PERFORMANCE

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Teacher In-Charge

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Date :

Examiner

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Practical No. 1

TOPIC : Limits and continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to a} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\dfrac{\cos 5x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4) $\lim_{x \to \infty} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

5) Examine the continuity of the following function

i) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 < x \le \pi/2 \\ \dfrac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

ii) $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x \le 9 \end{cases} \quad \text{at } x = 3 \text{ \& } x = 6$$

(6) Find value of K so that the function f(x) is continuous at the indicated point.

i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ = K, & x = 0 \end{cases}$ at $x = b$

ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} \\ = K \end{cases}$ at $x = 0$

iii) $f(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x}, & x \neq \pi/3 \\ = K, & x = \pi/3 \end{cases}$ at $x = \pi/3$

(7) Discuss the continuity of the following function which of these function have removable discontinuity? Redefine function so as to remove the discontinuity?

i) $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x}, & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}, & x \neq 0 \\ = \pi/60, & x = 0 \end{cases}$ at $x = 0$

⑧ If $f(x) = \frac{(e^{x^2} - \cos x)}{x^2}$

for $x \neq 0$ is continuous at $x = 0$ find $f(0)$.

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⑨ If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$

is continuous at $x = \pi/2$ find $f(\pi/2)$

~~Sol^n: If li~~

Solutions :

① $\lim\limits_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$= \lim\limits_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$.

$= \lim\limits_{x \to a} \frac{(a+2x-3x)\sqrt{3a+x} + 2\sqrt{x}}{3a+x-4x \sqrt{a+2x} + 3\sqrt{3x}}$

$= \lim\limits_{x \to a} \frac{(a-x)}{(3a-3x)} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim\limits_{x \to a} \frac{(a-x)\sqrt{3a+x} + 2\sqrt{x}}{3(a-x)\sqrt{a+2x} + \sqrt{3x}}$

$= \lim\limits_{x \to a} \frac{1}{3} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \frac{\sqrt{3a+a} + 2\sqrt{a}}{3\sqrt{a+2a} + \sqrt{3a}}$

$= \frac{\sqrt{4a} + 2\sqrt{a}}{3\sqrt{3a} + \sqrt{3a}}$

$= \frac{2\sqrt{a} + 2\sqrt{a}}{3 \times 2\sqrt{3a}} = \frac{2\sqrt{a}}{3\sqrt{3a}} = \frac{2}{3\sqrt{3}}$

(2) $\lim_{y \to 0} \frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}$

$= \lim_{y \to 0} \frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \propto \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}$

$= \lim_{y \to 0} \frac{(a+y-a)}{(y\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y \to 0} \frac{y}{y(\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y \to 0} \frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$= \lim_{y \to 0} \frac{1}{\sqrt{a}(2\sqrt{a})}$

$= \frac{1}{2a}$

(3) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

$x - \pi/6 = h \quad x = h + \pi/6 \quad h \to 0$

$= \lim_{h \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}(h+\pi/6)}{\pi - 6(h+\pi/6)}$

$= \lim_{x \to 0} \frac{(\cos h \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6) - \sqrt{3}(\sin h \cos \pi/6 + \cos h \sin \pi/6)}{-6h}$

$= \lim_{h \to 0} \frac{\left[\left(\cos h \frac{\sqrt{3}}{2}\right) - \left(\sin h \frac{1}{2}\right)\right] - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{2}\right)}{-6h}$

$= \lim_{h \to 0} \frac{\left(\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2}\right) - \left(\sin \frac{3h}{2} + \cos \frac{\sqrt{3}}{2} h\right)}{-6h}$

$$= \lim_{h\to 0} \frac{\frac{-\sin 4h}{2}}{-6h}$$

$$= \lim_{h\to 0} \frac{\sin 2h}{6h}$$

$$= \frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

4) $$\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$= \lim_{x\to\infty} \frac{(x^2+5-x^2+3)\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{(x^2+3-x^2-1)\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}$$

$$= \lim_{x\to\infty} \frac{8\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{2\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}$$

$$= \lim_{x\to\infty} \frac{4\left(\sqrt{x^2+3}\right) + \left(\sqrt{x^2+1}\right)}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$= \lim_{x\to\infty} \frac{4\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

$$= 4$$

(5) $\because f(\pi/2) = \frac{\sin(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$

1) $f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ define

2) $\lim_{x\to \pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

put $x - \pi/2 = h$

$\therefore x = \pi/2 + h$

as $x \to \pi/2$ $h \to 0^+$

$$= \lim_{h\to 0^+} \frac{\cos(\pi/2+h)}{\pi - 2(\pi/2+h)}$$

$$= \lim_{h\to 0^+} \frac{-\sin h}{\pi - \pi - 2h}$$

$$= \lim_{h\to 0^+} -\frac{\sin h}{-2h} = 1/2$$

$$\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x \to \pi/2^-} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}} = \lim_{x \to \pi/2^-} \frac{2 \sin x \cos x}{\sqrt{2}\ \sin x}$$

$$= \frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x = 0$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x = \pi/2$

5) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \leq x \leq 6$
$= \frac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

at $x = 3$ & $x = 6$

at $x = 3$

$$f(3) = \frac{x^2-9}{x-3} = 0$$

f at $x = 3$ define

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}$$

$f(x) = x+3 = 3+3 = 6$

f is define at $x = 3$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$$

= LHL = RHL

f is continuous at $x = 3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}$

$\lim_{x\to 6^+} = \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} = \frac{(x-3)(x+3)}{(x+3)} = \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

LHL $\neq$ RHL

function is not continuous

i) 6) $f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & x<0 \\ = K & x=0 \end{array}\right\}$ at $x=0$

Sol^n: f is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$2\lim_{x\to 0} \frac{2\sin^2 x}{x^2} = K$ $\therefore 2\lim_{x\to 0} \left(\frac{\sin_2^2 x}{x}\right)^2 = K$

$2(2)^2 = K$

$K = 8$

(7) $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ & x = 0 \end{cases}$ at $x=0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 \frac{3}{2}x}{x\tan x}$$

$$= \frac{2\sin^2 \frac{3}{2}x \times x^2 / x}{x^2} \cdot \frac{\tan x}{x^2} \times x^2$$

$$= 2 \lim_{x\to 0} \left(\frac{3}{2}\right)^2 / 1$$

$$= 2 \times \frac{9}{4} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = 9/2 \quad 9 = f(0)$

$\therefore f$ is not ds at $x=0$

Redifine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9/2 & x=0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

7) If $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$ } at $x=0$

$= \pi/6$ $x = 0$

at $x = 0$

$\lim_{x \to 0} (e^{3x}-1) \sin\left(\frac{\pi x}{180}\right) / x^2$

$= 3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin \pi x/180}{x}$

$3 \log e \ \pi/180 = \pi/60 = f(0)$

f is continuous at $x = 0$

(8) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$= \dfrac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= (e^{x^2} - 1) + (1 - \cos x) / x^2$

$\dfrac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} 2 \dfrac{\sin^2 x/2}{x^2}$

$= \log e + 2 \left(\dfrac{\sin x/2}{x}\right)^2$

Multiply with 2 on Numerator & Denominat

$= 1 + 2 \times 1/4 = 3/2 = f(0)$

(9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1 - \sin x}}{\cos 2x}$ $x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$

$= \dfrac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$

$= \dfrac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$

$$= \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times\sqrt{2}\,2} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

6 ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= k$ $x = 0$ $\Big\}$ at $x=0$

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

using

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

& $\cot^2 x = 1/\tan^2 x$

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

we know, that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$= e$

$\therefore k = e$

PRACTICAL No. 2

Topic : Derivative

1) Show that the following function defined from R to R are defferentiate

1) cot x 2) cosec x 3) sec x

2) If $f(x) = 4x+1 \quad x \le 2$
$= x^2+5 \quad x > 0$ at $x=2$ then
find f is differentiable or not.

3) If $f(x) = 4x+7 \quad x < 3$
$= x^2+3x+ \quad x \ge 3$ at $x=3$ then
find f is differentiable or not

4) If $f(x) = 8x-5 \quad x \le 2$
$= 3x^2-4x+7 \quad x < 2$ at $x=2$ then
find f is differentiable or not ?

Sol^n :

1) 1) cot x

$f(x) = \cot x$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \tan a}$$

put $x - a = h$, $x = a + h$ as $x \to a$, $h \to 0$

$$D \cdot f(x) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula : $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B) \; (1 + \tan A . \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} - \frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$$Df(a) = -\cos^2 a$$

$\therefore f$ is differentiable $\forall a \in \mathbb{R}$

d) cosec x

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$ $\quad x = a+h$ as $x \to a$ $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula :
sin $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-\frac{1}{2} \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec(x) - \sec(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$ put $x-a=h$, $x = a+h$

as $x\to a$ $h\to 0$

$$Df(x) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-h-a}{2}\right)}{\cos a \cos(a+h) \times -h/2}$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

Practical No : 3.

Topic : Application derivative

Q.1 Find the internal in which function is increasing of decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$.

Q.2 Find the intervals in which fun is concave upwards and concave downwards.

i) $y = 3x^2 - 2x^3$

Let $f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1 - 2x)$

$f(x)$ is concave upwards iff $f''(x) > 0$

$6(1-2x) > 0$

$\cancel{1-2x > 0}$

$x > +1/2$

$\therefore x \in (-\infty, +1/2)$

$f(x)$ is concave downward iff $f''(x) < 0$

$$6(1-2x) < 0$$
$$x < +\tfrac{1}{2}$$
$$x < \tfrac{1}{2}$$

$$\therefore x \in \left(\tfrac{1}{2}, \infty\right)$$

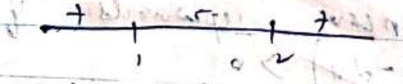
ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7.$

Let,

$$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$$
$$f'(x) = 4x^3 - 18x^2 + 24x + 5$$
$$f''(x) = 12x^2 - 36x + 24$$
$$= 12(x^2 - 3x + 2)$$
$$= 12[x^2 - 2x - x + 2]$$
$$= 12[x(x-2) - 1(x-2)]$$
$$= 12(x-2)(x-1)$$

$f(x)$ is concave upwards iff $f''(x) > 0$

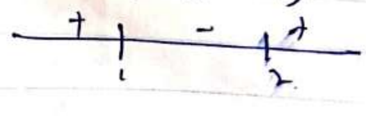
$$12(x-2)(x-1) > 0$$
$$x > 2, 1$$



$$\therefore x \in (-\infty, 1) \cup (2, \infty)$$

$f(x)$ is concave downwards iff

$$f''(x) < 0$$
$$12(x-2)(x-1) < 0$$



$x \in (1,2)$

iii) $y = x^3 - 27x + 5$

Let $f(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f(x)$ is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f(x)$ is concave downwards iff $f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (-\infty, 0)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

Let $f(x) = y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f(x)$ is concave upward iff $f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x > \frac{18}{12}$

$x > 3/2$

$\therefore x \in (3/2, \infty)$

$f(x)$ is concave downwards iff $f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < 18/12$

$x < 3/2$

$\therefore x \in (-\infty, 3/2)$

Practical : 4

Topic : Application of Derivative & Newton's Method.

1) Find maximum & minimum values of following functions.

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider $f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4} = 2 + \frac{96}{16} = 2 + 6 = 8 > 0$

$\therefore$ f has ~~max~~ minimum value at $x = 2$.

$\therefore f(x) = 2^2 + \frac{16}{2^2} = 4 + \frac{16}{4} = 4 + 4 = 8$

$f''(-2) = 2 + \frac{96}{(-2)^3} = 2 + \frac{96}{16} = 2 + 6 = 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$ & $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider $f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\frac{15x^4}{15x^2} = 1$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(-1) = -30 \times -1 + 60(-1)^3$

$= 30 - 60 = -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$f(-1) = 3-5(-1)^3+3(-1)^5$
$= 3-5\times-1-3$
$= 3+5-3 = 5$

$\therefore f$ has maximum value 5 at $x=-1$
& has the minimum value 1 at $x=1$

iv) $f(x) = 2x^3-3x^2-12x+1$ in $[-2,3]$
$f'(x) = 6x^2-6x-12$
consider $f'(x) = 0$
$6x^2-6x-12=0$
$6(x^2-x-2)=0$
$x(x-2)+1(x-2)=0$
$(x-2)(x+1)=0$
$x=2$ or $x=-1$

$\therefore f''(x) = 12x-6$
$f''(2) = 12\times2-6 = 18 > 0$
$\therefore f$ has minimum value of $x=2$
$\therefore f(2) = 2(2)^3-3(2)^2-12\times2+1$
$= 16-12-24+1$
$= -19 < 0$

$f''(-1) = 12\times-1-6 = -18 < 0$
f has maximum value at $x=-1$
$f(-1) = 2(-1)^3-3(-1)^2-12\times-1+1$
$= 8$

$\therefore f$ has maximum value 8 at $x=-1$ &
f has minimum value -19 at $x=2$

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iii) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2,]$

$f'(x) = 3x^2 - 6x$

consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6 \times 0 - 6 = -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$f(0) = 0^3 - 3(0)^2 + 1$

$= 1$

f has minimum value at $x = 2$

$f(2) = 2^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= -3$

f has maximum value 1 at $x = 0$

f has minimum value -3 at $x = 2$

Q.4 Find the root of following eq^n by Newton method (Take 4 interation only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

$f'(x) = 3x^2 - 6x - 55$

By Newton Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0 + \frac{9.5}{55}$$

$$x_1 = 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$

$$= 0.0050 - 0.0879 - 9.416 + 9.5$$

$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$

$$= -55.9393$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$= 0.1712$

The root of the eq^n is 0.1712

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = \cancel{2^3}\ 8 - 8 - 9 = -9$

$f(3) = 27 - 12 - 9 = 27 - 21 = 6$

Let $x_0 = 3$ be the initial approximation

By Newtons Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$

$$= 0.598$$

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$$f'(x_1) = 3(2.7392)^2 - 4$$
$$= 22.5096 - 4$$
$$= 18.5096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$
$$= 19.8386 - 10.8284 - 9$$
$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 21.9851 - 4$$
$$= 17.9851$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$
$$= 19.7158 - 10.806 - 9$$
$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$
$$= 21.8942 - 4$$
$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$
$$= 2.7065$$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 1.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= -2.2$

Let $x_0 = 2$ be initial approx

By Newton's law

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$= 1.5777$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 6.6755$

$f'(x) = 3(1.5777)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

Ali
07/01/2022

Integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

Substitute $x+1 = t$

$dx = dt$ where $t = x+1$

$= \int \frac{1}{\sqrt{t^2-4}} \, dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln\left(|x+\sqrt{x^2-a^2}|\right)$

$= \ln\left(|t+\sqrt{t^2-4}|\right) + c$

$$= \ln\left(|x+1+\sqrt{(x+1)^2-4}|\right)+c$$

$$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)+c$$

ii) $\int (4e^{3x}+1)\,dx$

$$= \int 4e^{3x}\,dx + \int 1\,dx$$

$$= \cancel{4\int e}\ \frac{4e^{3x}}{3} + x + c$$

$$= \frac{4e^{3x}}{3} + x + c$$

iii) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$$

$$= \int 2x^2\,dx - \int 3\sin x\,dx + \int 5x^{1/2}\,dx$$

$$= \frac{2x^3 + 10x^{3/2}}{3} + 3\cos x + c$$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \frac{2\sqrt[3]{x}}{7} + 2\sqrt[3]{x} + 8\sqrt{x} + c$

(v) $\int t^7 \times \sin(2t^4)\,dt$

put $x = t^4$

$dx = 4t^3\,dt$

$dt = \frac{dx}{x+3}$

$= \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$= \frac{1}{4}\left(x\int \sin 2x - \int\left[\int \sin 2x \frac{-dx}{dx}\right)\right)$

$= \frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + c$

Resubstituting $x = t^4$

$I = -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16}\sin(2t^4) + c$

vii) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3}\, dx = dt$

$I = \frac{1}{-2} \int \frac{-2}{x^3} \cdot \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2} \int \sin t \, dt$

$= -\frac{1}{2} \left[-\cos t \right] + c$

$= \frac{1}{2} \cos t + c$

Resubstituting $t = \frac{1}{x^2}$

$\therefore I = \frac{1}{2} \cos \frac{1}{x^2} + c$

$$\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$$

$$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$$

put $t = \sin x$

$dt = \cos x\, dx$

$$I = \int \frac{dt}{\sqrt[3]{t^2}}$$

$$I = \int \frac{dt}{t^{2/3}}$$

$$= 3t^{1/3} + c$$

$$= 3(\sin x)^{1/3} + c$$

$$= 3\sqrt[3]{\sin x} + c$$

1x) $\int e^{\cos^2 x} \cdot \sin 2x \, dx$

$I = \int e^{\cos^2 x} \cdot \sin 2x \, dx$

Let $\cos^2 x = t$

$-2 \cos x \sin x \, dx = dt$

$-2 \sin 2x \, dx = dt$

$I = \int - \sin 2x \, e^{\cos^2 x} \, dx$

$= -e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

(x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

Let $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x)\, dx = dt$

$3(x^2 - 2x)\, dx = \frac{dt}{3}$

$I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$

Resubstituting $t = x^3 - 3x^2 + 1$

$I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

AK
03/01/2020

Practical 6.

(1) $l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$x = t - \sin t \quad \therefore \frac{dx}{dt} = 1 - \cos t$

$y = t - \cos t \quad \therefore 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \sqrt{4\sin^2 t/2}$

$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad \therefore \sin^2 t/_2 = \frac{1-\cos t}{2}$

$= \int_0^{2\pi} 2\sin t/_2\, dt$

$= \left(-4\cos(t/_2)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

(1)

$x = \frac{1}{6} y^3 + \frac{1}{2} y \qquad y = (1, 2)$

$\frac{dx}{dy} = y^2/2 = \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$

$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}} \, dy$

$= \int_1^2 \frac{y^4+1}{2y^2} \, dy$

$= \frac{1}{2} \int_1^2 y^2 \, dy + \frac{1}{2} \int_1^2 y^{-2} \, dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-1}}{1} \right]_1^2$

$= \frac{1}{2} \left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right]$

$= \frac{17}{12}$

(3) $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0$, $b = 2$, $n = 4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2} dx = 0.5\Big[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\Big]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$$

$$= 1.779.$$

(ii) $\int_0^4 x^2 dx$

$L = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\,dx = \frac{1}{3}\left[16+4(10)+8\right]$$
$$= \frac{64}{3}$$

$$\therefore \int_0^4 x^2\,dx = 21.533$$

(iii) $$\int_0^{\pi/3} \sqrt{\sin x}\; dx \quad \text{with } n = 6$$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4166	0.50	0.70	0.080	0.87	0.99

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{\pi}{54} \times 12.1163$$

$$= 0.7049$$

ii) $y = \sqrt{4-x^2} \qquad x \in [-2, 2]$

$$L = \int_0^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$y = \sqrt{4-x^2} \qquad \therefore \frac{dy}{dt} = 2\int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\; dx$$

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$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$$

$$= 4\left(\sin^{-1}\left(x/2\right)\right)_0^2$$

$$= 2\pi$$

iii) $y = x^{3/2}$ in $[0.4]$

$f(x) = 3/2\, x^{1/2}$

$[f'(x)]^2 = 9/4\, x$

$$L = \int_a^b \sqrt{1+(f_1(x))^2}\,dx$$

$$= \int \sqrt{1+9/4\, x}\,dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\,dx$$

$$= 1/2 \int_0^4 \sqrt{4+9x}\,dx$$

$$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$$

$$= 1/27 \left[(4+9x)^{3/2}\right]_0^4$$

$$= \cancel{1/27} \left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= 1/27\ (4)^{3/2} - (40)^{3/2}$$

(N)

$x = 3\sin t \qquad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\,dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\,dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\,dt$$

$$= \int_0^{2\pi} 3\,dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

Practical No. 7.

(1) $x\frac{dy}{dx} + y = e^x$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$e^x(x) = 1/x$$

$$If = e^{\int(x)}\,dx$$
$$= e^{\int 1/x}\,dx$$
$$= e^{\pi} x$$
$$If = x.$$

$$y(If) = \int \theta(x)\,(If)\,dx + c$$
$$= \int \frac{e^x}{x} \cdot x . dx + c$$
$$= \int e^x\,dx + c$$

$$xy = e^x + c.$$

(x) $e^{x}\frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + \frac{2e^{x}}{e^{x}}y = \frac{1}{e^{x}}$ $(\div \text{ by } e^{x})$

$\frac{dy}{dx} + 2y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = e^{x}.$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$\int P(x)\,dx$

$If = e^{\int 2\,dx}$

$= e^{2x}.$

$I(If)f = \int Q(x)\,(IF)\,dx + c$

$y \cdot e^{2x} \cdot \int e^{-x+2x}\,dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

(xi) $x\frac{dy}{dx} = \cos\frac{x}{x} - 2y$

$x\frac{dy}{dx} = \cos\frac{x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

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$p(x) = 2/x \quad Q(x) = \cos x / x^2$

$$IF = e^{\int p(x)\,dx}$$
$$= e^{\int 2/x\,dx}$$
$$= \ln x^2$$

$$y(IF) = \int Q(x)(IF)\,dx + c$$
$$= \int \cos x + c$$
$$x^2 y = \sin x + c$$

(14) $x \dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$

$p(x) = 3/x \quad Q(x) = \sin x / x^3$

$$= e^{\int p(x)\,dx}$$
$$= e^{\int 3/x\,dx}$$
$$= e^{3/x}\,dx$$
$$= e^{\ln x^3}$$
$$IF = x^3$$

$$y(IF) = \int Q(x)(IF)\,dx + c$$
$$= \int \frac{\sin x}{x^3} x^3\,dx + c$$
$$= \int \sin x\,dx + c$$

$$x^3 y = -\cos x + c$$

(v) $$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$$

$$If = e^{\int P(x)\,dx}$$

$$= e^{\int 2\,dx} = e^{2x}$$

$$y\,(If) = \int Q(x)\,(If)\,dx + c$$

$$= \int 2xe^{-2x}\,e^{2x}\,dx + c$$

$$= \int 2x\,dx + c$$

$$ye^{2x} = x^2 + c$$

(vi) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \, \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy$$

$$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y}{\tan y} dy$$

$$\log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x - \tan y| + c$$

$$\tan x \cdot \tan y = e^c$$

(viii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = u$

Differentiating BTS

$x-y+1 = u$

$$1 - \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{1 - du}{dx} = dy/dx$$

$$1 - \frac{du}{dx} = \sin^2 u$$

$$\frac{du}{dx} = 1 - \sin^2 u$$

$$\frac{du}{dx} = \cos^2 u$$

$$\frac{du}{\cos^2 u} = dx$$

$$\int \sec^2 u \, du = \int dx$$

$$\tan u = x + c$$

$$\tan(x+y-1) = x + c.$$

(viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

Put $2x + 3y = u$

$$2 + \frac{3\,dy}{dx} = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{du}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{du}{dx} - 2\right) = \frac{1}{3}\frac{(u-1)}{(u+2)}$$

$$\frac{du}{dx} = \frac{u-1}{u+2} + 2$$

$$\frac{du}{dx} = \frac{u-1+2u+4}{u+2}$$

$$= \frac{2u+3}{u+2}$$

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$$= \frac{3(u+1)}{u+2}$$

$$\int \left(\frac{u+2}{u+1}\right) du = 3\,dx$$

$$\int \frac{u+1}{u}\, du = t \int \frac{1}{u+1}\, du = 3x$$

$$u + \log|u| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c.$$

AB
10/1/2020

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Practical No. 8

Topic : Euler's method

1) $\frac{dy}{dx} = y + e^x = e$

$f(x,y) = y + e^x - 2$ $y_0 = 2$, $x_0 = 0$ $h = 0.5$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2927	5.3615

$y_{n+1} = y_n + hf(x_n, y_n)$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	8.3615	7.8431	9.28305
4	2	9.2831		

$\therefore$ By Euler's formula
$y(2) = 9.2831$

Q2) $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$, $y_0 = 0$. $x_0 = 0$ $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h\, f(x_n, y_n)$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	161.04	0.408
2	0.4	0.408	1.1664	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

∴ By Euler's formula

$y(1) = 1.2942$

Q.3 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h \; f(x_n \cdot y_n)$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

$y(1) = ?$

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Q.4 $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$, $x_0 = 1$ $h = 0.25$

for $h = 0.5$

Using Euler's iteration formula.

$y_{n+1} = y_n + h\, f(x_n = y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

∴ By Euler's formula

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.35
3	1.75	6.3359	10.1715	8.90
4	2	8.9048		

By Euler's formula

$y(2) = 8.9048$

Q.5 $\frac{dy}{dx} = \sqrt{xy} + 2$ $y = 1$ $x_0 = 1$ $h = 0.2$

Using Euler's formula iteration

$$y_{n+1} = y_n + h \quad f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

∴ By Euler's formula

$y(1.2) = 1.6$

AK
13/01/2020

Limits and partial order derivatives

i) $\lim_{(x,y)\to(4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5}$$

$$= \frac{-61}{9}$$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(0+1)\left((2)^2+0-4(2)\right)}{2+0}$$

$= \frac{1(4+0-8)}{2}$

$= \frac{-4}{2}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $\Rightarrow$ 0

$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)\cancel{(x-yz)}}{\cancel{(x-yz)}\, x^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On applying limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q.2

i) $f(x,y) = xye^{x^2}+y^2$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2}+y^2)$

$= ye^{x^2}+y^2(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2}+y^2)$

$= xe^{x^2}+y^2(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$\frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_y = 2x^3y - 3x^2 + 3y^2$

Q.3)

(1) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)}$

$= \frac{2+2y^2 - 0}{(1+y^2)^2}$

$$\frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{(1+y^2)(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(2+0)^2}$$

$$= 0$$

Q.4 (1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{d}{dx}\left(-x^2 y - 2x(xy^2 - xy)\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{d}{dx}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad \text{--- (1)}$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} \quad \text{— (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right) \quad \text{— (3)}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2\frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^4 - 4xy + 2x^2}{x^4} \quad \text{— (4)}$$

from (3) & (4);

$f_{xy} - f_{yx}$

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$.

$$f_x = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right) \qquad f_y = \frac{\partial}{\partial y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1} \qquad = 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\,\frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(x^2+1)}{(x^2+1)^2}\right)$$

$$6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \text{— (1)}$$

$$f_{yy} = \frac{d}{dy}(6x^2y) \quad \text{—} (2)$$

$$= 6x^2$$

$$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0$$

$$= 12xy \quad - (3)$$

$$f_{yx} = \frac{d}{dx}(6x^2y)$$

$$= 12xy$$

from (3) & (4) $\quad \cancel{\text{—} (4)}$

$$f_{xy} = f_{yx}$$

iii) $f(x,y) = \sin(x,y) + e^{x+y}$

$$f(x) = y\cos(xy) + e^{x+y} \quad (1)$$

$$= y\cos(xy) + e^{x+y}$$

$$\therefore f(xx) = \frac{d}{dx}\left(y\cos(xy) + e^{x+y}\right)$$

$$= -y\sin(xy)\cdot y + e^{x+y} \quad (1)$$

$$= -y^2\sin(xy) + e^{x+y}$$

$$f_{yy} = \frac{d}{dx}\left(x\cos(xy) + xe^{x+y}\right)$$

$$= -x^2\sin(xy) + e^{x+y} \quad - (2)$$

$$f_{xy} = -y^2\sin(xy) + \cos(xy) + e^{x+y} \quad - (3)$$

$$f_{yx} = \frac{d}{dx}\left(x\cos(xy) + e^{x+y}\right)$$

$f_{yx} = -x^2 \sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f_{xy} \neq f_{yx}$

ii) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\therefore f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}$ $\qquad f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \dfrac{1}{\sqrt{2}}$ $\qquad f_y$ at $(1,1) = \dfrac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x-1) + \dfrac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}x + \dfrac{1}{\sqrt{2}}y - \dfrac{2}{\sqrt{2}}$

$= \dfrac{x+y}{\sqrt{2}}$

ii) $f(x,y) = \cancel{\ } 1-x+y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y \cdot \cos x$ $\qquad f_y = \sin x$

f_x at $\left(\frac{\pi}{2}, 0\right) = 1$ $\qquad f_y$ at $\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x,a) + f_y(a,b)(y,b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y)$

$\therefore L(x,y) = 1 - x + y$

ii) $f(x,y) = \log x + \log y$

$f(1,1) = \log(1) + \log(1)$

$f_x = \frac{1}{x}$ $\qquad f_y = \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= x - 1 + y - 1$

$= x + y - 2$

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Practical No. 10

ii) $f(x,y) = x + 2y - 3$ $\quad a(1,-1), \; u = +3i - j$

→ Here,

$u = 3i - j$ is not a unit vector

$\bar{u} = 3\hat{i} - \hat{j}$

$|u| = \sqrt{10}$

∴ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i - j)$

$= \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}\right) i \left(\frac{-1}{\sqrt{10}}\right)$

Now,

$f'(a+hu) = f\left[(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right]$

$= 1 + \frac{3h}{\sqrt{10}} + 2\left(-1, \frac{-h}{\sqrt{10}}\right) - 3$

$= -4 + \frac{h}{\sqrt{10}}$

$$f(a) = \lim_{h\to 0} f(a+hu) - f(a)$$

$$= \lim_{h\to 0} \frac{-4+\frac{h}{\sqrt{10}} - (-4)}{h}$$

$$= \lim_{h\to 0} \frac{h}{h\sqrt{10}} = \frac{1}{\sqrt{10}}$$

ii) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $v = i + 5j$

→ Here,

$v = i+5j$ is not a unit vector

$|v| = i + 5j$

$|\bar{v}| = \sqrt{26}$

$\therefore$ Unit vector along v is $\frac{\bar{v}}{|\bar{v}|} = \frac{1}{\sqrt{26}}(i+5j)$

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now,

$$f(a+hu) = f\left((3,4)+h\left(\frac{1}{\sqrt{26}},\frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3+\frac{h}{\sqrt{26}}\right), 4+\frac{5h}{\sqrt{26}}\right)$$

$$\left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right)+1$$

$$= 16+\frac{25h^2}{26}+\frac{40h}{\sqrt{26}}-12-\frac{4h}{\sqrt{26}}+1$$

$$= \frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5$$

$$f(a) = \lim_{h\to 0}\frac{f(a+hu)-f(a)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5-5}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}\left(\frac{25h}{26}+\frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \cancel{\frac{25(0)}{26}}+\frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

$a = (1, 2)$, $u = 3i + 4j$

$f(x, y) = 2x + 3y$

Here,

$v = 3i + 4j$ is not a unit vector

$\bar{v} = 3\hat{i} + 4\hat{j}$

$|v| = \sqrt{25} = 5$

$\therefore$ Unit vector along $v = \frac{\bar{v}}{|v|} = \frac{1}{5}(3\hat{i} + 4\hat{j})$

$= \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

Now,

$f(a+hu) = f\left[(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right]$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$

$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + 18h$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{18h}{5h}$

Q.

ii)

$= \frac{18}{5}$

(ii) $f(x,y) = x^y + y^x$

$f_x = y(x^{y-1}) + y^x \log y \qquad a = (1,1)$

$f_y = x(y^{x-1}) + x^y \log x$

$\Delta f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, x y^{x-1} + x^y \log x)$

$\Delta f(x,y)$ at $(1,1)$

$= (1,1)$

(iii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{(1+x^2)}$

$f_y = 2y \tan^{-1} x$

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$$\Delta f(x,y) = (f_x, f_y)$$

$$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$$

$\Delta f(x,y)$ at $(1,-1)$

$$= \left(\frac{(-1)^2}{1+(1)^2}, 2(-1)\tan^{-1}(1)\right)$$

$$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$$

ii) $f(x,y,z) = xyz - e^{x+y+z}$ at $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$$\Delta f(x,y,z) = (f_x, f_y, f_z)$$

$$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$$

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Q.3
(i) $x^2 \cos xy + e^{xy} = 2$

$f(x,y) = x^2 \cos y + e^{xy} - 2$

$f_x = 2x \cos y + y e^{xy}$

$f_y = -x^2 \sin y + x e^{xy}$

$(x_0, y_0) = (1, 0)$

f_x at $(1,0) = 2(1) \cos 0 + 0$

f_y at $(1,0) = -(1)^2 \sin(0) + (e)^{1(0)}$

$= 1$

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x + y - 2 = 0$

Now, for eq^n of Normal;

$\cancel{b}x + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0$

$d = -1$

$x + 2y - 1 = 0$ Eq^n of Normal.

Q.4 i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$
$= 2x + z$

$f_y = -2z + 3$

$f_z = -2y + x$

$(x_0, y_0, z_0) = (2,1,0) \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2 \times 2 + 0 = 4$

$f_y(x_0, y_0, z_0) = 3$

$f_z(x_0, y_0, z_0) = -2 \times 1 + 2 = 0$

eqⁿ of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4x + 3y - 11 = 0$

This is required eq^n of tangent
eq^n of normal at $(4, 3, -11)$

$$\left(\frac{x - x_0}{f_y}\right) = \frac{y - y_0}{f_y} = \frac{z - z_0}{-f_x}$$

$$\left(\frac{x - 2}{4}\right) = \frac{y - 1}{3} = \frac{2 - 0}{-11}$$

$$f(x, y) = \cancel{3y}\ 3x^2 + y^2 - 3xy + 6x - 4y$$

$$f_x = 6x - 3y + 6$$
$$f_y = 2y - 3x - 4$$
$$f_x = 0$$

$$6x - 3y + 6 = 0$$
$$3(2x - y + 2) = 0 \qquad -①$$
$$2x - y = -2$$

$$f_y = 0 \qquad - ②$$
$$2y - 3x = 4$$

Multiply eq^n ① & ②

$$4x - 2y = -4$$
$$2y - 3x = -4$$
$$x = 0$$

Substitute

$$2x - y = -2$$
$$-y = -2$$
$$y = 2$$

Critical points are $(0, 2)$

$$r = (f(x)) = 6$$

$$t = (f(y))_y = 2$$

$$s = f_{xy} = -3$$

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then $r > 0$

$6 \times 2 - (3)^2$

$= 12 - 9$

$= 3 > 0$

f has maximum at $(0, 2)$

$3x^2 + y^2 - xy + 6x - 4y$ at $(0, 4)$

$= 4 - 8$

$= -4$

(iii) $f(x, y) = x^2 - y^2 + 2x + 8y + 70$

$f_x + 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$2(x+1) = 0$

$x = -1$

$f_y = 0 \quad \therefore -2y + 8 = 0$

$-2(y-4) = 0$

$y = 4$

Critical point is $(-1,4)$

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$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

$\therefore f$ has minimum at $(-1,4)$

$f(x,y)$ at $(-1,4)$

$= x^2 - y^2 + 2x + 8y - 70$

$= (-1)^2 - (4)^2 + 2 \times -1 + 8 \times 4 - 70$

$= -88 + 33$

$= -55$

07/02/2020