Indian Institute of Technology Jodhpur

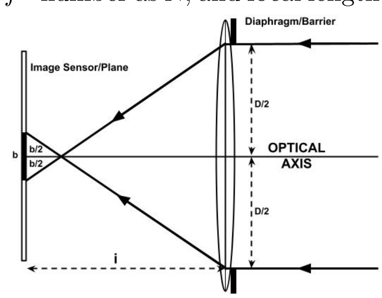
CSL7360: Computer Vision, Minor 2 Exam

Date: March 22, 2024, Max Marks: 30 Max Time: 60 minutes

1. Consider an image of size 1×3. Assume that each pixel can move only in the x-direction. Therefore, the optical flow at each point has only one component (u). The other component is zero (v) as there is no motion in vertical direction. Assume that that image gradients (I_x) and temporal gradients (I_t) at each pixel are given to you. Define the cost function for the Horn and Schunck algorithm for optical flow estimation for this image. Find the differentiation of the cost function with respect to the optical flow variables. Show that the vector $\mathsf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ containing the optimal flow at all three pixels satisfies a system of linear equations $\mathsf{Au} = \mathsf{b}$ for some matrix $\mathsf{A} \in \mathbb{R}^{3\times3}$ and a vector matrix $\mathsf{b} \in \mathbb{R}^{3\times1}$. Find the matrix A and the vector b. [8 **Marks**]

2. Consider a stereo camera with the given baseline $b = 5$ where both the cameras having the same intrinsic matrices $\mathsf{K}_\ell = \mathsf{K}_r = \mathsf{K} = \begin{bmatrix} \alpha & 0 & 200 \\ 0 & \alpha & 200 \\ 0 & 0 & 1 \end{bmatrix}$. Assume that the world coordinate system is aligned with the left camera coordinate system and the right camera coordinate system is a simple translation of the left camera system along x-axis by b units. Consider a pixel $\mathsf{p}_\ell = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ in the left camera and its corresponding pixel $\mathsf{p}_r = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$ in the right camera. Prove either $\mathsf{p}_r^\top \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{\alpha} \\ 0 & \frac{5}{\alpha} & 0 \end{bmatrix} \mathsf{p}_\ell = 0$. or $\mathsf{p}_\ell^\top \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{\alpha} \\ 0 & \frac{5}{\alpha} & 0 \end{bmatrix} \mathsf{p}_r = 0$. [7 **Marks**]

3. Consider a lens-based camera shown in the Figure, with aperture size D, $f-$number as N, and focal length of the lens as f. The image sensor is placed at a distance i behind the lens perpendicular to the optical axis but parallel to the lens, such that i>f. Assume that all the light rays from a scene point P reach the lens parallel to the optical axis. The optical axis passes through the center of the lens, and the lens is perpendicular to the optical axis. We observe that the lens is not able to focus these light rays from P onto a single point on the image plane leading to a blur circle of diameter b on the image sensor. Assume a thin lens.



Ques: Derive the equation for the blur circle diameter b **for this setup** showing all the steps. However, **the final expression for b should only contain i, f, and N**. **[7 Marks]**

4. Without using any matrix inverses, find the intrinsic matrix K and the values a, b, c in the following projection matrix P for a pin-hole camera, given the following rotation matrix R and translation vector t obtained while calibrating the camera. **[8 Marks]**

$$\mathsf{P} = \begin{bmatrix} 1 & -1 & 0 & a \\ 1 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}, \mathsf{R} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathsf{t} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}.$$