1. Testing for Autocorrelation Using the LM Test

We estimated two linear models for inflation using three (Model3) and six (Model4) lags, and tested the residuals for autocorrelation of order 1 and 12 using the Breusch-Godfrey LM test. The null hypothesis of the test is that there is no autocorrelation. For Model3, the LM statistics were 0.244 ($p = 0.6214$) at lag 1 and 13.184 ($p = 0.3558$) at lag 12. For Model4, the LM statistics were 0.0185 ($p = 0.8918$) at lag 1 and 7.829 ($p = 0.7984$) at lag 12. In all cases, we fail to reject the null hypothesis, suggesting that the residuals do not exhibit significant autocorrelation.

2. Testing for Heteroskedasticity and Estimating HAC Standard Errors

To assess the presence of heteroskedasticity in the regression models, we applied the Breusch-Pagan test. The null hypothesis of this test is that the variance of the residuals is constant (i.e., homoskedasticity). For both model3 and model4, the test returned p-values of 0.0153 and 0.0191, respectively, which are below the 5% significance level. Thus, we reject the null hypothesis and conclude that heteroskedasticity is present in both models. To address this issue, we re-estimated the models using heteroskedasticity and autocorrelation consistent (HAC) standard errors via the Newey-West estimator. The adjusted standard errors differed from the original ones, with some increasing and others decreasing across both models. These changes slightly influenced the significance of certain coefficients but did not substantially alter the overall interpretation. This adjustment ensures more reliable inference in the presence of heteroskedasticity.

Table 1: Comparison of Standard Errors Before and After HAC Adjustment

Model	**Coefficient**	**SE Before**	**SE After (HAC)**
Model 3	Intercept	0.4417	0.4251
	Inflation Lag 1	0.1262	0.1504
	Inflation Lag 2	0.1512	0.1671
	Inflation Lag 3	0.1081	0.1105
Model 4	Intercept	0.3870	0.3055
	Inflation Lag 1	0.1455	0.1204
	Inflation Lag 2	0.2104	0.1989
	Inflation Lag 3	0.1940	0.1507
	Inflation Lag 4	0.1695	0.0799
	Inflation Lag 5	0.1455	0.1406
	Inflation Lag 6	0.1033	0.0846

3. Plotting the Residuals for Model3 and Model4, and Their ACF and PACF

We analyzed the residuals and their ACF and PACF plots for both `model3` and `model4`. The residuals in both models are randomly scattered around zero, indicating no apparent patterns or trends. Furthermore, all spikes in the ACF and PACF plots fall within the blue lines, indicating that the autocorrelation is not statistically significant at any of the lags.

This suggests that the residuals do not exhibit serial correlation, meaning that the models have adequately captured the time-dependent structure in the data. In other words, the models fit the data well, and there is no need for further adjustments related to autocorrelation.

4. Removing Seasonality from the EUStockMarkets Dataset

Seasonality was removed from each series in the `EuStockMarkets` dataset using multiplicative decomposition. This method was chosen as the amplitude of seasonal fluctuations increased over time, indicating a multiplicative structure. Each series was decomposed using `decompose(type = "multiplicative")`, and seasonality was removed by dividing the original series by the seasonal component. The deseasonalised series were then combined into a multivariate time series.

5. ARIMA Model Estimation and Stationarity

Using the `auto.arima` function, we estimated a model for each seasonally adjusted index. For DAX, SMI, and CAC, the selected model was ARIMA(5,2,0), which means the series had to be differenced twice to achieve stationarity. For FTSE, the selected model was ARIMA(0,1,1) with drift, requiring only one differencing step. Since all models involve differencing ($d > 0$), this indicates that the original series were not stationary.

6. Building a Stationary Dataset and Testing for Unit Roots

After adjusting each series for seasonality using multiplicative decomposition, the resulting dataset was differenced once to eliminate trend components, yielding `EuStockMarketsDIFF`. The Augmented Dickey-Fuller tests on all four series (DAX, SMI, CAC, FTSE) showed p-values < 0.01, confirming stationarity after differencing. Lag order selection using `VARselect()` suggested lag 1 based on HQ and SC criteria, while AIC and FPE indicated lag 9.

7. Estimating the VAR Model and Interpreting the Coefficients

We estimated the VAR model with 9 lags, based on the Akaike Information Criterion (AIC), using the variables DAX, SMI, CAC, and FTSE. The correlation matrix shows strong relationships between the variables, with DAX and SMI having a correlation of 0.7459, and DAX and CAC at 0.7496. The covariance matrix further confirms the interdependence of these variables. All roots of the characteristic polynomial lie within the unit circle, indicating model stability. The log likelihood value is -33385.381, with a sample size of 1850. The coefficients from the model, which require further analysis for statistical significance and interpretation, will quantify the interdependencies between these stock indices over time.

8. Granger Causality Test and Model Refinement

We applied the Granger causality test to examine the relationships between the variables in the VAR model. The null hypothesis for the test is that a variable does not Granger cause another. The results show that the null hypothesis is rejected for all variables (DAX, SMI, CAC, FTSE), as the p-values for the Granger causality tests are all below 0.05, indicating that each variable Granger causes at least one of the others. Based on these results, all variables should be kept in the model.

9. Forecast for the Next 25 Months

The forecast for the next 25 months for each variable (DAX, SMI, CAC, FTSE) was generated using the VAR model with the code provided. The last actual values for each index were extracted, and forecasts were computed and combined with the original data. These forecasted values were then plotted for the last 100 data points, with actual values shown in blue and forecasted values in red, as illustrated in the code for plotting each index.

Forecasts for the Next 25 Months

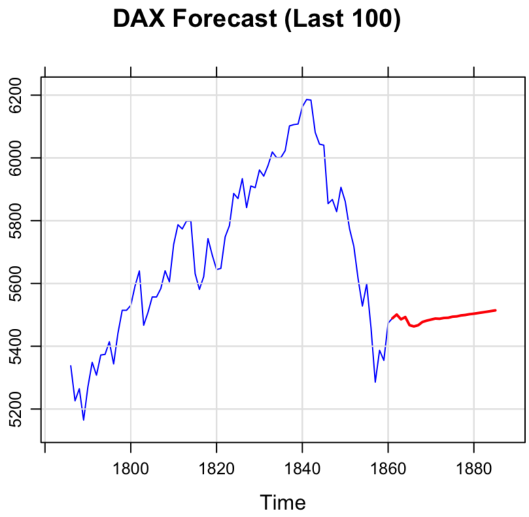

Figure 1: Forecast for DAX_Adj

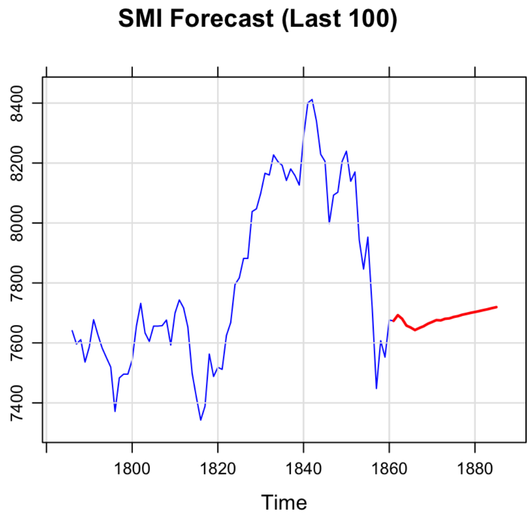

Figure 2: Forecast for SMI_Adj

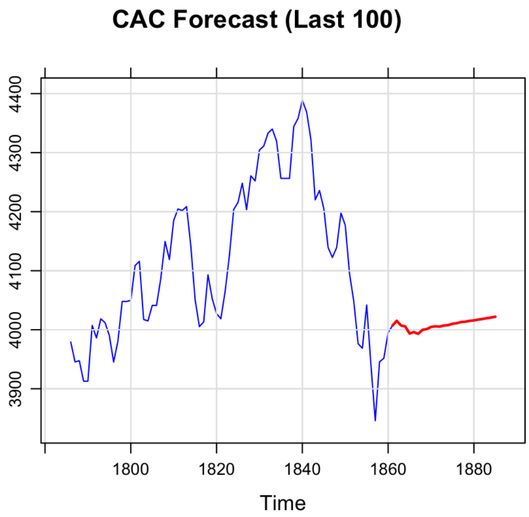

Figure 3: Forecast for CAC_Adj

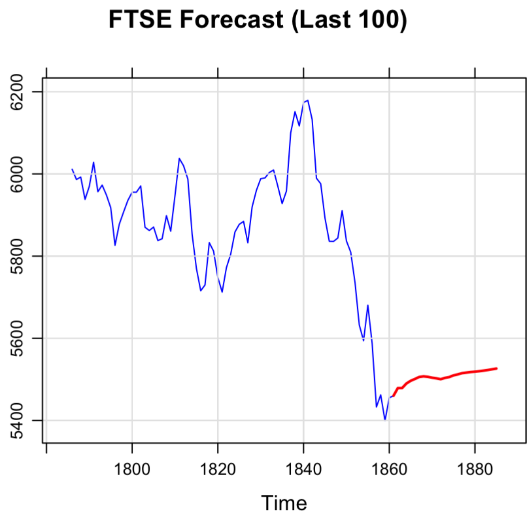

Figure 4: Forecast for FTSE_Adj