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Interbank markets allow credit institutions to exchange capital for purposes of liquidity management. These markets are among the most liquid markets in the financial system. However, liquidity of interbank markets dropped during the 2007–2008 financial crisis, and such a lack of liquidity influenced the entire economic system. In this paper, we analyse transaction data from the e-MID market which is the only electronic interbank market in the Euro Area and US, over a period of 11 years (1999–2009). We adapt a method developed to detect statistically validated links in a network, in order to reveal preferential trading in a directed network. Preferential trading between banks is detected by comparing empirically observed trading relationships with a null hypothesis that assumes random trading among banks doing a heterogeneous number of transactions. Preferential trading patterns are revealed at time windows of 3-maintenance periods. We show that preferential trading is observed throughout the whole period of analysis and that the number of preferential trading links does not show any significant trend in time, in spite of a decreasing trend in the number of pairs of banks making transactions. We observe that preferential trading connections typically involve large trading volumes. During the crisis, we also observe that transactions occurring between banks with a preferential connection occur at larger interest rates than the complement set—an effect that is not observed before the crisis.

1. Introduction

Interbank markets play a key role in banks liquidity management by allowing credit institutions to exchange capital to overcome short-term liquidity shocks. The interest rates determined in this market represent the marginal cost of capital for credit institutions. Variations in interbank rates are rapidly transmitted to the entire term structure, affecting borrowing conditions for households and firms. Interbank dynamics thus influence the whole economic system. In normal times, interbank markets are among the most liquid in the financial sector. Due to the short-term nature of the exchanged deposits banks have accepted non-collateralized loans and both liquidity and credit risks were perceived as negligible. During the 2007–2008 financial crisis though, liquidity in the interbank

market has considerably dried up, even at short maturities, and an increasing dispersion in the credit conditions of different banks has emerged. A number of studies have investigated how movements in borrowing costs depend on bank's specific characteristics such as their size and creditworthiness (Gabrieli 2009, Angelini *et al.* 2011), or on banks ability to exploit changing market microstructure conditions (Gabbi *et al.* 2012).

The combination of trust evaporation arising from informational asymmetries and increased counterparty risk have been identified as important factors in the dry up of liquidity in interbank exchanges such as the e-Mid (Cassola *et al.* 2008, Heider *et al.* 2009), the Fedfunds and the EONIA (Afonso *et al.* 2011, Acharya and Merrouche 2013).

Several authors have suggested that the interbank market freeze was the result of a high degree of financial interrelation and interaction among market participants (Shin 2010, Gai *et al.* 2011, Billio *et al.* 2012) and have called for the adoption of

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network analysis to fully understand the stability of the banking sector (Haldane 2009).

A substantial number of papers have adopted computational techniques to assess the possible extent of contagion via interbank liabilities, starting either from real exposure data (see Upper 2011 for an extensive review) or from agent-based models simulated data. Agent-based models have addressed complementary issues including: the relationship between the network structure of the interbank market and its resilience to different kind of shocks (Iori *et al.* 2006, Gai *et al.* 2011, Battiston *et al.* 2012, Lenzu and Tedeschi 2012, Co-Pierre 2013, Ladley 2013); the effects of assets fire-sale (Nier *et al.* 2007); roll-over risk and portfolio overlaps (Anand *et al.* 2012, Caccioli *et al.* 2012); feedback loops between the macroeconomy and the financial sector (Anand *et al.* 2012, Grilli *et al.* 2012).

On the empirical side, the network literature has aimed at characterizing the observed topology of the interbank market checking for regularities and stylized facts. Iori *et al.* (2008) analyse the microstructure of the e-Mid market over the period 1999–2002 and observe a fairly random distribution of links without preferential lending or intermediaries. Iazzetta and Manna (2009) performs an empirical analysis of the e-Mid market between 1990 and 2008 and report an increasing high degree of banks concentration with fewer banks acting as global hubs for the whole network. Bech and Atalay (2008) shows that the network resulting from Fedfunds exchanges is sparse, exhibits small-world phenomena and is disassortative. In addition, reciprocity and centrality measures are predictors of interest rate loans. Gabrieli (2012) also tests whether measures of centrality can explain the heterogeneous patterns in the interest rates paid on the e-Mid overnight market. Craig and Von Peter (2010) analyses tiering in the German interbank market and identifies a core-periphery structure. Martinez-Jaramillo *et al.* (2012) analyses the Mexican banking network combining data from the payment systems and interbank exposures. The authors provide a number of non-topological measures for the analysis of banking networks and their systemic risk. In addition, the authors show that the measure of interconnectedness of a bank is not necessarily related with its asset size.

On the theoretical side, network models of the banking sector focus their attention on how the incentives of single agents to form linkages, and their degree of risk aversion, affect the resulting network structure (Duffie *et al.* 2005, Babus 2011, Kovarik and Van der Leij 2012). Issues considered include how agents' behavioural rules and moral hazard affect the stability of the resulting network (Acemoglu *et al.* 2013, Castiglionesi and Navarro 2008).

While economic network theory predicts that the worsening of credit condition may affect the way banks choose their trading counterparties, identifying regularities in the way trading partnerships are formed is a relatively new, but growing, area of empirical research. Lasting interbank relationships in overnight interbank markets have been observed both in over-the-counter markets such the Portuguese (Cocco *et al.* 2009) and the German ones (Brauning 2011) and in transparent markets such as the e-MID (electronic Market for Interbank Deposit).

The first paper to explore interbank relationship lending is Cocco *et al.* (2009). Looking at data from the Portuguese interbank market, from January 1997 to March 2001, the authors find that both lenders and borrowers obtain more favourable rates when they trade with their preferred counterparties. In general, small borrowers tend to establish relationships with larger banks as lenders and vice versa. Borrowers with higher default risk tend to rely more on interbank lending. These risky borrowers pay higher interest rates, but achieve better prices if they trade with those banks with whom they have established strong relationships. In Brauning (2011), interbank transactions among German banks, from March 2006 to November 2007, extracted from the TARGET payment system are analysed. The author finds that, before August 2007, lenders imposed higher charges to banks with whom they had a relationship. However, at the peak of the crisis, lenders provided lower rates to their relationship counterparties, compared with other institutions. The author's interpretation of this result is that during normal times borrowers pay a premium to secure relationships, however, during periods of financial distress, relationship borrowers receive better rates from their closer counterparties due to the better information on the borrower creditworthiness at the lender disposal.

In Affinito (2011), the focus is on e-MID monthly data, spanning from June 1998 to April 2009. The author shows that stable interbank relationships were established prior to the subprime crisis, and lasted during its subsequent unfolding. Using the same data-set, in Temizsoy *et al.* (2013) no impact of relationship lending on rates is found before the financial crisis. During the crisis though, and in particular, after the Lehman Brothers collapse in September 2008, having relationships benefited both lenders and borrowers. Nonetheless, while the impact of trading with a preferential counterparty was substantial for transactions initiated as a borrower quote, it was negligible for transactions initiated as a lender quote. In Liedorp *et al.* (2010), the authors examine bank to bank relationships in the Dutch interbank market and show that market participants affect each other's riskiness through trading connections.

In all the above papers, the strength of lending relationships is measured by the concentration of lending/borrowing activity between banks. More precisely, for every lender and borrower, a preference index is computed, equal to the ratio of total funds that a lender (borrower) has lent to (borrowed from) a borrower (borrower) during a given period, over the total amount of funds that the lender (borrower) has lent in (borrowed from) the interbank market during the same period. This measure nonetheless does not take into account the heterogeneity of the banking system and the fact that large banks may have no alternative than to trade with each other if they need to exchange large volumes.

In this paper, we compare the trading relationships empirically observed in the e-MID market with a null hypothesis of random trading among banks performing a heterogeneous number of transactions as borrower and/or lender. We do this in two ways. First, we compare empirical results with a directed weighted configuration model obtained via sequential edge local rewiring of the real transactions. The weights are given in terms of the number of transactions done by banks as lenders
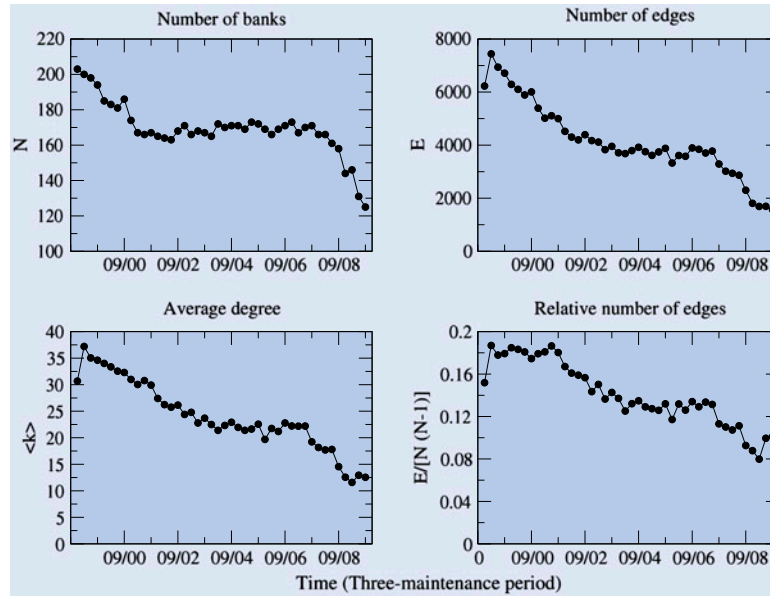
Figure 1. Summary statistics of the trading network for the period from 25 January 1999 to 7 December 2009. Top-left panel: Number of nodes (banks). Top-right panel: edges. Bottom-left panel: average out degree (equal to average in-degree). Bottom-right panel: edge density, that is the ratio between the observed number of directed edges and its maximum possible number, i.e. $N(N-1)$. We consider here the whole set of e-MID transactions.

and/or borrowers. Then, we provide a probabilistic description of the null hypothesis using the theoretical framework of the so-called statistically validated networks. We show that the reshuffling procedure can be efficiently described by the probabilistic model and therefore, one can investigate the whole system using the statistical validated networks approach.

Our working hypothesis is that bilateral trades whose intensity is preserved in the randomized ensemble can be traced back/explained as consequences of the heterogeneity in banks activity. If system heterogeneity cannot explain alone the number of trades among specific pairs of banks we will take this as evidence of trust (or distrust) among such banks leading to preferential relationships. Our specific objective is to provide empirically based answers to the following questions: does preferential trading and/or avoidance of trade exist between pairs of banks? If so, are these occurrences stable in time for a given pair of banks, and is there any change in preferential trading during (or leading up to) the financial crisis of 2007/2008?

Our results confirm that during crisis periods, credit relationships between banks are polarized in the two opposite and complementary directions: (i) relative increase of the trustworthy relationships and (ii) elimination of some of the not-trustworthy ones. We also observe that preferential trading connections typically involve larger trading volumes and that transactions occurring between banks with a preferential connection occur at higher interest rates after Lehman's bankruptcy. This last observation is consistent with the results of Cocco *et al.* (2009) and suggests that it is mostly the riskier banks that rely on relationship lending, particularly, during periods of financial distress.

The remainder of the paper is organized as follows: in section 2, we will present the e-MID data-set. In section 3, we will illustrate the methodologies we will use to investigate the system. In section 4, we will present our results and in section 5, we will draw our conclusions.

2. Data

Interbank markets can be organized in different ways: physically on the floor, by telephone calls or on electronic platforms. In Europe, interbank trades are executed in all these ways. The only electronic market for Interbank Deposits in the Euro Area and US is called e-MID. It was founded in Italy in 1990 for Italian Lira transactions and became denominated in Euros in 1999. When the financial crisis started, the market players were 246, members from 16 EU countries: Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, UK, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland and Portugal.

The number of transactions decreases, whereas the volume increases until the beginning of the financial crisis. According to the European Central Bank e-MID accounted, before the crisis, for 17% of total turnover in unsecured money market in the Euro Area. The last report on money markets (European Central Bank 2011), recorded around 10% of the total overnight turnovers.

Trading in e-MID starts at 8 am and ends at 6 pm. Contracts of different maturities, from one day to a year can be traded but the overnight segment (defined as the trade for a transfer of funds to be effected on the day of the trade and to return on the subsequent business day at 9 am) represents more than 90% of the transactions.

One distinctive feature of the platform is that it is fully transparent. Trades are public in terms of maturity, rate, volume and time. Buy and sell proposals appear on the platform with the identity of the bank posting them (the quoter may choose to post a trade anonymously but this option is rarely used). Market participants can choose their counterparties. An operator willing to trade can pick a quote and manifest his wish to close the trade while the quoter has the option to reject an aggression.

The database is composed by the records of all transactions registered in the period from 25 January 1999 to 7 December 2009. Each line contains a code labelling the quoting bank, i.e. the bank that proposes a transaction, and the aggressor bank, i.e. the bank that accepts a proposed transaction. The rate the lending bank will receive is expressed per year; the volume of the transaction is expressed in millions of Euros. A label indicates the side of the aggressor bank, i.e. whether the latter is lending/selling (Sell) or borrowing/buying (Buy) capitals to or from the quoting bank. Other labels indicate the dates and the exact time of the transaction and the maturity of the contract. We consider only the overnight (ON) and the overnight long (ONL) contracts. The latter is the version of the ON when more than one night/day is present between two consecutive business day. The banks are reported together with a code representing their country and, for Italian banks, a label that encodes their size, as measured in terms of total assets.

The period of time in which credit institutions have to comply with the minimum reserve requirements is called the reserve maintenance period. During each reserve maintenance period, minimum reserve levels are calculated on the basis of banks' own balance sheet. Each reserve maintenance period is usually equivalent to one calendar month, i.e. about 23 trading days. In the investigations, we will present hereafter, we have aggregated the maintenance periods in groups of three. In fact, these aggregated periods better capture the natural economic cycles that are usually organized on a nearly three-monthly basis. We, therefore, will consider 44 3-maintenance periods ranging from 25 January 1999 to 7 December 2009.† In these aggregated time period, the main crisis events occur at the 3-maintenance period 35 (13 June 2007/11 September 2007) and 3-maintenance period 40 (10 September 2008/9 December 2008), which corresponds to the Lehman Brothers bankruptcy.

We model the credit relationships of bank loans in the e-MID market as a network in which nodes are banks and a directional link is set from bank i to bank j if i lent money to j, the weight of the link being the number of times in which that event occurred in the considered time period.

In figure 1, we show the number of nodes (top-left panel), edges (top-right panel), average out degree (equal to average in-degree) (bottom-left panel) and edge density (bottom-right panel) for the period from 25 January 1999 to 7 December 2009. The four plots of figure 2 show the total number of transactions, the mean number of lending transactions per bank and the mean number of transactions per existing link. A general decline of trading activities in this market is observed in connection with the 2007 freezing of the interbank market, according to the number of active banks (top-left panel of figure 1) the number of edges between banks (top-right panel of figure 1) and total number of transactions (top-left panel of figure 2). Overall, these results indicate that the 2007–2008 crisis events had a measurable impact on the e-MID market, which experienced an overall decline of its trading relationships. Hereafter, we will investigate these effects.

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†It should be noted that the first 3-maintenance period covers the time period from 25 January 1999 to 23 March 1999, thus, involving a number of trading days which is reduced to a factor of about one-third with respect to all the other 3-maintenance periods.

3. Methods

Our aim is to asses the statistical significance of the observed interbank transaction records in order to reveal preferential credit relationships amongst banks during market activity. In order to access the statistical significance of the observed interbank credit relationships it is crucial to compare the empirically observed networks against an appropriate null hypothesis. Here, we will investigate a shuffling procedure applied to empirical data and we will compare the numerical results obtained in terms of a probabilistic description based on the hypergeometric distribution. Using the shuffling procedure allows one to include, in the null hypothesis, the constraint that a bank cannot lend money to itself. However, this procedure is computationally demanding and provides results with a precision that scales like the inverse of the square root of the number of nodes of the shuffled network. On the other hand, the probabilistic description based on the hypergeometric distribution provides analytical results that are only approximated because the probabilistic description does not avoid the possibility that a bank can lend money to itself. In this section, we discuss in details both approaches and show that the probabilistic description of the null hypothesis well describes the numerical outcomes of the shuffling procedure, due to the sparsity of the network.

3.1. A null hypothesis for the number of loans between two banks

For each link in the network, we perform a statistical test to check whether two banks preferentially traded in a given 3-maintenance period. Our test is done by using a recently proposed method (Carollo *et al.* 2013, Jiang *et al.* 2013) that is a directional variant of the method presented in Tumminello *et al.* (2011a,b). The statistical test is implemented as follows. For each 3-maintenance period, we define N_T as the total number of trades among banks in the system and focus on two banks i and j to check whether i preferentially lent money to j, that is, j preferentially borrowed money from i. Let us call n_l^i the number of times bank i lent money to any other banks, and n_b^j the number of times bank j borrowed money from any other bank. Assuming that n_{lb}^{ij} is number of times bank i lent money to j then the probability of observing such n_{lb}^{ij} trades, assuming that j borrows money randomly and i lends money randomly, is given by the hypergeometric distribution

$$H\left(n_{lb}^{ij}|N_T, n_l^i, n_b^j\right) = \frac{\binom{n_l^i}{n_{lb}^{ij}}\binom{N_T - n_l^i}{n_b^j - n_{lb}^{ij}}}{\binom{N_T}{n_b^j}}. \quad (1)$$

We use this probability to associate a *p*-value with the observed number n_{lb}^{ij} of trades from bank i to bank j as $p(n_{lb}^{ij}) = \sum_{X=n_{lb}^{ij}}^{Min[n_l^i, n_b^j]} H(X|N_T, n_l^i, n_b^j)$, that is, the probability of observing by chance a number of trades from i to j equal to n_{lb}^{ij} or larger. This *p*-value is calculated by taking the sum of probabilities over the right tail of the hypergeometric distribution. Therefore, a 'small'‡ *p*-value statistically indicates that the link

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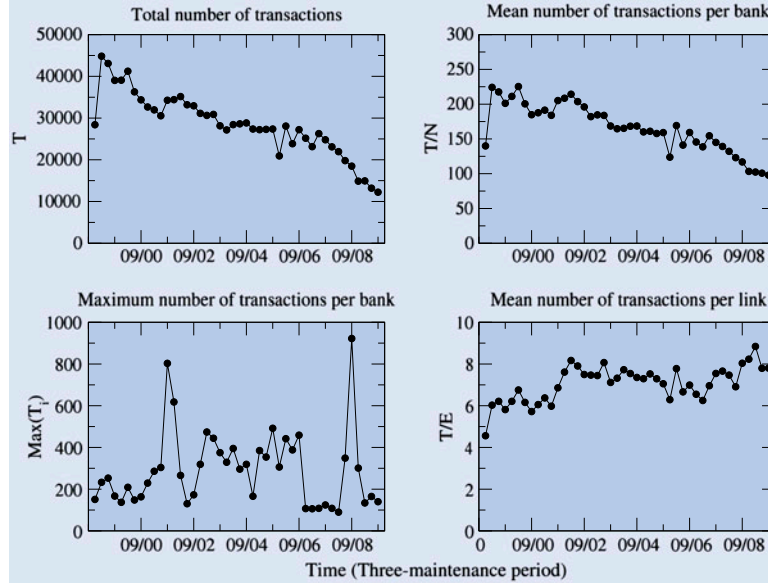‡This point will be quantitatively discussed later in this section.

Figure 2. Summary statistics of transactions. Top-left panel: total number of transactions. Top-right panel: mean number of transactions per bank. Bottom-left panel: maximum number of transactions per bank. Bottom-right panel: mean number of transactions per link. We consider here the whole set of e-MID transactions.
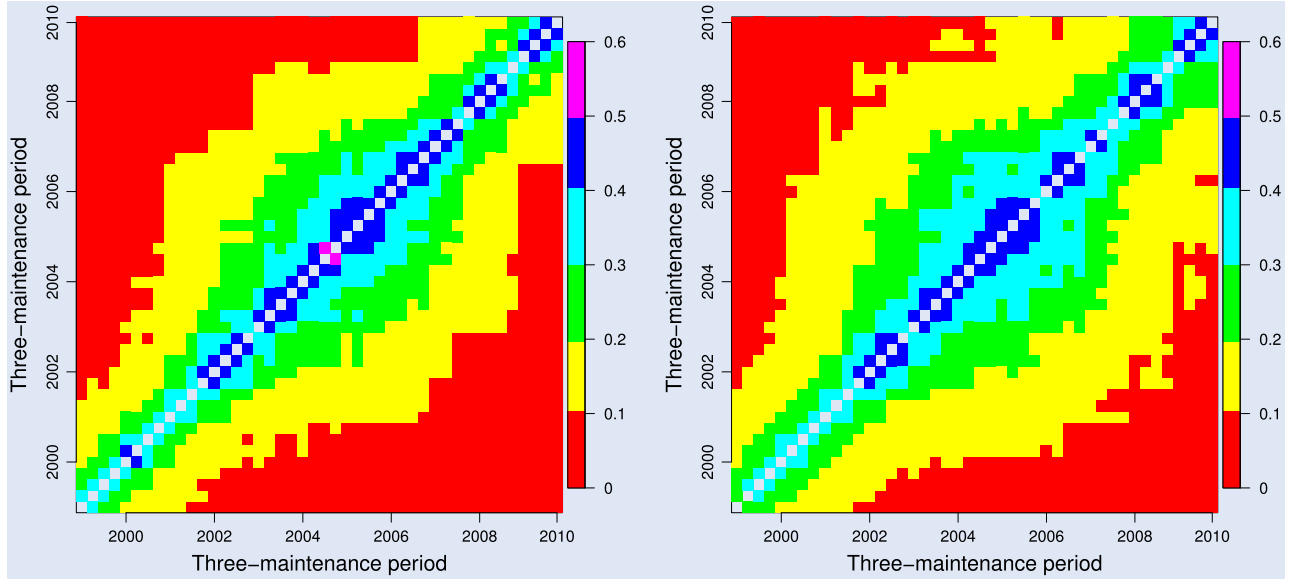

Figure 3. Contour plots of the weighted Jaccard index between all pairs of 3-maintenance periods. The left panel shows the weighted Jaccard index between original networks, and the right panel provides the same measure for rewired networks. The rewiring procedure preserves the total in and out strength of each node.

from i (the lender) to j (the borrower) is over-represented, in terms of number of loans, with respect to the null hypothesis of random trading. Analogously, we can statistically validate a link between i and j that is under-represented with respect to a random null hypothesis by taking into account the left tail of the hypergeometric distribution. In this case, one should compute the left-tail p-value as $p(n_{lb}^{ij}) = \sum_{X=0}^{n_{lb}^{ij}} H(X|N_T, n_l^i, n_b^j)$.

The hypergeometric distribution can be used to describe variable n_{lb}^{ij} because the problem can be mapped into an urn model (Feller 1968). Let's consider an urn with n_b^j blue marbles and $N_T - n_b^j$ red marbles. The probability $p(n_{lb}^{ij})$ is the

probability that, randomly picking n_l^i marbles from the urn without replacement, n_{lb}^{ij} of them are blue coloured, and the hypergeometric distribution is the one that provides $p(n_{lb}^{ij})$. Please notice that the symmetry properties of hypergeometric distribution allows one to obtain the same probability $p(n_{lb}^{ij})$ by reversing the role played by lender and borrower. Specifically, $p(n_{lb}^{ij})$ can also be obtained if we consider an urn with n_l^i blue marbles and $N_T - n_l^i$ red marbles, and n_{lb}^{ij} represents the number of blue marbles among n_b^j randomly drawn marbles. It is to notice that mapping the problem of randomizing bank loans into such an urn model is done at the cost that we get rid of
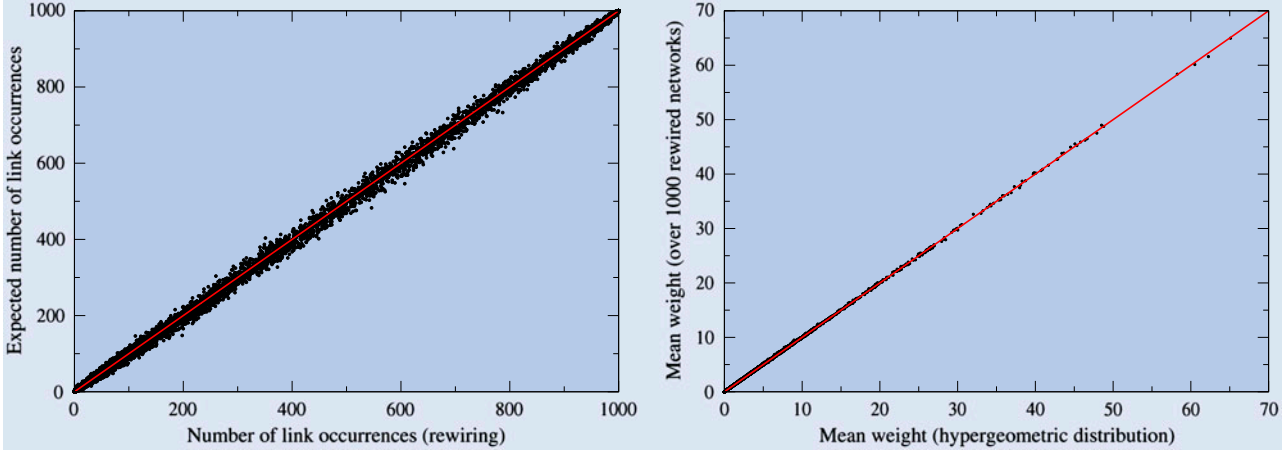
Figure 4. Left panel: scatter plot of the expected number of times in which a link is expected in the random case, according to equation (8) with $N_s = 1000$ vs. the number of times in which a link has been observed in the 1000 shuffling experiments. Right panel: scatter plot of the expected number of transactions between each (directed) pair of banks according to the hypergeometric distribution ($n_l^i n_b^j / N_T$ between lender i and borrower j) vs. the average number of trades observed between each pair of banks over the 1000 shuffling experiments. The data-set used to perform the shuffling experiments is the list of transactions occurred in the 3-maintenance period from 24 December 2004 to 23 March 2005.

the constraint that a bank cannot lend money to itself. This fact makes our analytical solution for the random system just an approximation of what we would obtain by randomly rewiring data and forcing the condition of no self-loans to be respected, as discussed in the next subsection.

If we were just interested in calculating p-values of overrepresentation only for all the directed edges, E in our network, then we should run E statistical tests. To avoid a large number of false positive validated links, due to the large number E of statistical tests, it is advisable to consider a method to control the family-wise error rate. This control can be done by applying the so-called Bonferroni correction (Miller 1981). This correction requires that the univariate level of statistical significance, e.g. $p_u = 0.01$, is corrected in presence of multiple tests. Specifically, the multivariate level is set to $p_m = p_u/E = 0.01/E$. The Bonferroni correction is the most conservative multiple hypothesis test correction.

In the following, we set $p_u = 0.01$ and say that bank i preferentially lent money to bank j if the estimated $p(n_{lb}^{ij})$ is less than $p_m = 0.01/E$. In this case, we will set a link from bank i to bank j in a 'filtered' network that is named Bonferroni network (Tumminello *et al.* 2011a).

3.1.1. Simultaneous test of over- and underexpression.
In many situations, it is convenient to study simultaneously over- and underexpressed links in a given network. This clearly affects the way in which the statistical threshold should be corrected for multiple comparison, because it changes the total number of tests. Below, we show a simple way to compute the correct statistical threshold p_m in this case.

Let us consider a specific time period—e.g. a 3-maintenance period a—and call E_a the number of pairs of banks that actually traded at least once in that time period; call L_a (B_a) the number of banks that have done at least one trade as a lender (borrower) in the time window, and call N_a^{BL} the number of banks that

have done at least one trade as a lender and at least one trade as a borrower in the a-th time period.

The number of tests for overexpression that we need to run is E_a. In general, $E_a \leq L_a \times B_a - N_a^{BL}$. In fact, the quantity $L_a \times B_a - N_a^{BL}$ gives the maximum expected number of links, given a certain number of lenders and borrowers. However, many of these links might not actually exist in the network, because trades between some bank pairs may not occur. Therefore, for some i and j, it is possible that $n_{lb}^{ij} = 0$, which, necessarily, cannot indicate an overrepresentation of loans from i to j, and therefore should not be tested for overrepresentation. However, $n_{lb}^{ij} = 0$ is an excellent candidate to test underrepresentation of loans from i to j—that is to test whether i avoids to lend money to j or j avoids to borrow money from i. This means that such cases should be included in the test for underrepresentation. So, in the case of underrepresentation, we have to run the test for all the pairs of active banks in the system ($L_a \times B_a$) minus the cases in which lender and borrower are the same bank, that is N_a^{BL}. Therefore, $L_a \times B_a - N_a^{BL}$ is the number of tests of underrepresentation that we have to run. As a result, the total number of tests, for both under- and overrepresentation, in a given time window, is

$$T_a = E_a + L_a \times B_a - N_a^{BL}. \qquad (2)$$

It is worth mentioning that a possible alternative would be $T_a = 2(L_a \times B_a - N_a^{BL})$. However, we believe that this choice is unnecessariliy conservative as it involves testing the null hypothesis in cases that are clearly not relevant for the overexpression analysis (those with $n_{lb}^{ij} = 0$).

3.2. A null hypothesis from shuffling and a probabilistic approach

For each time period labelled by time t a directed weighted graph can be defined over a set of banks V by considering their interbank transactions. More formally, each *observed* graph is
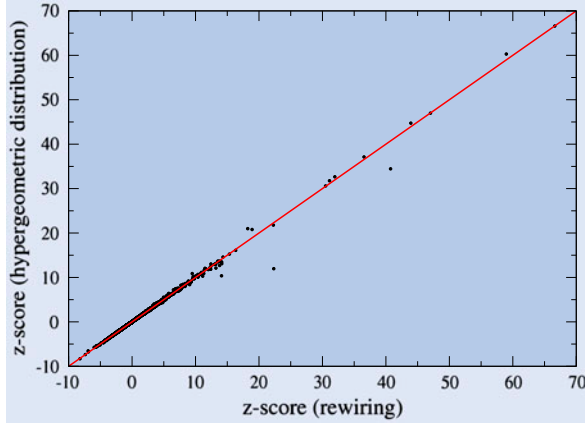
Figure 5. Scatter plot of the *z*-scores based on 1000 rewiring experiments vs. the *z*-scores obtained with the probabilistic modeling, for the 3-maintenance period from 24 December 2004 to 23 March 2005.

represented by the asymmetric weight matrix $W^o(t) = \{w_{i,j}^o(t)\}_{\forall i,j \in V}$ (with no self-interactions allowed) where $w_{i,j}^o(t)$ is the number of loans from bank i to bank j. It follows that $s_{i,out}^o(t) = \sum_j w_{i,j}^o(t)$, the out strength of bank i, is the total number of observed transactions in the period with bank i acting as a lender and $s_{i,in}^o(t) = \sum_j w_{j,i}^o(t)$ is the total number of observed transactions in the period with bank i acting as a borrower.

3.2.1. The directed weighted configuration model. As a null model, we now consider a random network characterized by a strength distribution equal to the one observed empirically for each time period (see Roberts and Coolen 2012 and references therein). For directed and weighted representations, we can construct a randomization using the edge swap procedure (that now conserves the vertex in-out strength sequence but not the in-out degree sequence) in the following way. Each weighted directed edge with weight $w_{i,j}$ is further inserted $w_{i,j} - 1$ times in the network and all edges have their weights set to 1. The resulting multigraph is then rewired as a directed unweighted graph where each edge now indicates a single transactions and the number of edges between i and j corresponds to their number of transactions. The rewired multigraph is then collapsed to a directed weighted graph via the reverse procedure (i.e. all m directed and unweighted edges between j and i are collapsed into a single edge with weight m) and the quantities of interest are then computed in this final graph. Note that, as it stands, this process only works for graphs with *integer* weights. Moreover, this reshuffling algorithm does not preserve the in- and out degree of nodes: only the strengths are preserved. This is because in order to redistribute the integer weights evenly between pairs a number of edges must be created. Of course, this would not be the case if all weights were equal in value. An advantage of the multigraph rewiring phase used here is that more edge swaps are permissible (resulting in statistical bias reduction) with respect to the unweighted case. An edge swap selects two ordered pairs $(x, y), (u, v)$ and swaps the endpoints (target nodes) while keeping the sources fixed such that two new

pairs will be inserted in the graph $(u, y), (x, v)$ and the original pairs deleted. For example, if any of the final edges in the transformation $(x, y)(u, v) \rightarrow (u, y)(x, v)$ are already in the set of edges this is not an allowed transformation in the directed unweighted case, whereas the multigraph rewiring side-steps this problem.

Statistically speaking, rewiring by local edge swaps generates a microcanonical ensemble. When applied to such integer-valued weighted graphs, the directed configuration model can generate an in and out strength-preserving ensemble of graphs. A sample of size n is chosen and for each graph and for all 3-maintenance period we generate a corresponding ensemble whose members are defined by the weight matrices $\{w_{i,j}^s(t)\}_{\forall s \in S}$ with $S = \{1, 2, \ldots, n\}$ an index set for the configuration model ensemble. These graphs, acting as one of possible null models, are characterized by $s_{i,x}^o(t) = s_{i,x}^s(t)$, $\forall i \in V$ and for $x \in \{in, out\}$ but are otherwise random. This fact allows one to investigate the extent at which banks' heterogeneity, in terms of number of transactions, determines the structure of the e-MID market and its persistence over time.

For each of the 44 3-maintenance periods, we generate the weighted directed network, in which weights are positive integers indicating the number of (directed) trades between two banks, and we calculate the weighted Jaccard index between each pair of these network. The weighted Jaccard index is a generalization of the usual Jaccard index that allows to take into account link weights:

$$J_W(net_1, net_2) = \frac{\sum_{i,j} Min[w_{i,j}^1, w_{i,j}^2]}{\sum_{i,j} Max[w_{i,j}^1, w_{i,j}^2]}, \qquad (3)$$

where $w_{i,j}^1$ is the weight of link $i \rightarrow j$ in the first network, net_1, and $w_{i,j}^2$ is the weight of link $i \rightarrow j$ in the second network, net_2. For an unweighted network, $J_W(net_1, net_2)$ reduces to usual Jaccard index:

$$J_U(net_1, net_2) = \frac{|E_1 \cap E_2|}{|E_1 \cup E_2|}, \qquad (4)$$

that is the number of directed links that belong to both network over the number of links in the union network. It is worth mentioning that the weighted Jaccard of equation (3) can be interpreted as a Tanimoto coefficient (Tanimoto 1968) if: (*i*) the length $\mathcal{L}$ of the vectors used in the definition of Tanimoto coefficient is set to the maximum number of directional transactions amongst all bank pairs and (*ii*) the vector $V_{ij} = \{v_{ij}^a\}$, $a = 1, \cdots, \mathcal{L}$ associated with the two banks, i and j, doing t_{ij}^q transactions in network q is defined as:

$$v_{ij}^a = \begin{cases} 1 & a \leq t_{ij}^q \\ 0 & a > t_{ij}^q \end{cases} \qquad (5)$$

In the left panel of figure 3, we show the contour plot of the weighted Jaccard index for each pair of 3-maintenance periods. The right panel of figure 3 shows a contour plot of the weighted Jaccard index between the randomly rewired† networks obtained in each 3-maintenance period. The left panel of the figure shows that network structure is rather persistent over time. However, the right panel shows that such a persistence is still present after a strength-preserving rewiring. Therefore, the persistence is mostly due to the fact that the heterogeneity

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†Each rewired network is obtained through 10^6 link swaps.

of banks is maintained over time. In fact, the contour plots reported in the two panels are very similar, and the only quantities that are not randomized in rewired networks are the in strength and out strength (in terms of number of transactions) of banks. This result indicates that banks' heterogeneity must be suitably considered in any analysis that aims at revealing preferential trading patterns.

3.2.2. Comparison of probabilistic predictions and network shuffling.

To establish a connection between the probability that a link occurs in n out of N_s shuffled data-sets and the probabilistic description of link weights based on the hypergeometric distribution we focus on a lender l and a borrower b and on a certain maintenance period, e.g. a 3-maintenance period. We assume that l (b) acted as a lender (borrower) n_l (n_b) times and that l lent money to b in n_{lb} occasions. Finally, N_T is the total number of loans among all the banks, in the given maintenance period.

The probability that l lends money to b at least in one shuffled data-set can be calculated as $p = 1 - P_{lm}(0)$, where $P_{lm}(0)$ is the probability of observing no loan from l to m according to the hypergeometric distribution, $H(n_{lb} = 0|n_b, n_l, N_T)$. We have:

$$p = 1 - P_{lm}(0) = 1 - H(n_{lb} = 0|n_b, n_l, N_T)$$

$$= 1 - \frac{\binom{n_l}{n_{lb}} \cdot \binom{N_T - n_l}{n_b - n_{lb}}}{\binom{N_T}{n_b}}\Bigg|_{n_{lb}=0} = 1 - \frac{\binom{N_T - n_l}{n_b}}{\binom{N_T}{n_b}}, \tag{6}$$

where it is implicitly assumed that $N_T - n_l \geq n_b$. If this is not the case, then $P_{lm}(0) = 0$ and $p = 1$. The probability p is the probability that a link from l to b occurs in the network associated with a shuffled data-set. Therefore, the probability that a link occurs in n out of N_s shuffled data-sets is given by the probability mass function of a Binomial distribution with parameters p (probability of success), n (number of successes) and N_s (number of trials):

$$P(n) = \binom{N_s}{n} p^n (1 - p)^{N_s - n}. \tag{7}$$

According to this probability, the expected value of n is given by

$$E(n) = N_s \, p = N_s \left[1 - \frac{\binom{N_T - n_l}{n_b}}{\binom{N_T}{n_b}} \right], \tag{8}$$

while the variance is

$$\sigma^2 = N_s \, p \, (1 - p) = N_s \, \frac{\binom{N_T - n_l}{n_b}}{\binom{N_T}{n_b}} \left[1 - \frac{\binom{N_T - n_l}{n_b}}{\binom{N_T}{n_b}} \right]. \tag{9}$$

These equations can be used to compare the null hypothesis obtained by rewiring the original network and the null hypothesis described by the hypergeometric distribution. Indeed, from a theoretical point of view, a null hypothesis of random

transaction among banks based on shuffling the data according to the edge swapping algorithm and a null hypothesis based on the hypergeometric distribution are not exactly equivalent. In fact, the hypergeometric distribution does not forbid self-links, that is, trades can occur between a bank and itself. However, in cases where links are rather sparse, as in the present investigation, the probability that such an event occurs by chance is very small. This observation suggests that our approximate approach provides a good description of sparse networks.

To prove such a conclusion, we show the results of two distinct although complementary investigations. On one side, the number of independent randomly rewired networks (out of a total of 1000) in which a directed link occurs between two given banks, despite the weight, is tested against the Binomial distribution $P(n)$ described above. None of the resulting p-values is below the Bonferroni threshold, and therefore, we cannot reject the hypothesis that the results from shuffling follow the indicated binomial distribution, and are in agreement with the underling hypergeometric distribution. In left panel of figure 4, we show the scatter plot of expected value of links obtained from equation (8) vs. the number of link occurrences detected in the shuffling experiments. In the second investigation, we take into account link weights. In the right panel of figure 4, we report a scatter plot of the expected number of transactions between each (directed) pair of banks according to the hypergeometric distribution ($n_l^i \, n_b^j / N_T$ between lender i and borrower j) and the average number of trades observed between each pair of banks over the 1000 shuffling experiments. Also these results confirm that the probabilistic modelling well describes the reshuffling procedure.

3.2.3. z-Score.

In both the random edge swapping algorithm preserving the strength of each network node and the probabilistic description of the null model of e-MID, it might be useful to consider z-transformed variables. In fact, an analytical form of the z-score can be associated with the number of transactions, n_{lb}^o, observed between a lender, l, and a borrower, b, as follows:

$$z_{lb} = \frac{n_{lb}^o - E(n_{lb})}{\sigma(n_{lb})} = \frac{n_{lb} - n_l \, n_b / N_T}{\sqrt{\frac{n_l \, n_b}{N_T} (1 - n_l / N_T) (1 - n_b / N_T)}}, \tag{10}$$

where the approximation $N_T \simeq N_T - 1$, has been used. The right term of the above equation can be interpreted as a correlation measure between two binary vectors of N_T components with n_l and n_b non-zero elements and intersection n_{lb}. The correlation coefficient between these two binary vectors (Tumminello *et al.* 2011b) is

$$\rho_{lb} = \frac{n_{lb} - n_l \, n_b / N_T}{\sqrt{n_l \, n_b \, (1 - n_l / N_T) (1 - n_b / N_T)}}. \tag{11}$$

It follows that:

$$z_{lb} = \sqrt{N_T} \, \rho_{lb}. \tag{12}$$

The z-score can be used to investigate the consistency between the two approaches. Specifically, for each link, the weights observed in the 1000 rewired networks are considered, and the average weight is compared with the expected value
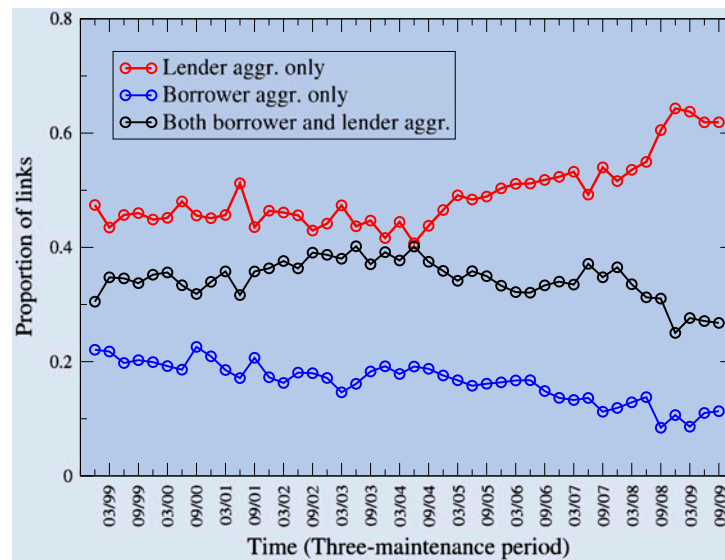
Figure 6. Time evolution of the fraction of directed links where the lender acts only as aggressor (red line), the borrower acts as aggressor only (blue line) and both the lender and borrower act as aggressors (black line). The analysis is performed on the Italian segment of the e-MID market.
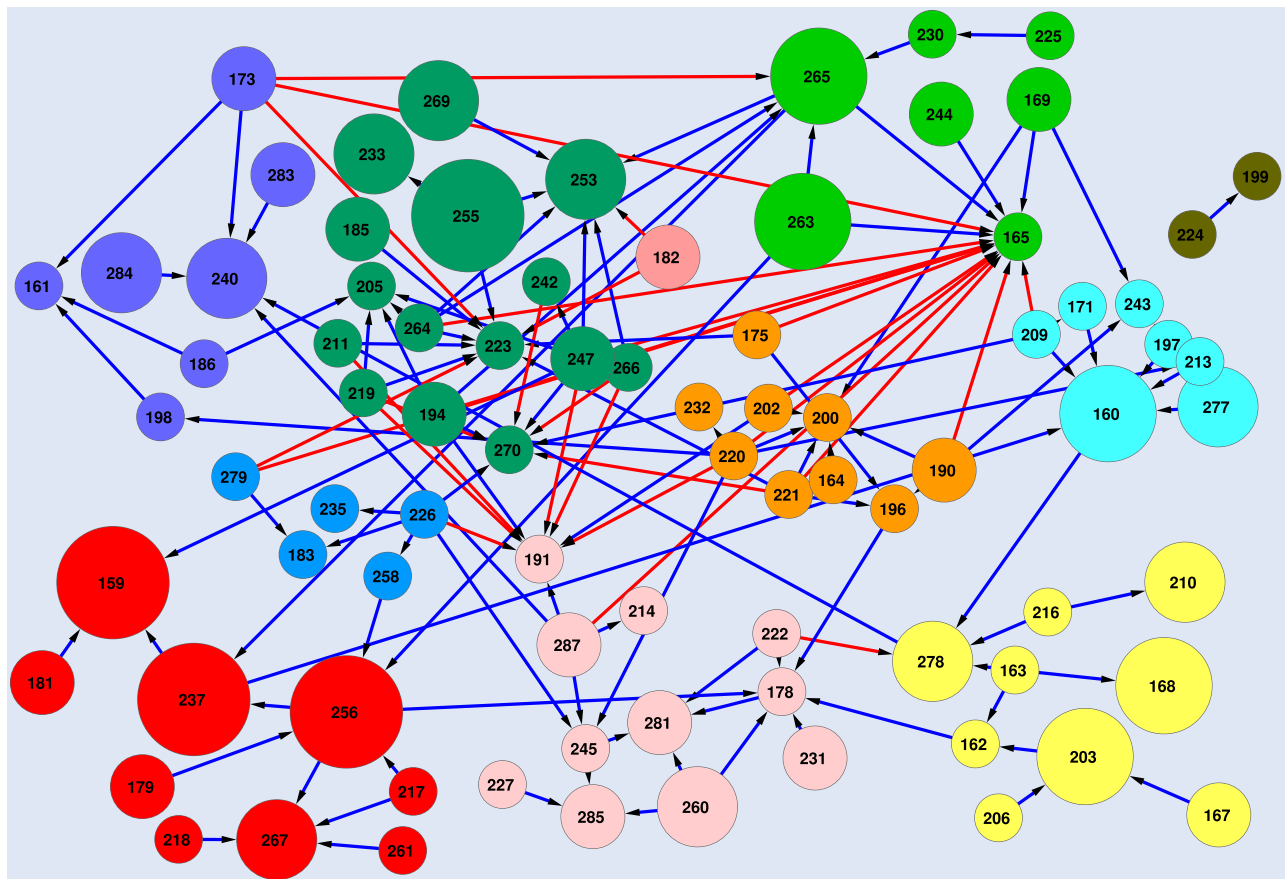

Figure 7. Bonferroni network for the 3-maintenance period 40 (10 September 2008/9 December 2008) of the lender-aggressor data-set. The different colours indicate the node membership to the partitions detected by using the Radatool algorithm (unweighted option) (Duch and Arenas 2005). Red links are underexpressed links, while blue links are overexpressed ones. The analysis is performed on the Italian segment of the e-MID market.

provided by the hypergeometric distribution, through a one-sample *t*-test. Due to the limited number of rewired networks, 1000, cases in which the sum of all the weights over all of the 1000 networks was less than 5 have been excluded from this analysis. Also, in this case, none of the *p*-values is below the Bonferroni threshold, indicating that the hypergeometric
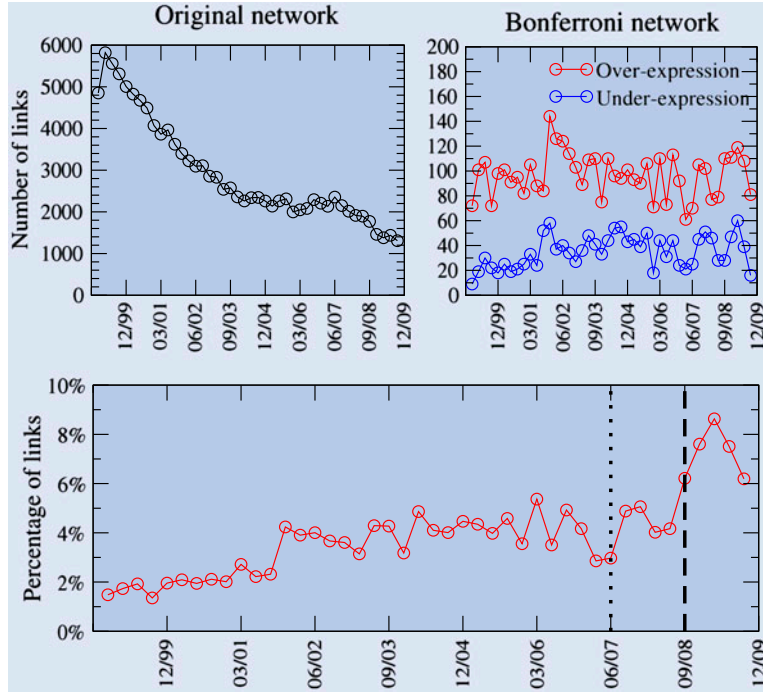
Figure 8. In the top-left panel, we show the number of links observed in the original network. In the top-right panel, the number of overexpressed links (red) and underexpressed links (blue) observed in the Bonferroni network is reported. In the bottom panel, we show the ratio between the number of overexpressed links observed in the Bonferroni and in the original network. The dotted line refers to the August 2007 market freezing, while the dashed line refers to the Lehman's bankruptcy. These data refer to the lender-aggressor data-set. The analysis is performed on the Italian segment of the e-MID market.
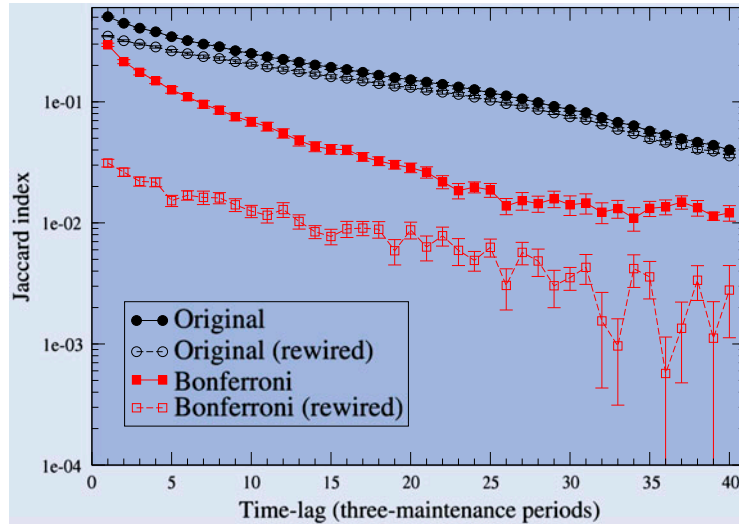

Figure 9. Average-lagged Jaccard index between the network of the 3-maintenance period t with respect to the network of the 3-maintenance period $t + L$, where L is the time-lag ranging from 1 to 40. The data shown in the figure are the average values and standard deviations (error bars) taken over all the 3-maintenance periods, for the lender-aggressor data-set. We show results relative to the original network (black) and the Bonferroni network (red). Open circles and squares refer to randomly rewired networks. The rewiring procedure preserves the node degree. These data refer to the lender-aggressor data-set. The analysis is performed on the Italian segment of the e-MID market.

distribution provides a good description of link weights obtained according to the edge swapping algorithm.

In figure 5, a scatter plot between the *z*-scores based on the 1000 rewired networks and those obtained under the hypothesis of an underlying hypergeometric distribution (z_{lb} given above) is shown for the period 24 December 2004 to 23 March 2005 (which correspond to a three maintenance period). Also this figure shows an excellent agreement between the two approaches.

4. Empirical results

A credit institution that first come to the market with a proposal to lend or borrow is called the quoter, and those who can pick a quote and exercise a loan proposal is called the aggressor. We will consider three different sets of data, depending on the type of the aggressor: the *lender-aggressor* data-set where the lender acts as aggressor in all transactions, the
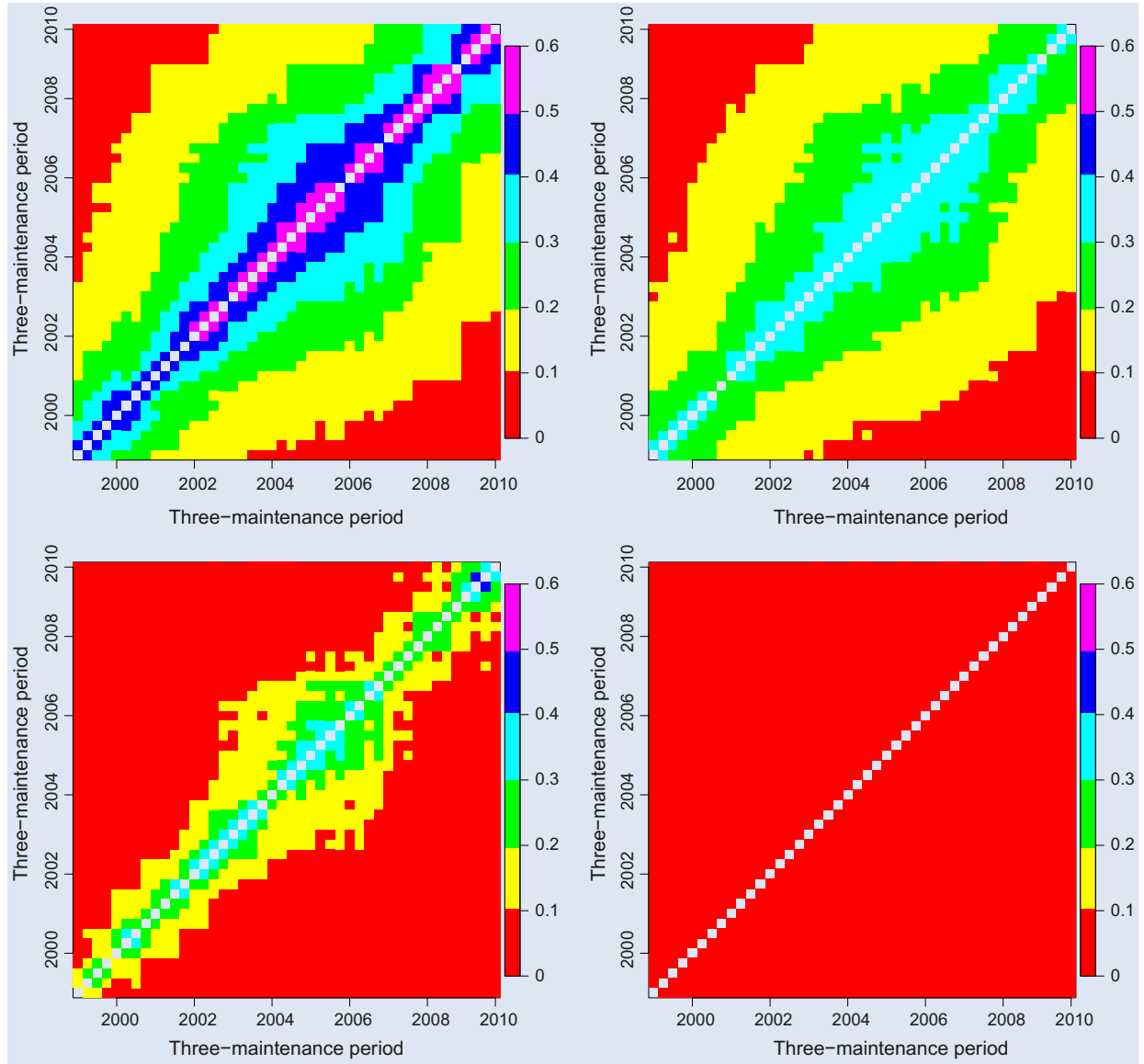
Figure 10. Contour plots of the Jaccard index between all pairs of 3-maintenance periods. The left panels show the Jaccard index between original networks (top) and between Bonferroni networks (bottom). The right panels provide the same measure for rewired networks. The rewiring procedure preserves the degree of each node. These data refer to the lender-aggressor data-set. We consider only the overexpressed links of the Bonferroni network. The analysis is performed on the Italian segment of the e-MID market.

borrower-aggressor data-set where the borrower acts as aggressor in all transactions and the *whole* data-set where we include all transactions independently of the side of the lenders and borrowers. The role of the aggressor in the market is asymmetric. Specifically, we observe that on average $70.6 \pm 5.9\%$ of transactions are of the lender aggressor type. When we consider volume, we observe that on average $72.4 \pm 7.5\%$ of transactions is of the lender aggressor type.

4.1. The e-MID market

In table 1, we show the number of transactions (second column) and the exchanged volume (seventh column) between banks in each of the 44 3-maintenance periods for the whole data-set. The decline in the number of transactions, shown in the top-left panel of figure 2 is evident. In the third, fourth, fifth and sixth column, we show the percentage of transactions amongst Italian banks, between Italian and foreign banks, between foreign and Italian banks and amongst foreign banks, respectively. The

same information is reported in the eighth to eleventh column for the exchanged volume. It is worth noticing that most of the transactions are amongst Italian banks. A significant part of the exchanged volume is relative to transactions occurring amongst Italian banks or amongst foreign banks. In fact, the volume exchanged between Italian and foreign banks is on average 12.9% of the whole exchanged volume. Based on these observation, we will consider hereafter only the Italian segment of the e-MID market.

4.2. The Italian segment of the e-MID Market

In figure 6, we show the dynamics of the fraction of directed links where the lender acts only as aggressor (red line), the borrower acts by aggressor only (blue line) and both the lender and borrower act as aggressors (black line). The figure shows that there exists a dynamics in the type of pair relationships in the network. In particular, the directed links where the lender
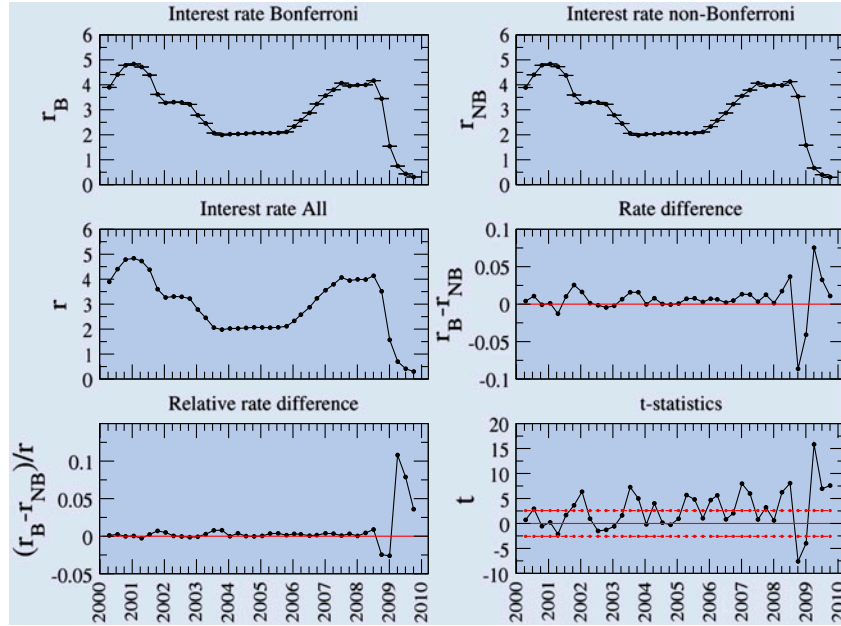
Figure 11. For each 3-maintenance period, the figure shows the average interest rates r_B observed in the transactions between banks connected in the Bonferroni network (top-left panel) and the average interest rates r_{NB} relative to transactions which are not included in the Bonferroni network (top-right panel). Averages are computed over the transactions occurred in a 3-maintenance period. The error bars are the corresponding standard deviations. The middle-left panel shows the average interest rate r of all transactions occurred in a 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $r_B - r_{NB}$, while the bottom-left panel shows the relative difference $(r_B - r_{NB})/r$. In the bottom-right panel of the figure, it is shown the unequal sample sizes and unequal variance t-statistics for r_B and r_{NB}. These data refer to the lender-aggressor data-set. We consider only the overexpressed links of the Bonferroni network. The analysis is performed on the Italian segment of the e-MID market.

Figure 12. For each 3-maintenance period, the figure shows the average interest rates r_B observed in the transactions between banks connected on the Bonferroni network (top-left panel) and the average interest rates r_{NB} relative to transactions which are not included in the Bonferroni network (top-right panel). Averages are computed over the transactions occurred in a 3-maintenance period. The error bars are the corresponding standard deviations. The middle-left panel shows the average interest rate r of all transactions occurred in each 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $r_B - r_{NB}$, while the bottom-left panel shows the relative difference $(r_B - r_{NB})/r$. In the bottom-right panel of the figure, it is shown the unequal sample sizes and unequal variance t-statistics for r_B and r_{NB}. These data refer to the borrower-aggressor data-set. We consider only the overexpressed links of the Bonferroni network. The analysis is performed on the Italian segment of the e-MID market.
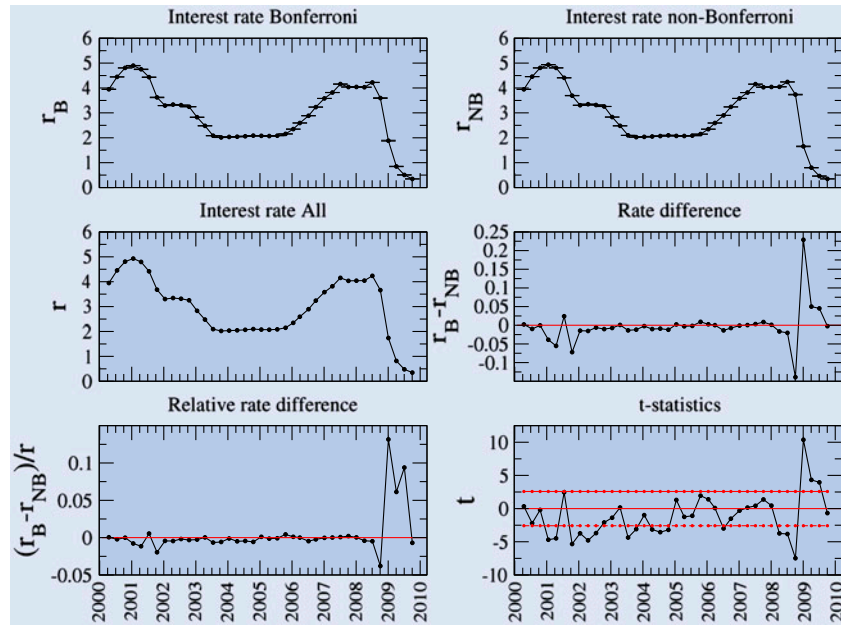
Table 1. Summary statistics of the number of transactions (second column) and of the exchanged volume (seventh column) between banks in each of the 44 3-maintenance periods in the case of the lender-aggressor data-set. In the third, fourth, fifth and sixth column, we show the percentage of transactions amongst Italian banks, between Italian and foreign banks, between foreign and Italian banks and amongst foreign banks, respectively. The same information is reported in the eighth to eleventh column for the exchanged volume.

3-period	Trades ALL number	Trades IT-IT %	Trades IT-F %	Trades F-IT %	Trades F-F %	Volume ALL MEur	Volume IT-IT %	Volume IT-F %	Volume F-IT %	Volume F-F %
01	28 391	100	0	0	0	434 834.38	100	0	0	0
02	44 837	100	0	0	0	633 682.49	100	0	0	0
03	43 092	100	0	0	0	592 231.16	100	0	0	0
04	39 038	99	1	0	0	598 948.43	98	1	0	0
05	39 071	100	0	0	0	677 539.28	99	1	0	0
06	41 234	99	1	0	0	848 558.77	96	2	1	1
07	36 286	98	1	1	0	729 806.35	93	1	5	0
08	34 361	97	1	2	0	793 091.44	87	2	9	2
09	32 647	97	1	2	0	724 048.63	83	2	10	6
10	32 650	97	2	1	1	757 021.86	79	8	4	9
11	30 539	96	2	1	0	740 106.92	80	8	4	7
12	34 265	94	4	1	1	826 929.53	74	15	2	9
13	34 414	93	5	1	1	866 837.01	69	18	2	11
14	35 143	95	4	1	1	861 243.7	74	13	5	8
15	33 187	95	4	1	1	886 143.52	73	13	4	10
16	32 931	92	6	1	1	935 672.46	66	19	2	13
17	31 135	92	3	2	2	925 252.34	67	10	6	17
18	30 642	90	6	2	2	961 911.7	62	18	4	15
19	30 880	91	4	2	3	1.0214383×10^6	59	14	5	22
20	28 143	91	4	2	3	1.00813273×10^6	53	17	5	25
21	27 158	91	4	2	3	977063.7	59	11	5	25
22	28 430	87	4	3	6	1.27433051×10^6	49	11	6	35
23	28 613	88	3	4	5	1.23952144×10^6	50	9	8	33
24	28 828	86	3	5	6	1.29990264×10^6	49	9	12	30
25	27 379	85	3	5	7	1.26115423×10^6	47	8	9	36
26	27 210	85	6	2	6	1.22476130×10^6	50	13	5	32
27	27 295	86	3	4	7	1.26343835×10^6	49	6	9	36
28	27 368	85	3	5	7	1.22835554×10^6	45	7	9	39
29	21 339	84	2	6	8	1.1188875×10^6	41	5	11	43
30	28 086	83	4	6	8	1.45326181×10^6	42	7	11	41
31	23 848	84	3	5	8	1.309460780×10^6	44	9	7	40
32	27 222	83	4	4	9	1.43526061×10^6	37	11	7	44
33	25 180	80	4	5	11	1.56399547×10^6	31	9	8	52
34	23 143	81	5	4	10	1.38353729×10^6	33	11	5	51
35	26 264	84	2	7	7	1.32691567×10^6	39	6	13	43
36	24 800	87	3	3	7	1.0552792×10^6	46	8	7	39
37	23 103	89	1	4	6	848 769.64	48	4	10	38
38	21 943	90	2	3	5	799 662.5	52	7	8	33
39	19 801	88	1	4	6	815 732.04	50	3	11	36
40	18 472	94	1	1	4	595 271	66	2	4	28
41	14 886	96	1	1	3	385 821.9	66	1	2	31
42	14 942	96	1	1	3	388 590.39	70	2	4	24
43	13 201	98	0	0	1	238 483.3	90	0	2	7
44	12 234	98	0	1	1	269 520.4	80	1	5	14

acts only as aggressor starts increasing from the beginning of 2005. A specular decline in the ratio of directed links where the borrower acts only as aggressor and when the banks act as both borrower and lender-aggressor is also observed. This implies that, starting from 2005, the ratio of borrowers who only act as quoters increases in turn and the markets become more and more borrower driven. This effect may be interpreted as an early indication that liquidity becomes scarcer in the interbank market, forcing borrowers to come forward and compete for loans long before the crises fully develops.

4.2.1. Preferential links. Further insights about the system can be obtained by considering the statistically validated networks discussed in section 3.1. Specifically, we will consider here only the statistically validated networks obtained by using the Bonferroni correction for multiple comparison. In figure 7, we show the Bonferroni network for the 3-maintenance period 40 (10 September 2008/9 December 2008), restricted to the italian banks only and the lender-aggressor data-set. The nodes in the networks are coloured according to the node membership to the partitions detected by using the Radatool
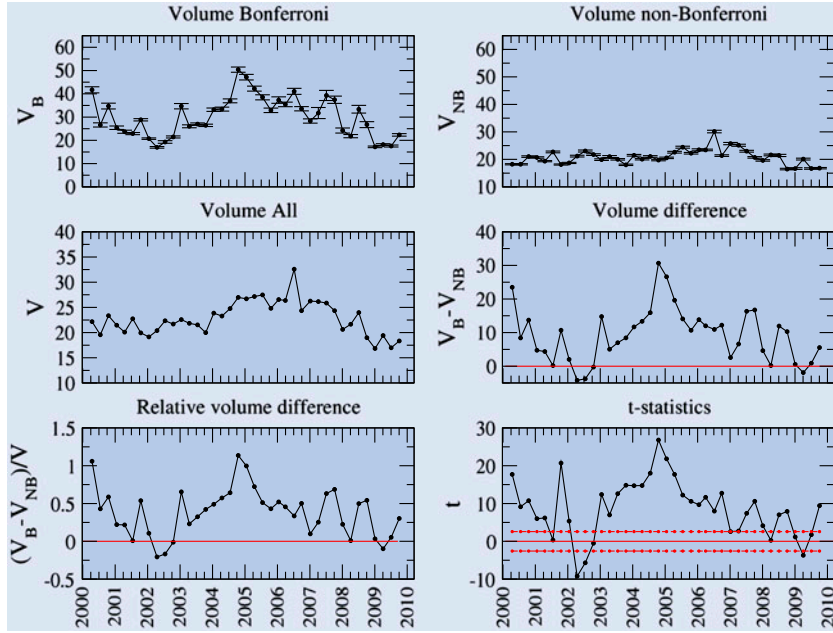
Figure 13. For each 3-maintenance period, the figure shows the average volume per transaction V_B exchanged in the transactions between banks connected in the Bonferroni network (top-left panel) and the average volume per transaction V_{NB} relative to transactions not included in the Bonferroni network (top-right panel). Averages are computed over the transactions occurred in a 3-maintenance period. The error bars are the corresponding standard deviations. The middle-left panel shows the average volume per transaction V of all transactions occurred in a 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $V_B - V_{NB}$, while the bottom-left panel shows the relative difference $(V_B - V_{NB})/V$. In the bottom-right panel of the figure, it is shown the unequal sample sizes and unequal variance t-statistics for V_B and V_{NB}. These data refer to the lender-aggressor data-set. We consider only the overexpressed links of the Bonferroni network. The analysis is performed on the Italian segment of the e-MID market.

algorithm (unweighted option) (Duch and Arenas 2005). The links are coloured according to the fact that they are over-expressed (blue) or underexpressed (red). In this network, it is worth noticing that underexpressed links usually connect nodes belonging to different partitions. Such feature is indeed observed in the networks relative to all the 44 3-maintenance periods. The role of the underexpressed links is currently under investigations and will be presented in a forthcoming paper.

For the lender-aggressor data-set, in the top-left panel of figure 8, we show the number of links observed in the original network and in the top-right panel the number of overexpressed (red) and underexpressed (blue) links in the Bonferroni net-work. The panels show a decline of the number of links in the original networks and an approximately constant profile in the Bonferroni network. In the bottom panel of figure 8, we show the ratio between the number of overexpressed links observed in the Bonferroni and in the original network. When consider-ing the ratio one clearly sees that the fraction of overexpressed links increases after the Lehman's bankruptcy (dashed line in the bottom panel of the figure). This is also observed in the ratio of underexpressed links (not shown). Similar results occur for the borrower-aggressor data-set. The Bonferroni network therefore reveals a change in the structure of the relationships between banks after the Lehman's crisis.

4.2.2. Stability of links. We have investigated the stability of the network over the 44 3-maintenance periods in order to see whether the crisis events introduce major changes in the network. Specifically, we have computed the Jaccard index between the networks, as defined in section 3.2.1. In figure 9,

we show the (lagged) Jaccard index between the network of the 3-maintenance period t with respect to the network of the 3-maintenance period $t + L$, where L is a lag ranging from 1 to 40. The data shown in the figure are the averages and stan-dard deviations (error bars) taken over all the 3-maintenance periods, for the lender-aggressor data-set. We show the re-sults relative to the original network (black circle) and the Bonferroni network (red squares).

Surprisingly, the (lagged) Jaccard index between the orig-inal networks is higher than the Jaccard index between the Bonferroni networks. Naively, one would have expected the opposite, on the ground that links non-explained by a random null hypothesis are supposed to be more persistent. Our work-ing hypothesis is that such difference is due to the different role of the degree distribution of the two networks. In fact, as we will see, the topology of the original network is largely determined by the degree distribution while this is not true in the Bonferroni network.

In order to test our working hypothesis, let us now compare the Jaccard index of the original and Bonferroni networks with that measured for surrogate networks generated by using a rewiring procedure that maintains the degree of each node. The rewiring procedure is different from the one used in section 3.2, where the strength and not the degree of each node was pre-served. In figure 9, we also show the rewired original network (black open circle) and the rewired Bonferroni network (red open squares). We observe that the value of the Jaccard index at different lags is very similar in the original network and the reshuffled ones suggesting that the reshuffling procedure does not manage to destroy correlations imposed by the degree pre-serving contrains. Degree heterogeneity in the system appears
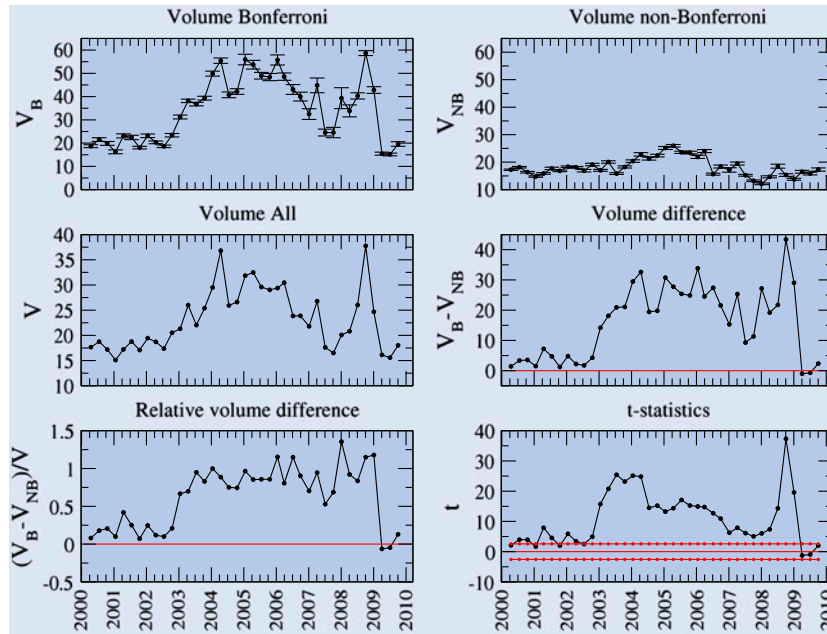
Figure 14.   For each 3-maintenance period, the figure shows the average volume per transaction V_B exchanged in the transactions between banks which are nodes of the Bonferroni network (top-left panel) and the average volume per transaction V_{NB} relative to transactions which are not included in the Bonferroni network (top-right panel). Averages are computed over the transactions occurred in a 3-maintenance period. The error bars are the corresponding standard deviations. The middle-left panel shows the average volume per transaction V of all transactions occurred in a 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $V_B - V_{NB}$, while the bottom-left panel shows the relative difference $(V_B - V_{NB})/V$. In the bottom-right panel of the figure, it is shown the unequal sample sizes and unequal variance t-statistics for V_B and V_{NB}. These data refer to the borrower-aggressor data-set. We consider only the overexpressed links of the Bonferroni network. The analysis is performed on the Italian segment of the e-MID market.

to be the main responsible for the stability of a (still relatively small) ratio of links in the real networks. In the Bonferroni network, instead, the randomization succeeds in redistributing the links, reducing the Jaccard index at lag one by an order of magnitude. This suggests that stable links in the Bonferroni network are not imposed by the constrains of the system but result from banks deliberate choices of counterparties.

This effect appears even more clearly in figure 10, where we show the contour plots of the Jaccard index between all pairs of 3-maintenance periods. The left panels show the **Jaccard index** in the original network (top) and in the Bonferroni network (bottom). The right panels provide the same measure for rewired networks. One can see that both the original and Bonferroni networks display larger stability in the central periods while stability fades away towards the end of the investigated periods, i.e. after the Lehman events. In general, the stability of the empirical networks is larger than that of the rewired networks. However, it is evident that the rewiring does not completely destroy the network structure of the original network, as it is the case for the Bonferroni network. This shows again that the degree heterogeneity has a crucial role in determining the stability of the empirical interbank networks.

4.2.3.   Volumes and interest rates of the banks belonging to the Bonferroni networks.   There are several possible reasons that explain why banks tend to have preferential relationships amongst them. One first feature we want to investigate is related to the interest rates associated to each transaction. For the lender-aggressor data-set and for each 3-maintenance period in our data-set, in figure 13, we show the average interest

rates r_B observed in the transactions between banks connected in the Bonferroni network (top-left panel) and the average interest rates r_{NB} relative to transactions which are not included in the Bonferroni network (left panel). Averages are computed over the transactions occurred in each 3-maintenance period. Error bars are the corresponding standard deviations.† The middle-left panel shows the average interest rate r of all transactions occurred in a 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $r_B - r_{NB}$, while the bottom-left panel shows the relative difference $(r_B - r_{NB})/r$. The middle-right panel indicates that there is no peculiarity in the interest rates of transactions occurring within or outside the Bonferroni network. Such difference starts to be significant at the end of 2008, i.e. after the Lehman crisis events. This is confirmed by looking at the bottom-left panel. In other words, after the Lehman bankruptcy event the cost of the credit for the preferential links increased. A more quantitative analysis of this effect is shown in the bottom-right panel of the figure where we show the t-statistics computed for the case of samples with unequal size and different variance. The two horizontal lines above and below zero represent the threshold corresponding to a 1% p-value, that is, any point over the top line or below the bottom line indicates that the null hypothesis of equal mean should be rejected at a 1% significance level.

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†In each 3-maintenance period, the analysis of interest rates and volumes only includes banks that traded both through links belonging to the Bonferroni network and through links not included in the Bonferroni network.

Similar outcomes are also observed when considering the borrower-aggressor data-set, as shown in figure 12. Both figures 11 and 12 show that the crisis of Lehman, in September 2008, varied the relationship between the average interest rate, r_B, of transactions that occurred along preferential links and the interest rate r_{NB} of the other transactions. Specifically, in both figures, we observe that r_B becomes smaller than r_{NB} soon after the Lehman crisis. From an economic point of view, this observation may be explained along the following line of thinking. Seeking a large amount of borrowing in correspondence with the crisis, some banks initially exploited the usual/preferential lines of credit (transactions in the Bonferroni network), and therefore, did not experience significant variations in the interest rate. However, seeking large amounts of borrowing is likely to entail a negative perception from the other market participants, which, in turn, will lend to these banks at higher interest rates (transactions outside the Bonferroni network). After a few maintenance periods, the liquidity of some of these banks deteriorated, and they had problems to gather credit outside their preferential connections, where, however, they could borrow at higher interest rates. As a result, the average interest rate in the Bonferroni network increased. On the other hand, most of the transactions that occurred outside the Bonferroni network were among 'healthy banks', and the interest rate did not increase. So, the difference $r_B - r_{NB}$ switched to positive values.

Overall, our results also suggest that the determinants of preferential links are mainly independent of the role played by the banks in the transactions, i.e. whether a bank acts as a quoter or an aggressor. In particular, it does not appear to be the case that banks pick preferentially the counterpart that offer the best available rate. The results rather suggest that the riskier banks, who pay the highest rates, are the ones that most depend on relationship lending.

In a second investigation, we consider the volume exchanged in the transactions. For the lender-aggressor data-set and for each 3-maintenance period in our data-set, in figure 13, we show the average volumes V_B exchanged, per transaction, between banks that are connected in the Bonferroni network (top-left panel), and the average volume V_{NB} exchanged per transaction between banks that are not connected in the Bonferroni network (top-right panel). Averages are computed over the transactions occurred in a 3-maintenance period. The error bars are the corresponding standard deviations. The middle-left panel shows the exchanged volume V of all transactions occurred in a 3-maintenance period, without distinguishing whether or not these transactions are included in the Bonferroni network. The middle-right panel shows the difference $V_B - V_{NB}$, while the bottom-left panel show the relative difference $(V_B - V_{NB})/V$. The middle-right panel indicates that the difference $V_B - V_{NB}$ is almost always positive; thus, indicating that in the Bonferroni network we find transaction on average larger than those observed outside the Bonferroni network. Moreover, the bottom-left panel shows that the relative amount transacted within the Bonferroni network has slightly decreased after the crisis. Again, a more quantitative analysis of this effect is shown in the bottom-right panel of figure 13, where we show the t-stastics computed for the case when one has unequal sample sizes and unequal variances.

Analogous results are shown in figure 14 for the borrower-aggressor data-set, showing again a similar picture to the one observed in lender-aggressor data-set. It should be noticed that in this case during the period of crisis, the relative volume difference of the borrower-aggressor data-set (bottom-left panel of figure 14) remains high from August 2007 to March 2009.

Our results indicate that preferential links facilitated large transaction volumes, particularly, before the subprime crisis. After the crisis, although large sums were still traded within the Bonferroni network, possibly thanks to better information on each other's trustworthiness acquired via previous trading relationships, their volume decreased and lenders required a higher interest rate premium than before.

5. Conclusions

In this paper, we have compared the empirically observed transactions in the e-MID market with a null hypothesis assuming random trading between banks that perform a heterogeneous number of transactions as borrowers and lenders. Specifically, we have compared empirical observations with a directed weighted configuration model obtained via sequential edge local rewiring of the real data. The weights are given in terms of the number of transactions done between two linked banks in which one bank was the lender and the other one was the borrower. We have also provided an analytical probabilistic description of the null hypothesis within the theoretical framework of statistically validated networks. The result that the weighted configuration model can be efficiently described by an analytical probabilistic model motivated us to investigate the properties of statistically validated networks of the e-MID interbank market and their evolution over the investigated time period of more than 10 years.

We have detected preferential trading among banks over time, studied its dynamics over the time window 1999–2009 and characterized it in terms of the associated interest rate and transaction volume. The average transaction volume between banks linked through a preferential trading relationship was typically higher than the average transaction volume of the remaining transactions, and this observation was persistent over time, even during the crisis. Before the crisis, we did not observe a significant difference, in terms of interest rate, between transactions within and outside the statistically validated networks. However, during the crisis, the average interest rate of transactions between banks who had established preferential trading relationship resulted to be higher than the average interest rate of the remaining transactions. These results suggest that, before the crisis, preferential trading was associated with a trustworthy relationship between two banks, which allowed for larger trading volumes at no extra cost. On the other hand, during the crisis, while the trustworthy relationship persisted to some extent, and still allowed larger trading volumes for the borrower aggressor transactions, this came at a price of larger interest rates. Such an interpretation is also supported by the fact that the number of preferential trading relationships remained, besides fluctuations, constant over time, while the total number of connections among banks displayed an apparent decreasing trend.

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