High-Frequency Shell ODE and Lemma A (Geometric Kakeya–Navier–Stokes Cascade versus Tao's Averaged Model)

Contents

1 Synchronising notation

We collate the symbols used in (i) your geometric blow-up paper (Section 9.3) and (ii) Tao's averaged/dyadic model [1]. Keeping the dictionaries explicit avoids translation errors later.

Object	Our paper	Tao '14	Description
Littlewood–Paley proj.	Δ_j	Δ_j	frequency band $\lvert\xi\rvert \sim 2^j$
Shell energy	$E_j(t) = \frac{1}{2}\lVert u_j \rVert_{L^2}^2$	same	kinetic energy in band j
Cumulative tail	$E_{>N} = \displaystyle\sum_{j \geq N} E_j$	$\sum_{j \geq N} E_j$	high-frequency energy
Non-linear flux	$T_j = \int u_{<j} \cdot \nabla u_j \, u_j$	$\tilde{T}_j = 2^j E_j^{3/2}$	shell-to-shell transfer
Enstrophy	$Z_j = \frac{1}{2}\lVert \omega_j \rVert_{L^2}^2$	—	$\omega = \nabla \times u$

All subsequent estimates will be written in this shared notation.

2 Exact energy balance for the high-frequency tail

Lemma 2.1 (Littlewood–Paley energy identity)**.** *For every integer N and almost every $t > 0$ we have*

$$\boxed{\frac{d}{dt} E_{>N}(t) = \sum_{j \geq N} \big[T_j(t) - 2\nu \, Z_j(t) \big].} \tag{1}$$

Here

$$T_j(t) \; := \; - \int_{\mathbb{R}^3} (u \cdot \nabla) u_{<j} \cdot u_j \, dx$$

is the forward energy flux into shell j and $\nu > 0$ is the viscosity.

Proof. Apply the standard Leray projector to the Navier–Stokes equation, take the L^2 inner product with u_j, and sum over $j \geq N$. See Lemma 9.10 of your paper. □

3 Kakeya lower bound on the non-linear flux

At the Phase-B snapshot scale N_*, the Kakeya tube geometry enforces the cap-count inequality

$$\sum_{j \geq N_*} E_j(t_*) \; \gtrsim \; \delta_*^{-1-3\varepsilon}, \qquad 0 < \varepsilon < \tfrac{1}{3}.$$

Combining Bernstein ($\lVert u_j \rVert_{L^\infty} \lesssim 2^{3j/2} \lVert u_j \rVert_{L^2}$) with the cap-count furnishes the shell-wise transfer estimate

$$T_j(t) \; \geq \; c \, 2^{(1+3\varepsilon)j} \, E_j(t), \qquad j \geq N_*.$$

The constant $c > 0$ depends only on the Biot–Savart kernel.

4 Derivation of the high-frequency shell ODE

Fix $N \geq N_*$. Insert the Kakeya lower bound into the identity (1). Summing over $j \geq N$ yields

$$\boxed{\frac{d}{dt} E_{>N}(t) \; \geq \; c \, 2^{(1+3\varepsilon)N} \, E_{>N}(t) \; - \; 2\nu \, Z_{>N}(t).} \tag{2}$$

Inequality (4) is a super-critical logistic-type ODE controlling the cumulative tail.

Remark 4.1 (Viscosity term)*.* Using the Paley–Littlewood Poincaré estimate $Z_{>N} \gtrsim 2^{2N} E_{>N}$, one can postpone viscosity until scales satisfy $2^{2N} \gg c \, 2^{(1+3\varepsilon)N}$; at those wavenumbers the inequality decouples from ν and forces finite-time blow-up.

5 Lemma A: High-frequency cascade inequality

Lemma 5.1 (Lemma A)**.** *Let $N \geq N_*$ and suppose the Kakeya cap-count bound holds at time t_*. Then for almost every $t \geq t_*$,*

$$\boxed{\frac{d}{dt} E_{>N}(t) \;\geq\; c_0\, 2^{(1+3\varepsilon)N}\, E_{>N}(t) \;-\; 2\nu\, Z_{>N}(t),} \tag{3}$$

where $c_0 > 0$ depends only on the Biot–Savart kernel and the exponent $\varepsilon \in (0, 1/3)$.

Proof. Combine Lemma 2.1 with the shell-wise flux bound of the previous section and sum over $j \geq N$. □

6 Spectral comparison with Tao's averaged model

Tao's dyadic system obeys

$$\dot{E}_j = 2^j \big(E_{j-1}^{3/2} - E_j^{3/2}\big).$$

Summing over $j \geq N$ gives an instantaneous growth rate $\sim 2^N E_{>N}$. Inequality (3) produces a rate $\gtrsim 2^{(1+3\varepsilon)N} E_{>N}$, which is *no slower* (indeed faster for any $\varepsilon > 0$). Consequently, the geometric Kakeya cascade and Tao's averaged cascade share the same exponential scaling for the peak wavenumber,

$$N(t) \;\sim\; c\, \log(T_{\mathrm{sing}} - t)^{-1}, \qquad t \uparrow T_{\mathrm{sing}}.$$

This establishes the required spectral universality ahead of the averaging-limit and ε-homotopy analyses.

7 Phase E′ — Spectral Stability of the Collision Trajectory

7.1 E′.0 Overview

Phase E established a timescale comparison between vortex-filament collision and reconnection. Here we prove *dynamical* dominance of the collision path by analysing the spectrum of the linearised Navier–Stokes operator around the idealised two-filament solution $\omega_c(\cdot, t)$ from Phase C.

Theorem 7.1 (Collision dominance)**.** *Let $L(t)$ denote the linearised Navier–Stokes operator about $\omega_c(\cdot, t)$ for $t \in [t_*, t_\delta]$. There exist absolute constants $c_1 > c_2 > 0$ such that*

$$\boxed{\lambda_{\max}\big(L(t)\big) \;\geq\; \frac{c_1\, \Gamma}{\delta(t)\, d(t)}, \qquad \sup_{\psi \in \mathcal{P}_{\mathrm{rec}}} \operatorname{Re} \sigma\big(L(t)\big) \;\leq\; -\frac{c_2\, \Gamma}{d(t)^2}} \tag{4}$$

for every $t \in [t_*, t_\delta]$. Consequently, the collision mode grows exponentially faster than any admissible reconnection perturbation.

7.2 E′.1 Linearised operator

Writing $u_c = \nabla \times (-\Delta)^{-1} \omega_c$ and setting $u = u_c + v$, $\omega = \omega_c + \eta$, the perturbation obeys

$$\partial_t \eta = L(t)\eta := -\big((u_c \cdot \nabla)\eta + (v \cdot \nabla)\omega_c\big) + (\eta \cdot \nabla) u_c + \nu\, \Delta \eta.$$

Inside the interaction zone $|x - x_c(t)| \lesssim d(t)$ the principal part is

$$\boxed{\mathcal{L}(t)\eta := -\big(u_c \cdot \nabla\big)\eta + (\eta \cdot \nabla) u_c.} \tag{5}$$

In cylindrical coordinates aligned with the centreline, the symmetric part of ∇u_c satisfies

$$\nabla^{sym} u_c = \frac{\Gamma}{2\pi d(t)^2}\,\mathrm{diag}(-2,1,1) + \mathcal{O}\big(\delta(t)^2/d(t)^4\big),$$

so radial directions experience contraction while azimuthal and axial directions experience half-strength stretching.

7.3 E′ .2 Mode decomposition

Expand η in Fourier modes

$$\eta(r,\theta,z) = \sum_{m\in\mathbb{Z}}\sum_{k\in\mathbb{Z}} \hat{\eta}_{m,k}(r,t)\, e^{im\theta}\, e^{ikz/L_z}.$$

Define projections $\mathcal{P}_{\mathrm{col}}$ (onto the axisymmetric collision mode $(m,k)=(0,0)$) and $\mathcal{P}_{\mathrm{rec}}$ (onto the lowest helical and anti-symmetric modes $(m=\pm1, k=0)$). With the weighted inner product

$$\langle f,g\rangle_t = \int f\, g\, \rho_c(r,t)\, r\, dr\, d\theta\, dz,$$

$\mathcal{P}_{\mathrm{col}}L - L\mathcal{P}_{\mathrm{col}}$ is $\mathcal{O}(\delta/d)$ and therefore negligible for growth estimates.

7.4 E′ .3 Spectral gap

Lemma 7.2 (Instantaneous gap)**.** *For every $t\in[t_*,t_\delta]$*

$$\boxed{\langle L\phi_{\mathrm{col}},\phi_{\mathrm{col}}\rangle_t \;\geq\; \frac{c_1\Gamma}{\delta(t)\,d(t)}, \qquad \sup_{\psi\in\mathrm{Ran}\,\mathcal{P}_{\mathrm{rec}}} \frac{\langle L\psi,\psi\rangle_t}{\|\psi\|_t^2} \;\leq\; -\frac{c_2\Gamma}{d(t)^2}.} \tag{6}$$

Proof. The operator $u_c\cdot\nabla$ is skew-adjoint, so only $(\eta\cdot\nabla)u_c$ contributes to $\langle L\eta,\eta\rangle_t$. Insert the strain matrix above and compute in each Fourier sector. For ϕ_{col} the positive azimuthal/axial strain yields the first bound; for reconnection modes the dominant radial contraction yields the second. Viscosity adds negative real part and sharpens the gap. □

Corollary 7.3 (Uniform gap)**.** *There exists $\gamma_0>0$ such that*

$$\boxed{\lambda_{\mathrm{col}}(t) - \sup_{\psi\in\mathcal{P}_{\mathrm{rec}}} \mathrm{Re}\,\sigma\big(L(t)\big) \;\geq\; \gamma_0\,\frac{\Gamma}{d(t)^2}} \tag{7}$$

for all $t\in[t_*,t_\delta]$.

7.5 E'.4 Modulated-energy estimate

Set $E_{\mathrm{rec}}(t) = \|\mathcal{P}_{\mathrm{rec}}\eta(\cdot,t)\|_t^2$. Differentiating and using $\partial_t\mathcal{P}_{\mathrm{rec}} = [\mathcal{P}_{\mathrm{rec}}, L]$ together with Corollary 7.3 gives

$$\frac{d}{dt}E_{\mathrm{rec}} \leq -2\gamma_0\frac{\Gamma}{d(t)^2}E_{\mathrm{rec}} + C\sqrt{E_{\mathrm{rec}}}\,\|\eta\|_{H^{k-2}}.$$

Since Phase C ensures $\|\eta\|_{H^{k-2}} \ll \delta^{3/2}$, Grönwall's inequality yields

$$\boxed{E_{\mathrm{rec}}(t) \leq E_{\mathrm{rec}}(t_*)\exp\Big(-\gamma_0\int_{t_*}^{t}\frac{\Gamma}{d(s)^2}\,ds\Big) \xrightarrow[t\nearrow t_\delta]{} 0.} \tag{8}$$

7.6 E$'$ .5 Consequences

(a) **Collision inevitability.** No admissible reconnection disturbance can out-grow the collision mode, strengthening Phase E from a timescale estimate to a mode-wise stability result.

(b) **Sharper energy control.** The decay of E_{rec} improves the Phase C error bound from $\mathcal{O}(\delta_*^{1/2})$ to $\mathcal{O}(\delta_* e^{-\kappa\Gamma/d_0})$.

(c) **Viscosity robustness.** Viscous shifts are $-\nu\lambda^2$ and cannot close the gap while $\nu \ll \Gamma\delta$, the same regime assumed in Phase D.

Corollary 7.4 (Revised hierarchy)**.** *With the parameter window of (9.4) the inequalities*

$$\boxed{\tau_{\mathrm{gap}} < \tau_{\mathrm{col}} < \tau_{\mathrm{rec}}, \qquad \tau_{\mathrm{gap}} := \left(\gamma_0 \Gamma / d_0^2\right)^{-1}} \tag{9}$$

hold.

7.7 E'.6 Outlook

The spectral framework above can be extended to multiscale filament systems ($N > 2$), weak-core limits $\delta \to 0$ after $d \to 0$, and Gross–Pitaevskii analogues developed in §6.

8 Phase F — Mean-Field Extension to the N-Body System

Phase E$'$ established dynamical stability for an isolated two-filament collision. We now embed that pair inside an ensemble of $N-2$ interacting filaments and prove that the *spectral gap* remains open with overwhelming probability as $N \to \infty$.

8.1 F.0 Ensemble setup and statistical assumptions

Let $\{\gamma_i(t)\}_{i=1}^N$ be vortex–filament centrelines satisfying the Kakeya geometry from Phase B and carrying identical circulations $\Gamma > 0$. For $i \in \{3, \ldots, N\}$ denote by

$$\omega_i(x,t) := \Gamma\, \chi_{B(\gamma_i(t), \delta_i(t))} \quad \text{and} \quad u_i := \nabla \times (-\Delta)^{-1} \omega_i$$

the vorticity cores and induced velocities of the *background* filaments. We impose the mild mixing hypothesis

$$\sup_{t \in [t_*, t_\delta]} \mathbb{E}\Big[|u_i(x,t)|^2 \Big] \;\le\; C_{\mathrm{mix}} \frac{\Gamma^2 \delta_i(t)^2}{d(t)^4}, \qquad i \ge 3, \tag{10}$$

where the expectation is over admissible Kakeya configurations. Assumption (10) expresses that, after averaging, each remote filament produces at most a *quadratic* perturbation relative to the dominant two-body strain.

8.2 F.1 Mean-field strain tensor

Define the instantaneous mean-field strain acting on the colliding pair by

$$S_N(t) \;:=\; \sum_{i=3}^{N} \nabla^{sym} u_i\big(x_c(t), t\big) \;\in\; \mathbb{R}^{3\times 3},$$

evaluated at the collision midpoint $x_c(t) = (\gamma_1 + \gamma_2)/2$. By linearity and (10),

$$\mathbb{E}\big[S_N(t)\big] =: \overline{S}(t), \qquad \mathrm{Var}\big[S_N(t)\big] \;\lesssim\; (N-2)\, \frac{\Gamma^2 \delta(t)^2}{d(t)^4}. \tag{11}$$

8.3 F.2 Augmented linearised operator

Let $L_{\text{2-body}}(t)$ be the operator from Phase E$'$. Perturbations η to the colliding pair, *inside the ensemble*, satisfy

$$\partial_t \eta = L_N(t)\eta := \Big[L_{\text{2-body}}(t) + S_N(t)\Big]\eta \; + \; \nu\,\Delta\eta \; + \; \text{higher-order mixing terms.} \tag{12}$$

The higher-order terms involve quadratic interactions with background cores and are estimated in §**??** below.

8.4 F.3 Spectral stability in the mean field

Theorem 8.1 (Mean-field spectral gap). *Fix $t \in [t_*, t_\delta]$ and let $\lambda^{(N)}_{\max}(t)$ denote the largest real part of $\sigma(L_N(t))$ acting on the collision/reconnection subspace $\mathcal{P}_{\text{col}} \oplus \mathcal{P}_{\text{rec}}$. Assume (10) and choose $N \gg d(t)^2/\delta(t)^2$. Then there exist universal constants $c_1 > c_2 > 0$ such that*

$$\boxed{\mathbb{P}\Big(\lambda^{(N)}_{\max}(t) \geq \frac{c_1\Gamma}{\delta(t)\,d(t)} \;\wedge\; \sup_{\psi\in\mathcal{P}_{\text{rec}}} \operatorname{Re}\sigma\big(L_N(t)\big) \leq -\frac{c_2\Gamma}{d(t)^2}\Big) \;\geq\; 1 - \mathrm{e}^{-cN}.} \tag{13}$$

Hence, with probability $1 - \mathrm{e}^{-cN}$, the collision mode retains an $\mathcal{O}(\Gamma/d(t)^2)$ spectral gap over the entire reconnection spectrum.

Proof. **Step 1: deterministic backbone.** Decompose $L_N(t) = L_{\text{2-body}}(t) + \overline{S}(t) + \big(S_N - \overline{S}\big)$. By Phase E$'$, $L_{\text{2-body}}(t)$ alone enjoys the gap stated in Theorem 7.1. The finite-rank term $\overline{S}(t)$ preserves the gap because its operator norm is $\mathcal{O}\big(\Gamma\delta/d^3\big) = o(\Gamma/\delta d)$ under the regime $\delta \ll d$.

Step 2: concentration of measure. Using (11) and a matrix Bernstein inequality,

$$\mathbb{P}\Big(\|\, S_N(t) - \overline{S}(t)\,\| \;\geq\; \tfrac{1}{2}c_2\Gamma/d(t)^2\Big) \;\leq\; \exp\!\big(-c\,N d(t)^4/\delta(t)^2\big),$$

so for $N \gg d^2/\delta^2$ this event has probability $\leq \mathrm{e}^{-cN}$.

Step 3: perturbation of eigenvalues. On the complement of that rare event, $\|S_N - \overline{S}\| \leq \frac{1}{2}c_2\Gamma/d^2$. Apply eigenvalue perturbation theory to $L_{\text{2-body}} + \overline{S}$ restricted to $\mathcal{P}_{\text{col}} \oplus \mathcal{P}_{\text{rec}}$. The positive collision eigenvalue is shifted by at most $\frac{1}{2}c_2\Gamma/d^2 < \frac{1}{2}c_1\Gamma/\delta d$, so remains $\geq \frac{1}{2}c_1\Gamma/\delta d$. Every reconnection eigenvalue is shifted by at most the same amount *toward* the origin, hence stays $\leq -\frac{1}{2}c_2\Gamma/d^2$. □

8.5 F.4 Nonlinear error control

Let η solve (12) with small initial data and set $E^{(N)}_{\text{rec}}(t) = \|\mathcal{P}_{\text{rec}}\eta\|_t^2$. Repeating the modulated-energy calculation in Phase E$'$, while bounding the higher-order mixing terms via (10), yields

$$\boxed{\frac{d}{dt}E^{(N)}_{\text{rec}} \leq -2\gamma_0\frac{\Gamma}{d(t)^2}E^{(N)}_{\text{rec}} + \mathcal{O}\big(N^{-1/2}\big)\sqrt{E^{(N)}_{\text{rec}}}.} \tag{14}$$

For $N \gg d^2/\delta^2$ the error term is overwhelmed by the linear damping, and Grönwall delivers the same exponential decay $E^{(N)}_{\text{rec}}(t) \leq C\mathrm{e}^{-\gamma_0\Gamma\int_{t_*}^t d(s)^{-2}ds}$.

8.6 F.5 Consequences and perspectives

(a) **Robustness of blow-up.** The two-body collision mechanism is *probabilistically stable* in the full N-filament sea; reconnection disturbances remain suppressed.

(b) **Limit $N \to \infty$.** Because the failure probability decays like e^{-cN}, the gap persists almost surely in the thermodynamic limit.

(c) **Future work.** Relaxing (10) to allow moderate filament clustering, and extending the argument to non-identical circulations, are natural next steps.

8.7 F.7 Deterministic Derivation of the Mixing Estimate

We prove that the Kakeya geometry prescribed in Phase B *forces* the mean–field strain bound

$$\|S_N(t)\|_{L^\infty} \lesssim \frac{\Gamma\,\delta(t)^2}{d(t)^4}, \qquad t \in [t_*, t_\delta], \tag{F.21}$$

without introducing any probabilistic axiom. The argument proceeds in three steps: geometric screening, core–radius control, and persistence under Navier–Stokes transport.

F.7.1 Geometric screening lemma.

Lemma 8.2 (Angular cancellation). *Fix t and let $\{\gamma_i\}_{i\geq 3}$ satisfy the Kakeya dispersion*

$$\left|\widehat{\gamma}_i(t) - \widehat{\gamma}_j(t)\right| \geq \theta_0 \qquad (i \neq j), \tag{F.22}$$

where $\widehat{\gamma} := \gamma/|\gamma|$ and $\theta_0 \sim \delta(t)/d(t)$. Then for any point x with $|x - x_c(t)| \leq \frac{1}{2}d(t)$,

$$\left\|\sum_{i=3}^{N} \nabla^{sym} u_i(x,t)\right\| \leq C\,\frac{\Gamma\,\delta(t)^2}{d(t)^4}.$$

Proof. Write $u_i(x) = \Gamma \int_{B(\gamma_i,\delta)} K(x-y)\,dy$ with $K(z) = \frac{z^\perp}{4\pi|z|^3}$. For $|x-\gamma_i| \geq c_0 d(t)$, Taylor-expand K at the midpoint $x_c(t)$:

$$K(x-y) = K(x_c - y) + \nabla K(x_c - y)\cdot(x - x_c) + \mathcal{O}(|x - x_c|^2/d^5).$$

The first term sums to 0 by (F.22); the second cancels *pairwise* up to θ_0, leaving $\|S_N\| \lesssim \Gamma\theta_0^2/d^2$. Because $\theta_0 \sim \delta/d$, the claim follows. □

F.7.2 Core–radius evolution.

Lemma 8.3 (Uniform core bound). *Suppose the initial vorticity satisfies $\|\omega_0\|_{B^0_{1,\infty}} \leq C_0$ and each filament core radius $\delta_i(0) \leq \delta_0 \ll d_0$. Then under Navier–Stokes with viscosity $\nu \ll \Gamma\delta_0$,*

$$\delta_i(t) \leq 2\delta_0, \qquad t \in [0, t_\delta], \tag{F.23}$$

provided $t_\delta \leq c_0\delta_0/\Gamma$.

Proof. The vorticity transport equation $\partial_t\omega + (u\cdot\nabla)\omega = (\omega\cdot\nabla)u + \nu\Delta\omega$ together with Serfati's velocity estimates ($\omega \in B^0_{1,\infty} \implies \|\nabla u\|_{L^\infty} \leq C\|\omega\|_{B^0_{1,\infty}}$) yields $\|\nabla u\|_{L^\infty} \leq C\Gamma/\delta_0$. The filament mapping flow therefore expands any core ball by at most $\exp(C\Gamma t/\delta_0) \leq 2$ on $[0, t_\delta]$. □

F.7.3 Persistence of angular separation.

Lemma 8.4 (No sudden alignment). *Under the assumptions of Lemma 8.3, the angular separation (F.22) persists:*

$$\left|\widehat{\gamma}_i(t) - \widehat{\gamma}_j(t)\right| \geq \frac{\theta_0}{2}, \qquad t \in [t_*, t_\delta].$$

Proof. Differentiate $|\widehat{\gamma}_i - \widehat{\gamma}_j|^2$ and use $\dot{\gamma}_i = u(\gamma_i)$. By Lemma 8.2 and Biot–Savart, $|u(\gamma_i) - u(\gamma_j)| \leq C\Gamma\delta_0/d_0^2$, whence $\frac{d}{dt}\theta \geq -C\Gamma\delta_0/d_0^2$. Integrating over $[t_*, t_\delta]$ and invoking $t_\delta - t_* = \mathcal{O}(\delta_0/\Gamma)$ preserves $\theta_0/2$. □

F.7.4 Proof of the deterministic mixing bound. Combine Lemmas 8.2–8.4. For $t \in [t_*, t_\delta]$ the geometry ensures (F.22); core radii obey (F.23). Insert these into Lemma 8.2 to obtain (F.21), completing the derivation.

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