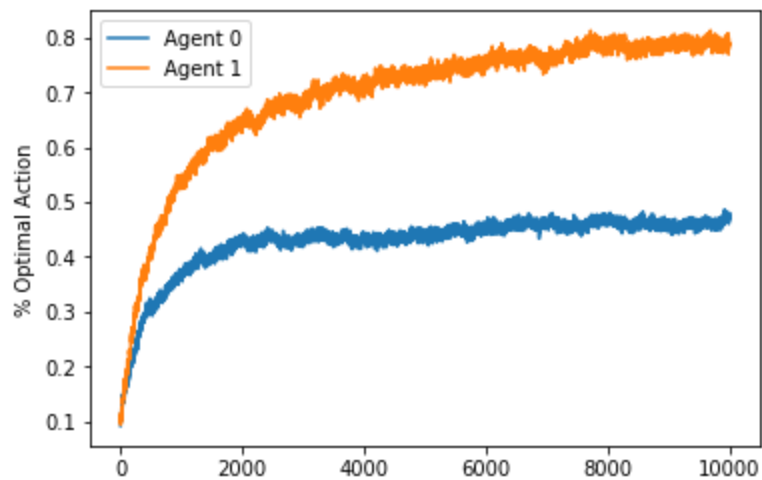
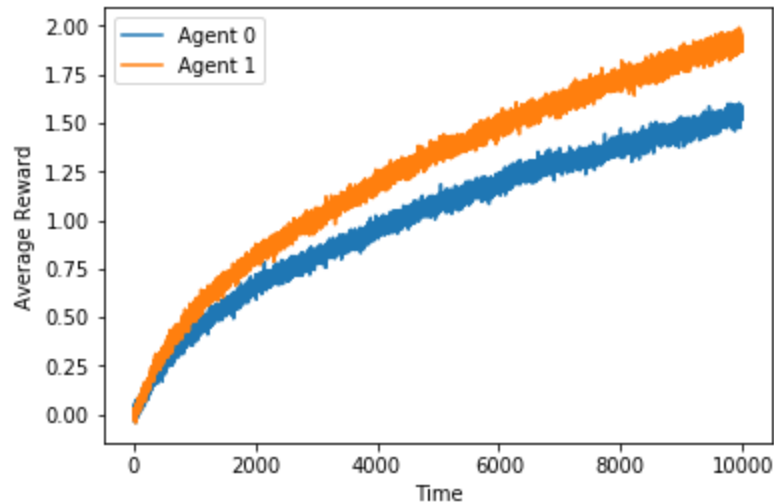


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RL HomeWork 1

Q1.



Agent 0 : Sample average.

Agent 1 : Constant alpha  $\alpha = 0.1$

Since more weight is given to the recent rewards in constant alpha than to the past rewards.

The constant alpha performs much better than the sample average.

Q2

The agent would tend to explore more in the starting, in the optimistic initial value case. During the initial phase it would try out each of the actions, which would result in decrease in the value of  $Q_t$ .

$$\therefore Q_t = Q_{t-1} + \alpha (R_t - Q_{t-1})$$

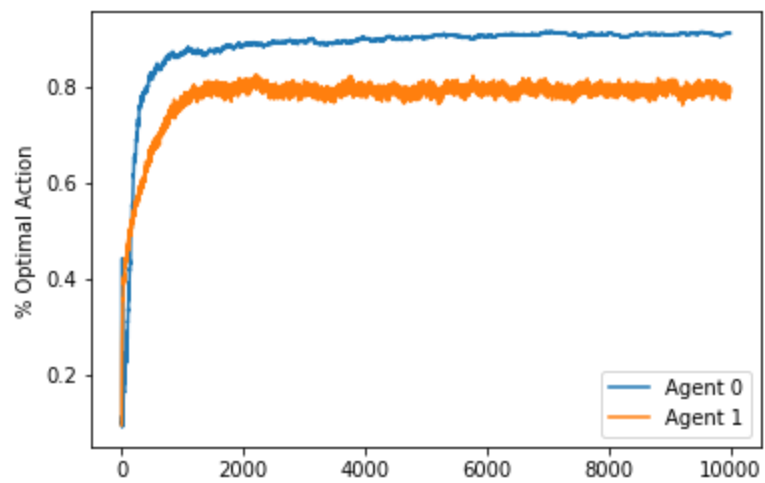
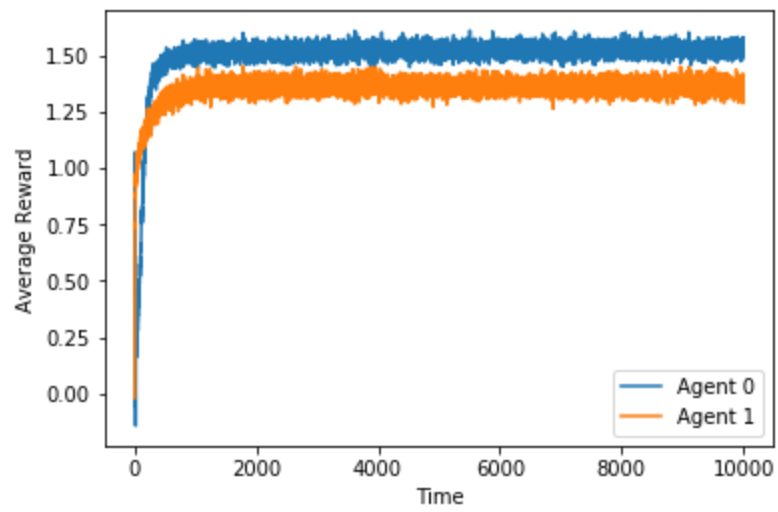
$Q_{t-1}$  is larger in the beginning,  $Q_t$  would always decrease. The agent would choose action with maximum  $Q_t$  which is an action not ~~done~~ even tried earlier.

All the  $K$  possible actions would have been chosen by the agent on an average after  $K$  turns. The agent would choose an action that would yield the maximum reward in  $(K+1)^{\text{th}}$  turn.

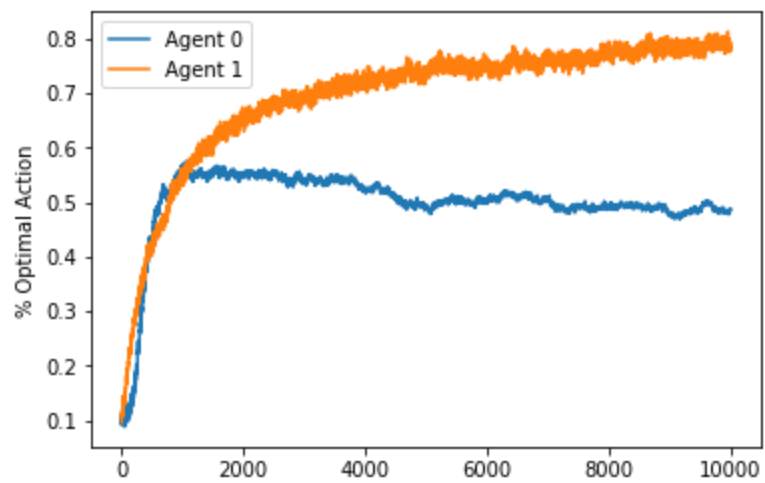
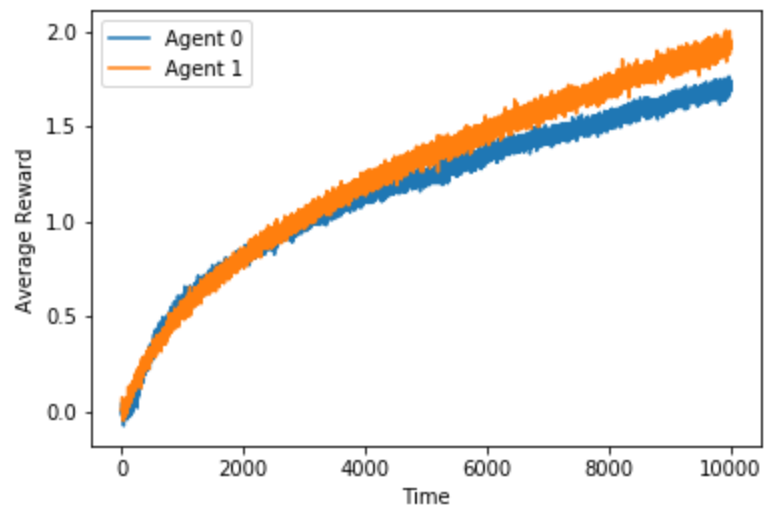
On an average this action would be the optimal action i.e.  $\arg\max(q^*)$

$\therefore$  An optimal action is chosen on the  $(k+1)^{\text{th}}$  step, which results in a spike in most ~~of~~ runs of the algorithm.

Stationary Environment:



Non Stationary Environment:



ln 3

$$\beta_n = \alpha / \bar{\sigma}_n$$

$$\bar{\sigma}_n = \bar{\sigma}_{n-1} + \alpha (1 - \bar{\sigma}_{n-1}) \quad \text{for } n > 0 \text{ with } \bar{\sigma}_0 = 0$$

$$Q_n = Q_{n-1} + \beta_n (R_n - Q_{n-1})$$

After substituting  $\beta_n$

$$Q_n = Q_{n-1} + \frac{\alpha}{\bar{\sigma}_n} (R_n - Q_{n-1})$$

after substituting  $\bar{\sigma}_n$

$$Q_n = Q_{n-1} + \frac{\alpha (R_n - Q_{n-1})}{\bar{\sigma}_{n-1} + \alpha (1 - \bar{\sigma}_{n-1})}$$

For  $n=1$

$$Q_1 = Q_0 + \frac{\alpha (R_1 - Q_0)}{\bar{\sigma}_0 + \alpha (1 - \bar{\sigma}_0)}$$

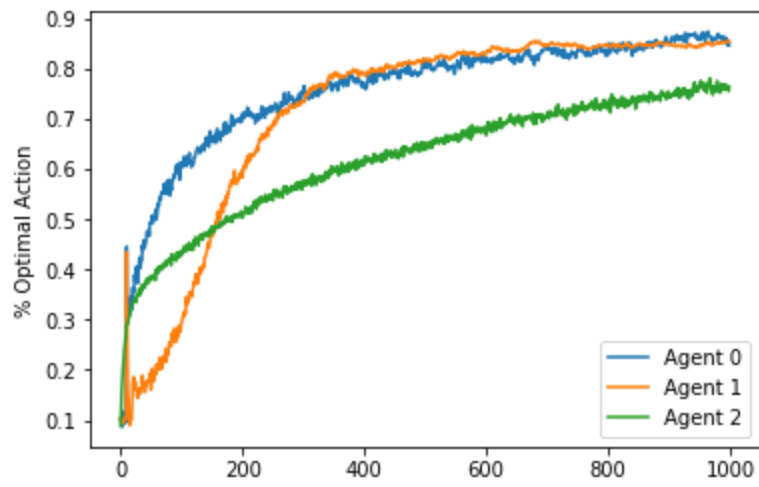
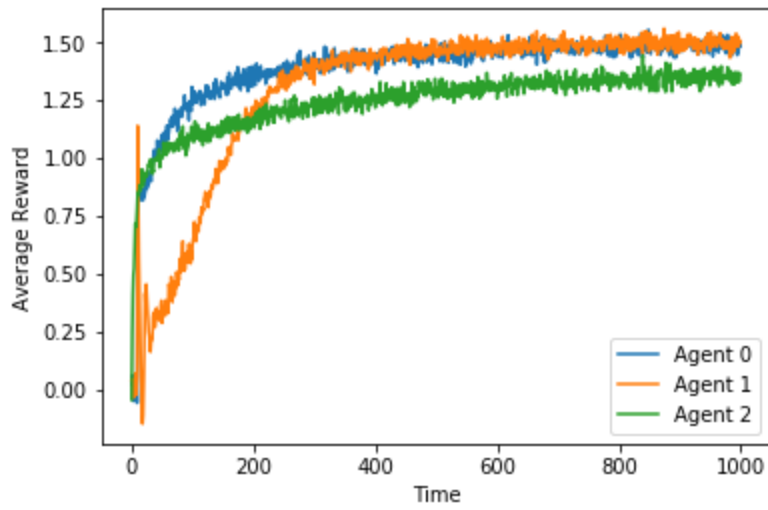
$$\bar{\sigma}_0 = 0 \text{ (given)}$$

$$\begin{aligned} Q_1 &= Q_0 + \frac{\alpha (R_1 - Q_0)}{0 + \alpha (1 - 0)} = Q_0 + \frac{\alpha}{\alpha} (R_1 - Q_0) \\ &= Q_0 + R_1 - Q_0 = R_1 \end{aligned}$$

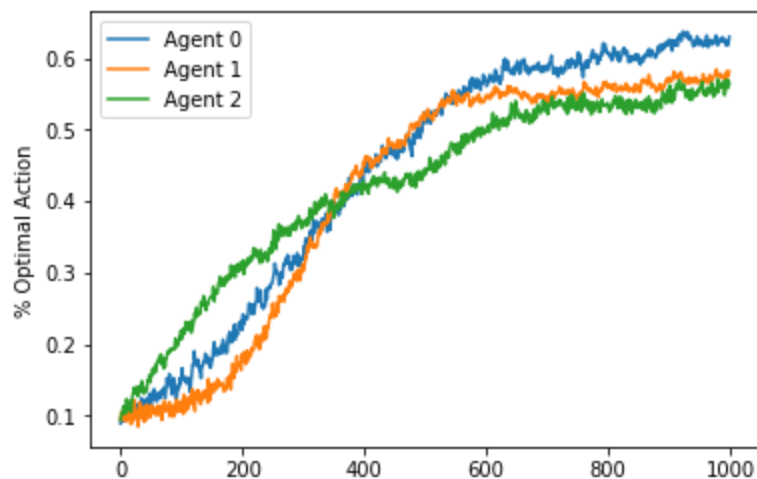
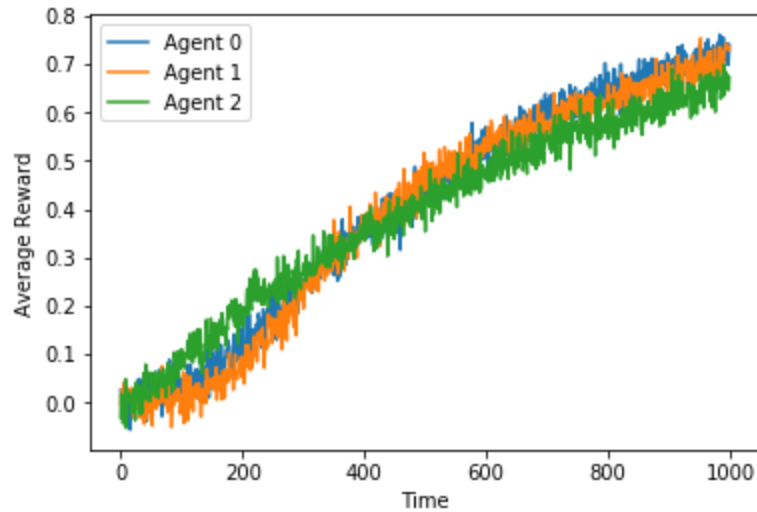
∴  $Q_n$  does not have an initial bias  
Therefore does not depend on  $Q_0$ .

Q4.

Stationary Environment



## Non Stationary



Agent 0 : UCB

Agent 1 : Optimistic Value

Agent 2: E-Greedy

UCB selects among non greedy actions as per their potential of being optimal, while taking into account two points. 1) How close their estimates are to being maximal 2) the uncertainties in those estimates. Hence ensuring that every action is selected once in a while leading to better result. That's why UCB outperforms Optimistic Value and E Greedy for the stationary case.

UCB performs worse in the early steps in the case of non stationary environment. But in the later time it starts performing better than the rest. In the early steps the uncertainty is more since  $N_t(a)$  is close to 0 but after some steps of times, some certainty is achieved. The term of uncertainty that is the uncertainty factor also assumes stationarity.