Lab 2

1. Calculate with pen-and-paper the LU decomposition of the matrix

$$A_1 = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}$$

2. Calculate with pen-and-paper the LU decomposition of the matrix

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3. Calculate with pen-and-paper the LU decomposition of the matrix

$$A_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

4. When a zero appears in a pivot position, the LU factorization is not possible. (We would require to use a pivot matrix P, so that A = PLU). Show directly why the following LU equations are impossible

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix} \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$$

5. Use the LU decomposition from question 1 to solve the linear system A x = b for the unknown vector x, where

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

6. Write a Python function `Ludecompose(A)` that takes as input a two-dimensional numpy array `A` and return either the LU decomposition of `A` or NaN if `A` does not have a LU decomposition. Use the function to verify the results from questions 1–3.

7. Use `numpy.linalg.solve` to verfiy the result from question 5.

8. Use `numpy.linalg.qr` to calculate the QR decompositions of the matrices A_1, A_2, A_3

9. Use `numpy.linalg.eig` to calculate the eigendecompositions of the matrices A_1, A_2, A_3

10. Use `numpy.linalg.svd` to calculate the singular value decompositions of A_1, A_2, A_3