Gradient Boosting

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Today's lecture

- Another way to get non-linear models in a linear form—adaptive basis function models.
- A general algorithm for greedy function approximation—gradient boosting machine.

Motivation

Recap: Adaboost

FINAL CLASSIFIER $G(x) = \mathrm{sign}\left[\sum_{m=1}^{M} lpha_m G_m(x) ight]$ Weighted Sample \cdots $G_M(x)$ Weighted Sample $G_3(x)$ Weighted Sample $G_2(x)$ Training Sample $G_1(x)$

AdaBoost: Algorithm

Given training set $\mathcal{D} = \{(x_1, y_1), \dots, (x_n, y_n)\}.$

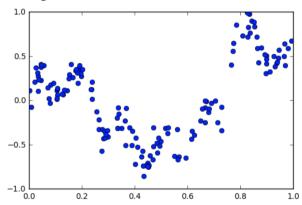
- 1 Initialize observation weights $w_i = 1, i = 1, 2, ..., n$.
- 2 For m = 1 to M:
 - Base learner fits weighted training data and returns $G_m(x)$
 - Compute weighted empirical 0-1 risk:

$$\operatorname{err}_m = \frac{1}{W} \sum_{i=1}^n w_i \mathbb{1}(y_i \neq G_m(x_i))$$
 where $W = \sum_{i=1}^n w_i$.

- Compute classifier weight: $\alpha_m = \ln\left(\frac{1 \text{err}_m}{\text{err}_m}\right)$.
- **9** Update example weight: $w_i \leftarrow w_i \cdot \exp\left[\alpha_m \mathbf{1}(y_i \neq G_m(x_i))\right]$
- **3** Return voted classifier: $G(x) = \text{sign}\left[\sum_{m=1}^{M} \alpha_m G_m(x)\right]$. Why not learn G(x) directly?

Nonlinear Regression

• How do we fit the following data?



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Linear Model with Basis Functions

• Fit a linear combination of transformations of the input:

$$f(x) = \sum_{m=1}^{M} v_m h_m(x),$$

where g_m 's are called **basis functions** (or feature functions in ML):

$$g_1,\ldots,g_M:\mathcal{X}\to\mathsf{R}$$

- Example: polynomial regression where $h_m(x) = x^m$.
- Can we use this model for classification?
- Can fit this using standard methods for linear models (e.g. least squares, lasso, ridge, etc.)
 - Note that h_m 's are fixed and known, i.e. chosen ahead of time.

Adaptive Basis Function Model

- What if we want to learn the basis functions? (hence adaptive)
- Base hypothesis space \mathcal{H} consisting of functions $h: \mathcal{X} \to \mathsf{R}$.
- An adaptive basis function expansion over \mathcal{H} is an ensemble model:

$$f(x) = \sum_{m=1}^{M} v_m h_m(x), \tag{1}$$

where $v_m \in \mathbb{R}$ and $h_m \in \mathcal{H}$.

Combined hypothesis space:

$$\mathfrak{F}_{M} = \left\{ \sum_{m=1}^{M} v_{m} h_{m}(x) \mid v_{m} \in \mathbb{R}, h_{m} \in \mathfrak{H}, m = 1, \dots, M \right\}$$

• What are the learnable?

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Empirical Risk Minimization

• What's our learning objective?

$$\hat{f} = \arg\min_{f \in \mathcal{F}_M} \frac{1}{n} \sum_{i=1}^n \ell(y_i, f(x_i)),$$

for some loss function ℓ .

• Write ERM objective function as

$$J(v_1,...,v_M,h_1,...,h_M) = \frac{1}{n} \sum_{i=1}^n \ell\left(y_i, \sum_{m=1}^M v_m h_m(x)\right).$$

• How to optimize J? i.e. how to learn?

Gradient-Based Methods

• Suppose our base hypothesis space is parameterized by $\Theta = \mathbb{R}^b$:

$$J(v_1,\ldots,v_M,\theta_1,\ldots,\theta_M)=\frac{1}{n}\sum_{i=1}^n\ell\left(y_i,\sum_{m=1}^Mv_mh(x;\theta_m)\right).$$

- Can we optimize it with SGD?
 - Can we differentiate J w.r.t. v_m 's and θ_m 's?
- For some hypothesis spaces and typical loss functions, yes!
 - Neural networks fall into this category! (h_1, \ldots, h_M) are neurons of last hidden layer.)

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What if Gradient Based Methods Don't Apply?

What if base hypothesis space ${\mathcal H}$ consists of decision trees?

- Can we even parameterize trees with $\Theta = R^b$?
- Even if we could, predictions would not change continuously w.r.t. $\theta \in \Theta$, so certainly not differentiable.

What about a greedy algorithm similar to Adaboost?

- Applies to non-parametric or non-differentiable basis functions.
- But is it optimizing our objective using some loss function?

Today we'll discuss gradient boosting.

- Gradient descent in the function space.
- It applies whenever
 - our loss function is [sub]differentiable w.r.t. training predictions $f(x_i)$, and
 - we can do regression with the base hypothesis space \mathcal{H} .

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History

Kearns, Valiant (1989): Can weak learners (e.g., 51% accuracy) be transformed to strong learners (e.g., 99.9% accuracy)?

Schapire (1990) & Freund (1995): Yes, weak learners can be iteratively improved to a strong learner.

Freund, Schapire (1996): And here is a practical algorithm—Adaboost.

Breiman (1996 & 1998): Yes, it works! Boosting is the best off-the-shelf classifier in the world.

(Attempts to explain why Adaboost works and improvements)

Friedman, Hastie, Tibshirani (2000): Actually, boosting fits an additive model.

Friedman (2001): Furthermore, it can be considered as gradient descent in the function space.

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Forward Stagewise Additive Modeling

Forward Stagewise Additive Modeling (FSAM)

Goal fit model $f(x) = \sum_{m=1}^{M} v_m h_m(x)$ given some loss function.

Approach Greedily fit one function at a time without adjusting previous functions, hence "forward stagewise".

• After m-1 stages, we have

$$f_{m-1} = \sum_{i=1}^{m-1} v_i h_i.$$

• In m'th round, we want to find $h_m \in \mathcal{H}$ (i.e. a basis function) and $v_m > 0$ such that

$$f_m = \underbrace{f_{m-1}}_{\text{fixed}} + v_m h_m$$

improves objective function value by as much as possible.

Forward Stagewise Additive Modeling for ERM

Let's plug in our objective function.

- Initialize $f_0(x) = 0$.
- 2 For m=1 to M:
 - Compute:

$$(v_m, h_m) = \underset{v \in \mathbb{R}, h \in \mathcal{H}}{\operatorname{arg\,min}} \frac{1}{n} \sum_{i=1}^n \ell \left(y_i, f_{m-1}(x_i) \underbrace{+vh(x_i)}_{\text{new piece}} \right).$$

- **2** Set $f_m = f_{m-1} + v_m h_m$.
- \odot Return: f_M .

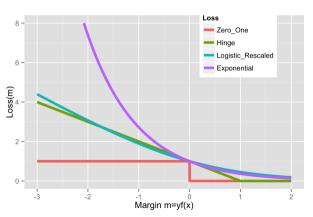
Recap: margin-based classifier

Binary classification

- Outcome space $\mathcal{Y} = \{-1, 1\}$
- Action space A = R (model outoput)
- Score function $f: \mathcal{X} \to \mathcal{A}$.
- Margin for example (x, y) is m = yf(x).
 - $m > 0 \iff$ classification correct
 - Larger m is better.
- Concept check: What are margin-based loss functions we've seen?

Exponential Loss

• Introduce the **exponential loss**: $\ell(y, f(x)) = \exp\left(-\underbrace{yf(x)}_{\text{margin}}\right)$.



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Forward Stagewise Additive Modeling with exponential loss

Recall that we want to do FSAM with exponential loss.

- Initialize $f_0(x) = 0$.
- ② For m = 1 to M:
 - Compute:

$$(v_m, h_m) = \underset{v \in \mathbb{R}, h \in \mathcal{H}}{\arg\min} \frac{1}{n} \sum_{i=1}^n \ell_{\exp} \left(y_i, f_{m-1}(x_i) \underbrace{+vh(x_i)}_{\text{new piece}} \right).$$

- **2** Set $f_m = f_{m-1} + v_m h_m$.
- \odot Return: f_M .

FSAM with Exponential Loss: objective function

- Base hypothesis: $\mathcal{H} = \{h: \mathcal{X} \to \{-1, 1\}\}.$
- Objective function in the *m*'th round:

$$J(v,h) = \sum_{i=1}^{n} \exp\left[-y_i \left(f_{m-1}(x_i) + vh(x_i)\right)\right]$$
 (2)

$$= \sum_{i=1}^{n} w_i^m \exp\left[-y_i v h(x_i)\right] \qquad \qquad w_i^m \stackrel{\text{def}}{=} \exp\left[-y_i f_{m-1}(x_i)\right] \qquad (3)$$

$$= \sum_{i=1}^{n} w_i^m \left[\mathbb{I}(y_i = h(x_i)) e^{-v} + \mathbb{I}(y_i \neq h(x_i)) e^{v} \right] \quad h(x_i) \in \{1, -1\}$$
 (4)

$$= \sum_{i=1}^{n} w_{i}^{m} \left[(e^{v} - e^{-v}) \mathbb{I} (y_{i} \neq h(x_{i})) + e^{-v} \right] \qquad \qquad \mathbb{I} (y_{i} = h(x_{i})) = 1 - \mathbb{I} (y_{i} \neq h(x_{i}))$$

(5)

FSAM with Exponential Loss: basis function

• Objective function in the *m*'th round:

$$J(v,h) = \sum_{i=1}^{n} w_i^m \left[(e^v - e^{-v}) \mathbb{I}(y_i \neq h(x_i)) + e^{-v} \right].$$
 (6)

• If v > 0, then

$$\underset{h \in \mathcal{H}}{\operatorname{arg\,min}} J(v, h) = \underset{h \in \mathcal{H}}{\operatorname{arg\,min}} \sum_{i=1}^{n} w_{i}^{m} \mathbb{I}(y_{i} \neq h(x_{i}))$$
(7)

$$h_m = \arg\min_{h \in \mathcal{H}} \sum_{i=1}^n w_i^m \mathbb{I}(y_i \neq h(x_i))$$
(8)

$$= \arg\min_{h \in \mathcal{H}} \frac{1}{\sum_{i=1}^{n} w_i^m} \sum_{i=1}^{n} w_i^m \mathbb{I}(y_i \neq h(x_i)) \quad \text{multiply by a positive constant}$$

(9)

i.e. h_m is the minimizer of the weighted zero-one loss.

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FSAM with Exponential Loss: classifier weights

• Define the weighted zero-one error:

$$\operatorname{err}_{m} = \frac{\sum_{i=1}^{n} w_{i}^{m} \mathbb{I}(y_{i} \neq h(x_{i}))}{\sum_{i=1}^{n} w_{i}^{m}}.$$
 (10)

• Exercise: show that the optimal v is:

$$v_m = \frac{1}{2} \log \frac{1 - \operatorname{err}_m}{\operatorname{err}_m} \tag{11}$$

- Same as the classifier weights in Adaboost modulo a constant.
- If $err_m < 0.5$ (better than chance), then $v_m > 0$.

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FSAM with Exponential Loss: example weights

• Weights in the next round:

$$w_i^{m+1} \stackrel{\text{def}}{=} \exp\left[-y_i f_m(x_i)\right]$$

$$= w_i^m \exp\left[-y_i v_m h_m(x_i)\right]$$

$$f_m(x_i) = f_{m-1}(x_i) + v_m h_m(x_i)$$
(12)

$$= w_i^m \exp\left[-v_m \mathbb{I}(v_i = h_m(x_i)) + v_m \mathbb{I}(v_i \neq h_m(x_i))\right]$$
(14)

$$= w_i^m \exp[2v_m \mathbb{I}(y_i \neq h_m(x_i))] \exp^{-v_m}$$
 (15)

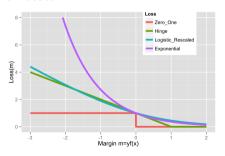
- The constant scaler will cancel out during normalization.
- $2v_m = \alpha_m$ in Adaboost.

Why Exponential Loss

- $\ell_{\exp}(y, f(x)) = \exp(-yf(x))$.
- Exercise: show that the optimal estimate is

$$f^*(x) = \frac{1}{2} \log \frac{p(y=1 \mid x)}{p(y=0 \mid x)}.$$
 (16)

• How is it different from other losses?



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AdaBoost / Exponential Loss: Robustness Issues

- Exponential loss puts a high penalty on misclassified examples.
 - \Longrightarrow not robust to outliers / noise.
- Empirically, AdaBoost has degraded performance in situations with
 - high Bayes error rate (intrinsic randomness in the label)
- Logistic/Log loss performs better in settings with high Bayes error.
- Exponential loss has some computational advantages over log loss though.

We've seen

- Use basis function to obtain nonlinear models: $f(x) = \sum_{i=1}^{M} v_m h_m(x)$ with known h_m 's.
- Adaptive basis function models: $f(x) = \sum_{i=1}^{M} v_m h_m(x)$ with unknown h_m 's.
- Forward stagewise additive modeling: greedily fit h_m 's to minimize the average loss.

But,

- We only know how to do FSAM for certain loss functions.
- Need to derive new algorithms for different loss functions.

Next, how to do FSAM in general.

Gradient Boosting / "Anyboost"

FSAM with squared loss

• Objective function at m'th round:

$$J(v,h) = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \left[f_{m-1}(x_i) \underbrace{+vh(x_i)}_{\text{new piece}} \right] \right)^2$$

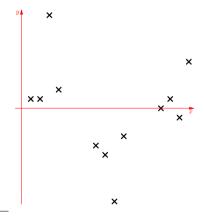
- If \mathcal{H} is closed under rescaling (i.e. if $h \in \mathcal{H}$, then $vh \in \mathcal{H}$ for all $h \in \mathbb{R}$), then don't need v.
- Take v = 1 and minimize

$$J(h) = \frac{1}{n} \sum_{i=1}^{n} \left(\left[\underbrace{y_i - f_{m-1}(x_i)}_{\text{residual}} \right] - h(x_i) \right)^2$$

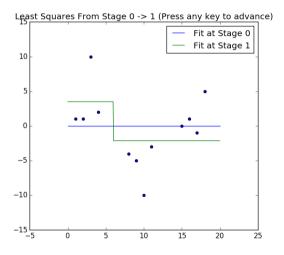
- This is just fitting the residuals with least-squares regression!
- Example base hypothesis space: regression stumps.

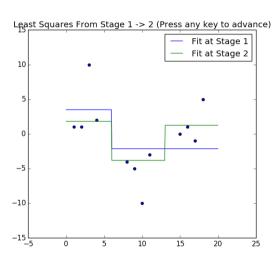
L^2 Boosting with Decision Stumps: Demo

- Consider FSAM with L^2 loss (i.e. L^2 Boosting)
- For base hypothesis space of regression stumps

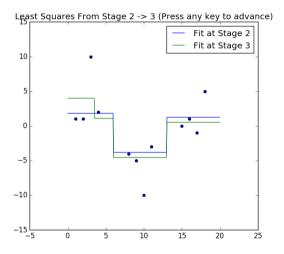


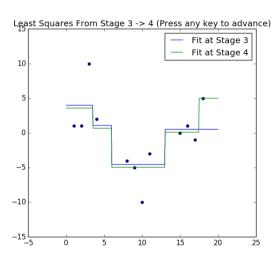
L^2 Boosting with Decision Stumps: Results



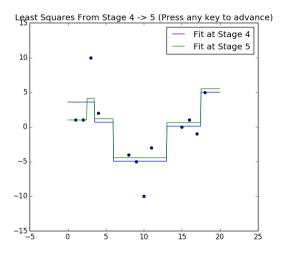


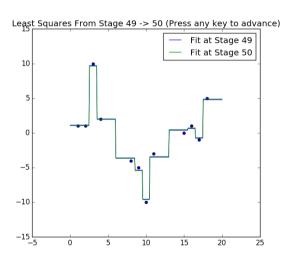
L^2 Boosting with Decision Stumps: Results





L^2 Boosting with Decision Stumps: Results





Interpret the residual

- Objective: $J(f) = \frac{1}{n} \sum_{i=1}^{n} (y_i f(x_i))^2$.
- What is the residual at $x = x_i$?

$$\frac{\partial}{\partial f(x_i)}J(f) = -2(y_i - f(x_i)) \tag{17}$$

- Gradient w.r.t. f: how should the output of f change to minimize the squared loss.
- Residual is the negative gradient (modulo some constant).
- At each boosting round, we learn a function $h \in \mathcal{H}$ to fit the residual.

$$f \leftarrow f + vh$$
 FSAM / boosting (18)

$$f \leftarrow f - \alpha \nabla_f J(f)$$
 gradient descent (19)

- h approximates the gradient (step direction).
- v is the step size.

"Functional" Gradient Descent

We want to minimize

$$J(f) = \sum_{i=1}^{n} \ell(y_i, f(x_i)).$$

- In some sense, we want to take the gradient w.r.t. f.
- J(f) only depends on f at the n training points.
- Define "parameters"

$$f = (f(x_1), \ldots, f(x_n))^T$$

and write the objective function as

$$J(\mathsf{f}) = \sum_{i=1}^{n} \ell(y_{i}, \mathsf{f}_{i}).$$

Functional Gradient Descent: Unconstrained Step Direction

Consider gradient descent on

$$J(\mathsf{f}) = \sum_{i=1}^{n} \ell(y_i, \mathsf{f}_i).$$

• The negative gradient step direction at f is

$$-g = -\nabla_{\mathbf{f}} J(\mathbf{f})$$

=
$$-(\partial_{\mathbf{f}_{1}} \ell(y_{1}, \mathbf{f}_{1}), \dots, \partial_{\mathbf{f}_{n}} \ell(y_{n}, \mathbf{f}_{n}))$$

which we can easily calculate.

- $-g \in \mathbb{R}^n$ is the direction we want to change each of our n predictions on training data.
- With gradient descent, our final predictor will be an additive model: $f_0 + \sum_{m=1}^{M} v_t(-g_t)$.

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Functional Gradient Descent: Projection Step

Unconstrained step direction is

$$-\mathsf{g} = -\nabla_{\mathbf{f}} J(\mathsf{f}) = -\left(\partial_{\mathsf{f}_1} \ell\left(y_1, \mathsf{f}_1\right), \ldots, \partial_{\mathsf{f}_n} \ell\left(y_n, \mathsf{f}_n\right)\right).$$

- Also called the "pseudo-residuals". (For squared loss, they're exactly the residuals.)
- Problem: only know how to update at n points. How do we take a gradient step in \mathcal{H} ?
- Solution: approximate by the closest base hypothesis $h \in \mathcal{H}$ (in the ℓ^2 sense):

$$\min_{h \in \mathcal{H}} \sum_{i=1}^{n} \left(-g_i - h(x_i) \right)^2.$$
 least square regression (20)

• Take the $h \in \mathcal{H}$ that best approximates -g as our step direction.

Explain by figure

Recap

Objective function:

$$J(f) = \sum_{i=1}^{n} \ell(y_i, f(x_i)).$$
 (21)

• Unconstrained gradient $g \in \mathbb{R}^n$ w.r.t. $\mathbf{f} = (f(x_1), \dots, f(x_n))^T$:

$$g = \nabla_{\mathbf{f}} J(f) = (\partial_{f_1} \ell(y_1, f_1), \dots, \partial_{f_n} \ell(y_n, f_n)). \tag{22}$$

• Projected negative gradient $h \in \mathcal{H}$:

$$h = \arg\min_{h \in \mathcal{H}} \sum_{i=1}^{n} (-g_i - h(x_i))^2.$$
 (23)

Gradient descent:

$$f \leftarrow f + \mathbf{v}h \tag{24}$$

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Functional Gradient Descent: hyperparameters

• Choose a step size by line search.

$$v_m = \underset{v>0}{\arg\min} \sum_{i=1}^n \ell\{y_i, f_{m-1}(x_i) + vh_m(x_i)\}.$$

• Regularization through **shrinkage**:

$$f_m \leftarrow f_{m-1} + \lambda v_m h_m \quad \text{where } \lambda \in [0, 1].$$
 (25)

- Typically choose $\lambda = 1$.
- Choose *M*, i.e. when to stop.
 - Tune on validation set.

The Gradient Boosting Machine Ingredients (Recap)

- Take any loss function [sub]differentiable w.r.t. the prediction $f(x_i)$
- Choose a base hypothesis space for regression.
- Choose number of steps (or a stopping criterion).
- Choose step size methodology.
- Then you're good to go!

BinomialBoost: Gradient Boosting with Logistic Loss

• Recall the logistic loss for classification, with $\mathcal{Y} = \{-1, 1\}$:

$$\ell(y, f(x)) = \log\left(1 + e^{-yf(x)}\right)$$

• Pseudoresidual for i'th example is negative derivative of loss w.r.t. prediction:

$$r_i = -\frac{\partial}{\partial f(x_i)} \ell(y_i, f(x_i)) \tag{26}$$

$$= -\frac{\partial}{\partial f(x_i)} \left[\log \left(1 + e^{-y_i f(x_i)} \right) \right] \tag{27}$$

$$=\frac{y_i e^{-y_i f(x_i)}}{1 + e^{-y_i f(x_i)}} \tag{28}$$

$$=\frac{y_i}{1+e^{y_i f(x_i)}}\tag{29}$$

BinomialBoost: Gradient Boosting with Logistic Loss

• Pseudoresidual for *i*th example:

$$r_i = -\frac{\partial}{\partial f(x_i)} \left[\log \left(1 + e^{-y_i f(x_i)} \right) \right] = \frac{y_i}{1 + e^{y_i f(x_i)}}$$

• So if $f_{m-1}(x)$ is prediction after m-1 rounds, step direction for m'th round is

$$h_m = \underset{h \in \mathcal{H}}{\operatorname{arg\,min}} \sum_{i=1}^n \left[\left(\frac{y_i}{1 + e^{y_i f_{m-1}(x_i)}} \right) - h(x_i) \right]^2.$$

• And $f_m(x) = f_{m-1}(x) + vh_m(x)$.

Gradient Tree Boosting

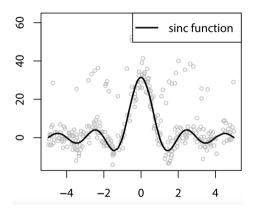
One common form of gradient boosting machine takes

$$\mathcal{H} = \{\text{regression trees of size } S\},$$

where S is the number of terminal nodes.

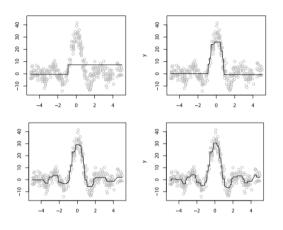
- S = 2 gives decision stumps
- HTF recommends $4 \leqslant S \leqslant 8$ (but more recent results use much larger trees)
- Software packages:
 - Gradient tree boosting is implemented by the gbm package for R
 - \bullet as ${\tt GradientBoostingClassifier}$ and ${\tt GradientBoostingRegressor}$ in ${\tt sklearn}$
 - xgboost and lightGBM are state of the art for speed and performance

Sinc Function: Our Dataset



From Natekin and Knoll's "Gradient boosting machines, a tutorial"

Minimizing Square Loss with Ensemble of Decision Stumps

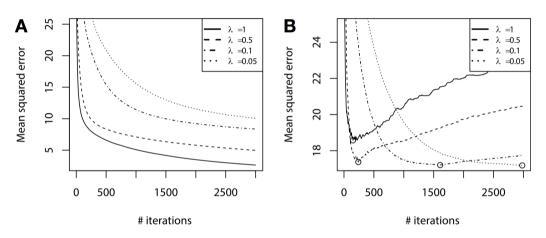


Decision stumps with 1,10,50, and 100 steps, step size $\lambda=1.$

Figure 3 from Natekin and Knoll's "Gradient boosting machines, a tutorial"

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Step Size as Regularization



Performance vs rounds of boosting and step size. (Left is training set, right is validation set)

Figure 5 from Natekin and Knoll's "Gradient boosting machines, a tutorial"

Rule of Thumb

- The smaller the step size, the more steps you'll need.
- But never seems to make results worse, and often better.
- So set your step size as small as you have patience for.

Gradient Boosting in Practice

Prevent overfitting

- Boosting is resistant to overfitting. Some explanations:
 - Implicit feature selection: greedily selects the best feature (weak learner)
 - As training goes on, impact of change is localized.
- But it can of course overfit. Common regularization methods:
 - Shrinkage (small learning rate)
 - Stochastic gradient boosting (row subsampling)
 - Feature subsampling (column subsampling)

Stochastic Gradient Boosting

- For each stage,
 - choose random subset of data for computing projected gradient step.
- Why do this?
 - Introduce randomization thus may help overfitting.
 - Faster; often better than gradient descent given the same computation resource.
- We can view this is a minibatch method.
 - Estimate the "true" step direction using a subset of data.

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Column / Feature Subsampling

- Similar to random forest, randomly choose a subset of features for each round.
- XGBoost paper says: "According to user feedback, using column sub-sampling prevents overfitting even more so than the traditional row sub-sampling."
- Speeds up computation.

Newton Step Direction

• For GBM, we find the closest $h \in \mathcal{F}$ to the negative gradient

$$-\mathbf{g} = -\nabla_{\mathbf{f}} J(\mathbf{f}).$$

- This is a "first order" method.
- Newton's method is a "second order method":
 - Find 2nd order (quadratic) approximation to J at f.
 - Requires computing gradient and Hessian of J.
 - Newton step direction points towards minimizer of the quadratic.
 - Minimizer of quadratic is easy to find in closed form
- Boosting methods with projected Newton step direction:
 - LogitBoost (logistic loss function)
 - XGBoost (any loss uses regression trees for base classifier)

Generically, second order Taylor expansion of J at f in direction r

$$J(f+r) = J(f) + \left[\nabla_f J(f)\right]^T r + \frac{1}{2} r^T \left[\nabla_f^2 J(f)\right] r$$

• For $J(f) = \sum_{i=1}^{n} \ell(y_i, f_i)$,

$$J(f+r) = \sum_{i=1}^{n} \left[\ell(y_i, f_i) + g_i r_i + \frac{1}{2} h_i r_i^2 \right],$$

where $g_i = \partial_{f_i} \ell(y_i, f_i)$ and $h_i = \partial_{f_i}^2 \ell(y_i, f_i)$.

- Can find r that minimizes J(f+r) in closed form.
- ullet Can take step direction to be "projection" of r into base hypothesis space ${\mathcal H}.$

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Summary

- Motivating idea of boosting: combine weak learners to produce a strong learner.
- The statistical view: boosting is fitting an additive model (greedily).
- The numerical optimization view: boosting makes local improvement iteratively—gradient descent in the function space.
- Gradient boosting is a generic framework
 - Any differentiable loss function
 - Classification, regression, ranking, multiclass etc.
 - Scalable, e.g., XGBoost