Midterm 1: Takehome section S320/S520

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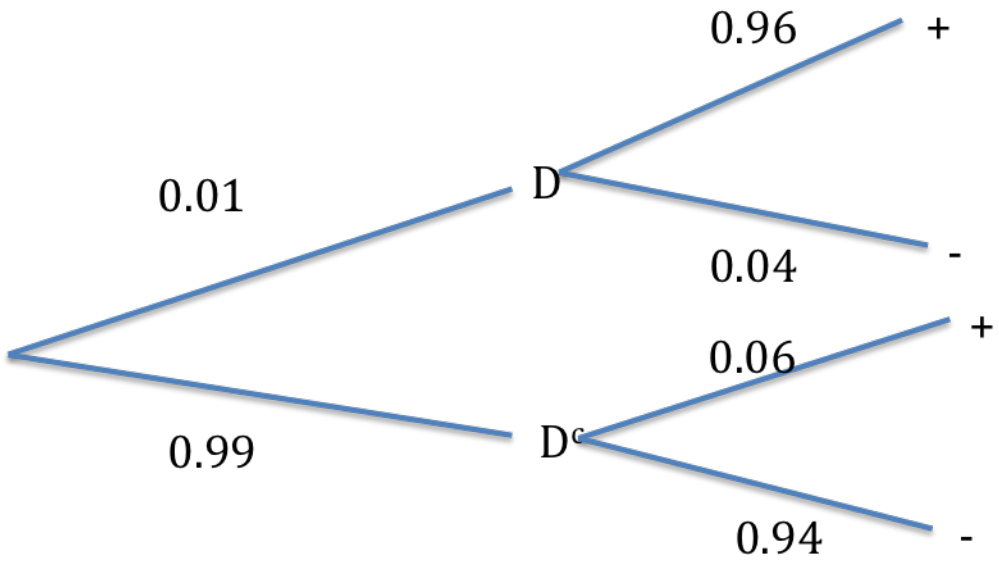
1. (3 points.) About 1 percent of men in their fifties have or will develop prostate cancer (within a certain timeframe.) Under one method for screening for prostate cancer: Out of men in their fifties that have or will develop prostate cancer, 96% test positive (the rest test negative.) Out of men in their fifties that do not have and will not develop prostate cancer, 94% test negative (the rest test positive.) I randomly select a man in his fifties who has both been screened using this method and tested positive. Based on the information above, what is the probability he has or will develop prostate cancer



Answer:

The probability that a randomly selected man who has tested positive, will develop cancer is

$$P(D|+) = \frac{P(D \cap +)}{P(+)} = \frac{P(D \cap +)}{P(D \cap +) + P(D^c \cap +)}$$

$$P(D|+) = \frac{0.01 * 0.96}{(0.01 * 0.96 + 0.99 * 0.06)} = 0.139$$

```
p = (0.01*0.96)/((0.01*0.96+0.99*0.06))
signif(p,3)
```

```
## [1] 0.139
```

2. Let X, Y , and Z be independent standard normal random variables.

a. (1 point.) Find P(1 < X < 2)

```
signif(pnorm(2)-pnorm(1),3)
```

```
## [1] 0.136
```

b. (1 point.) Find P(1 < X^2 < 2)

Answer:

```
#P (1 < X^2 < 2) = P (X^2 < 2)-P (X^2 < 1)
              #  = P (-1.414 < X < 1.414) - P (-1 < X < 1)
signif((2*pnorm(1.414))-(2*pnorm(1)),3)
```

```
## [1] 0.16
```

c. (1 point.) Find P(1 < X + Y + Z < 2)

Answer:

```
# X+Y+Z ~ Normal (meanX + meanY + meanZ,varX + varY + varZ)
#mean of X+Y+Z = 0
#variance of X+Y+Z = 3
signif(pnorm(2,0,sqrt(3))-pnorm(1,0,sqrt(3)),3)
```

```
## [1] 0.158
```

3. An American roulette wheel has 38 numbers, all equally likely. A roulette player repeatedly bets on '00'. The probability of the wheel landing on '00' is 1/38. If this happens, the player wins $35. Otherwise, the player loses $1.

a. (2 points.) The player bets on '00' for 200 spins. Let X be a random variable representing the number of times the player wins. What are the expected value and standard deviation of X?

Answer: We can say that getting '00' is the success event and getting any other number is a failure. The probability of success is 1/38 or 0.026. The probability of a failure is 37/38 or 0.974. If X is the random variable that represents the number of times the player wins. It ranges from 0 to 200. Each value of X however has a different probability given by dbinom. Then roulette(x,n,p) finds the probability for each win.

```
#P(X=0)
roulette = function(x,n,p){
    fx = dbinom(x,n,p)
  xfx = x*fx
  return(xfx)
}

#to find x^2*f(x)--To calculate Variance
```

```
rouletteX2 = function(x,n,p){
    fx = dbinom(x,n,p)
  x2fx = (x^2)*fx
  return(x2fx)
}

m = sum(roulette(0:200,200,0.026))
```

The mean value/Expected value is

```
signif(m,3)
```

```
## [1] 5.2
```

The standard deviation is

```
varx = sum(rouletteX2(0:200,200,0.026)) - m^2
signif(sqrt(varx),3)
```

```
## [1] 2.25
```

b. (1 point.) For the player to make a profit, the wheel must land on '00' on at least six of the 200 spins. What is the probability the player makes a profit?

Answer:

```
#We can write P(X>= 6) as 1-P(X<=5)
signif(1-pbinom(5,200,0.026),3)
```

```
## [1] 0.42
```

c. (2 points.) Suppose the player goes to the casino and bets on "00" 50,000 times. What is the probability the player makes a profit?

Answer: We can find the probability that the player makes a profit by finding the Expected Value of wins for 50000 spins.

```
#Profit = $35*E(X) - 1$*(50000-E(X)) where E(X) = sum(X*P(X))
#for X ranging between 0-50000
#Expected number of wins for 50000 spins
m = sum(roulette(0:50000,50000,0.026))
signif(m,3)
```

```
## [1] 1300
```

Equating profit to zero we can find the value of X for which the player begins to make profit.

```r
#X*35 = 50000-X
#36*X = 50000
X = 50000/36
ceiling(X)
```

```
## [1] 1389
```

The probability of profit is

```r
#x=seq(3000:4000)
#p = (1389*35) - (50000-1300) #all values of x >= 1389
#plot(x,p)
signif(1- pbinom(1388,50000,0.026),2)
```

```
## [1] 0.0069
```