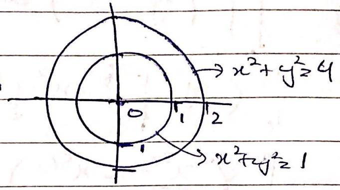
Exercise # 15.5

$\iint_\sigma f(x,y,z)\, dS.$

1. $f(x,y,z) = z^2$; σ is the portion of cone $z = \sqrt{x^2+y^2}$ b/w $z=1$ & $z=2$.

$f(x,y,g(x,y)) = x^2+y^2.$



$$= \iint (x^2+y^2) \sqrt{\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + 1^2}\, dA$$

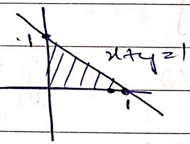
$$= \iint (x^2+y^2) \sqrt{\frac{\cancel{x^2+y^2}}{\cancel{x^2+y^2}} + 1}\, dA$$

$$= \int_0^{2\pi}\int_1^2 r^2 \cdot r \cdot \sqrt{2}\, dr\, d\theta \Rightarrow \sqrt{2}\int_0^{2\pi} \frac{r^4}{4}\Big|_1^2 d\theta$$

$$\Rightarrow \sqrt{2}\int_0^{2\pi} \frac{15}{4}\theta \Rightarrow \frac{15\sqrt{2}\,\pi}{4}.$$

2. $f(x,y,z) = xy$; $\sigma \Rightarrow x+y+z=1$ in 1^{st} quadrant

$x+y=1.$



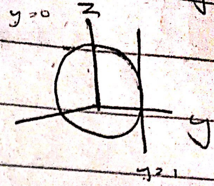
$$\int_0^1\int_0^{1-x} (xy)\sqrt{1+1+1}\, dy\, dx.$$

$$\Rightarrow \int_0^1 \sqrt{3}\, x\frac{y^2}{2}\Big|_0^{1-x} dx \Rightarrow \sqrt{3}\int_0^1 \frac{x(1-x)^2}{2}\, dx. \Rightarrow \frac{\sqrt{3}}{2}\int_0^1 (x-2x^2+x^3)\, dx$$

$$\Rightarrow \frac{\sqrt{3}}{2}\left\{\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right\}\Big|_0^1 \Rightarrow \frac{\sqrt{3}}{24}$$

Q.3 $f = x^2 y$, $\sigma \Rightarrow x^2 + z^2 = 1$ b/w $y = 0$ & $y = 1$

$$\int_0^{2\pi} \iint x^2$$



$$\int_0^1 \int_0 x^2 y \sqrt{\left(\frac{-x}{\sqrt{1-x^2}}\right)^2 + 0 + 1}\; dA$$

$$\int_0^{2\pi}\int_0^1 r^2\cos^2\theta \cdot r\sin\theta \sqrt{\frac{r^2\cos^2\theta}{(1-r^2\cos^2\theta)} + 1}\; \cancel{dx}\; r \cdot dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 r^4 \cos^2\sin\theta \sqrt{\frac{r^2\cancel{\cos^2\theta} + 1 - \cancel{r^2\cos^2\theta}}{1-r^2\cos^2\theta}}\; dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 r^{\cancel{4}}\cos^2\theta \sin\theta \frac{1}{\cancel{r}\sin\theta}\; dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 \frac{r^4}{4}\cos^2\theta \Rightarrow \int_0^{2\pi} \frac{1}{4}\cos^2\theta\, d\theta \Rightarrow \frac{1}{4}\int_0^{2\pi} \frac{\cos 2\theta + 1}{2}$$

$$\Rightarrow \frac{1}{8}\left\{\frac{\sin 2\theta}{2} + \theta\right\}_0^{2\pi} \Rightarrow \frac{1}{8}\{0 + 2\pi\}$$

$$\Rightarrow \frac{\pi}{4}.$$

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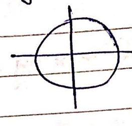
Q4. $f(x,y,z)=(x^2+y^2)z$; σ is portion of $x^2+y^2+z^2=4$ above $z=1$.

$f(x,y,g(x,y)) = (x^2+y^2)(\sqrt{4-x^2-y^2})$ $\qquad x^2+y^2=3$



$$\iint (x^2+y^2)(\sqrt{4-x^2-y^2})\sqrt{\left(\frac{-\cancel{2}x}{\sqrt{4-x^2-y^2}}\right)^2+\left(\frac{-y}{\,}\right)^2+1^2}$$

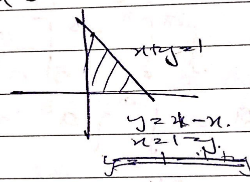
$$\int_0^{2\pi}\int_0^{\sqrt{3}} r^2\cdot\sqrt{4-r^2}\sqrt{\frac{r^2}{(\sqrt{4-r^2})}+1}\cdot r \, dr\, d\theta.$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} r^3\cancel{\sqrt{4-r^2}}\sqrt{\frac{\cancel{r^2}+4-\cancel{r^2}}{\cancel{4-r^2}}}\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} 2r^3 \cancel{\sqrt{2(2-r^2)}}\, dr\, d\theta \Rightarrow \int_0^{2\pi} \frac{r^4}{2}\Big|_0^{\sqrt{3}} d\theta$$

$$\Rightarrow \frac{9}{2}\theta\Big|_0^{2\pi} \Rightarrow 9\pi$$

Q5 $f = x-y-z$, $\sigma \Rightarrow$ ~~b/w~~ $x+y=1$ in 1st octant & b/w $z=0$ & $z=1$



$$\int_0^1\int_0^1 \{(1-y)-y-z\}\sqrt{(-1)^2+0+1^2}\, dy\, dz$$

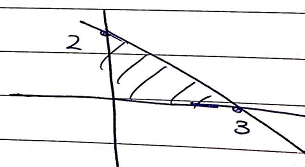
$$\int_0^1\int_0^1 (1-2y-z)\sqrt{2}\, dy\, dz$$

$$\sqrt{2}\int_0^1 (y-y^2-zy)\Big|_0^1 dz \Rightarrow \sqrt{2}\int_0^1 (\cancel{1}-\cancel{1}-z)\, dz$$

$$\sqrt{2}\cdot\frac{z^2}{2}\Big|_0^1 \Rightarrow \frac{-\sqrt{2}}{2} \Rightarrow \frac{-1}{\sqrt{2}}$$

Q6 $f = x+y$, $\sigma \Rightarrow z=6-2x-3y$ in 1st octant.

$0 = 6-2x-3y \Rightarrow 2x+3y=6.$



$$\int_0^3\int_0^{2-2/3x} (x+y)\sqrt{(-2)^2+(-3)^2+1}\, dy\, dx$$

$$\Rightarrow \int_0^1\int_0^{2-2/3x} \sqrt{14}(x+y)\, dy\, dx$$

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$2\cancel{4} - \frac{\cancel{8}4}{\cancel{3}\,\cancel{3}}x + \frac{\cancel{4}2}{\cancel{9}}x^2$

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$\Rightarrow \sqrt{14}\int_0^3 xy + \frac{y^2}{2}\Big|_0^{2-2/3x}\,dx. \Rightarrow \sqrt{14}\int_0^3 \left\{2x - \frac{2}{3}x^2 + \frac{4}{2} - \frac{4}{9\times 2}x^2\right\}dx$

$\Rightarrow \sqrt{14}\int_0^3 \left\{\frac{2x}{3} + 2 + \frac{8}{9}x^2\right\}dx \Rightarrow \sqrt{14}\left\{\frac{x^2}{3} + 2x - \frac{8}{27}x^3\right\}\Big|_0^3$

$\Rightarrow 7\sqrt{14}$

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Exercise 15.5

Q.19-20 Set up, but donot evaluate, an iterated integral equal to the given surface integral by projecting σ on a) the xy-plane b) the yz-plane & c) the xz-plane.

19 $\iint_\sigma xyz \, dS$, where σ is the portion of the plane $2x+3y+4z=12$ in first octant.

a) xy-plane :- $\quad 2x+3y=12$.

$$\Rightarrow \int_0^6 \int_0^{\frac{12-2x}{3}} xy\left(\frac{12-2x-3y}{4}\right)\sqrt{\left(\frac{2}{4}\right)^2+\left(\frac{3}{4}\right)^2+1^2}\, dy\, dx.$$

$$\Rightarrow \int_0^6 \int_0^{\frac{12-2x}{3}} \frac{\sqrt{29}}{16}\left(12xy-2x^2y-3xy^2\right) dy\, dx.$$

$$\Rightarrow \int_0^6 \frac{\sqrt{29}}{16}\left(\frac{\cancel{12}^{6}xy^2}{\cancel{2}} - \frac{\cancel{2}x^2y^2}{\cancel{2}} - \frac{\cancel{3}xy^3}{\cancel{3}}\right)\Bigg|_0^{\frac{12-2x}{3}} dx \qquad -\frac{x^2}{9}\left(144-24x+4\right.$$

$$\Rightarrow \int_0^6 \frac{\sqrt{29}}{16}\left\{\cancel{6}^{2}x\left(\frac{12-2x}{1\cancel{3}}\right)^2 - x^2\left(\frac{12-2x}{3}\right)^2 - x\left(\frac{12-2x}{3}\right)^3 \right\} dx.$$

$$\Rightarrow \frac{\sqrt{19}}{16}\left\{-\frac{(12-2x)^3}{3\times3} - \frac{144x^3}{3\times9} + \frac{24x^4}{9\times4} - \frac{4x^5}{9\times5} + \frac{(12-2x)^4}{9\times2\times4}\right\}\Bigg|_0^6$$

$$\Rightarrow \frac{\sqrt{29}}{16}\left[\left\{-\frac{(\cancel{12}-\cancel{12})^3}{9} - \frac{144(6)^3}{27} + \frac{24(6)^4}{36} - \frac{4(6)^5}{45} + 0\right\} - \left\{-\frac{(12)^3}{9} - \frac{(12)^4}{72}\right\}\right]$$

$$\Rightarrow \frac{\sqrt{29}}{16}\left[-1152 + 864 + \frac{3456}{5} + 192 + 288\right]$$

b) $\frac{\sqrt{29}}{4}\int_0^4\int_0^{\frac{12-3y}{4}} \frac{12-3y-4z}{2}\, dz\, dy.$

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20. $\iint_{\delta} xz\,ds$ where δ $x^2+y^2+z^2=a^2$ in first octant

a) $\iint x\left(\sqrt{a^2-x^2-y^2}\right)\sqrt{\left(\frac{-x}{\sqrt{a^2-x^2-y^2}}\right)^2+\left(\frac{-y}{\sqrt{a^2-x^2-y^2}}\right)^2+1}\;dA.$

$\iint x\cancel{\sqrt{a^2-x^2-y^2}}\sqrt{\frac{-(x^2+y^2)+a^2-x^2-y^2}{\cancel{a^2-x^2-y^2}}}\;dA$

$\iint x\sqrt{-2(x^2+y^2)+a^2}\;dA$

$\int_0^{\pi/2}\int_0^a r^2\cos\theta\sqrt{-2r^2+a^2}\;dr\,d\theta.$

$a\int_0^{\pi/2}\int_0^a r^2\cos\theta\;dr\,d\theta.$

Q 35–38 Evaluate $\iint_{\delta} f(x,y,z)\,dS.$

35. $f=xyz$; $r=u\cos v\,i+u\sin v\,j+3u\,k$.
$(1\le u\le 2,\; 0\le v\le \pi/2)$.

$\therefore \iint_{\delta} f(r(x),r(y),r(z))\cdot\left\|\frac{\partial r}{\partial u}\times\frac{\partial r}{\partial v}\right\|dA.$ x product.

$$\begin{vmatrix} i & j & k \\ \cos v & \sin v & 3 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$\Rightarrow i(-3u\cos v)-j(-3u\sin v)+k(u\cos^2 v+u\sin^2 v)$

$\Rightarrow -3u\cos v\,i+3u\sin v\,j+u\,k.$

$\Rightarrow \int_0^{\pi/2}\int_1^2 (u\cos v)(u\sin v)(3u)\left\|(\cos v\,i+\sin v\,j+3k)\times(-u\sin v+u\cos v)\right\|$

$\Rightarrow \int_0^{\pi/2}\int_1^2 3u^3\cos v\sin v\sqrt{(-3u\cos v)^2+(3u\sin v)^2+u^2}\;\cancel{\sqrt{(-u\sin v\cos v)^2+(u\cos v\sin v)^2}}\;du\,dv.$

$\Rightarrow \int_0^{\pi/2}\int_1^2 3u^4\cos v\sin v\sqrt{10}\;du\,dv.$

$\Rightarrow 3\sqrt{10}\int_0^{\pi/2}\cos v\sin v\cdot\frac{u^5}{5}\Big|_1^2 dv \Rightarrow 3\sqrt{10}\times\frac{31}{5}\int_0^{\pi/2}\frac{\sin 2v}{2}dv.$

$\Rightarrow \frac{93\sqrt{10}}{5}\;\frac{-\cos 2v}{4}\Big|_0^{\pi/2} \Rightarrow \frac{96\sqrt{10}}{5}\left(\frac{1}{4}+\frac{1}{4}\right)\Rightarrow \frac{93}{\sqrt{10}}$

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36. $f = \frac{x^2+z^2}{y}$; $r = 2\cos v\, i + uj + 2\sin v\, k$, $(1 \le u \le 3,\ 0 \le v \le 2\pi)$

$\left\| (1j) \times (-2\sin v\, i + 2\cos v\, k) \right\| = \left\| \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ -2\sin v & \cancel{2\cos}\ 0 & 2\cos v \end{vmatrix} \right\|$

$= \| 2\cos v\, i \cancel{+} - 2\sin v\, k \|$

$= \sqrt{(2\cos v)^2 + (-2\sin v)^2}$

$$\int_0^{2\pi}\int_1^3 \frac{(2\cos v)^2 + (2\sin v)^2}{4} \sqrt{4(\cos^2 v + \sin^2 v)}\ du\ dv.$$

$$\int_0^{2\pi}\int_1^3 \frac{4}{4} \times 2.\ du\ dv \Rightarrow 8\int_0^{2\pi} (\ln 3 - \ln 1)$$

$\Rightarrow 16\pi \ln 3.$

37. $f = \frac{1}{\sqrt{1+4x^2+4y^2}}$, $r = u\cos v\, i + u\sin v\, j + u^2 k$
$(0 \le u \le \sin v;\ 0 \le v \le \pi)$

$(\cos v\, i + \sin v\, j + 2uk) \times (-u\sin v\, i + u\cos v\, j) = \begin{vmatrix} i & j & k \\ \cos v & \cancel{u}\sin v & 2u \\ -u\sin v & u\cos v & 0 \end{vmatrix}$

$\Leftarrow i(2u^2\cos v) - j(-2u^2\sin v) + k(u\cos^2 v + u\sin^2 v)$

$$\int_0^{\pi}\int_0^{\sin v} \frac{1}{\sqrt{1+4(u\cos v)^2 + 4(u\sin v)^2}} \times \sqrt{(2u^2\cos v)^2 + (2u^2\sin v)^2 + u^2}\ du\,dv.$$

$$\int_0^{\pi}\int_0^{\sin v} \frac{1}{\sqrt{1+4u^2}} \sqrt{4u^4+u^2}\ du\,dv \Rightarrow \int_0^{\pi}\int_0^{\sin v} \frac{u}{\sqrt{\cancel{1+4u^2}}} \times \sqrt{\cancel{1+4u^2}}\ du\,d$$

$$\Rightarrow \int_0^{\pi}\int_0^{\sin v} u\ du\ dv \Rightarrow \int_0^{\pi} \frac{\sin^2 v}{2}\ dv. \Rightarrow \int_0^{\pi} \frac{-\cos 2v + 1}{4}\ dv$$

$$\Rightarrow \left\{ \frac{-\sin 2v}{8} + \frac{v}{4} \right\} \Big|_0^{\pi} \Rightarrow \frac{\cancel{4}\pi}{4} \cancel{+} \frac{\pi}{\cancel{4}} \Rightarrow$$

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Exercise # 15.8

~~Q.10 $f = x^3 i - x^3 j + y^3 k$ where $\sigma \Rightarrow x^2+y^2+z^2 = a^2$~~

~~$\iiint$ 0 → 0~~

Q 1-4 Verify Stokes Theorem (upward orientation).

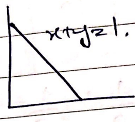
1. $F = (x-y)i + (y-z)j + (z-x)k$; $\sigma \Rightarrow x+y+z = 1$ in 1^{st} octant

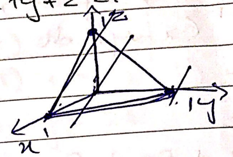
$z = 1-x-y$

$n = -(-i) - (-j) + k.$

~~$n = -(-z)i$~~

$n = i + j + k.$





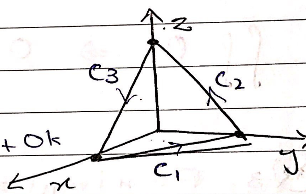
RHS :-

~~$\iint$~~ $\text{curl} F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x-y) & (y-z) & (z-x) \end{vmatrix} = i(0+1) - j(-1-0) + k(0+1) = i + j + k.$

$\iint \text{curl} F \cdot n \, dS = \int_0^1 \int_0^{1-x} (i+j+k)(i+j+k) \, dy \, dx.$

$= \int_0^1 \int_0^{1-x} (1+1+1) \, dy \, dx \Rightarrow \int_0^1 \int_0 3y \Big|_0^{1-x} dx$

$= 3 \int_0^1 \cancel{3}(1-x) \, dx \Rightarrow 3 \left\{ x - \frac{x^2}{2} \right\}_0^1 \Rightarrow \frac{3}{2}.$



LHS :-

for C_1 :- $(1,0,0) \to (0,1,0)$.

$x = 1$, $y = t$, $z = 0$. ; $r = i + tj + 0k$

$dr = 0i + dt \, j$

$\int_0^1 \cancel{(x-y)} \cdot 0 + (y-z) \, dt + 0$

$\Rightarrow \int_0^1 t \, dt \Rightarrow \frac{1}{2}.$

for C_2 :- $(0,1,0) \to (0,0,1)$

$x = 0$, $y = 1$, $z = t$

$r = j + tk$, $dr = dt \, k$,

$$\int_0^1 (z-x)\,dt \Rightarrow \int_0^1 (t-0)\,dt \Rightarrow \frac{1}{2}.$$

for C_3 :- $(0,0,1) \rightarrow (1,0,0)$.

$x=t,\quad y=0,\quad z=1$

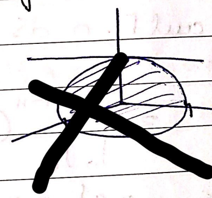
$$\int_0^1 (x-y)\,dt \Rightarrow \int_0^1 (t-0)\,dt \Rightarrow \frac{1}{2}.$$

$$C_1 + C_2 + C_3 \Rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \Rightarrow \frac{3}{2}$$

LHS = RHS $\quad$ proved

b) Q2 $F = x^2 i + y^2 j + z^2 k.\quad \sigma \Rightarrow z = \sqrt{x^2+y^2}$ below $z=1$

$$n = -\left(\frac{x}{\sqrt{x^2+y^2}}\right) i - j\left(\frac{y}{\sqrt{x^2+y^2}}\right) + k.$$



$$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= i(0) - j(0-0) + k(0).$$

$$\iint 0 \Rightarrow 0.$$

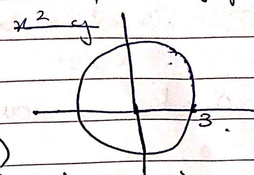
Q3. $F = xi + yj + zk,\quad \sigma \Rightarrow$ upper $z = \sqrt{a^2-x^2-y^2}$

$$n = -\left(\frac{-x}{\sqrt{a^2-x^2-y^2}}\right) i - j\left(\frac{-y}{\sqrt{a^2-x^2-y^2}}\right) + k,$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = i(0-0) - j(0-0) + k(0-0).$$

curl $= 0$ hence ans $= 0$

Q.4 $F = (z-y)i + (z+x)j - (x+y)k$, $\sigma \Rightarrow z = 9 - x^2 - y^2$
above xy-plane



LHS :- $x^2 + y^2 = 9$.

$n = -(-2x)i - j(-2y) + k$.

$= 2xi + 2yj + k$.

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (z-y) & (z+x) & -(x+y) \end{vmatrix} = i(-1-1) - j(-1-1) + k(1+1).$$

$= -2i + 2j + 2k$.

$$w = \iint (2xi + 2yj + k)(-2i + 2j + 2k)\, dA.$$

$$= \iint (\cancel{x} - 4x + 4y + 2)\, dA.$$

$$= \int_0^{2\pi}\int_0^3 (-4\cancel{r}\cos\theta + 4r\sin\theta + 2)\, r\, dr\, d\theta.$$

$$= \int_0^{2\pi} \left(\frac{-4r^3}{3}\cos\theta + \frac{4r^3}{3}\sin\theta + \frac{\cancel{2}r^2}{\cancel{2}} \right)\Bigg|_0^3 d\theta$$

$$= \int_0^{2\pi} \left(-\overset{36}{\cancel{48}}\cos\theta + \overset{36}{\cancel{48}}\sin\theta + 9 \right) d\theta.$$

$$= (-36\sin\theta - 36\cos\theta + 9\theta)\Big|_0^{2\pi}$$

$$= -36 + 18\pi + 36 \Rightarrow \boxed{18\pi}$$

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Q. Use Stokes's Theorem to evaluate $\oint_C F \cdot dr$

Q5. $F = z^2 i + 2x j - y^3 k$, $C: x^2+y^2=1$ in xy plane w/ counter clockwise down & +ve z axis.

$n = k$, curl $F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z^2 & 2x & -y^3 \end{vmatrix} = i(-3y^2) - j(-2z) + k(2)$

$= -3y^2 i + 2zj + 2k$

$\iint (-3y^2 i + 2zj + 2k)(k)\, dS$

$\int_0^{2\pi}\int_0^1 2r\, dr\, d\theta \Rightarrow \cancel{2}\int_0^{2\pi} \frac{1}{\cancel{2}}\, d\theta \Rightarrow 2\pi.$

Q6 $F = xz\, i + 3x^2y^2 j + yx\, k$; $C: z = y$. from fig 15.8.2

$n = 0i + 1j - k = j - k.$

curl $F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz & 3x^2y^2 & yx \end{vmatrix} = i(x-0) - j(y-x) + k(6xy^2 - 0)$

$= \cancel{xi}$

$= \iint (j-k)(xi + j(x-y) + k(6xy^2))\, dA.$

$= \int_0^1\int_0^3 (x - y - 6xy^2)\, dy\, dx \Rightarrow \int_0^1 \left(xy - \frac{y^2}{2} - \frac{6xy^3}{3}\right)\Big|_0^3 dx$

$= \int_0^1 \left(3x - \frac{9}{2} - 54x\right) dx \Rightarrow \int -\frac{51x^2}{2} - \frac{9x}{2}\Big|_0^1$

$= -30$

Q.37 $F = 3zi + 4xj + 2yk$. from fig 15.8.3.

$z = 4 - x^2 - y^2$, $x^2 + y^2 \leq 4$.

$n = 2xi + 2yj + k.$

curl $F = \begin{vmatrix} i & j & k \\ F_x & F_y & F_z \\ 3z & 4x & 2y \end{vmatrix} = i(2-0) - j(0-3) + k(4-0)$

$= 2i + 3j + 4k.$

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$\Rightarrow \iint (2xi + 2yj + k)(2i + 3j + 4k)\, dA.$

$\Rightarrow \iint (4x + 6y + 4)\, dA. \Rightarrow \int_0^{2\pi}\int_0^{2} (4r\cos\theta + 6r\sin\theta + 4)\, r\, dr\, d\theta.$

$= \int_0^{2\pi} \left(\frac{4r^3\cos\theta}{3} + \frac{\cancel{6}^{2} r^3 \sin\theta}{\cancel{3}_1} + \frac{\cancel{4}^{2} r^2}{\cancel{2}_1} \right)\Big|_0^2 d\theta$

$= \int_0^{2\pi} \left(\frac{32}{3}\cos\theta + 16\sin\theta + 8 \right) d\theta$

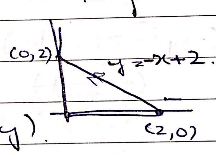
$= \left(\frac{32}{3}\sin\theta - 16\cos\theta + 8\theta \right)\Big|_0^{2\pi}$

$= -16 + 16\pi + 16 \Rightarrow 16\pi$

48. $F = -3y^2 i + 4z j + 6x k$ C: $z = \frac{1}{2}y$ w/ $(2,0,0)$ $(0,2,1)$ $(0,0,0)$ w/ counterclockwise looking down the z-axis.

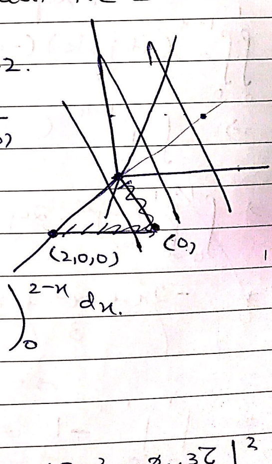
$n = -\frac{1}{2} j + k.$

$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -3y^2 & 4z & 6x \end{vmatrix}$



$= i(0-4) - j(6-0) + k(6y).$

$= -4i - 6j + 6yk.$



$\int_0^2 \int_0^{2-x} \left(-\frac{1}{2} j + k\right)(-4i - 6j + 6yk)\, dy\, dx$

$\int_0^2 \int_0^{2-x} (3 + 6y)\, dy\, dx \Rightarrow \int_0^2 \left(3y + \frac{6y^2}{2}\right)\Big|_0^{2-x} dx.$

$\int_0^2 \left(3(2-x) + 3(2-x)^2\right) dx$

$\int_0^2 (6 - 3x + 12 - 12x + 3x^2)\, dx \Rightarrow \left\{ 18x - \frac{15x^2}{2} + \frac{\cancel{3}x^3}{\cancel{3}} \right\}\Big|_0^2$

$\Rightarrow 14$