Lab - 2

1) LU decomposition

$$A_1 = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} = U$$

$$L = AU^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

ie,
$$A_1 = \underbrace{\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}}_{U}$$

2) LU decomposition

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{RREF}(A_2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = U$$

$$L = AU^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3) $A_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$U = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{bmatrix}$ $R_2 \rightarrow R_2 + \frac{R}{2}$ $R_3 \rightarrow R_3 + \frac{2}{3}$

$L = AU^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$

$A_3 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{bmatrix}}_{U}$

4) $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$

Here 0 appears in a pivot position
∴ a permutation matrix is required.

$A = P^{-1}LU$

$\boxed{PA = LU}$

normal LU:

$(A) = (L)(U) = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$

$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{pmatrix} d & e \\ ld & le+f \end{pmatrix}$

$d = 0$; $e = 1$

$l \cdot d = 2$; $le + f = 3$

$l \cdot 0 = 2$

$0 = 2 \Rightarrow$ contradiction.

∴ normal LU decomposition is not possible.

$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $L = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $U = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

5) Use LU decomposition to solve linear eqn.

$Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 \\ 6 & 7 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$A = LU$

$$U = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \quad L = AU^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

$Ax = b$

$\hookrightarrow A = LU$

$\Rightarrow L \cdot Ux = b$

$L \cdot y = b$; where $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \Rightarrow Ux$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$y_1 + 0y_2 = 1 \Rightarrow \boxed{y_1 = 1}$

$3y_1 + y_2 = 1 \Rightarrow y_2 = 1 - 3y_2$

$\boxed{y_2 = -2}$

$Ux = y$

$$\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$2x_1 + x_2 = 1$

$0x_1 + 4x_2 = -2$

$$\left.\begin{aligned} x_2 &= -\tfrac{1}{2} \\ x_1 &= \tfrac{3}{4} \end{aligned}\right\}$$ is the soln to the system of linear equation $Ax = b$