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3) $C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 0 & 4 \\ 4 & 5 & 0 \\ 0 & 4 & 5 \end{bmatrix}$

$C.D = \begin{bmatrix} 13 & 22 & 19 \\ 19 & 13 & 22 \\ 22 & 19 & 13 \end{bmatrix}$

C.D is also circulant

C.D has constant diagonals.

sums of rows are same $= \underline{\underline{55}}$

4) $a = [0, 1, 2]$

$b = [3, 1, 2]$

$A = a_0 I_{3\times3} + A_2 P_{3\times3} + a_3 P^2_{3\times3}$

ie, $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

b.A $\Rightarrow$ gives convolution of a and b

$[3, 1, 2] \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

$\Rightarrow [4 \;\; 7 \;\; 7]$ is the cyclic convolution of two vectors a and b

Blue bird

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5) $x = [0, 1, 0, 1]$

$y = [0, 1, 2, 3]$

$Y = y_0 I + y_1 P + y_2 P^2 + y_3 P^4$

$$Y = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

$x \cdot y \Rightarrow$ circular convolution

$$\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 & 2 \end{bmatrix}$$

$[4, 2, 4, 2]$ is the circular convolution of two vectors x and y.

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6) Fourier matrix $\boxed{F_2}$

$$F_2 = \begin{pmatrix} F_{00} & F_{01} \\ F_{10} & F_{11} \end{pmatrix} \quad ; \quad F_{JK} = \frac{1}{\sqrt{n}} e^{-2\pi i \frac{J \cdot K}{2}}$$

$$F_{00} = \frac{1}{\sqrt{2}} e^{-2\pi i \frac{0 \cdot 0}{2}} = \frac{1}{\sqrt{2}}$$

$$F_{01} = \frac{1}{\sqrt{2}} e^{-2\pi i \, 0 \cdot 1/2} = \frac{1}{\sqrt{2}}$$

$$F_{10} = \frac{1}{\sqrt{2}} e^{-2\pi i \frac{1 \cdot 0}{2}} = \frac{1}{\sqrt{2}}$$

$$F_{11} = \frac{1}{\sqrt{2}} e^{-2\pi i \frac{1 \cdot 1}{2}} = \frac{e^{-\pi i}}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$F_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

9) Eigen decomposition of $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= \lambda^2 - 4\lambda + 3 = 0$$

$$\boxed{\lambda = 3, 1}$$

$$\lambda = 1 \Rightarrow \begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix} x = 0$$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x = 0 \quad , \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} x = 0$

$Ax = 0 \Rightarrow$ nullspace

$x_1 + x_2 = 0$

$x_2 = x_2$

$x_1 = -x_2$

$x_2 = x_2$

ie, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\therefore$ eigon space $= \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} = V_1$

$\lambda = 3 \Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} x = 0$

$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} x = 0$

$-x_1 + x_2 = 0$

$x_2 = x_2$

$x_1 = x_2$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

eig. space : $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \rightarrow V_2$

Eigen decomposition :

$$\lambda = X \Lambda X^{-1}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\cancel{X = \begin{bmatrix} x_1 & x_2 \end{bmatrix}} \quad X = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

$$x_1 = |v_1| = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$x_2 = |v_2| = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$X^{-1} = \begin{bmatrix} \frac{1}{4\sqrt{2}} & -1/4\sqrt{2} \\ -1/4\sqrt{2} & -1/4\sqrt{2} \end{bmatrix}$$

ie, $X \Lambda X^{-1}$

$$= \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4\sqrt{2} & -1/4\sqrt{2} \\ -1/4\sqrt{2} & -1/4\sqrt{2} \end{bmatrix}$$