Exercise : 2.2

Question # 1

$$A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\det(A) = \det(A^T)$$

L.H.S:-

$$\det(A) = -8 - 3 = -11$$

R.H.S:-

$$\det(A^T) = \det\begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}^T$$

$$= \det\begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\det(A^T) = -8 - 3 = -11$$

$= -11.$

Hence proved

Question # 3:-

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}$$

$$\det(A) = \det(A^T)$$

L.H.S:-

$$\det(A) = 2\begin{vmatrix} 2 & 4 \\ -3 & 6 \end{vmatrix} - 1\begin{vmatrix} -1 & 3 \\ -3 & 6 \end{vmatrix} + 5\begin{vmatrix} -1 & 3 \\ 2 & 4 \end{vmatrix}$$

$\det(A) = 2(12+12) - 1(-6+9) + 5(-4-6)$

$= 48 - 3 - 50$

$\det(A) = -5$

R.H.S:- $\det(A^T)$

$$A^T = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 2 & -3 \\ 3 & 4 & 6 \end{bmatrix}$$

$$\det(A^T) = 2\begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ 4 & 6 \end{vmatrix} + 3\begin{vmatrix} 1 & 5 \\ 2 & -3 \end{vmatrix}$$

$= 2(12+12) + 1(6-20) + 3(-3-10)$

$= 48 - 14 - 39$

$\det(A^T) = -5$

hence proved!

Question # 5:-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(1)\begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(1)(-5) = -5$$

Question # 7:-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(-1)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(-1)(1) = -1.$$

Question # 9:- (Det using Row Echlon form)

$$A = \begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

$$\det(A) = 3\begin{vmatrix} 7 & -2 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} -6 & 9 \\ 1 & 5 \end{vmatrix}$$

$$= 3(35+2) + 2(-30-9)$$

$$= 111 - 78 = 33.$$

$$\Rightarrow 1(1)\begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(1)(-5) = -5$$

Question # 7:-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix} 0 & \cancel{0}1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(-1)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 1(-1)(1) = -1.$$

Question # 9:- (Det using Row Echlon form).

$$A = \begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

$$\det(A) = 3\begin{vmatrix} 7 & -2 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} -6 & 9 \\ 1 & 5 \end{vmatrix}$$

$$= 3(35+2) + 2(-30-9)$$

$$= 111 - 78 = 33.$$

using cofactor and Row operation:-

$R_1 \div 3$

$$\Rightarrow 3 \begin{bmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

$\rightarrow R_2 + 2R_1$

$$3 \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow 3(1) \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix}$$

$$\Rightarrow 3(15-4) \Rightarrow 3(11) = 33$$

Question # 11:-

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$\Rightarrow R_1 \longleftrightarrow R_2$

$\Rightarrow$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$\rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\rightarrow (1) \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

$\rightarrow R_2 - 2R_1$

$$(4) \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$\rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} \Rightarrow 1(-4-2)$$

$$\Rightarrow -6 .= Det(A)$$

Question # 13:-

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & -7 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow (1)\begin{vmatrix} -1 & 2 & 6 & 8 \\ -7 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$\Rightarrow R_2 - 7R_1$

$$\begin{array}{r} 49 \\ \times 2 \\ \hline 98 \end{array}$$

$$\Rightarrow (1)\begin{vmatrix} -1 & 2 & 6 & 8 \\ 0 & -13 & -42 & -56 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow (1)(-1)\begin{vmatrix} -13 & -42 & -56 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$R_1 + 7R_2$

$$\Rightarrow (1)(-1)\begin{vmatrix} 1 & -35 & -49 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

~~÷ R_2 by 2.~~

$R_2 - 2R_1$

$$\Rightarrow (1)(-1)\begin{vmatrix} 1 & -35 & -49 \\ 0 & 71 & 98 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow (1)(-1)(1)\begin{vmatrix} 71 & 98 \\ 1 & 1 \end{vmatrix}$$

$$\Rightarrow (1)(-1)(1)(71 - 98)$$

$$\Rightarrow (-1)(-27) = 27.$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6$$

Question # 15.

$$\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$$

$$= d\begin{vmatrix} h & i \\ b & c \end{vmatrix} - g\begin{vmatrix} e & f \\ b & c \end{vmatrix} + a\begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$= d(hc - ib) - g(ce - fb) + a(ei - hf)$$

$$= dhc - dib - gce + gfb + aei - ahf.$$

$R_1 \longleftrightarrow R_3$

$$\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6.$$

Question #17:-

$$\begin{bmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{bmatrix}$$

$\Rightarrow$ $\div$ R_1 with 3

$$3 \begin{bmatrix} a & b & c \\ -d & -e & -f \\ 4g & 4h & 4i \end{bmatrix}$$

$\Rightarrow$ $\div$ R_3 by 4

$$3(4) \begin{bmatrix} a & b & c \\ -d & -e & -f \\ g & h & i \end{bmatrix}$$

$\div$ $(-)$ to R_2

$$= -(3)(4) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$\Rightarrow -(3)(4)(-6)$

$\Rightarrow 72.$

Question # 19:-

$$\begin{bmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{bmatrix}$$

$R_1 - R_3$

$$\Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$\Rightarrow$ -6

Question # 21:-

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$$

$\Rightarrow R_1 \div (-3)$

$$(-3) \begin{vmatrix} a & b & c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$$

$\Rightarrow R_3 - 4R_2$

$$(-3) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \Rightarrow -3(6) = -18$$

Question # 27:-

$$\begin{bmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{bmatrix} = -2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$\rightarrow$ $\cancel{R_1 + R_2}$ $C_1 + C_2$

$$\begin{bmatrix} 2a_1 & a_1-b_1 & c_1 \\ 2a_2 & a_2-b_2 & c_2 \\ 2a_3 & a_3-b_3 & c_3 \end{bmatrix}$$

$\rightarrow$ $\cancel{R_2 - 1/2\,a\,R_1}$ $C_2 - 1/2\,C_1$

$$\begin{bmatrix} 2a_1 & -b_1 & c_1 \\ 2a_2 & -b_2 & c_2 \\ 2a_3 & -b_3 & c_3 \end{bmatrix}$$

$\rightarrow$ 2 common from R_1 and $-$ from R_2

$$-2 \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Hence proved!

Question # 23.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

L.H.S:-

$R_3 - aR_2$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 0 & b^2-ab & c^2-ca \end{vmatrix}$$

$R_2 - aR_1$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b(b-a) & c(c-a) \end{vmatrix}$$

$$\Rightarrow 1 \begin{vmatrix} b-a & c-a \\ b(b-a) & c(c-a) \end{vmatrix}$$

$$= 1\left[c(b-a)(c-a) - b(c-a)(b-a)\right]$$

$$\Rightarrow (b-a)(c-a)(c-b)$$

Hence proved!

Question # 29:-

$$A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

By $C_1 \div$ by 2

$$\begin{bmatrix} -2 & 4 & 1 & 4 \\ 3 & 1 & 5 & 1 \\ 1 & 5 & 6 & 5 \\ 4 & -3 & 4 & -3 \end{bmatrix}$$

As C_2 and C_3 are equivalent

$\det(A) = 0$

Exercise: 2.3

Question # 1:-

$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, \quad K = 2$

$\det(KA) = K^n \det(A)$

L.H.S:-

$\Rightarrow \det \begin{bmatrix} -2 & 4 \\ 6 & 8 \end{bmatrix}$

$\Rightarrow -16 - 24 \Rightarrow -40$

R.H.S:-

$= 2^2 \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$= 4(-4-6) \Rightarrow 4(-10)$

$= -40$

Hence proved!

Question # 3:-

$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}, \quad K = -2$

L.H.S:- $\det(KA)$

$= \det \begin{bmatrix} -4 & 2 & -6 \\ -6 & -4 & -2 \\ -2 & -8 & -10 \end{bmatrix}$

$$= -4\begin{vmatrix} -4 & -2 \\ -8 & -10 \end{vmatrix} + 6\begin{vmatrix} 2 & -6 \\ -8 & -10 \end{vmatrix} - 2\begin{vmatrix} 2 & -6 \\ -4 & -2 \end{vmatrix}$$

$\Rightarrow -4(40-16)+6(-20-48)-2(-4-24)$

$\Rightarrow -4(24)+6(-68)-2(-28)$

$\Rightarrow -96 - 408 + 56$

$\Rightarrow -448.$

R.H.S:- $K^n \det(A)$.

$$= (-2)^3\left[2\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} - 3\begin{vmatrix} -1 & 3 \\ 4 & 5 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}\right]$$

$\Rightarrow (-8)[2(10-4)-3(-5-12)+1(-1-6)]$

$\Rightarrow (-8)[12+51-7]$

$\Rightarrow -448.$

Question # 5:-

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

det (AB) = det BA.

$$AB = \begin{bmatrix} 2+7 & -2+1 & 6+2 \\ 3+28 & -3+4 & 9+8 \\ 10 & 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 9 & -1 & 8 \\ 31 & 1 & 17 \\ 10 & 0 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 1-4 & 6 \\ 14+3 & 7+4 & 4 \\ 10 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & 6 \\ 17 & 11 & 4 \\ 10 & 5 & 2 \end{bmatrix}$$

L.H.S:- det (AB)

$$\Rightarrow 9\begin{vmatrix} 1 & 17 \\ 0 & 2 \end{vmatrix} - 31\begin{vmatrix} -1 & 8 \\ 0 & 2 \end{vmatrix} + 10\begin{vmatrix} -1 & 8 \\ 1 & 17 \end{vmatrix}$$

$$\Rightarrow 9(2) - 31(-2) + 10(-17-8)$$

$$\Rightarrow 18 + 62 - 250$$

$$\Rightarrow -170$$

R.H.S:- det (BA)

$$\Rightarrow -1\begin{vmatrix} 11 & 4 \\ 5 & 2 \end{vmatrix} - 17\begin{vmatrix} -3 & 6 \\ 5 & 2 \end{vmatrix} + 10\begin{vmatrix} -3 & 6 \\ 11 & 4 \end{vmatrix}$$

$$\Rightarrow -1(22-20) - 17(-6-30) + 10(-12-66)$$

$$\Rightarrow -2 + 612 - 780$$

$$\Rightarrow -170$$

hence ~~dont~~ prove!

det (A+B) = det(A) + det(B)

L.H.S:-

$$A+B = \begin{bmatrix} 3 & 0 & 3 \\ 10 & 5 & 2 \\ 5 & 0 & 3 \end{bmatrix}$$

$$\det(A+B) = 3\begin{vmatrix} 5 & 2 \\ 0 & 3 \end{vmatrix} - 10\begin{vmatrix} 0 & 3 \\ 0 & 3 \end{vmatrix} + 5\begin{vmatrix} 0 & 3 \\ 5 & 2 \end{vmatrix}$$

$$= 3(15) - 10(0) + 5(-15)$$

$$= 45 - 75$$

$$= -30$$

R.H.S:-

$$\det(A) = 2\begin{vmatrix} 4 & 0 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$$

$$= 2(8) - 3(2)$$

$$= 16 - 6 = 10$$

$$\det(B) = 1\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} - 7\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix} + 5\begin{vmatrix} -1 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= 1(1) - 7(-1) + 5(-2-3)$$

$$= 1 + 7 - 25$$

$$= -17$$

$$\det(A) + \det(B) = -7$$

$$\det(A+B) \neq \det(A) + \det(B)$$

Question# 7:-

$$A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\det(A) = 2\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix}$$

$$= 2(-3) + 1(15-20) + 2(-5)$$

$$= -6 - 5 - 10$$

$$\det(A) = -21 \rightarrow \det(A) \neq 0$$

INVERTIBLE.

Question #9

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2\begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} \Rightarrow 2(2) = 4$$

INVERTIBLE.

Question # 11:-

$$A = \begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix}$$

$C_3 \div$ by 2

$$\begin{bmatrix} 4 & 2 & 4 \\ -2 & 1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

Two column $\Rightarrow$ are same so Not Invertible because $\det(A) = 0$

Question # 13:-

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$$

Invertible because diagnol is non-zero so, $\det(A) \neq 0$

Question # 15:-

$$A = \begin{bmatrix} K-3 & -2 \\ -2 & K-2 \end{bmatrix}$$

$$\det(A) = (K-3)(K-2)-4$$
$$= K^2-2K-3K+6-4$$
$$= K^2-5K+2$$

$$\det(A) \neq 0$$
$$K^2-5K+2 \neq 0$$
$$K \neq \frac{5 \pm \sqrt{17}}{2}$$

Question # 17:-

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ K & 3 & 2 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} + K\begin{vmatrix} 2 & 4 \\ 1 & 6 \end{vmatrix}$$
$$= 1(2-18)-3(4-12)+K(12-4)$$
$$\det(A) = 16+24+8K$$
$$\because \det(A) \neq 0$$
$$40+8K \neq 0$$
$$\boxed{K \neq -5}$$

Question # 19:-

$$\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\det(A) = 2\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix}$$

$$= 2(-3-0) + 1(15-20) + 2(0-5)$$

$$= -6-5-10$$

$$\det(A) = -21$$

$$\det(A) \neq 0$$

Invertible

$$A^{-1} = \frac{1}{|A|} \text{ adj of } A$$

$$\cancel{A^{-1} = \frac{1}{-21} \begin{bmatrix} 2 & -1 & 2 \\ 5 & -1 & 4 \\ 5 & 0 & 3 \end{bmatrix}}$$

Adj of A:-

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} = -3$$

$$A_{12} = (-1)^{3} \begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} = -1(15-20) = 5$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix} = -4+2 = -2$$

$$A_{21} = (-1)^3 \begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} = -1(15-20) = 5$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = 6-10 = -4$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & 5 \\ 2 & 4 \end{vmatrix} = -1 \cdot (8-10) = 2$$

$$A_{31} = (-1)^4 \begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix} = (0+5) = 5$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix} = -1(5) = -5$$

$$A_{3+3} = (-1)^6 \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} = (-2+5) = 3$$

$$\text{Adj of } A = \begin{vmatrix} -3 & 5 & -2 \\ 5 & -4 & 2 \\ 5 & -5 & 3 \end{vmatrix}^T$$

Adj of A
$$\begin{vmatrix} -3 & 5 & 5 \\ 5 & -4 & -5 \\ -2 & 2 & 3 \end{vmatrix}$$

$$A^{-1} = \frac{1}{-21} \begin{vmatrix} -3 & 5 & 5 \\ 5 & 4 & -5 \\ -2 & 2 & 3 \end{vmatrix}$$

Question # 21:-

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\det(A) = 2 \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix}$$

$$\det(A) = 2(2) = 4$$

Invertible.

Adj of A:-

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2$$

$$A_{12} = (-1)^3 \begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 0$$

$$A_{13} = (-1)^4 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$A_{21} = (-1)^3 \begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 6$

$A_{22} = (-1)^4 \begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 4$

$A_{23} = (-1)^5 \begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = (-1)^4 \begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 9-5 = 4$

$A_{32} = (-1)^5 \begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 6$

$A_{33} = (-1)^6 \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2$

$\text{Adj of } A = \begin{vmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{vmatrix}^T$

$\text{Adj of } A = \begin{vmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 2 & 2 \end{vmatrix}$

$A^{-1} = \frac{1}{4} \begin{vmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 2 & 2 \end{vmatrix}$

$$A^{-1} = \begin{vmatrix} 1/2 & 3/2 & 1 \\ 0 & 1 & 3/2 \\ 0 & 1/2 & 1/2 \end{vmatrix}$$

Question # 25:-

$$4x + 5y = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

$$|D| = \begin{vmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 11\begin{vmatrix} 5 & 0 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} 5 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= 4(2-10) - 11(10) + (10)$$

$$= -32 - 110 + 10$$

$$|D| = -132.$$

$$D_x = \begin{vmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}$$

$$|D_x| = 2\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 3\begin{vmatrix} 5 & 0 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} 5 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= 2(2-10) - 3(10) + (10) \Rightarrow -36$$

$$D_y = \begin{vmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$|D_y| = 4\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} - 11\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix}$$

$$= 4(6-2) - 11(4) + (4)$$

$$= 16 - 44 + 4$$

$$= -24$$

$$D_z = \begin{vmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{vmatrix}$$

$$|D_z| = 4\begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} - 11\begin{vmatrix} 5 & 2 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= 4(1-15) - 11(5-10) + 1(15-2)$$

$$= -56 + 55 + 13$$

$$= 12$$

$$x = \frac{|D_x|}{|D|} \qquad y = \frac{|D_y|}{|D|} \qquad z = \frac{|D_z|}{|D|}$$

$$x = \frac{-36}{-132} \qquad y = \frac{-24}{-132}, \qquad z = \frac{12}{-132}$$

$$\boxed{x = \frac{3}{11}}, \quad \boxed{y = \frac{2}{13\cancel{2}}}, \quad \boxed{z = \frac{1}{11}}$$

Question # 27:-

$x_1 - 3x_2 + x_3 = 4$

$2x_1 - x_2 = -2$

$4x_1 - 3x_3 = 0$

$$D = \begin{vmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{vmatrix}$$

$$|D| = 1\begin{vmatrix} -1 & 0 \\ 0 & -3 \end{vmatrix} - 2\begin{vmatrix} -3 & 1 \\ 0 & -3 \end{vmatrix} + 4\begin{vmatrix} -3 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= 3 - 18 + 4$$

$$|D| = -11$$

$$D_x = \begin{vmatrix} 4 & 3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{vmatrix}$$

$$|D_x| = 4\begin{vmatrix} -1 & 0 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ 0 & -3 \end{vmatrix}$$

$$= 4(3) + 2(-9)$$

$$= 12 - 18$$

$$|D_x| = -6$$

$$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{vmatrix}$$

$$|D_y| = 1\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} - 2\begin{vmatrix} 4 & 1 \\ 0 & -3 \end{vmatrix} + 4\begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix}$$

$$= 1(6) - 2(-12) + 4(2)$$

$$= 6 + 24 + 8$$

$$|D_y| = 38$$

$$D_z = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix}$$

$$|D_z| = 1\begin{vmatrix} -1 & -2 \\ 0 & 0 \end{vmatrix} - 2\begin{vmatrix} -3 & 4 \\ 0 & 0 \end{vmatrix} + 4\begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix}$$

$$= 0 - 0 + 4(6+4)$$

$$|D_z| = 40$$

$$x = \frac{|D_x|}{|D|}, \quad y = \frac{|D_y|}{|D|}, \quad z = \frac{|D_z|}{|D|}$$

$$\boxed{x = \frac{-6}{-11}}, \quad \boxed{y = \frac{38}{-11}}, \quad \boxed{z = \frac{40}{-11}}$$

Question # 29:-

$$3x_1 + x_2 + x_3 = 4$$
$$-x_1 + 7x_2 - 2x_3 = 1$$
$$2x_1 + 6x_2 - x_3 = 5$$

$$D = \begin{vmatrix} 3 & 1 & 1 \\ -1 & 7 & -2 \\ 2 & 6 & -1 \end{vmatrix}$$

$$|D| = 3\begin{vmatrix} 7 & -2 \\ 6 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 6 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 7 & -2 \end{vmatrix}$$
$$= 3(-7+12) + 1(-1-6) + 2(-2-7)$$
$$= 3(5) + (-7) + 2(-9)$$
$$= 15 - 7 - 18 = -10.$$

$$D_x = \begin{vmatrix} 4 & 1 & 1 \\ 1 & 7 & -2 \\ 5 & 6 & -1 \end{vmatrix}$$

$$|D_x| = 4\begin{vmatrix} 7 & -2 \\ 6 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 6 & -1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 7 & -2 \end{vmatrix}$$
$$= 4(5) - 1(-7) + 5(-9)$$
$$= 20 + 7 - 45$$
$$= -18$$

$$D_y = \begin{vmatrix} 3 & 4 & 1 \\ -1 & 1 & -2 \\ 2 & 5 & -1 \end{vmatrix}$$

$$|D_y| = 3\begin{vmatrix} 1 & -2 \\ 5 & -1 \end{vmatrix} + 1\begin{vmatrix} 4 & 1 \\ 5 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 3(-1+10) + (-4-5) + 2(-8-1)$$

$$= 27 - 9 - 18$$

$$= 0$$

$$D_z = \begin{vmatrix} 3 & 1 & 4 \\ -1 & 7 & 1 \\ 2 & 6 & 5 \end{vmatrix}$$

$$|D_z| = 3\begin{vmatrix} 7 & 1 \\ 6 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 4 \\ 6 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 4 \\ 7 & 1 \end{vmatrix}$$

$$= 3(35-6) + 1(5-24) + 2(1-28)$$

$$= 87 + (-19) - 54$$

$$= 14$$

$$x = \frac{-18}{-10}, \quad y = 0, \quad z = \frac{14}{-10}$$

$$\boxed{x = +\frac{9}{5}} \quad \boxed{y = 0} \quad \boxed{z = -\frac{7}{5}}$$

$\det(KA) = K^n \det(A)$

Question # 33

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \det(A) = -7$$

a) $\det(3A)$ b) $\det(A^{-1})$ c) $\det(2A^{-1})$
d) $\det((2A)^{-1})$ e) $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

a) $\det(3A)$

$\det(KA) = K^n \det(A)$

$\Rightarrow 3^n \det(A)$

$\Rightarrow 3^3 (-7)$

$\Rightarrow 27 \times (-7)$

$\Rightarrow -189$.

b) $\det(A^{-1}) = \dfrac{1}{\det(A)}$

$\Rightarrow \dfrac{-1}{7}$.

c) $\det(2A^{-1}) = K^n \det(A^{-1})$

$\Rightarrow 2^3 \left(\frac{-1}{7}\right)$

$\Rightarrow -\frac{8}{7}$

d) $\det((2A)^{-1}) = \frac{1}{\det(2A)} \Rightarrow \frac{1}{K^n \det(A)}$

$\Rightarrow \frac{1}{2^3(-7)} = -\frac{1}{56}$.

e) $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} \Rightarrow \det(A^T)$

$\Rightarrow -7$.

Ans