Name (NUID)

**Program Structures & Algorithms**

**Spring 2021**

**Assignment No. N**

* **Task**
* **Output**
* **Relationship Conclusion:**
* **Evidence to support the conclusion:**
* **Graphical representation:**
* **Unit tests result:**

**Task:**

Imagine a drunken man who, starting out leaning against a lamp post in the middle of an open space, takes a series of steps of the same length: 1 meter. The direction of these steps is randomly chosen from North, South, East or West. **After n steps, how far (***d***) is the man from the lamp post?** Note that *d* is the Euclidean distance of the man from the lamppost.

It turns out that there is a relationship between *d*and *n* which is typically applicable to many different types of stochastic (randomized) experiments. My task is to implement the code for the experiment and, most importantly, to **deduce the relationship between them**.

**Output:**

After my implementation and experiments of running the test cases I have reported the result values for each of them and tabulated in the excel sheet attached which has helped me deduce the relationship between n and d in this problem statement.

**Relationship Conclusion:**

According to the problem, the drunken man starting out leaning against a lamppost in middle of an open space takes series of steps each of 1 meter, in random directions – north east, west or south, i.e.., moves can be (+-1, 0) or (0, +-1) accordingly.

Let s1 be the first step taken, s2 be the second step taken, s3 be the third step taken and so on .. till sn be the total number of steps n taken by the drunken man.

Then normally we have the total distance he travelled d as,

d=s1+s2+s3…. +sn

But since the man does not walk linear in one direction , we need to include his moves accordingly, that d can either be positive or negative, depending on whether the he ends up to the right or left of 0 in the Cartesian plane (north, south east or west).

N

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The distance traveled after N steps will vary each time, we repeat the experiment, so what we want to know is, if we repeat the experiment many times, how far the man will have traveled *on an average*. Let's call that distance traveled "davg".

davg=avg(s1+s2+s3…+sn)

Here , there is a probability case where the davg can be negative since the man moves randomly and moves can be (+-1,0) or(0,+-1), we have probable scenario as below :

|  |  |  |
| --- | --- | --- |
| s1 | s2 | s1s2 |
| 1 | 1 | 1 |
| 1 | -1 | -1 |
| -1 | 1 | -1 |
| -1 | -1 | 1 |

Since s1s2 is equally likely to be +1 or -1, Avg(s1s2) = 0. The same is true of all of Avg(s1s3), Avg(s1sn), Avg(s2s3), Avg(s2sn) and all of the other terms containing two different steps. Then:

Squaring the values gives us:

Avg(d^2) =Avg (s1^2 + s2^2+ s3^2 + ... + sN^2) + 2 Avg (s1s2 + s1s3 + ... s1sN + s2s3 + ... s2sN + ...)

= (1 + 1 + 1 + ... +1) + 2 (0 + 0 + ... + 0 + 0 + ...) = N

The average of the square of the distance is equal to N. If we take the square root of this equation, we realize that:

sqrt(d^2) =sqrt(N)

Approx., d=sqrt(N)

Since sqrt(d2) is something like the average positive distance davg away from 0 after N steps (technically, it's called the "root-mean-squared" distance), we expect that after N steps, the man will be roughly  be sqrt(N) steps away from where it started.

So, for 100 steps, we expect the man to have moved roughly 9 avg distance from 0 in either direction. Also, we have reported such values as below in 10 experiments for test cases as well that holds true to this relation.

|  |  |
| --- | --- |
| **Steps (N)** | **MeanDistance(d)** |
| 1 | 1 |
| 50 | 4.481589718 |
| 100 | 9.281894317 |
| 150 | 7.279477761 |

**Passed all the testcases – attached files.**

**Graphical representation and Unit tests result are also in the attached file-Output excel file .**