Lecture 1-Brushing Up Matrices

Column Space contains all vectors

Few Comments

- Most of the lectures follow the book namely **Linear Algebra and Learning from Data (2019)** by **Gilbert Strang**

- Dr. Sundar Ram K will take lab sessions
- Ms. Anuja Kunjumol will assist Dr. Sundar in lab sessions
- Faculty Assistants : **Ms. Anuja Kunjumol @ am.sc.r4cse23022@am.students.amrita.edu and Ms. Anna N Kurian @ am.en.r4cse22025@am.students.amrita.edu (Uploading regular theory class attendances at AUMS; send attendance sheet at her email)** and **Ms. Honeymol O (Conducting Quiz)**

Evaluation Pattern

Assessment	Weightage (%)
Assignment (along with Viva)	15
Online Quiz 1 (30mins)	10
Online Quiz 2 (30mins)	10
Lab Exam 1	15
Midterm	20
End Term	30

Few Comments

The order we will follow to conduct test, etc. in the following order:

1. Online Quiz 1
2. Online Quiz 2
3. Midterm Exam
4. Assignment (along with Viva)
5. Lab Exam
6. Endterm Exam

Today’s Discussion

- Matrix multiplication with vectors Ax
- Matrix multiplication with matrices
- Column space C(A)
- Rank r of a Matrix
- A=CR

Source: Section I.1 in Linear Algebra and Learning from Data (2019) by **Gilbert Strang**

Matrix Multiplication with vectors

Let A= $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix}$ $and\ X = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$

Row multiplication =(row).(column)=2x1+x2+3x3..etc.

AX= $[x1]\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ + $[x2]\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}$ + $[x3]\begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$ = combination of colms a1, a2 and a3 (Column multiplication)

Look on that vector-wise. x1 multiplies first column 2,3,5, x2 times second column, 1, 1, 7, and x3 times third column, 3, 4, 12

Combination of vectors, and it produces a vector. And here, 3 by 3 matrix on our vectors are in R^3. And most vectors will be in R^3 or R^n

All Ax gives us a bunch of vectors. And that collection of vectors is called the column space of A. It is a space too.

All Ax=Column space of A= C(A)

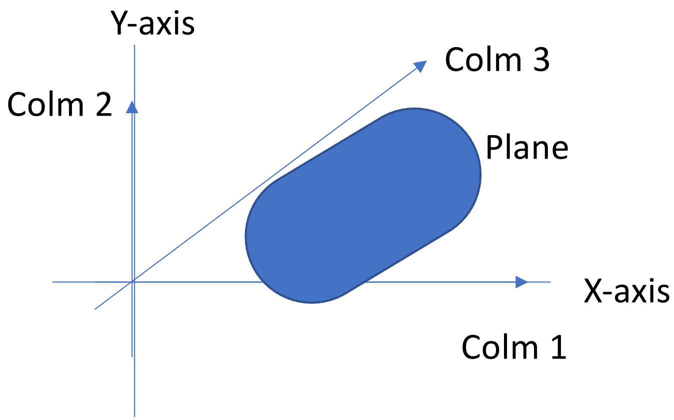
Thus, *Ax* is a linear combination of the columns of *A.* This is fundamental

b=(b1, b2,b3) is in the column space of A exactly when Ax=b has a solution (x1,x2, x3)

Matrix Multiplication with vectors

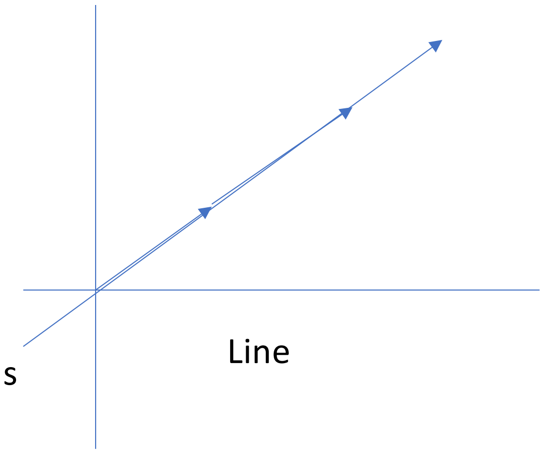
- A= $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix}$ $and\ X = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$
- Let matrix A, and we take all x's and we imagine all the outputs
- If we take 1 vector x, we get a vector output. It takes a vector to a vector
- A (3x3) matrix is a column space, its columns are independent, its rows are going to be independent, it's going to be invertible
- The above matrix is plane because the 3rd column is the sum of colm 1 and colm 2
- See the diagram in the next foil

Matrix Multiplication with vectors



$$A=\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix}$$

A has two independent columns as 3rd column is sum of colum 1 and colum 2. A has two basis since A has two independent columns and its rank r=2



$$\text{Let } B = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8 \\ 1 & 3 & 8 \end{bmatrix}$$

a matrix whose column space would only be a line.
All 1s? let us have 3, 3, 3, 8, 8, 8
C(B)= line and rank =1

The rank is the dimension of the column-- any combination x1 of that plus x2 of that plus x3 of that is along that line

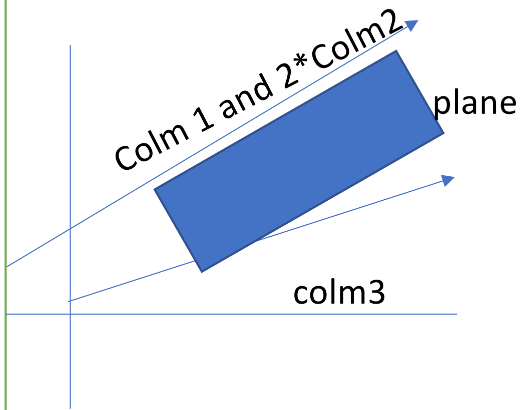
If we take any multiple of column one, we've got a whole line, any multiple of column two.
When we put the two together, it fills in the plane
This is a matrix of rank. So, Rank A= 2

Matrix Multiplication with vectors

- What's the rank of this matrix A ? Because it's got two independent columns, but the third column is dependent and a combination of the others
- Matrices are the building blocks of linear algebra, they're the building blocks of data science. They're rank one matrices
- And let us show you a special way to write those rank one matrices. Take the matrix as the column vector 1, 1, 1 times the row vector 1, 3, 8. So it's a column times a row. That's a rank one matrix
- B = $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8 \\ 1 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1\ 3\ 8] = u.v^T$
- C(B)= line → (3x1) (1x3)= (3x3). Rank B =1=number of indep. Colms
- What is basis? A basis is independent columns
- All three together would not be a basis. But they must be not just independent, but they must fill the space-- their combinations must fill the space

Matrix Multiplication with vectors

- Let A= $\begin{bmatrix} 2 & 4 & 1 \\ 3 & 6 & 1 \\ 5 & 10 & 7 \end{bmatrix}$



Look at the first column. It's not 0s. If it was all 0s, we wouldn't want that in a basis. First vector is the basis

On the second column. If that column was 4, 6, 10, would we put it in the basis? No

But 1, 1, 7, is in a different direction

the rank is two-- the column rank

The number of independent columns is two

Matrix Multiplication with vectors

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

A(3x3) C(3x2) R(2x3) ← Basis of row Space

↑ Basis of Colm Space

Column rank=row rank = 2

Basis of number of indep. colms i.e., their combination to fill up the space

Row space of matrix A = all combinations of rows is row space

Row rank is dimension of row space i.e., $C(A^T)$ = row space of A = Column space of A^T

Hence A=CR

Column space of A is also called range

Back-up Slides

Matrix Multiplication with vectors

- Random sampling of a matrix. So how could you sample a matrix? So you have a matrix
- You just look at A times x. Let x be a random vector. Rank of C, so it's got m rows and one column. It's a vector
- what space is it in? Column space. Ax is in the column space
- So, if you want a random vector in the column space, we wouldn't suggest to just randomly pick one of the columns
- Better to take a mixture of columns by taking a random vector x, and looking at Ax
- And if you wanted 100 random vectors, you'd take a 100 random x's, and that would give you a pretty good idea, in many cases, of what the column space looks like
- That would be enough to work with often
- Is ABCx-- is that in the column space of A?
- Suppose we have matrices A, B, and C, and a vector x, and we take their product
- Does that give me something in the column space of A?

Matrix Multiplication with vectors

- ABCx≈ A(BCx)
- A times something. Putting parentheses in the right place is the key to linear algebra. And there it is.
- Let AB $[rows\ A]$ [Column B] = row times column = r.C
- Combination of AB = k-times columns x k-times rows
- i.e., (col 1).(row1)+ (col 2).(row2)+… +(col k).(row k)= Sum of (col k) of A and (row k) of B

- How many multiplications are there?
- (mxn) (nxp)=(mxp)
- i.e., mnp number of column multiplications is equivalent to nx(mp)=nmp=number of rows multiplications