Computer Vision Quiz 2 Solution

1. (a) Let us assume a lens with a focal length of 80mm. Suppose, there is an object point P_o and a 2D image sensor facing it, with a gap of 500mm between them. Where should you place the lens between them such that all light rays from P_o are projected exactly onto the same point on the image sensor? Find the image distance d_i and object point distance d_o from the lens placed between them such that the above condition is satisfied. Assume all distances to be positive whether in front of or behind the lens. [Hint: solution to quadratic equation $ax^2 + bx + c = 0$ is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$] [6 **Marks**]

Ans:

From the question, $d_i + d_o = 500$mm

Using Gaussian lens law

$1/d_i + 1/d_o = 1/f$

$1/d_i + 1/(500-d_i) = 1/80$

$d_i^2 - 500d_i + 40000 = 0$

Solving using $ax^2 + bx + c = 0$ is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$]

Either

$d_i = 400$ and $d_o = 100$ Or **$d_i = 100$ and $d_o = 400$**

(b) What is the aperture size for the camera with a focal length of 60mm and f-number 1.5? [2 **Marks**]

Ans: aperture size D can be obtained using D = (focal length f)/ (f number N)

D= 60/1.5 = 40mm

2. Consider the essential matrix E between the two images of a static scene obtained from two different viewpoints using a single single calibrated camera. Let $\ell_1, \ell_2 \in \mathbb{R}^3$ be the two epipolar lines with respect to the first and second camera frames. Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be the two corresponding pixels in the images and let $\mathbf{x}_1 = \mathsf{K}^{-1}\hat{\mathbf{p}}_1$ and $\mathbf{x}_2 = \mathsf{K}^{-1}\hat{\mathbf{p}}_2$ be the respective normalized coordinates. Let $ax + by + c = 0$ be the representation of ℓ_2 where (x, y) is any point on ℓ_2. You can refer to the below diagram for a reference. Show that $\begin{bmatrix} a & b & c \end{bmatrix}^\top = \mathsf{E}\mathbf{x}_1$. [8 **Marks**]

Epipolar constraint: $x_2^T E x_1 = 0$ ………………………...eq(1)

Since x_2 lies on l_2 ,

$x_2^T l_2 = 0$ …………………eq(2)

From eq(1) and eq(2),
l_2=E x_1

Hence, **$[a\ b\ c]^T = E\ x_1$**