## Multiclass Classification and Structured Prediction

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# Logistics

#### Section

- Review of lecture
  - Averaged perceptron

### Tutorial

- Multiclass concept check
- Review subgradient
- Review bias and variance of estimators

Overview

### Motivation

- So far, most algorithms we've learned are designed for binary classification.
- Many real-world problems have more than two classes.
- What are some potential issues when we have a large number of classes?

## Today's lecture

- Recap: how to reduce multiclass classification to binary classification?
- How do we generalize binary classification algorithm to the multiclass setting?
- Example of very large output space: structured prediction.

Reduction to Binary Classification

# One-vs-All / One-vs-Rest

### Setting

- Input space:  $\mathfrak{X}$
- Output space:  $\mathcal{Y} = \{1, \dots, k\}$

### **Training**

- Train k binary classifiers, one for each class:  $h_1, \ldots, h_k : \mathcal{X} \to \mathbf{R}$ .
- Classifier  $h_i$  distinguishes class i (+1) from the rest (-1).

### Prediction

Majority vote:

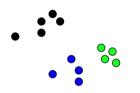
$$h(x) = \arg\max_{i \in \{1, \dots, k\}} h_i(x)$$

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Ties can be broken arbitrarily.

# OvA: 3-class example

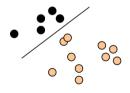
#### Consider a dataset with three classes:



**Assumption**: each class is linearly separable from the rest.

Ideal case: only target class has positive score.

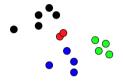
Train OvA classifiers:





# OvA: 4-class non-separable example

#### Consider a dataset with four classes:



Cannot separate red points from the rest. Which classes might have low accuracy?

### Train OvA classifiers:



# All vs All / One vs One / All pairs

### Setting

- Input space:  $\mathfrak X$ 
  - Output space:  $\mathcal{Y} = \{1, \dots, k\}$

## Training

- Train  $\binom{k}{2}$  binary classifiers, one for each pair:  $h_{ij}: \mathcal{X} \to \mathbf{R}$  for  $i \in [1, k]$  and  $j \in [i+1, k]$ .
- Classifier  $h_{ij}$  distinguishes class i (+1) from class j (-1).

### Prediction

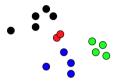
• Majority vote (each class gets k-1 votes)

$$h(x) = \underset{i \in \{1, \dots, k\}}{\operatorname{arg\,max}} \sum_{j \neq i} \underbrace{h_{ij}(x) \mathbb{I}\{i < j\}}_{\operatorname{class } i \text{ is } s + 1} - \underbrace{h_{ji}(x) \mathbb{I}\{j < i\}}_{\operatorname{class } i \text{ is } s - 1}$$

- Tournament
- Ties can be broken arbitrarily.

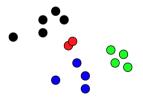
## AvA: four-class example

Consider a dataset with four classes:



**Assumption**: each pair of classes are linearly separable. More expressive than OvA.

What's the decision region for the red class?



		OvA	AvA
computation	train test	$O(kB_{train}(n)) \ O(kB_{test})$	$O(k^2 B_{train}(n/k)) \ O(k^2 B_{test})$
challenges	train		small training set
	test	calibration / scale tie breaking	

Lack theoretical justification but simple to implement and works well in practice (when # classes is small).

Question: When would you prefer AvA / OvA?

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### Code word for labels

Using the reduction approach, can you train fewer than k binary classifiers?

Key idea: Encode labels as binary codes and predict the code bits directly.

OvA encoding:

class	$h_1$	h <sub>2</sub>	h <sub>3</sub>	h <sub>4</sub>
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

OvA uses k bits to encode each label, what's the minimal number of bits you can use?

# Error correcting output codes (ECOC)

### Example: 8 classes, 6-bit code

0	0	1	0	0
0	_		•	U
-	0	0	0	0
1	1	0	1	0
1	0	0	0	0
1	0	0	1	0
0	1	1	0	1
0	1	0	0	0
1	0	1	0	0
֡	1 1 0 0	1 0 1 0 0 1 0 1	1 0 0 1 0 0 0 1 1 0 1 0	1     0     0     0       1     0     0     1       0     1     1     0       0     1     0     0

Training Binary classifier  $h_i$ :

• +1: classes whose *i*-th bit is 1

• -1: classes whose *i*-th bit is 0

Prediction Closest label in terms of Hamming distance.

$h_1$	$h_2$	h <sub>3</sub>	h <sub>4</sub>	$h_5$	h <sub>6</sub>
0	1	1	0	1	1

Code design Want good binary classifiers.

# Error correcting output codes: summary

- Computationally more efficient than OvA (a special case of ECOC). Better for large k.
- Why not use the minimal number of bits  $(\log_2 k)$ ?
  - If the minimum Hamming distance between any pair of code word is d, then it can correct  $\left\lfloor \frac{d-1}{2} \right\rfloor$  errors.
  - In plain words, if rows are far from each other, ECOC is robust to errors.
- Trade-off between code distance and binary classification performance.
- Nice theoretical results [Allwein et al., 2000] (also incoporates AvA).

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### Reduction-based approaches:

- Reducing multiclass classification to binary classification: OvA, AvA, ECOC.
- Key is to design "natural" binary classification problems without large computation cost.

### But,

- Unclear how to generalize to extremely large # of classes.
- ImageNet: >20k labels; Wikipedia: >1M categories.

Next, generalize previous algorithms to multiclass settings.

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Linear Multiclass Predictors

- Base Hypothesis Space:  $\mathcal{H} = \{h : \mathcal{X} \to R\}$  (score functions).
- Multiclass Hypothesis Space (for k classes):

$$\mathcal{F} = \left\{ x \mapsto rg \max_{i} h_{i}(x) \mid h_{1}, \dots, h_{k} \in \mathcal{H} \right\}$$

- $h_i(x)$  scores how likely x is to be from class i.
- OvA objective:  $h_i(x) > 0$  for x with label i and  $h_i(x) < 0$  for x with all other labels.
- At test time, for (x, i) we only need

$$h_i(x) > h_j(x) \qquad \forall j \neq i.$$
 (1)

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# Multiclass perceptron

- Base linear predictors:  $h_i(x) = w_i^T x \ (w \in \mathbb{R}^d)$ .
- Multiclass perceptron:

```
Given a multiclass dataset \mathcal{D} = \{(x, y)\};
Initialize w \leftarrow 0:
for iter = 1, 2, \dots, T do
    for (x, y) \in \mathcal{D} do
         \hat{y} = \operatorname{arg\,max}_{v' \in \mathcal{Y}} w_{v'}^T x;
        if \hat{v} \neq v then // We've made a mistake
              w_v \leftarrow w_v + x; // Move the target-class scorer towards x
             w_{\hat{v}} \leftarrow w_{\hat{v}} - x; // Move the wrong-class scorer away from x
         end
    end
end
```

• (Geometric interpretation)

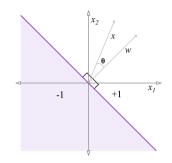
# Side note: Linear Binary Classifier Review

- Input Space:  $\mathfrak{X} = \mathbf{R}^d$
- Output Space:  $\mathcal{Y} = \{-1, 1\}$
- Linear classifier score function:

$$f(x) = \langle w, x \rangle = w^T x$$

- Final classification prediction: sign(f(x))
- Geometrically, when are sign(f(x)) = +1 and sign(f(x)) = -1?

# Side note: Linear Binary Classifier Review



Suppose ||w|| > 0 and ||x|| > 0:

$$f(x) = \langle w, x \rangle = ||w|| ||x|| \cos \theta$$

$$f(x) > 0 \iff \cos \theta > 0 \iff \theta \in (-90^{\circ}, 90^{\circ})$$

$$f(x) < 0 \iff \cos \theta < 0 \iff \theta \notin [-90^{\circ}, 90^{\circ}]$$

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# Rewrite the scoring function

- Remember that we want to scale to very large # of classes and reuse algorithms and analysis for binary classification
  - $\implies$  a single weight vector is desired
- How to rewrite the equation such that we have one w instead of k?

$$w_i^T x = w^T \psi(x, i) \tag{2}$$

$$h_i(x) = h(x, i) \tag{3}$$

- Encode labels in the feature space.
- Score for each label  $\rightarrow$  score for the "compatibility" of a label and an input.

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## The Multivector Construction

### How to construct the feature map $\psi$ ?

• What if we stack  $w_i$ 's together (e.g.,  $x \in \mathbb{R}^2$ ,  $y = \{1, 2, 3\}$ )

$$w = \left(\underbrace{-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}}_{w_1}, \underbrace{0, 1}_{w_2}, \underbrace{\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}}_{w_3}\right)$$

• And then do the following:  $\Psi: \mathbf{R}^2 \times \{1,2,3\} \to \mathbf{R}^6$  defined by

$$\Psi(x,1) := (x_1, x_2, 0, 0, 0, 0)$$

$$\Psi(x,2) := (0,0,x_1,x_2,0,0)$$

$$\Psi(x,3) := (0,0,0,0,x_1,x_2)$$

• Then  $\langle w, \Psi(x,y) \rangle = \langle w_v, x \rangle$ , which is what we want.

Multiclass perceptron using the multivector construction.

```
Given a multiclass dataset \mathcal{D} = \{(x, y)\};
Initialize w \leftarrow 0:
for iter = 1, 2, \dots, T do
      for (x, y) \in \mathcal{D} do
           \hat{y} = \operatorname{arg\,max}_{v' \in \mathcal{Y}} w^T \psi(x, y'); // Equivalent to \operatorname{arg\,max}_{v' \in \mathcal{Y}} w_{v'}^T x
           if \hat{v} \neq v then // We've made a mistake
            w \leftarrow w + \psi(x,y); // Move the scorer towards \psi(x,y)
w \leftarrow w - \psi(x,\hat{y}); // Move the scorer away from \psi(x,\hat{y})
            end
      end
end
```

Exercise: What is the base binary classification problem in multiclass perceptron?

Geometric interpretation

## Feature templates

Toy multiclass example: Part-of-speech classification

- $\mathfrak{X} = \{\text{All possible words}\}\$
- $y = \{NOUN, VERB, ADJECTIVE, ...\}$ .
- Features of  $x \in \mathcal{X}$ : [The word itself], ENDS\_IN\_ly, ENDS\_IN\_ness, ...

How to construct the feature vector?

- Multivector construction:  $w \in \mathbb{R}^{d \times k}$ —doesn't scale.
- Feature templates: directly design features for each class.

$$\Psi(x,y) = (\psi_1(x,y), \psi_2(x,y), \psi_3(x,y), \dots, \psi_d(x,y))$$
(4)

• Size can be bounded by d.

## Feature templates

### Sample training data:

The boy grabbed the apple and ran away quickly .

### Feature templates:

$$\begin{array}{lll} \psi_1(x,y) &=& 1(x=\operatorname{apple}\,\operatorname{AND}\,y=\operatorname{NOUN})\\ \psi_2(x,y) &=& 1(x=\operatorname{run}\,\operatorname{AND}\,y=\operatorname{NOUN})\\ \psi_3(x,y) &=& 1(x=\operatorname{run}\,\operatorname{AND}\,y=\operatorname{VERB})\\ \psi_4(x,y) &=& 1(x\,\operatorname{ENDS\_IN\_ly}\,\operatorname{AND}\,y=\operatorname{ADVERB})\\ &\dots \end{array}$$

- E.g.,  $\Psi(x = \text{run}, y = \text{NOUN}) = (0, 1, 0, 0, ...)$
- After training, what's  $w_1$ ,  $w_2$ ,  $w_3$ ,  $w_4$ ?
- No need to include feature templates unseen in training data.

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## Feature templates: implementation

- Flexible, e.g., neighboring words, suffix/prefix.
- "Read off" features from the training data.
- Often sparse—efficient in practice, e.g., NLP problems.
- Can use a hash function: template  $\rightarrow \{1, 2, ..., d\}$ .

### Review

### Ingredients in multiclass classification:

- Scoring functions for each class (similar to ranking).
- Represent labels in the input space  $\implies$  single weight vector.

#### We've seen

- How to generalize the perceptron algorithm to multiclass setting.
- Very simple idea. Was popular in NLP for structured prediction (e.g., tagging, parsing).

### Next,

- How to generalize SVM to the multiclass setting.
- Concept check: Why might one prefer SVM / perceptron?

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# Margin for Multiclass

Binary • Margin for  $(x^{(n)}, y^{(n)})$ :

$$y^{(n)}w^Tx^{(n)} \tag{5}$$

• Want margin to be large and positive ( $w^T x^{(n)}$  has same sign as  $y^{(n)}$ )

Multiclass

• Class-specific margin for  $(x^{(n)}, v^{(n)})$ :

$$h(x^{(n)}, y^{(n)}) - h(x^{(n)}, y).$$
 (6)

- Difference between scores of the correct class and each other class
- Want margin to be large and positive for all  $y \neq y^{(n)}$ .

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# Multiclass SVM: separable case

### Binary

$$\min_{w} \quad \frac{1}{2} \|w\|^2 \tag{7}$$

s.t. 
$$\underbrace{y^{(n)}w^Tx^{(n)}}_{\text{margin}} \geqslant 1 \quad \forall (x^{(n)}, y^{(n)}) \in \mathcal{D}$$
 (8)

Multiclass As in the binary case, take 1 as our target margin.

$$m_{n,y}(w) \stackrel{\text{def}}{=} \underbrace{\left\langle w, \Psi(x^{(n)}, y^{(n)}) \right\rangle}_{\text{score of correct class}} - \underbrace{\left\langle w, \Psi(x^{(n)}, y) \right\rangle}_{\text{score of other class}}$$
(9)

$$\min_{w} \quad \frac{1}{2} \|w\|^2 \tag{10}$$

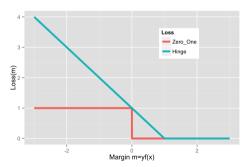
s.t. 
$$m_{n,y}(w) \ge 1 \quad \forall (x^{(n)}, y^{(n)}) \in \mathcal{D}, y \ne y^{(n)}$$
 (11)

Exercise: write the objective for the non-separable case

# Recap: hingle loss for binary classification

• Hinge loss: a convex upperbound on the 0-1 loss

$$\ell_{\mathsf{hinge}}(y, \hat{y}) = \mathsf{max}(0, 1 - yh(x)) \tag{12}$$



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## Generalized hinge loss

• What's the zero-one loss for multiclass classification?

$$\Delta(y, y') = \mathbb{I}\left\{y = y'\right\} \tag{13}$$

- In general, can also have different cost for each class.
- Upper bound on  $\Delta(y, y')$ .

$$\hat{y} \stackrel{\text{def}}{=} \underset{y' \in \mathcal{Y}}{\operatorname{arg\,max}} \langle w, \Psi(x, y') \rangle \tag{14}$$

$$\Longrightarrow \langle w, \Psi(x, y) \rangle \leqslant \langle w, \Psi(x, \hat{y}) \rangle \tag{15}$$

$$\Longrightarrow \Delta(y,\hat{y}) \leqslant \Delta(y,\hat{y}) + \langle w, (\Psi(x,\hat{y}) - \Psi(x,y)) \rangle \qquad \text{ When are they equal?} \tag{16}$$

Generalized hinge loss:

$$\ell_{\mathsf{hinge}}(y, \hat{y}) \stackrel{\mathsf{def}}{=} \max_{y' \in \mathcal{Y}} \left( \Delta(y, y') + \left\langle w, \left( \Psi(x, y') - \Psi(x, y) \right) \right\rangle \right) \tag{17}$$

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# Multiclass SVM with Hinge Loss

• Recall the hinge loss formulation for binary SVM (without the bias term):

$$\min_{w \in \mathbf{R}^d} \frac{1}{2} ||w||^2 + C \sum_{n=1}^N \max \left( 0, 1 - \underbrace{y^{(n)} w^T x^{(n)}}_{\text{margin}} \right).$$

• The multiclass objective:

$$\min_{w \in \mathbf{R}^d} \frac{1}{2} ||w||^2 + C \sum_{n=1}^N \max_{y' \in \mathcal{Y}} \left( \Delta(y, y') - \underbrace{\left\langle w, \left( \Psi(x, y) - \Psi(x, y') \right) \right\rangle}_{\text{margin}} \right)$$

- $\Delta(y, y')$  as target margin for each class.
- If margin  $m_{n,y'}(w)$  meets or exceeds its target  $\Delta(y^{(n)},y')$   $\forall y \in \mathcal{Y}$ , then no loss on example n.

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# Recap: What Have We Got?

- Problem: Multiclass classification  $\mathcal{Y} = \{1, ..., k\}$
- Solution 1: One-vs-All
  - Train k models:  $h_1(x), \ldots, h_k(x) : \mathfrak{X} \to \mathbb{R}$ .
  - Predict with  $\arg \max_{y \in \mathcal{Y}} h_y(x)$ .
  - Gave simple example where this fails for linear classifiers
- Solution 2: Multiclass loss
  - Train one model:  $h(x,y): \mathfrak{X} \times \mathcal{Y} \to \mathbf{R}$ .
  - Prediction involves solving  $\arg\max_{y \in \mathcal{Y}} h(x, y)$ .

### Does it work better in practice?

- Paper by Rifkin & Klautau: "In Defense of One-Vs-All Classification" (2004)
  - Extensive experiments, carefully done
    - albeit on relatively small UCI datasets
  - Suggests one-vs-all works just as well in practice
    - (or at least, the advantages claimed by earlier papers for multiclass methods were not compelling)
- Compared
  - many multiclass frameworks (including the one we discuss)
  - one-vs-all for SVMs with RBF kernel
  - one-vs-all for square loss with RBF kernel (for classification!)
- All performed roughly the same

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# Why Are We Bothering with Multiclass?

- The framework we have developed for multiclass
  - compatibility features / scoring functions
  - multiclass margin
  - target margin / multiclass loss
- Generalizes to situations where k is very large and one-vs-all is intractable.
- Key idea is that we can generalize across outputs y by using features of y.

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Introduction to Structured Prediction

# Example: Part-of-speech (POS) Tagging

• Given a sentence, give a part of speech tag for each word:

X	[START]	He	eats	apples
	<i>x</i> <sub>0</sub>	<i>×</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	<i>X</i> 3
У	[START]	Pronoun	Verb	Noun
	<i>y</i> <sub>0</sub>	$y_1$	<i>y</i> <sub>2</sub>	<i>у</i> з

- $\mathcal{V} = \{\text{all English words}\} \cup \{[\text{START}], "."\}$
- $\mathfrak{X} = \mathcal{V}^n$ , n = 1, 2, 3, ... [Word sequences of any length]
- $\mathcal{P} = \{START, Pronoun, Verb, Noun, Adjective\}$
- $y = \mathcal{P}^n$ , n = 1, 2, 3, ...[Part of speech sequence of any length]

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# Multiclass Hypothesis Space

- Discrete output space: y(x)
  - Very large but has structure, e.g., linear chain (sequence labeling), tree (parsing)
  - Size depends on input x
- Base Hypothesis Space:  $\mathcal{H} = \{h : \mathcal{X} \times \mathcal{Y} \to \mathbf{R}\}$ 
  - h(x,y) gives compatibility score between input x and output y
- Multiclass hypothesis space

$$\mathcal{F} = \left\{ x \mapsto \operatorname*{arg\,max}_{y \in \mathcal{Y}} h(x, y) \mid h \in \mathcal{H} \right\}$$

- Final prediction function is an  $f \in \mathcal{F}$ .
- For each  $f \in \mathcal{F}$  there is an underlying compatibility score function  $h \in \mathcal{H}$ .

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### Structured Prediction

Part-of-speech tagging

Multiclass hypothesis space:

$$h(x,y) = w^{T} \Psi(x,y) \tag{18}$$

$$\mathcal{F} = \left\{ x \mapsto \operatorname*{arg\,max}_{y \in \mathcal{Y}} h(x, y) \mid h \in \mathcal{H} \right\}$$
 (19)

- A special case of multiclass classification
- How to design the feature map  $\Psi$ ? What are the considerations?

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# Unary features

- A unary feature only depends on
  - the label at a single position,  $y_i$ , and x
- Example:

$$\begin{array}{lcl} \varphi_1(x,y_i) &=& 1(x_i=\mathsf{runs})1(y_i=\mathsf{Verb}) \\ \varphi_2(x,y_i) &=& 1(x_i=\mathsf{runs})1(y_i=\mathsf{Noun}) \\ \varphi_3(x,y_i) &=& 1(x_{i-1}=\mathsf{He})1(x_i=\mathsf{runs})1(y_i=\mathsf{Verb}) \end{array}$$

- A markov feature only depends on
  - two adjacent labels,  $y_{i-1}$  and  $y_i$ , and x
- Example:

$$\theta_1(x, y_{i-1}, y_i) = 1(y_{i-1} = \text{Pronoun})1(y_i = \text{Verb})$$
  
 $\theta_2(x, y_{i-1}, y_i) = 1(y_{i-1} = \text{Pronoun})1(y_i = \text{Noun})$ 

- Reminiscent of Markov models in the output space
- Possible to have higher-order features

### Local Feature Vector and Compatibility Score

• At each position *i* in sequence, define the **local feature vector** (unary and markov):

$$\Psi_{i}(x, y_{i-1}, y_{i}) = (\phi_{1}(x, y_{i}), \phi_{2}(x, y_{i}), \dots, \\
\theta_{1}(x, y_{i-1}, y_{i}), \theta_{2}(x, y_{i-1}, y_{i}), \dots)$$

- And local compatibility score at position  $i: \langle w, \Psi_i(x, y_{i-1}, y_i) \rangle$ .
- The compatibility score for (x, y) is the sum of local compatibility scores:

$$\sum_{i} \langle w, \Psi_{i}(x, y_{i-1}, y_{i}) \rangle = \left\langle w, \sum_{i} \Psi_{i}(x, y_{i-1}, y_{i}) \right\rangle = \left\langle w, \Psi(x, y) \right\rangle, \tag{20}$$

where we define the sequence feature vector by

$$\Psi(x,y) = \sum_{i} \Psi_i(x,y_{i-1},y_i).$$
 decomposable

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```
Given a dataset \mathcal{D} = \{(x, v)\}:
Initialize w \leftarrow 0:
for iter = 1, 2, \dots, T do
      for (x, y) \in \mathcal{D} do
            \hat{y} = \operatorname{arg\,max}_{\mathbf{v}' \in \mathbf{V}(\mathbf{x})} w^T \psi(\mathbf{x}, \mathbf{y}');
            if \hat{v} \neq v then // We've made a mistake
             w \leftarrow w + \Psi(x,y); // Move the scorer towards \psi(x,y)
w \leftarrow w - \Psi(x,\hat{y}); // Move the scorer away from \psi(x,\hat{y})
            end
      end
end
```

Identical to the multiclass perceptron algorithm except the arg max is now over the structured output space y(x).

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### Structured hinge loss

Recall the generalized hinge loss

$$\ell_{\mathsf{hinge}}(y, \hat{y}) \stackrel{\mathsf{def}}{=} \max_{y' \in \mathcal{Y}(x)} \left( \Delta(y, y') + \left\langle w, \left( \Psi(x, y') - \Psi(x, y) \right) \right\rangle \right) \tag{21}$$

- What is  $\Delta(y, y')$  for two sequences?
- Hamming loss is common:

$$\Delta(y, y') = \frac{1}{L} \sum_{i=1}^{L} 1(y_i \neq y_i')$$

where L is the sequence length.

• Can generalize to the cost-sensitive version using  $\delta(y_i, y_i')$ 

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#### Structured SVM

#### Exercise:

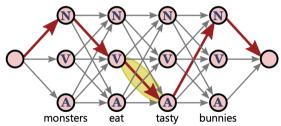
- Write down the objective of structured SVM using the structured hinge loss.
- Stochastic sub-gradient descent for structured SVM (similar to HW3 P3)
- Compare with the structured perceptron algorithm

### The argmax problem for sequences

Problem To compute predictions, we need to find  $\arg\max_{y\in\mathcal{Y}(x)}\langle w,\Psi(x,y)\rangle$ , and  $|\mathcal{Y}(x)|$  is exponentially large.

Observation  $\Psi(x,y)$  decomposes to  $\sum_i \Psi_i(x,y)$ .

Solution Dynamic programming (similar to the Viterbi algorithm)



What's the running time?

Figure by Daumé III. A course in machine learning. Figure 17.1.

### The argmax problem in general

#### Efficient problem-specific algorithms:

problem	structure	algorithm
constituent parsing	binary trees with context-free features	CYK
dependency parsing	spanning trees with edge features	Chu-Liu-Edmonds
image segmentation	2d with adjacent-pixel features	graph cuts

#### General algorithm:

• Integer linear programming (ILP)

$$\max_{z} a^{T} z$$
 s.t. linear constraints on  $z$  (22)

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- z: indicator of substructures, e.g.,  $\mathbb{I}\{y_i = \text{article and } y_{i+1} = \text{noun}\}$
- constraints: z must correspond to a valid structure

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#### Conclusion

#### Multiclass algorithms

- Reduce to binary classification, e.g., OvA, AvA, ECCO
  - Good enough for simple multiclass problems
- Generalize binary classification algorithms using multiclass loss
  - Useful for problems with extremely large output space, e.g., structured prediction
  - Related problems: ranking, multi-label classification