INDIAN INSTITUTE OF TECHNOLOGY (INDIAN SCHOOL OF MINES), DHANBAD
Monsoon Mid-Semester Examination, Session 2017-18

Examination: **III B.Tech + Dual (CSE)** Time: **2 Hours**
Subject: **Discrete Mathematics (CSC13103)** Max. Marks: **60**

Instructions: (i) Answer ALL questions.
(ii) Attempt all sub-parts of a same question at same place and in order.
(iii) Assume suitable data wherever not provided.

Q. No.	Question	Mark
1. (a)	There are two restaurants next to each other. One has a sign that says, "Good food is not cheap," and the other has a sign that says, "Cheap food is not good." A customer read these sings and understood that these two signs saying the same thing. Justify customer's understanding using propositional calculus.	6
(b)	Thirty cars were assembled in a factory. The options available were a radio, an air conditioner, and white-wall tires. It is known that 15 of the cars have radios, 8 of them have air conditioners, and 6 of them have white-wall tires. Moreover, 3 of them have all three options. Calculate at least how many cars do not have any options at all.	6
2. (a)	Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\sim Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \sim S(x))$, and $\exists x \sim P(x)$ are true, then $\exists x \sim R(x)$ is also true, where the domains of quantifiers are the same.	6
(b)	Let p, q, and r stand for the following propositions: p: It is raining. q: I have a headache. r: I attend the lecture. The proposition $P = (\neg p \wedge \neg q) \rightarrow r$ is rendered in English as follows: "If it is not raining and I do not have a headache, then I attend the lecture." i. Write in English the negation of P. ii. Write in English the contrapositive of P.	3+3
3. (a)	Use mathematical induction to prove that $(\cos(x) + i\sin(x))^n = \cos(nx) + i\sin(nx)$, where x is a complex number and n is a positive integer.	6
(b)	If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$, and $A_3 = \{5\}$, define relation R on A by $x\ R\ y$ if x and y are in the same subset of A_i, for $1 \leq i \leq 3$. Is R an equivalence relation? Justify your answer.	6
4. (a)	Let $(A, \vert)$ be a poset in which "$\vert$" be the relation on A defined as "x divides y", written as $x \vert y$. Draw the Hasse diagram and determine which of the following posets is lattice and why? i. $A = \{1, 2, 3, 4, 6, 12\}$ ii. $A = \{1, 2, 3, 4, 6\}$	3+3
(b)	Let $f_1(x) = x + 4$, $f_2(x) = x - 4$, and $f_3(x) = 4x$ for $x \in \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Find $f_1 \circ f_3$; $f_3 \circ f_1$; and $f_1 \circ f_3 \circ f_2$.	6
5.	For $n \geq 0$, let $S = \{1, 2, 3, \ldots, n\}$ (when $n = 0$, $S = \emptyset$), and let a_n denotes the number of subsets of S that contain no consecutive integers. Find and solve a recurrence relation for a_n.	6+6