DIP Assignment 3

Q. 8)

Part 1

Rotating the image in spatial domain by θ.

To prove:

If $g(x,y) \xleftrightarrow{FT} G(f_x, f_y)$

then $g(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta) \xleftrightarrow{FT} G(f_x\cos\theta + f_y\sin\theta, -f_x\sin\theta + f_y\cos\theta)$

(i.e. if image is rotated by θ in spatial domain then the fourier transform of image is also rotated by θ)

Proof: we define:

a New coordinate system with (x', y')

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$= \begin{bmatrix} x'\cos\theta - y'\sin\theta \\ x'\sin\theta + y'\sin\theta \end{bmatrix}$$

$$F(g(x', y')) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x', y')\, e^{-j2\pi(xf_x + yf_y)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x', y')\, e^{-j2\pi(x'f_x\cos\theta - y'f_x\sin\theta + x'f_y\sin\theta + y'f_y\cos\theta)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x', y')\, e^{-j2\pi[x'(f_x\cos\theta + f_y\sin\theta) + y'(-f_x\sin\theta + f_y\cos\theta)]}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x', y')\, e^{-j2\pi[x'(f_x\cos\theta + f_y\sin\theta) + y'(-f_x\sin\theta + f_y\cos\theta)]}\, dx'\, dy'$$

$$= G(f_x\cos\theta + f_y\sin\theta, -f_x\sin\theta + f_y\cos\theta)$$

$$\left(\because dx'dy' = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial x} \\ \frac{\partial x'}{\partial y} & \frac{\partial y'}{\partial y} \end{vmatrix} dx\,dy = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} dx\,dy = dx\,dy\right)$$

Part 2

To prove: If $g(x) \xleftrightarrow{FT} G(s)$

$$g(x-a) \xleftrightarrow{FT} e^{-j2\pi a s} G(s)$$

Proof:

$$G(g(x-a)) = \int_{-\infty}^{\infty} g(x-a)\, e^{-j2\pi s x} dx = \int_{-\infty}^{\infty} g(x-a)\, e^{-j2\pi s(x-a)} e^{-j2\pi s a} \, d(x-a)$$

$$= e^{-j2\pi a s} G(s)$$

⇒ A shift in position in spatial domain gives rise to a phase change in another domain.