Q1 $u_1 - u_2 - u_3$

Cost function $E = \lambda \sum_{i=1}^{3} (I_x(i)\, u_i + I_t(i))^2 + (u_1-u_2)^2 + (u_2-u_1)^2 + (u_2-u_3)^2 + (u_3-u_2)^2$

$$\frac{\partial E}{\partial u_1} = 2\lambda(I_x(1)u_1 + I_t(1))I_x(1) + 2(u_1-u_2) - 2(u_2-u_1) = 0$$

$$= 2\lambda I_x^2(1) u_1 + 2\lambda I_x(1) I_t(1) + 4u_1 - 4u_2 = 0$$

$$\Rightarrow (4 + 2\lambda I_x^2(1))u_1 - 4u_2 = -2\lambda I_x(1) I_t(1) \quad \text{— (1)}$$

$$\frac{\partial E}{\partial u_2} = 2\lambda(I_x(2)u_2 + I_t(1))I_x(2) - 2(u_1-u_2) + 2(u_2-u_1) + 2(u_2-u_3) - 2(u_3-u_2) = 0$$

$$= 2\lambda I_x^2(2) u_2 + 2\lambda I_t(1) I_x(2) + 8u_2 - 4u_1 - 4u_3 = 0$$

$$\Rightarrow (8 + 2\lambda I_x^2(2))u_2 - 4u_1 - 4u_3 = -2\lambda I_x(2) I_t(2) \quad \text{— (2)}$$

$$\frac{\partial E}{\partial E_3} = 2\lambda(I_x(3)u_3 + I_t(3))I_x(3) - 2u_2 + 2u_3 + 2u_3 - 2u_2 = 0$$

$$\Rightarrow (4 + \lambda I_x^2(3))u_3 - 4u_2 = -2\lambda I_x(3) I_t(3) \quad \text{— (3)}$$

using eq. (1), (2), (3)

$$\begin{bmatrix} 4+2\lambda I_x^2(1) & -4 & 0 \\ -4 & 8+2\lambda I_x^2(2) & -4 \\ 0 & -4 & 4+2\lambda I_x^2(3) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -2\lambda I_x(1) I_t(1) \\ -2\lambda I_x(2) I_t(2) \\ -2\lambda I_x(3) I_t(3) \end{bmatrix}$$

$$A u = b$$

Ans 2 Here, $R = I$ and $t = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow$ Essential matrix $E = [t]_x R = [t]_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -b \\ 0 & b & 0 \end{bmatrix}$

Now, we know that $p_2^T F p_1 = 0$ where F is the fundamental matrix

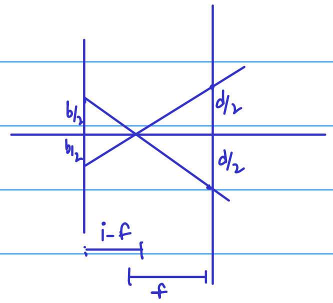
also $F = (K^{-1})^T E K \Rightarrow p_2^T (K^{-1})^T [t]_x K^{-1} p_1 = 0$

Now, $K = \begin{bmatrix} \alpha & 0 & 200 \\ 0 & \alpha & 200 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow K^{-1} = \begin{bmatrix} 1/\alpha & 0 & -200/\alpha \\ 0 & 1/\alpha & -200/\alpha \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow (K^{-1})^T [t]_x K^{-1} = \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\alpha & 0 \\ -\frac{200}{\alpha} & -\frac{200}{\alpha} & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -b \\ 0 & b & 0 \end{bmatrix} \begin{bmatrix} 1/\alpha & 0 & -200/\alpha \\ 0 & 1/\alpha & -200/\alpha \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\alpha & 0 \\ -\frac{200}{\alpha} & -\frac{200}{\alpha} & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -b \\ 0 & \frac{b}{\alpha} & -\frac{200b}{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{b}{\alpha} \\ 0 & \frac{b}{\alpha} & 0 \end{bmatrix}$$

$$\Rightarrow p_2^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/\alpha \\ 0 & 1/\alpha & 0 \end{bmatrix} p_1 = 0.$$

(3)



$$\frac{b/2}{d/2} = \frac{i-f}{f} \Rightarrow b = \frac{di}{f} - \frac{fd}{f} = b = \frac{i}{N} - \frac{f}{N}$$

$$\boxed{b = \frac{i-f}{N}}$$

(4) $P = [KR \;\; Kt]$

$$KR = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha_u/\sqrt{2} & -\alpha_u/\sqrt{2} & u_0 \\ \alpha_v/\sqrt{2} & \alpha_v/\sqrt{2} & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha_u = \sqrt{2}, \quad \alpha_v = \sqrt{2}, \quad u_0 = 0, \quad v_0 = 0$$

$$Kt = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\Rightarrow a = b = 1, \quad c = 0$$