10. Arlen is planning a dinner party at which he will be able to accommodate seven guests. From past experience, he knows that each person invited to the party will accept his invitation with probability 0.5. He also knows that each person who accepts will actually attend with probability 0.8. Suppose that Arlen invites twelve people. Assuming that they behave independently of one another, what is the probability that he will end up with more guests than he can accommodate?

14. The Association for Research and Enlightenment (ARE) in Virginia Beach, VA, offers daily demonstrations of a standard technique for testing extrasensory perception (ESP). A "sender" is seated before a box on which one of five symbols (plus, square, star, circle, wave) can be illuminated. A random mechanism selects symbols in such a way that each symbol is equally likely to be illuminated. When a symbol is illuminated, the sender concentrates on it and a "receiver" attempts to identify which symbol has been selected. The receiver indicates a symbol on the receiver's box, which sends a signal to the sender's box that cues it to select and illuminate another symbol. This process of illuminating, sending, and receiving a symbol is repeated 25 times. Each selection of a symbol to be illuminated is independent of the others. The receiver's score (for a set of 25 trials) is the number of symbols that s/he correctly identifies. For the purpose of this exercise, please suppose that ESP does not exist.

(a) How many symbols should we expect the receiver to identify correctly?

(b) The ARE considers a score of more than 7 matches to be indicative of ESP. What is the probability that the receiver will provide such an indication?

(c) The ARE provides all audience members with scoring sheets and invites them to act as receivers. Suppose that, as on August 31, 2002, there are 21 people in attendance: 1 volunteer sender, 1 volunteer receiver, and 19 additional receivers in the audience. What is the probability that at least one of the 20 receivers will attain a score indicative of ESP?

2. Suppose that X is a continuous random variable with probability density function (pdf) f defined as follows:

$$f(x) = \left\{ \begin{array}{lll} 0 & \text{if} & x < 1 \\ 2(x-1) & \text{if} & 1 \le x \le 2 \\ 0 & \text{if} & x > 2 \end{array} \right\}.$$

(a) Graph f.

(b) Verify that f is a pdf.

(c) Compute $P(1.50 < X < 1.75)$.

First since f(x)>0
f is a pdf if integral of f(x) between 1 and 2

3. Consider the function $f : \Re \to \Re$ defined by

$$f(x) = \left\{ \begin{array}{ll} 0 & x < 0 \\ cx & 0 < x < 1.5 \\ c(3-x) & 1.5 < x < 3 \\ 0 & x > 3 \end{array} \right\},$$

where c is an undetermined constant.

(a) For what value of c is f a probability density function?

(b) Suppose that a continuous random variable X has probability density function f. Compute EX. (Hint: Draw a picture of the pdf.)

(c) Compute $P(X > 2)$.

(d) Suppose that $Y \sim \text{Uniform}(0, 3)$. Which random variable has the larger variance, X or Y? (Hint: Draw a picture of the two pdfs.)

(e) Determine and graph the cumulative distribution function of X.

4. Imagine th...

5. Consider an unfair six-sided die. Let X be a discrete random variable representing the result of a roll of the die. The probability mass function of X is

$$f(x) = \begin{cases} 0.1 & x = 1 \\ 0.1 & x = 2 \\ 0.3 & x = 3 \\ 0.3 & x = 4 \\ 0.1 & x = 5 \\ 0.1 & x = 6 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $F(x)$, the cumulative distribution function of X, for all $x \in (-\infty, \infty)$.

(b) Find the expected value and the variance of X.

(c) Suppose I roll the die ten times (all independently.) Let Y be the sum of the ten die rolls. What are the expected value and the variance of Y?

6. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 2k & 0 \leq x < 3 \\ 3k & 3 \leq x < 5 \\ 0 & \text{otherwise.} \end{cases}$$

where k is a constant.

(a) Find k.

(b) Find $F(4)$, the cumulative distribution function at $x = 4$.

(c) Find the expected value of X.