1. **ODE Algorithms and Error Estimates**
   Consider the equation
$$\frac{dy}{dx} = -xy \tag{1}$$
   with initial condition y(0) = 1.

   Obtain the numerical solution of the above ODE using the following methods a) Euler Method b) Runge-Kutta 2nd Order, c) Runge-Kutta 4th Order and d) Backward Euler Method. In particular, verify that the errors (difference between numerical and exact solutions) decrease according to the expected power of the explicit algorithms only and discuss the difference between local error and global error.

2. **Stellar Orbits in Kepler Potential**
   The equation that governs the motion of a star in r-θ plane under a Kepler potential is given by
$$\frac{d^2u}{d\theta^2} + u = \frac{1}{a(1-e^2)} \tag{2}$$
   where $u = 1/r$, a and e are constants. Solve the above equation using values a = 5.0 and e = 0.8, plot the trajectory of the star with a) Euler Method and b) Runge Kutta 2nd Order Method. Is the motion of the star what you expect i.e., a closed ellipse with eccentricity e and semi-major axis a ?. Justify the choices of initial conditions taken and any deviation from the expected orbit. *Hint:* The analytic relation between radius r and θ for the above equation can be given as
$$r(\theta) = \frac{a(1-e^2)}{1+e\cos(\theta-\theta_0)},$$
   where $\theta - \theta_0$ is called the true anomaly.