The solutions should be handed in no later than 48 hours after downloading the exam.
Data for the exam can be downloaded from Canvas.
It is not permitted to get help from other persons. Credits can be given for partially solved problems. Similar to the assignments, you should hand in a pdf-file (handwritten solutions should be scanned and inserted in the pdf) together with Matlab code. Write your solutions neatly, explain your calculations and specify what Matlab-scripts you have used. The pdf and your m-files should also be submitted through the canvas page.

1. *Consider the camera*

$$P = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix}.$$

a) *Compute the projections of the 3D scene points* $(1,0,0)$, $(0,1,0)$ *and* $(0,0,1)$.
Also compute the projection of a point that is infinitely far away in the direction $(1,0,-3)$. *If possible, your answers should be in* $\mathbb{R}^2$. *(0.4)*

b) *What is the camera center and principal axis of this camera?* *(0.6)*

2. *Consider the homographies* $\mathbb{P}^2 \to \mathbb{P}^2$ *given by*

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } H_2 = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ -1 & 2 & 0 \end{pmatrix}.$$

a) *What type of projective transformations are these?*
(Euclidean, similarity, affine or projective. Motivate your answer.) *(0.3)*

b) *Transform the points* $(1,0)$, $(1,1)$ *and* $(2,1)$ *using* H_1 *and* H_2. *Which of the transformed points can be interpreted as regular points in* $\mathbb{R}^2$*? What is the interpretation of the other transformed points?* *(0.3)*

c) *Show that if*

$$H_1\boldsymbol{x} \sim H_2\boldsymbol{x},$$

then

$$H\boldsymbol{x} = \lambda\boldsymbol{x},$$

where $H = H_2^{-1}H_1$. *That is,* $\boldsymbol{x}$ *is an eigenvector of* H. *(0.1)*

d) *Find all points for which the transformation using* H_1 *is the same as that of* H_2. *(0.3)*

3. *Suppose that two calibrated cameras have the essential matrix*

$$E = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

a) *Determine which of the three following 2D point pairs could be the projections of the same 3D point?* *(0.3)*

	1	*2*	*3*
Image 1:	$\boldsymbol{x}_1 = (1, 1)$	$\boldsymbol{x}_2 = (0, 2)$	$\boldsymbol{x}_3 = (2, -2)$
Image 2:	$\boldsymbol{x}'_1 = (-1, -1)$	$\boldsymbol{x}'_2 = (2, 1)$	$\boldsymbol{x}'_3 = (-1, 1)$

b) *Compute the two epipoles.* *(0.2)*

c) *The matrix E has the singular value decomposition*

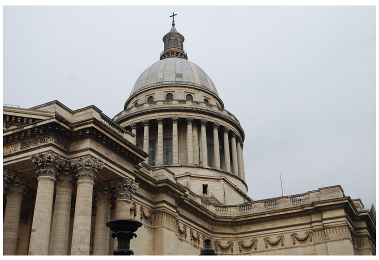
$$E = \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix}}_{S} \underbrace{\begin{pmatrix} & 1 & \\ -1 & & \\ & & 1 \end{pmatrix}}_{V^T}.$$

Use this to compute the four possible camera pairs from E. *(0.2)*

d) *The point pair $\boldsymbol{x}_4 = (1, 0)$ and $\boldsymbol{x}'_4 = (-2, 0)$ is a real match. Use this to select the real solution from* ***d****).* *(0.2)*

e) *What kind of motion is the camera undergoing?* *(0.1)*

4. *Below we show an image (*`ex4.jpg`*) of a building together with a 3D model. In this exercise we will estimate the camera matrix P from 2D-3D point correspondences. Unfortunately the matches contain some outliers, so RANSAC will be necessary.*



a) *How many degrees of freedom does an uncalibrated pinhole camera have?*
How many point matches do you need to be able to compute the camera matrix? *(0.2)*

b) *Suppose that the number of mismatched points is roughly 25%. If you use a minimal set of correspondences, how many RANSAC iterations do you need to find an outlier-free set of point correspondences with 99.99% probability?* *(0.3)*

c) *Write a function that computes a camera matrix from a minimal number of correspondences using DLT. Use RANSAC with this function to determine the camera matrix from the matches in* `ex4.mat`*. A point is considered to be an inlier if its projection is less than 5 pixels from the corresponding image point. Plot the 3D model and the camera center and principal axis in a 3D plot. (Don't forget to make sure that the principal axis point towards the visible 3D points)* *(0.5)*

5. *Consider the three cameras*

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad P_3 = \begin{pmatrix} -1 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix},$$

a) *Let F_{ij} denote the fundamental matrix from camera P_i to P_j. Compute the three possible fundamental matrices F_{12}, F_{13} and F_{23}.* *(0.6)*

b) *Let $\mathbf{e}_{12}$ and $\mathbf{e}_{21}$ denote the right and left epipoles of F_{12}, i.e. $F_{12}\mathbf{e}_{12} = F_{12}^T\mathbf{e}_{21} = 0$. Compute the epipoles and verify that the following equations are satisfied:* *(0.2)*

$$\mathbf{e}_{23}^T F_{12} \mathbf{e}_{13} = \mathbf{e}_{32}^T F_{13} \mathbf{e}_{12} = \mathbf{e}_{31}^T F_{23} \mathbf{e}_{21} = 0$$

c) *Give a geometric interpretation of why these equations hold for any three cameras. (0.2)*

6. *Consider the three cameras from exercise* **5.** *There exist a 3D plane that yields the homography*

$$H_{23} = \begin{pmatrix} 0 & 0 & 2 \\ -2 & 1 & 1 \\ -2 & 0 & 0 \end{pmatrix}$$

between the second and third image, i.e. $\boldsymbol{x}_3 \sim H_{23}\boldsymbol{x}_2$.

a) *One point on the plane projects onto $\boldsymbol{x}_1 = (1, 1)$ in the first image. Determine the corresponding points $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ in the second and third image.* *(0.4)*

b) *The 3D plane also induces a homography from the first to the second image, i.e. H_{12}. Determine this homography.* *(0.2)*

c) *The file* `ex6.mat` *contains the camera matrices P_1, P_2 and P_3 for the three images in Figure 1, as well has the homography H_{23} mapping from the second to third image. Compute the homography H_{12} and use it to overlay the second image onto the first, completing the Arnold poster.* *(0.4)*

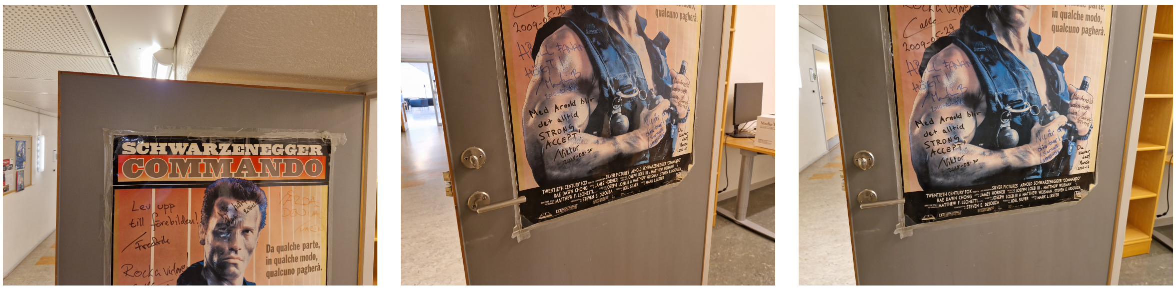


Figure 1: The images `ex6_1.jpg`, `ex6_2.jpg` and `ex6_3.jpg`.

Good Luck!