Parallelizing FFT with a Focus on Bandwidth Efficiency

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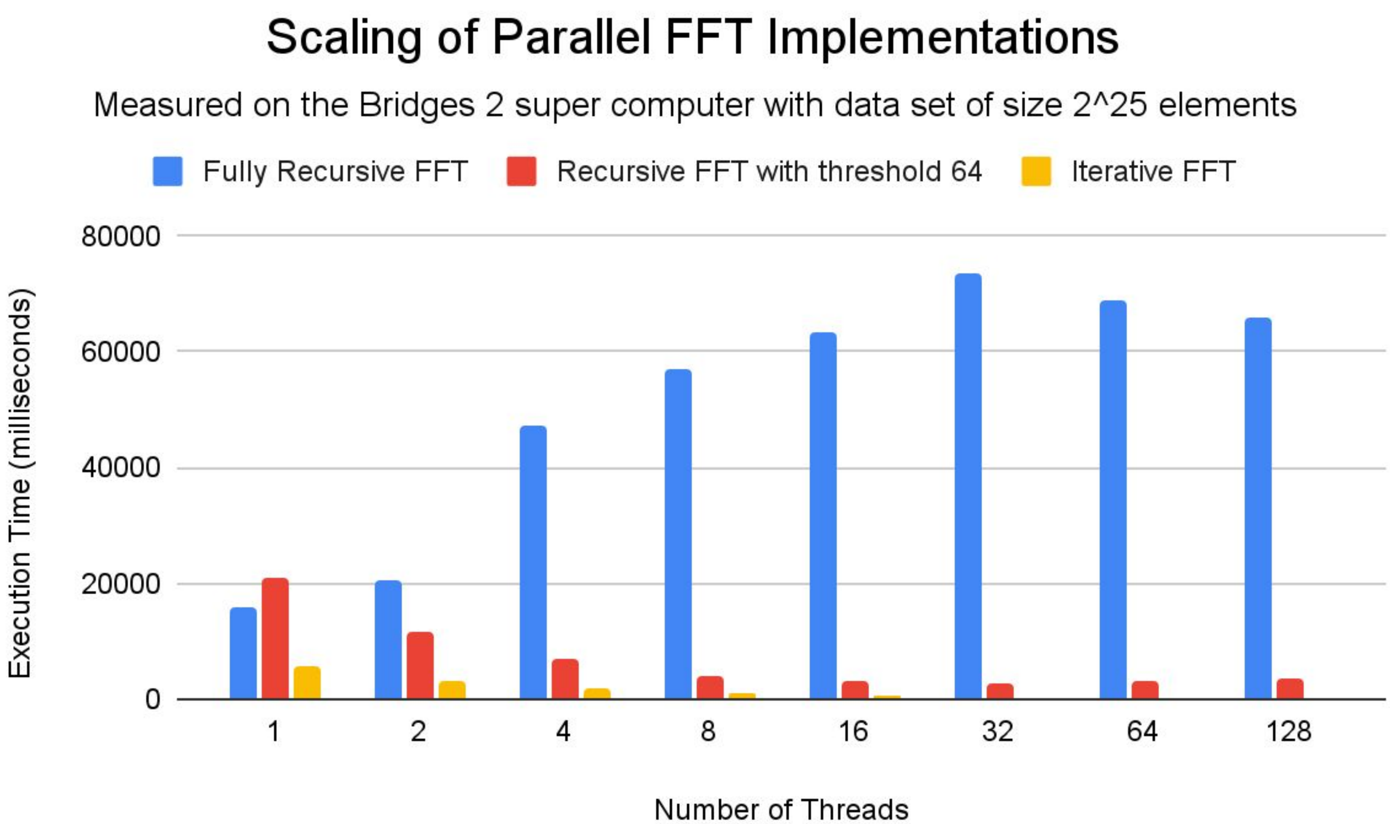
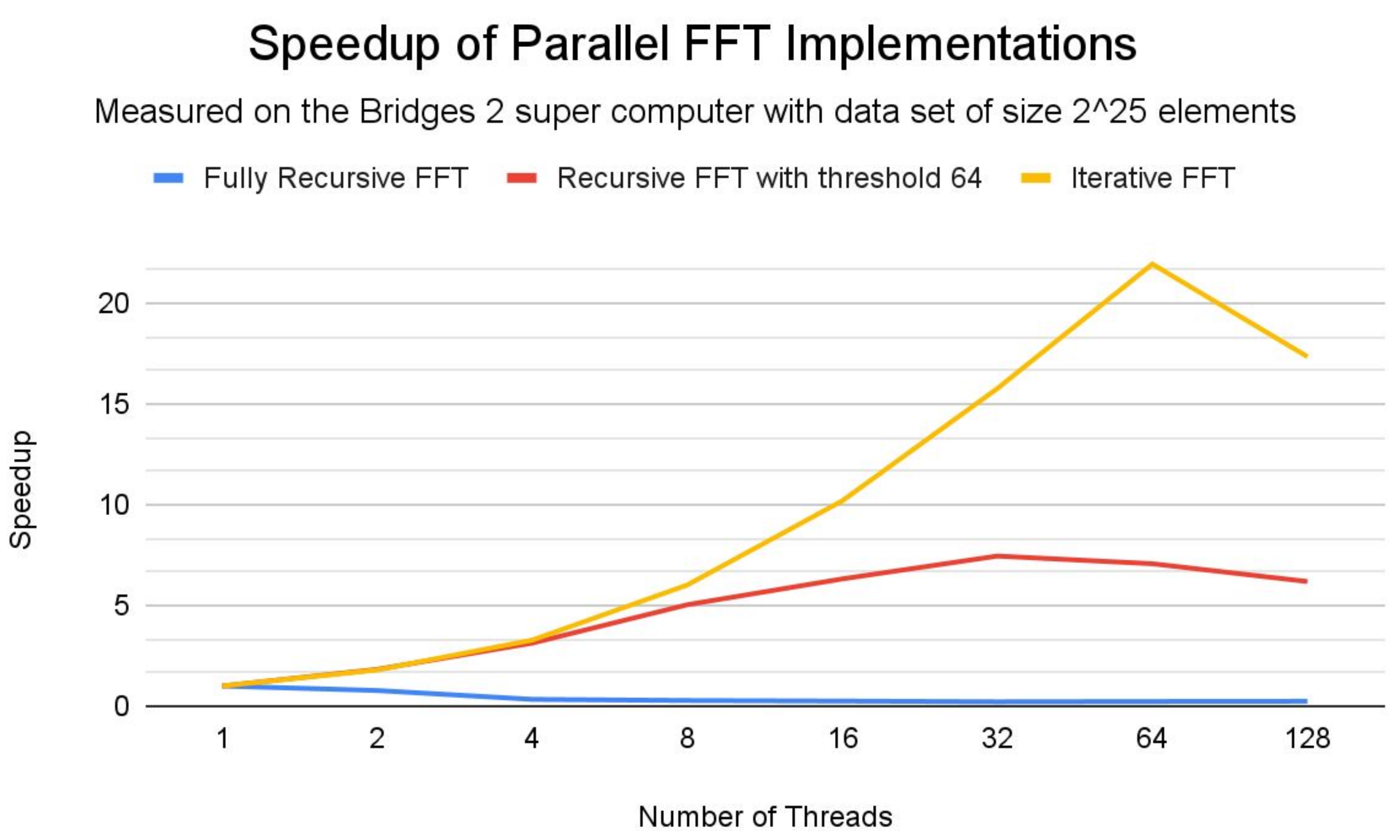
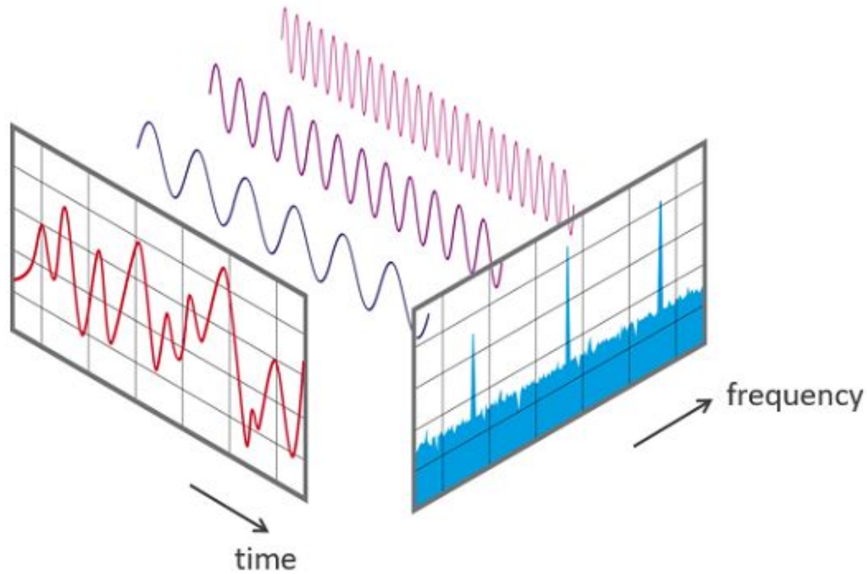




FFT and our Approach

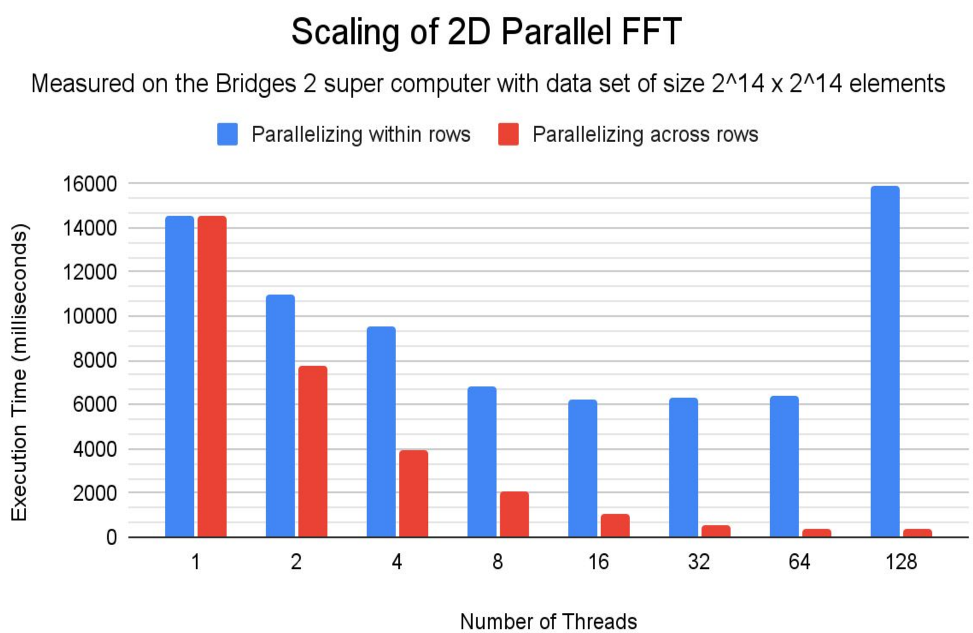
- What is Fast Fourier Transform (FFT)?
- Discrete Fourier Transform
- FFT (Fully Recursive, Recursive with Threshold, and Iterative)
- 2D FFT
- Image Compression

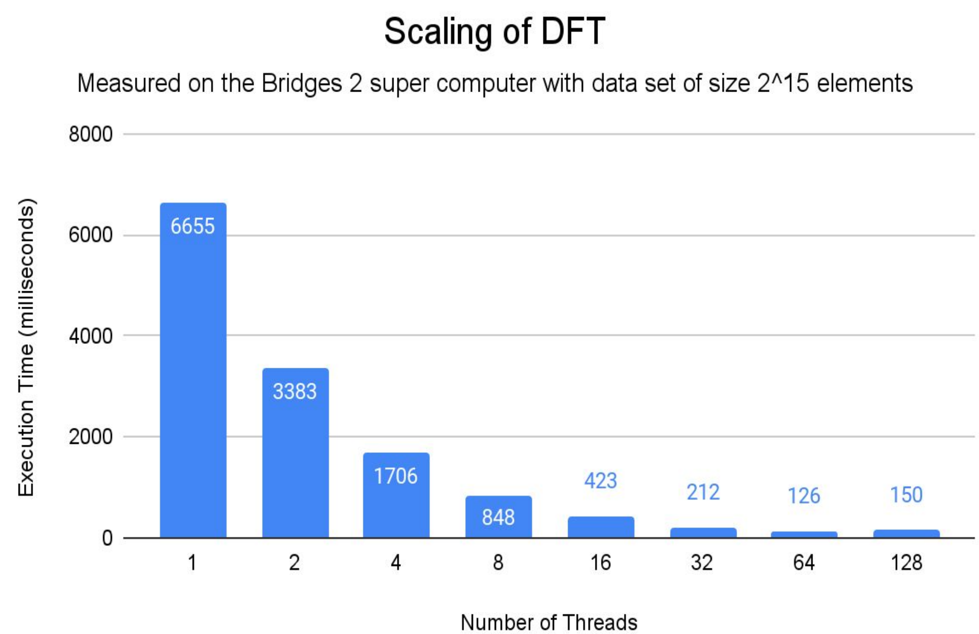
$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk}$$

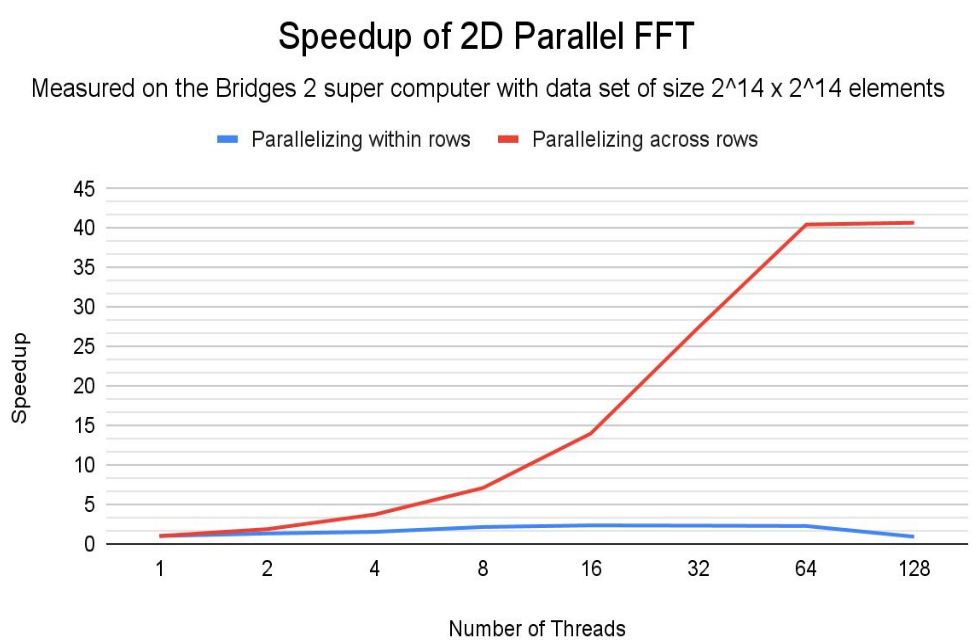


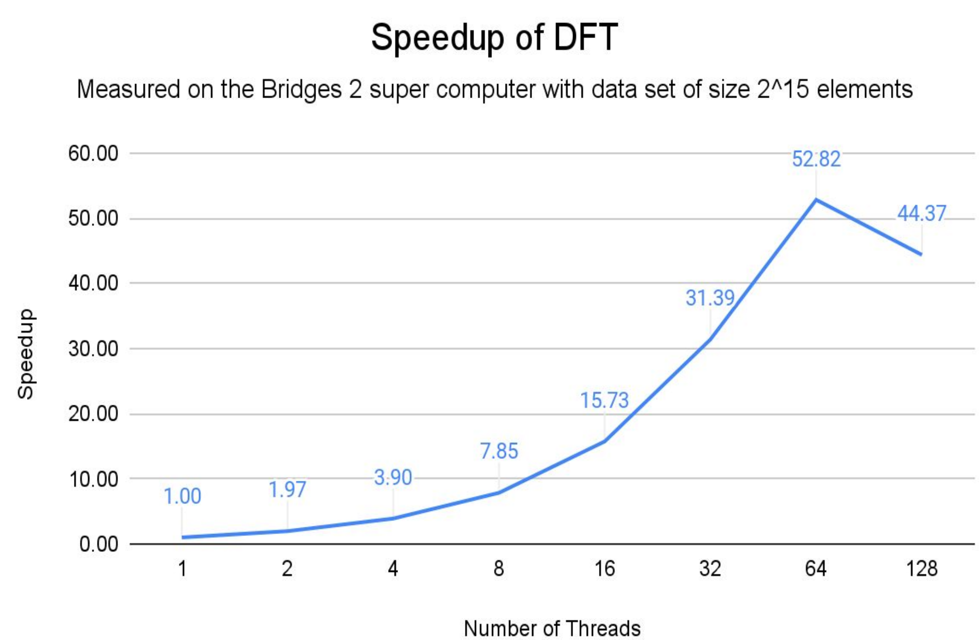
Main Optimizations

- Parallel pre-computation of Twiddle Factors
    - Loop collapse
- Switching to DFT in Recursive FFT at Threshold
- Iterative Algorithmic Modifications
- Bit Reversal allowing Butterfly Networks
- Chunking of Data that fits in L1/L2 Cache
- Matrix Transpose enabling better Spatial Locality
- Exploiting more available parallelism


Scaling of 2D Parallel FFT
Measured on the Bridges 2 super computer with data set of size 2^14 x 2^14 elements
Parallelizing within rows
Parallelizing across rows
Execution Time (milliseconds)
0
2000
4000
6000
8000
10000
12000
14000
16000
1
2
4
8
16
32
64
128
Number of Threads


Scaling of DFT
Measured on the Bridges 2 super computer with data set of size 2^15 elements
Execution Time (milliseconds)
0
2000
4000
6000
8000
6655
3383
1706
848
423
212
126
150
1
2
4
8
16
32
64
128
Number of Threads

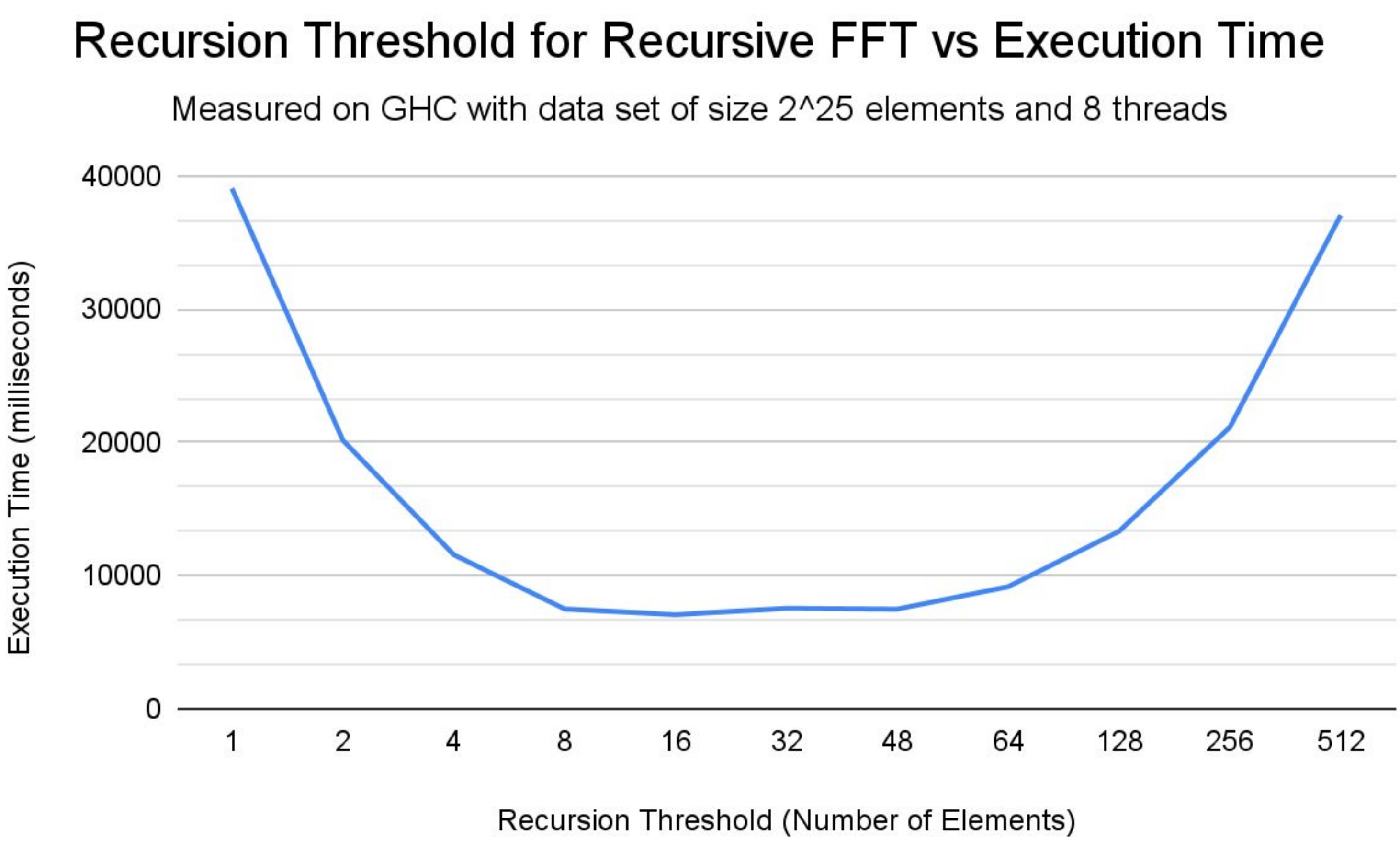

Speedup of 2D Parallel FFT
Measured on the Bridges 2 super computer with data set of size 2^14 x 2^14 elements
Parallelizing within rows
Parallelizing across rows
Speedup
0
5
10
15
20
25
30
35
40
45
1
2
4
8
16
32
64
128
Number of Threads

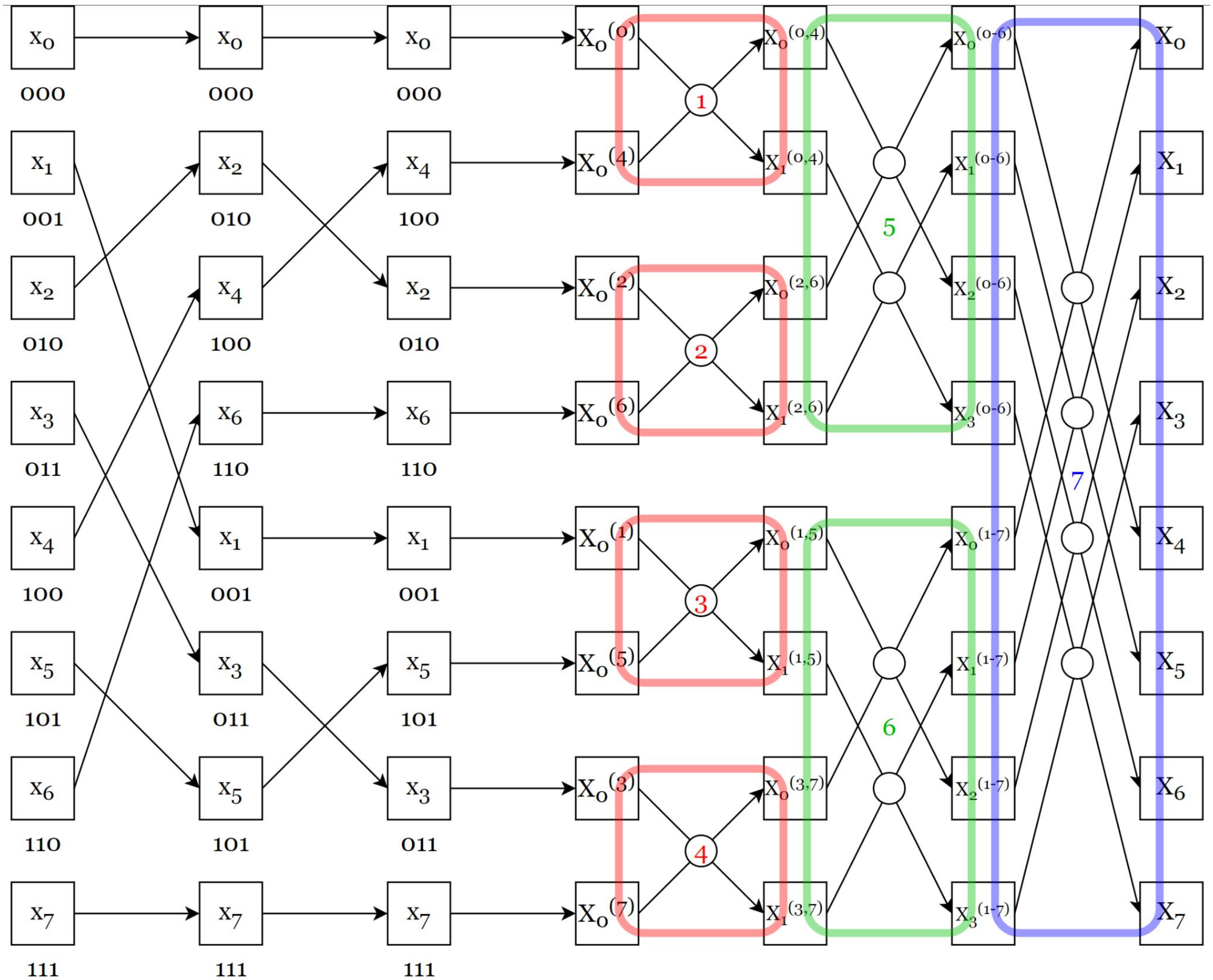

Speedup of DFT
Measured on the Bridges 2 super computer with data set of size 2^15 elements
Speedup
0.00
10.00
20.00
30.00
40.00
50.00
60.00
1.00
1.97
3.90
7.85
15.73
31.39
52.82
44.37
1
2
4
8
16
32
64
128
Number of Threads

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of even-indexed part of } x_n} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of odd-indexed part of } x_n}$$

$$\begin{aligned} X_k &= E_k + e^{-\frac{2\pi i}{N} k} O_k \\ X_{k+\frac{N}{2}} &= E_k - e^{-\frac{2\pi i}{N} k} O_k \end{aligned}$$

- Sequential access within a recursive call
- Needless copying causes poor locality
- High overhead due to recursion at small sizes
- Better scaling and constant factors for DFT

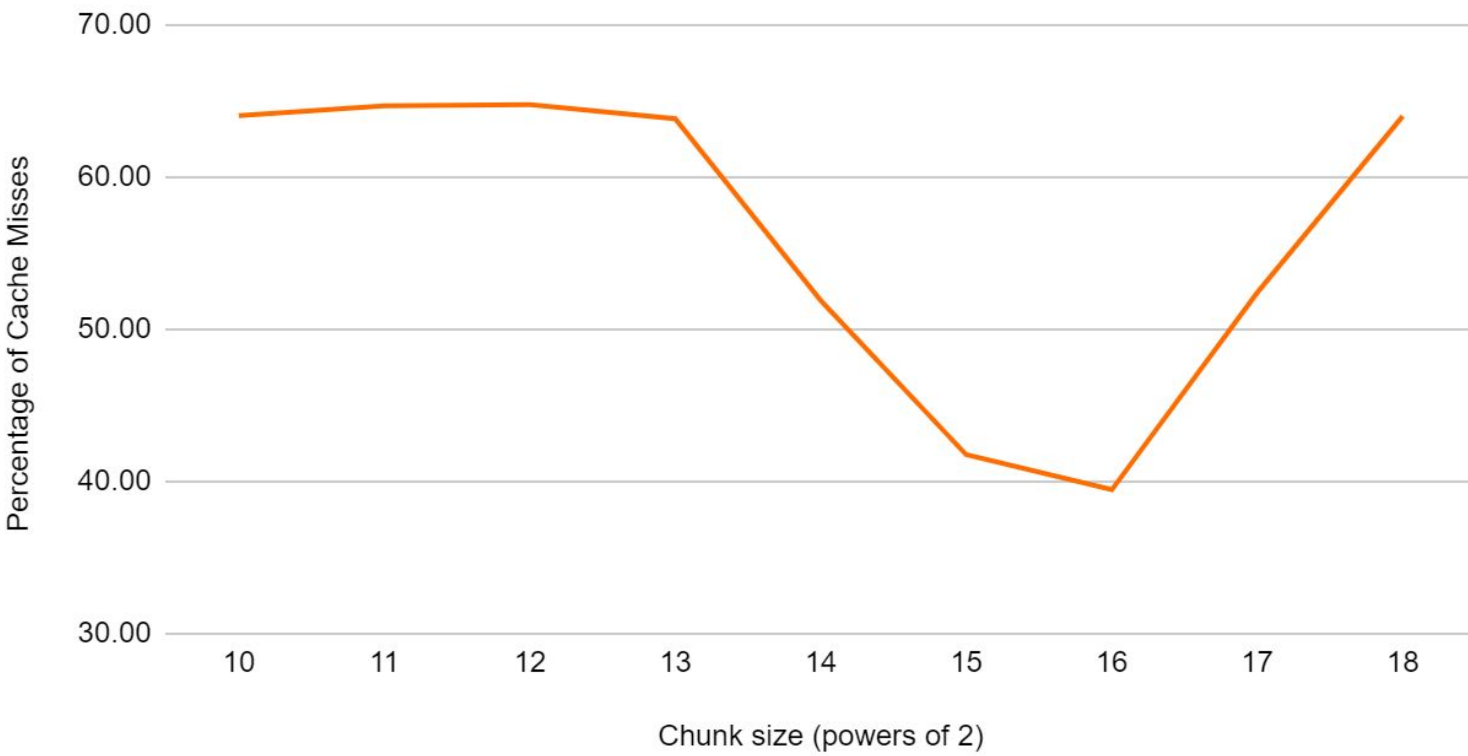




- Recursive Shuffle
  - Sort by reverse of the bits of the index
  - Bit reversal is a bijection and swaps are independent of each other
- Chunking (4 elements in cache)
  - 1 -> 2 -> 3 -> 4 -> 5 -> 6 -> 7
  - 5 reuses data brought in by 1 and 2
  - 1 -> 2 -> 5 -> 2 -> 4 -> 6 -> 7
  - N log(N/C) instead of N log(N)

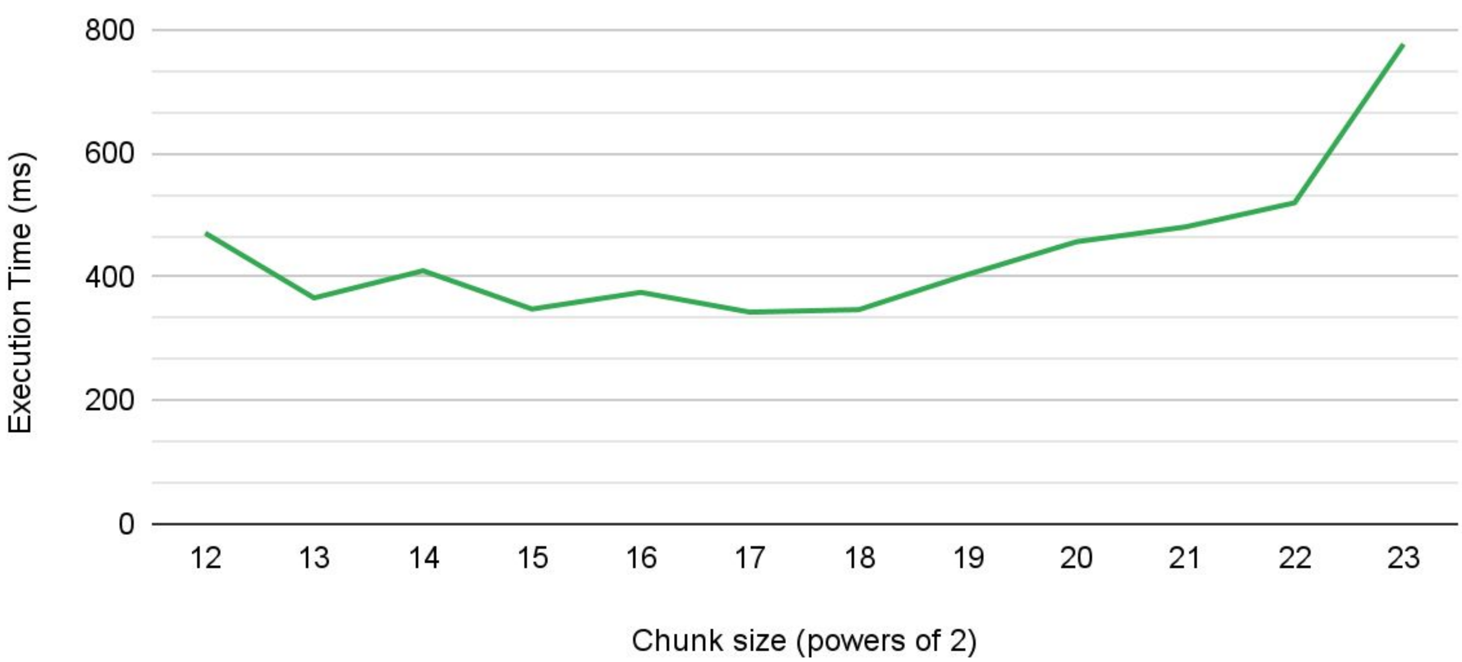
Number of Iterations in Cache vs % Cache Misses for Iterative FFT

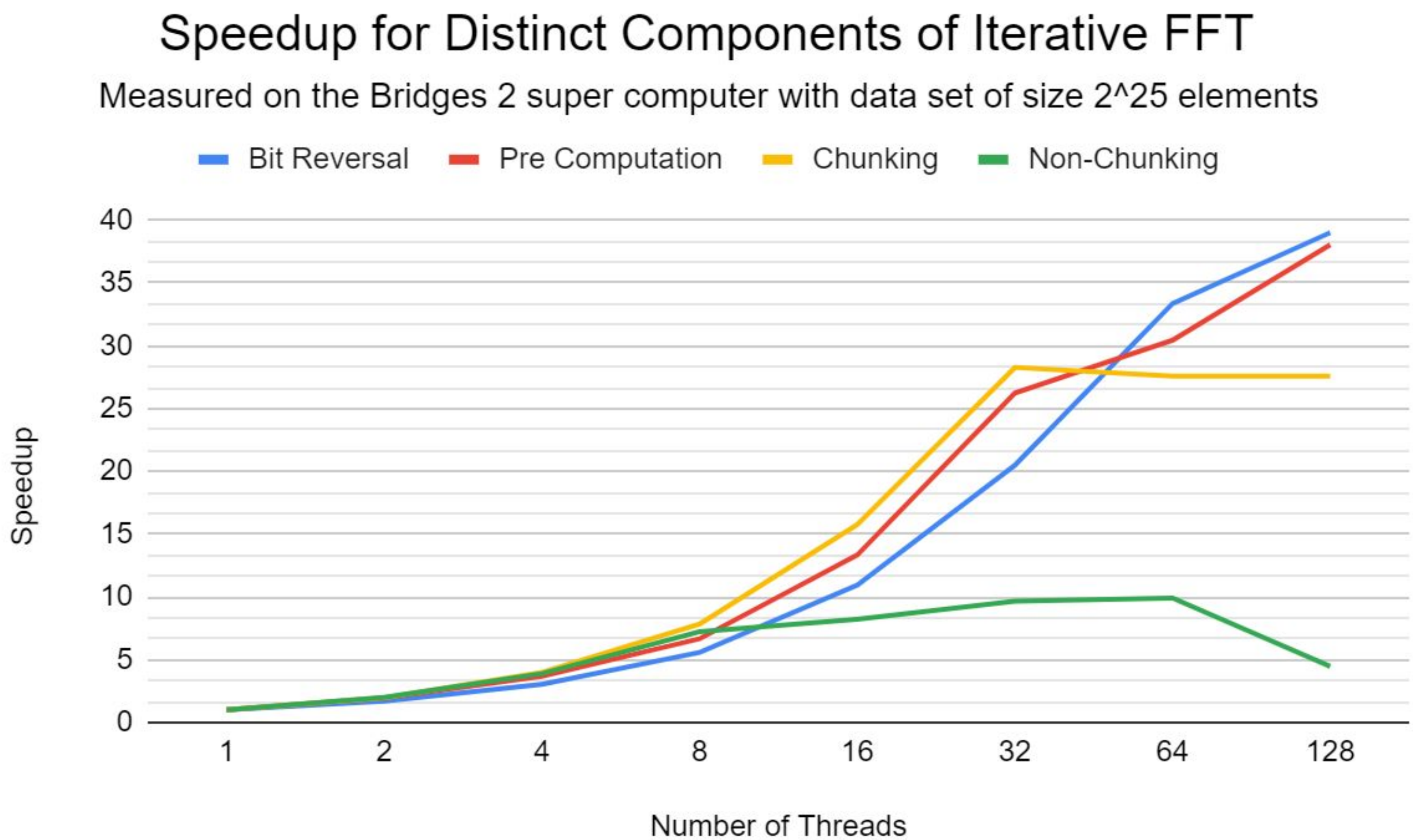
Measured on GHC with dataset of size 2^25 elements and 8 threads



Number of Iterations in Cache vs Execution Time For Iterative FFT

Measured on GHC with data set of size 2^25 elements and 8 threads





- Chunking is effective
- Non-chunking still bandwidth bound
- Bit reversal (bit shifting) and pre computation (std::polar) are more compute bound

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2-Dimensional FFT

- Steps
  - Perform in-place 1D FFT on the rows
  - Transpose the matrix
  - Perform in-place 1D FFT on the rows (which were the columns pre-transpose)
  - Transpose the matrix
- Parallel across rows or within row

Image Compression

- 2D FFT on each color obtaining the Fourier Coefficients
- Take top x% of coefficients by magnitude and remove the rest
- Store remaining coefficients and their indices
- Convert back to matrix by adding zeros for missing data
- Perform Inverse FFT on the matrix to obtain image