1. In the class, we have seen that for the Harris corner detection algorithm, we have to maximize $E(u, v)$ (defined below) with respect to (u, v) to find potential corner points in an image $I(x, y)$.

$$E(u,v) = \sum_{x=-1}^{1} \sum_{y=-1}^{1} (I(x+u, y+v) - I(x,y))^2.$$

Show that the quadratic approximation of $E(u, v)$ with respect to (u, v) at the origin is defined as below:

$$E(u,v) = \begin{bmatrix} u & v \end{bmatrix} \left(\sum_{x=-1}^{1} \sum_{y=-1}^{1} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}.$$

Here, $I_x = \frac{\partial I(x,y)}{\partial x}$ and $I_y = \frac{\partial I(x,y)}{\partial y}$ represents the image gradients. The eigenvalues of the matrix $\mathbf{M} = \sum_{x=-1}^{1} \sum_{y=-1}^{1} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ plays a critical role to determine corner points and eigenvectors tells the direction of the edges. Consider the gradients $I_x = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & -4 & 0 \end{bmatrix}$ and $I_y = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$ in the 3×3 neighborhood region of a pixel. Determine if this pixel is a corner, edge, or flat. [10 **Marks**]

Ans:

Ans 1: (a) $E(u,v) = \sum\sum (I(x+u, y+v) - I(x,y))^2$

using Taylor series expansion, we can approximate $I(x+u, y+v)$ as

$I(x+u, y+v) \simeq I(x,y) + u I_x + v I_y$ (1st order approximation)

$\Rightarrow$

$E(u,v) = \sum\sum (u I_x + v I_y)^2$

$= \sum\sum (u^2 I_x^2 + v^2 I_y^2 + 2uv I_x I_y)$

$= \sum\sum \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$

$= \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$

$M = \sum\sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$

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(b) $I_x = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & -4 & 0 \end{bmatrix}$, $I_y = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$

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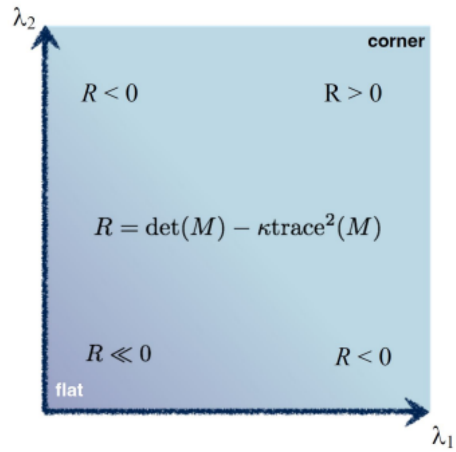
$M = \begin{bmatrix} 16+16 & 0 \\ 0 & 16+16 \end{bmatrix} = \begin{bmatrix} 32 & 0 \\ 0 & 32 \end{bmatrix}$ $\lambda_1 \simeq \lambda_2 >> 0$

corner.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$



$\det M = \lambda_1 \lambda_2$

$\operatorname{trace} M = \lambda_1 + \lambda_2$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

2. Consider the Hough transform for detecting linear edges in an image. Use the polar representation ($x \cos\theta + y \sin\theta = \rho$) of the line parameterized by (θ, ρ) as discussed in the class. Discretize the parameter space such that $\theta \in \{0°, 45°, 90°\}$ and $\rho \in \{0, \sqrt{2}\}$. Consider that there are four edge points $(1,1), (2,0), (0,2), (2,2)$ in an image. Find the voting in the parameter space. Also, determine for which pair of parameters, you get the maximum number of votes. [5 **Marks**]

Ans:

Ans 2: $x\cos\theta + y\sin\theta = \rho$

$$A = \begin{array}{c|c|c|l} \theta=0° & \theta=45° & \theta=90° & \\ \hline & & & \rho=0 \\ & & & \rho=\sqrt{2} \end{array}$$

Accumulation Array 1

Pixel $(1,1) \rightarrow A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ 1

Pixel $(2,0) \rightarrow A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1+1 & 0 \end{bmatrix}$ 1

Pixel $(0,2) \rightarrow A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1+1+1 & 0 \end{bmatrix}$ 1

Pixel $(2,2) \rightarrow A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3+0 & 0 \end{bmatrix} \Rightarrow \theta = 45°, \rho = \sqrt{2}$ 1

Maximum votes.

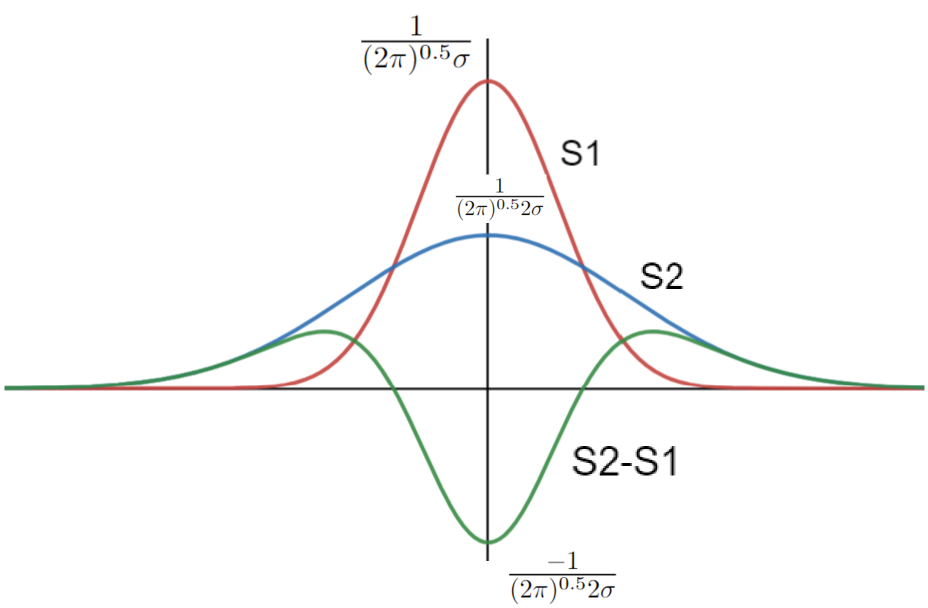
3. Suppose you have a 1D signal that has been transformed to the frequency domain and is represented using the frequency components having frequencies -3u, -2u, -u, 0, u, 2u, and 3u, where u is a positive integer. Now, suppose you design a zero mean Gaussian frequency domain filter with standard deviation σ, such that multiplying the signal with the filter in the frequency domain results in the complete removal of the $\pm$2u and $\pm$3u frequency components of S. Fill in the blank spaces in the following condition that should be satisfied for the above requirement and explain why? $\square < 3\sigma < \square$ [3 **Marks**]

Ans: <u>u</u> < 3σ < <u>2u</u>

Reason : Since in a Gaussian Distribution, ~99% of the non zero amplitude lies within 3σ of the mean, we want the frequencies>=+-2u to lie outside this region, so that after multiplying the signal with the Gaussian filter in the frequency domain, such frequency components will get zeroed out.

4. a) Plot two 1D Gaussian signals S1 and S2 having standard deviations σ and 2σ, respectively, and centered at the origin, and also plot the difference S2-S1 signal. Assume σ to be a positive integer. Clearly label the different signals in the graph. Mark the values of all the signals at the origin. The difference signal can be considered as an approximation of what type of filter and how will you identify edges in the output obtained by applying this filter? [6 **Marks**]

Ans:



S1,S2,S2-S1 can also be shown in separate plots.

Values of the signal at the origin: S1 is $1/((2\pi)^{0.5}\sigma)$ at origin,
S2 is $1/((2\pi)^{0.5}2\sigma)$ at origin,
S2-S1 is $-1/((2\pi)^{0.5}2\sigma)$ at origin.
“Difference signal can be considered as an approximation of the Laplacian of Gaussian filter”.
“Edges can be found out by identifying the zero-crossings in the output after applying the Laplacian of Gaussian filter”

b) Apply the filter F_1 to image I using correlation to get output O_1. There exists another filter F_2, which when applied to O_1, gives an output O_2 that is the same as the output O_3 produced by directly applying the Scharr filter to I. Find F_2 and compute O_1, O_2 and O_3. Use zero-padding and full output size for the correlation operation so that you can apply the filters to every pixel in the image. Do not normalize the output of any filtering operation. [6 **Marks**]

$$I = \begin{bmatrix} 0 & 10 & 20 \\ 0 & 10 & 20 \\ 0 & 10 & 20 \end{bmatrix} \; F_1 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Ans:

$$O_1 = \begin{bmatrix} -10 & -20 & 10 \\ -10 & -20 & 10 \\ -10 & -20 & 10 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 3 \\ 10 \\ 3 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} -130 & -260 & 130 \\ -160 & -320 & 160 \\ -130 & -260 & 130 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} -130 & -260 & 130 \\ -160 & -320 & 160 \\ -130 & -260 & 130 \end{bmatrix}$$