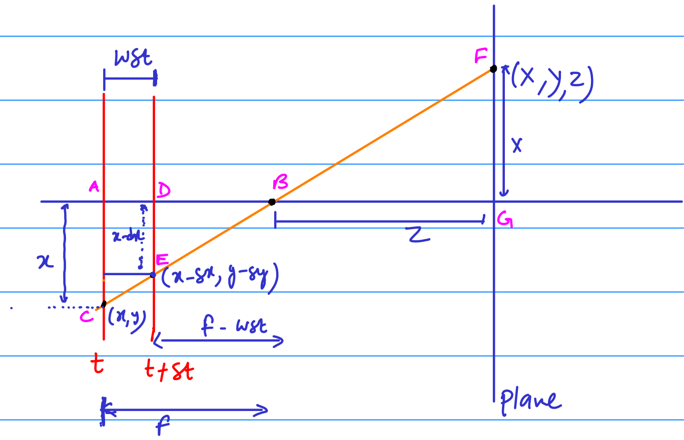
Ans 1



from the similarity of $\triangle ABC$ & $\triangle GBF$

$$\frac{x}{X} = \frac{f}{Z} \quad \text{—— (1)}$$

from the similarity of $\triangle DBE$ & $\triangle GBF$

$$\frac{x - \delta x}{X} = \frac{f - W\delta t}{Z} \quad \text{—— (2)}$$

from (1) & (2)

$$\frac{\delta x}{X} = \frac{W}{Z}\delta t$$

$$\Rightarrow \frac{\delta x}{\delta t} = \frac{W}{Z}X$$

$$\Rightarrow u = \frac{W}{Z}x$$

Similarly $\quad v = \frac{W}{Z}y$

(2) let $E = \lambda \sum_{i,j} (I_x u_{ij} + I_y v_{ij} + I_t)^2 + \sum_{i,j} (u_{ij} - \bar{u}_{ij})^2 + \sum_{i,j} (v_{ij} - \bar{v}_{ij})^2$

(a)
$$\frac{\partial E}{\partial u_{ke}} = 2\lambda (I_x u_{ke} + I_y v_{ke} + I_t) I_x + 2(u_{ke} - \bar{u}_{ke}) = 0$$

$$\frac{\partial E}{\partial v_{ke}} = 2\lambda (I_x u_{ke} + I_y v_{ke} + I_t) I_y + 2(v_{ke} - \bar{v}_{ke}) = 0$$

These are the same equations that we obtain in the original Horn and Schunck Algo rest of the steps are same as in the derivation shared in classroom.

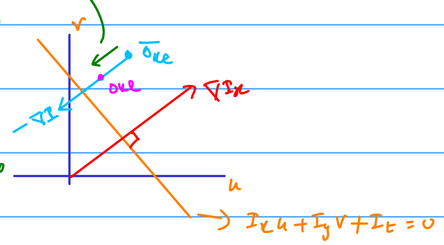
Final solution:

$$\begin{cases} u_{ke} = \bar{u}_{ke} - \alpha I_x \\ v_{ke} = \bar{v}_{ke} - \alpha I_y \end{cases} \qquad \alpha = \frac{\lambda (I_x \bar{u}_{ke} + I_y \bar{v}_{ke} + I_t)}{1 + \lambda (I_x^2 + I_y^2)}$$

$$\begin{bmatrix} u_{ke} \\ v_{ke} \end{bmatrix} = \begin{bmatrix} \bar{u}_{ke} \\ \bar{v}_{ke} \end{bmatrix} - \alpha \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$

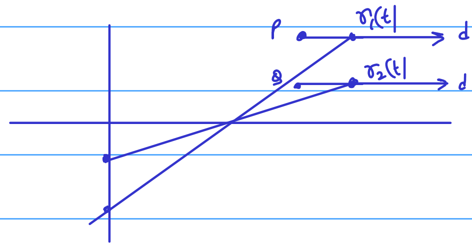
$$O_{ke} = \bar{O}_{ke} - \alpha \nabla I$$

(b) moving closer to the optimal solution that lie on the line $I_x u + I_y v + I_t = 0$



(c) Last part was left as an exercise the Section 2 students as well.

③



$$r_1(t) = p + td = \begin{bmatrix} p_x + td_x \\ p_y + td_y \\ p_z + td_z \end{bmatrix}$$

$$r_2(t) = q + td = \begin{bmatrix} q_x + td_x \\ q_y + td_y \\ q_z + td_z \end{bmatrix}$$

$$\text{Proj of } r_1(t) = \left(\frac{f(p_x + td_x)}{p_z + td_z}, \frac{f(p_y + td_y)}{p_z + td_z} \right) \quad \text{as } t \to \infty \quad = \left(\frac{fd_x}{d_z}, \frac{fd_y}{d_z} \right)$$

$$\text{Proj of } r_2(t) = \left(\frac{f(q_x + td_x)}{q_z + td_z}, \frac{f(q_y + td_y)}{q_z + td_z} \right) \quad \text{as } t \to \infty \quad = \left(\frac{fd_x}{d_z}, \frac{fd_y}{d_z} \right)$$

Same

④ Follow example 9.7 of Hartley & Zisserman

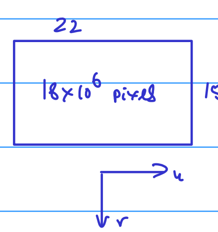
⑤ Since camera is moving in xy plane $t = \begin{bmatrix} t_x \\ t_y \\ 0 \end{bmatrix}$

also rotations are about only z-axes $\Rightarrow$ $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ $E = [t]_x R.$

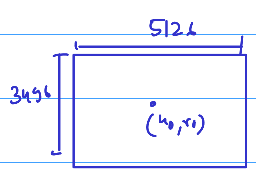
Now put $[t]_x$, R & give E in this equation to find t_x, t_y & θ in terms of a, b, c, d.

⑥ $f = 50mm$



$\left(\frac{18 \times 10^6}{22 \times 15} \right)$ pixels / mm²

$k_u = k_v = \sqrt{\frac{18 \times 10^6}{15 \times 22}} = 233$ pix/mm



$k_u = 233$ pix/mm

$\Rightarrow$ $233 \times 22 = 5126$ pix in u direction

$u_0 = \frac{5126}{2} = 2563$

$k_v = 233$ pix/mm $\Rightarrow$

$15 \times 233 = 3495$ pix in v direction

$v_0 = \frac{3496}{2} = 1748$ pix

P_1 (2m, 2m, 10m) — x y z

$u = \frac{k_u f x}{z} + u_0 = \frac{233 \times 50 \times 2}{10} + 2563$

$= 4893$

$v = \frac{k_v f y}{z} + v_0 = \frac{233 \times 50 \times 2}{10} + 1748$

$= 4078$

$(u, v) = (4893, 4078)$

$\rightarrow$ not visible