation 6:

$$Q_2 = \frac{\dot{m}}{\rho} = \frac{50 \text{ g/min}}{33.27 \text{ g/ft}^3} = 1.50\ (\pm 0.0037) \text{ cfm} \tag{7}$$

Rearranging equation 5, in terms of D_2 the impeller diameter during the operating conditions of the lab (i.e. $n_2 = 2000$ rpm and Q_2 from equation 7).

$$D_2 = \sqrt[3]{\frac{Q_2\ n_1}{Q_1\ n_2}} \times D_1 = \sqrt[3]{\frac{(1.50\text{ cfm}) \times (4200\text{ rpm})}{(25\text{ cfm}) \times (2000\text{ rpm})}} \times (2.5\text{ in}) = 1.25\ (\pm 0.0037)\text{ in} \tag{8}$$

The power consumption [1] of a fan can be expressed as:

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 \tag{9}$$

where P is the fan power, ρ is air density, subscript 1 refer to manufacturers testing conditions at standard conditions, and subscript 2 refers to the laboratory operating conditions.

For the operating conditions $\rho_2 = 33.27$ g/ft^3 computed in equation 6. To determine ρ_1 of at standard conditions, ideal gas law can be utilized:

$$\rho_1 = \frac{m}{V} = \frac{P}{RT} = \frac{101.325\text{ kPa}}{(0.2870\text{ kJ/kgK}) \times (273.15\text{ K})} = 1.29\ (\pm 0.0029)\ kg/m^3 \tag{10}$$

By unit conversion, $\rho_1 = 36.6$ g/ft^3. Rearranging equation 9 to determine P_2 the power out of the propeller during the operating conditions:

$$P_2 = \frac{(33.27\text{ g/ft}^3)}{(36.6\text{ g/ft}^3)}\left(\frac{2000\text{ rpm}}{4200\text{ rpm}}\right)^3\left(\frac{1.25\text{ in}}{2.5\text{ in}}\right)^5 \times (0.001\text{ hp}) = 3.09132457 \times 10^{-6}\ (\pm 1.0 \times 10^{-8})\text{ hp} \tag{11}$$

By unit conversion the rate of work done $\dot{W} = P_2 = 3.09132457 \times 10^{-6}$ hp $= 0.00227$ Watts. During each trial in part 2, the fan was operating for a duration of 5 minutes on average. To determine the overall work done by the propeller throughout the 5-minute intervals for each trial:

$$W = \dot{W}\Sigma t_{on} = P_2\Sigma t_{on} = (0.00227\text{ W}) \times (300\text{ s}) = 0.681\ (\pm 0.0022)\text{ J} \tag{12}$$

This is negligible compared to the energy added by the heaters, during these intervals, which ranges from 17.50 kJ to 61.90 kJ: on the high end, the work done by the propellers is only 0.00388% of the heat added.

To calculate the rate of temperature rise an energy balance can be utilized by equating the work done by the propeller to the internal energy of an adiabatic system.

$$Q + W = \Delta U = mc_v\Delta T \tag{13}$$

Considering an adiabatic system, Q can be assumed to equal 0. Therefore the rate of work done is equal to rate of internal energy change.

$$\dot{W} = P_2 = mc_v \left(\frac{dT}{dt}\right) \tag{14}$$

Rearranging equation 14 for $\left(\frac{dT}{dt}\right)$ to compute the temperature rise:

$$\left(\frac{dT}{dt}\right) = \frac{\dot{W}}{mc_v} = \frac{(0.00227 \text{ W})}{(m_i) \times (0.3175 \text{ J/gK})} \tag{15}$$

where m_i is the mass during trial a, b, c, and d respectively computed in part 1 and $c_v = 0.3175$ J/gK is the literature value [2] of constant volume specific capacity of air computed using interpolation. It is essential to note, that the actual c_v values from part 2 calculations are not utilized because they are not consistent within the acceptable range. Therefore the rate of temperature rise using mass and c_V from each trial would be:

$$\frac{dT}{dt}_a = 2.07 \times 10^{-4}\ (\pm 8.3 \times 10^{-7}) \text{ K/s}, \qquad \frac{dT}{dt}_b = 1.39 \times 10^{-4}\ (\pm 5.4 \times 10^{-7}) \text{ K/s},$$

$$\frac{dT}{dt}_c = 2.23 \times 10^{-4}\ (\pm 7.7 \times 10^{-7}) \text{ K/s}, \qquad \frac{dT}{dt}_d = 1.44 \times 10^{-4}\ (\pm 5.2 \times 10^{-7}) \text{ K/s}$$

Thus, if the temperature of the tank were increased only by adding energy in the form of work due to the propeller, the amount of time it would take to heat up the tank from its initial temperatures to final temperatures would be on the order of a full day (found by dividing the difference in temperature by the rates of temperature increase above). Thus further shows that work done by the propeller is negligible, since the heater causes this temperature increase in under 2 minutes.

4 Discussion

The results and calculations demonstrate the application of the first law of thermodynamics. There are several sources of error and uncertainties that could have affected the accuracy and precision of the calculations and introduced random and systematic errors to lab.

Firstly, instruments like the pressure gauge were not calibrated correctly: there was a non-zero offset between the gauge reading of $1 \pm$ psi and the reading on the LabVIEW software of 0 psi. This introduced a systematic discrepancy of about 10^{-1} psi between the real and observed readings of pressure on the gauge (calculated by the difference between the initial readings from LabVIEW and the pressure gauge). Furthermore, this experiment set-up consists of a mechanical lever to release the gas and empty the tank, which means that the air tank was not perfectly sealed, causing leakage that could have affected the pressure readings. However, these errors were not avoidable as the equipment could not be modified or calibrated by ourselves. Another critical error in this experiment design was the fact that calculations were based on single trials of different pressures and temperatures for Part 2. As such anomalies couldn't have been identified or eliminated. In the future, having more repetition could allow anomalies to be eliminated and minimize the impact of experimental uncertainties.

Secondly, throughout parts 1 and 3, air was assumed to behave like an ideal gas. However, real gases like air can deviate from ideal gases particularly at high pressures and low temperatures, because inter-molecular forces are not negligible, collisions are not perfectly elastic, and molecules would have rotational kinetic energy. As such, assuming the ideal gas behavior of air in part 1 to compute the mass of the system and in part 3 to determine the density of air could have lead to inaccuracies in calculations. Correspondingly for part 2, the heat loss through the walls of the air tank is computed using the heat conduction equation, which assumes that the system is under steady-state conditions. However, fluctuations in insulation or temperature distribution may lead to discrepancies in heat loss calculation. As such, the computed values of the constant volume specific heat capacity, c_V, are not consistent with the literature value: The accepted value for the specific heat capacity of air at constant volume is between 0.718 kJ/kgK and 0.721 kJ/kgK for the temperatures $40°C$ and $60°C$ [2] (note that the units J/g$°C$ used to report the values of c_V in Part 2 are equivalent to kJ/kgK). Thus, the values of c_V found experimentally in Part 2 are greater than the accepted value by a factor of 133 ± 36. Similarly, for calculations in part 3, the process was assumed to be adiabatic (i.e. no heat transfer/exchange with the surroundings), which is not true since we found that heat was lost at a rate of between about 58 and 203 Watts . And by considering the fan blade performance of the system to compute the work done by the propeller, it was assumed that the system was subject to ideal conditions and there exists a perfect similarity between the operating and testing conditions. Though, in practice, other factors like fluctuations/differences in ambient conditions, blade imperfections and turbulent flow may introduce deviations. As such these factors could lead to inaccuracies in calculations.

Nevertheless, it is still essential to iterate the strengths of this experiment. A major strength of this experiment is that two different instruments were utilized to record pressure which allowed the readings to be verified for consistency, and account for miscalibration and zero-offset errors. Additionally the instrumental uncertainties for this set-up were notably due to the high precision of the LabVIEW software which regularly monitored the temperature and pressure at small increments of time. This automated collection of data using LabVIEW reduced the potential of human error and time lag due to manual data collection. Furthermore the experiment was conducted in relatively well-controlled conditions (i.e. there weren't significant fluctuations in the room temperature and pressure).

5 Conclusion

Ultimately, the experimental results of this laboratory successfully demonstrate the application of the first law of thermodynamics within a pressurized air-tank system. Despite the limitations aforementioned, considering the strengths of this experiment, and the consistency with the first law of thermodynamics, the results and conclusions drawn from this experiment seem sufficient and reliable. The specific heat capacity of air at constant volume and the work done by the propeller were successfully determined, despite being outside the acceptable range which could have been due to assumptions and experimental uncertainties, which were unavoidable and beyond our control.

References

[1] The Engineering ToolBox (2003). *Fan Affinity Laws. [online] Available at: https://www.engineeringtoolbox.com/fan-affinity-laws-d*$_1 96.html.Accessed4Nov.2023.$

[2] Sanjeev Chandra. *Energy, Entropy, and Engines: An Introduction to Thermodynamics.* John Wiley & Sons, Ltd, 2016.

6 Appendix A: Uncertainty Calculations

6.1 Part 1

The uncertainty for the measured P, T, and $\dot{m}$ values were found by taking half of the smallest measurement value for each to get,

$$\Delta P = 0.05 \text{ psig}, \; \Delta T = 0.05 \text{ °C}, \; \Delta \dot{m} = 0.005 \text{ g/min}$$

The uncertainty for m was then found to be,

$$\Delta m = m\sqrt{\left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta P}{P}\right)^2 + \left(\frac{\Delta \dot{m}}{\dot{m}}\right)^2}$$

This was done for each of the trials to give,

$$\Delta m_{Left,a} = 0.067g, \; \Delta m_{Left,b} = 0.099g, \; \Delta m_{Left,c} = 0.062g, \; \Delta m_{Left,d} = 0.096g$$

6.2 Part 2

The uncertainty for for $\dot{Q}$ was found to be,

$$\Delta \dot{Q} = \dot{Q}\sqrt{\left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta r_1}{r_1}\right)^2 + \left(\frac{\Delta r_2}{r_2}\right)^2}$$

This was done for each trial to give,

$$\Delta \dot{Q}_a = 3.9 \text{ W}, \; \Delta \dot{Q}_b = 3.9 \text{ W}, \; \Delta \dot{Q}_c = 9.7 \text{ W}, \; \Delta \dot{Q}_d = 9.7 \text{ W}$$

The uncertainty for the measured Q value was found by taking half of the smallest measurement value to get,

$$\Delta Q = 0.05 \text{ kJ}$$

The uncertainty for c_v was found to be,

$$\Delta c_v = c_v\sqrt{\left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta Q}{Q}\right)^2 + \left(\frac{\Delta m}{m}\right)^2}$$

This was done for each trial to get,

$$\Delta c_{v,a} = 0.31\frac{J}{g°C}, \ \Delta c_{v,b} = 0.21\frac{J}{g°C}, \ \Delta c_{v,c} = 0.36\frac{J}{g°C}, \ \Delta c_{v,d} = 0.23\frac{J}{g°C}$$

6.3 Part 3

The uncertainties for ρ_1, Q_2, D_2, P_2, W determined to be,

$$\Delta\rho_1 = \rho_1\sqrt{\left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta P}{P}\right)^2} = 0.0029 \ kg/m^3$$

$$\Delta Q_2 = Q_2\sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\Delta\dot{m}}{\dot{m}}\right)^2} = 0.0037 \ cfm$$

$$\Delta D_2 = D_2\sqrt{\left(\frac{\Delta Q_2}{Q_2}\right)^2} = 0.0031 \ in$$

$$\Delta P_2 = P_2\sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\Delta D_2}{D_2}\right)^2} = 1.0 \times 10^{-8} \ hp$$

$$\Delta W = W\sqrt{\left(\frac{\Delta P_2}{P_2}\right)^2} = 0.0022 \ J$$

The uncertainty for $\frac{dT}{dt}$ was determined to be,

$$\Delta\frac{dT}{dt} = \frac{dT}{dt}\sqrt{\left(\frac{\Delta P_2}{P_2}\right)^2 + \left(\frac{\Delta m}{m}\right)^2} = \ K/s$$

This was done for each trial to get,

$$\Delta\frac{dT}{dt}_a = 7.7\times10^{-7}K/s, \ \Delta\frac{dT}{dt}_b = 5.2\times10^{-7}K/s, \ \Delta\frac{dT}{dt}_c = 8.3\times10^{-7}K/s, \ \Delta\frac{dT}{dt}_d = 5.4\times10^{-7}K/s$$

7 Appendix B: Figures for Part 2

Figures 3, 4, 5, and 6 display the graphs from data collection in part 2 of the lab.

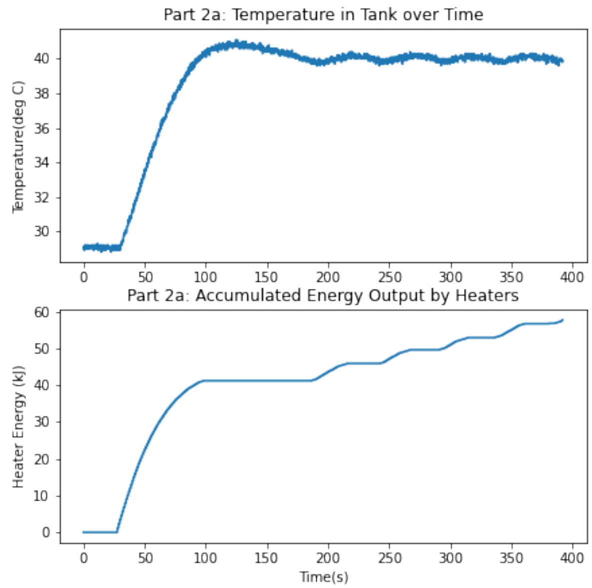

Figure 3: Heating 40 psig of Air to 40° C

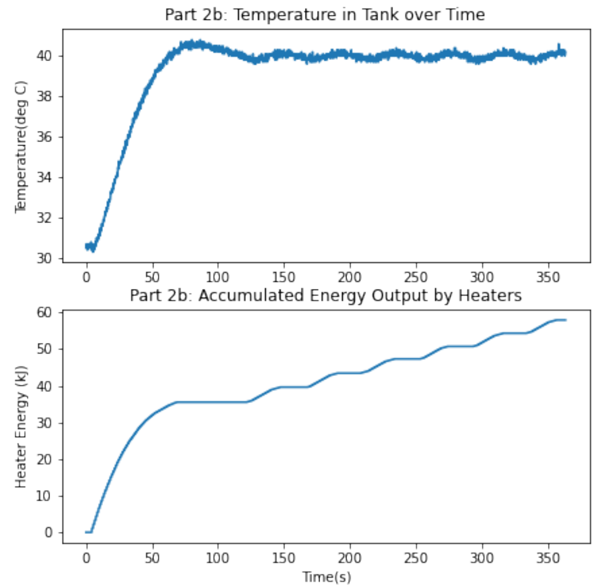

Figure 4: Heating 70 psig of Air to 40° C

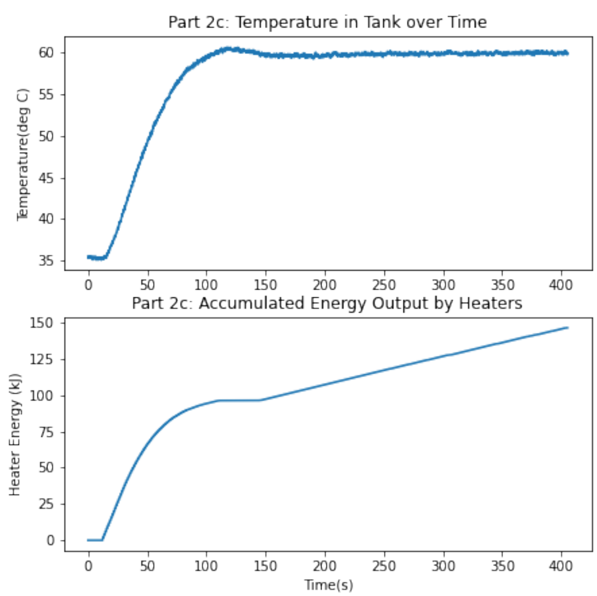

Figure 5: Heating 40 psig of Air to 60° C

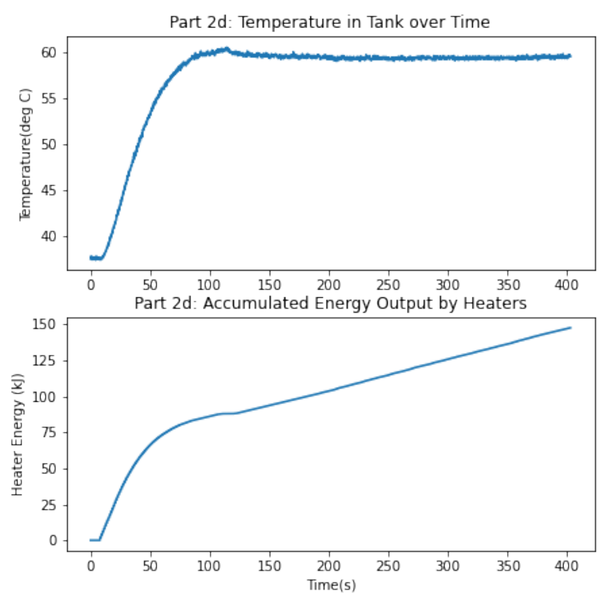

Figure 6: Heating 70 psig of Air to 60° C