# Digital Logic Design (EE227)

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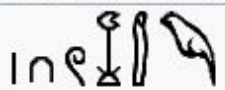
Decimal to Binary and Binary to Decimal Conversions

| Name | Base | Sample | Approx. first appearance |
|---|---|---|---|
| Babylonian numerals | 60 | 𒐕 𒐖 𒐗 𒐘 𒐙 𒐚 𒐛 𒐜 𒐝 𒐞 𒐟 | 3100 BC |
| Egyptian numerals | 10 |  or | 3000 BC |
| Aegean numerals | 10 | 𐄇 𐄈 𐄉 𐄊 𐄋 𐄌 𐄍 𐄎 𐄏 𐄐 | c1500 BC |
| Maya numerals | 20 | | <15th century |
| Muisca numerals | 20 | | <15th century |
| Indian Numerals | 10 | Tamil 0 ௧ ௨ ௩ ௪ ௫ ௬ ௭ ௮ ௯ ௰ Devanagari 0 १ २ ३ ४ ५ ६ ७ ८ ९ | 750 BC – 690 BC |
| Chinese numerals, Japanese numerals, Korean numerals (Sino-Korean) | 10 | 〇/零 一 二 三 四 五 六 七 八 九 十 | |

## Decimal Numbers

The position of each digit in a weighted number system is assigned a weight based on the **base** or **radix** of the system. The radix of decimal numbers is ten, because only ten symbols (0 through 9) are used to represent any number.

The column weights of decimal numbers are powers of ten that increase from right to left beginning with $10^0 = 1$:

$\ldots 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.$

For fractional decimal numbers, the column weights are negative powers of ten that decrease from left to right:

$10^2\ 10^1\ 10^0.\ 10^{-1}\ 10^{-2}\ 10^{-3}\ 10^{-4}\ \ldots$

**https://en.wikipedia.org/wiki/List_of_numeral_systems**

## Decimal Numbers

Decimal numbers can be expressed as the sum of the products of each digit times the column value for that digit. Thus, the number 9240 can be expressed as

$$(9 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (0 \times 10^0)$$

or

$$9 \times 1{,}000 + 2 \times 100 + 4 \times 10 + 0 \times 1$$

**Example** Express the number 480.52 as the sum of values of each digit.

**Solution**

$$480.52 = (4 \times 10^2) + (8 \times 10^1) + (0 \times 10^0) + (5 \times 10^{-1}) + (2 \times 10^{-2})$$

## Binary Numbers

For digital systems, the binary number system is used. Binary has a radix of two and uses the digits 0 and 1 to represent quantities.

The column weights of binary numbers are powers of two that increase from right to left beginning with $2^0 = 1$:

$$\ldots 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0.$$

For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

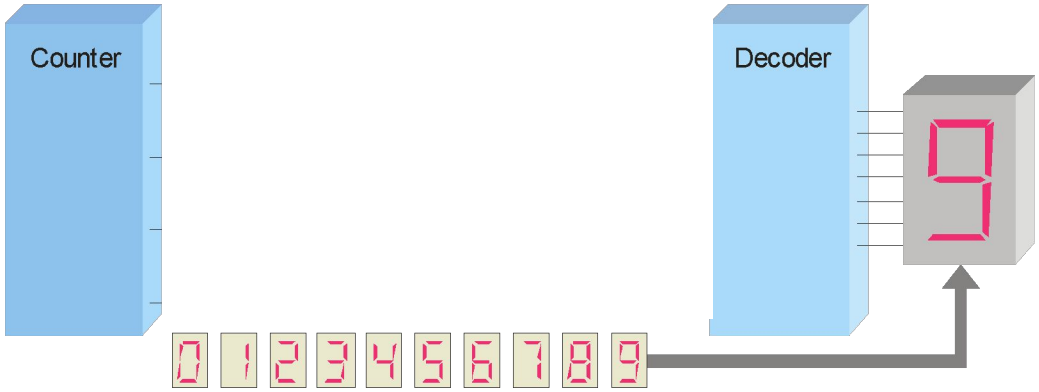
$$2^2\ 2^1\ 2^0.\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ \ldots$$

# Binary Numbers

A binary counting sequence for numbers from zero to fifteen is shown.

Notice the pattern of zeros and ones in each column.

Digital counters frequently have this same pattern of digits:




| Decimal Number | Binary Number |
|---|---|
| 0 | 0 0 0 0 |
| 1 | 0 0 0 1 |
| 2 | 0 0 1 0 |
| 3 | 0 0 1 1 |
| 4 | 0 1 0 0 |
| 5 | 0 1 0 1 |
| 6 | 0 1 1 0 |
| 7 | 0 1 1 1 |
| 8 | 1 0 0 0 |
| 9 | 1 0 0 1 |
| 10 | 1 0 1 0 |
| 11 | 1 0 1 1 |
| 12 | 1 1 0 0 |
| 13 | 1 1 0 1 |
| 14 | 1 1 1 0 |
| 15 | 1 1 1 1 |

## Binary Conversions

The decimal equivalent of a binary number can be determined by adding the column values of all of the bits that are 1 and discarding all of the bits that are 0.

**Example** Convert the binary number 100101.01 to decimal.

**Solution** Start by writing the column weights; then add the weights that correspond to each 1 in the number.

| $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ . | $2^{-1}$ | $2^{-2}$ |
|---|---|---|---|---|---|---|---|
| 32 | 16 | 8 | 4 | 2 | 1 . | ½ | ¼ |
| 1 | 0 | 0 | 1 | 0 | 1. | 0 | 1 |

32 +4 +1 +¼ = 37¼

## Binary Conversions

You can convert a decimal whole number to binary by reversing the procedure. Write the decimal weight of each column and place 1's in the columns that sum to the decimal number.

**Example** Convert the decimal number 49 to binary.

**Solution** The column weights double in each position to the right. Write down column weights until the last number is larger than the one you want to convert.

| $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$. |
|---|---|---|---|---|---|---|
| 64 | 32 | 16 | 8 | 4 | 2 | 1. |
| 0 | 1 | 1 | 0 | 0 | 0 | 1. |

## Binary Conversions

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

**Example** Convert the decimal fraction 0.188 to binary by repeatedly multiplying the fractional results by 2.

**Solution**

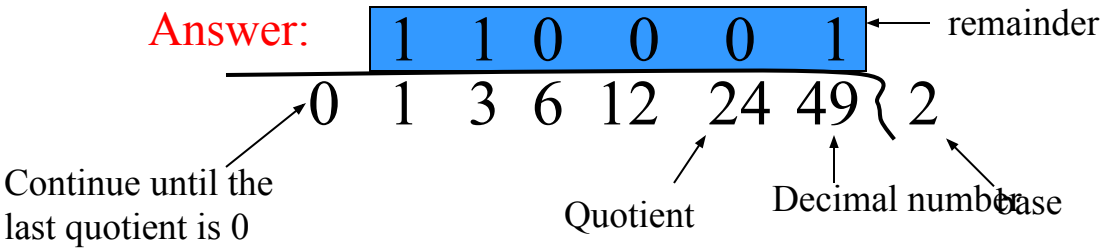
$0.188 \times 2 = 0.376$ carry = 0
$0.376 \times 2 = 0.752$ carry = 0
$0.752 \times 2 = 1.504$ carry = 1
$0.504 \times 2 = 1.008$ carry = 1
$0.008 \times 2 = 0.016$ carry = 0

MSB

Answer = .00110 (for five significant digits)

## Binary Conversions

You can convert decimal to any other base by repeatedly dividing by the base. For binary, repeatedly divide by 2:

**Example** Convert the decimal number 49 to binary by repeatedly dividing by 2.

**Solution** You can do this by "reverse division" and the answer will read from left to right. Put quotients to the left and remainders on top.