Indian Institute of Technology Jodhpur
CSL7360: Computer Vision
Practice Questions Set 2

1. Consider a pin-hole camera facing (in z-direction) a plane perpendicular to the z-axis. The image plane is moving at a speed of W. Consider a point (X, Y, Z) on the plane and its projection (x, y) in the image plane. Find the optical flow of the pixel (x, y).

2. Consider the modified cost function for the Horn and Schunck optical flow estimation algorithm.
$$\min_{\mathsf{u},\mathsf{v}} \lambda \sum_{i,j} (I_x u_{ij} + I_y v_{ij} + I_t)^2 + \sum_{i,j} (u_{ij} - \bar{u}_{ij})^2 + \sum_{i,j} (v_{ij} - \bar{v}_{ij})^2.$$
Here,
$$\begin{aligned} \bar{u}_{ij} &= \frac{1}{4}\left[(u_{i,j} - u_{i+1,j})^2 + (u_{i,j} - u_{i,j+1})^2 + (u_{i,j} - u_{i-1,j})^2 + (u_{i,j} - u_{i,j-1})^2\right] \\ \bar{v}_{ij} &= \frac{1}{4}\left[(v_{i,j} - v_{i+1,j})^2 + (v_{i,j} - v_{i,j+1})^2 + (v_{i,j} - v_{i-1,j})^2 + (v_{i,j} - v_{i,j-1})^2\right] \end{aligned}$$
Design an iterative algorithm for finding optimal optical flow that minimizes above cost function. Also, show that after each iteration, you move closer to the optimal solution. Show, that the optimal u and v can be obtained by solving a linear system if equations.

3. Consider the pinhole camera with focal length f. Now, consider a line in the direction $\mathsf{d} = (d_x, d_y, d_z) \neq \mathsf{0}$ passing from the point $P = (p_x, p_y, p_z)$. Consider another line in the direction d passing from the point $Q = (q_x, q_y, q_z)$. You can parameterize these lines as $\ell_1(t) = P + t\mathsf{d}$ and $\ell_2(t) = Q + t\mathsf{d}$, where $t \in \mathbb{R}$ parameterizes these two lines where $\ell_1(t)$ and $\ell_2(t)$ represent 3D points on these lines for some value of t. Now, show that the projection of the 3D points $\ell_1(t)$ and $\ell_2(t)$ on the image plane will be the same for $t \to \infty$.

4. Consider the projections p and q (homogeneous representation) of a 3D point X in two frames of a static scene captured by two different cameras with intrinsic matrices K_1 and K_2. Assume that the world-coordinate system is aligned with the first camera coordinate system. The relative orientation of the second camera with respect to the first camera is (R, t). Show that $\mathsf{q} = \mathsf{K}_2\mathsf{R}\mathsf{K}_1^{-1}\mathsf{p} + \frac{1}{\lambda}\mathsf{K}_2\mathsf{t}$. Here, λ is the distance of X from the camera centre measured along the principal axis of the first camera.

5. Suppose we know that the camera always moves on the XY plane. Show that:
   (a) The essential matrix $\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ is of special form $\mathbf{E} = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{bmatrix}$ for some $a, b, c, d \in \mathbb{R}$.
   (b) Find a solution to $\mathbf{R}$ and $\mathbf{t}$ in terms of a, b, c, d.

6. Consider an 18 Mega-pixel camera with $f = 50$ mm and $k_u = k_v$. Here, k_u and k_v are the pixel densities (in pixels/m) along the directions u and v, respectively. The size of the sensor array is 22mm $\times$ 15mm. Find the value of the parameters k_u and k_v. Consider two points (2m, 2m, 10m) and (8m, 8m , 20m) in a scene measured with respect to the camera coordinate system. Find the corresponding locations of these points in the image plane with respect to the coordinate system located at the left-upper corner of the image plane. Here, 'm' denotes meter and 'mm' denotes millimeter.