UNIVERSITY OF TORONTO
Engineering Science
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Midterm Review
Heat Transfer

(Covering Chapter 16 & 17 from the Textbook)

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Contents

1 Chapter 16 - Mechanisms of Heat Transfer

1.1 Introduction

- A thermodynamic analysis is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the rates of such energy transfers is the science of heat transfer.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two media reach the same temperature.
- Heat can be transferred in three different modes: conduction, convection, and radiation. All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.

1.2 Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons. Rate of Heat Conduction depends on: geometry, material and temperature difference.

Fourier's Law of Heat Conduction

Under **steady-steady state heat conduction**, the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area but is inversely proportional to the thickness of the layer.

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} \qquad \text{(W)} \tag{1}$$

where, the constant of proportionality k is the thermal conductivity of the material $(W/m \cdot K)$, which is a measure of the ability of a material to conduct heat, $\frac{dT}{dx}$ is the temperature gradient, which is the slope of the temperature curve on a T-x diagram (the rate of change of T with x), at location x. The heat transfer area A is always normal to the direction of heat transfer. Note that the thickness of the wall has no effect on the area.

Thermal Conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

- A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.
- The kinetic theory of gases predicts, and experiments confirm, that the thermal conductivity of gases is proportional to the square root of the thermodynamic temperature T and inversely proportional to the square root of the molar mass M.
- The thermal conductivities of gases at 1 atm pressure are listed in Table A-23. However, they can also be used at pressures other than 1 atm since the thermal conductivity of gases is independent of pressure in a wide range of pressures encountered in practice.
- The thermal conductivity of a substance is normally highest in the solid phase and lowest in the gas phase.

Thermal Diffusivity The product ρc_p, which is often encountered in heat transfer analysis, is called the heat capacity of a material. Both the specific heat c p and the heat capacity ρc_p represent the heat storage capability of a material. But c_p expresses it per unit mass, whereas ρc_p expresses it per unit volume, as can be noticed from their units $J/kg{\cdot}K$ and $J/m^3{\cdot}K$, respectively. Another material property that appears in the transient heat conduction analysis is the thermal diffusivity, which represents how fast heat diffuses through a material and is defined as

Thermal Diffusivity

$$\alpha = \frac{Heat\ Conduction}{Heat\ Storage} = \frac{k}{\rho c_p} \qquad (\mathrm{m^2/s}) \tag{2}$$

Note that the thermal conductivity k represents how well a material conducts heat, and the heat capacity ρc_p represents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the heat conducted through the material to the heat stored per unit volume. A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

1.3 Convection

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. Convection = Conduction (right at the interface) + Advection (due to bulk motion of the fluid)

Rate of Convection Heat Transfer

The rate of convection heat transfer is observed to be proportional to the temperature difference, and it is conveniently expressed by Newton's law of cooling as:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W) \tag{3}$$

where h is the convection heat transfer coefficient in $W/m^2 \cdot K$, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface. Note that at the surface, the fluid temperature equals the surface temperature of the solid.

The convection heat transfer coefficient h is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

1.4 Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium. In fact, heat transfer by radiation is fastest (it occurs at the speed of light), and it suffers no attenuation in a vacuum.

Stefan-Boltzmann Law

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T s (in K or R) is given by the Stefan–Boltzmann law as:

$$\dot{Q}_{emit,max} = \sigma A_s T_s^4 \qquad \text{(W)} \tag{4}$$

where $\sigma = 5.670 \times 10^{-8}\ W/m^2 \cdot K^4$ is the Stefan–Boltzmann constant. The idealized surface that emits radiation at this maximum rate is called a blackbody, and the radiation emitted by a blackbody is called blackbody radiation.

The property emissivity, whose value is in the range $0 \leq \varepsilon \leq 1$, is a measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$. Another important radiation property of a surface is its absorptivity α, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range $0 \leq \alpha \leq 1$. A blackbody absorbs all radiation incident on it. That is, a blackbody is a perfect absorber ($\alpha = 1$) as well as a perfect emitter. In general, both ε and α of a surface depend on the temperature and the wavelength of the radiation.

Blackbody Radiation for All Real surfaces

The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature and is expressed by equation 5.

$$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4 \qquad \text{(W)} \tag{5}$$

where ε is the emissivity of the surface.

Kirchhoff's Law of Radiation

The kirchhoff's law of radiation relates the emissivity the absorptivity of a surface at a given temperature and wavelength are equal.

$$\dot{Q}_{reflected} = (1-\alpha)\dot{Q}_{incident} \tag{6}$$

$$\dot{Q}_{absorbed} = \alpha \cdot \dot{Q}_{incident} \tag{7}$$

$$\dot{Q}_{emitted} = \varepsilon \sigma A_s T_s^4 \tag{8}$$

$$\therefore \quad Q_{emitted} = \dot{Q}_{absorbed} \qquad \varepsilon \sigma A_s T_s^4 = \alpha \sigma A_s T_s^4$$

$$\therefore \varepsilon = \alpha$$

Net Radiation by Small Surface Enclosed in a Completely Large Surface

When a surface of emissivity ε and surface area A_s at a thermodynamic temperature T_s is completely enclosed by a much larger (or black) surface at thermodynamic temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4) \qquad (W) \tag{9}$$

In this special case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

1.5 Simultaneous Heat Transfer Mechanisms

Medium	No. of Modes	Mechanisms
Opaque Solid	1 Mode	Conduction
Semitransparent Solid	2 Modes	Conduction and Radiation
Still Fluid	2 Modes	Conduction and Radiation
Flowing Fluid	2 Modes	Convection and Radiation
Gas	2 Modes	Radiation and (Conduction or Convection)
Vacuum	1 Mode	Radiation

Table 1: Heat Transfer Mechanisms for Different Mediums

2 Chapter 17 - Steady Heat Conduction

2.1 Steady Heat Conduction in Plane Walls

Rate of Energy Change

The rate of energy change within a plane wall can be given by the rate of energy flowing into the wall minus the rate of energy flowing out of the wall.

$$Q_{in} - Q_{out} = \frac{dE_{Wall}}{dt} \tag{10}$$

Under steady state conditions $\frac{dE_{Wall}}{dt} = 0$

Fourier's Law

As learned in previous chapter (chapter 16), the conductivity of a plane wall can be given by the fourier's law:

$$\dot{Q}_{cond,wall} = -kA(\frac{dT}{dx}) \tag{11}$$

Under steady state conditions, if the rate of change of energy is zero, then $\dot{Q}_{cond,wall} = constant$ which means that $\frac{dT}{dx} = constant$. Using integration by separating variables:

$$\int_{x=0}^{x=L} \dot{Q}_{cond,wall}dx = \int_{T=T_1}^{T=T_2} kAdT \tag{12}$$

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \tag{13}$$

2.1.1 Thermal Resistance Concept

R is the thermal resistance of a medium which depends on a the geometry and thermal properties of the medium.

Thermal Resistance for Conduction

$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx} = \frac{T_1 - T_2}{R_{cond}} \tag{14}$$

$$R_{Wall} = \frac{L}{kA} \qquad \text{(K/W)} \tag{15}$$

Thermal Resistance for Convection

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = \frac{T_s - T_\infty}{R_{conv}} \tag{16}$$

$$R_{conv} = \frac{1}{hA_s} \qquad \text{(K/W)} \tag{17}$$

Thermal Resistance for Radiation

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \tag{18}$$

$$R_{rad} = \frac{1}{h_{rad}A_s} \qquad \text{(K/W)} \tag{19}$$

Use the equation below to avoid complications with radiation:

$$h_{combined} = h_{rad} + h_{conv} \tag{20}$$

2.1.2 Thermal Resistance Network

For a one dimension heat transfer through a plane wall, we can consider the rate of heat convection into the wall is equal to the rate of heat conduction within the wall which is also equivalent to the rate of heat convection outside the wall.

Thermal Networks

You can use a thermal network to simplify the process of finding the rate of heat transfer

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad \text{(W)} \tag{21}$$

$$R_{total} = R_{conv,1} + R_{cond} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \tag{22}$$

2.1.3 Multilayer Plane Walls

Thermal Networks

You can use a thermal network to simplify the process of finding the rate of heat transfer

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad \text{(W)} \tag{23}$$

$$R_{total} = R_{conv,1} + R_{cond,Wall\ 1} + R_{cond,Wall\ 2} R_{conv,2} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \tag{24}$$

2.2 Thermal Contact Resistance

2.3 Generalized Thermal Resistance Networks

Question 1) Assume steady state conditions and account for tip conversion

3 Chapter 16 - Practice Problems

Assigned Problems for Chapter 16: 19, 23, 26, 28, 32, 36, 40, 41, 43, 44, 51, 53, 57, 58, 60, 64, 68, 70, 73, 76, 78, 79

3.1 Question 19

Question 19 Problem

The inner and outer surfaces of a 4-m×7-m brick wall of thickness 30 cm and thermal conductivity 0.69 W/mK are maintained at temperatures of $20C$ and 5° C, respectively. Determine the rate of heat transfer through the wall in W.

Solution:

A brick wall is an opaque solid, so therefore we can assume that the heat trasnfer occurs at steady state and the only mechanism of heat transfer present is conduction.

$$A_s = 4\ m \times 7\ m = 28\ \text{m}^2 \tag{25}$$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{5°C - 20°C}{0.3\ m} = \frac{15°C}{0.3\ m} = -50\ (°K/m) \tag{26}$$

Substituting the value for thermal conductivity and the A_s and $\frac{dT}{dx}$, in equation 1 the rate of heat transfer can be determined:

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} = -0.69\ (W/mK) \times 28\ \text{m}^2 \times -50\ (°K/m) = 966\ \text{W} \tag{27}$$

3.2 Question 23

Question 23 Problem

An aluminum pan whose thermal conductivity is 237 W/mK has a flat bottom with diameter 15 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 800 W. If the inner surface of the bottom of the pan is at 105° C, determine the temperature of the outer surface of the bottom of the pan.

Solution:

Given the heat transfer occurs at a steady state and assume that the only mechanism of heat transfer is conduction.

$$A = \pi r^2 = \pi(0.075\ m)^2 = 0.0177\ \mathrm{m}^2 \tag{28}$$

$$\dot{Q_{cond}} = kA\frac{T_1 - T_2}{L} \tag{29}$$

Rearranging equation 1 for the outer temperature:

$$T_1 = \frac{\dot{Q_{cond}}L}{kA} + T_2 \tag{30}$$

Substituting the given variables, the outer temperature can be determined:

$$T_1 = \frac{(800\ \mathrm{W})(0.004\ \mathrm{m})}{(237\ \mathrm{W/mK})(0.0177\ \mathrm{m}^2)} + 105 = 105.76°\ \mathrm{C} \tag{31}$$

3.3 Question 26

Question 26 Problem

One way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical rectangular samples of the material and to heavily insulate the four outer edges. Thermocouples attached to the inner and outer surfaces of the samples record the temperatures. During an experiment, two 0.5-cm-thick samples 10 cm × 10 cm in size are used. When steady operation is reached, the heater is observed to draw 25 W of electric power, and the temperature of each sample is observed to drop from 82° C at the inner surface to 74° C at the outer surface. Determine the thermal conductivity of the material at the average temperature.

Solution:

Assume both the samples are in steady state and the only mechanism of heat transfer is by conduction. Because both samples are drawing out the heat from the heater, each heater draws out:

$$\dot{Q}_{sample} = \frac{25\text{ W}}{2} = 12.5\text{ W} \tag{32}$$

The temperature drop observed by the surface is:

$$T_1 - T_2 = 82\text{ °C} - 74\text{ °C} = 8\text{ °C} \tag{33}$$

Assuming both the samples are made from the same material and have the same thermal conductivity, equation 1 can be rearranged for k.

$$k = \frac{\dot{Q}_{cond}L}{A(T_1 - T_2)} = \frac{(12.5\text{ W}) \times (0.0005\text{ m})}{(0.01\text{ m}^2)(8\text{ °C})} = 0.7812\text{ W/mk} \tag{34}$$