Lab 3

1. Create the circulant matrix with a top row of (3, 5, 7) in Python.
2. Write a Python function `iscirculant(M)` that takes as input a 2-D NumPy array and returns a boolean indicating if this array represents a circulant matrix.
3. Check that the product of the following two circulant matrices is again a circulant matrix:

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 5 & 0 & 4 \\ 4 & 5 & 0 \\ 0 & 4 & 5 \end{bmatrix}$$

4. Find the cyclic convolution of the vectors a = (0, 1, 2) and b = (3, 1, 2), first by hand and then by using matrix multiplication in Python.
5. Find the cyclic convolution of the vectors x = (0, 1, 0, 1) and y = (0, 1, 2, 3), first by hand and then by using matrix multiplication in Python.
6. Calculate the Fourier matrix for n = 2 by hand.
7. Verify the last question in Python.
8. Calculate the Fourier matrix for n = 4 directly in Python.
9. Calculate the eigendecomposition of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ by hand.
10. Calculate the eigendecomposition of a circulant matrix C, where the first row of C is (0, 1, 2, 3) in Python.