NOOR-US-SAHAR
19K-0224
BCS-5J
Assignment #02
Multivariable Calculus

Questions
Exercise 14.3

Question: 02

$$\int_0^{\pi}\int_0^{1+\cos\theta} r \, dr \, d\theta$$

$$\int_0^{\pi} \frac{r^2}{2} \Big|_0^{1+\cos\theta} d\theta$$

$$\int_0^{\pi} \frac{(1+\cos\theta)^2}{2} d\theta$$

$$\int_0^{\pi} \frac{1}{2} + \cancel{2}\cos\theta + \frac{\cos^2\theta}{2} \Big) d\theta$$

$$\int_0^{\pi}\left(\frac{1}{2} + \cos\theta + \frac{1}{2}\cdot\frac{1+\cos 2\theta}{2}\right) d\theta$$

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$$= \int_0^{\pi} \left(\frac{1}{2} + \cos\theta + \frac{1}{4} + \frac{\cos 2\theta}{2} \right) d\theta$$

$$= \frac{3}{4}\theta + \sin\theta + \frac{1}{8}\sin 2\theta \Big|_0^{\pi}$$

$$= \frac{3\pi}{4}$$

Answer

3. $$\int_0^{\pi/2} \int_0^{a\sin\theta} r^2 \, dr \, d\theta$$

$$\int_0^{\pi/2} \frac{r^3}{3} \Big|_0^{a\sin\theta} d\theta$$

$$\int_0^{\pi/2} \frac{a^3 \sin^3\theta}{3} \, d\theta$$

$$\int_0^{\pi/2} \frac{a^3 (\sin^2\theta \ \sin\theta)}{3} \, d\theta$$

$$\int_0^{\pi/2} \frac{a^3 (1-\cos^2\theta) \sin\theta}{3} \, d\theta$$

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$$= \int_0^{\pi/2} \frac{a^3 \sin\theta \, d\theta}{3} - \int_0^{\pi/2} \frac{a^3 \cos^2\theta \sin\theta \, d\theta}{3}$$

$$= \frac{a^3}{3} \left| -\cos\theta \right|_0^{\pi/2} - \frac{a^3}{3} \left| -\frac{\cos^3\theta}{3} \right|_0^{\pi/2}$$

$$= \frac{a^3}{3}(1) - \frac{a^3}{9}(1)$$

$$= \frac{2a^3}{9}$$ Answer

6. $$\int_0^{\pi/2} \int_0^{\cos\theta} r^3 \, dr \, d\theta$$

$$\int_0^{\pi/2} \frac{r^4}{4} \Big|_0^{\cos\theta} d\theta$$

$$\int_0^{\pi/2} \frac{\cos^4\theta}{4} \, d\theta$$

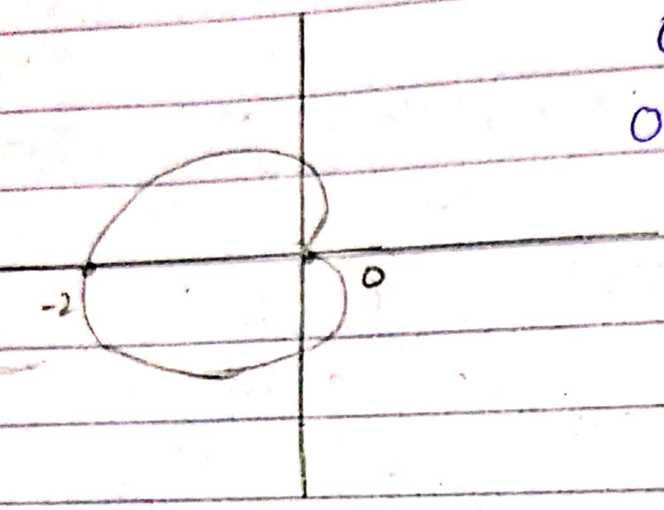
$$\int_0^{\pi/2} \frac{\cos^3\theta}{4} \cdot \cos\theta \, d\theta$$

$$\frac{3\pi}{64}$$ Answer

7. $r = 1 - \cos\theta$

$0 \le \theta \le 2\pi$

$0 \le r \le 1 - \cos\theta$



$$\int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$\int_0^{2\pi} \frac{r^2}{2}\Big|_0^{1-\cos\theta} d\theta$$

$$\int_0^{2\pi} \frac{1}{2} - \cos\theta + \frac{1}{4} + \frac{\cos 2\theta}{4} \Big\} d\theta$$

$$\frac{3}{4}\theta - \sin\theta + \frac{\sin 2\theta}{8}\Big|_0^{2\pi}$$

$$\frac{3}{2}\pi$$

Answer

9. $r = 1$ & $r = \sin 2\theta$

$\pi/4 \le \theta \le \pi/2$

$$\int_{\pi/4}^{\pi/2} \int_{1}^{\sin 2\theta} r \, dr \, d\theta$$

$$\int_{\pi/4}^{\pi/2} r^2 \Big|_{1}^{\sin 2\theta} d\theta$$

$$\cancel{\int_{\pi/4}^{\pi/2} \frac{\sin^2 2\theta}{2} d\theta}$$

$$\int_{\pi/4}^{\pi/2} \frac{(\sin 2\theta)^2}{2} - \frac{1}{2} \, d\theta$$

$$\frac{1}{16}\pi - \left[\frac{\pi}{4} - \frac{\pi}{8} \right]$$

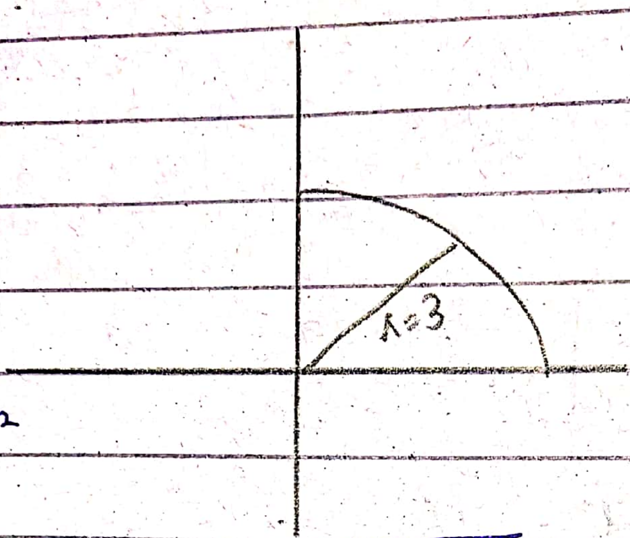
$$-\frac{\pi}{16}$$

~~Answer~~ Answer.

24. $\iint_R \sqrt{9-x^2-y^2}$ $\quad x^2+y^2=9$

$0 \le r \le 3$

$0 \le \theta \le \pi/2$



$x^2+y^2=r^2$

$\cancel{\int_0^3\int_0^{\pi/2}}$ $\int_0^{\pi/2}\int_0^3 \sqrt{9-r^2}\, r\,dr\,d\theta$

$9-r^2=u$

$-2r\,dr=du$

$r\,dr=-\frac{du}{2}$

$$\int_0^{\pi/2}\int_0^3 u^{1/2} \cdot -\frac{du}{2}\,d\theta$$

$$\int_0^{\pi/2} \frac{1}{3} u^{3/2}\,d\theta$$

$$\int_0^{\pi/2} -\frac{1}{3}\left|(9-r^2)^{3/2}\right|_0^3\,d\theta$$

$$\int_0^{\pi/2} -3\,d\theta$$

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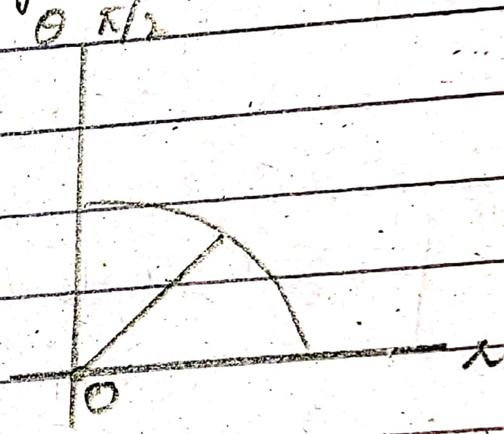
$$-9\theta \Big|_0^{\pi/2}$$

$$-\frac{9\pi}{2}$$ Answer.

29. $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx.$$



$$x^2+y^2=r^2$$

$$\sqrt{2x-x^2}=y$$

$$x^2+y^2=2x$$

$$r^2=2x$$

$$r^2=2r\cos\theta$$

$$r=2\cos\theta$$

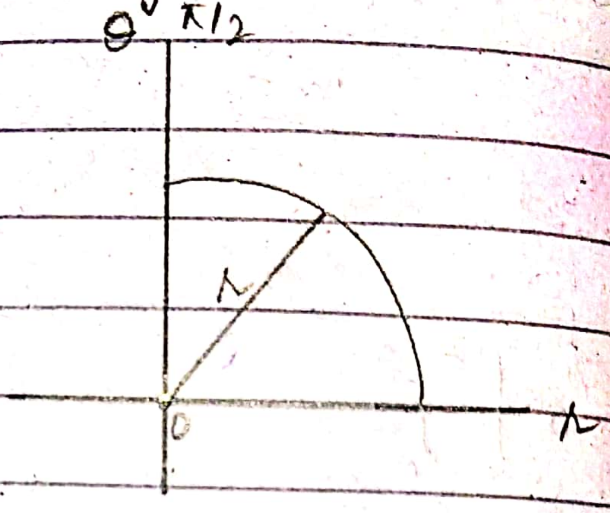
$$0\le\theta\le\pi/2$$

$$\int_0^{\pi/2}\int_0^{2\cos\theta} r^2\, dr\, d\theta \quad = \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^{2\cos\theta} d\theta$$

$$\int_0^{\pi/2} \frac{8\cos^3\theta}{3}\, d\theta$$

$$= \frac{16}{9}$$ Answer.

30. $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\, dx\, dy$



$x^2+y^2 = r^2$

$\sqrt{1-y^2} = x$

$x^2+y^2=1$

$r=1$

$0 \le \theta \le \pi/2$

$0 \le r \le 1$

$\int_0^{\pi/2} \int_0^1 \cos r^2 \cdot r\, dr\, d\theta$

let $r^2 = u$

$2r\,dr = du$

$r\,dr = \frac{1}{2} du$

$\int_0^{\pi/2} \frac{1}{2} \int_0^1 \cos u \, du\, d\theta$

$u=0 \quad r=0$

$u=1 \quad r=1$

$\frac{1}{2} \int_0^{\pi/2} \sin u \Big|_0^1 \, d\theta$

$\frac{1}{2} \sin 1 \int_0^{\pi/2} d\theta$

$\frac{1}{2} \sin 1 \quad \theta \Big|_0^{\pi/2}$

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