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$x_1 - x_3 = 0$

$x_1 = x_3$

let $x_2 = s$, $x_3 = t$ so, $x_1 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

eigen space: $\{(0,1,0), (1,0,1)\}$.

EXERCISE 5.2

In Exercise 1-4, show that A and B are not similar matrices.

1. $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$

Ans.

$|A| = 2 - 3$

$|A| = -1$

$|B| = -2$

$|A| \neq |B|$

3. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans. $(\lambda I - B) = 0$

$$\begin{vmatrix} \lambda - 1 & -2 & 0 \\ -1/2 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 1 \end{vmatrix} = 0$$

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$$(\lambda-1)\{(\lambda-1)(\lambda-1)\}+2\left(-\tfrac{1}{2}(\lambda-1)\right)=0$$

$$(\lambda-1)^3-(\lambda-1)$$

$$(\lambda-1)\{\lambda^2-2\lambda+1-1\}=0$$

$$(\lambda-1)\{\lambda(\lambda-2)\}=0$$

$$\lambda(\lambda-1)(\lambda-2)=0$$ characteristic equation.

$\lambda=0,\ \lambda=1,\ \lambda=2$ eigenvalues

For $\lambda=0$

$$\begin{bmatrix} -1 & -2 & 0 \\ -1/2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} =$$

$$x_1+2x_2=0$$

$$x_1=-2x_2$$

let $x_2=t$

$x_1=-2t,\ x_2=t,\ x_3=0$

$$v_1 = t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow v_1=(-2,1,0)$$

For $\lambda=1$

$$\begin{bmatrix} 0 & -2 & 0 \\ -1/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$-2x_2=0 \Rightarrow x_2=0$

$-x_1/2=0 \Rightarrow x_1=0$

let $x_3=t$

$$v_2 = t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$v_2=(0,0,1)$

For $\lambda=2$

$$\begin{bmatrix} 1 & -2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$

let $x_2 = t$

$x_1 = 2t \ , \ x_2 = t$

$x_3 = 0$

$$v_3 = t\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \ , \Rightarrow v_3 = (2, 1, 0)$$

$$P = \begin{bmatrix} -2 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}\left[\begin{array}{ccc:ccc} -2 & 0 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -1 & -1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -1 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1/2 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1/4 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1/4 & 1/2 & 0 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} -1/4 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 0 \end{bmatrix}$$

$P^{-1}AP$

$$P^{-1}AP = \begin{bmatrix} -1/4 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1/4 & 0 & 1/4 \\ 0 & 0 & 1 \\ 1/4 & 1 & 7/4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & -1/2 & -1 \\ 0 & 1 & 0 \\ 1/2 & 1 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/4 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 7/4 & 3/2 \end{bmatrix}$$

$B \neq P^{-1}AP$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 1/2 & 1/4 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 7/4 & 3/2 \end{bmatrix}$$

In Exercise 5-8, find a matrix P that diagnelize and check your work by computing $P^{-1}AP$.

5. $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

Ans. $(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda - 1 & 0 \\ -6 & \lambda + 1 \end{bmatrix} = 0$$

$\det' = (\lambda^2 - 1) = 0$ characteristic equ.

$\lambda = \pm 1$ eigen values

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$$

$-6x_1 + 2x_2 = 0$

$+3x_1 = x_2$

$x_1 = x_2/3$ let $x_2 = t$

$x_1 = t/3$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \qquad v_1 = (1/3, 1)$$

For $\lambda = -2$

$$\begin{bmatrix} -2 & 0 \\ -6 & 0 \end{bmatrix}$$

$-2x_1 = 0 \Rightarrow x_1 = 0$ let $x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow v_2 = (0, 1)$$

$$\boxed{P = \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{Ans}}$$

$P^{-1} = ?$

$$\left[\begin{array}{cc|cc} 1/3 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 3 & 0 \\ -3 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = D$$

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7. $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Ans. $(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda-2 & 0 & +2 \\ 0 & \lambda-3 & 0 \\ 0 & 0 & \lambda-3 \end{bmatrix} = 0$$

$(\lambda-2)(\lambda-3)^2 = 0$ characteristic equ.

$\lambda = 2, 3$ eigenvalues.

For $\lambda = 2$

$$\begin{bmatrix} 0 & 0 & +2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$+2x_3 = 0 \Rightarrow x_3 = 0$

$-x_2 = 0 \Rightarrow x_2 = 0$

let $x_1 = t$

so $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \overset{t}{=} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow v_1 = (1, 0, 0)$

For $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & +2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 2x_3 = 0$

$x_1 = -2x_3$

let $x_3 = t$ $\quad x_1 = -2t$

let $x_2 = s$.

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$v_2 = (2, 0, 1)$

$v_3 = (0, 1, 0)$

$$\boxed{P = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{Ans}}$$

$P^{-1} = ?$

$$\left[\begin{array}{ccc:ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$D = P^{-1}AP$

$$D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 3 \\ 0 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

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9. Let $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$

(a) Find the eigenvalues of A.
(b) For each eigenvalue λ, find the rank of the matrix $\lambda I - A$
(c) Is A diagonalizable? Justify your conclusion.

Ans. λ $(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda-4 & 0 & -1 \\ -2 & \lambda-3 & -2 \\ -1 & 0 & \lambda-4 \end{bmatrix}$$

$$\det = (\lambda-4)(\lambda-3)(\lambda-4) - 1(\lambda-3) = 0$$
$$(\lambda-3)((\lambda-4)^2-1) = 0$$
$$(\lambda-3)(\lambda^2-8\lambda+15) = 0$$
$$(\lambda-3)(\lambda^2-3\lambda-5\lambda+15) = 0$$
$$(\lambda-3)(\lambda(\lambda-3)-5(\lambda-3)) = 0$$
$$(\lambda-3)^2(\lambda-5) = 0$$

$\boxed{\text{(a)} \quad \lambda = 5, 3 \quad \text{eigen values of } A}$

For $\lambda = 3$

$$\begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{(b)} \quad \text{rank}(3I - A) = 1}$

$x_1 + x_3 = 0 \Rightarrow x_1 = -x_3$ let $x_3 = t$

let $x_2 = s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$v_1 = (0, 1, 0)$, $v_2 = (-1, 0, 1)$

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For $\lambda = 5$

$$\begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -\cancel{3}4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{(b)\ \text{rank}(5I - A) = 2}$

$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$

let $x_3 = t$

$x_2 = 2x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$v_1 = (0, 1, 0)$

$v_2 = (-1, 0, 1)$

$v_3 = (1, 2, 1)$

(c) Since there 3 vectors and the dimension of A matrix is also 3, so it will form invertible P matrix and YES it can be diagnolize.

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In Exercise 11-14, find geometric and algebraic multiplicity of each eigenvalue of the matrix A, and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A, and find $P^{-1}AP$

11. $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

Ans. $(\lambda I - A) = 0$ For eigenvalues.

$$\begin{bmatrix} \lambda+1 & -4 & 2 \\ 3 & \lambda-4 & 0 \\ 3 & -1 & \lambda-3 \end{bmatrix}$$

$$(\lambda+1)(\lambda-4)(\lambda-3)+4(\lambda-3)(3)+2(-3-3(\lambda-4))=0$$
$$(\lambda+1)(\lambda-4)(\lambda-3)+12(\lambda-3)-6-6(\lambda-4)=0$$
$$(\lambda^2-3\lambda-4)(\lambda-3)\cancel{+12\lambda}\ 12\lambda-36-6-6\lambda+24=0$$
$$\lambda^3-3\lambda^2-4\lambda-3\lambda^2+9\lambda+12+12\lambda-18-6\lambda=0$$
$$\lambda^3-6\lambda^2+11\lambda-6=0$$
$$\lambda=1,2,3$$

For $\lambda=1$

$$\begin{bmatrix} 2 & -4 & 2 \\ 3 & -3 & 0 \\ 3 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 3 & -3 & 0 \\ 3 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & -3 \\ 0 & 5 & -5 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 5 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_3$ let $x_3 = t$

$x_2 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$v_1 = (1, 1, 1)$

geometric multiplicity = 1

algebraic multiplicity = 1

For $\lambda = 2$

$$\begin{bmatrix} 3 & -4 & 2 \\ 3 & -2 & 0 \\ 3 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4/3 & 2/3 \\ 3 & -2 & 0 \\ 3 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4/3 & 2/3 \\ 0 & 2 & -2 \\ 0 & 3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4/3 & 2/3 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 2x_3/3$

$x_2 = x_3$ let $x_3 = t$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 2/3 \\ 1 \\ 1 \end{bmatrix}$$

$v_2 = (2/3, 1, 1)$

Geometric multiplicity = 1

Algebraic multiplicity = 1

For $\lambda = 3$

$$\begin{bmatrix} 4 & -4 & 2 \\ 3 & -1 & 0 \\ 3 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1/2 \\ 3 & -1 & 0 \\ 3 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 2 & -3/2 \\ 0 & 2 & -3/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 1 & -3/4 \\ 0 & 2 & -3/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/4 \\ 0 & 1 & -3/4 \\ 0 & 0 & 0 \end{bmatrix}$$

let! $x_3 = t$

$x_1 = x_3/4$

$x_2 = 3x_3/4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1/4 \\ 3/4 \\ 1 \end{bmatrix}$$

$v_3 = (1/4, 3/4, 1)$

Geometric multiplicity = 1

Algebraic multiplicity = 1

It can be diagonalized because for every eigenvalue algebraic and geometric multiplicity is equal.

$$P = \begin{bmatrix} 1 & 2/3 & 1/4 \\ 1 & 1 & 3/4 \\ 1 & 1 & 1 \end{bmatrix}$$

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$P^{-1} = ?$

$$\left[\begin{array}{ccc:ccc} 1 & 2/3 & 1/4 & 1 & 0 & 0 \\ 1 & 1 & 3/4 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 2/3 & 1/4 & 1 & 0 & 0 \\ 0 & 1/3 & 1/2 & -1 & 1 & 0 \\ 0 & 1/3 & 3/4 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 2/3 & 1/4 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & -3 & 3 & 0 \\ 0 & 1/3 & 3/4 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -3/4 & 3 & -2 & 0 \\ 0 & 1 & 3/2 & -3 & 3 & 0 \\ 0 & 0 & 1/4 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -3/4 & 3 & -2 & 0 \\ 0 & 1 & 3/2 & -3 & 3 & 0 \\ 0 & 0 & 1 & 0 & -4 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 3 & -5 & 3 \\ 0 & 1 & 0 & -3 & +9 & -6 \\ 0 & 0 & 1 & 0 & -4 & 4 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 3 & -5 & 3 \\ -3 & 9 & -6 \\ 0 & -4 & 4 \end{bmatrix}$$

$D = P^{-1}AP$

$$D = \begin{bmatrix} 3 & -5 & 3 \\ -3 & 9 & -6 \\ 0 & -4 & 4 \end{bmatrix} \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2/3 & 1/4 \\ 1 & 1 & 3/4 \\ 1 & 1 & 1 \end{bmatrix}$$

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$$D = \begin{bmatrix} +3 & -5 & 3 \\ -6 & 18 & -12 \\ a & -12 & 12 \end{bmatrix} \begin{bmatrix} 1 & 2/3 & 1/4 \\ 1 & 1 & 3/4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\boxed{D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$$

13. $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

Ans. $\lambda I - A = 0$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ -3 & 0 & \lambda-1 \end{bmatrix}$$

$$\lambda(\lambda)(\lambda-1) = 0$$
$$\lambda^2 = 0 \text{ , } \lambda = 1$$
$$\lambda = 0, 0, 1$$

For $\lambda = 0$ Algebraic multiplicity $= 2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -3 & 0 & -1 \end{bmatrix}$$

Same for $\lambda = 0$

$\therefore -3x_1 = x_3$ let $x_2 = s$ and $x_3 = t$

so, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1/3 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ Geometric multiplicity $= 2$

$v_1 = (-1/3, 0, 1)$

$v_2 = (0, 1, 0)$

For $\lambda = 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 0 \end{bmatrix}$$

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$x_1 = 0$, $x_2 = 0$, let $x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$v_3 = (0, 0, 1)$

Algebraic multiplicity = 1

Geometric multiplicity = 1

It can be diagonalize, because geometric and algebraic multiplicity is equal for every eigenvalue.

$$P = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Ans.

$P^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} -1/3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$D = P^{-1}AP$

$$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In eachpart of Exercise 15-16, the characteristic equation of a matrix A is given. Find the size of the matrix and the possible dimension of it's eigenspaces.

15 (a) $(\lambda-1)(\lambda+3)(\lambda-5)=0$

Ans (i) A is 3x3 matrix

(ii) $\lambda=1$ (Dimension: 1)

$\lambda=-3$ (Dimension: 1)

$\lambda=5$ (Dimension: 1)

(b) $\lambda^2(\lambda-1)(\lambda-2)^3=0$

Ans. (i) A is 6x6 matrix

(ii) ~~Dimens~~ Possible dimension of eigen space of

$\lambda=0$ can be 1 or 2

$\lambda=1$ must be 1

$\lambda=2$ can be 1, 2, or 3

In Exercise 17-18, use the method of Example 6 to compute the matrix A^{10}.

17. $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Ans. $\lambda I - A = 0$

$$\begin{bmatrix} \lambda & -3 \\ -2 & \lambda+1 \end{bmatrix}$$

$\lambda(\lambda+1)-6=0$

$\lambda^2+\lambda-6=0$

$\cancel{\lambda^2+3\lambda-2\lambda}$

$\lambda^2+3\lambda-2\lambda-6=0$

$\lambda(\lambda+3)-2(\lambda+3)=0$

$(\lambda-2)(\lambda+3)=0$ characteristic equation

$\lambda=2,-3$

For $\lambda=2$

$$\begin{bmatrix} 2 & -3 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}$$

$2x_1=3x_2$ let $x_2=t$

$x_1=\frac{3t}{2}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=t\begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$$ $v_1=(3/2,1)$

For $\lambda=-3$

$$\begin{bmatrix} -3 & -3 \\ -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$x_1=-x_2$ let $x_2=t$

$x_1=-t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$ $v_2=(-1,1)$

$$P=\begin{bmatrix} 3/2 & -1 \\ 1 & 1 \end{bmatrix}$$

$D=P^{-1}AP$ $\quad P^{-1}=\dfrac{\begin{bmatrix} 1 & 1 \\ -1 & 3/2 \end{bmatrix}}{5/2}$

$$P^{-1}=\begin{bmatrix} 2/5 & 2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

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$D = P^{-1}AP$

$$D = \begin{bmatrix} 2/5 & 2/5 \\ -2/5 & 3/5 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3/2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 4/5 & 4/5 \\ 6/5 & -9/5 \end{bmatrix} \begin{bmatrix} 3/2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

$A^{10} = ?$

So, $D^{10} = \begin{bmatrix} 2^{10} & 0 \\ 0 & (-3)^{10} \end{bmatrix}$

$$D^{10} = \begin{bmatrix} 1024 & 0 \\ 0 & 59049 \end{bmatrix}$$

$A^n = PD^nP^{-1}$

$A^{10} = PD^{10}P^{-1}$

$$A^{10} = \begin{bmatrix} 3/2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1024 - 0 & \\ 0 & 59049 \end{bmatrix} \begin{bmatrix} 2/5 & 2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1536 & -59049 \\ 1024 & 59049 \end{bmatrix} \begin{bmatrix} 2/5 & 2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 24,234 & -34815 \\ -23,210 & 35,839 \end{bmatrix}$$ Ans.

19. Let $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$

Ans. P^{-1}?

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 4 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 5 & 0 & 0 & 1 \\ 0 & 1 & -4 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & -5 & 1 \\ 0 & 1 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = P^{-1}AP$$

$$D = \begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 10 & -2 \\ -1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so, P diagonalize A

$A^{11} = ?$

$A^{11} = PD^{11}P^{-1}$

$$D^{11} = \begin{bmatrix} (-2)^{11} & 0 & 0 \\ 0 & (-1)^{11} & 0 \\ 0 & 0 & (1)^{11} \end{bmatrix}$$

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$$D^{11} = \begin{bmatrix} -2048 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{11} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} -2048 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^{11} = \begin{bmatrix} -2048 & -1 & 1 \\ 0 & 0 & 1 \\ -2048 & 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^{11} = \begin{bmatrix} -1 & 10237 & -2047 \\ 0 & 1 & 0 \\ 0 & 10245 & -2048 \end{bmatrix}$$ Ans.

CHAPTER #01 REMAINING

EXERCISE 1.8

In Exercise 1–2, find the domain and co-domain of the transformation $T_A(x) = Ax$.

1 (a) A has size 3×2.

Ans Domain : R^2
Co-domain : R^3

(b) A has size 2×3

Ans. Domain : R^3
Co-domain : R^2

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(c) A has size 3×3

Ans. Domain: R^3 Operator.
Co-domain: R^3

(d) A has size 1×6

Ans. Domain: R^6
Co-domain: R^1

In Exercise 3-4, find the domain and co-domain of the transformation defined by the equations.

3 (a)
$$w_1 = 4x_1 + 5x_2$$
$$w_2 = x_1 - 8x_2$$

Ans. Domain: R^2 Operator.
Co-domain: R^2

(b)
$$w_1 = 5x_1 - 7x_2$$
$$w_2 = 6x_1 + x_2$$
$$w_3 = 2x_1 + 3x_2$$

Ans. Domain: R^2.
Co-domain: R^3

In Ex 5-6, find the domain and co-domain of the transformation defined by the matrix product.

5(a)
$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Ans Domain: R^3
Co-domain: R^2

Date

5(b) $\begin{bmatrix} 2 & -1 \\ 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Ans. Domain: R^2
Co-domain: R^3

In Exercise 7-8, find the domain and co-domain of the transformation T defined by the formula.

7. (a) $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$

Ans. Domain: R^2
Co-domain: R^2,

(b) $T(x_1, x_2, x_3) = (4x_1 + x_2, x_1 + x_2)$

Ans. Domain: R^3
Co-domain: ~~R^3~~ R^2

In Exercise 9-10, find the domain and co-domain of the transformation T defined by the formula

9. $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 4x_1 \\ x_1 - x_2 \\ 3x_2 \end{bmatrix}$

Ans. Domain: R^2
Co-domain: R^3

In Exercise 11-12, find the standard matrix for the transformation defined by the equations.

11. (a) $w_1 = 2x_1 - 3x_2 + x_3$
$w_2 = 3x_1 + 5x_2 - x_3$

Ans. $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix}$

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(b) $w_1 = 7x_1 + 2x_2 - 8x_3$
$w_2 = \quad -x_2 + 5x_3$
$w_3 = 4x_1 + 7x_2 - x_3$

Ans. $\begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix}$ Ans

13 Find the standard matrix for the transformation T defined by the formula.

(a) $T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2)$

Ans. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$

(b) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$

Ans. $\begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$

(c) $T(x_1, x_2, x_3) = (0, 0, 0, 0, 0)$

Ans. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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(d) $T(x_1, x_2, x_3, x_4) = (x_4, x_1, x_3, x_2, x_1 - x_3)$

Ans:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

15. Find the standard matrix for the operator $T: R^3 \to R^3$ defined by

$$w_1 = 3x_1 + 5x_2 - x_3$$
$$w_2 = 4x_1 - x_2 + x_3$$
$$w_3 = 3x_1 + 2x_2 - x_3$$

and then compute $T(-1, 2, 4)$ by directly substituting in the equations and then by matrix multiplication.

$$\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$$

Directly substituting

$w_1 = 3(-1) + 5(2) - (4) \Rightarrow 3$

$w_2 = 4(-1) - (2) + (4) \Rightarrow -2$

$w_3 = 3(-1) + 2(2) + 4(-1) \Rightarrow -3$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$$

By multiplying

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

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