CSE455/555 - Intro to Pattern Recognition
Problem Set 1: Bayesian Decision Theory

Due Date: Friday, February 18, 2022 11:59PM

In this problem you will apply discriminant analysis to recognize the digits in the MNIST data set http://yann.lecun.com/exdb/mnist/. As a bonus problem you will construct ”Fisher digits”. You will train your model using the training data sets (”train-images-idx3ubyte.gz” and ”train-labels-idx1-ubyte.gz”) and test the performance using the test data set (”t10k-images-idx3-ubyte.gz” and ”t10k-labels-idx1-ubyte.gz”).

1 Task-1

The images are 28 x 28 pixels in gray-scale. The categories are 0, 1, ... 9. Concatenate the image rows into a 28 x 28 vector and treat this as your feature, and assume the feature vectors in each category in the training data (”train-images-idx3-ubyte.gz”) have Gaussian distribution. Draw the mean and standard deivation of those features for the 10 categories as 28 x 28 images using the training images (”train-images-idx3-ubyte.gz”). There should be 2 images for each of the 10 digits, one for mean and one for standard deviation. We call those ”mean digits” and ”standard deviation digits”.

2 Task-2

Classify the images in the testing data set (”t10k-images-idx3-ubyte.gz”) using 0-1 loss function and Bayesian Decision Rule and report the performance.
Before coding the discriminant functions, review Section 2.6 in textbook ”Pattern Classification by Richard O. Duda, Peter E. Hart and David G. Stork”.
Answer the question: Why it doesn’t perform as good as many other methods on LeCun’s web page?

3 Task-3[Optional]

Construct the ”Fisher digits” from the MNIST data set according to Sections 3.8.2 and 3.8.3 in textbook ”Pattern Classification by Richard O. Duda, Peter E. Hart and David G. Stork”. These two web pages on Fisher Faces might be helpful:

http://www.scholarpedia.org/article/Fisherfaces
https://www.bytefish.de/blog/fisherfaces.html.
Answer the following two questions about these sections:
(a) Why should the vector *w* maximizing Eq. (103) satisfy Eq. (104)?
(b) Why should the between-class scatter matrix in Eq. (115) is $\frac{n_1 \times n_2}{n}$ times the one in Eq. (102) in two-class case (i.e., c=2)?

In addition, convince ourselves that Eq. (125) is the quotient between two ”volumes” by referring the Wikipedia page on determinant https://en.wikipedia.org/wiki/Determinant.

4 Submission

Submit your solutions as one ipynb file through UBlearn. You can use Google Colab:
https://colab.research.google.com/notebooks/intro.ipynb
https://towardsdatascience.com/getting-started-with-google-colab-f2fff97f594c. The ipynb file should include your code, execution results, any explanations and answers to the questions. Use text cells to answer questions and add explanations. Markdown guide for text cells:
https://colab.research.google.com/notebooks/markdown_guide.ipynb#scrollTo=Lhfnlq1Surtk
https://colab.research.google.com/notebooks/basic_features_overview.ipynb#scrollTo=4hfV37gxpP_c
You can also add math to text cells using LaTeX. Just place the statement within a pair of $ signs. Please typeset your mathematics. Do not upload pictures of handwriting math formulas. Math typesetting help: https://www.codecogs.com/latex/eqneditor.php

5 Libraries

Basic libraries are allowed, such as gzip, pickle, math, numpy, matplotlib, etc.
Do not use any Python libraries/toolboxes, built-in functions, or external tools/libraries that directly perform classification. You have to code the discriminant functions by yourself. Do not use libraries like sklearn to perform the discriminant analysis.

6 Rubric

Total: 10 points + 2 bonus points

Task-1:
5 points: 3 points code, 2 points output images.
Task-2:
5 points: 3 points code,1 points performance report, 1 points question answers.
Task-3:
2 bonus points: 1 points code and result, 1 points question answers.

7 Acknowledgement

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