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DATA 598 - Deep Learning
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Homework 2

1 Edge Cases of Automatic Differentiation

1.1 Derivatives of a discontinuous function

- Define and plot a (mathematical) function $f : R \to R$ which is discontinuous at $\hat{x}$ with a jump discontinuity. Clearly show the point at which f is discontinuous and indicate whether it is right continuous or left continuous.
- Implement f as a DiffProg function in PyTorch so that PyTorch returns a derivative of 0 at $\hat{x}$, our point of discontinuity.
- Implement f again in DiffProg so that PyTorch now returns a derivative of -1728 at exactly the same point $\hat{x}$.

In [1]:

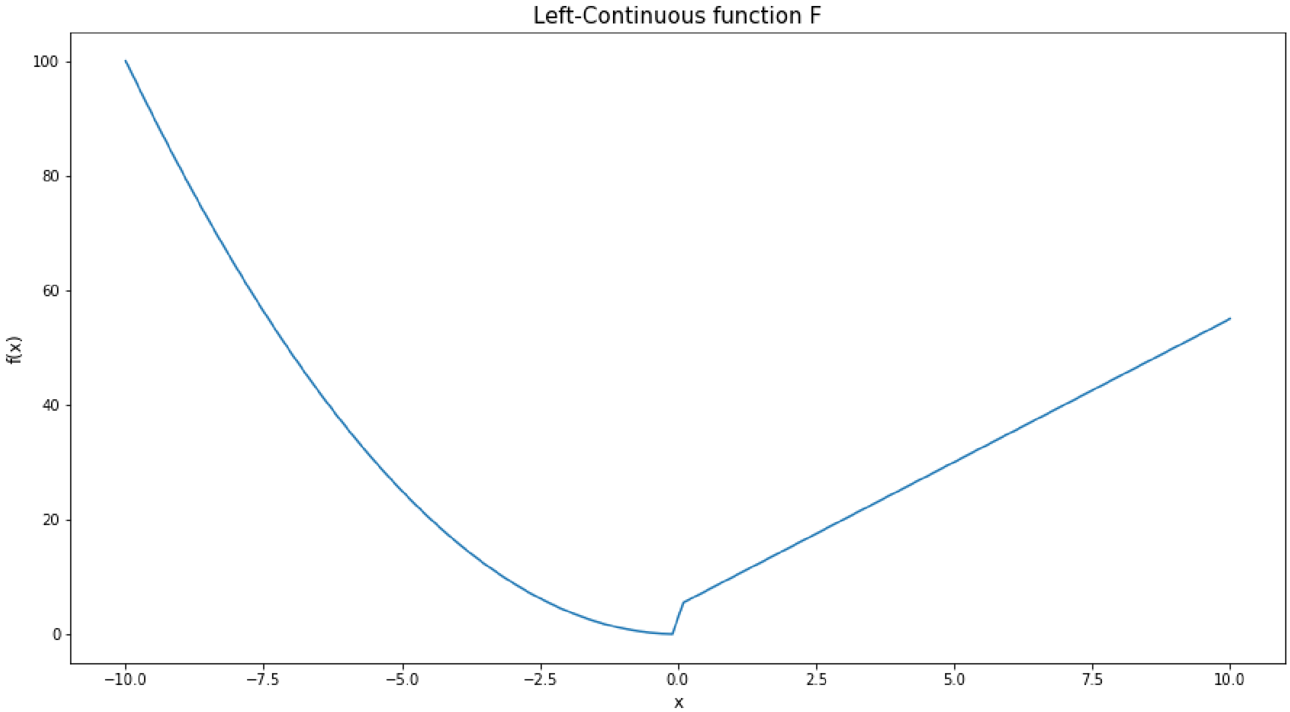
```
# Formatting notebok
from IPython.core.display import display, HTML
display(HTML("<style>.container { width:55% !important; }</style>"))

# Imports
import torch
import matplotlib.pyplot as plt
import numpy as np
```

In [2]:

```
def my_discont_func(x):
    """
    Left-continuous function f
    Input x is a torch scalar
    """
    if x > 0:
        return (x * 5 + 5)
    else:
        return torch.pow(x, 2)

x = torch.linspace(-10, 10, steps=100)
f = [my_discont_func(i) for i in x]

fig = plt.figure(figsize=(15, 8))
plt.plot(x, f)
plt.title('Left-Continuous function F', fontsize=15)
plt.xlabel('x', fontsize=12)
plt.ylabel('f(x)', fontsize=12)
```

Out[2]:

```
Text(0, 0.5, 'f(x)')
```



In [3]:

```
# Implement f as a DiffProg function correctly
def my_diffprog_func(x):
    """
    Left-continuous function f
    Input x is a torch scalar
    """
    if x > 0:
        return (x * 5 + 5)
    else:
        return torch.pow(x, 2)

# Check positive input
x = torch.rand(1, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

# Check negative input
x = -1 * torch.ones(1, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

# Check discontinuity point
x = torch.zeros_like(x, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: tensor([8.4651], grad_fn=<AddBackward0>), y_prime: tensor([5.])
y: tensor([1.], grad_fn=<PowBackward0>), y_prime: tensor([-2.])
y: tensor([0.], grad_fn=<PowBackward0>), y_prime: tensor([0.])
```

In [4]:

```
# Implement f as a DiffProg function s.t. x' at the discontinuity point evaluates to -1728
def my_diffprog_func(x):
    """
    Left-continuous function f
    Input x is a torch scalar
    Derivative at discontinuity point evaluates to -1728
    """
    if x > 0:
        return (x * 5 + 5)
    elif x < 0:
        return torch.pow(x, 2)
    else:
        return -1728 * x

# Check positive input
```

```python
x = torch.rand(1, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

# Check negative input
x = -1 * torch.ones(1, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

# Check discontinuity point
x = torch.zeros_like(x, requires_grad=True)
y = my_diffprog_func(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: tensor([8.8285], grad_fn=<AddBackward0>), y_prime: tensor([5.])
y: tensor([1.], grad_fn=<PowBackward0>), y_prime: tensor([-2.])
y: tensor([-0.], grad_fn=<MulBackward0>), y_prime: tensor([-1728.])
```

1.2 Inconsistent derivatives of a differentiable function

- Implement g as a DiffProg function in PyTorch so that PyTorch returns a derivative of 0 at $\hat{x} = 0$.
- Implement g again in DiffProg so that PyTorch now returns a derivative of 897 at exactly the same point $\hat{x} = 0$.

In [5]:

```python
def diffprog_g(x):
    """
    Continuous function g
    Input x is a torch scalar
    """
    return torch.pow(x, 2)

# Check discontinuity point
x = torch.zeros_like(x, requires_grad=True)
y = diffprog_g(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: tensor([0.], grad_fn=<PowBackward0>), y_prime: tensor([0.])
```

In [6]:

```python
def diffprog_g(x):
    """
    Continuous function g
    Input x is a torch scalar
    Derivative at x=0 evaluates to 897
    """
    if x != 0:
        return torch.pow(x, 2)
    else:
        return 897 * x

# Check discontinuity point
x = torch.zeros_like(x, requires_grad=True)
y = diffprog_g(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: tensor([0.], grad_fn=<MulBackward0>), y_prime: tensor([897.])
```

1.3 Derivatives with Loops: When is it Valid?

- Write a DiffProg function in PyTorch which takes an input x_0 and iteratively updates $x_{t+1} \leftarrow \frac{x_t}{2}$ until a stopping criterion $|x_t| < 10^{-6}$ is satisfied.
- Plot this function in the range [−1, 1]. Are the derivatives of this function well-defined everywhere?
- Find a point $\hat{x}$ such that implementing the stopping criterion as $|x_t| < 10^{-6}$ or $|x_t| \leq 10^{-6}$ changes the value of the derivative returned by PyTorch. Is the derivative mathematically well-defined at $\hat{x}$.
- Write out the (mathematical) function $\psi : R \to R$ which is implemented by this DiffProg function.

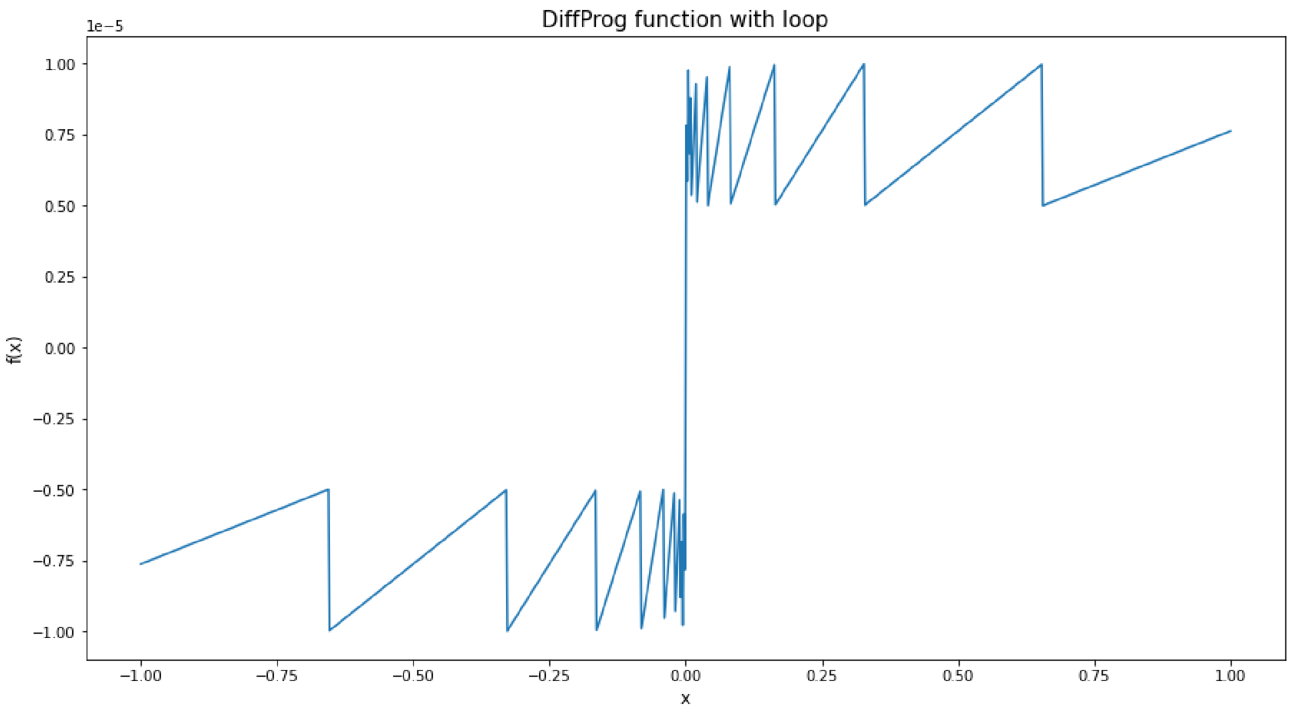
In [7]:

```python
def diffprog_loop(x_0):
    """
    Takes intial x_0 as torch scalar to start
    Updates until |x| < 10e-6
    """
```

```
    x = x_0
    while torch.abs(x) >= 10e-6:
        x = x / 2
    return x

x = torch.linspace(-1, 1, 1000)
f = [diffprog_loop(i) for i in x]
# f_prime = torch.autograd.grad(outputs=f, inputs=[x], allow_unused=True)[0]

fig = plt.figure(figsize=(15, 8))
plt.plot(x, f, label='f')
# plt.plot(x, f_prime, label="f'")
plt.title('DiffProg function with loop', fontsize=15)
plt.xlabel('x', fontsize=12)
plt.ylabel('f(x)', fontsize=12)
```

Out[7]:

```
Text(0, 0.5, 'f(x)')
```



This function is not continuous and has many points of discontinuity, so it is not differentiable almost anywhere.

In [8]:

```
x = torch.tensor(10e-6, requires_grad=True)
y = diffprog_loop(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: 4.999999873689376e-06, y_prime: 0.5
```

In [37]:

```
def diffprog_loop(x_0):
    """
    Takes intial x_0 as torch scalar to start
    Updates until x <= 10e-6
    """
    x = x_0
    while torch.abs(x) > 10e-6:
        print(x)
        x = x / 2
    return x

x = torch.tensor(10e-6, requires_grad=True)
y = diffprog_loop(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
y: 9.999999747378752e-06, y_prime: 1.0
```

At $\hat{x} = 10^{-6}$, the derivative produced by PyTorch changes from $f' = 0.5$ to $f' = 1$ when the stopping criterion changes.

The function for $\psi : \mathbb{R} \to \mathbb{R}$ with x_0 as the initial starting value is

$$x_t = \begin{cases} \frac{x}{2}, & \text{if } |x_t| \geq 10^{-6} \\ x_t & \text{otherwise} \end{cases}$$

1.4 When can we not use branches in differentiable programs?

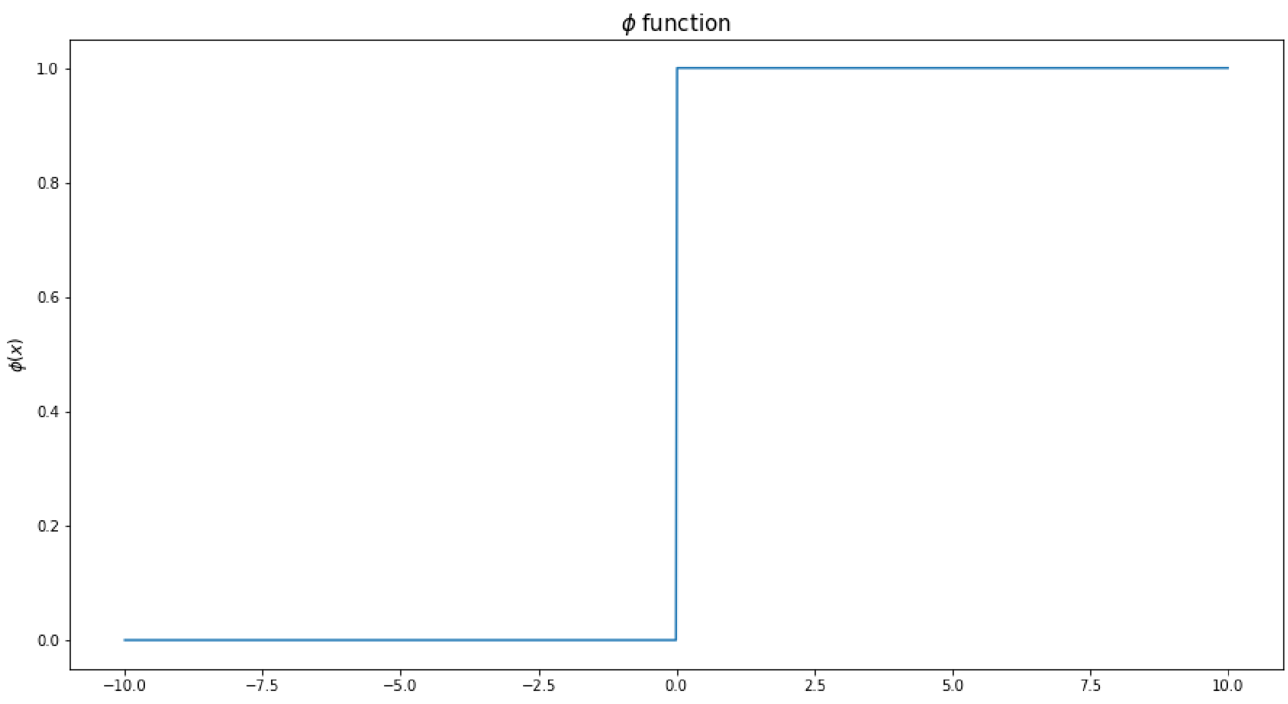
- Plot this function. Is it continuous? Is it differentiable almost everywhere?
- Implement this in PyTorch. Try to compute its derivatives. What do we get?
- Can we train a differentiable program containing this function as a component using stochastic gradient descent? Why or why not? Justify your answer in words.

In [10]:

```
def function_h(x):
    """
    Returns 1 if x is positive
    Else 0
    x is a torch scalar.
    """
    if x >= 0:
        return torch.ones_like(x, requires_grad=True)
    else:
        return torch.zeros_like(x, requires_grad=True)

x = torch.linspace(-10, 10, 1000, requires_grad=True)
function_vals = [function_h(i).item() for i in x]

fig = plt.figure(figsize=(15, 8))
plt.plot(x.detach().numpy(), function_vals)
plt.title('$\phi$ function', fontsize=15)
plt.xlabel('x', fontsize=12)
plt.ylabel('$\phi(x)$', fontsize=12)
```

Out[10]:

```
Text(0, 0.5, '$\\phi(x)$')
```



This function is discontinuous at $x = 0$ but is differentiable almost everywhere.

In [43]:

```
x = torch.rand(1, requires_grad=True)
print('x = ', x)
# call the function using the `apply` method
y = function_h(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

x = -1 * torch.rand(1, requires_grad=True)
print('x = ', x)
# call the function using the `apply` method
y = function_h(x)
```

```python
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')

x = torch.zeros(1, requires_grad=True)
print('x = ', x)
# call the function using the `apply` method
y = function_h(x)
y_prime = torch.autograd.grad(outputs=y, inputs=[x], allow_unused=True)[0]
print(f'y: {y}, y_prime: {y_prime}')
```

```
x =  tensor([0.3186], requires_grad=True)
y: tensor([1.], requires_grad=True), y_prime: None
x =  tensor([-0.8586], grad_fn=<MulBackward0>)
y: tensor([0.], requires_grad=True), y_prime: None
x =  tensor([0.], requires_grad=True)
y: tensor([1.], requires_grad=True), y_prime: None
```

We find that PyTorch is unable to compute the derivatives for this function since it is returning `None` type.

We cannot use this kind of function for gradient descent because while there are many minimums for the function, the derivative of it is 0 everywhere. Therefore trying to iterate over this gradient will be useless since there is no direction of "steepest" descent - there is no where to move.

2 Data Augmentation

Data augmentation can be applied at training time or testing time.

- Training time: in each iteration, we sample a minibatch, take one transformation per-image and use those instead to compute the minibatch stochastic gradient. The rest of the training loop continues as usual.
- Test time: we predict an output for an image x as follows. Take augmentations $x_1, x_2, \cdots, x_T$ of x. For each augmented image x_i, obtain prediction y_i. The combined prediction y for image x is obtained by taking a majority vote from $y_1, \cdots, y_T$. Note that the augmentations can only be used to compute the accuracy but not the loss.

In this exercise, we will try four combinations:

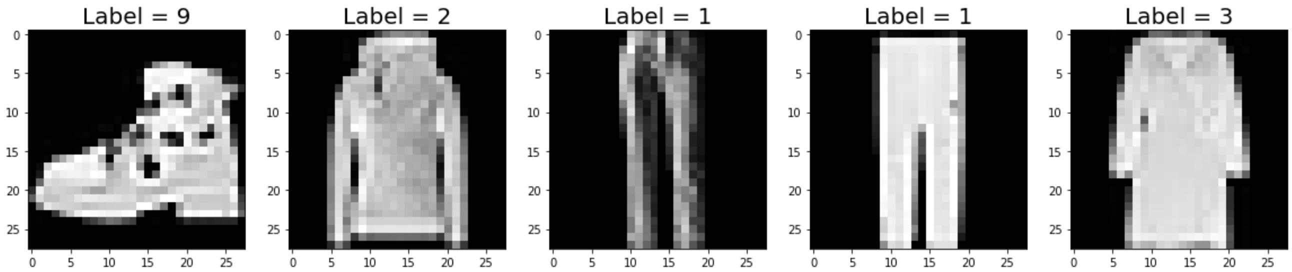
1. No data augmentation for training or testing
2. Use data augmentation for training but not for testing
3. Use data augmentation for testing but not for training
4. Use data augmentation for both training and testing

In [13]:

```python
from torchvision.datasets import FashionMNIST
import matplotlib.pyplot as plt

# download dataset (~117M in size)
train_dataset = FashionMNIST('./data', train=True, download=True)
X_train = train_dataset.data # torch tensor of type uint8
y_train = train_dataset.targets # torch tensor of type Long
test_dataset = FashionMNIST('./data', train=False, download=True)
X_test = test_dataset.data
y_test = test_dataset.targets

# choose a subsample of 10% of the data:
idxs_train = torch.from_numpy(
    np.random.choice(X_train.shape[0], replace=False, size=X_train.shape[0]//10)).long()
X_train, y_train = X_train[idxs_train], y_train[idxs_train]
# idxs_test = torch.from_numpy(
#     np.random.choice(X_test.shape[0], replace=False, size=X_test.shape[0]//10))
# X_test, y_test = X_test[idxs_test], y_test[idxs_test]

print(f'X_train.shape = {X_train.shape}')
print(f'n_train: {X_train.shape[0]}, n_test: {X_test.shape[0]}')
print(f'Image size: {X_train.shape[1:]}')

f, ax = plt.subplots(1, 5, figsize=(20, 4))
for i, idx in enumerate(np.random.choice(X_train.shape[0], 5)):
    ax[i].imshow(X_train[idx], cmap='gray', vmin=0, vmax=255)
    ax[i].set_title(f'Label = {y_train[idx]}', fontsize=20)

# Normalize dataset: pixel values lie between 0 and 255
# Normalize them so the pixelwise mean is zero and standard deviation is 1

X_train = X_train.float()  # convert to float32
X_train = X_train.view(-1, 784)
mean, std = X_train.mean(axis=0), X_train.std(axis=0)
```

```python
X_train = (X_train - mean[None, :]) / (std[None, :] + 1e-6)  # avoid divide by zero

X_test = X_test.float()
X_test = X_test.view(-1, 784)
X_test = (X_test - mean[None, :]) / (std[None, :] + 1e-6)
```

```
X_train.shape = torch.Size([6000, 28, 28])
n_train: 6000, n_test: 10000
Image size: torch.Size([28, 28])
```



In [14]:

```python
from torch.nn.functional import cross_entropy

def compute_objective(net, X, y):
    """ Compute the multinomial logistic loss.
        net is a module
        X of shape (n, d) and y of shape (n,)
    """
    # send
    score = net(X)
    # PyTorch's function cross_entropy computes the multinomial logistic loss
    return cross_entropy(input=score, target=y, reduction='mean')

@torch.no_grad()
def compute_accuracy(net, X, y, augment=False, augmentations=8):
    """ Compute the classification accuracy
        If augment flag is True, then transform the X images augmentations number of times
        and compute accuracy based on majority vote from augmented images.
        X of shape (n, d) and y of shape (n,)
    """
    if augment:
        augment_predictions = torch.empty((X.shape[0], augmentations), dtype=torch.float32)
        for i in range(augmentations):
            # Get augmentation
            X_transformed = transform_selected_data(X)
            # Get score
            score = net(X_transformed)
            # Get prediction
            pred = torch.argmax(score, axis=1)  # returns 1d tensor
            augment_predictions[:, i] = pred

        # Take majority vote
        predictions = torch.mode(augment_predictions, axis=1)[0]
    else:
        score = net(X)
        predictions = torch.argmax(score, axis=1)  # class with highest score is predicted

    return (predictions == y).sum() * 1.0 / y.shape[0]

@torch.no_grad()
def compute_logs(net, augment=False, augmentations=8, verbose=False):
    train_loss = compute_objective(net, X_train, y_train)
    test_loss = compute_objective(net, X_test, y_test)

    # Do not report train accuracy on augmented images
    train_accuracy = compute_accuracy(net, X_train, y_train)
    # Can choose to augment test images and get test accuracy
    test_accuracy = compute_accuracy(net, X_test, y_test, augment, augmentations)
    if verbose:
        print(('Train Loss = {:.3f}, Train Accuracy = {:.3f}, ' +
               'Test Loss = {:.3f}, Test Accuracy = {:.3f}').format(
                train_loss.item(), train_accuracy.item(),
                test_loss.item(), test_accuracy.item())
    )
    return (train_loss, train_accuracy, test_loss, test_accuracy)

def minibatch_sgd_one_pass(net, X, y, learning_rate, batch_size, transform_flag=False, verbose=False):
    num_examples = X.shape[0]
    average_loss = 0.0
```

```python
    num_updates = int(round(num_examples / batch_size))
    for i in range(num_updates):
        idxs = np.random.choice(num_examples, size=(batch_size,))
        # Randomly transform the images in the batch
        if transform_flag:
            X_transformed = transform_selected_data(X[idxs])
        else:
            X_transformed = X[idxs]

        objective = compute_objective(net, X_transformed, y[idxs])
        average_loss = 0.99 * average_loss + 0.01 * objective.item()

        if verbose and (i+1) % 100 == 0:
            print(average_loss)

        gradients = torch.autograd.grad(outputs=objective, inputs=net.parameters())
        # perform SGD update. IMPORTANT: Make the update inplace!
        with torch.no_grad():
            for (w, g) in zip(net.parameters(), gradients):
                w -= learning_rate * g
    return net
```

In [15]:

```python
class MyConvNet(torch.nn.Module):
    def __init__(self, num_classes=10):
        super().__init__()
        self.conv_ensemble_1 = torch.nn.Sequential(
            torch.nn.Conv2d(1, 16, kernel_size=5, padding=2),
            torch.nn.ReLU(),
            torch.nn.MaxPool2d(2))
        self.conv_ensemble_2 = torch.nn.Sequential(
            torch.nn.Conv2d(16, 32, kernel_size=5, padding=2),
            torch.nn.ReLU(),
            torch.nn.MaxPool2d(2))
        self.fully_connected_layer = torch.nn.Linear(7*7*32, 10)

    def forward(self, x):
        x = x.view(-1, 1, 28, 28)  # reshape input; convolutions need a channel
        out = self.conv_ensemble_1(x)  # first convolution + relu + pooling
        out = self.conv_ensemble_2(out) # second convolution + relu + pooling
        out = out.view(out.shape[0], -1)  # flatten output
        out = self.fully_connected_layer(out)  # output layer
        return out
```

In [16]:

```python
import torchvision.transforms as transforms

def transform_selected_data(X):
    # X is of shape (B, 784), where B is the batch_size
    X = X.view(-1, 28, 28)  # reshape to 28x28
    transform1 = transforms.RandomResizedCrop((28, 28), scale=(0.75, 1.0), ratio=(0.75, 1.33), interpolation=2)
    transform2 = transforms.RandomRotation((-10, 10))
    X_transformed = transform2(transform1(X))
    return X_transformed.view(-1, 784) # reshape into a vector
```

In [19]:

```python
def compute_cnn_results(
    model, X_train, y_train, # CNN & tensors
    learning_rate, batch_size, max_iters, # Learning rate, batch size, max iterations paramters
    transform_flag=False, # Flag to transform the training images
    augment_flag=False, num_augmentations=8 # flag to augment the test data & how many augmentations
):
    """
    Train the passed CNN on X_train and optionally transform the training images.
    Optionally augment the testing images.
    Outputs a list of logged outputs
    """
    log = []
    print('Iteration 0', end=', ')
    # Don't transform or augment for 0th log
    log.append(
        compute_logs(model, verbose=True)
    )

    for j in range(MAX_ITERS):
        model = minibatch_sgd_one_pass(
            model, X_train, y_train,
            learning_rate, batch_size,
            transform_flag=transform_flag, verbose=False
```

```python
        )
        print(f'Iteration {j+1}', end=', ')
        log.append(
            compute_logs(model, augment=augment_flag, augmentations=num_augmentations)
        )
        # End when model perfectly interpolates training data (training acc == 100%)
        if log[-1][1] == 1.0:
            break

    return np.asarray(log)
```

In [20]:

```python
model = MyConvNet(num_classes=10)
LEARNING_RATE = 0.04
BATCH_SIZE = 16
MAX_ITERS = 100

logs = []

# No augmentations on training or testing
log1 = compute_cnn_results(
    model, X_train, y_train,
    LEARNING_RATE, BATCH_SIZE, MAX_ITERS,
    transform_flag=False, augment_flag=False
)
logs.append(log1)

# Augment training but not testing
log2 = compute_cnn_results(
    model, X_train, y_train,
    LEARNING_RATE, BATCH_SIZE, MAX_ITERS,
    transform_flag=True, augment_flag=False
)
logs.append(log2)

# Augment testing but not training
log3 = compute_cnn_results(
    model, X_train, y_train,
    LEARNING_RATE, BATCH_SIZE, MAX_ITERS,
    transform_flag=False, augment_flag=True
)
logs.append(log3)

# Augment training and testing
log4 = compute_cnn_results(
    model, X_train, y_train,
    LEARNING_RATE, BATCH_SIZE, MAX_ITERS,
    transform_flag=True, augment_flag=True
)
logs.append(log4)
```

```
Iteration 0, Train Loss = 2.287, Train Accuracy = 0.137, Test Loss = 4.767, Test Accuracy = 0.128
Iteration 1, Iteration 2, Iteration 3, Iteration 4, Iteration 5, Iteration 6, Iteration 7, Iteration 8, Iteration 9, Ite
ration 10, Iteration 11, Iteration 12, Iteration 13, Iteration 14, Iteration 15, Iteration 16, Iteration 17, Iteration 1
8, Iteration 19, Iteration 20, Iteration 21, Iteration 22, Iteration 23, Iteration 24, Iteration 25, Iteration 26, Itera
tion 27, Iteration 28, Iteration 29, Iteration 30, Iteration 31, Iteration 0, Train Loss = 0.004, Train Accuracy = 1.00
0, Test Loss = 52.032, Test Accuracy = 0.864
Iteration 1, Iteration 2, Iteration 3, Iteration 4, Iteration 5, Iteration 6, Iteration 7, Iteration 8, Iteration 9, Ite
ration 10, Iteration 11, Iteration 12, Iteration 13, Iteration 14, Iteration 15, Iteration 16, Iteration 17, Iteration 1
8, Iteration 19, Iteration 20, Iteration 21, Iteration 22, Iteration 23, Iteration 24, Iteration 25, Iteration 26, Itera
tion 27, Iteration 28, Iteration 29, Iteration 30, Iteration 31, Iteration 32, Iteration 33, Iteration 34, Iteration 35,
Iteration 36, Iteration 37, Iteration 38, Iteration 39, Iteration 40, Iteration 41, Iteration 42, Iteration 43, Iteratio
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eration 53, Iteration 54, Iteration 55, Iteration 56, Iteration 57, Iteration 58, Iteration 59, Iteration 60, Iteration
61, Iteration 62, Iteration 63, Iteration 64, Iteration 65, Iteration 66, Iteration 67, Iteration 68, Iteration 69, Iter
ation 70, Iteration 71, Iteration 72, Iteration 73, Iteration 74, Iteration 75, Iteration 76, Iteration 77, Iteration 7
8, Iteration 79, Iteration 80, Iteration 81, Iteration 82, Iteration 83, Iteration 84, Iteration 85, Iteration 86, Itera
tion 87, Iteration 88, Iteration 89, Iteration 90, Iteration 91, Iteration 92, Iteration 93, Iteration 94, Iteration 95,
Iteration 96, Iteration 97, Iteration 98, Iteration 99, Iteration 100, Iteration 0, Train Loss = 0.148, Train Accuracy =
0.949, Test Loss = 141.652, Test Accuracy = 0.867
Iteration 1, Iteration 2, Iteration 3, Iteration 4, Iteration 5, Iteration 6, Iteration 7, Iteration 8, Iteration 9, Ite
ration 10, Iteration 11, Iteration 12, Iteration 0, Train Loss = 0.005, Train Accuracy = 1.000, Test Loss = 341.585, Tes
t Accuracy = 0.881
Iteration 1, Iteration 2, Iteration 3, Iteration 4, Iteration 5, Iteration 6, Iteration 7, Iteration 8, Iteration 9, Ite
ration 10, Iteration 11, Iteration 12, Iteration 13, Iteration 14, Iteration 15, Iteration 16, Iteration 17, Iteration 1
8, Iteration 19, Iteration 20, Iteration 21, Iteration 22, Iteration 23, Iteration 24, Iteration 25, Iteration 26, Itera
tion 27, Iteration 28, Iteration 29, Iteration 30, Iteration 31, Iteration 32, Iteration 33, Iteration 34, Iteration 35,
Iteration 36, Iteration 37, Iteration 38, Iteration 39, Iteration 40, Iteration 41, Iteration 42, Iteration 43, Iteratio
n 44, Iteration 45, Iteration 46, Iteration 47, Iteration 48, Iteration 49, Iteration 50, Iteration 51, Iteration 52, It
eration 53, Iteration 54, Iteration 55, Iteration 56, Iteration 57, Iteration 58, Iteration 59, Iteration 60, Iteration
61, Iteration 62, Iteration 63, Iteration 64, Iteration 65, Iteration 66, Iteration 67, Iteration 68, Iteration 69, Iter
ation 70, Iteration 71, Iteration 72, Iteration 73, Iteration 74, Iteration 75, Iteration 76, Iteration 77, Iteration 7
8, Iteration 79, Iteration 80, Iteration 81, Iteration 82, Iteration 83, Iteration 84, Iteration 85, Iteration 86, Itera
tion 87, Iteration 88, Iteration 89, Iteration 90, Iteration 91, Iteration 92, Iteration 93, Iteration 94, Iteration 95,
Iteration 96, Iteration 97, Iteration 98, Iteration 99, Iteration 100,
```

Deliverable 1: Report final test accuracies

In [31]:

```python
for log in logs:
    print('Final test accuracy: ', str(log[-1, 3]))
```

```
Final test accuracy:  0.8645
Final test accuracy:  0.8674
Final test accuracy:  0.8686
Final test accuracy:  0.869
```

Deliverable 2: Plot 4 Models

In [28]:

```python
fig, ax = plt.subplots(2, 2, figsize=(15, 8))

# Plot no augmentations
ax[0][0].plot(logs[0][:, 0], label='No augs')
ax[0][1].plot(logs[0][:, 2], label='No augs')
ax[1][0].plot(logs[0][:, 1], label='No augs')
ax[1][1].plot(logs[0][:, 3], label='No augs')
# Plot train augmentations
ax[0][0].plot(logs[1][:, 0], label='Train augs')
ax[0][1].plot(logs[1][:, 2], label='Train augs')
ax[1][0].plot(logs[1][:, 1], label='Train augs')
ax[1][1].plot(logs[1][:, 3], label='Train augs')
# Plot test augmentations
ax[0][0].plot(logs[2][:, 0], label='Test augs')
ax[0][1].plot(logs[2][:, 2], label='Test augs')
ax[1][0].plot(logs[2][:, 1], label='Test augs')
ax[1][1].plot(logs[2][:, 3], label='Test augs')
# Plot both augmentations
ax[0][0].plot(logs[3][:, 0], label='Both augs')
ax[0][1].plot(logs[3][:, 2], label='Both augs')
ax[1][0].plot(logs[3][:, 1], label='Both augs')
ax[1][1].plot(logs[3][:, 3], label='Both augs')

ax[1][1].set_xlabel('# Passes', fontsize=12)
ax[1][0].set_xlabel('# Passes', fontsize=12)
ax[0][0].set_ylabel('Loss', fontsize=12)
ax[1][0].set_ylabel('Accuracy', fontsize=12)
ax[0][0].set_title('Train Loss', fontsize=12)
ax[0][1].set_title('Test Loss', fontsize=12)
ax[1][0].set_title('Train Accuracy', fontsize=12)
```

```
ax[1][1].set_title('Test Accuracy', fontsize=12)
ax[0][0].legend(fontsize=12)
```

Out[28]:

```
<matplotlib.legend.Legend at 0x7f7a1403b9d0>
```