Q1 - 24 January - Shift 1

$$\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+\ldots+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$$ is equal to

(1) n^2+n

(2) n

(3) $\frac{n(n+1)}{2}$

(4) n^2

Space for your notes:

Q2 - 24 January - Shift 2

The set of all values of a for which $\operatorname{Lim}_{x\to a}([x-5]-[2x+2])=0$, where $[\propto]$ denotes the greater integer less than or equal to $\propto$ is equal to

(1) $(-7.5,-6.5)$ (2) $(-7.5,-6.5]$
(3) $[-7.5,-6.5]$ (4) $[-7.5,-6.5)$

Space for your notes:

Q3 - 29 January - Shift 1

Let $x=2$ be a root of the equation $x^2+px+q=0$

and $f(x)=\begin{cases}\frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4}, & x\neq 2p\\ 0, & x=2p\end{cases}$

Then $\lim_{x\to 2p^+}[f(x)]$

where [.] denotes greatest integer function, is

(1) 2 (2) 1
(3) 0 (4) –1

Space for your notes:

Q4 - 30 January - Shift 2

Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases}$,

$g(x)=\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x\neq -1\\ 1, & x=-1\end{cases}$ and $h(x)=2[x]-f(x)$,

where [x] is the greatest integer $\leq$ x. Then the value of $\lim\limits_{x\to 1} g(h(x-1))$ is

(1) 1
(2) sin(1)
(3) –1
(4) 0

Space for your notes:

Q5 - 31 January - Shift 2

Hence $y\in\left(-\infty,\frac{-21}{4}\right]\cup[0,\infty)$

$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

(1) is equal to 9
(2) is equal to 27
(3) does not exist
(4) is equal to $\frac{27}{2}$

Space for your notes:

Answer Key

(As per Official NTA Key released on 2 Feb)

Q1 (2) **Q2** (1) **Q3** (3) **Q4** (1)

Q5 (2)

Q1 (2)

$$\lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+\ldots\ldots+n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$$

$$=\lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t}+\ldots\ldots+1\right)^{\sin^2 t}$$

$$=n$$

Q2 (1)

$$\lim_{x \to a}([x-5]-[2x+2])=0$$

$$\lim_{x \to a}([x]-5-[2x]-2)=0$$

$$\lim_{x \to a}([x]-[2x])=7$$

$[a]-[2a]=7$

$a \in I, \quad a=-7$

$a \notin I, \quad a=I+f$

Now, $[a]-[2a]=7$

$-I-[2f]=7$

Case-I: $f \in \left(0, \frac{1}{2}\right)$

$2f \in (0, 1)$

$-I=7$

$I=-7 \Rightarrow a \in (-7, -6.5)$

Case-II: $f \in \left(\frac{1}{2}, 1\right)$

$2f \in (1, 2)$

$-I-1=7$

$I=-8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

Q3 (3)

$$\lim_{x\to 2p^+}\left(\frac{1-\cos\left(x^2-4px+q^2+8q+16\right)}{\left(x^2-4px+q^2+8q+16\right)^2}\right)\left(\frac{\left(x^2-4px+q^2+8q+16\right)^2}{(x-2p)^2}\right)$$

$$\lim_{h\to 0}\frac{1}{2}\left(\frac{(2p+h)^2-4p(2p+h)+q^2+82+16}{h^2}\right)^2=\frac{1}{2}$$

Using L'Hospital's

$$\lim_{x\to 2p^+}[f(x)]=0$$

Q4 (1)

$$\text{LHL}=\lim_{k\to 0}g\left(h(-k)\right) \quad ,k>0$$

$$=\lim_{k\to 0}g(-2+1) \quad \because f(x)=-1\ \forall\ x<0$$

$$=g(-1)=1$$

$$\text{RHL}=\lim_{k\to 0}g\left(h(k)\right) \quad ,k>0$$

$$=\lim_{k\to 0}g(-1) \quad ,\because f(x)=1,\ \forall\ x>0$$

$$=1$$

Q5 (2)

$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

$$\lim_{x\to\infty} x^3\times\left\{\frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right\}}\right\}$$

$$=\frac{\left(2\sqrt{3}\right)^6+0}{2^6+0}=3^3=(27)$$