Q1 - 24 June - Shift 1

The remainder when 3^{2022} is divided by 5 is

(A) 1
(B) 2
(C) 3
(D) 4

Space for your notes:

Q2 - 24 June - Shift 2

The remainder on dividing $1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$ by 50 is ______.

Space for your notes:

Q3 - 25 June - Shift 1

Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3\cdot2C_2 + 5\cdot3C_3 + ...$ upto 10 terms

$$= \frac{\alpha\times2^{11}}{2^{\beta}-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....\text{upto 10 terms}\right)$$

then the value of $\alpha + \beta$ is equal to

Space for your notes:

Q4 - 25 June - Shift 2

The coefficient of x^{101} in the expression

$$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+.....x^{500},$$

$x > 0$, is

(A) ${}^{501}C_{101}(5)^{399}$
(B) ${}^{501}C_{101}(5)^{400}$
(C) ${}^{501}C_{100}(5)^{400}$
(D) ${}^{500}C_{101}(5)^{399}$

Space for your notes:

Q5 - 25 June - Shift 2

If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot 3^9$, then β is equal to ______

Space for your notes:

Q6 - 26 June - Shift 1

The remainder when $(2021)^{2023}$ is divided by 7 is :

(A) 1 (B) 2 (C) 5 (D) 6

Space for your notes:

Q7 - 26 June - Shift 2

If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+\ldots+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$, m and n are coprime, then m + n is equal to _____ .

Space for your notes:

Q8 - 27 June - Shift 1

If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}+\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k l$, where l, k $\in$ N and l is co-prime to 5, then k is equal to ______.

Space for your notes:

Q9 - 27 June - Shift 2

If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is :

Space for your notes:

Q10 - 28 June - Shift 1

If

$$\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)},$$

Where $\alpha \in R$, then the value of 16α is equal to

(A) 1411
(B) 1320
(C) 1615
(D) 1855

Space for your notes:

Q11 - 28 June - Shift 1

The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 \cdot \ell$, where ℓ is an odd integer, is ________.

Space for your notes:

Q12 - 28 June - Shift 2

The term independent of x in the expression of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is

(A) $\frac{7}{40}$
(B) $\frac{33}{200}$
(C) $\frac{39}{200}$
(D) $\frac{11}{50}$

Space for your notes:

Q13 - 29 June - Shift 1

If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to :

(A) 6 (B) 7

(C) 8 (D) 9

Space for your notes:

Q14 - 29 June - Shift 2

Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\,\alpha$ and $6^n - 5n - 1 = 25\,\beta$, then $\alpha - \beta$ is equal to:

(A) $1 + {}^nC_2\,(8-5) + {}^nC_3\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-1} - 5^{n-1})$

(B) $1 + {}^nC_3\,(8-5) + {}^nC_4\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-2} - 5^{n-2})$

(C) ${}^nC_3\,(8-5) + {}^nC_4\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-2} - 5^{n-2})$

(D) ${}^nC_4\,(8-5) + {}^nC_5\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-3} - 5^{n-3})$

Space for your notes:

Q15 - 29 June - Shift 2

Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to__.

Space for your notes:

Answer Key

Q1 (D)	**Q2** (4)	**Q3** (286)	**Q4** (A)
Q5 (83)	**Q6** (C)	**Q7** (102)	**Q8** (5)
Q9 (57)	**Q10** (A)	**Q11** (2)	**Q12** (B)
Q13 (D)	**Q14** (C)	**Q15** (5)	

Q1 (D)

$3^{2022} = 9^{1011} = (10-1)^{1011} = 10\,m - 1 = 10\,m - 5 + 4$

$= 5\,(2m-1) + 4$ (m is integer)

Remainder = 4

Q2 (4)

$$\frac{1.\left(3^{2022}-1\right)}{2} = \frac{9^{1011}-1}{2}$$

$$= \frac{(10-1)^{1011}-1}{2}$$

$$= \frac{100\lambda + 10110 - 1 - 1}{2}$$

$$= 50\lambda + \frac{10108}{2}$$

$$= 50\lambda + 5054$$

$$= 50\lambda + 50 \times 101 + 4$$

$$\text{Rem }(50) = 4.$$

Q3 (286)

$(1 + x)^{10} = C_0 + C_1x + C_2x^2 + ... + C_{10}x^{10}$

Differentiating

$10(1 + x)^9 = C_1 + 2C_2x + 3C_3x^2 + ... + 10C_{10}x^9$

replace $x \to x^2$

$10(1+x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 +....+ 10C_{10}x^{18}$

$10\cdot x(1+x^2)^9 = C_1x+2C_2x^3+ 3C_3x^5+....+ 10C_{10}x^{19}$

Differentiating

$10\left((1+x^2)^9\cdot 1 + x\cdot 9(1+x^2)^8 2x\right)$

$= C_1x+2\,C_2\cdot 3x^3+ 3\cdot 5\cdot C_3x^4+....+ 10\cdot 19C_{10}x^{18}$

putting $x = 1$

$10(2^9+18\cdot 2^8)$

$= C_1 + 3\cdot 2\cdot C_2 + 5\cdot 3\cdot C_3 +....+ 19\cdot 10\cdot C_{10}$

$C_1 + 3\cdot 2\cdot C_2 + ... + 19\cdot 10\cdot C_{10}$

$= 10 \cdot 2^9 \cdot 10 = 100 \cdot 2^9$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \underset{\substack{\uparrow \\ 10^{th}\text{ term}}}{\frac{C_9}{11}} + \underset{\substack{\uparrow \\ 11^{th}\text{ term}}}{\frac{C_{10}}{11}} = \frac{2^{11}-1}{11}$$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_9}{11} = \frac{2^{11}-2}{11}$$

Now, $100\cdot 2^9 = \frac{\alpha\cdot 2^{11}}{2^\beta - 1}\left(\frac{2^{11}-2}{11}\right)$

Eqn. of form $y = k\,(2^x - 1)$.

It has infinite solutions even if we take $x, y \in N$.

Q4 (A)

$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+....+x^{500}$

$=\dfrac{(5+x)^{501}-x^{501}}{(5+x)-x}=\dfrac{(5+x)^{501}-x^{501}}{5}$

$\Rightarrow$ coefficient x^{101} in given expression

$=\dfrac{{}^{501}C_{101}5^{400}}{5}={}^{501}C_{101}5^{399}$

Q5 (83)

$T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\dfrac{3}{x}\right)^r$

$={}^{10}C_r2^{10-r}3^rx^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get $\beta = 83$

Q6 (C)

$(2021)^{2023}=(7\lambda-2)^{2023}$

$={}^{2023}C_0(7A)^{2023}-\ldots\ldots{}^{2023}C_{2023}2^{2023}$

$=7t-2^{2023}$

$\therefore\ -2^{2023}=-2\times2^{2022}$

$=-2\times\left(2^3\right)^{674}$

$=-2\left(1+7\mu\right)^{674}$

$=-\left(7\alpha+2\right)$

$\Rightarrow$ remainder $=-2$ or $+5$

Q7 (102)

$$^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots\ldots {}^{59}C_{19} + {}^{60}C_{20}$$

$$\left(\frac{1}{41}+1\right){}^{41}C_1 + {}^{42}C_2 + \ldots\ldots$$

$$\left[\frac{42}{41}\left(\frac{2}{42}\right)+1\right]{}^{42}C_2 + {}^{43}C_3 + \ldots\ldots$$

$$\left(\frac{2}{41}+1\right){}^{42}C_2 + {}^{43}C_3 + \ldots\ldots$$

$$\left(\frac{43}{41}\times\frac{3}{43}+1\right){}^{43}C_3 + {}^{44}C_4 + \ldots\ldots$$

$$\frac{3+41}{41}.\,{}^{43}C_3 + \ldots\ldots$$

Similarly :

$$\frac{20+41}{41}$$

$$\Rightarrow m = 61 \; ; \; n = 41$$

$$m + n = 102$$

Q8 (5)

$$\left(\frac{\sqrt{x}}{5^{1/4}}+\frac{\sqrt{5}}{x^{1/3}}\right)^{60}$$

$$T_{r+1}={}^{60}C_r\left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r}\left(\frac{5^{1/2}}{x^{1/3}}\right)r$$

$$={}^{60}C_r 5\frac{3r-60}{4}.x\frac{180-5r}{6}$$

$$\frac{180-5r}{6}=10\Rightarrow r=24$$

$$\text{Coeff. of } x^{10}={}^{60}C_{24}5^3=\frac{\lfloor 60}{\lfloor 24 \lfloor 36}5^3$$

$$\text{Powers of 5 in} = {}^{60}C_{24}.5^3=\frac{5^{14}}{5^4\times 5^8}\times 5^3=5^5$$

Q9 (57)

coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n}\rightarrow {}^7C_0$	1
$x^{6n-5}\rightarrow 2\cdot{}^7C_1$	$1+14$
$x^{5n-10}\rightarrow 2^2\cdot{}^7C_2$	$1+14+84$
$x^{4n-15}\rightarrow 2^3\cdot{}^7C_3$	$1+14+84+280$
$x^{3n-20}\rightarrow 2^4\cdot{}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25}\rightarrow 2^5\cdot{}^7C_5$	

$3n-20\geq 0 \cap 2n-25<0\cap n\in I$

$\therefore \quad 7\leq n\leq 12$

Sum $=7+8+9+10+11+12=57$

Q10 (A)

$$\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$$

$$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + \ldots + {}^{31}C_{31} \cdot {}^{31}C_{30}$$

$$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + \ldots + {}^{31}C_{30} \cdot {}^{31}C_0$$

$$= {}^{62}C_{30} \cdot$$

Similarly

$$\sum_{R=1}^{30} \left({}^{30}C_R \cdot {}^{30}C_{R-1} \right) = {}^{60}C_{29}$$

$${}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!32!} - \frac{60!}{29!31!}$$

$$= \frac{60!}{29!\,31!} \left\{ \frac{62 \cdot 61}{30 \cdot 32} - 1 \right\}$$

$$= \frac{60!}{30!31!} \left(\frac{2822}{32} \right)$$

$$\therefore\ 16\alpha = 16 \times \frac{2822}{32} = 1411$$

Q11 (2)

$$\left(2x^3 + \frac{3}{x^k} \right)^{12}$$

$$t_{r+1} = {}^{12}C_r \left(2x^3\right)^r \left(\frac{3}{x^k} \right)^{12-r}$$

$$x^{3r-(12-r)k} \rightarrow \text{constant}$$

$$\therefore 3r - 12k + rk = 0$$

$$\Rightarrow k = \frac{3r}{12-r}$$

$\therefore$ possible values of r are 3,6,8,9,10 and corresponding values of k are 1,3,6,9,15

Now ${}^{12}C_r = 220, 924, 495, 220, 66$

$\therefore$ possible values of k for which we will get 2^8 are

3, 6

Q12 (B)

$$\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$$

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

$$^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$$

General term is $^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}$

Now, term independent of x

$1\times$ coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$-1\times$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ +

$3\times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

for coefficient of x^0 $\quad 33-5r=0$ not possible

for coefficient of x^{-2} $\quad 33-5r=-2$

$35=5r\Rightarrow r=7$

for coefficient of x^{-3} $\quad 33-5r=-3$

$36=5r$ not possible

So term independent of x is

$$(-1)\ ^{11}C_7\left(\frac{5}{2}\right)^4\left(-\frac{1}{5}\right)^7=\frac{33}{200}$$

Q13 (D)

General term

$$T_{r+1} = \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3}(3)^{r_1}(-2)^{r_2}(5)^{r_3}(x)^{3r_1+2r_2-5r_3}$$

$3r_1 + 2r_2 - 5r_3 = 0$ ...(1)

$r_1 + r_2 + r_3 = 10$ ...(2)

from equation (1) and (2)

$r_1 + 2(10 - r_3) - 5r_3 = 0$

$r_1 + 20 = 7r_3$

$(r_1, r_2, r_3) = (1, 6, 3)$

constant term $= \frac{\lfloor 10}{\lfloor 1 \lfloor 6 \lfloor 3}(3)^1(-2)^6(5)^3$

$= 2^9.3^2.5^4.7^1$

$l = 9$

Q14 (C)

$$\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + \ldots$$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + \ldots.$

option (3) will be the answer.

Q15 (5)

$$T_{r+1} = (-1)^r.{}^{15}C_r.2^{15-r}\ x^{\frac{15-2r}{5}}$$

$m = {}^{15}C_{10}\, 2^5$

$n = -1$

so $mn^2 = {}^{15}C_5\, 2^5$