Exercise 14.7

6. $u = e^x \quad v = ye^{-x}$

$x = \ln u \qquad y = v \cdot e^x$

$y = v \cdot u$

$$J(u,v) = \begin{vmatrix} \partial x / \partial u & \partial x / \partial v \\ \partial y / \partial u & \partial y / \partial v \end{vmatrix}$$

$\frac{\partial x}{\partial u} = \frac{1}{u} \qquad \frac{\partial x}{\partial v} = 0$

$\frac{\partial y}{\partial u} = v \qquad \frac{\partial y}{\partial v} = u$

$$J(u,v) = \begin{vmatrix} 1/u & 0 \\ v & u \end{vmatrix}$$

$= 1 - 0$

$J(u,v) = 1$

Answer

10. $x = u - uv \qquad y = uv - uvw \qquad z = uvw$

$$J(u,v,w) = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v & \partial x/\partial w \\ \partial y/\partial u & \partial y/\partial v & \partial y/\partial w \\ \partial z/\partial u & \partial z/\partial v & \partial z/\partial w \end{vmatrix}$$

$\frac{\partial x}{\partial u} = 1 - v \qquad \frac{\partial x}{\partial v} = -u \qquad \frac{\partial x}{\partial w} = 0$

$\frac{\partial y}{\partial u} = v - vw \qquad \frac{\partial y}{\partial v} = u - uw \qquad \frac{\partial y}{\partial w} = -uv$

$\frac{\partial z}{\partial u} = vw \qquad \frac{\partial z}{\partial v} = uw \qquad \frac{\partial z}{\partial w} = uv$

$$J(u,v,w) = \begin{vmatrix} 1-v & -u & 0 \\ v-wv & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$$

$$= 1-v \begin{vmatrix} u-uw & -uv \\ uw & uv \end{vmatrix} + u \begin{vmatrix} v-vw & -uv \\ vw & uv \end{vmatrix}$$

$= 1-v\,[u^2v - u^2vw + u^2vw] + u\,[uv^2 - v^2uw + uv^2w]$

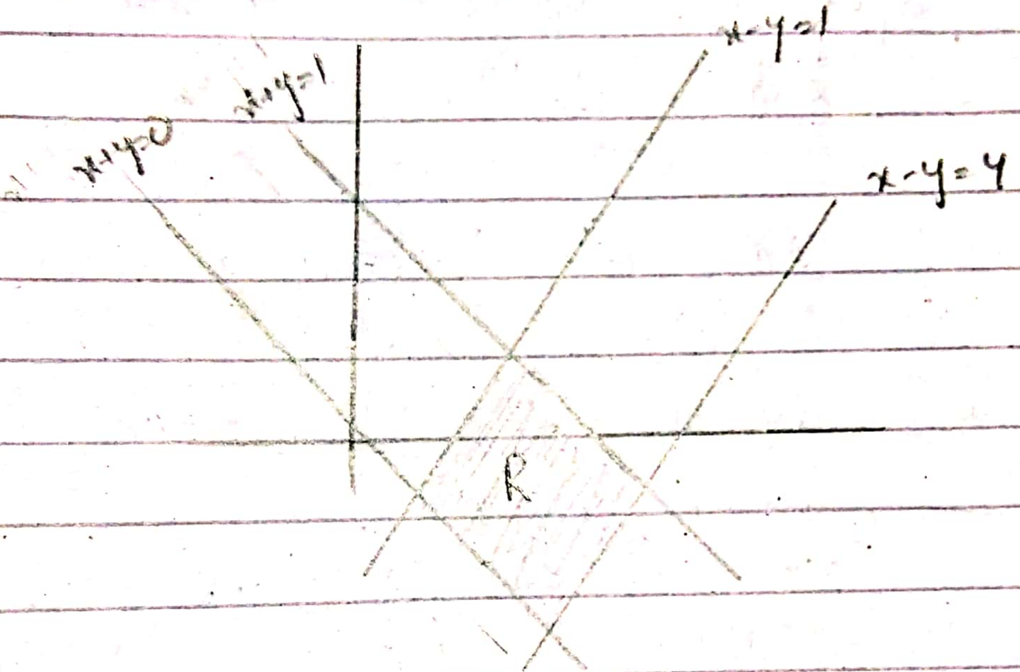
$= u^2v - u^2v^2 + u^2v^2$

$J(u,v,w) = u^2v$

Answer

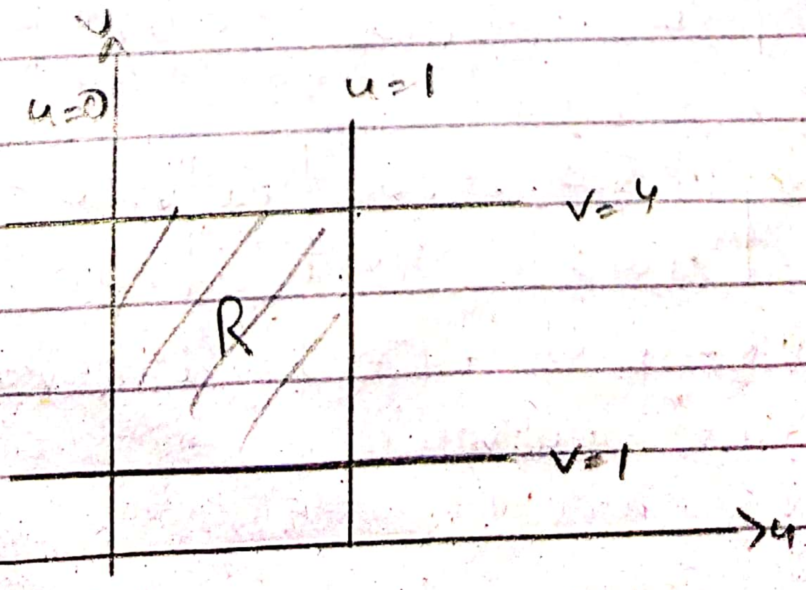
22. $\iint_R (x-y)\, e^{x^2-y^2}\, dA$

$x+y=0$, $x+y=1$, $x-y=1$, $x-y=4$



Let $x+y=u$ & $x-y=v$

$0 \leq u \leq 1$ $\quad$ $1 \leq v \leq 4$



$$f(x,y) = x-y \; e^{x^2-y^2}$$

$$f(x,y) = v \; e^{u.v}$$

$(a+b)(a-b) = a^2-b^2$

$(x+y)(x-y) = x^2-y^2$

$u = x+y$

$v = x-y$

$x = \frac{u+v}{2}$ $\quad \frac{\partial x}{\partial u} = \frac{1}{2}$ $\quad \frac{\partial x}{\partial v} = \frac{1}{2}$

$y = \frac{u-v}{2}$ $\quad \frac{\partial y}{\partial u} = \frac{1}{2}$ $\quad \frac{\partial y}{\partial v} = -\frac{1}{2}$

$$J(u,v) = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix}$$

$$J(u,v) = \frac{-1}{4} - \frac{1}{4} = \frac{-2}{4} = \frac{-1}{2}$$

$$\cancel{\int_0^1 \int_1^4 v \cdot e^{u.v} \, du \, dv}$$

$$\int_1^4 \int_0^1 v \cdot e^{u.v} \left| \frac{-1}{2} \right| du \, dv$$

$$\frac{1}{2} \int_1^4 e^{u.v} \Big|_0^1 dv$$

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$$\frac{1}{2}\int_{1}^{4} (e^{v} - e^{0})\,dv$$

$$\frac{1}{2}\int_{1}^{4} e^{v}\,dv \quad - \frac{1}{2}\int_{1}^{4} dv$$

$$\frac{1}{2}\Big| e^{v} \Big|_{1}^{4} \quad - \frac{1}{2}\Big| v \Big|_{1}^{4}$$

$$\frac{1}{2}(e^{4} - e^{1}) - \frac{1}{2}(4-1)$$

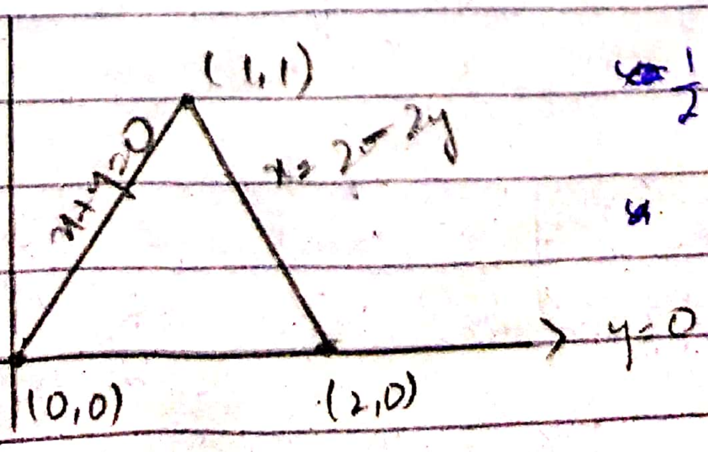
$$\frac{1}{2}(e^{4} - e - 3)$$

Answer.

Q.23 $\iint_R \sin\frac{1}{2}(x+y)\,\cos\frac{1}{2}(x-y)\,dA$

$u = \frac{1}{2}(x+y) \quad v = \frac{1}{2}(x-y)$

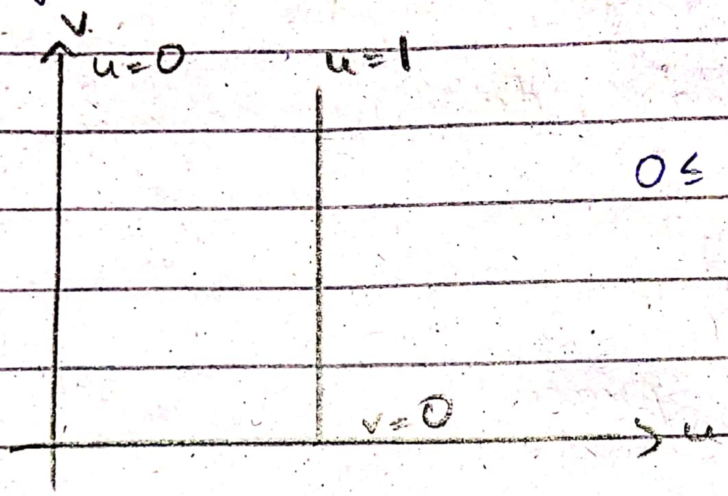
Vertices = $(0,0), (2,0), (1,1)$



$\frac{1}{2}(0) \le u \le \frac{1}{2}(2)$

$0 \le u \le 1$

$y=0$ so $v=0$



$0 \le v \le u$

$u = \frac{1}{2}x + 1/2y$

$v = \frac{1}{2}x - 1/2y$

$x = u+v$

$y = u-v$

$\frac{\partial x}{\partial u} = 1 \qquad \frac{\partial x}{\partial v} = 1$

$\frac{\partial y}{\partial u} = 1 \qquad \frac{\partial y}{\partial v} = -1$

$J(u,v) = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$

$= -2$

$\int_0^1 \int_0^u \sin u \cos v \; 2 \, dv \, du$

$$\int_0^1 2\sin u \left| \cancel{\cos v}\ \sin v \Big|_0^u \right\} du$$

$$2\int_0^1 \sin^2 u \ du$$

0.545

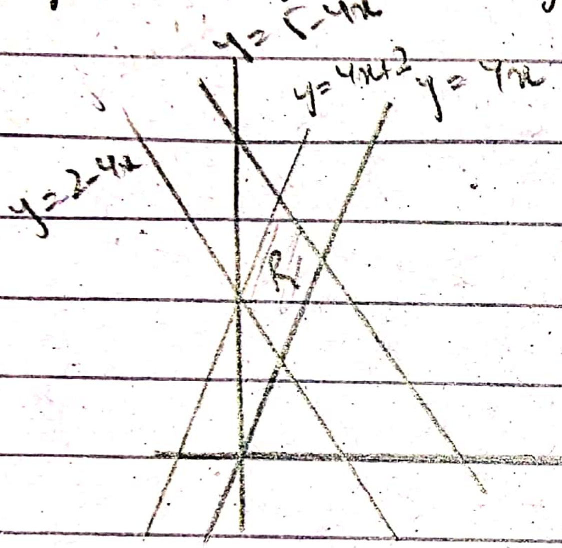
Answer

25. $$\iint_R \sqrt{16x^2 + 9y^2} \ dA$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

35. $\displaystyle\iint_R \frac{y-4x}{y+4x}\, dA$

① $y = 4x$, ② $y = 4x+2$ ③ $y = 2-4x$ ④ $y = 5-4x$



$u = y - 4x$

$v = y + 4x$

Eq ① ⇒ $y - 4x = 0$

Eq ② ⇒ $y - 4x = 2$

$0 \le u \le 2$

Eq ③ ⇒ $y + 4x = 2$

Eq ④ ⇒ $y + 4x = 5$

$2 \le v \le 5$

$-\frac{1}{2}u + \frac{1}{2}v = 4x$

$\frac{1}{8}v - \frac{1}{8}u = x$

$u = y - 4x$

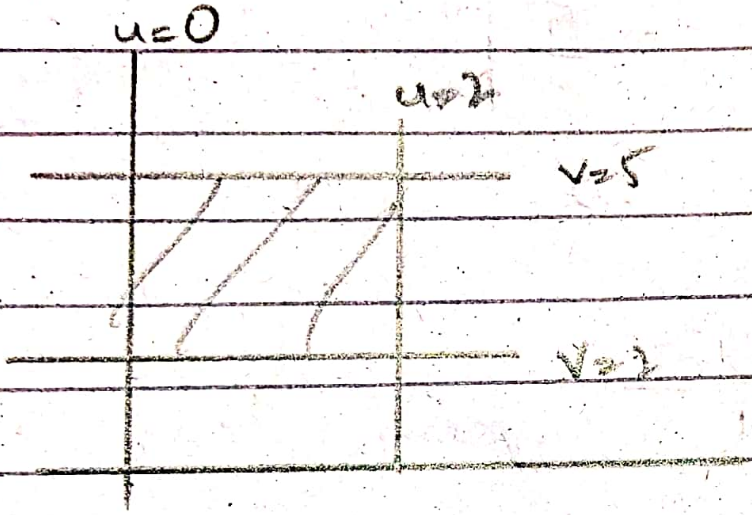
$v = y + 4x$

$y = \frac{1}{2}(u+v)$

$u = \frac{1}{2}(u+v) - 4x$

$u - \frac{1}{2}u - \frac{1}{2}v = -4x$

$$x = \frac{1}{8}(v - u)$$



$$\frac{\partial x}{\partial u} = -\frac{1}{8} \qquad \frac{\partial x}{\partial v} = \frac{1}{8}$$

$$\frac{\partial y}{\partial u} = \frac{1}{2} \qquad \frac{\partial y}{\partial v} = \frac{1}{2}$$

$$J(u,v) = \begin{vmatrix} -1/8 & 1/8 \\ 1/2 & 1/2 \end{vmatrix}$$

$$J(u,v) = -\frac{1}{8}$$

$$\int_0^2 \int_2^5 \frac{u}{v} \cdot \frac{1}{8} \, dv \, du$$

$$= \frac{1}{8} \int_0^2 u \Big| \ln v \Big|_2^5 \, du$$

$$\frac{\ln 5 - \ln 2}{8} \int_0^2 u \, du$$

$$\frac{\ln 5 - \ln 2}{8} \cdot \frac{u^2}{2} \Big|_0^2$$

$$\frac{\ln 5 - \ln 2}{8} \cdot 2$$

$$\frac{\ln 5 - \ln 2}{4}$$ Answer

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