

# Portfolio Analysis Results Report

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## 1 Introduction

This report presents the key findings from an analysis of selected stocks and portfolio strategies, based on the execution of the R script `final.R`. The analysis covers descriptive statistics of individual stocks, Capital Asset Pricing Model (CAPM) estimations, and the out-of-sample performance evaluation of Global Minimum Variance (GMV) and Mean-Variance (MV) portfolios, including the impact of transaction costs. The numerical results presented herein are from the script execution on May 16, 2025.

## 2 Data Overview

The analysis utilizes historical monthly stock returns and risk-free rates from `SP100.RData`, and Fama-French factor data (specifically market excess return) from `FF3.RData`. The stocks selected for this analysis are: MDT, MMM, MO, MRK, and MS.

## 3 Analysis and Results

### 3.1 Task 1: Descriptive Statistics for Individual Stocks

The R script calculated key descriptive statistics for each of the five selected stocks. These statistics provide insights into their historical return and risk characteristics.

Table 1: Summary Statistics for Individual Stocks (Monthly)

Stock	Mean Return	Standard Deviation	Sharpe Ratio
MDT	0.005982	0.060808	0.074083
MMM	0.008067	0.062516	0.105428
MO	0.014888	0.067650	0.198549
MRK	0.006797	0.067884	0.078450
MS	0.009213	0.099037	0.078089

#### Comments:

- **Mean Return:** MO exhibited the highest average monthly return (1.49%), while MDT had the lowest (0.60%).

- **Standard Deviation (Risk):** MS was the riskiest stock with the highest standard deviation (9.90%), while MDT was the least volatile (6.08%).
- **Sharpe Ratio (Risk-Adjusted Return):** MO offered the best risk-adjusted return (Sharpe Ratio of 0.1985), indicating a higher return per unit of risk compared to the other stocks. MDT had the lowest Sharpe Ratio (0.0741).

### 3.2 Task 2: Wealth Evolution for Individual Stocks

This task visualizes the growth of a hypothetical \$1 investment in each of the selected stocks over the observed period.

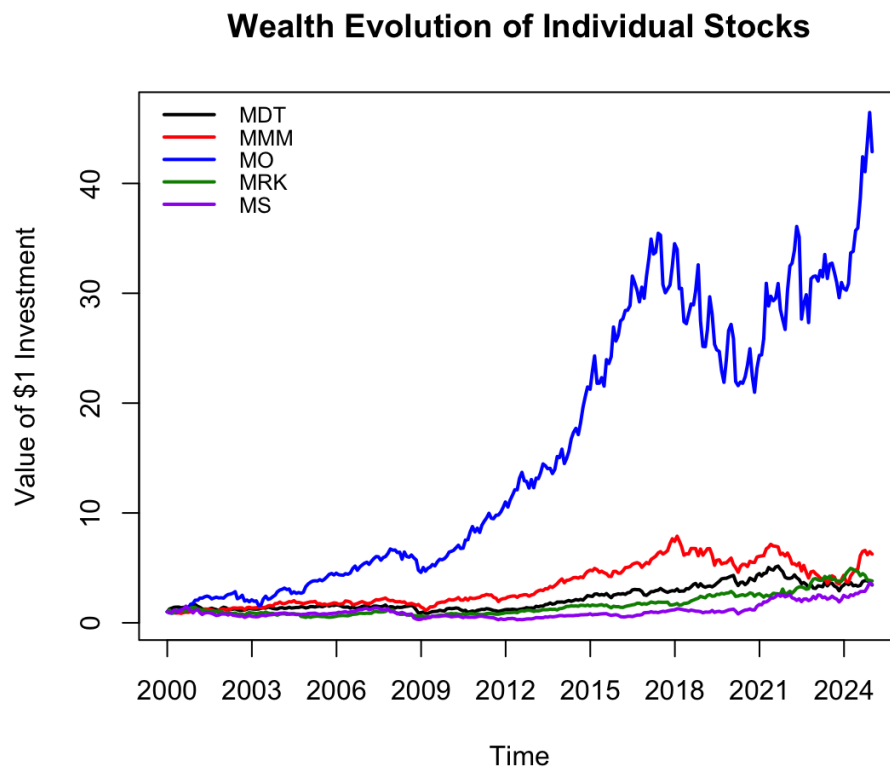


Figure 1: Growth of \$1 invested in MDT, MMM, MO, MRK, and MS over the sample period. The y-axis represents the value of the investment, and the x-axis represents time.

#### Comments:

- The plot would illustrate the cumulative performance of each stock.
- Observe which stock(s) achieved the highest terminal wealth, indicating superior long-term performance over the period.
- Note any significant divergences in performance paths or periods of high volatility for specific stocks.

### 3.3 Task 3: CAPM Expected Returns and Comparison

The Capital Asset Pricing Model (CAPM) was used to estimate expected returns for each stock. These estimations were performed separately for the first half (H1) and the second half (H2) of the dataset to assess the stability of CAPM parameters.

Table 2: CAPM Expected Monthly Returns

Stock	Expected Return (H1)	Expected Return (H2)
MDT	0.002260	0.009673
MMM	0.002343	0.011938
MO	0.002132	0.007112
MRK	0.002144	0.005370
MS	0.003096	0.016452

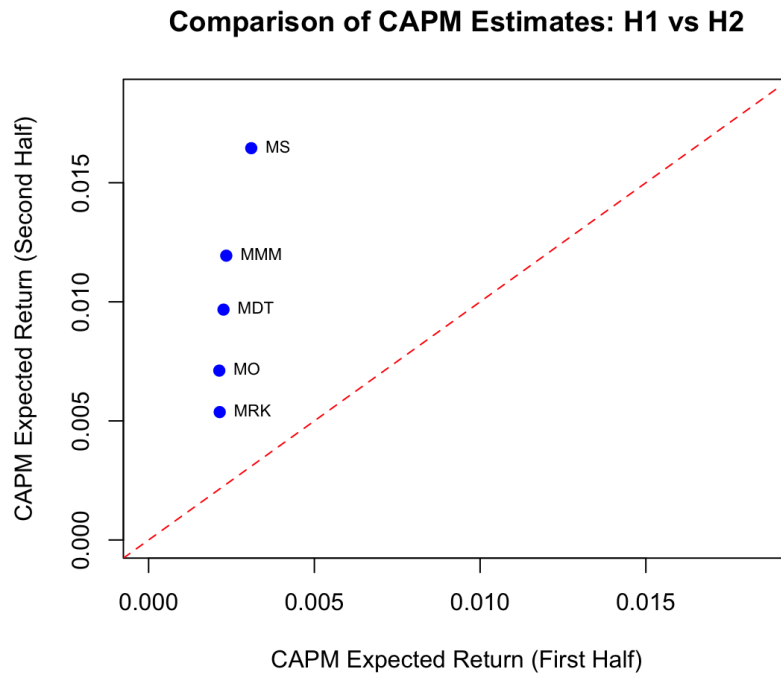


Figure 2: Scatter plot of CAPM expected monthly returns estimated in the first half (H1, x-axis) versus the second half (H2, y-axis) of the data. The red dashed line is the 45-degree line of equality.

#### Comments:

- Table 2 shows the CAPM-based expected returns. All stocks show a notable increase in their CAPM expected returns from the first half (H1) to the second half (H2) of the dataset. This suggests a significant change in market conditions, perceived risk, or market beta for these stocks between the two periods.
- Figure 2 visually confirms this shift. Points would generally be above the 45-degree line, indicating higher H2 estimates compared to H1. The magnitude of the shift

varies by stock, with MS and MMM showing particularly large increases.

### 3.4 Task 4: Portfolio Construction with Expanding Window

Global Minimum Variance (GMV) and Mean-Variance (MV) portfolios were constructed using an expanding window approach. This means that portfolio weights were re-calculated annually (with 20 rebalances performed), using all available historical data up to each rebalancing point. The initial estimation window was 60 months. The MV portfolio targeted the average mean excess return of the constituent stocks at each rebalancing point. The calculated weights from this task were then used for out-of-sample performance evaluation.

### 3.5 Task 5: Out-of-Sample Performance Evaluation

The performance of the GMV and MV portfolios was evaluated using out-of-sample data. This provides a more realistic assessment of how the strategies might have performed in practice. Performance was assessed both gross (before transaction costs) and net (after applying a 0.1% one-way transaction cost at each annual rebalance).

Table 3: Consolidated Out-of-Sample Portfolio Performance Metrics (Monthly)

Portfolio Strategy	Mean	StdDev	Skewness	Kurtosis	Sharpe Ratio
GMV Gross	0.007832	0.044256	-0.302482	3.404323	0.147570
MV Gross	0.007880	0.044248	-0.293762	3.448928	0.148712
GMV Net	0.007823	0.044256	-0.302113	3.404374	0.147373
MV Net	0.007870	0.044248	-0.293292	3.448647	0.148492

#### Comments:

- **Mean Return:** The MV portfolios (Gross and Net) had slightly higher average monthly returns (approx. 0.788% and 0.787% respectively) compared to the GMV portfolios (approx. 0.783% and 0.782%).
- **Standard Deviation (Risk):** The standard deviations are very similar across all four portfolio strategies (around 4.42% - 4.43%), with the MV portfolios showing marginally lower volatility than GMV in this specific run. This is somewhat counterintuitive as GMV aims to minimize variance, suggesting the target return for MV might have coincidentally led to a similar risk profile in this out-of-sample period.
- **Sharpe Ratio (Risk-Adjusted Return):**
  - The MV portfolios (Gross and Net) achieved slightly higher Sharpe Ratios (0.1487 and 0.1485 respectively) than the GMV portfolios (0.1476 and 0.1474). This indicates a marginally better risk-adjusted return for the MV strategy in this out-of-sample period.
- **Impact of Transaction Costs:** Transaction costs had a very small impact on mean returns and Sharpe Ratios for both strategies, reducing them only slightly.

This suggests that the annual rebalancing frequency and the resulting portfolio turnover did not lead to substantial transaction costs with the assumed 0.1% rate.

- **Skewness and Kurtosis:** All portfolios exhibit slight negative skewness (returns distribution skewed to the left, indicating a slightly higher probability of larger negative returns than larger positive ones). Kurtosis values are slightly above 3 (leptokurtic), suggesting slightly fatter tails than a normal distribution.

### 3.6 Task 6: Plot Portfolio Wealth Evolution (Out-of-Sample)

This task visualizes the growth of a \$1 investment in the out-of-sample GMV and MV portfolios, both gross and net of transaction costs.

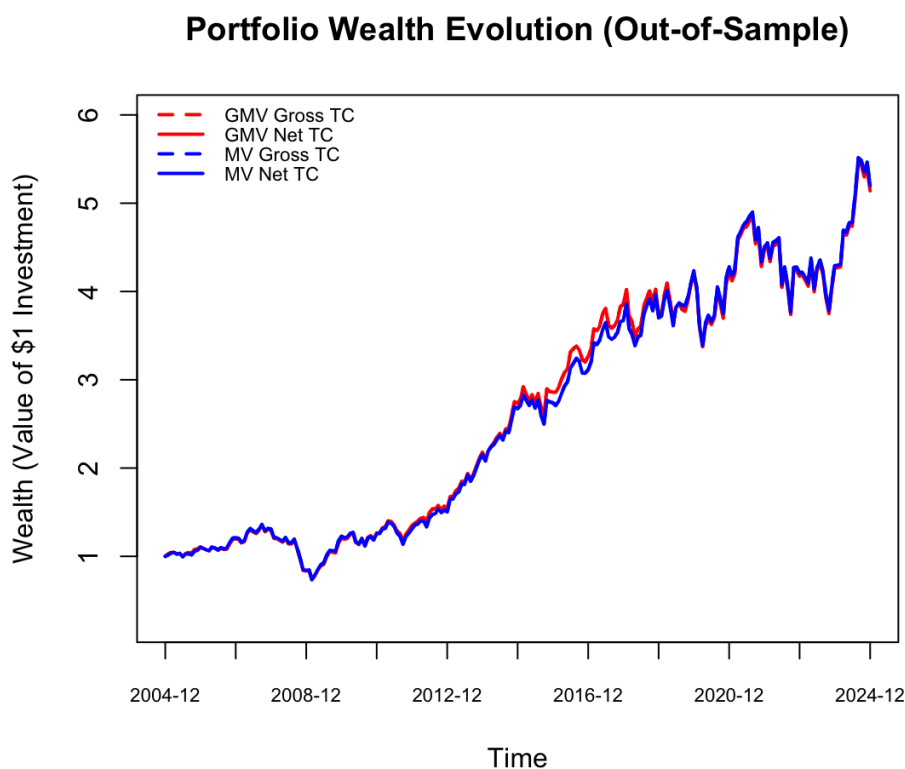


Figure 3: Growth of \$1 invested in GMV (Gross and Net TC) and MV (Gross and Net TC) portfolios during the out-of-sample period. The y-axis represents the value of the investment, and the x-axis represents time.

#### Comments:

- The plot would show the cumulative wealth generated by each of the four portfolio strategies.
- Given the very similar performance metrics in Table 3, the lines for all four strategies are expected to be very close to each other.
- The gap between the "Gross" and "Net" lines for each strategy would visually represent the impact of transaction costs, which is expected to be minimal based

on the numerical results.

- It would be interesting to observe if one strategy consistently, albeit marginally, outperformed the other or if their paths intertwined closely.

## 4 Conclusion

The analysis, based on the R script execution with the current dataset, provided insights into the historical performance of five selected stocks and two portfolio construction strategies (GMV and MV). Key findings from this specific run include:

- Individual stocks showed varied risk and return profiles, with MO offering the highest historical Sharpe ratio.
- CAPM expected return estimates increased substantially for all stocks from the first half of the dataset to the second, indicating changing market dynamics or risk perceptions.
- In the out-of-sample evaluation, the MV portfolio strategy yielded marginally higher mean returns and Sharpe Ratios than the GMV strategy. Both strategies exhibited very similar levels of risk.
- Transaction costs, at the assumed rate and annual rebalancing frequency, had a minimal impact on the performance of both portfolio strategies.
- All portfolios displayed slight negative skewness and leptokurtosis in their out-of-sample return distributions.

The visualizations of wealth evolution for both individual stocks and the constructed portfolios (once generated and inserted) will further aid in understanding the long-term performance implications.