1. **Flux Limiters : Shock Capturing Schemes**
   We have seen in the eariler problem of advecting square pulse that higher-order schemes gives rise to oscillations at the sharp gradient. The following technique will help resolve the issue and also ensure flux conservation.

   - Explain what is Total Variation Diminising Scheme? Define Slope Limiters and Flux Limiters. Is there any difference between the two limiters?
   - The general expression of the flux to be estimated at the left interface (i-1/2) for a flux conserving TVD scheme is given by
     $$f_{i-1/2}^{n+1/2} = \frac{1}{2}u_{i-1/2}\left[(1+\theta_{i-1/2})q_{i-1}^n + (1-\theta_{i-1/2})q_i^n\right] + \frac{1}{2}|u_{i-1/2}|\left(1 - \left|\frac{u_{i-1/2}\Delta t}{\Delta x}\right|\right)\Phi(r_{i-1/2}^n)(q_i^n - q_{i-1}^n)$$
   - Adopt the following Flux limiter funtions and see how the advection solution of square pulse is affected. Use the same grid definition and advection velocity $\lambda = 3.0$.

Name	$\Phi(r)$
Donner-Cell	0
Lax-Wendroff	1.0
Beam-Warming	r
Fromm	$\frac{1}{2}$ (1+r)
Minmod	minmod(1,r)
Superbee	max(0,min(1,2r),min(2,r))
MC	max(0,min((1+r)/2,2,2r))
van Leer	$(r + |r|)/(1 + |r|)$

2. **1D Isothermal Shock Tube**
   In this example, we will develop an algorithm for solving a set of 1D Isothermal Hydrodynamic equation. To do that adopt the following steps :

   - Write the 1D Isothermal Hydroynamic Equations in its Conservative form using Primitive variables ρ, v and P. Assume an Isothermal Sound Speed c_0.
   - Adopt the appropriate CFL time condition assuming the signal speed as c_0 and ensure Δt is chosen appropriately with a CFL factor 0.5.
   - Solve the continuity Equation numerically taking into account any choice of above Flux Limiter.
   - Solve the momentum conservation equation in two steps: a) Solve the transport step by advecting momentum ρv numerically using same Flux limter as adopted for continuity equation. b) Solve the Source step using the second update of velocity.
   - Test the above algorithm for a simple top-hat profile with 100 physical grid cells (+ boundaries), $\Delta x = 1$, $c_0 = 1$. Adopt closed boundaries (reflective) with following initial conditions

     $$\rho(x,t=0) = 2\Theta(50 - x) + 1 \quad v(x,t=0) = 0$$

     where, $\Theta(x)$ is a top-hat function with value 1 for $x > 0$ and 0 otherwise. Integrate upto time t = 30.