Lund University

Center for Mathematical Sciences

FMAN95

Computer Vision

Home Take Exam

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Exercise 1

We have a camera $P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix}$.

a

We have $3D$ points: $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, and $X_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$. We get projections

$$x_1 \sim PX_1 = P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}.$$

Therefore $x_1 \sim \begin{pmatrix} 2, 2, 2 \end{pmatrix}$ (which can be interpreted as $\begin{pmatrix} 1, 1 \end{pmatrix}$ in $\mathbb{R}^2$).

$$x_2 \sim PX_2 = P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Therefore $x_2 \sim \begin{pmatrix} 0, 0, 1 \end{pmatrix}$ (which can be interpreted as $\begin{pmatrix} 0, 0 \end{pmatrix}$ in $\mathbb{R}^2$).

$$x_3 \sim PX_3 = P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Therefore $x_3 \sim \begin{pmatrix} 1, 2, 1 \end{pmatrix}$ (which can be interpreted as $\begin{pmatrix} 1, 2 \end{pmatrix}$ in $\mathbb{R}^2$).

Since X_4 is a vanishing point, we have $X_4 = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix}$ as the homogeneous coordinates. To get the projection we

$$x_4 \sim PX_4 = P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}.$$

Therefore $x_4 \sim \Big(-1, -2, -1\Big)$ (which can be interpreted as $\Big(1, 2\Big)$ in $\mathbb{R}^2$).

b

The camera center C of P is given by the nullspace of P which we find by solving the system.

$$\begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Since we have 3 equations and 4 unknowns Gaussian elimination gives the solution parameterization

$$\begin{cases} 2X + Z & = 0 \\ X - Y + Z + W & = 0 \\ 2X + Y + Z & = 0 \end{cases} \Leftrightarrow \begin{cases} X & = s \\ Y & = 0 \\ Z & = -2s \\ W & = s \end{cases}, \ s \in \mathbb{R}. \tag{1}$$

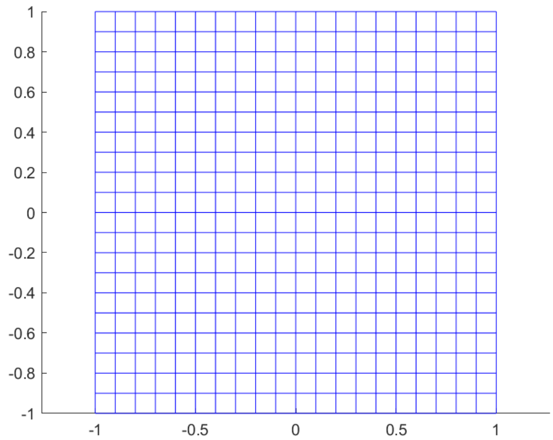
This gives the camera center $C = \Big(1, 0, -2\Big)$.
If we write the camera matrix P as $[M|p^4]$, then the (inhomogeneous) direction of the principal axis toward the front of the camera is given by $det(M)m^3$ where m^3 is the last row of M. In matlab we can use,
$v(:) = sign(det(P(:, 1:3))) * P(3, 1:3);$
$v(:) = v(:)./norm(v(:));$
taken from **plotcams.m** function from assignments. And we get $v = \Big(-0.82, -0.41, -0.41\Big)$.

Exercise 2

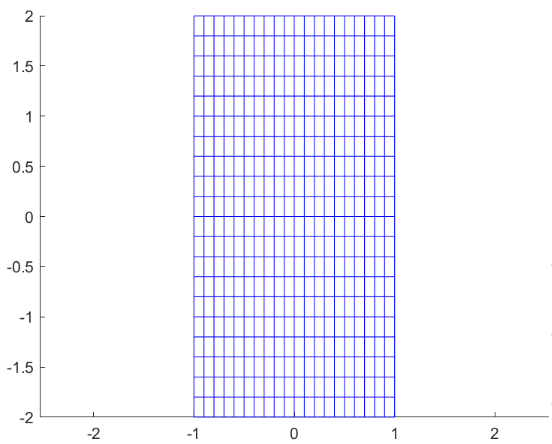
We have $H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $H_2 = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix}$.

a

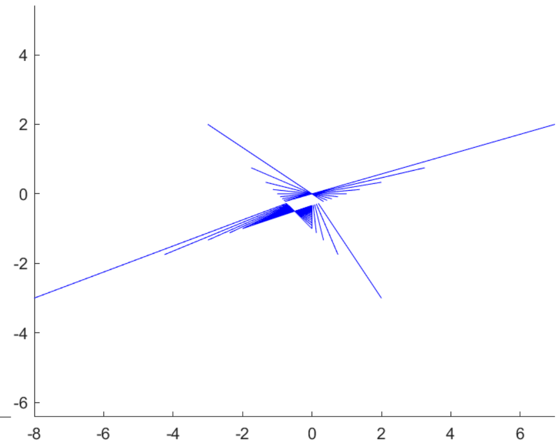
I took the data from **Computer Exercise 3** from **Assignment 1** to plot the transformations and analyze them. I got this results:



(a) Initial plot.



(b) H1 transform.



(c) H2 transform.

Figure 1: Plots for transformations

As we can see from the plots, H_1 transformation preserves angles, parallel lines are mapped to parallel lines, hence it is a **Similarity transformation**. However, H_2 only preserves lines, and is a **General projective transformation**.
Another way to classify the transformations is to look at the transformation matrices $H_1 = \begin{pmatrix} sR & t \\ 0 & 1 \end{pmatrix}$, whereas H_2 has a 0 at the last coordinate and does not have a form of any special transformations.

b

First let's transform the points with H_1 transformation. We get:

$$x_1 = H_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Which can be interpreted as a regular point $\Big(1, 0\Big)$ in $\mathbb{R}^2$.

$$x_2 = H_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Which can be interpreted as a regular point $\Big(1, 2\Big)$ in $\mathbb{R}^2$.

$$x_3 = H_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}.$$

Which can be interpreted as a regular point $\Big(2, 2\Big)$ in $\mathbb{R}^2$.
Now let's do the same with H_2 transformation. We get:

$$x_1 = H_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}.$$

Which can be interpreted as a regular point $\Big(1, 0\Big)$ in $\mathbb{R}^2$.

$$x_2 = H_2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}.$$

Which can be interpreted as a regular point $\Big(-2, -1\Big)$ in $\mathbb{R}^2$.

$$x_3 = H_2 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}.$$

Which is a vanishing point that we can interpret as a point infinitely far away in the direction $\Big(-2, -1\Big)$.

C

$$H_1 x \sim H_2 x \Leftrightarrow H_2^{-1} H_1 x \sim H_2^{-1} H_2 x = x \Leftrightarrow Hx = \lambda x, \text{where } H = H_2^{-1} H_1.$$

Hence, x is an eigenvector of H.

d

From the last task we got that for all eigenvectors of H the transformation using H_1 is the same as that of H_2. So we need to find the eigenvectors.
I did it in matlab using the function $eig()$. I got

$$V = \begin{pmatrix} 0.71 & 0.71 & 0.89 \\ 0 & 0 & 0.45 \\ -0.71 & 0.71 & 0 \end{pmatrix}.$$

Where each column represents the eigenvector of H. Which can be interpreted as $\Big(-1, 0\Big)$, $\Big(1, 0\Big)$ in $\mathbb{R}^2$. And for the last vector we get a vanishing point that we can interpret as a point infinitely far away in the direction $\Big(0.89, 0.45\Big)$.

Exercise 3

Two calibrated cameras have the essential matrix $E = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

a

Given the essential matrix E, which relates corresponding points between two images through the relation $x'^T E x = 0$, where x and x' are corresponding points in the two images.

$$x_1'^T E x_1 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = 0.$$

Hence the first $2D$ point pairs **could** be the projections of the same $3D$ point.

$$x_2'^T E x_2 = \begin{pmatrix} 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = -4.$$

Hence the second $2D$ point pairs **could not** be the projections of the same $3D$ point.

$$x_3'^T E x_3 = \begin{pmatrix} 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0.$$

Hence the third $2D$ point pairs **could** be the projections of the same $3D$ point.

b

We know that $e_2^T E = 0$ and $Ee_1 = 0$. Hence

$$\begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow e_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow e_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

c

Done in matlab:

$$W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ u_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$P_2 = \begin{bmatrix} UWV^T & u_3 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \text{ or } \begin{bmatrix} UWV^T & -u_3 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\text{or } \begin{bmatrix} UW^TV^T & u_3 \end{bmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \text{ or } \begin{bmatrix} UW^TV^T & -u_3 \end{bmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

d

Since $x_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $x_4' = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$ are a real match $\Rightarrow$ there is a $3D$ point $X = \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$ that satisfies both camera equations.

$$x_4 \sim P_1 X,\ x_4' \sim P_2 X.$$

From the first equation we get:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}.$$

Therefore we can say $X = \begin{pmatrix} 1 \\ 0 \\ 1 \\ s \end{pmatrix}$.

Now we need to see which camera satisfies the second equation.

First solution:

$$P_2X(s) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1+s \end{pmatrix} = \lambda x_4' = \lambda \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

Hence, $s = -1.5$.

To determine if a point is in front of a camera we will compute its depth with respect to that camera.

$$depth(P_2, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{sign(1) * (-0.5)}{-1.5} = 1/3.$$

$$depth(P_1, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{1}{-1.5}.$$

Hence the point $X(s)$ is **not** in front of **both** cameras.

Second solution:

$$P_2X(s) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1-s \end{pmatrix} = \lambda x_4' = \lambda \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

Hence, $s = 1.5$.

$$depth(P_2, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{sign(1) * (-0.5)}{1.5} = -1/3.$$

Hence the point $X(s)$ is **not** in front of **both** cameras.

Third solution:

$$P_2X(s) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1+s \end{pmatrix} = \lambda x_4' = \lambda \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

Hence, $s = -0.5$.

$$depth(P_2, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{sign(1) * (0.5)}{-0.5} = -1.$$

Hence the point $X(s)$ is **not** in front of **both** cameras.
Fourth solution:

$$P_2X(s) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1-s \end{pmatrix} = \lambda x_4' = \lambda \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

Hence, $s = 0.5$.

$$depth(P_2, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{sign(1) * (0.5)}{0.5} = 1.$$

$$depth(P_1, \mathbf{X}) = \frac{sign(det(A))\lambda}{||A_3||s} = \frac{1}{0.5} = 2.$$

Hence the point $X(s)$ is in front of **both** cameras.

We get that $P_2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$ is the real solution.

e

We see that the camera P_2 has been rotated around x and y-axis by 180°. and moved one unit along the negative z-axis.

Exercise 4

a

The 3×4 matrix P has 12 elements, but the scale is arbitrary and therefore it only has 11 degrees of freedom.
There are $3n$ equations (3 for each point projection), therefore we need

$$3n \geq 11 + n \Leftrightarrow n \geq 6$$

points in order for the problem to be well defined.

b

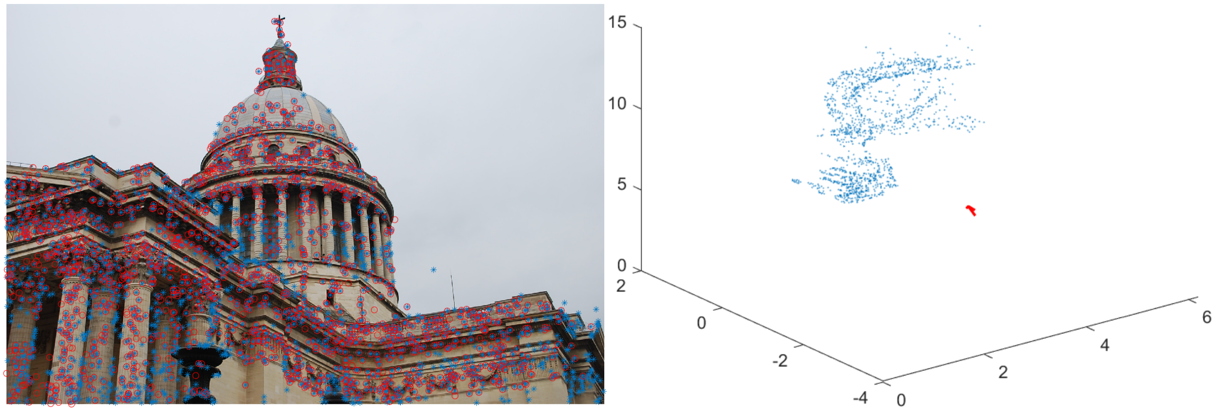
The probability of selecting an inlier set is $(1 - 0.25)^6 = 0.75^6$. Therefore the probability of failing to do so is $1 - 0.75^6$. Now suppose we sample consensus sets n times. Finding an inlier set at least ones is the complement event of failing to find an inlier set all n times.

The probablility of failing n times is $(1-0.75^6)^n$. We should therefore have

$$(1-0.75^6)^n < 1-0.9999 \Rightarrow n\log(1-0.75^6) \leq \log(1-0.9999) \Rightarrow n \geq \frac{\log(1-0.9999)}{\log(1-0.75^6)} \approx 46.9.$$

Therefore $n = 47$ works.

c



(a) Reprojections. (b) 3D plot.

Figure 2: Plots for Exercise 4c

Exercise 5

We have three matrices

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = [I\ 0],\ P_2 = \begin{pmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix},\ P_3 = \begin{pmatrix} -1 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}.$$

a

The fundamental matrix F_{12} will be (Canonical cameras):

$$F_{12} = [t]_\times A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix}.$$

The fundamental matrix F_{13} will be (Canonical cameras):

$$F_{13} = [t]_\times A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & -2 \\ -1 & 0 & -1 \\ 0 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & -1 \end{pmatrix}.$$

The fundamental matrix F_{23} will be (General cameras):
In this case the first camera is not $[I\ 0]$, that's why the formula will be different. We first need to compute the camera center of the first camera. We do so by computing its nullspace.

$$\begin{pmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\begin{cases} x + 4z + w & = 0 \\ y + 2z & = 0 \\ z + w & = 0 \end{cases} \Leftrightarrow \begin{cases} x & = 3s \\ y & = 2s \\ z & = -s \\ w & = s \end{cases}, s \in \mathbb{R}. \tag{2}$$

We get $C_2 = \begin{pmatrix} 3 \\ 2 \\ -1 \\ 1 \end{pmatrix}$.

Then we need to compute the second epipole e_{32}:

$$e_{32} = P_3 C_2 = \begin{pmatrix} -1 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}.$$

We also need the pseudo-inverse of P_2, which I computed in matlab.

$$F_{23} = [e_{32}]_\times P_3 P_2^+ = \begin{pmatrix} 0 & 4 & -2 \\ -4 & 0 & -2 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.4 & -0.4 & -0.6 \\ -0.4 & 0.73 & 0.27 \\ 0.2 & 0.13 & -0.13 \\ -0.2 & -0.13 & 1.13 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 4 & -2 \\ -4 & 0 & -2 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1.4 & 0.73 & 2.27 \\ -0.6 & 0.27 & 0.73 \\ 0.8 & -1.47 & -0.53 \end{pmatrix} = \begin{pmatrix} -4 & 4 & 4 \\ 4 & 0 & -8 \\ -4 & 2 & 6 \end{pmatrix}.$$

b

Calculated in matlab.
$e23'F12e13 = 0$

$e32'F13e12 = 1.1102e - 16$
$e31'F23e21 = -8.8818e - 16.$

C

Geometrically, the fundamental matrix F maps a point in one image to its corresponding epipolar line in the other image. The epipole e represents the projection of the camera center onto the other camera's image plane.
When we take the dot product of the epipole and the fundamental matrix corresponding to the same cameras, the result is always zero. This represents the geometric constraint that the epipole lies on the corresponding epipolar line.

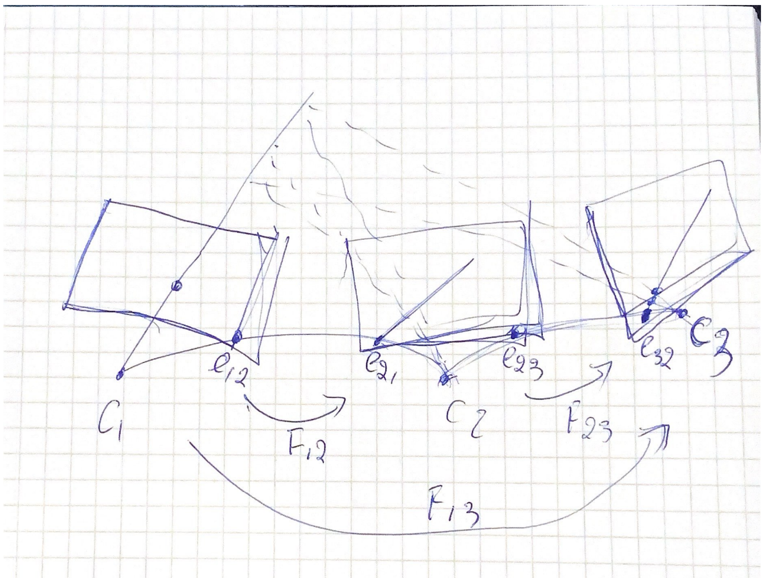


Figure 3: Epipolar Geometry - Hand Sketch

Exercise 6

$$H_{23} = \begin{pmatrix} 0 & 0 & 2 \\ -2 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix}, x_3 \sim H_{23}x_2.$$

a

We got that

$$x_2^T F_{12} x_1 = 0,\ x_3^T F_{13} x_1 = 0,\ x_3 \sim H_{23} x_2.$$

$$\begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix} \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix} \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix} = -3x_2 + 4y_2 + 3z_2 = 0.$$

$$x_3 \sim H_{23}x_2 = \begin{pmatrix} 0 & 0 & 2 \\ -2 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 2z_2 \\ -2x_2 + y_2 + z_2 \\ -2x_2 \end{pmatrix}.$$

$$\begin{pmatrix} 2z_2 & -2x_2 + y_2 + z_2 & -2x_2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2z_2 & -2x_2 + y_2 + z_2 & -2x_2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} =$$

$$= -4x_2 + 2y_2 + 2z_2 + 4x_2 = 2y_2 + 2z_2 = 0.$$

So far we've got

$$\begin{cases} -3x_2 + 4y_2 + 3z_2 & = 0 \\ 2y_2 + 2z_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} x_2 & = -\frac{s}{3} \\ y_2 & = -s \\ z_2 & = s \end{cases}, s \in \mathbb{R}. \tag{3}$$

Therefore $x_2 = \begin{pmatrix} -1/3 \\ -1 \end{pmatrix}$. In which case $x_3 \sim \begin{pmatrix} 2z_2 \\ -2x_2 + y_2 + z_2 \\ -2x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2/3 \\ 2/3 \end{pmatrix}$. Dividing by last coordinate we get $x_3 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

b

Done in matlab.

$$H_{12} = \begin{pmatrix} -0.73 & 0.09 & 0.67 \\ 0.03 & 0.03 & 0.03 \\ -0.04 & -0.04 & 1.64 \end{pmatrix}.$$

C

I first combined the second and third image with H_{23} homography and later combined the result with the first image. I computed the homography the same way I did for Assignment 4. I computed matching points for first image and the resulting image with vlfeat and used RANSAC to get inliers. After getting the bestH I stitched the images together. I did not use the camera matrices at all, and I couldn't make it work to look better.

Figure 4: Arnold poster