Tutorial - 4

(1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3, b = 2$ $\quad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ according to master's theorem:
$T(n) = \theta(n^2)$

(2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

$\therefore$ according to masters theorem $T(n) = \theta(n^2 \log n)$

(3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

According to masters theorem $T(n) = \theta(2^n)$

(4) $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$\therefore$ Master's theorem is not applicable as a function

(5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$ $\quad f(n) = n$

$n^{\log_b a} = n^{\log_2 16} = n^2$, $f(n) < n^2$

$T(n) = \theta(n^2)$

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(6) $T(n) = 2T(n/2) + n\log n$

$a=2, b=2, f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) > n$

Acc to masters $T(n) = \Theta(n\log n)$

(8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2, b=4, f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

Acc to masters Method $T(n) = \Theta(n^{0.51})$

(9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

as $a<1$ $\therefore$ Master's Method not applicable

(10) $T(n) = 16T(n/4) + n!$

$a=16, b=4$ $f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According to masters,

$T(n) = \Theta(n!)$

(11) $T(n) = 4T(n/2) + \log n$

$a=4, b=2$ $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to master's $T(n) = \Theta(n^2)$

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(12) $T(n) = \text{Sqrt}(n) + n/2 + \log n$

$\therefore$ Master's Not applicable as a is not a constant

(13) $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ acc to master's method theorem,

$T(n) = O(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, b = 3, f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a = 4, b = 2, f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

According to master's method, $T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

Acc to master's method, $T(n) = \Theta(n\log n)$

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(17)

$$T(n) = 3T(n/3) + (n/2)$$

$a=3, b=3, f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\theta(n) = \theta(n/2)$

$T(n) = \theta(n \log n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a=6, b=3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$\therefore$ acc to master's method $T(n) = \theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$a=4, b=2, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

According to masters theorem $T(n) = \theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Master's theorem is not applicable as $f(n)$ is not increasing function.

(21) $T(n) = 7T(n/3) + n^2$

$a=7, b=3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2 \Rightarrow T(n) = \theta(n^2)$ (by master's method)

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem isn't applicable since regularity condition is isolated in case 3.

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