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ASSIGNMENT 5.

Question 1:

(a) floors: 27. has 37 offices per floor

$27 \times 37 = 999$

999 offices in a building.

(b) Colours = 12.

diff versions: 2 (gender)

sizes = 3.

$12 \times 2 \times 3 = 72$. shirts in total

Question 2.

(a) 26 letters.

$26 \times 26 \times 26 = 17576$.

Each initial can be any of the 26 alphabets

(b) $26 \times 25 \times 24$ / ${}^{26}P_3$ => product rule

=> first initial can be of any 26 alphabets

=> 2nd initial can be of any 25 "

=> 3rd from any 24 "

$\boxed{15600}$ initials possible.

Question#3:-

(a) Hexadecimal digits - 0,1,2,3,4,5,6,7,8,9,A,B,C, D,E,F.

= 16 digits in total

10 digits : $(16)^{10}$
26 digits : $(16)^{26}$
58 digits : $(16)^{26}$

Product rule in all three.

$16^{10} + 16^{28} + 16^{58}$ => sum rule.

$\boxed{6.9017 \times 10^{69}}$ different WEP keys are possible

(b) a string of any four lowercase letters.

$26 \times 26 \times 26 \times 26 = (26)^4 = 456976$.

a string w/o any x.

$25 \; 25 \; 25 \; 25 = (25)^4 = 390625$.

any usual string - string w/o letter x.

$456976 - 390625$

$\boxed{66351}$ possible strings

Question#4:-

(a) Each value of the first set can have any of two values of the second set

$2 \times 2 \times 2 \times 2 \cdots n$.

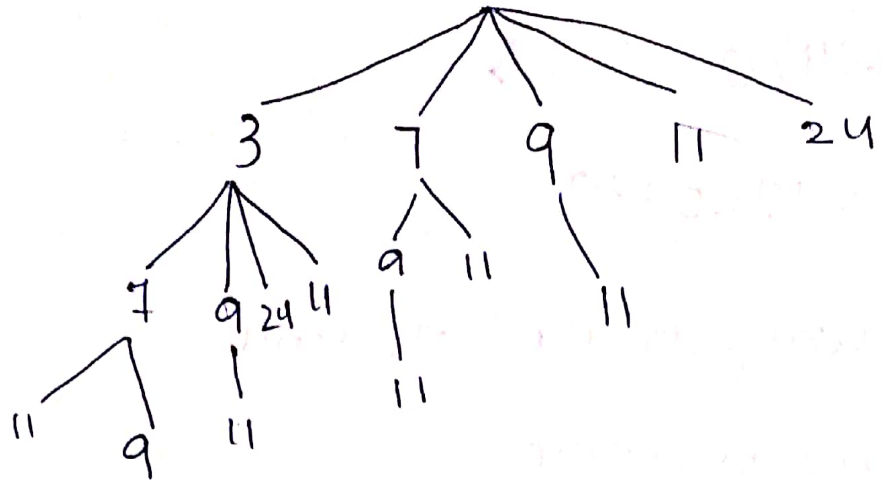
2^m.

(b) 1st element has to choose from second element has remaing 4 to choose from 3rd element has remaing 3. so on.

$5 \times 4 \times 3 \times 2 \times 1 / 5!$

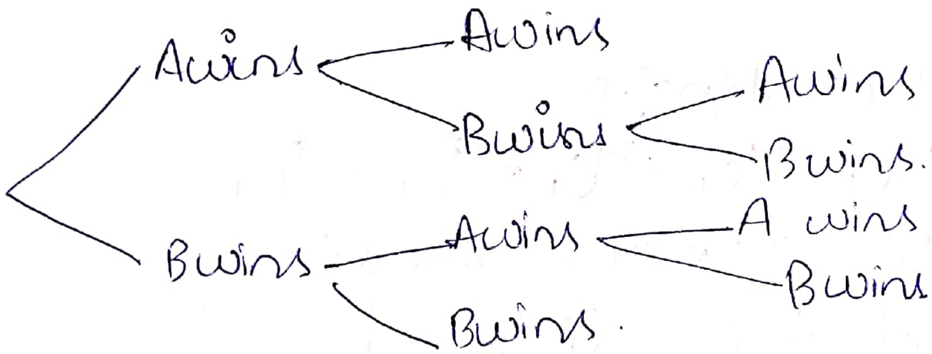
$\boxed{120}$

Question # 5:-



Possible subsets:- {3,7,11}, {3,9,11}, {3,7,9}, {3,7}, {3,9}, {3,11}, {3,24}, {7,9}, {7,11}, {7,9,11}, {9,11}.

(b)



AA , ABA , ABB
BAA , BAB , BB.
Six ways in which the tournament can occur.

Question # 6:-

(a) $^{n}C_{1} = {}^{8}C_{3}$ (combinations) => 56 ways.

(b) $^{12}C_{6}$ => 924 ways to choose for 6 courses.

(c) $^{9}C_{5}$ => 126 ways to select a 5 member team.

Question #7:-

(a) $^{20}C_5 \times 5! \Rightarrow 1860480$

or

$P(20,5) \Rightarrow {}^{20}P_5 \Rightarrow 1860480$

(b) $^{16}P_4 \Rightarrow 43680$ ways to select students.

(c) $^{15}P_2 \Rightarrow 210$ ways to choose.

Question #8:-

(a) $^5C_1 \times {}^3C_1 \times {}^4C_1 \times {}^4C_3$.

(b) Diff facial combinations:-

$^{15}C_1 \times {}^{48}C_1 \times {}^{24}C_1 \times {}^{34}C_1 \times {}^{28}C_1 \times {}^{28}C_1$

$15 \times 48 \times 24 \times 34 \times 28 \times 28$

$\boxed{460615680 \text{ diff faces}}$

Question #9:-

(a) begin with 3 zeros:-

$\underbrace{0\ 0\ 0}_{\text{fixed}}\ \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$.

$(2)^7 \Rightarrow 128$ strings of length 10

that begin with 3 0's end with 2 0's.

$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}\ \underbrace{0\ 0}_{\text{fixed}}$

(2^8)

$\boxed{256 \text{ strings}}$ of length 10 that end with

2 0's begins with 4 zero and end with 2 0's.

$\underbrace{\underline{0}\ \underline{0}\ \underline{0}}_{\text{fixed}}\ \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}\ \underbrace{\underline{0}\ \underline{0}}_{\text{fixed}}$

$(2)^5 = 32$

$A \cup B = A + B - A \cap B$

$728 + 256 - 32$

$\boxed{1352}$ possible strings

(b) Strings that begin w/0

$\underset{\text{fixed}}{0}\ \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$

$2^4 = \boxed{16}$

Strings that end with two 1's.

$2 \times 2 \times 2\ \underbrace{\underline{1}\ \underline{1}}_{\text{fixed}} \rightarrow 2^3 = \boxed{8}$

Strings that begin w/0 AND end w/ two 1's

$\underset{\text{fixed}}{0}\ \underline{2} \times \underline{2}\ \underbrace{\underline{1}\ \underline{1}}_{\text{fixed}}$

$A \cup B = A + B - A \cap B$

$= 16 + 8 - 4$

$= 24 - 4$

$= \boxed{20 \text{ strings}}$

Question #10:-

ⓐ The first letter of each ~~letter~~ last name are pigeonholes and the letter of the alphabets are pigeons. By the generalized pigeonhole principle $\left\lceil \frac{30}{26} \right\rceil = 2$. So there are atleast two students, have last names that begin with same letter.

ⓑ Assuming no one has more than 1000,000 hairs on the head of any person and that the population of Newyork was 8,008 278 in 2010 . . . . .

Solution:-

By generalised Pigeonhol principle, $\left\lceil \frac{8008278}{1000000} \right\rceil = 9$

ⓒ The 38 time periods are the pigeonholes and the 677 classes are the pigeon. By generalised pigeonhole principle there is atleast on time period in which atleast $\left\lceil \frac{677}{38} \right\rceil = 18$ classes are meeting. Since each class must meet in a different room. We need 18 rooms.

Question # 11:-

(a) coefficient of x^5 in $(1+x)^{11}$

$^{11}C_r (1)^{11-r} (x)^r$

~~r = 5 th term~~ $\because r=5 \rightarrow$ 6Th term

$= {^{11}C_5} (1)^{11-5} (x)^5$

$= 462x^5$

$\boxed{\text{coeff} = 462}$

(b) $^{24}C_r (2a)^{24-r} (-b)^r$

$\because r = 17 \rightarrow$ 18th term

$= {^{24}C_{17}} \times (2a)^{24-17} (-b)^{17}$

$= 346104 \times 128a^7 \times -b^{17}$

$\boxed{\text{coeff} = -44301312}$

Question # 12:.

(a) $a|b$ is written as $b = ak$ (1) for some integer $K \in \mathbb{Z}$

$b|c$ is written as $c = bl$ for some integer $l \in \mathbb{Z}$.

$c = b.l$

$c = (aK).l \rightarrow$ from eq (1)

$c = a(K.l)$

$\underbrace{c = a.m}$ where $m = K.l \in \mathbb{Z}$.

def of divisibility

$\rightarrow$ translates to $a|c = m$

(b) $a|b$ is written as $b = ak$ for some integer $k \in \mathbb{Z}$
$a|c$ is written as $c = al$ for some integer $l \in \mathbb{Z}$
$b+c$ is written as
$b+c = ak + al \rightarrow$ (1) and (2)

$b+c = a(k+l)$
$b+c = a \cdot m$
$\Rightarrow a|(b+c) = m$.

Question #13:

(a) for $n=1 \rightarrow 2^1 - 1 \Rightarrow 1 \rightarrow$ not a prime.
for $n=2 \rightarrow 2^2 - 1 \Rightarrow 3 \Rightarrow$ is a prime.
for $n=3 \rightarrow 2^3 - 1 \Rightarrow 7 \rightarrow$ is a prime.
for $n=4 \rightarrow 2^4 - 1 \Rightarrow 15 \rightarrow$ not a prime
for $n=5 \rightarrow 2^5 - 1 \Rightarrow 31 \rightarrow$ is a prime.
for $n=6 \rightarrow 2^6 - 1 \Rightarrow 63 \rightarrow$ not a prime
for $n=7 \rightarrow 2^7 - 1 \Rightarrow 127 \rightarrow$ is a prime
This satisfies the conditions as $n=7$ so $n>5$
and 127 is a prime number.

(b) Suppose that $p|a$ and $p|(a+1)$
thereby $p|a$ is written as $a = pk$ for
some integer $k \in \mathbb{Z}$.
$p|(a+1)$ is written as $a+1 = pl$ for some integer
$l \in \mathbb{Z}$.

$a+1 = pl \rightarrow (2)$

$1 = pl - a$

$1 = pl - pK \rightarrow$ from (1).

$1 = p(l-K) \rightarrow \quad p|1 = l-K$ wher

$l-K \in \mathbb{Z}$.

Question # 14:-

(a) $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$

S.B.S.

$(\sqrt{a+b})^2 = (\sqrt{a}+\sqrt{b})^2$

$a+b = (\sqrt{a})^2 + 2(\sqrt{a})(\sqrt{b}) + (\sqrt{b})^2$

$a+b = a + 2\sqrt{a}\sqrt{b} + b$

$0 = 2\sqrt{a}\sqrt{b}$.

If L.H.S = R.H.S $\quad a$ or $b = 0$.

If we suppose $a=9$ and $b=0$

$\sqrt{9+0} = \sqrt{9} + \sqrt{0}$

$3 = 3$

L.H.S = R.H.S

If we stand true for supposition.

(b) If $\underbrace{|x| > 1}_{p}$ when $\underbrace{x>1 \text{ or } x<-1}_{q}$ for all $x \in R$

contraposition:-

$\sim q \rightarrow \sim p$.

If $x \le 1$ and $x \ge -1$, then $|x| \le 1$

$\therefore -1 \le x \le 1$

$\therefore |x| \le 1$

Question # 15:-

(a) Proof by counter Example:-

Let rational num: $\sqrt{2}$ and $\sqrt{8}$

Product $= \sqrt{2} \times \sqrt{8}$

$= \sqrt{16} \Rightarrow \pm 4.$

These are rational num. Hence statement is disprove.

(b) Proof by contradiction:-

→ The sum of a rational and an irrational num is rational.

Let r be a rational

and ⇒ (1) $r = \frac{a}{b}$ for some integer a[b ∈ ℤ and b ≠ 0

Let s be an irrational number.

(2) $r + s = \frac{c}{d}$ for some integer c and d ∈ ℤ d ≠ 10

$r + s = \frac{c}{d}$

$\frac{a}{b} + s = \frac{c}{d} \rightarrow (1)$

$s = \frac{c}{d} - \frac{a}{b}$

$s = \frac{cb - ad}{bd}$ since all are integer then product will be integer - subtraction is also integer so, $s = K / L$ where.

$K = cb - ad$ ∈ ℤ

$L = bd$ for ∈ ℤ.

Question# 16:-

a) let $n = 2$

$n+2$

$2+2$

4

4 is not a prime

$\therefore$ The statement is disproved!

b) Proof the contradiction:-

Suppose, the set of prime numbers is finite then, some prime numbers p is the largest of all prime numbers.

$2, 3, 5, 7, 11 \ldots p.$

Let N be the product of all prime numbers plus 1.

$N = (2 \cdot 3 \cdot 5 \cdot 7 \cdot 11) \ldots p) + 1.$

Then $N > 1$ and also divisible by some prime number q. If q is prime number then it must be equal to one of the prime number.

i.e: $2, 3, 5, 7, 11 \ldots p.$

Thus the def of divides $2+3+5+7+11 \ldots p.$

Question # 17:-

(a) If n and m are odd then their sum is also odd.

let $n = 2K+1 \quad K \in \mathbb{Z}$

$m = 2l+1 \quad l \in \mathbb{Z}$

$n+m = (2K+1)+(2l+1)$

$= 2K+2l+2$

$= 2(K+l+1)$

where $(K+l+1) = \cancel{m}$ r for any integer
⇒ the result is also even by × by 2.
This contradicts our supposition that n+m is odd.

(b) $\sim q \to \sim p$.

If m and n are both even and m and n are not both odd, then m+n is odd.

let m be even.

$m = 2a$

Let n be odd

$n = 2b+1$.

$m+n = 2a+2b+1$

$= 2(a+b)+1 \qquad \because a+b = r$

$= 2r+1$

Thus the def, makes m+n is odd, If we do vice versa the result will be odd.

Question # 18:-

a) suppose

$6 - 7\sqrt{2}$ is rational

Then by definition,

$6 - 7\sqrt{2}$ can be expressed as

where a and b are some integers $\in \mathbb{Z}$

and $b \neq 0$

$$6 - 7\sqrt{2} = a/b$$

$$7\sqrt{2} = \frac{-a}{b} + b$$

$$7\sqrt{2} = \frac{6b - a}{b}$$

$$\cancel{7}\sqrt{2} = \frac{6b - a}{7b} \quad \text{and } b \neq 0$$

Since, a and b are integers the 6b, 7b are also integers. 6b-a is also integer.

$$\sqrt{2} = \frac{m}{n} \quad \text{where } m = 6b - a$$
$$n = 7b.$$

This make $\sqrt{2}$ rational number by def which contradicts the fact of mathematics as $\sqrt{2}$ is an irrational number.

b) Assume that.

$\sqrt{2} + \sqrt{3}$ is rational.

$$\Rightarrow \sqrt{2} + \sqrt{3} = \frac{a}{b} \quad \text{where } b \neq 0.$$

$$\Rightarrow (\sqrt{2} + \sqrt{3})^2 = (a/b)^2$$

$$2 + 3 + 2\sqrt{6} = a^2/b^2.$$

$$2\sqrt{6} = \frac{a^2}{b^2} - 5$$

(h)

$$2\sqrt{6} = \frac{a^2-5b^2}{b^2}$$

$$\sqrt{6} = \frac{a^2-5b^2}{2b^2}$$

$a^2, 5b^2, 2b^2$ are all integers and a and b are also integers -

$\Rightarrow \quad \sqrt{6} = \frac{m}{n} \qquad \because m = a^2-5b^2$

$n = 2b^2$.

This contradicts the mathematical fact as $\sqrt{6}$ is irrational. Therefore our supposition is false

Question # 19:-

(a)

i) $6 \times 6 = 36$.

$(1,5), (2,4), (3,3), (4,2), (5,1)$

$P(\text{sum of } 6) = \frac{P(E)}{P(S)} = \frac{5}{36}$.

ii) $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$

$P(\text{sum} = 7) = \frac{6}{36} = \boxed{\frac{1}{36}}$

iii) $(4,1), (4,2), (4,3), (4,4), (4,4), (2,4), (3,4)$

$P(\text{layer number is } 4) = \frac{7}{36}$.

(b)

Outcomes in sample space.
Integers 1 through 99.

{11, 22, 33, 44, 55, 66, 77, 88, 99}

Probability $= \frac{9}{99} = \boxed{\frac{1}{11}}$

Question # 20 :-

(a)

Let $P(n) : 1^2 + 2^2 + 3^2 + \cdots n^2 = (n(n+1)(2n+1))/6$

Basic step:-

$n = 1$

$1^2 \Rightarrow 1.$

R.H.S:

$(1(1+1)(2(1)+1))/6$

$\frac{2 \times 3}{6} \Rightarrow 6/6$

$1.$

L.HS = R.H.S
proved.

Let $n = K$.

$1^2 + 2^2 + 3^2 \cdots K^2 = (K(K+1)(2K+1))/6$ — (1)

to prove $P(K+1)$

$1^2 + 2^2 + 3^2 \cdots (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

L.H.S:-

$$\rightarrow \underbrace{1^2+2^2+3^2+\cdots K^2}+(K+1)^2.$$

$$\rightarrow \frac{K(K+1)(2K+1)}{6}+(K+1)^2$$

$$\rightarrow (K+1)\left[\frac{K(2K+1)}{6}+(K+1)\right]$$

$$\rightarrow (K+1)\left[\frac{K(2K+1)+6(K+1)}{6}\right]$$

$$\rightarrow (K+1)\left[\frac{2K^2+K+6K+6}{6}\right]$$

$$\rightarrow (K+1)\left[\frac{2K^2+7K+6}{6}\right]$$

$$\rightarrow \frac{(K+1)(2K^2+7K+6)}{6}$$

$$\rightarrow \frac{(K+1)(2K+3)(K+2)}{6}$$

$$\rightarrow \frac{(K+1)(2K+3)(K+2)}{6}$$

L.H.S = R.H.S

proved.

(b) let $P(n): 1+2+2^2+\dots 2^n = 2^{n+1}-1 \quad n \geq 0$

Basic step:-

for $n=0$

$1+2+2^2 \dots 2^0 = 2^{0+1}-1$

$1 = 2-1$

$1 = 1$

proved.

let $n=K$:-

$1+2+2^2 \dots 2^K = 2^{K+1}-1.$

$n=K+1$:-

$\because 2^n = 2^{n+1}-1$

$2^{K+1} = 2^{K+1+1}-1$

$2^{K+1} = 2^{K+2}-1$

add $(2^{K+2}$ ...

Add 2^{K+1} on both sides.

$= 2^{K+1} - 1 + 2^{K+1}$

$= 2(2^{K+1}) - 1$

$= 2^1 \times 2^{K+1} - 1$

$= 2^{K+1+1} - 1$

$= 2^{K+2} - 1$

L.H.S = R.H.S