Averages, Equations & Problems on Ages

Averages

Basic Concepts

- $Average = \frac{Sum\ of\ Observations}{Total\ number\ of\ observations}$
- If the value of each unit in a class is increased by some value x, then the average of the class also increases by x.
- If the value of each unit in a class decreases by some value x, then the average of the class also decreases by x.
- The average of any number series or group is always between its smallest and the largest value.
- When a person leaves the group, and replacement is made of that person then:
  - ✓ If the average age increases,

    Age of new person = Age of separated person + (increase in the average × total number of persons)

  - ✓ If the average age decreases,

    Age of new person = Age of separated person – (decrease in the average × total number of persons)

- When a person joins the group then:
  - ✓ If the average age increases,

    Age of new person = Previous average + (increase in average × total members including new member).

  - ✓ If the average age decreases,

    Age of new person = Previous average – (decrease in average × total members including new member).

- There are two batches A and B in a class. To find the average of the whole class we use the formula:

Batch A	**Batch B**
Number of students = a	Number of Students = b
Average of batch A = x	Average of batch B = y

Average of whole class (Batch 1 and Batch 2) = $\mathbf{\frac{(ax + by)}{(a + b)}}$

- Average of 'n' consecutive Natural Numbers = $\frac{n+1}{2}$
- Average of the square of consecutive n natural numbers = $\frac{(n+1)(2n+1)}{6}$
- Average of cubes of consecutive n natural numbers = $\frac{n(n+1)^2}{4}$
- Average of n consecutive even numbers = (n+1)
- Average of consecutive even numbers till n = $\frac{n}{2} + 1$
- Average of n consecutive odd numbers = n
- Average of consecutive odd numbers till n = $\frac{n+1}{2}$
- Sum of 1st n even consecutive natural numbers = n (n+1)
- Sum of 1st n odd consecutive natural numbers is = n^2

EXAMPLES

1. The average of 4 terms is 20 and the 1st term is 1/3 of the remaining terms. What will be the first number?
   (a) 30 (b) 20 (c) 60 (d) 80
2. For 9 innings, Boman has an average of 75 runs. In the tenth inning, he scores 100 runs, thus increasing his average. His new average is.
   (a) 75 (b) 100 (c) 72 (d) 77.5
3. In a family of 8, the men eat on average 72kg of food and women eat on an average 50kg of food. The men and women are equal in number. A hungry woman named Neetu joined the family for dinner and the average consumption became 67. How much did Neetu eat (in kgs)?
   (a) 115 (b) 80 (c) 90 (d) 85
4. If the average number of 8 terms is given to be 40 and the average of first 6 terms is given to be 35. What is the average of the remaining 2 terms?
   (a) 30 (b)55 (c)40 (d)42
5. In a class, the average marks of 40 students were calculated to be 52.15. It was later discovered that the marks of a student were taken to be 49, instead of 85. Find the real average of the class.
   (a) 53.05 (b) 53.15 (c) 52.85 (d) 52.95

Answer keys

1. b	2. d	3. a	4. b	5. a

PROBLEMS ON AGES

EXAMPLES

1. The sum of ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?
   (a) 4 years (b) 8 years (c) 10 years (d) None of these
2. Shankar is 5 years younger than Ron. Four years later, Ron will be twice as old as Shankar. Find the present age of Ron.
   (a) 1 (b) 2 (c)6 (d)4
3. Present ages of Sameer and Anand are in the ratio of 5 : 4 respectively. Three years hence, the ratio of their ages will become 11: 9 respectively. What is Anand's present age in years?
   (a) 24 (b) 27 (c) 40 (d) Cannot be determined
4. The age of father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. The ratio of their present ages is?
   (a) 5 : 2 (b) 7 : 3 (c) 9 : 2 (d) 13 : 4
5. A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?
   (a) 32 years (b) 36 years (c) 40 years (d) 48 years

Answer keys

1. a	2. c	3. a	4. b	5. c

Equations

- An Equation is defined to be a mathematical statement of equality.
- *Simple Equation:*
  A simple equation in one unknown x is in the form:
  $ax + b = 0$, where a, b are known as constants and $a \neq 0$
- A simple equation has only one root.
- The general form of a linear equation in two unknowns x and y is $ax + by + c = 0$, where a and b are non-zero coefficients.
- Two equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ form a pair of simultaneous equations in x and y. A value for each unknown which satisfies both equations at the same time gives the roots/ solution of the equation.

- *Quadratic Equations:*

  An equation in the form $ax2 + bx + c = 0$, where x is a variable and a, b and c are constants with $a \neq 0$.

  Example: $x^2+5x+6=0$

  The solutions of a quadratic equation are called its roots.

  Roots of a Quadratic Equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXAMPLES

1. The length of a rectangle is 8cm more than its breadth. If the perimeter of the rectangle is 68 cm, find its length.
   (a) 21cm (b) 14cm (c)8cm (d)20cm
2. The price of 10 chairs is equal to that of 4 tables. The price of 15 chairs and 2 tables together is Rs. 4000. The total price of 12 chairs and 3 tables is:
   (a) Rs. 3500 (b) Rs. 3750 (c) Rs. 3840 (d) Rs. 3900
3. There are two examinations rooms A and B. If 10 students are sent from A to B, then the number of students in each room is the same. If 20 candidates are sent from B to A, then the number of students in A is double the number of students in B. The number of students in room A is:
   (a) 20 (b) 80 (c) 100 (d)200
4. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of children been half, each child would have got 16 notebooks. Total how many notebooks were distributed?
   (a) 256 (b) 432 (c)512 (d) 640
5. Solve the equation for x: $\boldsymbol{6x - 27 + 3x = 4 + 9 - x}$
   (a) 4 (b) 5 (c) 6 (d) −4

Answer keys

1. a	2. d	3. c	4. c	5. a

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