# PARTIAL DERIVATIVES

Date

## EXERCISE 13.1

1-8 These exercises are concerned with functions of two variables.

1\. Let $f(x,y) = x^2y+1$. Find.

(e) $f(3a,a)$

Ans. $f(3a,a) = (3a)^2(a) + 1$
$f(3a,a) = 9a^3+1$ Ans

(f) $af(ab, a-b)$

Ans. $f(ab, a-b) = (ab)^2(a-b)+1$
$= a^3b^2 - a^2b^3+1$ Ans

3\. Let $f(x,y) = xy+3$. Find.

(a) $f(x+y, x-y)$

Ans. $f(x+y, x-y) = (x+y)(x-y)+3$
$= x^2-y^2+3$. Ans.

(b) $f(xy, 3x^2y^3)$

Ans $f(xy, 3x^2y^3) = 3x^3y^4+3$

5\. Find $g(u(x,y), v(x,y))$ if $g(x,y) = y\sin(x^2y)$, $u(x,y) = x^2y^3$, and $v(x,y) = \pi xy$

Ans. $u(x,y) = x^2y^3$
$v(x,y) = \pi xy$
$g(u(x,y), v(x,y)) = (\pi xy)\sin((x^2y^3)^2(\pi xy))$
$g(u(x,y), v(x,y)) = \pi xy \sin(\pi x^5y^7)$ Ans.

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1. Let $F(x,y) = x + 3x^2y^2$, $x(t) = t^2$ and $y(t) = t^3$. Find

(a) $F(x(t), y(t))$

Ans. $x(t) = t^2$, $y(t) = t^3$

$F(x(t), y(t)) = t^2 + 3(t^2)^2(t^3)^2$

$F(x(t), y(t)) = 3t^{10} + t^2$ Ans.

(b). $F(x(2), y(2))$

Ans. $x(2) = 4$, $y(2) = 8$

$F(x(2), y(2)) = 4 + 3(4)^2(8)^2$

$F(x(2), y(2)) = 3076$ Ans

17-20 These exercises involve functions of three variables.

17. Let $F(x,y,z) = xy^2z^3 + 3$. Find.

(a) $F(2,1,2)$

Ans. $F(2,1,2) = 19$ Ans.

(c) $F(0,0,0)$

Ans $F(0,0,0) = 3$ Ans.

e) $F(t, t^2, -t)$

Ans. $F(t, t^2, -t) = t(t^2)^2(-t)^3 + 3$

$= -t^8 + 3$

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19. Find $F(f(x), g(y), h(z))$ if $F(x,y,z) = y e^{xyz}$, $f(x) = x^2$, $g(y) = y+1$, and $h(z) = z^2$.

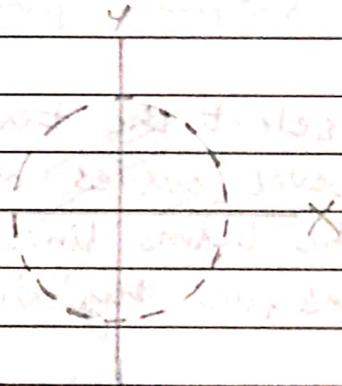
Ans- $f(x) = x^2$, $g(y) = y+1$, $h(z) = z^2$

$$F(f(x), g(y), h(z)) = (y+1) e^{x^2(y+1)z^2}$$

23-26 Sketch the domain of $f$. Use solid lines for portions of the boundary included in the domain and dashed lines for portions not included.

23. $f(x,y) = \ln(1 - x^2 - y^2)$

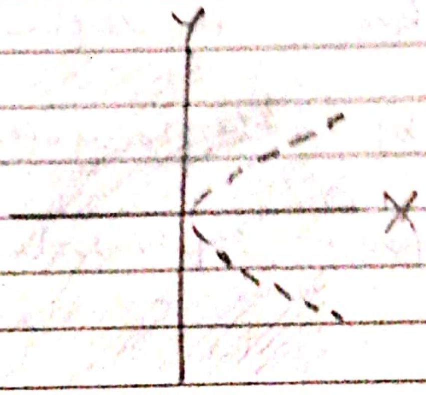
Ans. $1 - x^2 - y^2 > 0 \Rightarrow 1 > x^2 + y^2$




25. $f(x,y) = \dfrac{1}{x - y^2}$

Ans ~~$x - y^2 > 0$~~ $x - y^2 \neq 0$

$x \neq y^2$

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27–28 Define the domain of in words.

27. (a) $f(x,y) = xe^{-\sqrt{y+2}}$

$y+2 \geq 0 \quad y \geq -2$

Ans. All points above and equal to $y=-2$

(b) $f(x,y,z) = \sqrt{25-x^2-y^2-z^2}$

Ans. $25 - x^2-y^2-z^2 > 0$

$25 \geq x^2+y^2+z^2$

All points on ~~the~~ or within the sphere $x^2+y^2+z^2=25$.

(c) $f(x,y,z) = e^{xyz}$

Ans.

All points on 3-space graph.

43–44 In each part, select the term that best describes the level curves of the function f. Choose from the terms lines, circle, noncircular ellipses, parabolas, or hyperbolas.

43. (a) $f(x,y) = 5x^2-5y^2$

Ans. Hyperbolas

b) $f(x,y) = y-4x^2$

Ans. parabolas.

c) $f(x,y) = x^2+3y^2$

Ans non-circular ellipses.

d) $f(x,y) = 3x^2$

Ans lines

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51-56 sketch the level curve $z = k$ for the specified values of $k$.

51. $z = x^2+y^2$ ; $k = 0, 1, 2, 3, 4$

Ans.

$z = k$

$x^2+y^2 = k$

$k = 0$

$x^2+y^2 = 0$

$k = 1$

$x^2+y^2 = 1$

$k = 2$

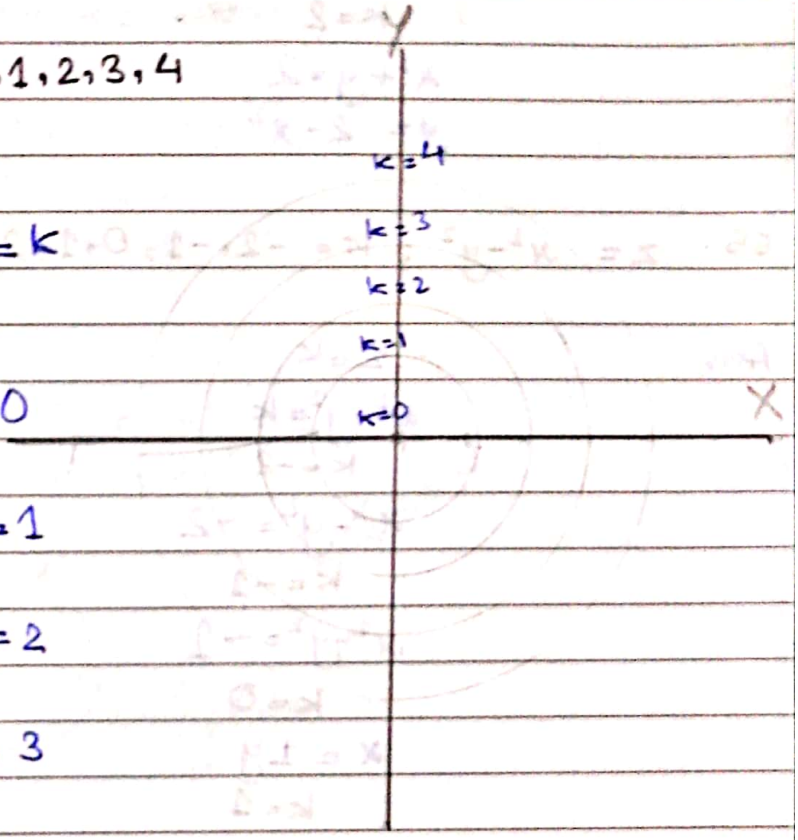
$x^2+y^2 = 2$

$k = 3$

$x^2+y^2 = 3$

$k = 4$

$x^2+y^2 = 4$




53. $z = x^2+y$ ; $k = -2, -1, 0, 1, 2$

Ans.

$z = k$

$x^2+y = k$

$k = -2$

$x^2+y = -2$

$y = -2-x^2$

$k = -1$

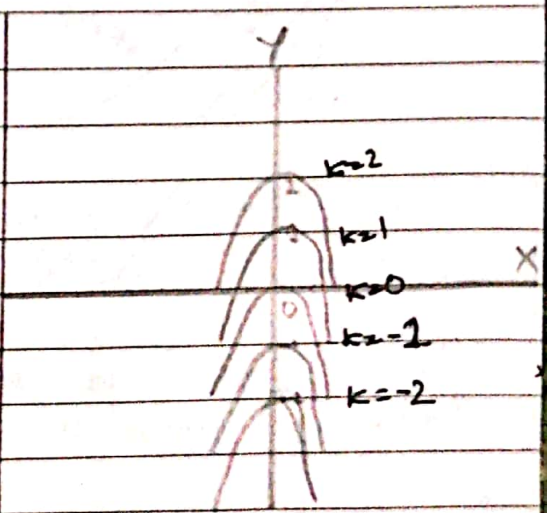
$x^2+y = -1$

$y = -1-x^2$

$k = 0$

$x^2+y = 0$

$y = -x^2$

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$k=1$

$x^2+y=1$

$y=1-x^2$

$k=2$

$x^2+y=2$

$y=2-x^2$

55. $z=x^2-y^2$ ; $k=-2,-1,0,1,2$

Ans.

$z=k$

$x^2-y^2=k$

$k=-2$

$x^2-y^2=-2$

$k=-1$

$x^2-y^2=-1$

$k=0$

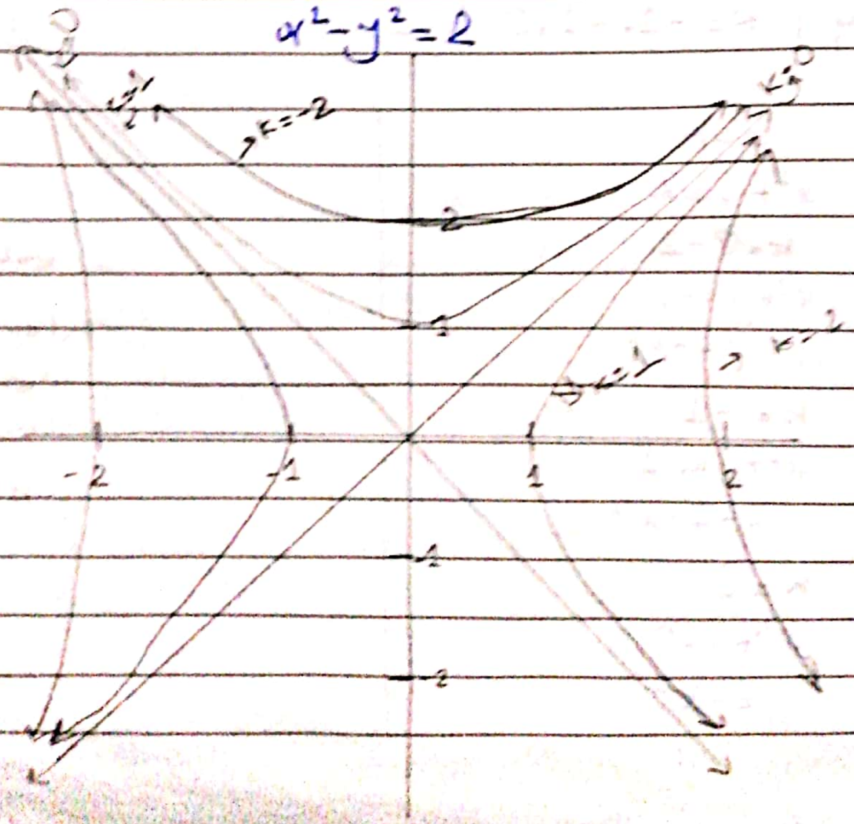
$x=\pm y$

$k=1$

$x^2-y^2=1$

$k=2$

$x^2-y^2=2$



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57–60 Sketch the level surface. $F(x,y,z) = k$

57. $f(x,y,z) = 4x^2 + y^2 + 4z^2$ ; $k = 16$

Ans.

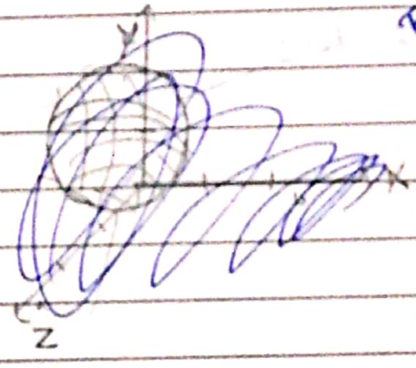
P $k = f(x,y,z)$

$4x^2 + y^2 + 4z^2 = 16$

$$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{4} = 1$$

$$\frac{x^2}{(2)^2} + \frac{y^2}{(4)^2} + \frac{z^2}{(2)^2} = 1$$

Ellipsoid




65. Let $f(x,y) = x^2 - 2x^3 + 3xy$. Find an equation of the level curve that passes through the point.

(a) $(-1, 1)$

Ans. $x^2 - 2x^3 + 3xy = (-1)^2 - 2(-1)^3 + 3(1)(-1)$

$x^2 - 2x^3 + 3xy = 0$ Ans.

67. Let $f(x,y,z) = x^2 + y^2 - z$. Find an equation of the level surface that passes through the point.

(a) $(1, -2, 0)$

Ans. $x^2 + y^2 - z = 5$ Ans

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# EXERCISE 13.2

1–6 Use limit Laws and continuity properties to evaluate the limit.

1. $$\lim_{(x,y)\to(1,3)} (4xy^2 - x^3)$$

Ans. $$= \lim_{(x,y)\to(1,3)} (4xy^2 - x^3)$$

$$= 4(1)(3)^2 - (1^3)$$

$$= 35 \quad \text{Ans}$$

3. $$\lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y}$$

Ans. $$= \lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y}$$

$$= \frac{(-1)(2)^3}{(-1)+(2)}$$

$$= -8 \quad \text{Ans.}$$

5. $$\lim_{(x,y)\to(0,0)} \ln(1 + x^2y^3)$$

Ans $$= \lim_{(x,y)\to(0,0)} \ln(1 + x^2y^3)$$

$$= \ln(1 + 0^2 0^3)$$

$$= 0 \quad \text{Ans.}$$

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7-8 Show that the limit does not exist by considering the limits as $(x,y) \to (0,0)$ along the co-ordinate axes.

(a) $$\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}$$

Ans So along y-axis $x=0$

parametric equations.

$y=t$ , $x=0$ $t \to 0$ when $y \to 0$

~~when $y \to 0$ as $t \to 0$~~

$$\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2} = \lim_{t\to 0} \frac{3}{0^2+2t^2}$$

$$\lim_{(x,y)\to(0,0)} 3/x^2+2y^2 = \frac{3}{0}$$

$= \infty$ limit does not exist

same on along x-axis.

Hence limit does not exist on co-ordinate axes

b) $$\lim_{(x,y)\to(0,0)} \frac{x+y}{2x^2+y^2}$$

Ans. So along x-axis $y=0$

$x=t$ , $y=0$

$t \to 0$ when $x \to 0$

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{2x^2+y^2} = \lim_{t\to 0} \frac{x+y}{2x^2+y^2}$$

$= +\infty$ limit does not exist.

same along y-axis.

Hence, limit does not exist on co-ordinate axes

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9-12 Evaluate the limit using the substitution $z = x^2+y^2$ and observing that $z \to 0^+$ if and only if $(x,y) \to (0,0)$

9. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

Ans. After substitution of $z = x^2+y^2$

$z \to 0^+$ when $(x,y) \to (0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{z\to 0^+} \left(\frac{\sin z}{z}\right)$$

$\therefore$ As we know $\dfrac{\sin\theta}{\theta} = 1$

So, $= \lim\limits_{z\to 0^+} \dfrac{\sin z}{z}$

$= 1$ Ans.

10. $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-\cos(x^2+y^2)}{x^2+y^2}$

given $z = x^2+y^2$

Ans. $\therefore$ $z \to 0^+$ when $(x,y) \to (0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = \lim_{z\to 0^+} \frac{1-\cos(z)}{z}$$

Applying L-Hopital rule.

$= \lim\limits_{z\to 0^+} \dfrac{0+\sin(z)}{1}$

$= 0$. Ans.

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11. $\lim_{(x,y)\to(0,0)} e^{-1/(x^2+y^2)}$

Ans given $z = x^2+y^2$

$z \to 0^+$ when $(x,y)\to(0,0)$

$$\lim_{(x,y)\to(0,0)} e^{-1/x^2+y^2} = \lim_{z\to 0^+} e^{-1/z}$$

$$= e^{-\infty}$$

$$= 0 \text{ Ans.}$$

13-22 Determine whether the limit exists. If so find it's value.

13. $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

Ans $= \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)(x^2+y^2)}{(x^2+y^2}$

$$= \lim_{(x,y)\to(0,0)} x^2-y^2$$

$= 0$ Ans. Limit exists.

15. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$

Ans. $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$ $\Rightarrow$ 0/0 so L'Hopital rule

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Applying L-Hopital rule.

$$= \lim_{(x,y)\to(0,0)} \frac{(1)(1)}{6x+4y}$$

$= +\infty$ limit does not exist

17/ $\lim_{(x,y,z)\to(2,-1,2)} \frac{xz^2}{\sqrt{x^2+y^2+z^2}}$

Ans. $= \frac{(2)(2)^2}{\sqrt{(2)^2+(-1)^2+(2)^2}}$

$= \frac{8}{\sqrt{9}}$

$= 8/3$ Ans limit exist

19. $\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}}$

Ans.

$= \lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}}$ $\boxed{0/0 \text{ L-Hop.}}$

~~Applying L-Hopital rule.~~

$= \lim_{(x,y,z)\to(0,0,0)} \cos(x^2+y^2+z^2)(2x)(2y)(2z)$

substituting $t = x^2+y^2+z$

$t \to 0^+$ when $(x,y,z)\to(0,0,0)$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = \lim_{t\to 0^+} \frac{\sin(t)}{\sqrt{t}}$$

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Applying L-Hopital rule.

$$= \lim_{t \to 0^+} \frac{\cos(t)}{1/2\, t^{-1/2}}$$

$$= \lim_{t \to 0^+} 2\sqrt{t} \cos t$$

$= 0$ Ans. Limit exists.

21. $\lim_{(x,y,z) \to (0,0,0)} \frac{e^{\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}}$

Ans substituting $t = x^2+y^2+z^2$
$t \to 0^+$ when $(x,y,z) \to (0,0,0)$

$$= \lim_{t \to 0^+} \frac{e^{\sqrt{t}}}{t}$$

$= \infty$ Limit does not exists.

23-26 Evaluate the limits by converting to polar co-ordinates.

23. $\lim_{(x,y) \to (0,0)} \sqrt{x^2+y^2} \ln(x^2+y^2)$

Ans $r = x^2+y^2$ $\quad r \to 0^+$ when $(x,y) \to (0,0)$
$x = r\cos\theta$
$y = r\sin\theta$
$\theta = \tan^{-1}(y/x)$

$$= \lim_{r \to 0^+} r \ln(r^2)$$

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$$= \lim_{r \to 0^+} 2r\ln(r)$$

$$= \lim_{r \to 0^+} \frac{2\ln(r)}{1/r}$$

$\infty/\infty$ Apply L-Hopital rules

$$= 2\lim_{r \to 0^+} \frac{1/r}{-r^{-2}}$$

$$= 2\lim_{r \to 0^+} -r$$

$= 0$ Ans.

25. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{x^2+y^2}}$

Ans.

$x = r\cos\theta$
$y = r\sin\theta$
$r = x^2+y^2$
$\theta = \tan^{-1}(y/x)$

$r \to 0^+$ when $(x,y) \to (0,0)$

$$= \lim_{r \to 0^+} \frac{r^4 \cos^2\theta \sin^2\theta}{r}$$

$$= \lim_{r \to 0^+} r^3 \cos^2\theta\sin^2\theta$$

$= 0$ Ans.

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34. (a) show that the value of $\frac{x^3y}{2x^6+y^2}$ approaches 0 as $(x,y)\to(0,0)$ along any straight line $y=mx$, or along any parabola $y=kx^2$

Ans. parametric equ. $y=mx$

Let $y=mt$, $x=t$
so $t\to 0$ when $x\to 0$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = \lim_{t\to 0} \frac{mt^4}{2t^6+m^2t^2}$$

$$= \lim_{t\to 0} \frac{mt^2}{2t^4+m^2}$$

$= 0$ Ans. (Limit exists)

Now, let $y=kx^2$
parametric equations.
$x=t$, $y=kt^2$
$t\to 0$ when $x\to 0$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = \lim_{t\to 0} \frac{kt^5}{2t^6+k^2t^4}$$

$$= \lim_{t\to 0} \frac{kt}{2t^2+k^2}$$

$= 0$ (limit exists.

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35. Show that the value of $\frac{xyz}{x^2+y^4+z^4}$ approaches 0 as $(x,y,z) \to (0,0,0)$ along any line $x=at, y=bt, z=ct$

$= \lim_{(x,y,z)\to(0,0,0)} \cancel{xyz(abc} \; abct^3$

$t \to 0^+$ when $(x,y,z) \to (0,0,0)$

$= \lim_{t\to 0^+} \frac{abct^3}{a^2t^2+b^4t^4+c^4t^4}$

$= \lim_{t\to 0^+} \frac{t^3(abc)}{t^2(a^2+b^4t^2+c^4t^2)}$

$= \lim_{t\to 0^+} \frac{t(abc)}{a^2+b^4t^2+c^4t^2}$

$= 0$ Ans. (Limit exists)

38. Let $f(x,y) = \begin{cases} \frac{\sin(x^2+y^2)}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 1 & ,(x,y) = (0,0) \end{cases}$

Show that $f$ is continuous at $(0,0)$

Ans. $= \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

$z = x^2+y^2$

$z \to 0^+$ when $(x,y) \to (0,0)$

$= \lim_{z\to 0^+} \frac{\sin(z)}{z}$

$\boxed{\because \text{As we know } \frac{\sin\theta}{\theta} = 1}$

So,

$= 1$

$f(0,0) = 1$ and $\lim_{(x,y)\to(0,0)} f(x,y)$ is equal.

$f(0,0) = \lim_{(x,y)\to(0,0)} f(x,y)$ Hence, $f(x,y)$ is continuous at $(0,0)$.

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39-40 Determine whether $f(x,y)$ has a removable discontinuity at $(0,0)$

39. $f(x,y) = \dfrac{x^2}{x^2+y^2}$

Ans. $= \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2}$

~~Applying L-Hopital rule.~~

$=$ substituting $y = mx$

$x = t,\ y = mt$

$= \lim\limits_{t\to 0} \dfrac{t^2}{m^2t^2+t^2}$

$= \lim\limits_{t\to 0} \dfrac{1}{1+m^2}$

As the value of $f(x,y)$ depends on slope so limit does not exist and is not continuous.

There is no removable discontinuity.

40. $f(x) = \begin{cases} x^2+7y^2, & \text{if } (x,y) \neq (0,0) \\ -4, & \text{if } (x,y) = (0,0) \end{cases}$

Ans. $= \lim\limits_{(x,y)\to(0,0)} x^2+7y^2$

$= 0$ limit exists

It is a removable discontinuity.

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# EXERCISE 13.3

1. Let $F(x,y) = 3x^3y^2$. Find

(a) $f_x(x,y)$

Ans. $f_x(x,y) = 9x^2y^2$ Ans

(c) $f_x(1,y)$

Ans. $f_x(x,y) = 9x^2y^2$
$f_x(1,y) = 9y^2$ Ans.

e) $f_y(1,y)$

Ans $f_y(x,y) = 6x^3y$
$f_y(1,y) = 6y$ Ans

g) $f_x(1,2)$

Ans. $f_x(x,y) = 9x^2y^2$
$f_x(1,2) = 36$ Ans.

3-10 Evaluate the indicated partial derivatives.

3. $z = 9x^2y - 3x^5y$ ; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

Ans $\frac{\partial z}{\partial x} = 18xy - 15x^4y$

$\frac{\partial z}{\partial y} = 9x^2 - 3x^5$

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5. $z = (x^2+5x-2y)^8$; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

Ans $\frac{\partial z}{\partial x} = 8(x^2+5x-2y)^7(2x+5)$

$\frac{\partial z}{\partial x} = (16x+40)(x^2+5x-2y)^7$ Ans

$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7(-2)$

$\frac{\partial z}{\partial y} = -16(x^2+5x-2y)^7$ Ans.

7. $\frac{\partial}{\partial p}(e^{-7p/q})$, $\frac{\partial}{\partial q}(e^{-7p/q})$

Ans. $= \frac{\partial}{\partial p}(e^{-7p/q})$

$= (e^{-7p/q})(-7/q)$

$= \frac{-7e^{-7p/q}}{q}$ Ans

$= \frac{\partial}{\partial q}(e^{-7p/q})$

$= (e^{-7p/q})(7p/q^2)$

$= \frac{7pe^{-7p/q}}{q^2}$ Ans.

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9. $z = \sin(5x^3y + 7xy^2)$; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

Ans $\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)(15x^2y + 7y^2)$

$\frac{\partial z}{\partial x} = (15x^2y + 7y^2)\cos(5x^3y + 7xy^2)$

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$\frac{\partial z}{\partial y} = \cos(5x^3y + 7xy^2)(5x^3 + 14xy)$

$\frac{\partial z}{\partial y} = (5x^3 + 14xy)\cos(5x^3y + 7xy^2)$

11. Let $F(x,y) = \sqrt{3x+2y}$

(a) Find the slope of the surface $z = F(x,y)$ in the x-direction at the point $(4,2)$.

Ans. $F_x(4,2) = ?$

$F_x(x,y) = \frac{1}{2\sqrt{3x+2y}}(3)$

$F_x(4,2) = \frac{3}{2(\sqrt{3(4)+2(2)})}$

$F_x(4,2) = \frac{3}{8}$ Ans.

b) Find the slope of the surface $z = F(x,y)$ in the y-direction at the point $(4,2)$.

Ans. $F_y(4,2) = ?$

$F_y(x,y) = \frac{1}{2\sqrt{3x+2y}}(2)$ => $F_y(4,2) = \frac{1}{4}$ Ans

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13. Let $z = \sin(y^2 - 4x)$

(a) Find the rate of change of z with respect to x at the point (2,1) with y held fixed.

Ans. $f_x(2,1) = ?$

$f_x(x,y) = \cos(y^2 - 4x)(-4)$

$f_x(2,1) \quad \cos(1 - 8)(-4)$

$f_x(2,1) = -4\cos(7)$ Ans

b) Find the rate of change of z with respect to y at the point (2,1) with x held fixed.

Ans. $f_y(2,1) = ?$

$f_y(x,y) = \cos(y^2 - 4x)(2y)$

$f_y(2,1) = \cos(1 - 8)(2)$

$f_y(2,1) = 2\cos(7)$ Ans.

25-30 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

25. $z = 4e^{x^2y^3}$

Ans $\frac{\partial z}{\partial x} = 4e^{x^2y^3}(2xy^3)$

$\frac{\partial z}{\partial x} = 8xy^3e^{x^2y^3}$ Ans

$\frac{\partial z}{\partial y} = 4e^{x^2y^3}(3x^2y^2)$

$\frac{\partial z}{\partial y} = 12x^2y^2e^{x^2y^3}$ Ans.

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27. $z = x^3 \ln(1 + xy^{-3/5})$

Ans $\frac{\partial z}{\partial x} = \frac{x^3}{(1+xy^{-3/5})}(y^{-3/5}) + 3x^2 \ln(1+xy^{-3/5})$

$\frac{\partial z}{\partial x} = \frac{x^3}{y^{3/5}(1+xy^{-3/5})} + 3x^2 \ln(1+xy^{-3/5})$

$\frac{\partial z}{\partial x} = \frac{x^3}{y^{3/5}\frac{(y^{3/5}+x)}{y^{3/5}}} + 3x^2 \ln(1+xy^{-3/5})$

$\frac{\partial z}{\partial x} = \frac{x^3}{(y^{3/5}+x)} + 3x^2 \ln(1+xy^{-3/5})$

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$\frac{\partial z}{\partial y} = \frac{x^3}{(1+xy^{-3/5})}(x(-3/5y^{-8/5}))$

$\frac{\partial z}{\partial y} = \frac{-3x^4y^{-8/5}}{5(1+xy^{-3/5})}$

$\frac{\partial z}{\partial y} = \frac{-3x^4}{5y^{8/5}\frac{(y^{3/5}+x)}{y^{3/5}}}$

$\frac{\partial z}{\partial y} = \frac{-3x^4}{5y(y^{3/5}+x)}$ $\Rightarrow \frac{\partial z}{\partial y} = \frac{-3x^4}{5(y^{8/5}+xy)}$ Ans.

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29. $z = \dfrac{xy}{x^2+y^2}$

$$\boxed{d\left(\frac{u}{v}\right) = \frac{v\,d(u) - u\,d(v)}{v^2}}$$

Ans. $\dfrac{\partial z}{\partial x} = \dfrac{(x^2+y^2)(y) - (xy)(2x)}{(x^2+y^2)^2}$

$$\frac{\partial z}{\partial x} = \frac{y(x^2+y^2-2x^2)}{(x^2+y^2)^2}$$

$$\frac{\partial z}{\partial x} = \frac{y(y^2-x^2)}{(y^2+x^2)^2} \quad \text{Ans.}$$

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$$\frac{\partial z}{\partial y} = \frac{(x^2+y^2)(x) - (xy)(2y)}{(x^2+y^2)^2}$$

$$\frac{\partial z}{\partial y} = \frac{x(x^2+y^2-2y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial z}{\partial y} = \frac{x(x^2-y^2)}{(x^2+y^2)^2} \quad \text{Ans}$$

31-36 Find $f_x(x,y)$ and $f_y(x,y)$

31. $f(x,y) = \sqrt{3x^5y - 7x^3y}$

Ans. $f_x(x,y) = \dfrac{1}{2\sqrt{3x^5y-7x^3y}}\left(15x^4y - 21x^2y\right)$

$$f_x(x,y) = \frac{(15x^4y - 21x^2y)}{2\sqrt{3x^5y - 7x^3y}}$$

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$$f_y(x,y) = \frac{1}{2\sqrt{3x^5y-7x^3y}}(3x^5-7x^3)$$

$$f_y(x,y) = \frac{3x^5-7x^3}{2\sqrt{3x^5y-7x^3y}} \quad \text{Ans.}$$

33. $$f(x,y) = y^{-3/2}\tan^{-1}(x/y)$$

Ans. $$f_x(x,y) = y^{-3/2}\left(\frac{1}{1+(x/y)^2}\right)$$

$$f_x(x,y) = \frac{1}{y^{3/2}\frac{(y^2+x^2)}{y^2}}(1/y)$$

$$f_x(x,y) = \frac{y\sqrt{y}\,(1/y)}{y^2+x^2}$$

$$f_x(x,y) = \frac{1}{(y^2+x^2)\sqrt{y}} \quad \text{Ans}$$

$$f_y(x,y) = y^{-3/2}\frac{(-xy^{-2})}{1+(x/y)^2} + \tan^{-1}(x/y)\left(-\frac{3}{2}y^{-5/2}\right)$$

$$f_y(x,y) = \frac{-xy^{-7/2}}{\frac{y^2+x^2}{y^2}} + -\frac{3}{2}y^{-5/2}\tan^{-1}(x/y)$$

$$f_y(x,y) = \frac{-xy^{-3/2}}{y^2+x^2} - \frac{3}{2}y^{-5/2}(\tan^{-1}(x/y)) \quad \text{Ans}$$

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35. $f(x,y) = (y^2 \tan x)^{-4/3}$

Ans $\frac{\partial}{\partial x}$ $f_x(x,y) = \frac{-4}{3}(y^2\tan x)^{-7/3}(y^2\sec^2 x)$

$f_x(x,y) = -\frac{4}{3}(y^2 \tan x)^{-7/3}(y^2\sec^2 x)$ Ans

$f_y(x,y) = \frac{-4}{3}(y^2\tan x)^{-7/3}(2y\tan x)$

$= \frac{-8}{3}$

$f_y(x,y) = \frac{-8}{3}(y^2\tan x)^{-7/3}(y\tan x)$ Ans

37-40. Evaluate the indicated partial derivatives.

37. $F(x,y) = 9 - x^2 - 7y^3$; $f_x(3,1)$, $f_y(3,1)$

Ans. $f_x(x,y) = -2x$

$F_x(3,1) = -6$ Ans

$f_y(x,y) = -21y^2$

$f_y(3,1) = -21$ Ans.

+39. $z = \sqrt{x^2+4y^2}$; $\frac{\partial z}{\partial x}(1,2)$, $\frac{\partial z}{\partial y}(1,2)$

Ans. $\frac{\partial z}{\partial x} = \frac{1(2x)}{2\sqrt{x^2+4y^2}}$

$\frac{\partial z}{\partial x}(1,2) = \frac{1}{\sqrt{1+16}} \Rightarrow \frac{\partial z}{\partial x}(1,2) = 1/\sqrt{17}$ Ans.

$\frac{\partial z}{\partial y} = \frac{1(8y)}{2\sqrt{x^2+4y^2}} \Rightarrow \frac{\partial z}{\partial y}(1,2) = \frac{8(2)}{2\sqrt{17}}$

$\frac{\partial z}{\partial y}(1,2) = 8/\sqrt{17}$ Ans.

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41. Let $f(x,y,z) = x^2y^4z^3 + xy + z^2 + 1$. Find

(a) $f_x(x,y,z)$

Ans. $F_x(x,y,z) = 2xy^4z^3 + y$

(c) $f_z(x,y,z)$

Ans. $f_z(x,y,z) = 3x^2y^4z^2 + 2z$

e) $f_y(1,2,z)$

Ans. $f_y(x,y,z) = 4x^2y^3z^3 + x$

$f_y(1,2,z) = 4(1)^2(2)^3z^3 + (1)$

$f_y(1,2,z) = 32z^3 + 1$ Ans

43-46 Find $f_x$, $f_y$, and $f_z$.

43. $f(x,y,z) = z\ln(x^2y\cos z)$

Ans. $f_x(x,y,z) = \dfrac{z}{x^2y\cos z}(2xy\cos(z))$

$f_x(x,y,z) \quad \dfrac{2z}{x}$ Ans.

$f_y(x,y,z) = \dfrac{z}{x^2y\cos(z)}(x^2\cos z)$

$f_y(x,y,z) = \dfrac{z}{y}$ Ans.

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$f_z = \frac{z}{x^2y\cos(z)}(-x^2y\sin z) + \ln(x^2y\cos z)$

$f_z(x,y,z) = -z\tan(z) + \ln(x^2y\cos z)$ Ans

45. $f(x,y,z) = \tan^{-1}\left(\frac{1}{xy^2z^3}\right)$

Ans $\cancel{f_x(x,y,z) = \frac{y^2z^3}{-1}}$

$\cancel{f_x(x,y,z) = \frac{-x^{-2}}{y^2z^3\left(\frac{xy^2z^3+1}{xy^2z^3}\right)}}$

$\cancel{f_x(x,y,z) = \frac{1}{x(xy^2z^3+1)}}$

$f_x(x,y,z) = \frac{-x^{-2}}{y^2z^3\left(\frac{x^2y^4z^6+1}{x^2y^4z^6}\right)}$

$f_x(x,y,z) = \frac{-y^2z^3}{(x^2y^4z^6+1)}$ Ans.

$f_y(x,y,z) = \frac{-2y^{-3}}{xz^3\left(\frac{x^2y^4z^6+1}{x^2y^4z^6}\right)}$

$f_y(x,y,z) = \frac{-2xyz^3}{(x^2y^4z^6+1)}$ Ans.

$f_z(x,y,z) = \frac{-3z^{-4}}{xy^2\left(\frac{x^2y^4z^6+1}{x^2y^4z^6}\right)}$

$f_z(x,y,z) = \frac{-3xy^2z^2}{(x^2y^4z^6+1)}$ Ans.

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47-50. Find $\partial w/\partial x$, $\partial w/\partial y$ and $\partial w/\partial z$.

47. $w = ye^z \sin xz$

Ans. $\frac{\partial w}{\partial x} = y\left(e^z \cos(xz)(z)\right)$

$\frac{\partial w}{\partial x} = yz\, e^z \cos(xz)$ Ans

$\frac{\partial w}{\partial y} = e^z \sin(xz)$ Ans.

$\frac{\partial w}{\partial z} = y\left(e^z \cos(xz) x + e^z \sin(xz)\right)$

$\frac{\partial w}{\partial z} = xye^z \cos(xz) + ye^z \sin(xz)$ Ans.

49. $w = \sqrt{x^2+y^2+z^2}$

Ans. $\frac{\partial w}{\partial x} = \frac{1\,(2x)}{2\sqrt{x^2+y^2+z^2}} \Rightarrow \frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}$ Ans.

$\frac{\partial w}{\partial y} = \frac{1\;2(y)}{2\sqrt{x^2+y^2+z^2}} \Rightarrow \frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}$ Ans

$\frac{\partial w}{\partial z} = \frac{1\;2z}{2\sqrt{x^2+y^2+z^2}} \Rightarrow \frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}$ Ans

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