# INFO 6205 Program Structures & Algorithms
# Assignment – 4

## Task:

1. For weighted quick union, store the depth rather than the size.
2. For weighted quick union with path compression, do two loops, so that all intermediate nodes point to the root, not just the alternates.

For both, code the alternative and benchmark it against the implementation in the repository.

**Output:** I have created new files WQU_Depth and WQUPC_2 where I have implemented the alternatives to weighted quick union to store depth instead of size (as part1) and weighted quick union with path compression where all intermediate nodes point to the root and not just the alternates respectively. Also, I have created UF_Benchmark to conduct benchmark the implementation for these scenarios.

## Relationship Conclusion:

After I ran my code for test runs n=10 with random pairs of number in WQU_Depth and WQUPC_2 to check the correctness I have obtained following results:

**Weighted Quick Union (store depth)**

```
Number of sites n = 10
union (9, 7)
union (0, 6)
union (0, 3)
union (7, 2)
union (6, 4)
union (9, 2)
union (4, 8)
union (0, 4)
union (0, 7)
union (1, 5)
union (2, 1)
Weighted Quick Union with Depth:
  count: 1
  parents: [0, 0, 9, 0, 0, 1, 0, 9, 0, 0]
  depths: [2, 1, 0, 0, 0, 0, 0, 0, 0, 1]
0: 0, 2
1: 0, 1
2: 9, 0
3: 0, 0
4: 0, 0
5: 1, 0
6: 0, 0
7: 9, 0
8: 0, 0
```

```
9: 0, 1
With n=10 sites, generated= 11 connections
```

### Weighted Quick Union with Path Compression

```
For Number of sites n = 10
union (7, 6)
union (6, 8)
union (1, 5)
union (1, 8)
union (5, 0)
union (1, 6)
union (9, 2)
union (6, 4)
union (0, 3)
union (8, 1)
union (5, 9)
WQUPC_2: weighted quick union with path compression
 count: 1
 parents: [7, 7, 9, 7, 7, 1, 7, 7, 7, 7]
 sizes: [1, 2, 1, 1, 1, 1, 1, 10, 1, 2]
0: 7, 1
1: 7, 2
2: 9, 1
3: 7, 1
4: 7, 1
5: 1, 1
6: 7, 1
7: 7, 10
8: 7, 1
9: 7, 2
With n=10 sites, generates = 11 connections
```

### Benchmark Results: (for all cases mentioned above)

I have conducted benchmarking test for range n =100 to n = 10,000 with 10 runs for each n I did this for all five scenarios (weighted quick union (depth) ,weighted quick union (size), height weighted quick union, weighted quick union with one pass path compression (grandparent fix), and weighted quick union with two pass path compression (connecting intermediate nodes to root). For each value of n, I have implemented method to get the maximum depth of the tree at the end of all components connected.
When I ran the benchmark implementation for runs=10, n=100 to 10000 I got below results:

```
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 100time taken to connect all nodes in weighted quick union(with depth) = 0.08337
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
```

```
For nodes= 100 time taken to connect all nodes in weighted quick union(with size) = 0.05269
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 100 time taken to connect all nodes in height weighted quick union = 0.06068
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes=100time taken to connect all nodes in weighted quick union with one pass path compression =
0.11285
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 100time taken to connect all nodes in weighted quick union with two pass path compression =
0.05775
```

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```
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 200time taken to connect all nodes in weighted quick union(with depth) = 0.11873
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 200 time taken to connect all nodes in weighted quick union(with size) = 0.061419999999999995
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 200 time taken to connect all nodes in height weighted quick union = 0.06908
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes=200time taken to connect all nodes in weighted quick union with one pass path compression =
0.08549
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 200time taken to connect all nodes in weighted quick union with two pass path compression =
0.06938
```

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```
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 400time taken to connect all nodes in weighted quick union(with depth) = 0.15542
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 400 time taken to connect all nodes in weighted quick union(with size) = 0.12795
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 400 time taken to connect all nodes in height weighted quick union = 0.12729
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
```

```
For nodes=400time taken to connect all nodes in weighted quick union with one pass path compression =
0.18995
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 400time taken to connect all nodes in weighted quick union with two pass path compression =
0.20815999999999998

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 800time taken to connect all nodes in weighted quick union(with depth) = 0.28826999999999997
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 800 time taken to connect all nodes in weighted quick union(with size) = 0.27864
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 800 time taken to connect all nodes in height weighted quick union = 0.25606
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes=800time taken to connect all nodes in weighted quick union with one pass path compression =
0.31081
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 800time taken to connect all nodes in weighted quick union with two pass path compression =
0.27876

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 1600time taken to connect all nodes in weighted quick union(with depth) = 0.36059
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 1600 time taken to connect all nodes in weighted quick union(with size) = 0.57192
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 1600 time taken to connect all nodes in height weighted quick union = 0.28869
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes=1600time taken to connect all nodes in weighted quick union with one pass path compression =
0.36468
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10
runs
For nodes= 1600time taken to connect all nodes in weighted quick union with two pass path compression =
0.28332
```

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 3200time taken to connect all nodes in weighted quick union(with depth) = 0.62783
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 3200 time taken to connect all nodes in weighted quick union(with size) = 0.49812
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 3200 time taken to connect all nodes in height weighted quick union = 0.45653
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes=3200time taken to connect all nodes in weighted quick union with one pass path compression = 0.65029
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 3200time taken to connect all nodes in weighted quick union with two pass path compression = 0.63495

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 6400time taken to connect all nodes in weighted quick union(with depth) = 1.2379799999999999
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 6400 time taken to connect all nodes in weighted quick union(with size) = 0.77683
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 6400 time taken to connect all nodes in height weighted quick union = 0.66579
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes=6400time taken to connect all nodes in weighted quick union with one pass path compression = 0.8676999999999999
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 6400time taken to connect all nodes in weighted quick union with two pass path compression = 0.73131

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 12800time taken to connect all nodes in weighted quick union(with depth) = 2.38966
```

```
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 12800 time taken to connect all nodes in weighted quick union(with size) = 1.72041
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 12800 time taken to connect all nodes in height weighted quick union = 1.82497
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes=12800time taken to connect all nodes in weighted quick union with one pass path compression = 2.34954
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 12800time taken to connect all nodes in weighted quick union with two pass path compression = 1.7246099999999998

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 25600time taken to connect all nodes in weighted quick union(with depth) = 5.1566
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 25600 time taken to connect all nodes in weighted quick union(with size) = 3.92665
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 25600 time taken to connect all nodes in height weighted quick union = 3.38732
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes=25600time taken to connect all nodes in weighted quick union with one pass path compression = 5.7174
2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 25600time taken to connect all nodes in weighted quick union with two pass path compression = 3.93851

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2021-03-01 21:34:52 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 51200time taken to connect all nodes in weighted quick union(with depth) = 11.5735
2021-03-01 21:34:53 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 51200 time taken to connect all nodes in weighted quick union(with size) = 7.357559999999999
2021-03-01 21:34:53 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 51200 time taken to connect all nodes in height weighted quick union = 7.25294
```

```
2021-03-01 21:34:53 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes=51200time taken to connect all nodes in weighted quick union with one pass path compression = 9.951690000000001
2021-03-01 21:34:53 INFO  Benchmark_Timer - Begin run: Benchmark Test for Union Find Experiment with 10 runs
For nodes= 51200time taken to connect all nodes in weighted quick union with two pass path compression = 7.45143

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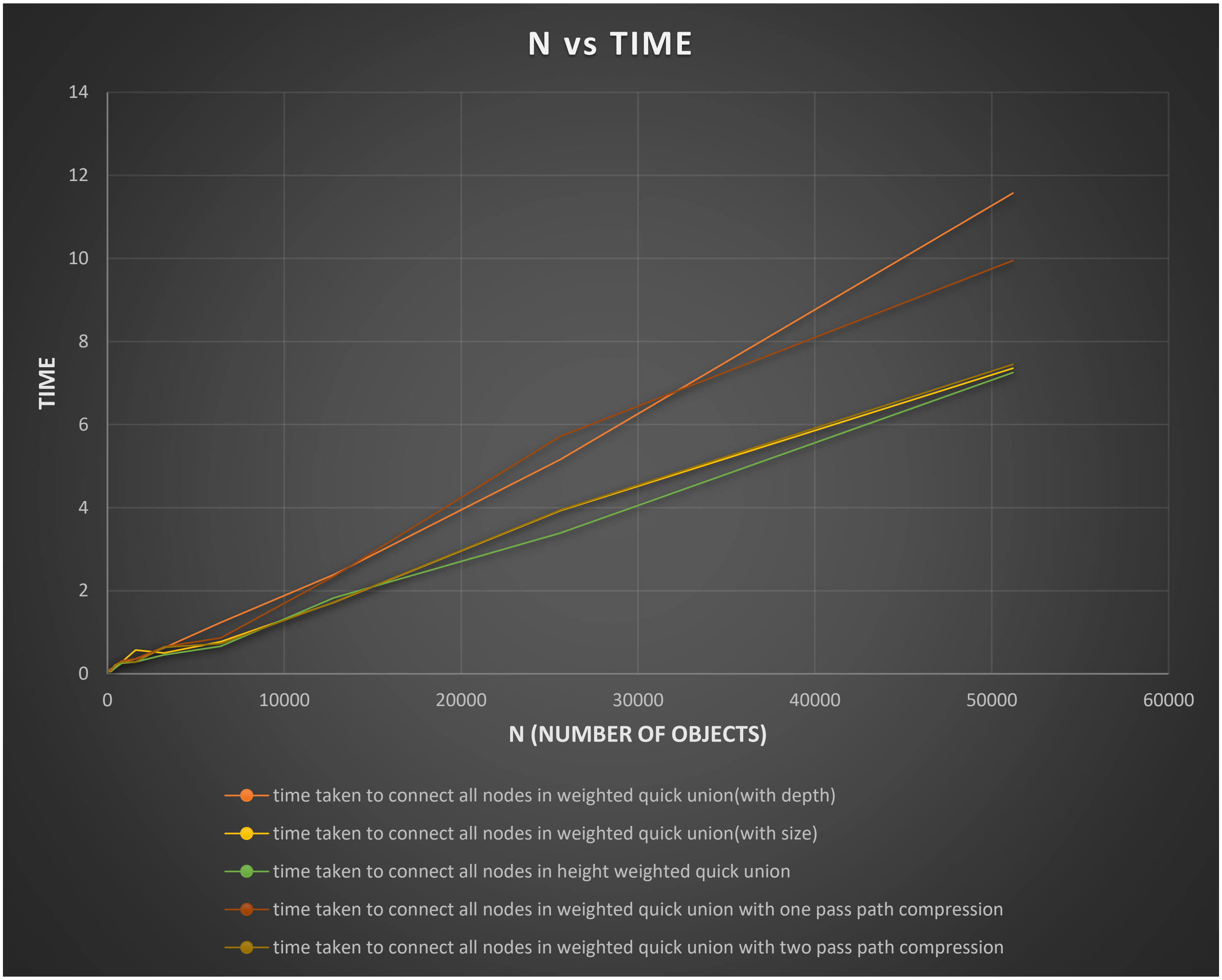
```

**The summary of benchmark results:**

| N(number of objects) | time taken to connect all nodes in weighted quick union(with depth) | time taken to connect all nodes in weighted quick union(with size) | time taken to connect all nodes in height weighted quick union | time taken to connect all nodes in weighted quick union with one pass path compression | time taken to connect all nodes in weighted quick union with two pass path compression |
|---:|---:|---:|---:|---:|---:|
| 100 | 0.08337 | 0.05269 | 0.06068 | 0.11285 | 0.05775 |
| 200 | 0.11873 | 0.06141 | 0.06908 | 0.08549 | 0.06938 |
| 400 | 0.15542 | 0.12795 | 0.12729 | 0.18995 | 0.20815 |
| 800 | 0.28826 | 0.27864 | 0.25606 | 0.31081 | 0.27876 |
| 1600 | 0.36059 | 0.57192 | 0.28869 | 0.36468 | 0.28332 |
| 3200 | 0.62783 | 0.49812 | 0.45653 | 0.65029 | 0.63495 |
| 6400 | 1.23797 | 0.77683 | 0.66579 | 0.86769 | 0.73131 |
| 12800 | 2.38966 | 1.72041 | 1.82497 | 2.34954 | 1.7246 |
| 25600 | 5.1566 | 3.92665 | 3.38732 | 5.7174 | 3.93851 |
| 51200 | 11.5735 | 7.35755 | 7.25294 | 9.95169 | 7.45143 |

**The graphical representation of the above benchmark results is show below:**



**Following output is results obtained to get the average depth for each case:**

```
For nodes: 100 the average depth in weighted quick union = 4
For nodes: 100 the average depth in height weighted quick union =  3
For nodes: 100 the average depth in weighted quick union with one pass path compression = 1
For nodes: 100 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 200 the average depth in weighted quick union = 3
For nodes: 200 the average depth in height weighted quick union =  4
For nodes: 200 the average depth in weighted quick union with one pass path compression = 2
For nodes: 200 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 400 the average depth in weighted quick union = 4
For nodes: 400 the average depth in height weighted quick union =  4
For nodes: 400 the average depth in weighted quick union with one pass path compression = 1
For nodes: 400 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 800 the average depth in weighted quick union = 5
For nodes: 800 the average depth in height weighted quick union =  5
For nodes: 800 the average depth in weighted quick union with one pass path compression = 2
For nodes: 800 the average depth in weighted quick union with two pass path compression =  1

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For nodes: 1600 the average depth in weighted quick union = 5
For nodes: 1600 the average depth in height weighted quick union =  4
For nodes: 1600 the average depth in weighted quick union with one pass path compression = 2
For nodes: 1600 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 3200 the average depth in weighted quick union = 5
For nodes: 3200 the average depth in height weighted quick union =  6
For nodes: 3200 the average depth in weighted quick union with one pass path compression = 2
For nodes: 3200 the average depth in weighted quick union with two pass path compression =  2

--------------------------------------------------------------------------------------------------------

For nodes: 6400 the average depth in weighted quick union = 5
For nodes: 6400 the average depth in height weighted quick union =  6
For nodes: 6400 the average depth in weighted quick union with one pass path compression = 2
For nodes: 6400 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 12800 the average depth in weighted quick union = 6
For nodes: 12800 the average depth in height weighted quick union =  6
For nodes: 12800 the average depth in weighted quick union with one pass path compression = 2
For nodes: 12800 the average depth in weighted quick union with two pass path compression =  2

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For nodes: 25600 the average depth in weighted quick union = 6
For nodes: 25600 the average depth in height weighted quick union =  6
For nodes: 25600 the average depth in weighted quick union with one pass path compression = 2
For nodes: 25600 the average depth in weighted quick union with two pass path compression =  2

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```

For nodes: 51200 the average depth in weighted quick union = 7
For nodes: 51200 the average depth in height weighted quick union = 6
For nodes: 51200 the average depth in weighted quick union with one pass path compression = 2
For nodes: 51200 the average depth in weighted quick union with two pass path compression = 1

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**Following is the results of average maximum depth for each cases:**

| N(number of objects) | the average depth in weighted quick union(with depth) | the average depth in weighted quick union with one pass path compression | the average depth in weighted quick union with two pass path compression | the average depth in height weighted quick union |
|---:|---:|---:|---:|---:|
| 100 | 4 | 1 | 2 | 3 |
| 200 | 3 | 2 | 2 | 4 |
| 400 | 4 | 1 | 2 | 4 |
| 800 | 5 | 2 | 1 | 5 |
| 1600 | 5 | 2 | 2 | 4 |
| 3200 | 5 | 2 | 2 | 6 |
| 6400 | 5 | 2 | 2 | 6 |
| 12800 | 6 | 2 | 2 | 6 |
| 25600 | 6 | 2 | 2 | 6 |
| 51200 | 7 | 2 | 1 | 6 |

**From the above experiment result we can observe that:**

The benchmarking results for height weighted quick unions and weighted quick unions are almost same as they have same upper bounds. For instance, in weighted quick union considering the size, if a tree has a height of 1, it must have at least two nodes in it because to get a height of one, two nodes must be connected. Similarly, for a height of two, it must have at least 4 nodes because to get a height of 2, two trees of height 2 must be connected and so on.

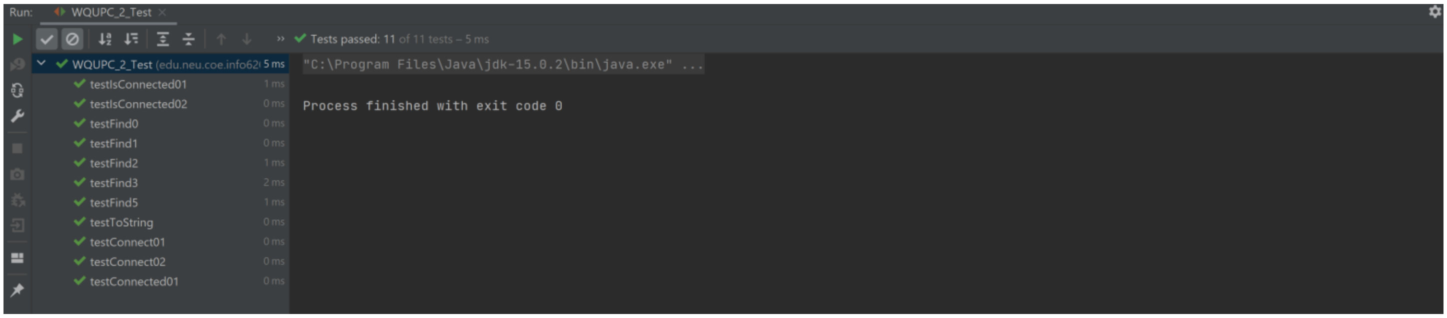
We generalize it by saying a tree of height n must have at least 2n nodes in it. That is a tree with n number of nodes has a height of at most log n. Since the height is log n in both types of weighted unions, the find and union operation have a performance of O(log (n)) because during the find operation, we are traversing the height which is at most log n in both cases. For union of n nodes, the performance is O (n log(n)).

Path Compassion tree is much lower than Tree without Path Compassion. That is because almost all nodes are directly connected to the root, which greatly reduces the height of the tree. On the contrary, the height of trees without PC is relatively high, so with the growth of N, the average height of trees without PC becomes higher and higher.

Without PC, the WQU with “Size” tree is little bit taller than “Depth” tree. This is because, in WQU(Depth), it is always possible to ensure that the lower trees are connected to the higher trees. However, in WQU(Size), a small size tree does not mean that the height of the tree is also low, so it is not guaranteed that each union tree is the lowest situation.

The Depth of WQUPC (two pass) and WQUPC (one pass) is similar. All is approximately around 2.

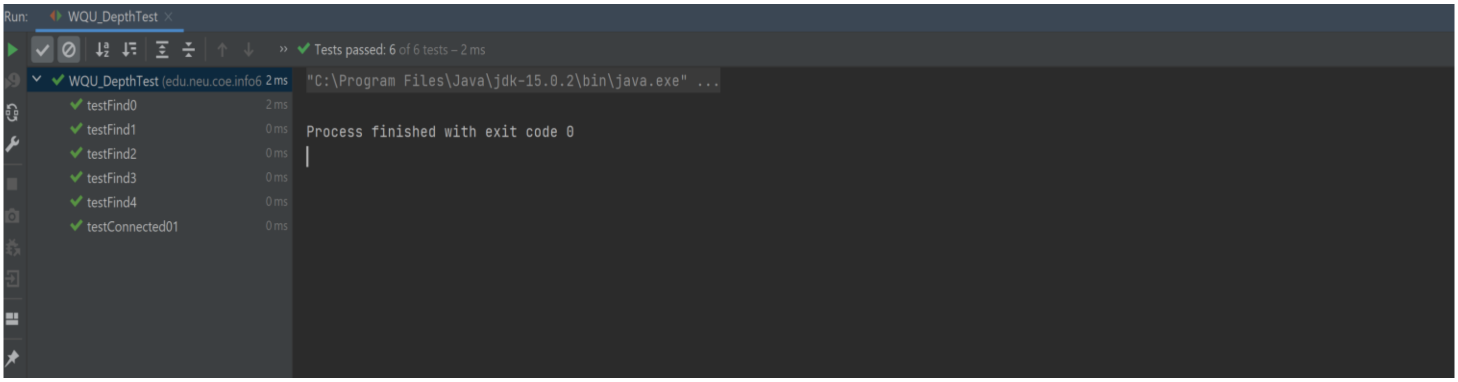
## Screenshot of Tests run:



```
Tests passed: 11 of 11 tests – 5 ms
WQUPC_2_Test (edu.neu.coe.info62( 5 ms
    testIsConnected01   1 ms
    testIsConnected02   0 ms
    testFind0           0 ms
    testFind1           0 ms
    testFind2           1 ms
    testFind3           2 ms
    testFind5           1 ms
    testToString        0 ms
    testConnect01       0 ms
    testConnect02       0 ms
    testConnected01     0 ms
```

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" ...

Process finished with exit code 0
```



```
Tests passed: 6 of 6 tests – 2 ms
WQU_DepthTest (edu.neu.coe.info6 2 ms
    testFind0           2 ms
    testFind1           0 ms
    testFind2           0 ms
    testFind3           0 ms
    testFind4           0 ms
    testConnected01     0 ms
```

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" ...

Process finished with exit code 0
```