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Algorithm:

Finite set of instructions to solve a problem.

Design methods / strategies:

1. Divide & conquer
2. Backtracking
3. Dynamic Programming
4. Branch & Bound

Analysis strategies:

1. Time complexity
2. Space complexity.

Real life Algorithm Usage:

1. Google Map ← Shortest path algorithm (eg: Dijkstra, Bellmann Ford, Floyd warshall)
2. Shopping sites ← Recommendation algorithms (eg: apriori)
3. Youtube ← Recommendation algorithm

Algorithm v/s Program:

Algorithm	Program
1. Design Phase	1. Implementation Phase
2. Domain specific knowledge	2. Programmer
3. Natural language; no specific syntax	3. In programming language, ~~no~~ specific syntax
4. Analysis	4. Testing

Q. What are the main properties of an algorithm?

↳ 1. Finite
2. Unambiguous
3. Effective / Efficient
4. input
5. output

1* **Input:**
For an algorithm, it has 0 or more inputs.

2* **Output:**
Should have atleast 1 output

3* **Unambiguous:**
- Should not have any confusion
- An algorithm should be enclosed within start/begin and stop/end statements.

eg: algorithm for sum of 2 numbers:

Start
Step 1: Read a } or Step 1: Read a, b
Step 2: Read b }
Step 3: sum :- a+b
Step 4: print sum
End

4* **Finite:**
Number of steps are countable.

5* **Effective/Efficient:**
Avoid unnecessary steps.

Q. What is the need of an algorithm?

↳ For time saving. It provides a plan on how to make the program.

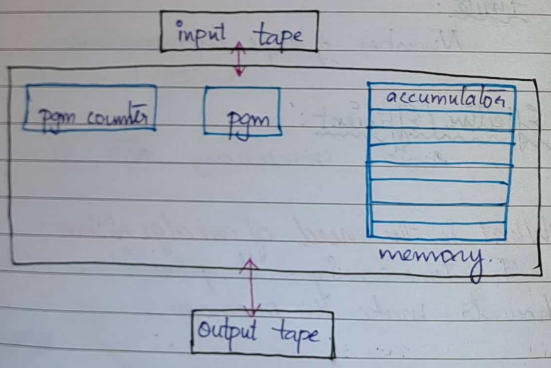
Q. Algorithm for largest number among 3 numbers?

Start
Step 1: Read a, b, c
Step 2: If a > b then max = a else max = b
Step 3: If c > max then max = c
Step 4: print max
End

Analysis Of Algorithms:

1. Factors included in the Analysis → depends on f(n)
2. Types of Analysis → priori, posteriori
3. Asymptotic Analysis → Big Oh, Big Omega, Big Theta

RAM Machine / Model:



We need RAM Machine for finding f(n) (for time complexity)

We are counting the number of primitive operations in the pgm or algorithm. It is basically RAM model.

I/p & O/p tape are groups of cells. →

Based on the ~~map~~ mapping of i/p tape & o/p tape we get ∄ f(n) [the number of primitive operations]

Say,

eqn ⇒ $5n^2 + 6n + 2$

Q. What is the role of n^2, n & 2 in the value?

Let n = 1

Role of n^2 = $\frac{5}{5\times1^2+6\times1+2} \times 100 = \frac{5}{13} \times 100 = 38.46\%$

Role of n = $\frac{6}{13} \times 100 = 46.15\%$

Role of 2 = $\frac{2}{13} \times 100 = 15.38\%$

n	Role of n^2	Role of n	Role of 2
1	38.46	46.15	15.38
100	9881	0.01	0.003
1000	⋮	⋮	⋮
⋮			

From this we can see n^2 has more role. From this we approximate time complexity to $O(n^2)$

2# Type of Analysis:

Priori	Posteriori
• Before execution	After execution
• Independent of hardware ~~memory, processing time~~	Dependent on hardware, memory & processing time.
• Algorithm	Program
• Approximation	Actual

3# Asymtotic Notations.

The approximation is the asymtotic notations.
Big Oh - worst case Big Theta - avg case
Big Omega - best case

Best case:
Takes the least time for execution.
Number of primitive operations is least.

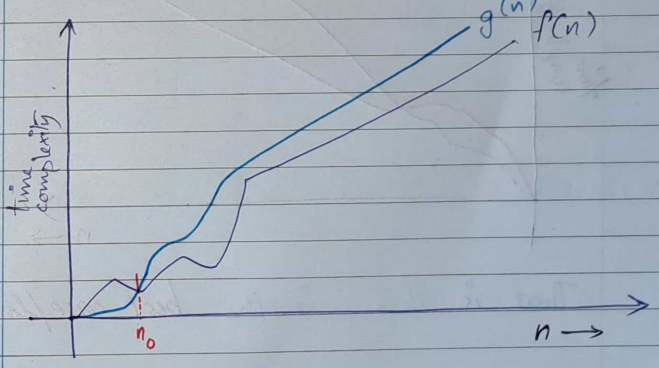
Worst case:
Maximum no: of primitive operations occurs.

Average case:
This is between best & worst case.

Graphical:

1. Big Oh:

$f(n) \leq c\,g(n)$



$n \geq n_0 \geq 1$

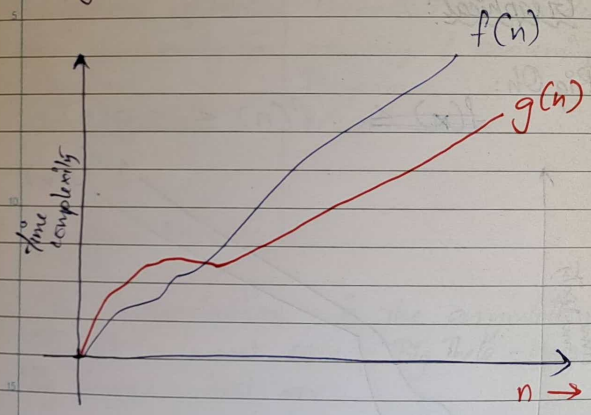
That is - this is max time taken.
That is: Worst case / upper bound.

2. Omega.

- $g(n) \leq cf(n)$

Or

$cg(n) \leq f(n)$



That is this is the best case/lower bound.

3. Theta

Average case.

$c_1 g(n) \leq f(n) \leq c_2 g(n)$

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$$f(n) = \Theta(g(n))$$

Q. Suppose you have two functions $f(n) = 4n+3$ and $g(n) = n$. Check whether $f(n) = O(g(n))$ is true or not? If true find c and n_0?

Ans: Two methods to solve:

1. Table method.

n	$f(n) = 4n+3$	$g(n) = n$	$2n$ $5n$ $7n$
1	7	1	2 5 7
2	11	2	4 10 14
3	15	3	6 15 21
4	19	4	8 20 28
5	23	5	10 25 35
6	27	6	12 30 42
7	31	7	14 35 49

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$f(n) \leq O(g(n))$ → $c \cdot g(n) = c \cdot n$

$4n+3 \leq c \cdot n$

When $c=1$

$4n+3 \leq n$

↳ Not possible. Proof: Hash table.

When $c=2$

$4n+3 \leq 2n$ → From table we can see it is not possible.

⋮

$4n+3 \leq 5n$ for $n \geq 3$

Select min: 'c' value. Hence we take $c=5$.

$\therefore c=5 \quad n_0 = 3$

Hence $f(n) = O(g(n))$

where $c=5$ and $n_0 = 3$

2. Shortcut method.

1. Take highest order ~~of~~ coefficient.

④$n+3$

2. Add 1

$\therefore c = 4+1 = 5$

Will not work for all problems

$4n+3 \leq c \cdot n$

$4n+3 \leq 5n$

$3 \leq 5n-4n$

$n=3 \longrightarrow n_0$

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$\therefore 4n+3 = O(n)$

where $c=5$

$n_0 = 3$

Q. $f(n) = n+4$, $g(n) = n$.

A:

①$n+4$

↳ $c = 1+1 = 2$

$n+4 \leq 2n$

$4 \leq n_0 \longrightarrow n_0 = 4$

$\therefore n+4 = O(n)$

where $c=2$

$n_0 = 4$

Q. $f(n) = n^2+n+3$, $g(n) = n^2$

A:

①n^2+n+3

↳ $c = 1+1 = 2$

$n^2+n+3 \leq 2n^2$

$n+3 \leq n^2$

$3 \leq n^2-n$

$0 \leq n^2-n-3$

$n = \frac{1+\sqrt{13}}{2}, \frac{1-\sqrt{13}}{2} = -1.30$

$= 2.30$ ✓

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$n_0 = \lceil 2.3 \rceil$

$= 3$

OR,

$n^2 - n = 3$

$n(n-1) \geq 3$

~~n=3 or n=~~

$n=1$ Not true

$2(2-1) = 2$ Not true

$3(3-1) = 3 \times 2 = 6 \geq 3$ → true

$\therefore n_0 = 3$

$\therefore$ ~~If~~ $n^2+n+3 = O(n^2)$

where $c = 2$

& $n_0 = 3$

Q. ~~Find~~ $f(n) = n^2$ & $g(n) = 2^n$. Check if $f(n) = O(g(n))$. If yes, find c & n_0?

A:

n	$f(n)=n^2$		$g(n)=2^n$
1	1	<	2
2	4	=	4
3	9	>	8
4	16	=	16
5	25	<	32
6	36	<	64

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at $c=1$ & $n_0 = 4$,

$f(n) = O(g(n))$

Q. $f(n) = 3n+2$ $g(n) = n$

A:

$c = 3+1 = 4$

~~If~~ $3n+2 \not\leq 4n$

~~$2 \leq n$~~

$4n \leq 3n+2$

$n \leq 2$

$n_0 = 2$

$\therefore f(n) = O(g(n))$ at $c = 4$ & $n_0 = 2$

Q. $f(n) = n^2$ $g(n) = n$

A:

n	$f(n)=n^2$	$g(n) = n$				
1	1	1	2	3	4	5
2	4	2	4	6	8	10
3	9	3	6	9	12	15
4	16	4	8	12	16	20
5	25	5	10	15	20	25
6	36	6	12	18	24	30
⋮	⋮	⋮				

This is not possible.

Hence $f(n) \neq O(g(n))$.

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Q. $f(n) = 3n+2$ and $g(n) = n$. Find Ω ?

A: Put $c = 1$

Then find n_0 value.

So,

$f(n) \geq c \cdot g(n)$

$f(n) \geq g(n)$

$3n+2 \geq n$

$2n \geq -2$

$n \geq -1$

$n_0 = 1$

n_0 has to be greater than 1.

n	$f(n) = 3n+2$		$g(n) = n$
1	5	$\geq$	1
2	8	$>$	2
3	11	$>$	3
4	14	$>$	4

Q. $f(n) = n$ & $g(n) = 3n+2$. Find Ω ?

A: $c = 1$

$f(n) \geq c g(n)$

$n \geq (3n+2)c$

$-2n \geq 2$

$n \geq -1$

So $n_0 = 1$

We can't take $c=1$ as it won't work.

n	$f(n) = n$	$g(n) = 3n+2$, $c = 1/5$
1	1	5
2	2	8
3	3	11
4	4	14
5	5	17
6	6	20

If $c = 1/10$

$n \geq c \cdot g(n)$

$n \geq \frac{1}{10} \cdot (3n+2)$

$10n \geq 3n+2$

$7n \geq 2$

$n \geq 2/7$

$n_0 = \lceil 2/7 \rceil$

$= \lceil 0.28 \rceil$

$= 1$

0.28
7 | 20
14
60
56
4

★ For logarithmic values we use floor ($\lfloor \rfloor$), instead of ciel ($\lceil \rceil$)

Q. $100n\log n = O(\frac{n\log n}{100})$. Is this true or false. If true find c?

A: $100n\log n \leq \frac{n\log n}{100} \cdot c$

$c = 10^4$

Then,

$100n\log n \leq 10^4 \times \frac{n\log n}{100}$

$100n\log n \leq 100\log n$

That is LHS = RHS. (= also works)

Hence $100n\log n = O(\frac{n\log n}{100})$ proved.

Q. $\sqrt{\log n} = O(\log\log n)$. Check whether it is true?

A:

$\sqrt{\log n} \leq c\log\log n$

$\log n^{1/2} \leq \log\log n$

But $\log n > \log\log n$

So $\log n^{1/2} > \log\log n$

Hence $\sqrt{\log n} \neq O(\log\log n)$

Q. If $0 < x < y$, then prove $n^x = O(n^y)$ g.t. is true or false?

A:

$n^x \leq n^y$

log on both sides

$\log n^x \leq \log n^y$

$x \log n \leq y \log n$

$x < y$.

So the condition is always true.

OR take an example of value for x & y & then prove.

Q. $2n \neq O(nk)$. Check whether true or false?

A: $2n \leq O(nk)$

When $k = 2$ $\quad 2n = O(2n)$

So the stmt $2n \neq O(nk)$ is false.

Q. $(n+k)^m = O(n^m)$

A: $m=2$, $k=3$

$(n+3)^2 = O(n^2)$

$n^2 + 6n + 9 = O(n^2)$

↓ highest order

$O(n^2) = O(n^2)$

LHS = RHS.

Hence proved.

Q. $2^{n+1} = O(2^n)$. True or False?

A:

$2^{n+1} \leq 2^n \cdot c$

$2^n \cdot 2 \leq 2^n \cdot c$

$\therefore c = 2$

Hence stmt is true.

Q. $2^{2n+1} = O(2^n)$? If true, find c value?

A:

$2^{2n+1} \leq c \cdot 2^n$

$2^n \cdot 2^n \cdot 2 \leq c \cdot 2^n$

$c \geq 2^{n+1}$ (c value has to be constant) ∴ Not possible.

Hence stmt is false.

Q. $X = \sum_{i=0}^{n} i^3$: $O(n^4)$, $O(n^5)$, $\Omega(n^3)$, $\Theta(n^5)$. Which of these are true?

A:

$X = 0^3 + 1^3 + 2^3 + 3^3 + \ldots + n^3$

$= \left[\frac{n(n+1)}{2}\right]^2$

$= \left[\frac{n^2+n}{2}\right]^2 = \frac{n^4 + 2n^3 + n^2}{2}$

↓ $O(n^4)$, $\Omega(n^4)$ and $\Theta(n^4)$

$O(n^4)$ is UB, so $O(n^5)$ would also become an UB

So correct options are: $O(n^4)$, $O(n^5)$, $\Omega(n^3)$.

$\Omega(n^4)$ is LB, and $\Omega(n^3)$ comes inside $\Omega(n^4)$ so this is also true.

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Q1. If $f(n) = O(g(n))$ then,

$2^{f(n)} = O(2^{g(n)})$ ~~Prove~~ True or False?

Q2. If $f(n) = O(g(n))$ then ~~$f(n) \times h(n)$~~

$f(n) * h(n) = O(g(n) \times h(n))$. True or False.

Answers:

1. ~~Let $f(n) = g(n) = 2n$~~

Let $f(n) = 2n$ $\quad g(n) = n$

$f(n) \leq c \cdot g(n)$

$2n \leq c \cdot n$

$c = 2$

Here I got it in one case but check for 3 cases like,

If $f(n) = n$ & $g(n) = 2n$ ($f(n) < g(n)$)

$f(n) = 2n$ & $g(n) = 2n$ ($f(n) = g(n)$)

$f(n) = 2n$ & $g(n) = n$ ($f(n) > g(n)$)

→ do all three cases

Now, To check: $2^{f(n)} \leq c_2 \cdot 2^{g(n)}$

$2^{2n} \leq c_2 \cdot 2^n$

$2^n \cdot 2^n \leq c_2 \cdot 2^n$

$c_2 = 2^n$

but this is not possible as c is constant & n is variable

Hence false.

2. $f(n) = O(g(n))$

ie; $f(n) \leq c_1 \cdot g(n)$

Now to check if:

$f(n) \cdot h(n) = O(g(n) \cdot h(n))$

ie; $f(n) \cdot h(n) \leq c_2 \cdot g(n) \cdot h(n)$

ie; $f(n) \leq c_2 \cdot g(n)$

↓ which is true when $c_1 = c_2$

Hence stmt is true.

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X. Q3: $f(n) = O(f(n))$. Is it true or false?

A: It is true. This is called Reflexive property.

$f(n) \leq c \cdot f(n)$

~~$f(n) < c \cdot f(n)$~~ $\rightarrow c \geq 1$

Q4: $f(n) = O(f(n/2))$

A: To check if $f(n) \leq c \cdot f(n/2)$

Say $f(n) = n^2$.

then $f(n/2) = (n/2)^2$

Now

$n^2 \leq c \cdot \frac{n^2}{4}$

So when $c \geq 4$, it is true.

Hence this is ~~always~~ true. (We are taking complexities so remove constants).

~~Q5: $f(n)$~~

$f(n) = 2^{2n}$

$f(n/2) = 2^{2 \cdot \frac{n}{2}} = 2^n$

$f(n) \leq c \cdot f(n/2)$

$\Rightarrow 2^{2n} \leq c \cdot 2^n$

$c = 2^n$ which is not possible

Hence this is False.

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Qs. $f(n) = 5n^2 + 6n + 8$. find Big Oh notation?

Ans: $O(5n^2 + 6n + 8)$

$= O(5n^2 + 6n)$ Removal of constants & lower powers.

$= O(5n^2)$

$= O(n^2)$

OR.

$O(5n^2 + 6n + 8)$

$\downarrow \quad \downarrow \quad \downarrow$ maximise

$O(5n^2 + 6n^2 + 8n^2)$

$O(19n^2)$

$O(n^2)$

ie; $c = 19$

$n_0 \geq 1$

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Guidelines for Asymptotic Notations :

1. Loop

```
for (i=1 ; i<=n ; i++) {
    //stmt
}
```

Loop executes n times

Time complexity = O(n)

2. Nested Loop

```
for(i=1 ; i<=n ; i++) {
    for(j=1 ; j<=n ; j++) {
        //stmt executes nxn times.
    }
}
```

executes n times

executes n^2 times

TC = $O(n^2)$

3. Consecutive statements

```
int x = 2 ;
int i ;
x = x+1 ;
for(i=1; i<=n; i++) {
    //stmt
}
for(i=1 ; i<=n ; i++){
    for(j=1; j<=n ; j++){
        //stmt
    }
}
```

1

n

n^2

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TC = O(1+n+n²)
= O(n²)

4. if-then-else stmts

```
scanf("%d", &n);          → 1
if (n==0){                → 1
    // stmt               → O(1)
}                         O(1+1) = O(1)
else {                    OR
    for(i=1; i<=n; i++){  → n   O(1+n) = O(n)
        // stmt;
    }
}
```

scanf("%d", &n) ↳ test case (if(x==0))

TC = O(1+1+n) ↳ else part.

= O(n)

We generally take worst case time complexity & just take greater value.

5. Logarithmic Time Complexity:

```
for(i=1; i<=n){
    // stmt
    i = i*2
}
```

i	stmt repeats
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
⋮	⋮
2^x	$x+1$

$2^x <= n$ → $2^{x-1} \le n$

$x+1 = \log_2 n$ → $x-1 = \log_2 n$

$x = \log_2 n + 1$

$TC = O(\log_2 n)$ ⟷ $O(\log_2 n)$

* if $i = i*3$ ↳ $O(\log_3 n)$

⋮

ie; $i = i*k$ ↳ $O(\log_k n)$

~~Q. Find time comp~~

6. Logarithmic extra

```
for(i=n; i>=1)
{
    // stmt;
    i = i/2;
}
```

i	iteration
n	1
$n/2$	2
$n/2^2$	3
⋮	⋮
$n/2^{x-1}$	x

$$\frac{n}{2^{x-1}} \geq 1$$

$$n \geq 2^{x-1}$$

$$\log n \geq x-1$$

$$\log_2 n + 1 \geq x.$$

$$O(\log_2 n)$$

$$\therefore TC = O(\log_2 n)$$

$\therefore$ $i = i/k$

$\hookrightarrow O(\log_k n)$

Q1. void fun(int n) {

$O(1)$ ← int i, j, k, count = 0; → steps @ $n/2$ so $O(n/2) = O(n)$

$O(n)$ ← for(i = n/2 ; i <= n; i++) {

for(j = 1 ; j + n/2 <= n; j++) {

$O(\log_2 n)$ ← for (k = 1; k <= n; k = k * 2) {

count++;

} } }

↳ logarithmic rule

1st loop: 2nd loop:

j	$j + n/2 \leq n$
1	↓
⋮	$j = n/2$
$n/2$	

$$O(n/2) = O(n).$$

$$1 + (O(n) \times O(n) \times O(\log_2 n))$$

$$= O(n^2 \log_2 n)$$

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Q1. void fun (int n) {
for (i = n/2 ; i <= n ; i++) { → O(n/2) = O(n)
for (j = 1 ; j <= n ; j = j * 2) { → O(log n)
for (k = 1 ; k <= n ; k++) { → O(n)
count ++;
}
}
}

A: O(n × log n × n) = O(n² log n)

Q2. void fun (int n) {
if (n <= 1) → O(1)
return;
int i, j; → O(1)
for (i = 1 ; i <= n ; i++) { → O(n)
for (j = 1 ; j <= n ; j++) { → O(1)
printf ("hello");
break;
}
}

A: O(n)

Q3. void fun (int n) {
int i = 1; → 1
while (i < n) { → log n
int j = n;
while (j > 0) { → log n
j = j / 2;
}
i = i * 2;
}
}

A: Same as waiting

for (i = 1 ; i < n ; i = i * 2) { → O(log n)
for (j = n ; j > 0 ; j = j / 2) { O(log n)
#stmt
}
}

TC = O(log n × log n) = O(log n)²

Q4. void fun (int n) {
int i, j; → 1
for (i = 1 ; i <= n/3 ; i++) { → O(n/3) = O(n)
for (j = 1 ; j <= n ; j += 4) { → O(n)
printf ("hello");
}
}

j	
1	1
1+4	2
1+4×2	3
1+4×3	4
1+4×(k-1) = n	k
k = (n-1)/4	

A: $O(n^2)$

2nd loop: j	iteration
1	1
1+4	2
1+4x2	3
1+4x3	4
⋮	⋮
1+4x(k-1)	k

$1 + 4x(k-1) \leq n$

$1 + 4k - 4 = n$

$4k - 3 = n$

$4k = n+3$

$k = \frac{n+3}{4}$

$\hookrightarrow O(n)$

TC = $1 + (n * n) = 1 + n^2 = O(n^2)$

Qs.
```
A() {
int
```
```
fun() {
    int i, j, k, n;
    for(i=1; i<=n; i++){
    for
```

Qs.
```
fun() {
int i, j, k, n;                          → 1
for(i=1; i<=n; i++) {                    → n
    for(j=1; j<=i; j++){
        for(k=1; k<=100; k++){
            printf("hello");
        }
    }
}
```

A:

i	j
1	1
2	1, 2
3	1, 2, 3
4	1, 2, 3, 4
⋮	⋮
n	1, 2, 3, ..., n

printf "hello" occurrence:

$100 (1+2+3+4+...+n)$

$\downarrow$

$\frac{n(n+1)}{2}$

$O(n^2)$

OR

i=1	i=2	i=3	... i=k
j=1	j=1,2 (2 times)	j=1,2,3 (3 times)	j=1,2,3,...,k
k=100	k=2x100	k=3x100	k=kx100

$TC = 100 + 2\times100 + 3\times100 + 4\times100 + \cdots + k\times100$

How many times printf("Hello") occurs.

$= 100\left(\frac{k\times(k+1)}{2}\right)$ (Because $i=n \Rightarrow$ termination so $k=n$)

$= O\left(\frac{n(n+1)}{2}\right)$

$= O(n^2)$

Q6.
```
fun() {
  int i, j, k, n;
  for(i=1; i<=n; i++){
    for(j=1; j<=i^2; j++){
      for(k=1; k<=n/2; k++){
        printf("Hello");
      }
    }
  }
}
```

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A.

i=1	i=2	i=3	...
j=1	j=1,2,3,4	j=3^2	
k=n/2 times	k=$4\times\frac{n}{2}$	k=$3^2\times\frac{n}{2}$	

$TC = \frac{n}{2} + 4\frac{n}{2} + 3^2\frac{n}{2} + \cdots + k^2\cdot\frac{n}{2}$

$= \frac{n}{2}\left(1^2+2^2+3^2+\cdots+k^2\right)$ ($k=n$: termination)

$= \frac{n}{2}\left(1^2+2^2+3^2+\cdots+n^2\right)$

$= \frac{n}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$= O(n^4)$

Q7.
```
fun(){
  int i, j, k;                          ---- 1
  for(i=n/2; i<=n; i++){                ---- n
    for(j=1; j<=n; j=2*j){              ---- log n
      for(k=1; k<=n; k=k*2){            ---- log n
        printf("hello");
      }
    }
  }
}
```

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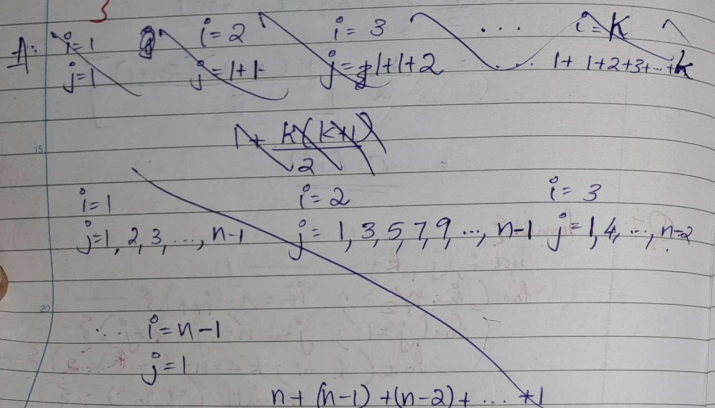
A: $O(1 + n\log^2 n)$

$= O(n\log^2 n)$

Q8.
```
fun(){
  for(i=1; i<=n; i++){
    for(j=1; j<=n; j=j+i){
      printf("hello");
    }
  }
}
```

A:



A:

$i=1$	$i=2$	$i=3$	$i=k$
$j=n$	$j=1$ to n	$j=1$ to n	$j=1$ to n
$j=j+1$	$j=j+i$	$j=j+i+3$	$j=j+k$
1, 2, 3, ...	1, 3, 5, 7, ...	1, 4, 7, 10, ...	
n times	$n/2$ times	$n/3$ times	n/k times

$$\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{k}$$

$$n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right]$$

$n(\log n)$

~~$\therefore O(n\log n) \cdot O(1)$~~

$\therefore O(n\log n)$

Comparison Of Growth Rate:

1. Log Method.
2. L Hospital Method. → applied in ∞/∞ or $0/0$
   ↳ if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty/\infty$ or $0/0$ then apply.

~~n~~

★ $1 \cdot \log n \quad n \quad n\log n \quad n^2 \quad n^3 \quad \cdots \quad n^n \quad 2^n \quad 3^n$

↓ least. → increasing order.

Q1. Compare:
1. $f(n) = n^3 \log n$
$g(n) = (n \log n)^{20}$

A: $\log(f(n)) = \log(n^3 \log n)$
$= \log n^3 + \log \log n$
$= 3 \log n + \log \log n$

$\log(g(n)) = \log(n \log n)^{20}$
$= 20 (\log(n \log n))$
$= 20 \log n + 20 \log \log n$

$\therefore \log(f(n)) < \log(g(n))$

$f(n)$ grows slower than $g(n)$

Q2. $f(n) = 4n^{\sqrt{n}}$
$g(n) = 2^{\sqrt{n} \log_2 n}$

A: $4n^{\sqrt{n}}$ $\quad$ $2^{\sqrt{n} \log n}$

$\log(f(n))$ $\quad$ $\log(g(n))$
$= \log(4n^{\sqrt{n}})$ $\quad$ $\log(2^{\sqrt{n} \log n})$
$= \log 4 + \sqrt{n} \log n$ $>$ $\sqrt{n} \log n \log_2 2^1$

$\therefore f(n) > g(n)$

Q3 $f(n) = \log 2^n$ $\quad$ $g(n) = \log 2^{2n}$

A: $g(n) \geq f(n)$

Q4. $f(n) = n^{\log n}$ $\quad$ $g(n) = 2^{\sqrt{n}}$

A: $\log(f(n))$ $\quad$ $\log(g(n))$
$= \log(n^{\log n})$ $\quad$ $= \log(2^{\sqrt{n}})$
$= \log n \log n$ $\quad$ $= \sqrt{n} \log 2 \to 1$
$= \log^2 n$ $\quad$ $= \sqrt{n}$

Again log.

$\log(\log n)^2$ $\quad$ $\log(n^{1/2})$
$2 \log \log n$ $<$ $\frac{1}{2} \log n$

$\therefore g(n) > f(n)$

Q5. $f(n) = 2^{\log n}$ $\quad$ $g(n) = n^{\sqrt{n}}$

A:

$\log(f(n))$ $\quad$ $\log(n^{\sqrt{n}})$
$= \log(2^{\log n})$ $\quad$ $\sqrt{n} \log n$
$= \log n \cdot \log 2^1$ $<$

$\therefore g(n) > f(n)$

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L-Hospital Rule:

$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{\cos x}{1}$$

$$= \lim_{x\to 0} \cos x$$

$$= \cos 0 = 1$$

Evaluate,

1. $\lim_{x\to 0} \frac{1-\cos x}{x}$

$$\lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0} \frac{\sin x}{1}$$

$$= \sin 0$$

$$= 0$$

2. $\lim_{x\to 0} \frac{3x+\sin 6x}{4x^2-3\sin x}$

A:

$$\lim_{x\to 0} \frac{\frac{d}{dx}(3x+\sin 6x)}{\frac{d}{dx}(4x^2-\sin x)}$$

$$\lim_{x\to 0} \frac{3+6\cdot\cos 6x}{8x-\cos x}$$

$$\frac{3+6}{-1} = -9$$

3. $\lim_{x\to 3} \frac{x-3}{x^2-x-6}$

A:

$$\lim_{x\to 3} \frac{\frac{d}{dx}(x-3)}{\frac{d}{dx}(x^2-x-6)}$$

$$= \lim_{x\to 3} \frac{1}{2x-1}$$

$$= \frac{1}{6-1} = \frac{1}{5}$$

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4. $\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$

A: $\lim_{x \to a} \frac{d/dx(\cos x - \cos a)}{d/dx(x - a)}$

$= \lim_{x \to a} \frac{-\sin x - 0}{1}$

$= -\sin a$

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Date :

4. $\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$

A: $\lim_{x \to a} \frac{\frac{d}{dx}(\cos x - \cos a)}{\frac{d}{dx}(x - a)}$

$= \lim_{x \to a} \frac{-\sin x - 0}{1}$

$= -\sin a$

8/5/24/Wednesday.

Q1. $\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}$

A: $= \lim_{x \to 0} \frac{\frac{d}{dx}(1 - \cos 4x)}{\frac{d}{dx}(x^2)}$

$= \lim_{x \to 0} \frac{2 \times \sin 4x}{2x}$

$= \lim_{x \to 0} \frac{2 \sin 4x}{x}$ Again do L hospital rule

(Because it is not 0, ∞, constant)

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$= \lim_{x \to 0} \frac{\frac{d}{dx} 2 \sin 4x}{\frac{d}{dx} x}$

$= \lim_{x \to 0} 2 \cos 4x \times 4$

$= \lim_{x \to 0} 8 \cos 4x$ → a constant.

$= 8$

Q2: $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

A: $= \lim_{x \to 0} \frac{\frac{d}{dx}(\cos 2x - 1)}{\frac{d}{dx}(\cos x - 1)} = \lim_{x \to 0} \frac{\sin 2x \times 2}{\sin x}$

$= \lim_{x \to 0} \frac{2 \sin 2x}{\sin x}$

Again apply L hospital rule.

$= \lim_{x \to 0} \frac{4 \cos 2x}{\cos x} = 4$ ↓ constant

If @ end of L hospital rule you get (f is faster ↑)

1. ∞ → Ω (best case - Omega) → f > g
2. 0 → O (worst case - big Oh) → f < g
3. constant → Θ (avg) → f ± g are comparable.

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Q3. Compare x^m & e^{nx} ; $m>0$ & $n>0$ & m is an Integer.

A. $\lim_{x\to\infty} \frac{x^m}{e^{nx}}$ → in $\frac{\infty}{\infty}$ form so we can apply L hospital rule.

$$\lim_{x\to\infty} \frac{m x^{m-1}}{e^{nx}\cdot n} \to \frac{\infty}{\infty}$$

So apply L hospital rule again

$$\lim_{x\to\infty} \frac{m\cdot(m-1)\, x^{m-2}}{e^{nx}\cdot n^2} \to 2^{nd} \text{ iteration}$$

Again apply L hospital rule.

⋮

m^{th} iteration: $$\lim_{x\to\infty} \frac{m\cdot(m-1)(m-2)\cdots\times 2\times 1\cdot x^{m-m}}{e^{nx}\cdot n^m}$$

$$\lim_{x\to\infty} \frac{m!\times x^0}{e^{nx}\times n^m}$$

$$= \frac{m!}{n^m\cdot\infty} = \frac{m!}{n^m}\cdot\frac{1}{\infty}$$

$$= \frac{m!}{n^m}\cdot 0$$

$$= 0 \to \text{zero, so big Oh.}$$

So f<g. That is; x^m is lower than e^{nx}.

or

$$x^m = O(e^{nx})$$

Q4. Analyse the growth rate of $\ln(x^{20})$ and $\ln(x)$.

A: $\lim_{x\to\infty} \frac{\ln(x^{20})}{\ln(x)} \to \frac{\infty}{\infty}$ so L hospital rule,

$$= \lim_{x\to\infty} \frac{\frac{d}{dx}(\ln(x^{20}))}{\frac{d}{dx}(\ln(x))}$$

$$= \lim_{x\to\infty} \frac{\frac{20x^{19}}{x^{20}\,x}}{\frac{1}{x}} \qquad \lim_{x\to\infty}\frac{20}{x} = \frac{1}{x}$$

$$= \lim_{x\to\infty} 20$$

$$= 20 \to \text{constant}$$

So avg bound.

ie; f(n) & g(n) grow at same rate.

ie; $\ln(x^{20}) = \Theta(\ln(x))$

Q5. Compare e^x & x^2.

A: $\lim_{x\to\infty} \frac{e^x}{x^2} \to \frac{\infty}{\infty}$ so L hospital rule.

$$\lim_{x\to\infty} \frac{\frac{d}{dx}e^x}{\frac{d}{dx}x^2} = \lim_{x\to\infty} \frac{e^x}{2x}$$ Again L hospital rule

$$= \lim_{x\to\infty} \frac{e^x}{2} = \infty/2 = \infty$$

$\infty \rightarrow \Omega \rightarrow f > g$: f is ~~slower~~ faster than g.

ie; $e^x = \Omega(x^2)$

or $e^x > x^2$

e^x has faster growth rate.

Q6. Compare 3^x & 2^x ?

A: $\lim_{x\to\infty} \frac{3^x}{2^x} \rightarrow \frac{\infty}{\infty}$ can apply L hospital rule

$\lim_{x\to\infty} \left(\frac{3}{2}\right)^x$

$= \infty$ so Ω,

That is; $3^x = \Omega(2^x)$ or $3^x > 2^x$.

Q7. Compare x^2 & $\ln(x)$

A: $\lim_{x\to\infty} \frac{x^2}{\ln(x)} \rightarrow \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{1/x} = \lim_{x\to\infty} 2x^2$

$= 2\infty^2 = \infty$

So Ω.

$x^2 = \Omega(\ln(x))$

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Q8. Compare $\ln(x)$ & $x^{1/n}$?

A: $\lim_{x\to\infty} \frac{\ln(x)}{x^{1/n}}$

$\lim_{x\to\infty} \frac{1/x}{\frac{1}{n} x^{1/n-1}}$

$= \lim_{x\to\infty} \frac{1}{x} \div \frac{1}{n} \cdot x^{1/n-1}$

$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{n}{x^{1/n-1}}$

$= \lim_{x\to\infty} \frac{n}{x^{1/n}}$

$= \frac{n}{\infty} = 0$

So Big Oh.

ie; $\ln(x) = O(x^{1/n})$

09/05/24/Thursday.

Q9. Show that $\log x = O(x)$?

A: $f(n) = \log x$

$g(n) = x$.

$\lim_{x\to\infty} \frac{\log x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x}$

$= \frac{1}{\infty} = 0$

So big Oh; ie; $\log(x) = O(x)$

Hence proved.

Camlin

Q10. Show that $\sqrt{2}^{\log n} = O(\sqrt{n})$?

A: $f(n) = \sqrt{2}^{\log n}$ $\quad g(n) = \sqrt{n}$

$$\lim_{n\to\infty} \frac{\sqrt{2}^{\log n}}{\sqrt{n}}$$

$a^{\log b} = b^{\log a}$

$$\lim_{n\to\infty} \frac{n^{\log\sqrt{2}}}{\sqrt{n}}$$

[no idea how it got there ...

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = \frac{1}{\infty} = 0$$

So $\sqrt{2}^{\log n} = O(\sqrt{n})$

Q11. Compare the growth rate of $\log_2 n$ & $\sqrt{n}$?

A: $$\lim_{n\to\infty} \frac{\log_2 n}{\sqrt{n}}$$

↓ d/dx on num & denom

$$\lim_{n\to\infty} \frac{1/n}{1/2\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{2\sqrt{n}}{n}$$

$$= \lim_{n\to\infty} \frac{2}{\sqrt{n}}$$

$$= \frac{2}{\infty} = 0 \rightarrow \log_2(n) = O(\sqrt{n})$$

$\log n$ has slower growth rate

Q12. Compare growth rate of $f(n) = n!$ & $g(n) = 2^n$?

A: Here we cannot use L hospital rule unless we do something abt $n!$. So we use Stirling's Approximation ↓

$$n! = \sqrt{2\pi n}\left[\frac{n}{e}\right]^n \left[1 + \Theta\left(\frac{1}{n}\right)\right]$$

Now,

$$\lim_{n\to\infty} \frac{n!}{2^n}$$

$$= \lim_{n\to\infty} \frac{\sqrt{2\pi n}\left[\frac{n}{e}\right]^n \left[1 + \Theta \cdot \frac{1}{n}\right]}{2^n}$$

Don't apply derivative. Directly apply $n = \infty$

$$= \frac{\sqrt{2\pi\infty}\left[\frac{\infty}{e}\right]^{\infty}\left[1 + \Theta \cdot \frac{1}{\infty}\right]}{2^{\infty}} \quad (2^{\infty} \rightarrow \text{becomes } \infty)$$

$$= \infty \cdot \left[1 + \Theta \cdot \frac{1}{\infty}\right]$$

$$= \infty \cdot 1$$

$$= \infty$$

$\therefore n! = \Omega(2^n)$

Q13. Prove that $n! = O(n^n)$

A: $n! = (2\pi n)\left[\frac{n}{e}\right]^n [1 + \Theta \cdot \frac{1}{n}]$

$$\lim_{n\to\infty} \frac{n!}{n^n}$$

$$\lim_{n\to\infty} \frac{2\pi n \left[\frac{n}{e}\right]^n [1 + \Theta \cdot \frac{1}{n}]}{n^n}$$

$$= \frac{2\pi\infty \left[\frac{\infty}{e}\right]^\infty [1 + \Theta \cdot \frac{1}{\infty}]}{\infty^\infty}$$

$$= \frac{(1 + \Theta \cdot \frac{1}{\infty})}{\infty} = \frac{1}{\infty} = 0$$

So $n! = O(n^n)$

Hence proved.

Q14. Show that $10n^2 + 9 = O(n^2)$

A:

On hold...

No answer yet...

Q15. Show that $\frac{6n^3}{\log n + 1} = O(n^3)$?

A: $$\lim_{n\to\infty} \frac{\frac{6n^3}{\log n + 1}}{n^3} = \lim_{n\to\infty} \frac{6}{\log n + 1}$$

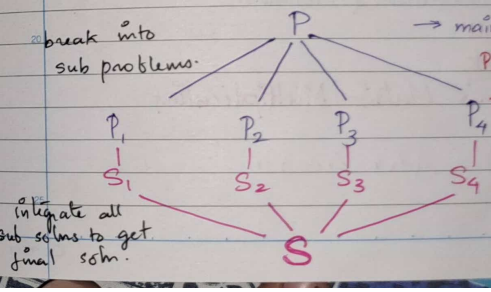
$$= 6/\infty = 0$$

Hence: $\frac{6n^3}{\log n + 1} = O(n^3)$

Recurrence Relation:

- Used for recurring functions.
- use of divide & conquer strategy.
- There are other methods as well. One such method is Dynamic Programming.

Divide & Conquer Strategy: (DAC)



break into sub problems.

→ main problem.

P_1, P_2, P_3, P_4 are → sub problems. They all have to be of the same type. eg: if P is a sorting prob, the P_1, P_2, P_3 and P_4 should be sorting prob.

integrate all sub solns to get final soln.

Guideline to check if Divide & Conquer Strategy can be used:

If a big problem can be divided into subproblems of same type, each having solutions which can be integrated together to form the final soln. Then we can apply DAC.

```
DAC(P):
    if (small(P)){
        S(P);
    }
    else if (big(P)){
        divide P into P1, P2, P3, ..., Pk
        Apply DAC(P1), DAC(P2), ..., DAC(Pk)
    }
    combine S1, S2, S3, ..., Sk to form S.
```

Example problems where we can apply DAC:

- Binary search
- Finding Maximum & Minimum
- Merge Sort
- Quick Sort
- Strassen's Matrix Multiplication

Recurrence Relation

Decreasing Function	Dividing Function	Root Function
$T(n-b)$	$T(n/b)$	$T(\sqrt{n})$
↓	↓	↓
1	1	2

smallest value possible:

Solution approaches:

1. Back Substitution
2. Recurrence tree / Tracing Method
3. Master Method.

15/5/24/Wednesday

Q.

```
void Test(int n)                    ---> T(n)
{
  if (n>0)                          ---> 1
  {
    printf("%d", n);                ---> 1
    Test(n-1);                      ---> T(n-1)
  }
}
```

A: First (This is a decreasing fn).
assume time for Test(n) = T(n)
then remaining as marked ...
Then,

$T(n) = 1 + 1 + T(n-1)$
$= (2) + T(n-1)$ ⟩ asymptotic (approximation)
$= T(n-1) + (1)$

ie; $T(n) = T(n-1) + 1$

Now write this in recurrence relation form:

smallest value for decreasing fn.

$$T(n) = \begin{cases} 1 & ; n=0 \\ T(n-1)+1 & ; n>0 \end{cases}$$

Solving using:

1. Back Substitution:

$T(n) = T(n-1) + 1$ —— ①
$T(n-1) = T(n-2) + 1$ (we get by applying (n-1) in 1)

Now back substitute this in ①,

$T(n) = [T(n-2) + 1] + 1$
$T(n) = T(n-2) + 2$ —— ②

Do such back substitutions for 3 or 4 iterations before generalizing.

Similarly $T(n-2) = T(n-3) + 1$ ⟩ back substitute

$\Rightarrow T(n) = [T(n-3) + 1] + 2 = T(n-3) + 3$

⋮

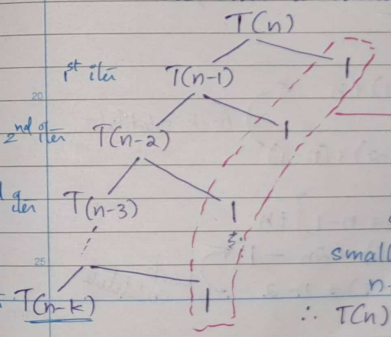
Now in k^{th} iteration,

$T(n) = [T(n-k)] + k$
↳ assume this to be smallest value possible for decreasing fn.
$= 1 + k$
↳ smallest value for decreasing fn.

From the recurrence relation we know that the smallest value of T(n) occurs when n = 0
ie;
$n - k = 0$
$n = k$.
ie; $T(n) = 1 + n$
↓ Asymptotically
$T(n) = n$
ie; $T(n) = O(n)$

2. Tree Method:

We have to draw the recurrence tree.



Now ignore the T branch.
only consider this part.
Add them.
$1 + 1 + 1 \ldots + 1^{k} = 1 \times k = k$
So $T(n) = k$
smallest time: $n = 0$ so,
$n - k = 0 \rightarrow n = k$
$\therefore T(n) = O(n)$

Q2.
```
void Test (int n)                          → T(n)
{
  if (n>0)                                 → 1
  {
    for (int i=0; i<n; i++)                → n
    {
      printf("%d", n);
    }
    Test(n-1);                             → T(n-1)
  }
}
```

A: $T(n) = 1 + n + T(n-1)$

↓ Asymptotically.

$T(n) = T(n-1) + n$

Recurrence Relation:

$$T(n) = \begin{cases} 1 & ; n = 0 \\ T(n-1) + n & ; n > 0 \end{cases}$$

Back Substitution:

1st — $T(n) = T(n-1) + n$ ← back substitute

$T(n-1) = T(n-2) + (n-1)$

$T(n) = [T(n-2) + n - 1] + n$

2nd — $T(n) = T(n-2) + 2n - 1$ ← back substitute

$T(n-2) = T(n-3) + n - 2$

3rd ← $T(n) = T(n-3) + n - 2 + 2n - 1$

$T(n) = T(n-3) + 3n - 3$ ← back substitute

$T(n-3) = T(n-4) + n - 3$

$T(n) = T(n-4) + n - 3 + 3n - 3$

$T(n) = T(n-4) + 4n - 6$

4th iter ← $T(n) = T(n-4) + 4n - (1+2+3)$

⋮

k^{th} iter: $T(n) = T(n-k) + kn - (1+2+3+\dots+k)$

$$T(n) = T(n-k) + kn - \left[\frac{k(k+1)}{2}\right]$$

$$= T(n-k) + kn - \frac{k^2+k}{2}$$

$T(n) = T(n-3) + n - 2 + 2n - 1$

3rd ← $T(n) = T(n-3) + 3n - 3$

back substitute

$T(n-3) = T(n-4) + n - 3$

$T(n) = T(n-4) + n - 3 + 3n - 3$

$T(n) = T(n-4) + 4n - 6$

4th iteration ← $T(n) = T(n-4) + 4n - (1+2+3)$

⋮

k^{th} iteration: $T(n) = T(n-k) + kn - (1+2+3+\ldots+k)$

$T(n) = T(n-k) + kn - \left[\frac{k(k+1)}{2}\right]$

$= T(n-k) + kn - \frac{k^2+k}{2}$

$n-k=0$

$= T(0) + n^2 - \frac{n^2+n}{2}$

$= 1 + \frac{2n^2-n^2+n}{2}$

$= 1 + \frac{n^2+n}{2}$

$= O(n^2)$

16/05/24/Thu.

OR.

$T(n) = T(n-1) + n$

$T(n) = T(n-2) + (n-1) + n$

$T(n) = T(n-3) + (n-2) + (n-1) + n$

So,

k^{th} → $T(n) = T(n-k) + (n-(k-1)) + \ldots + (n-1) + n$

Minimum value → 1 when $n = 0$.

So, $n - k = 0 \rightarrow n = k$.

$T(n) = 1 + (n-(n-1)) + \ldots + (n-1) + n$

$= 1 + (1) + \ldots + (n-2) + (n-1) + n$

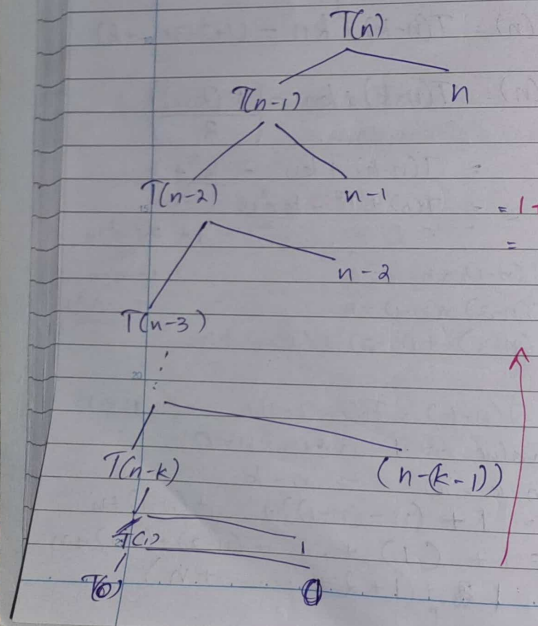
$= 1 + (1 + 2 + \ldots + n)$

$= 1 + \frac{n(n+1)}{2}$

$= 1 + \frac{n^2+n}{2}$

$= O(n^2)$

Trace Method:



$= 1 + 2 + 3 + \dots + n$

$= \frac{n(n+1)}{2}$

$= \frac{n^2+n}{2}$

$= O(n^2)$

Q3.
```
void Test(int n)                    ---> T(n)
{
  if(n>0)                           ---> 1
  {
    for(int i=1; i<n; i=(i*2))      }
    {                               } log2n
      printf("%d", n);              }
    }
    Test(n-1);                      ---> T(n-1)
  }
}
```

$T(n) = 1 + \log n + T(n-1)$

$T(n) = T(n-1) + \log n$

$$T(n) = \begin{cases} 1 & ; n=0 \\ T(n-1) + \log n & ; n>0 \end{cases}$$

Back Substitution:

$T(n) = T(n-1) + \log n$ —— ①

$T(n-1) = T(n-2) + \log(n-1)$

$T(n) = T(n-2) + \log(n-1) + \log n$ —— ②

$T(n-2) = T(n-3) + \log(n-2)$

$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$ —— ③

$\vdots$

K^{th} ← $T(n) = T(n-k) + \log(n-(k-i)) + \dots + \log(n-1) + \log n$

Minimum at $n=0$ for $T(n)$.

So, $n-k=0$

$n=k$.

$T(n) = T(0) + \log(n-(n-1)) + \dots + \log(n-1) + \log n$

$= 1 + \log(1) + \log 2 + \dots + \log(n-1) + \log n$

$\log ab = \log a + \log b$ → $= 1 + \log(1 \times 2 \times \dots \times (n-1) \times n)$

$= 1 + \log(n!)$

$= \log(n!)$

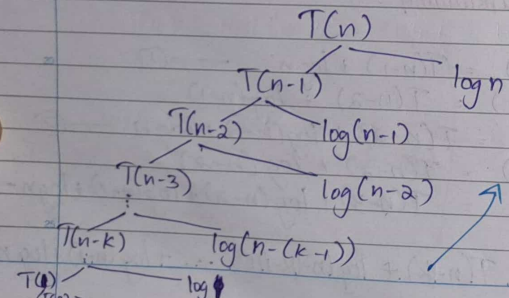
n^n → upper bound.

$= \log(n^n)$

$= n \log n$

$= O(n \log n)$

Tree Method:



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$\log 1 + \log 2 + \dots + \log(n)$

$= \log(1 \times 2 \times 3 \times \dots \times (n-1) \times n)$

$= \log(n!)$

$= \log(n^n)$

$= n \log n$

$= O(n \log n)$

Q4:

```
void Test(int n)          ---- T(n)
{
if (n>0)                  ---- 1
{
printf("%d", n);         ---- 1
Test(n-1);               ---- T(n-1)
Test(n-1);               ---- T(n-1)
}
}
```

A: $T(n) = 1 + 1 + T(n-1) + T(n-1)$

$= 2 + 2(T(n-1))$ ↓ asymptotically

$= T(n-1) + T(n-1) + 1$

$$T(n) = \begin{cases} 1 & ; n=0 \\ 2T(n-1)+1 & ; n>0 \end{cases}$$

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$$T(n) = 2T(n-1) + 1 \quad \text{—— (1)}$$

$$T(n-1) = 2T(n-2) + 1$$

~~$T(n) = 2T(n-2) + 2T(n-1) + 1$~~

$$T(n) = 2(2T(n-2)+1) + 1$$

$$T(n) = 4T(n-2) + 2 + 1 \quad \text{—— (2)}$$

$$T(n-2) = 2T(n-3) + 1$$

$$T(n) = 4(2T(n-3)+1) + 2 + 1$$

$$T(n) = 8T(n-3) + 4 + 2 + 1 \quad \text{—— (3)}$$

$$T(n-3) = 2T(n-4) + 1$$

$$T(n) = 8[2T(n-4)+1] + 4 + 2 + 1$$

$$T(n) = \underset{2^4}{16}T(n-4) + \underset{2^3}{8} + \underset{2^2}{4} + \underset{2^1}{2} + \underset{2^0}{1} \quad \text{—— (4)}$$

⋮

$$T(n) = 2^k T(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2^1 + 2^0$$

Minimum when $n-k=0$; $n=k$.

$T(0) = 1$

$$T(n) = 2^n T(n-n) + 2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0$$

$$= 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0 \rightarrow GP$$

$$\text{GP series sum} = \frac{a(r^k - 1)}{r-1}$$

(a → 1st term; k → no. of terms; r → common factor)

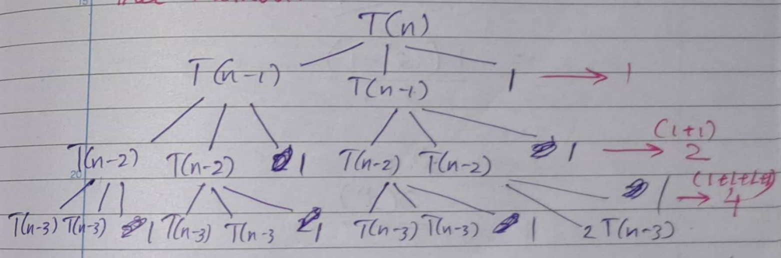
$a=1$; $r=2$; $k=n$

$$T(n) = \frac{1(2^n - 1)}{2-1}$$

$$= 2^n - 1$$

$$= O(2^n)$$

Tree Method



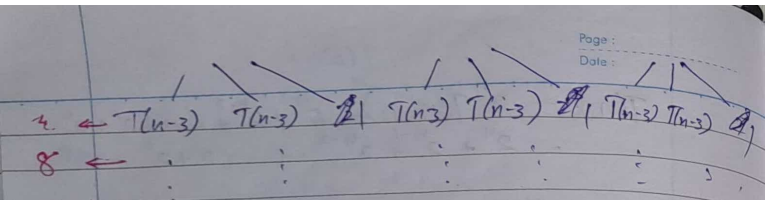
⋮

$$2^k T(n-k) \quad 2^{k-1} \dots$$

$$2^n + 2^{n-1} + 2^{n-2} + \dots + 2^3 + 2^2 + 2^1 + 2^0$$

$$= 2^n - 1 = O(2^n)$$

cont in next page



$T(n) = 2^0 + 2^1 + \dots + 2^{k-1}$

$= 2^0 + 2^1 + \dots + 2^{n-1}$

$= \frac{1(1-2^n)}{2-1}$

$= 1 - 2^n$

$= \frac{1(2^n - 1)}{2-1}$

$= 2^n - 1$

$= O(2^n)$

$\frac{a(r^n-1)}{r-1}$

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Master's Theorem For Decreasing Function :

$T(n) = T(n-1) + 1$	–	$O(n)$
$T(n) = T(n-1) + n$	–	$O(n^2)$
$T(n) = T(n-1) + \log n$	–	$O(n \log n)$
$T(n) = 2T(n-1) + 1$	–	$O(2^n)$
$T(n) = 3T(n-1) + 1$	–	$O(3^n)$
$T(n) = 2T(n-1) + n$	–	$O(n 2^n)$

★ $T(n) = 3T(n-1) + 1$

$T(n-1) = 3T(n-2) + 1$

$T(n) = 3^2 T(n-2) + 3 + 1$

$T(n-2) = 3T(n-3) + 1$

$T(n) = 3^3 T(n-3) + 3^2 + 3 + 1$

$\vdots$

$T(n) = 3^k T(n-k) + 3^{k-1} + \dots + 3^1 + 3^0$

$= 3^n \cdot T(0) + 3^{n-1} + \dots + 3^1 + 3^0$

$= 3^n + 3^{n-1} + \dots + 3^1 + 3^0$

$= \frac{1(3^n - 1)}{2}$

$= O(3^n)$

★ $T(n) = 2T(n-1) + n$ $\quad T(n-1) = 2T(n-2) + n$

$T(n) = 2^2 T(n-2) + 2n + n$

$T(n) = 2^3 T(n-3) + 2^2 n + 2n + n$

$\vdots$

$T(n) = 2^k T(n-k) + 2^{k-1} n + \dots + 2^1 n + 2^0 n$

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$T(n) = 2^n + 2^{n-1} + 2^{n-2} + \dots + 2n + n$

$= n(2^n + 2^{n-1} + \dots + 2^0)$

$= n \cdot \frac{2^n - 1}{2-1}$

$= n \cdot 2^n$

$= O(2^n \cdot n)$

Cases:

$T(n) = aT(n-b) + f(n)$

1. Case 1: if $a < 1$; $O(f(n))$
2. Case 2: if $a = 1$; $O(n * f(n))$
3. Case 3: if $a > 1$; $O(f(n) * a^{n/b})$

Recurrence Relations : Dividing Functions :

Q1.
```
void Test(int n)          ---- T(n)
{
  if(n>1)                 ---- 1
  printf("%d", n);        ---- 1
  Test(n/2);              ---- T(n/2)
}
```

$T(n) = 1 + 1 + T(n/2)$

$= 2 + T(n/2)$ } Asymptotically

$= T(n/2) + 1$

$$T(n) = \begin{cases} 1 & ; n = 1 \\ T(n/2) + 1 & ; n > 1 \end{cases}$$

Back Substitution:

$T(n) = T(n/2) + 1$ — (1)

$T(n/2) = T(n/2 \div 2) + 1$

$= T(n/2^2) + 1$

$T(n) = T(n/2^2) + 1 + 1$

$T(n) = T(n/2^2) + 2$ — (2)

$T(n/2^2) = T(n/2^3) + 1$

$T(n) = T(n/2^3) + 3$ — (3)

$\vdots$

$T(n) = T(n/2^k) + k$

——→ assume to be min.

$T(n) = 1 + k$

$T(n) = 1 + \log n$

$= \log n$

~~$T(n) = O(n)$~~

↳ @ ~~$k = n$~~

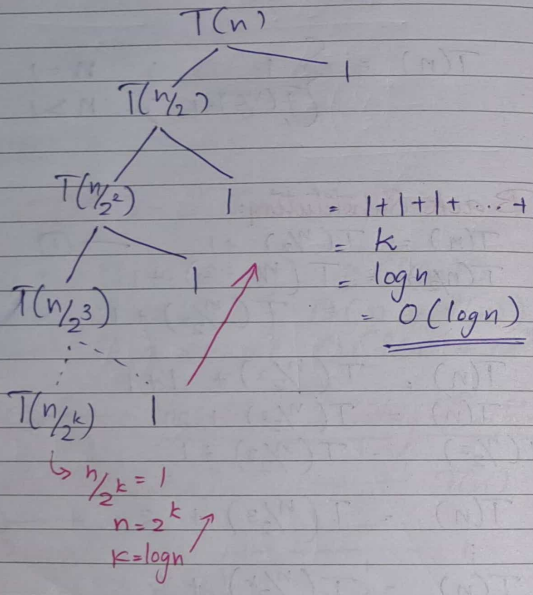
$n/2^k = 1$

$n = 2^k$

$k = \log n$

$T(n) = O(\log n)$

Tree Method:



$= 1+1+1+\dots+1$

$= k$

$= \log n$

$= O(\log n)$

↳ $n/2^k = 1$

$n = 2^k$

$k = \log n$

Q2. $T(n) = \begin{cases} 1 & ; n = 1 \\ T(n/2) + n & ; n > 1 \end{cases}$

1: Back Substitution:

$T(n) = T(n/2) + n$ —①

$T(n/2) = T(n/2^2) + n/2$

$T(n) = T(n/2^2) + n/2 + n$ —②

$T(n/2^2) = T(n/2^3) + n/2^2$

$T(n) = T(n/2^3) + n/2^2 + n/2 + n$ —③

⋮

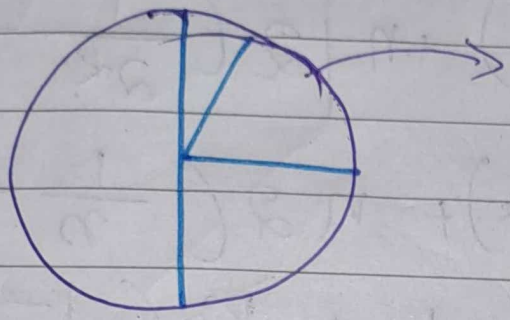
$T(n) = T(n/2^k) + n/2^{k-1} + \dots + n/2 + n$ → minimise.

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \log n$

~~$T(n) = 1 + \frac{n}{2^{\log n - 1}} + \dots + n/2 + n$~~

$T(n) = T(n/2^k) + \frac{n}{2^{k-1}} + \dots + n/2 + n$

$= T(n/2^k) + n\left[\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \dots + \frac{1}{2} + 1\right]$



circle considered as one.

So assuming

$$\frac{1}{2^{k-1}} + \frac{1}{2^k} + \dots + \frac{1}{2^2} + \frac{1}{2^1} + \frac{1}{2^0}$$

is equal to 1.

Then, $T(n) = T\left(\frac{n}{2^k}\right) + n(1)$

$= 1 + n \quad 2\left(\frac{1}{2^k} - 1\right)$

$= O(n)$

GP would be more complicated...

$a = \frac{1}{\quad}$ $r = \frac{1}{2}$ $k = k$ $\frac{1\left(\frac{1}{2}^k - 1\right)}{\frac{1}{2}}$

$GP = \frac{1\left(\left(\frac{1}{2}\right)^k - 1\right)}{\frac{1}{2} - 1} = \frac{-2\left(\left(\frac{1}{2}\right)^k - 1\right)}{= \frac{1}{2^k}}$

$= T(n/2^k) + n\left(\frac{1}{2^k}\right)$ $\quad n/2^k = 1$

$= 1 + n$ $\quad 2^k = n$

$= 1 + n$

$= T(n/2^k) + n\left[2\left(\frac{1}{2^k} - 1\right)\right]$

$= T(n/2^k) + n\,2\left(\frac{1}{n} - 1\right)$

$= 1 + n\left(\frac{1-n}{n}\right)$

$= O(n)$

Q3.
```
void Test(int n)          — T(n)
{
if(n>1)                   — 1
{
for(int i=0; i<n; i++)    } n
{
//stmt;
}
Test(n/2);                — T(n/2)
Test(n/2);                — T(n/2)
}
```

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Q4: $T(n) = 1 + n + 2T(n/2)$

$$T(n) = \begin{cases} 1 & ; n = 1 \\ 2T(n/2) + n & ; n > 1 \end{cases}$$

$T(n) = 2T(n/2) + n$ —— (1)

$T(n/2) = 2T(n/2^2) + n/2$

$T(n) = 2\left[2T(n/2^2) + n/2\right] + n$

$T(n) = 2^2 T(n/2^2) + n + n$ —— (2)

$T(n/2^2) = 2T(n/2^3) + n/2^2$

$T(n) = 2^2\left[2T(n/2^3) + \frac{n}{2^2}\right] + 2n$

$T(n) = 2^3 T(n/2^3) + 3n$ —— (3)

$\vdots$

$T(n) = 2^k T(n/2^k) + 2^{k-1}n + \cdots + 2n + n$

$T(n) = 2^k T(n/2^k) + 2^{k-1}n + 2^{k-2}n + \cdots + 2n + 2^0 n$

$T(n) = 2^k T(n/2^k) + k \cdot n$

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$T(n) = 2\left(T\left(\frac{n}{2^k}\right)\right)$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$

↳ minimised.

$\frac{n}{2^k} = 1$

$n = 2^k \qquad k = \log n$

$T(n) = n \cdot 1 + n\log n$

$T(n) = n\log n$

$T(n) = O(n \log n)$

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$T(n) = 2^k T(n/2^k) + n\left[2^{k-1} + 2^{k-2} + \dots + 2^1 + 2^0\right]$

$a = 1 \quad r = 2 \quad k = k$

$GP = \frac{a(r^k - 1)}{r - 1} = \frac{1(2^k - 1)}{2 - 1}$

$= 2^k - 1$

$T(n) = 2^k \left[T(n/2^k)\right] + n \cdot (2^k - 1)$

$T(n) = 2^k T(n/2^k) + kn$

↳ minimised.

$\frac{n}{2^k} = 1$

$n = 2^k \qquad k = \log n$

$T(n) = n \cdot 1 + n \log n$

$T(n) = n \log n$

$T(n) = O(n \log n)$

Camlin

22/05/24/Wednesday.

Master's Theorem for Dividing Function:

$T(n) = aT(n/b) + f(n)$

where:

$a \geq 1$ a should be a constant

$b > 1$

$f(n) = O(n^k \log^p n)$

1. Case 1: if $\log_b a > k$; then $O(n^{\log_b a})$

2. Case 2: if $\log_b a = k$
   - i) if $p > -1$; $O(n^k \log^{p+1} n)$
   - ii) if $p = -1$; $O(n^k \log \log n)$
   - iii) if $p < -1$; $O(n^k)$

3. Case 3: if $\log_b a < k$
   - i) if $p \geq 0$; $O(n^k \log^p n)$
   - ii) if $p < 0$; $O(n^k)$

Q. $T(n) = 2T(n/2) + 1$

A: $a = 2 \quad b = 2 \quad f(n) = 1$] write in form of $n^k \log^p n$

$f(n) = n^0 \log^0 n$

↓

So $k = 0 \quad p = 0$

$\log_b a = \log_2 2 = 1 > k = 0$ So case 1

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So $O(n^{\log_b a})$

$= O(n^1) = \underline{O(n)}$

Q2. $T(n) = 4T(n/2) + n$

A: $a = 4 \quad b = 2 \quad f(n) = n$

$= n^1 \log^0 n$

$k = 1 \quad p = 0$ ←

$\log_b a = \log_2 4 = 2 > k = 1$

So case 1,

$O(n^{\log_b a}) = \underline{O(n^2)}$

Q3. $T(n) = 8T(n/2) + n$

A: $a = 8 \quad b = 2 \quad f(n) = n$

$k = 1 \quad p = 0$ ← $= n^1 \log^0 n$

$\log_b a = \log_2 8 = 3 > k = 1$

So case 1, ~~$O(\log_b a$~~ $O(n^{\log_b a})$

$= \underline{O(n^3)}$

Q4. $T(n) = 9T(n/3) + 1$

A: $a = 9 \quad b = 3 \quad f(n) = 1$

$k = 0 \quad p = 0 \quad = n^0 \log^0 n$

$\log_b a = \log_3 9 = 2 > k = 0$

$O(n^{\log_b a}) = \underline{O(n^2)}$

Q5. $T(n) = 9T(n/3) + n$

A: $a = 9 \quad b = 3 \quad f(n) = n$

$k = 1 \quad p = 0 \quad = n^1 \log^0 n$

$\log_b a = \log_3 9 = 2 > k = 1$

So case 1,

$\underline{O(n^2)}$

Q6. $T(n) = 9T(n/3) + n^2$

A: $a = 9 \quad b = 3 \quad f(n) = n^2$

$k = 2 \quad p = 0 \quad = n^2 \log^0 n$

$\log_b a = \log_3 9 = 2 = k = 2$

So Case 2 ∴ $p > -1$

$O(n^k \log^{p+1} n)$

$= \underline{O(n^2 \log n)}$

Q7. $T(n) = 4T(n/2) + n$

A: $a = 4 \quad b = 2 \quad f(n) = n$

$k = 1 \quad p = 0 \quad = n^1 \log^0 n$

$\log_b a = \log_2 4 = 2 > k = 1$

So case 1, $O(n^2)$

Q8. $T(n) = 8T(n/2) + n \log n$

A: $a = 8 \quad b = 2 \quad f(n) = n \log n$

$k = 1 \quad p = 1 \quad = n^1 \log^1 n$

$\log_b a = \log_2 8 = 3 > k = 1$

Case 1 so, $\Rightarrow O(n^3)$

Q9. $T(n) = 2T(n/2) + n$

$a = 2 \quad b = 2 \quad f(n) = n$

$k = 1 \quad p = 0 \quad = n^1 \log^0 n$

$\log_b a = \log_2 2 = 1 = k = 1$

So case 2, $p = 0 > -1$ so $O(n^k \log^{p+1} n) = O(n \log n)$

Q10. $T(n) = 4T(n/2) + n^2$

A: $a = 4 \quad b = 2 \quad f(n) = n^2$

$k = 2 \quad p = 0 \quad = n^2 \log^0 n$

$\log_b a = \log_2 4 = 2 = k = 2$

So case 2, $p > -1$, $O(n^k \log^{p+1} n)$

$= O(n^2 \log n)$

Q11. $T(n) = 4T(n/2) + n^2 \log n$

A: $a = 4 \quad b = 2 \quad f(n) = n^2 \log^1 n$

$k = 2 \quad p = 1$

$\log_b a = \log_2 4 = 2 = k = 2$

So case 2, $p > -1$

$O(n^k \log^{p+1} n) = O(n^2 \log^2 n)$

Q12. $T(n) = 2T(n/2) + \frac{n}{\log n}$

A: $a = 2 \quad b = 2 \quad f(n) = n \log^{-1} n$

$k = 1 \quad p = -1$

$\log_b a = \log_2 2 = 1 = k = 1$

So case 2, $p = -1$,

$O(n^k \log\log n)$

$= O(n \log\log n)$

Q13. $T(n) = 2T(n/2) + n/\log^2 n$

A: $a=2 \quad b=2 \quad f(n) = n \log^{-2} n$

$k=1 \quad p=-2$

$\log_b a = \log_2 2 = 1 = k = 1$

So case 2, $p < -1$,

$O(n^k) = O(n)$

Q14. $T(n) = T(n/2) + n^2$

A: $a=1 \quad b=2 \quad f(n) = n^2 \log^0 n$

$k=2 \quad p=0$

$\log_b a = \log_2 1 = 0 < k=2$

So case 3, $p=0$

$O(n^k \log^p n) = O(n^2)$

Q15. $T(n) = 2T(n/2) + n^2 \log^2 n$

A: $a=2 \quad b=2 \quad f(n) = n^2 \log^2 n$

$k=2 \quad p=2$

$\log_2 2 = 1 < k=2$

So case 3, $p \geq 0$

$O(n^k \log^p n)$

$= O(n^2 \log^2 n)$

Q16. $T(n) = 4T(n/2) + n^3$

A: $a=4 \quad b=2 \quad f(n) = n^3 \log^0 n$

$k=3 \quad p=0$

$\log_2 4 = 2 < k=3$

So case 3, $p=0$

$O(n^k \log^p n) = O(n^3 \log^0 n)$

$= O(n^3)$

Q17. $T(n) = 4T(n/2) + n^3/\log n$

A: $a=4 \quad b=2 \quad f(n) = n^3 \log^{-1} n$

$k=3 \quad p=-1$

$\log_2 4 = 2 < k=3$

So case 3, $p < 0$ So $O(n^k)$

$= O(n^3)$

Page :
Date :

Q18. $T(n) = 4T(n/2) + (n \log n)^2$

A: $a = 4 \quad b = 2 \quad k = 2 \quad p = 2$

$\log_2 4 = 2 = k = 2$

So case 2, $p > -1$

So $O(n^k \log^{p+1} n)$

$= O(n^2 \log^3 n)$

Q19. $T(n) = 3T(n/2) + n^2$

A: $a = 3 \quad b = 2 \quad k = 2 \quad p = 0$

$\log_2 3 = 1.585 < 2 \; k = 2$

So case 3, $p = 0$ so $O(n^k \log^p n)$

$= O(n^2 \log^0 n)$

$= O(n^2)$

Q20. $T(n) = 3T(n/2) + n$

A: $a = 3 \quad b = 2 \quad k = 1 \quad p = 0$

$\log_2 3 = 1.585 > k = 1$

So case 1, $O(n^{\log_b a}) = O(n^{1.585})$

23/06/24 Thursday

21. $T(n) = 3^n T(n/2) + n^2$

A: Since $a = 3^n$ which makes it a non-constant, we can't apply master's theorem.
So we try using back substitution.
We don't need to do it...

22. $T(n) = 2T(n/4) + n^{0.51}$

A: $a = 2 \quad b = 4 \quad f(n) = n^{0.51}$
$k = 0.51 \quad p = 0$

$\log_b a = \log_4 2 = 0.5 < k = 0.51$

~~Case 2, $p \geq -1$~~
~~So case~~ ~~$O(n^k \log^{p+1} n)$~~

So, Case 3, $p = 0$

$O(n^k \log^p n)$

$O(n^{0.51})$

$O(n^{0.51})$

Q23. $T(n) = 64 T(n/8) - n^2 \log n$

A: Here $f(n)$ is negative. Not in form of $T(n) = aT(n/b) + f(n)$
So no master's theorem.

Q24. $T(n) = T(n/2) + n(2 - \cos n)$

A: Here also we cannot apply master's because $f(n)$ is not in the form of $n \log n$.

Recurrence Relations: Root Functions:

Q. ~~void (Test~~

```
void Test (int n) {          —— T(n)
{
  if (n > 2)                 —— 1
  {
    stmt;                    —— 1
    Test(√n);                —— T(√n)
  }
}
```

A: $T(n) = 1 + 1 + T(\sqrt{n})$

$= 2 + T(\sqrt{n})$

$= 1 + T(\sqrt{n})$

$$T(n) = \begin{cases} 1 & ; n = 2 \\ T(\sqrt{n}) + 1 & ; n > 2 \end{cases}$$

By back substitution,

$T(n) = T(\sqrt{n}) + 1$ —— ①

$T(\sqrt{n}) = T(\sqrt{\sqrt{n}}) + 1$

$T(n) = T((n^{1/2})^{1/2}) + 1 + 1$

$T(n) = T(n^{1/4}) + 2$ —— ②

$T(n^{1/4}) = T((n^{1/4})^{1/2}) + 1$

$T(n) = T(n^{1/8}) + 1 + 2$

$T(n) = T(n^{1/8}) + 3$ —— ③

$\vdots$

$T(n) = T(n^{1/2^k}) + k$

$$T(n) = \begin{cases} T(n^{1/2^k}) + k & ; n > 2 \\ 1 & ; n = 2 \end{cases}$$

smallest value of $n = 2$,

$T(2) + k$; ie, $n^{1/2^k} = 2$

$\frac{1}{2^k} \log n = \log 2$

$\frac{1}{2^k} \log n = 1$

$\log n = 2^k$

$\log \log n = k \log_2 2$

$k = \log \log n$

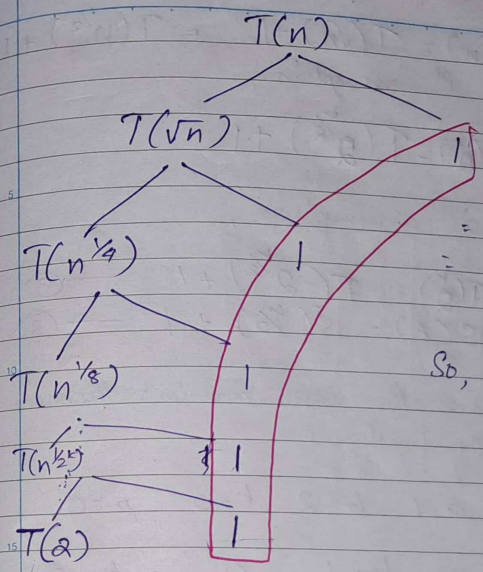
So, $T(2) + k$

$= 1 + k$

$= 1 + \log \log n$

$= O(\log \log n)$

By using Tree method :



$= 1 + 1 + \ldots + 1$ (k^{th})

$=$ upto $n^{1/2^k} = 2$

$k = \log \log n$

So, $O(\log \log n)$

This is not coming for exam

Master's Theorem for Root Function :

Assume 1. $n = 2^k$

2. the growth rate of $T(2^k) = k$.

so that of $T(2^{k/2}) = k/2$

$T(n) = T(\sqrt{n}) + 1 = T(n^{1/2}) + 1$

After 1st assumption,

$T(2^k) = T(2^{k/2}) + 1$

After 2nd assumption,

$T(2^k) = T(2^{k/2}) + 1$ — (2)

$S(k) = S(k/2) + 1$ — (3)

Now use the same master's theorem as Dividing function.

Same for this

→ Then $a=1$ $b=2$ $k=0$ $p=0$

$\log_b a = \log_2 1 = 0 = k = 0$

So case 2, $p = 0 > -1$

$O(n^k \log^{p+1} n) = O(k^0 \log^{0+1} n)$

$= O(\log k)$

$k = 2^k$

$k = \log n \rightarrow O(\log \log n)$

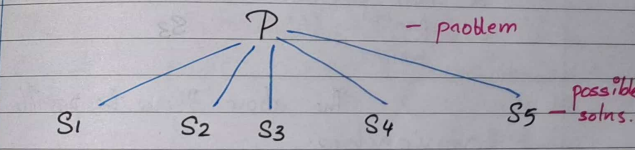
Camlin

Greedy Approach:

problem → size ~~of solns~~

```
Greedy (a, n)
for i=1 to n do
    x = select(a)
    if feasible(x), then
        solun = solun + x;
```

- This is another strategy like Divide & conquer.
- This is used for optimisation problems.



Now it checks among the possible solns, it checks which all are feasible. Then they choose the soln which is most optimal one amongst the feasible solns.

There is only 1 optimised soln.

Camlin

We can use branch and bound as well as dynamic programming methods to solve optimisation problems.

Fractional knap-sack problem:

Object	O_1	O_2	O_3	O_4	O_5	O_6	O_7
Profit	5	10	15	7	8	9	4
Weight	1	3	5	4	1	3	2.
P/W	5	3.3	3	1.75	8	3	2

Constraint: carry bag capacity limit = 15kg.
carry bag should have max. profit

here $n = 7$

Here you can select objects by:
1. Min weight — S_1
2. Max profit — S_2
3. $\frac{\text{profit}}{\text{weight}}$ ratio — S_3.

The above 3 are the possible soln approaches.

Now check each soln,

1# S_1: ↓ based on ↓ wt.

object	profit	weight (kg)	remaining wt. (kg)
O_1	5	1	15-1 = 14
O_5	8	1	14-1 = 13
O_7	4	2	13-2 = 11
O_2	10	3	11-3 = 8
O_6	9	3	8-3 = 5
O_4	7	4	5-4 = 1
O_3	$15 \times \frac{1}{5} = 3$	1	1-1 = 0

After O_4, we had only 1 kg left. So we took a fraction of O_3. That is,
wt of O_3 = 5kg ; 1 kg of O_3 = 3 profit.
↓
15 profit.

Total profit = 46

2# S_2 ↓ based on ↑ profit.

Object	profit	weight	remaining weight
O_3	15	5	15-5 = 10
O_2	10	3	10-3 = 7
O_6	9	3	7-3 = 4
O_5	8	1	4-1 = 3
O_4	$7 \times \frac{3}{4} = 5.25$	$4 \times \frac{3}{4} = 3$	3-3 = 0

Total profit = 47.25

3# S3:

Object	Profit	Weight	Remaining wt.
			15-1 = 14
O5	8	1	14-1 = 13
O1	5	1	13-3 = 10
O2	10	3	10-5 = 5
O3	15	5	5-3 = 2
O6	9	3	2-2 = 0
O7	4	2	

Total profit = 51

So,

S1 = 46
S2 = 47.25
S3 = 51 ✓

We choose S3 because it is most optimised.

In exam if this qn comes, directly do S3. No need to do S1 and S2 as those are there only for proof.

Do S1 if they ask specifically for it.

Camlin

Q1.

Objects	1	2	3	4
wt	3	4	5	6
profit	4	6	7	11

constraints = ~~wt~~ max wt = 9kg
max profit should be there.

A. Using P/W ratio.

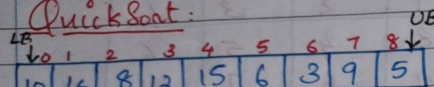
Objects	1	2	3	4
profit	4	6	7	11
wt	3	4	5	6
P/W	1.33	1.5	1.4	1.83

Object	profit	wt	rem. wt
4	11	6	9-6 = 3
2	$6 \times \frac{3}{4} = 4.5$	$4 \times \frac{3}{4} = 3$	3-3 = 0

Max profit = 11 + 4.5
= 15.5

Camlin

QuickSort:



We use divide and conquer strategy and fix the position of

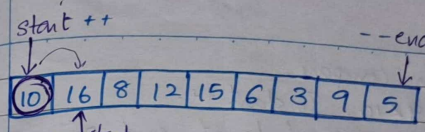
Step 1: Select a pivot element. It may be first, last, random, or median element.

Step 2: Partition step. In this elements less than pivot element should go to the left of pivot and those greater should go to its right.
Repeat this multiple times if you get the list sorted.

We need two pointers in this. First we fix A[0] as start pointer and A[8] as end pointer.
start pointer - either increments (+) or stops.
end pointer - either decrements (-) or stops.

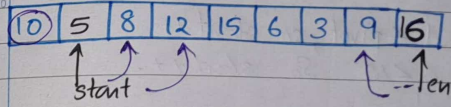
start - increment if element is lesser than pivot
end - decrement if element is greater than pivot.

- In the above example, say (10) is the pivot selected.

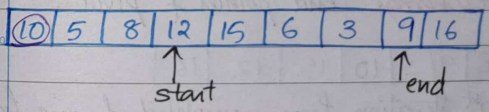


if pivot is the first element, then we increment the start pointer immediately.

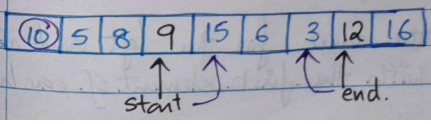
Now 16 > 10, so decrement end pointer. but at end pointer, 5 < 10. So now exchange start & end pointer values.



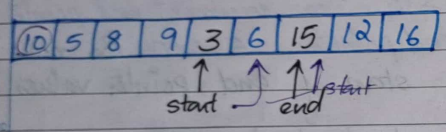
Now again check with start pointer. 5 < 10. So start ++.
Now A is 8 < 10, Again start ++.
Now it is 12 > 10. Go to end pointer.
16 > 10. So decrement end pointer.



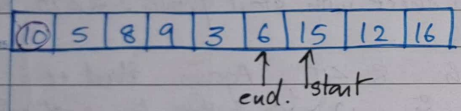
Now we exchange. Because end < pivot & start > pivot.



Now again compare.
9 < 10. So start ++.
Now 15 > 10. So look at end pointer.
12 > 10. So decrement end pointer. end--.
Now 3 < 10. That is end < pivot & start > pivot. So exchange.



Now start < pivot, so start ++
Now 6 < 10. So start ++.
Now end --.

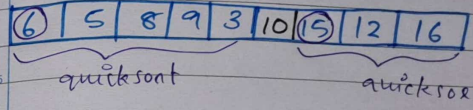


Since start pointer passed the end pointer,
Now exchange end pointer & pivot.

6	5	8	9	3	10	15	12	16

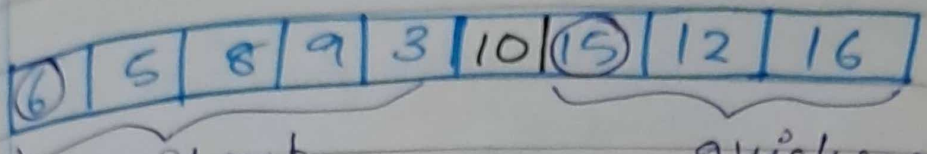
Now 1st iteration has completed & we have fixed position of pivot

Now do the same for the left and right subarrays with the first element of each subarray as the pivot (or as whatever you wish on pivot).



Repeat until you get sorted array.

subarray as the pivot (or as whatever you wish as pivot).

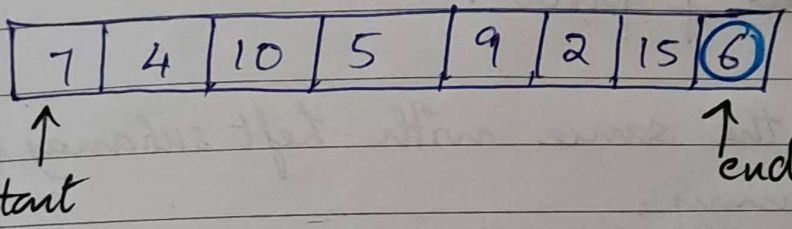


Repeat until you get sorted array.

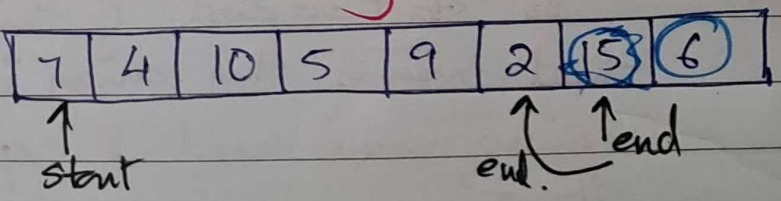
22/5/24/Wednesday.

Q. 7, 4, 10, 5, 9, 2, 15, 6. Take 6 as pivot & do quicksort?

A:

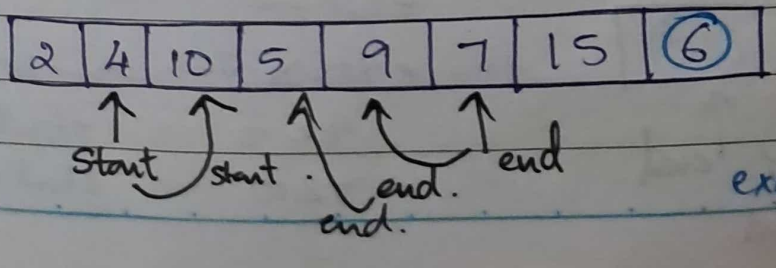


Here since pivot is at the end decrement end pointer first. Then do the same steps as done previously...

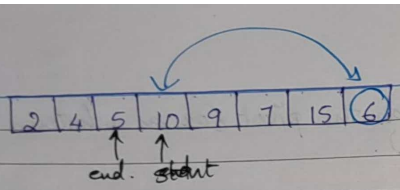


~~15~~ 15 > 7 so end--

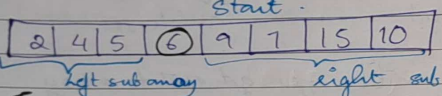
2 < 7 so exchange.



exchange 10 & 5.



So now exchange pivot with start.



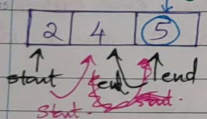
left sub array right sub array

First iteration is done.

Since pivot was the last element, exchange the start & pivot.

Now do the same with left subarray & right subarray.

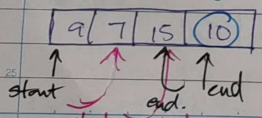
Left sub array:



last element is pivot.
end--
4 > 2 so start++

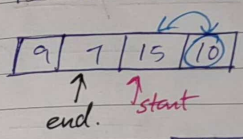
so start == end. If this happens, that means the array is sorted. So no more need to repeat the iterations.

Right sub array:

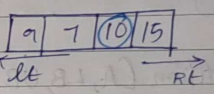


end
9 < 15 so
9 < 10 so start++
7 < 10 so start++

15 > 10 so end--

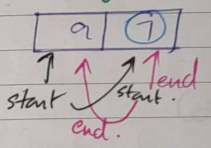


Now exchange start & pivot

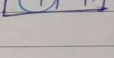


2nd iter done.

Lt sub array:

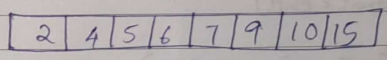


7 < 9 so end--
start++

3rd iter done 

Now exchange,

So final array.



* if the pivot was random in the first iter then it is random in remaining iters as well.

30/5/24 Thu.

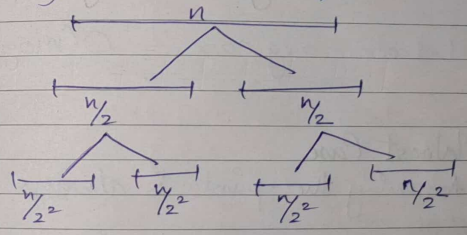
QuickSort Algorithm & Time Complexities

```
QuickSort(A, LB, UB)                          ——— T(n)
{
  if (LB < UB)                                ——— 1
  {
    loc = partition (A, LB, UB);              ——— partition time
    QuickSort (A, LB, loc-1);                 } depends on case.
    QuickSort (A, loc+1, UB);
  }
}

partition (A, LB, UB)
{
  pivot = A[LB];
  start = LB;
  end = UB;
  while(start < end)
  {
    while (A[start] <= pivot)
    {
      start ++;
    }
    while (A[end] > pivot)
    {
      end--;
    }
    if (start < end)
    {
      swap (A[start], A[end]);
    }
  }
  swap (A[LB], A[end]);
  return end;
}
```

Best Case & Average Case.

Assuming pivot position is always in the middle. That is in each iteration we are dividing into 2 equal parts.



So,

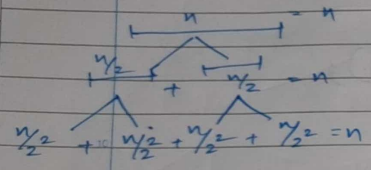
QuickSort (A, LB, loc-1); ——— $T(n/2)$
QuickSort (A, loc+1, UB); ——— $T(n/2)$

So,

$T(n) = 1 + \text{partitionTime} + 2T(n/2)$

↓

each iteration takes n time.



So, $T(n) = 1 + n + 2T(n/2)$

↓ asymptotically

$T(n) = 2T(n/2) + n$

time complexity (TC)

$a = 2 \quad b = 2 \quad \log_b a = 1 = k = 1 \quad p = 0 \Rightarrow p > -1$

$TC = n^k \log^{P+1} n = n \log n$

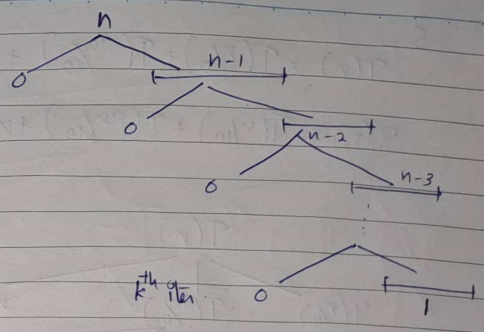
So bestcase = avg case = $O(n \log n)$

2# Worst Case

Assuming the pivot is always at the extreme end.

For calculating time complexity, lets consider it being at LB all the time...

(pivot)



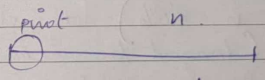
So time $= n + (n-1) + (n-2) + \cdots + 2 + 1$

$= \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$

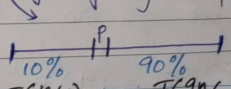
$= O(n^2)$

∴ Worst case time complexity = $O(n^2)$

3# Special Case - Unbalanced Tree.



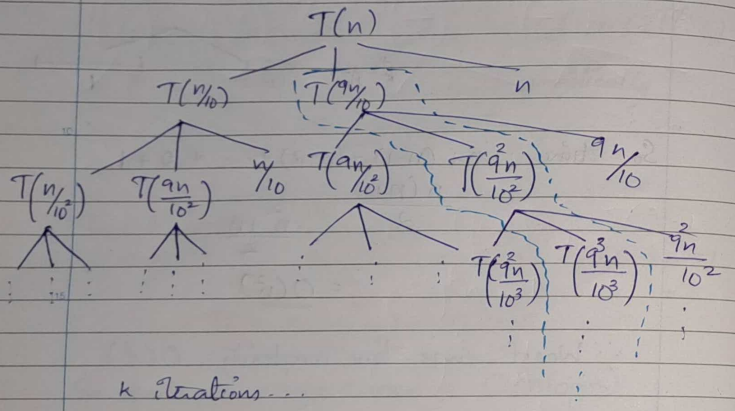
(after fitting the pivot's pos.



in every case you compare with pivot as n

So, $T(n) = T(n/10) + T(9n/10) + \text{partition time}$

$T(n) = T(n/10) + T(9n/10) + n$ → Recurrence Relation.



k iterations...

Since these are imbalanced trees (one part is 10% & other is 90%) so we take the longest branch (longest branch has the greatest value among all the branches).

So we go as ⇒ $\frac{9n}{10} \to \frac{9^2n}{10^2} \to \frac{9^3n}{10^3} \to \cdots$

So, K^{th} iter. $= \left(\frac{9}{10}\right)^k n = T\left[\left(\frac{9}{10}\right)^k n\right]$ //

Now equate to min: value min value = 1 @ $n/b = 1$ for $T(n/b)$.

ie; $\left(\frac{9}{10}\right)^k n = 1$

$\left(\frac{9}{10}\right)^k = \frac{1}{n}$

$n = \left(\frac{10}{9}\right)^k$

$k = \log_{10/9} n$

Time taken for : (add the non-TC branches)

1^{st} iter $= n$

2^{nd} iter $= \frac{n}{10} + \frac{9n}{10} = n$

3^{rd} iter $= \frac{n}{10^2} + \frac{9n}{10^2} + \frac{9n}{10^2} + \frac{9^2n}{10^2}$

$= n$

$\vdots$

k^{th} iter $= n$

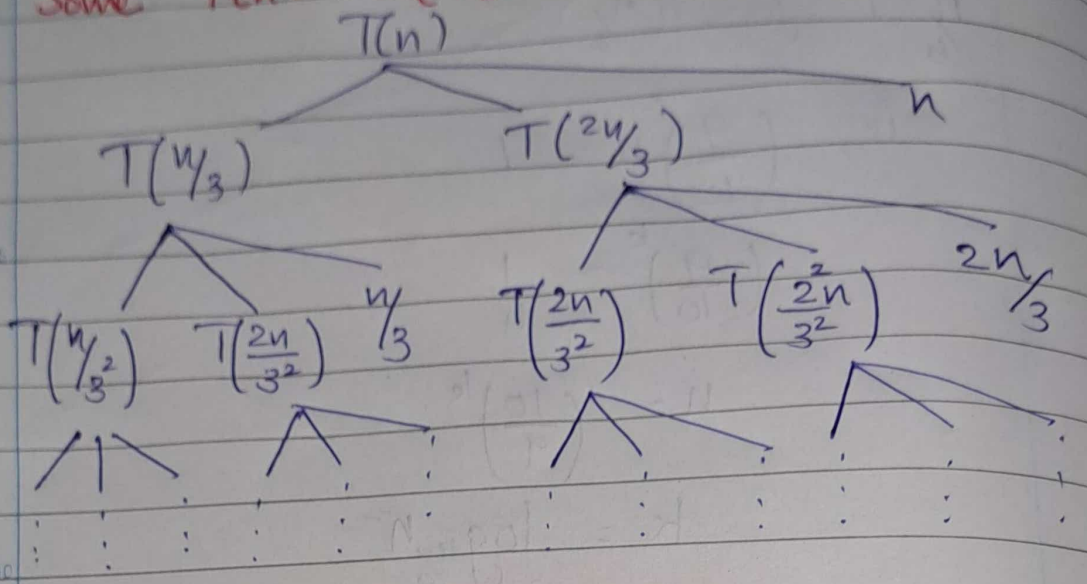
So each level time taken = n

There are k such levels, so,

Time complexity = $k \cdot n$

$TC = n \log_{10/9} n$

Q. Solve $T(n) = T(n/3) + T(2n/3) + n$



k levels ...

Taking longest branch, $T\left(\left(\frac{2}{3}\right)^k n\right) = T(1) = 1$

$$\left(\frac{2}{3}\right)^k n = 1$$

$$\left(\frac{2}{3}\right)^k = \frac{1}{n}$$

$$n = \left(\frac{3}{2}\right)^k$$

$$k = \log_{3/2} n$$

Similarly each iteration takes time $= n$

So time complexity $= n \cdot \log_{3/2} n$

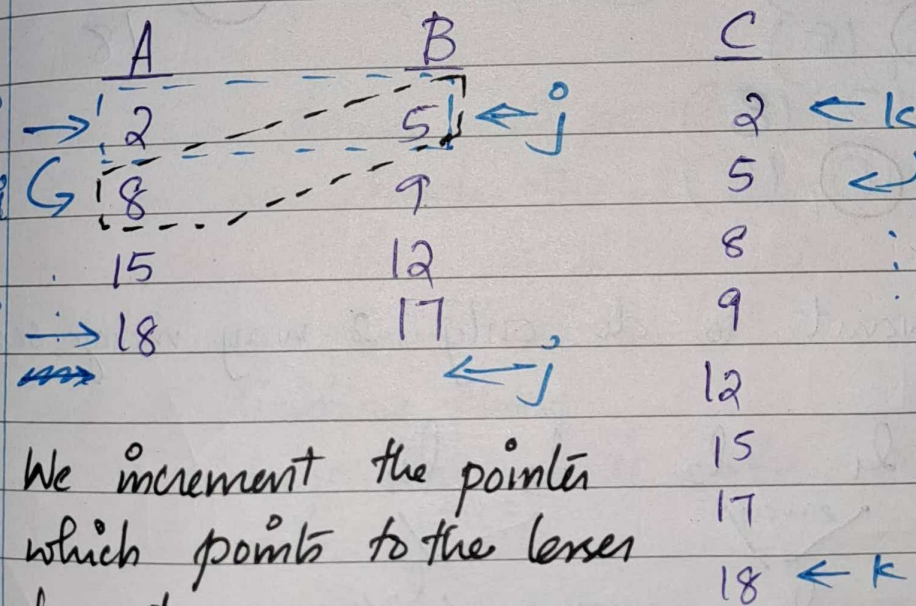
mid sem questions over...

(Unbalanced tree not present) ----

Merge Sort

- 2 Way Merge Sort → iterative
- Merge Sort → recursive.

- We use divide & conquer approach.
- We will have 2 sorted lists & we merge both & create another sorted list.



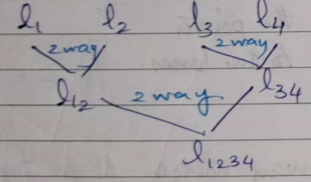
We increment the pointer which points to the lesser element.

This is 2 way merge. At a time we are comparing 2 elements so it is 2 way merge. If at a time 4 elements are compared then it is 4-way merge.

A	B	C	D	F
				2
4	3	8	2	3
6	5	10	4	4
12	9	16	18	4
				5
(4, 3, 8, ②)				6
(4, ③, 8, 4)				8
(④, 5, 8, 4)				9
(6, 5, 8, ④)	(⑫, —, 16, 18)			10
(6, ⑤, 8, 18)	(—, —, ⑯, 18)			12
(⑥, 9, 8, 18)	(—, —, —, ⑱)			16
(12, 9, ⑧, 18)				18
(12, ⑨, 10, 18)				
(12, —, ⑩, 18)				

- follow FIFO nxt

If you want to do only 2-way merge sort then,



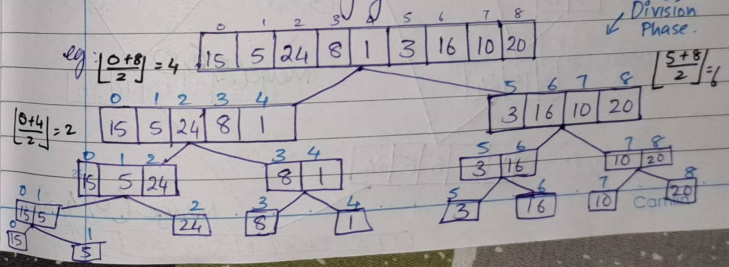
Merge Sort Algorithm:

```
MergeSort (A, lb, ub)
{
  if (lb < ub)
  {
    mid = ⌊(lb+ub)/2⌋      → take floor value.
    MergeSort (A, lb, mid);
    MergeSort (A, mid+1, ub);
    merge (A, lb, mid, ub);
  }
}
```

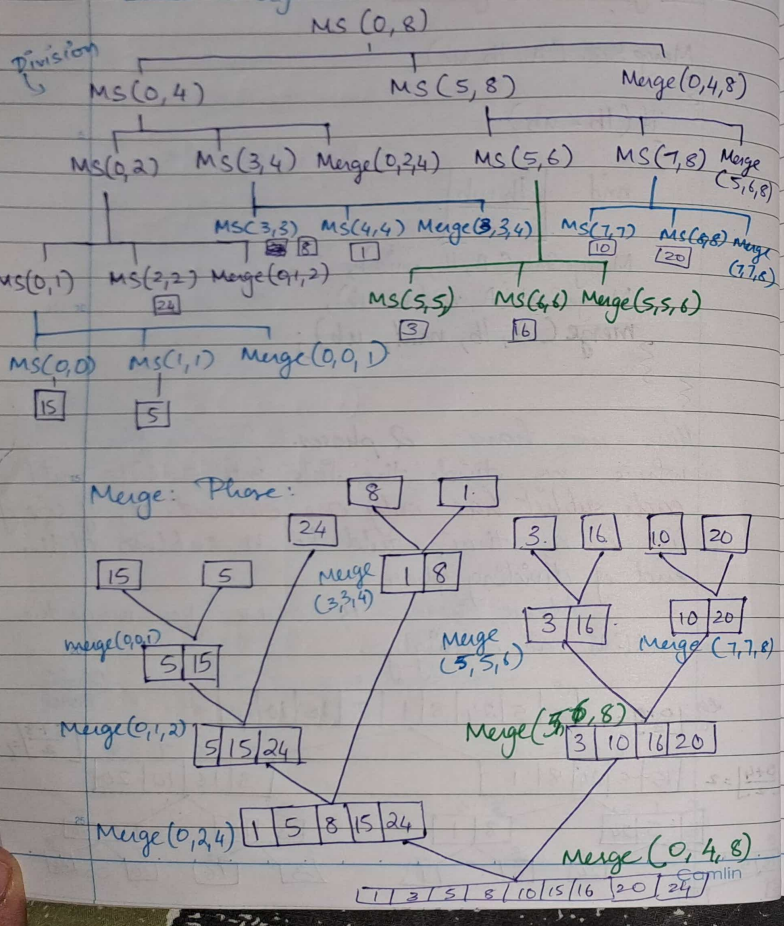
Here we have 2 phases. Dividing Phase where we divide the lists into sublists until each sublist has only one element. ie; if size of list is n, there would be n sublists at the end of dividing phase.

Merge Phase is where you merge the sublists accordingly.

eg:



Based on algorithm:



∴ Final array is: 1, 3, 5, 8, 10, 15, 16, 20, 24.

31/5/24 Friday.

Merge Algorithm

```
Merge(A, lb, mid, ub)
{
  i = lb;
  j = mid+1;
  k = lb;
  while(i<=mid && j<=ub)
  {
    if (a[i] <= a[j])
    {
      b[k] = a[i];
      i++;
      k++;
    }
    else
    {
      b[k] = a[j];
      j++;
      k++;
    }
```

```
if (i>mid)
{
  while (j<=ub)
  {
    b[k] = a[j];
    j++;
  }
  k++;
}
else
{
  while (i<=mid)
  {
    b[k] = a[i];
    i++;
    k++;
  }
}
for (k=lb; k<=ub; k++)
{
  a[k] = b[k];
}
```

```
MergeSort (A, lb, ub)  ——— T(n)
{
  if (lb < ub)  ——— 1
  {
    mid = ⌊(lb+ub)/2⌋  ——— 1
    MergeSort (A, lb, mid);  ——— T(n/2)
    MergeSort (A, mid+1, ub);  ——— T(n/2)
    Merge (A, lb, mid, ub);  ——— n
  }
}
```

For Merge algorithm, the time taken is always n because we are comparing each and every element and merging them one sublist at a time if initially all sublists have only one element. Like this n sublists are compared.

As for the MergeSort recursive calls, we always divide the list into half. So both the calls take $T(n/2)$ time.

Recurrence Relation ⇒

$$T(n) = 1 + 1 + T(n/2) + T(n/2) + n$$

$$\boxed{T(n) = 2T(n/2) + n}$$

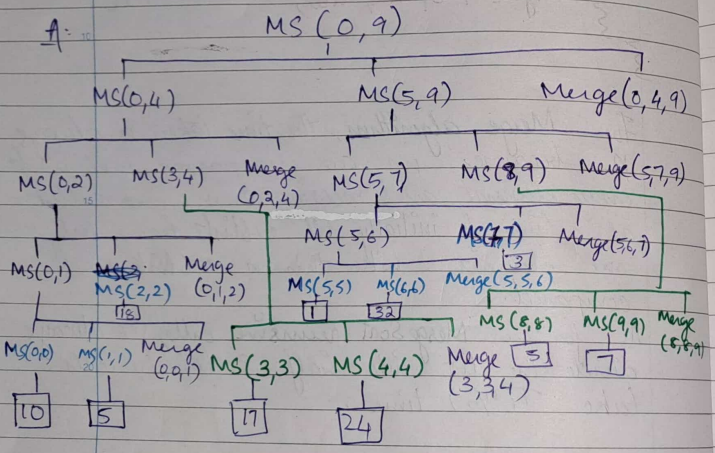
$$\boxed{TC = n \log n}$$

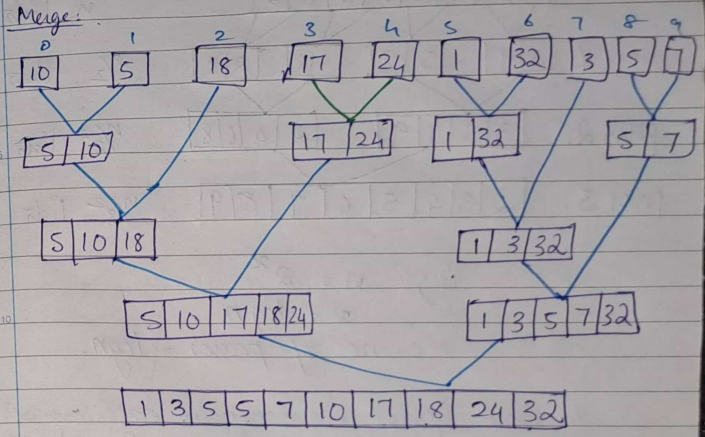
That is, MergeSort has same time complexity as best/avg case of quicksort.

This time complexity is same for best, avg and worst case of merge sort.

Q. Sort these elements using MergeSort?
{10, 5, 18, 17, 24, 1, 32, 3, 5, 7}

A:





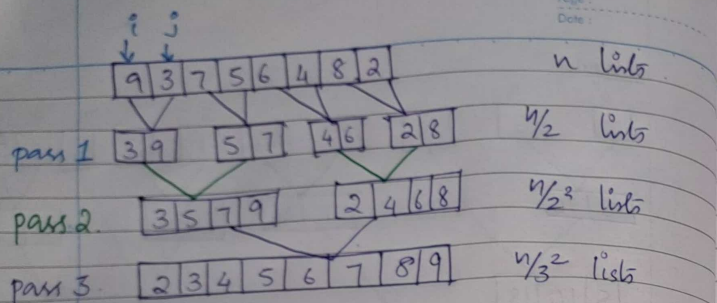
1/6/24 | Saturday

2 Way Merge Sort:

	0	1	2	3	4	5	6	7
A	9	3	7	5	6	4	8	2

Here there is no division phase. There is only merge phase present.

Here each element is considered as a list & the 2 way merge sort is done... Merge operation...



ie; $n = 2^x$

$x = \log n$

∴ no. of passes = $\log n$.

In each pass, we compare each and every element each time. So time in each pass = n.

∴ Time complexity = $n \cdot$ (no. of passes)

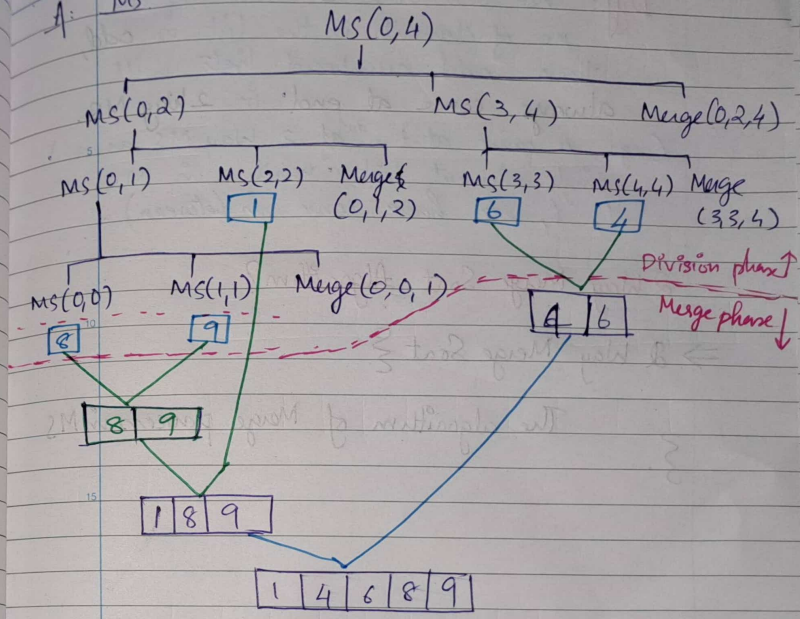
$= n \cdot \log n$

For best, avg, & worst cases.

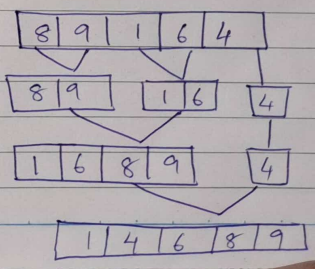
Q. Sort $\{8, 9, 1, 6, 4\}$ using 2 way Merge Sort & Merge Sort. Explain the diff: observed?

Camlin

A: MS



2 Way MS



Camlin

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Diff: We can observe that if the no: of elements in the list is odd, then odd numbered lists will always come at end in 2 Way MS. (eg: 4 came at the end of 2 Way MS as a single element list while in MS, 1, 6, etc have come in between).

2 Way Merge Sort Algorithm?

⇒ 2 Way Merge Sort {

The algorithm of Merge process in MS

}.

Diff: We can observe that if the no: of elements in the list is odd, then odd numbered lists will always come at end in 2 Way MS.
(Eg: 4 came at the end of 2 Way MS as a single element list while in MS, 1, 6, etc have come in between).

2 Way Merge Sort Algorithm?

⇒ 2 Way Merge Sort {

The algorithm of Merge process in MS

}

12/06/24/ Wednesday

Binary Search:

- Should be a sorted array.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	3	6	8	12	14	17	25	29	31	36	42	47	53	55	62

l ↑ h ↑

Case 1:

Key element = 42.
(keyelt)

Steps:

1. $mid = \left\lfloor \frac{l+h}{2} \right\rfloor$

l	h	mid
1	15	$\left\lfloor \frac{1+15}{2} \right\rfloor = 8$

2. A[8] → mid.
   Check if A[mid] == keyelt
   A[8] = 29 < keyelt = 49.
3. Keyelt < A[mid] → search right.
   else if keyelt > A[mid] → search left.
   A[8] = 29 < keyelt = 49.
   So search right.

search right →

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	3	6	8	12	14	17	25	29	31	36	42	47	53	55	62

l, mid, l, mid, h

4. new l = mid+1
   new h = h.

Do Binary Search in this again & repeat.

l	h	mid
1	15	8
9	15	$\lfloor \frac{9+15}{2} \rfloor = \lfloor \frac{24}{2} \rfloor = 12$

A[12] = 47 > Keyelt = 42

So search left.

Left:-

Newh = mid − 1

Newl = l

l	h	mid
1	15	8
9	15	12
9	11	10

$\lfloor \frac{9+11}{2} \rfloor = \lfloor \frac{20}{2} \rfloor = 10$

A[mid] = 36 < Keyelt = 42

So search right.

l = mid + 1 = 11

h = 11.

l	h	mid
1	15	8
9	15	12
9	11	10
11	11	11

A[mid] = 42 == Keyelt = 42.

Hence we got the element.

- We are following Divide & Conquer Strategy.

Case 2:

Keyelt = 12.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	8	12	14	17	25	29	31	36	42	47	53	55	62

l ↑ mid ↑ h

l	h	mid	
1	15	8	29 > 12 → Search lt.
1	7 (mid−1)	4	12 = 12.

We have found the element.

Case 3:
Key elt = 30

l	h	mid	~~mid~~
1	15	8	29 < 30, Search rt
9	15	12	47 > 30, search lt
9	11	10	36 > 30, Search lt
9	9	9	31 > 30, Search lt
9	8	h > l → not possible.	

must write this → step as well

But there is no more to go to.
That is, element is not found!
If h crosses l, element is not found!

Algorithm:

Iterative:

```
int BS (A, n, key)
{
    l=1, h=n;
    while (l <= h)
    {
        mid = (l+h)/2 ;
        if (key == A[mid])
            return mid;
        if (key < A[mid])
            h= mid - 1;
        else
            l = mid +1;
    }
    return 0;
}
```

Recursive:

```
RBS (l, h, key)
{
    mid = (l+h)/2 ;
    if (key == A[mid])
        return mid;
    if (key < A[mid])
        return RBS (l, mid-1, key);
    else
        return RBS (mid+1, h, key);
}
```

T(n)
1
1
1
T(n/2)
~~T(n/2)~~

if-else loop → only one is considered. asymptotic.

$T(n) = T(n/2) + 1$

$a = 1 \quad b = 2 \quad \log_b a = 0 \quad k = 0 \quad p = 0 > -1$

$O(n^k \log^{p+1} n)$

$TC = n^{\log_b a} = O(1) = O(\log n)$

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Strassen's Matrix Multiplication Method :

13/06/24 Thursday.

Strassen's Matrix Multiplication Method:

Matrix Multiplication
- General method.
- Strassen's method.

Using Divide & Conquer on General Method:

We divide matrices bigger than 2×2 into 2×2 matrices. Matrix should be sq matrix of even order if not sq matrix with even order then append zeroes.

eg. $A_{4\times4} \times B_{4\times4}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad A_{11}, A_{12}, A_{21}, A_{22}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \quad B_{11}, B_{12}, B_{21}, B_{22}$$

ie; $A_{4\times4} \times B_{4\times4} = A_{2\times2} \times B_{2\times2}$

$$= \begin{bmatrix} A_{11}\times B_{11} + A_{12}\times B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Algorithm for Divide & Conquer General Matrix Multiplication

Algorithm MM(A, B, n) —— T(n)

{

if (n <= 2) —— 1

use 4 formulas $(C_{11}, C_{12}, C_{21}, C_{22})$

else

{

find n/2 (till dividing it into smaller 2 prob of size 2×2)

MM($A_{11}, B_{11}, n/2$) (+) MM($A_{12}, B_{21}, n/2$);
MM($A_{11}, B_{12}, n/2$) (+) MM($A_{12}, B_{22}, n/2$);
MM($A_{21}, B_{11}, n/2$) (+) MM($A_{22}, B_{21}, n/2$);
MM($A_{21}, B_{12}, n/2$) (+) MM($A_{22}, B_{22}, n/2$); } $8T(n/2)$ +

↳ matrix addition $\rightarrow n^2$.

}

★ Time complexity of general matrix multiplication with no divide & conquer applied : $O(n^3)$

There are 3 loops used in matrix multiplication (ordinary method) ↓

```
for (i=0 ; i<n; i++){
    for (j=0; j<n; j++){
        for (k=0; k<n; k++){
            Cij = Σ_{k=1}^{n} Aik × Bkj ;
        }
    }
}
```

Recurrence relation for DAC Matrix Mult:

$$T(n) = \begin{cases} 8T(n/2) + n^2 \end{cases}$$

$$TC = \log_b a = \log_2 8 = 3 > 2$$

$$O(n^{\log_b a}) = O(n^3)$$

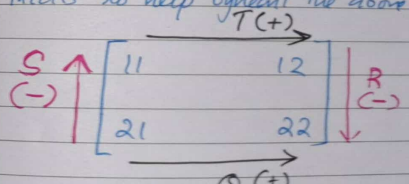
But Strassen's Matrix Mult has $TC < n^3$, hence we follow Strassen's matrix Mult method.

Strassen's Matrix Multiplication:

Formulas:

1. $P = (A_{11} + A_{22})(B_{11} + B_{22})$
2. $Q = B_{11}(A_{21} + A_{22})$
3. $R = A_{11}(B_{12} - B_{22})$
4. $S = A_{22}(B_{21} - B_{11})$
5. $T = B_{22}(A_{11} + A_{12})$
6. $U = (A_{21} - A_{11})(B_{11} + B_{12})$
7. $V = (A_{12} - A_{22})(B_{21} + B_{22})$
8. $C_{11} = P + S - T + V$
9. $C_{12} = R + T$
10. $C_{21} = Q + S$
11. $C_{22} = P + R - Q + U$

Tricks to help byheart the above formulas:



B A A B
11 11 22 22
(Q) (R) (S) (T)

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Time complexity :

for recurrence relation : count no: of matrix mults & additions

$$T(n) = 7T(n/2) + n^2$$

↳ 7 multiplications :

P-1 S-1 U-1
Q-1 T-1 V-1

$\log_2 7 > 2$

so,

$$TC = n^{\log_2 7} = \underline{O(n^{2.81})}$$

↑ this is less than ordinary DAC mult where it is n^3.

Time complexity:

for recurrence relation: (matrix)
const no: of mults & additions

$T(n) = 7T(n/2) + n^2$

↳ 7 multiplications:
P-1 S-1 U-1
Q-1 T-1 V-1

$\log_2 7 > 2$

so,

$TC = n^{\log_2 7} = O(n^{2.81})$

↑ this is less than ordinary DAC mult where it is n^3.

18/6/24 Tuesday

Q. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Multiply using Strassen's matrix multiplication method?

A: $A_{11} = 1$ $B_{11} = 5$
$A_{12} = 2$ $B_{12} = 6$
$A_{21} = 3$ $B_{21} = 7$
$A_{22} = 4$ $B_{22} = 8$

$P = (A_{11} + A_{22})(B_{11} + B_{22})$
$= (5+11)(1+4)(5+8)$
$= 5 \times 13$
$= 65$

$Q = B_{11}(A_{21} + A_{22})$
$= 5(3+4)$
$= 5 \times 7 = 35$

$R = A_{11}(B_{12} - B_{22})$
$= 1(6-8) = 1(-2)$
$= -2$

$S = A_{22}(B_{21} - B_{11})$
$= 4(7-5) = 4 \times 2 = 8$

$T = B_{22}(A_{11} + A_{12})$
$= 8(1+2) = 8 \times 3 = 24$

$U = (A_{21} - A_{11})(B_{11} + B_{12})$
$= (3-1)(5+6) = 2 \times 11 = 22$

$V = (A_{12} - A_{22})(B_{21} + B_{22})$
$= (2-4)(7+8) = -2(15)$
$= -30$

$-54+165 = 111+8$

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$C_{11} = P+S-T+V = 65+8-24+-30 = 19$

$C_{12} = R+T = -2+24 = 22$

$C_{21} = Q+S = 35+8 = 43$

$C_{22} = P+R-Q-U = 65+-2-35-22$
$= 50$

$\therefore A \times B = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

Method 1 f

Q. HW →

$$A = \left[\begin{array}{cc:cc} 4 & 2 & 0 & 1 \\ 3 & 1 & 2 & 5 \\ \hdashline 3 & 2 & 1 & 4 \\ 5 & 2 & 6 & 7 \end{array}\right] \quad \begin{matrix} A_{11} \\ A_{12} \\ A_{21} \\ A_{22} \end{matrix}$$

$$B = \left[\begin{array}{cc:cc} 2 & 1 & 3 & 2 \\ 5 & 4 & 2 & 3 \\ \hdashline 1 & 4 & 0 & 2 \\ 3 & 2 & 4 & 1 \end{array}\right] \quad \begin{matrix} B_{11} \\ B_{12} \\ B_{21} \\ B_{22} \end{matrix}$$

Methods

A:

$A_{11} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ ~~A~~

Later . . .

$A_{12} = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$

Camlin

Huffman coding:

B C C A B B D D A E C C B B A E D D C C

↓

to send this from sender to receiver, we convert it to bits & send.

↓

Method 1#

Convert into ASCII → 8 bits

characters in msg → 20

Total bits transferred = 20×8 = 160 bits

Method 2#

Count the freq & represent using that & then for identification use ASCII.

char.	count/freq	# bits
A	3	000
B	5	001
C	6	010
D	4	101
E	2	111

3×5 bits for this... We code each as 8 bit ASCII code. So 8×5 = 40 bits. Now no: of chars in 20, so 15×20 = 60 bits. So total bits = 60+40 = 100.

But no standardization for this...

Method 3#

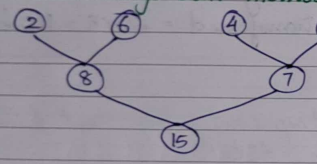
Huffmann Coding

char	freq/count
A	3
B	5
C	6
D	4
E	2

For the bits you use optimal merge pattern. (Merge sort usage)

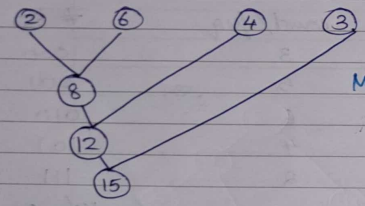
Various merge sort methods:

1#



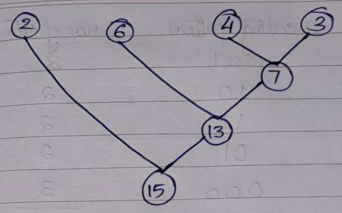
Merge cost = 8+7+15
= 30

2#



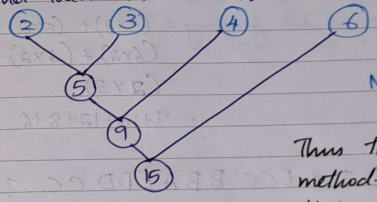
Merge cost = 8+12+15
= 35

3#



Merge cost = 7+13+15
= 35

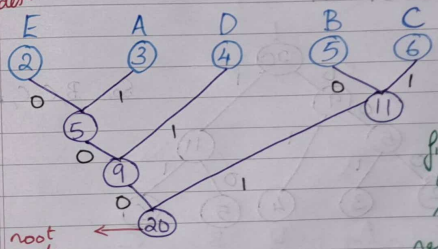
4# optimal merge sort method: arrange in ascending order and then in each iteration combine smallest nos only.



Merge cost = 5+9+15
= 29

Thus this is the most optimal method. We use this in Huffmann method.

leaf nodes →



root node.

Put
0 - left edge
1 - right edge

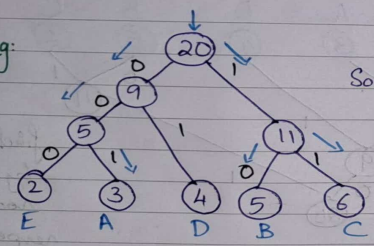
find bit representation by going from the root node to the respective leaf nodes.

char	freq	coding pattern	no: of bits
A	3	001	3
B	5	10	2
C	6	11	2
D	4	01	2
E	2	000	3
			12

In this method
char representation = 12 bits
total no: of bits for msg = (3x3) + (5x2) + (6x2) + (4x2) + (2x3)
= 9+10+12+8+6 = 45

BCC A B B D D A E C C B B A E D D C C
10111100110100101001000111110 10
00100001011111

Decoding:



So: B, C, C, A, ...

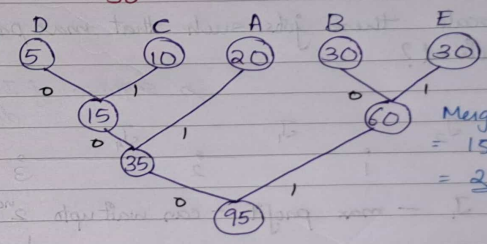
Thus we can decode directly using the tree. This coding-decoding method is called Huffman Coding. This follows Greedy ~~Alg~~ Approach.

Q. Using Huffman coding, code the msg, find code for each character and the optimal merge cost?

char	freq
A	20
B	30
C	10
D	5
E	30

msg: AABC EDADDC

A:



Merge cost
= 15+35+95+60
= 205

char	freq	code	# bits
A	20	01	2
B	30	10	2
C	10	001	3
D	5	000	3
E	30	11	2

AABCEDADDC ↓

0101100011100001000000001

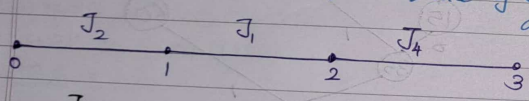
Job Sequencing With Deadlines using Greedy Approach

n=5

Jobs	J_1	J_2	J_3	J_4	J_5
Profit (Rs/-)	20	15	10	5	1
Deadlines (hrs)	2	2	1	3	3

Allocate these jobs such that max profit is received?

↗ sequence of Job scheduling doesn't matter.



J_1 – max profit – can wait upto 2nd hours
1
allocated.

J_2 – next most profit – can wait upto 2nd hours
1
allocated

J_3 – nxt most profit – can wait upto 1st hour
no time slot – not allocated.

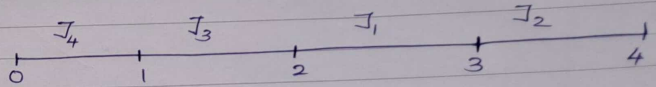
J_4 – nxt most profit – can wait upto 3rd hour I allocated.

Max profit = 20 + 15 + 5 = 40

Q.

Jobs	J_1	J_2	J_3	J_4	J_5	J_6	J_7
Profit	35	30	25	20	15	12	5
Deadline	3	4	4	2	3	1	2

A:



profit = 35 + 30 + 25 + 20

= 110

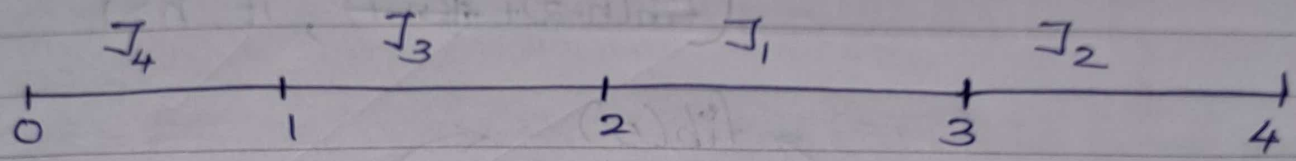
J_4 - next most profit - can wait upto 3rd hour I allocated.

Max profit = 20 + 15 + 5 = 40

Q.

Jobs	J_1	J_2	J_3	J_4	J_5	J_6	J_7
Profit	35	30	25	20	15	12	5
Deadline	3	4	4	2	3	1	2

A:



profit = 35 + 30 + 25 + 20

= 110

19/06/24 Wednesday.

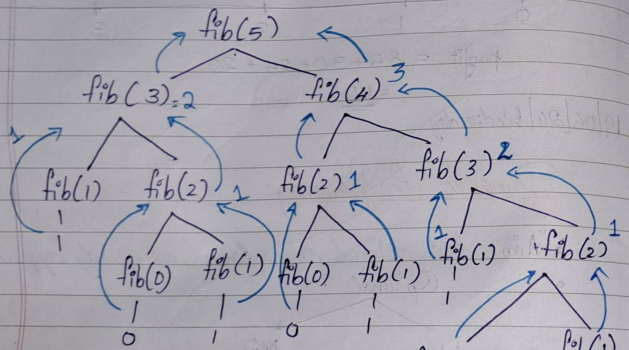
Dynamic Progsamming:

- Aim: Get optimal soln. Same as greedy approach.
- Optimal
  - Max
  - Min.
- If criteria or preconditions are present then we take greedy approach.

- We will many feasible solns and in the final stage we select the best soln. This is dynamic programming approach. There are no preconditions.

eg: Fibonacci Series: 0 1 1 2 3 5 8 ...

Method 1# $fib(n) = \begin{cases} 0 & ; \text{if } n=0 \\ 1 & ; \text{if } n=1 \\ fib(n-2)+fib(n-1) & , \text{if } n>1 \end{cases}$



fib(2) = 1 fib(3)=2 fib(4)=3

fib(5) = 1+0+1+0+1+1+0+1 or = 2+3

fib(5) = 5 = 5

Number of calls = 15.

top to bottom ↓ Method 2# Memoization / Tabulas method ← bottom to top

We use dynamic pgmming approach.

▷ First make a global array. Size = 5 (fib(5)).

-1	-1	-1	-1	-1	-1
0	1	2	3	4	5

Initial value = -1

going by tree (left first) (modified traversal)

fib(5) = -1 → we don't know value.
fib(3) = -1 → "
fib(4) = -1 → "
fib(1) = 1 → we know the value so we update.

-1	1	-1	-1	-1	-1
0	1	2	3	4	5

fib(2) = -1 → we don't know the value.
fib(0) = 0 → we update it.

0	1	-1	-1	-1	-1
0	1	2	3	4	

fib(2) = fib(0) + fib(1) = 1 → we update.

0	1	1	-1	-1	-1
0	1	2	3	4	5

fib(3) = fib(2) + fib(1) = 1+1 = 2 → we update

0	1	1	2	-1	-1
0	1	2	3	4	5

Now go to right subtree of fib(5),

fib(4) = -1 → we don't know

fib(2) = 1 } we know the values.

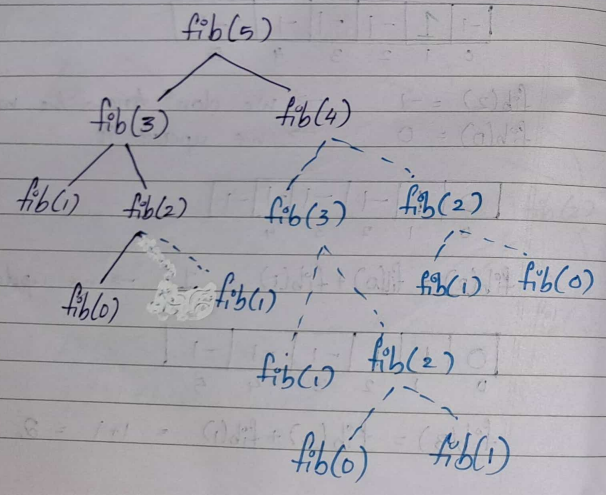
fib(3) = 2 }

fib(4) = fib(2) + fib(3)

= 1 + 2 = 3 → update.

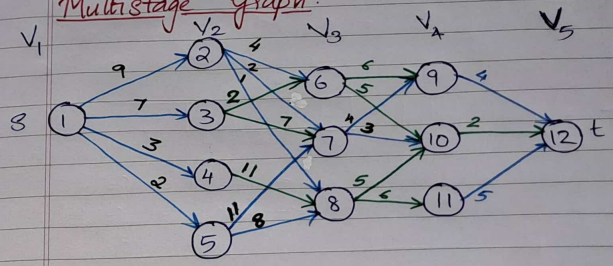
0	1	1	2	3	-1
0	1	2	3	4	5

fib(5) = fib(3) + fib(4) = 2+3 = 5 → update.



So here we only had 6 function calls.

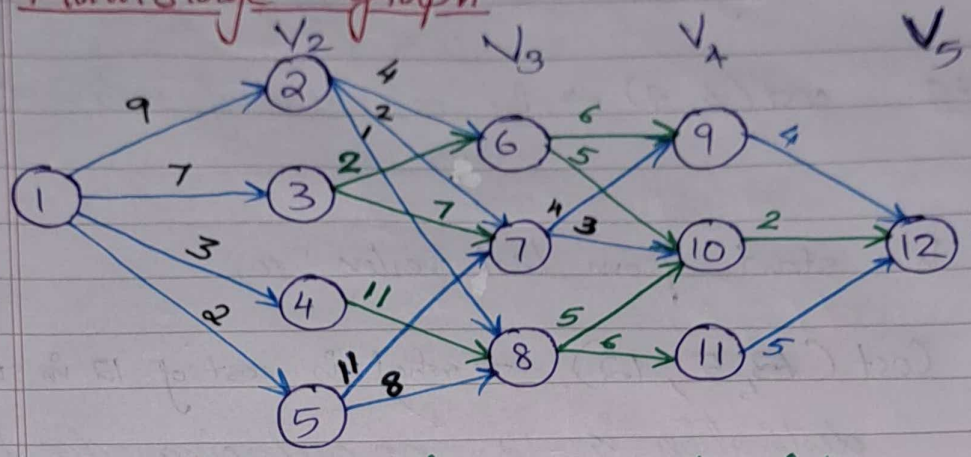
Multistage Graph:



We can solve this using Dynamic Programming. (In Greedy approach we could have used Dijkstra or Bellford etc).

So here we only had 6 function calls.

Multistage Graph:



Find shortest path from source to sink.

We can solve this using Dynamic Programming. (In Greedy approach we could have used Dijkstra or Bellford etc).

20/6/24
Thursday.

- Directed weighted graph.
- This is a resource allocation graph.
- It is a forward problem. but we start from last vertex.

dist wrt to vertex

v	1	2	3	4	5	6	7	8	9	10	11	12
cost	16	7	9	18	15	7	5	7	4	2	5	0
d	2/3	7	6	8	8	10	10	10	12	12	12	12

we write the vertex no.

From this table only we find best approach. This is the diff: b/w greedy approach method & DP method.

- cost (stage no:, vertex no:)
  - stage no: → in which stage
  - vertex no: → for which vertex the cost is being found.

eg: cost(4, 9) → the cost value of vertex 9 in stage 4.

So We start from last vertex so,

Step 1: Cost (5, 12) → what is cost of 12 in stage 5?
destination is 12, no outgoing edge from 12 so cost is 0.

Step 2: Now go back to stage 4, three vertices: (9), (10), (11) So,
cost (4, 9)
cost (4, 10)
cost (4, 11)
} find them.

cost(4,9) → only one outgoing edge.
→ cost from vertex 9 to vertex 12 is 4.
→ so d = 12 and cost = 4
destination vertex, cost of the only outgoing edge.
Similarly for cost(4,10) & cost(4,11),

Step 3: Now go to Stage 3,
cost(3,6)
cost(3,7)
cost(3,8)
} two outgoing edges for each.

If more than one outgoing edge then take minimum.

cost (3,6) = { cost(6, 9) + cost(9, 9) = 10 (we get this from table; from vertex, to vertex, stage, vertex)
cost(6, 10) + cost(4, 10) = 7 ✓
the two outgoing edges.

cost(3,6) = 7 due to vertex 10.
So cost = 7 & d = 10

cost (3,7):

cost(3,7) = { cost(7, 9) + cost(4,9) = 8 (outgoing edges. / from table)
cost(7, 10) + cost(4,10) = 5 ✓

cost(3,7) = 5 & d = 10

cost (3, 8):
cost(3,8) = { cost(8,10) + cost(4, 10) = 7 ✓
cost(8, 11) + cost(4, 11) = 11
cost(3,8) = 7 & d = 10

Step 4. Now go to stage 2

$$cost(2,2) = \begin{cases} cost(2,6) + cost(3,6) = 11 \\ cost(2,7) + cost(3,7) = 7 \checkmark \\ cost(2,8) + cost(3,8) = 8 \end{cases}$$

$d = 7 \quad cost = 7$

$$cost(2,3) = \begin{cases} cost(3,6) + cost(3,6) = 9 \checkmark \\ cost(3,7) + cost(3,7) = 12 \end{cases}$$

$d = 6 \quad cost = 9$

$cost(2,4) = 11+7$ & $d = 8$
$= 18$

$$cost(2,5) = \begin{cases} cost(5,7) + cost(3,7) = 16 \\ cost(5,8) + cost(3,8) = 15 \checkmark \end{cases}$$

$d = 8.$

Step 5. Stage 1,

$$cost(1,1) = \begin{cases} cost(1,2) + cost(2,2) = 16 \checkmark \\ cost(1,3) + cost(2,3) = 16 \\ cost(1,4) + cost(2,4) = 21 \\ cost(1,5) + cost(2,5) = 17 \end{cases}$$

Write both in this case.

Step 6. Now find shortest path using the table made.

1) $1 \longrightarrow 2 \longrightarrow 7 \longrightarrow 10 \longrightarrow 12$ (the d values)
cost = $9+2+3+2 = 16$.
(The costs are given in graph)

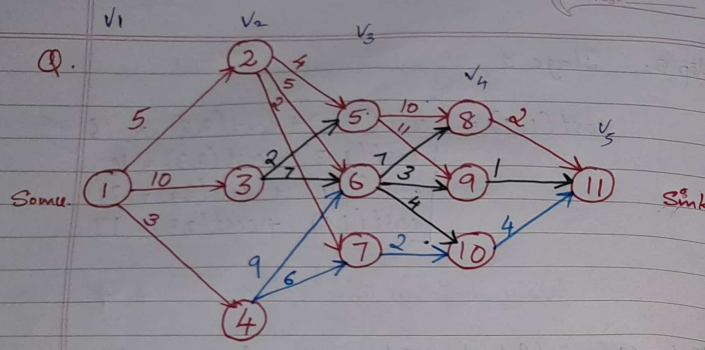
2) $1 \longrightarrow 3 \longrightarrow 6 \longrightarrow 10 \longrightarrow 12$
cost = $7+2+5+2 = 16$

There two paths are available as shortest path.

Q. Why is this called a forwarding problem even though we are backtracking?

A: All the edges & path are forward. We are only considering them forward edges so it is a forwarding path

Q.



A: v	1	2	3	4	5	6	7	8	9	10	11
cost	13	8	11	12	12	4	6	2	1	4	0
d.	2	7	6	7	8/9	9	10	11	11	11	11

$cost(3,7) = 6$

$cost(3,6) = \begin{cases} 9 & (8) \\ 4 & (9) \checkmark \\ 8 & (10) \end{cases}$ d.

$cost(3,5) = \begin{cases} 12 & (8) \\ 12 & (9) \end{cases}$

$cost(2,4) = \begin{cases} 13 & (6) \\ 6+6=12 & (7) \checkmark \end{cases}$

$cost(2,3) = \begin{cases} 2+12=14 & (5) \\ 7+4=11 & (6) \checkmark \end{cases}$

$cost(2,2) = \begin{cases} 4+12=16 & (5) \\ 5+4=9 & (6) \\ 2+6=8 & (7) \checkmark \end{cases}$

$cost(1,1) = \begin{cases} 5+8=13 & (2) \checkmark \\ 10+11=21 & (3) \\ 3+12=15 & (4) \end{cases}$

Path:

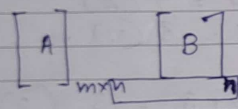
1. $1 \rightarrow 2 \rightarrow 7 \rightarrow 10 \rightarrow 11$

   $5+2+2+4 = 13$ → cost.

$$cost(i,j) = \min_{\substack{j,l \in E \\ l \in V_{i+1}}} \{ cost(j,l) + cost((i+1),l) \}$$

Matrix Chain Multiplication using DP:

Total no. of multiplications in matrix:



$= m \times n \times y$

eg: in $A_{2\times 2} \times B_{2\times 2}$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & 2 \\ 2 & 2 \end{bmatrix}$$

$= 2 \times 2 \times 2 = 8$ multiplication.

chain matrix multiplication:

$A_1 \times A_2 \times A_3$

$2\times 3 \quad 3\times 4 \quad 4\times 2$

Multiplication ways:

1. $(A_1 \times A_2) \times A_3$

$2\times 3 \quad 3\times 4 \quad 4\times 2$

Total multiplications = $(2\times 3\times 4) + (2\times 4\times 2)$

$= 24 + 16$

$= 40$

2. $A_1 \times (A_2 \times A_3)$

$2\times 3 \quad 3\times 4 \quad 4\times 2$

Total multiplication = $(3\times 4\times 2) + (2\times 3\times 2)$

$= 24 + 12$

$= 36$

So obviously 2nd method is better.

Using DP we can find the optimal chain matrix multiplication is better.

Generalization:

matrix → dimensions

$A_1 \rightarrow d_0 \times d_1$ $\quad c[1,2] \rightarrow$ cost of multiplying matrices 1 & 2.

$A_2 \rightarrow d_1 \times d_2$

$A_3 \rightarrow d_2 \times d_3$ $\quad c[3,3] \rightarrow$ cost of multiplying matrix 3 with itself (which is zero).

- $(A_1 \times A_2) \times A_3$

  $c[1,2] \quad c[3,3]$

  $c[1,2] + c[3,3] + d_0 \times d_2 \times d_3$

  $i \; k \quad k+1 \; j \quad d_{i-1} \; d_k \; d_j$

- $A_1 \times (A_2 \times A_3)$

  $c[1,1] \quad c[2,3]$

  $c[1,1] + c[2,3] + d_0 \times d_1 \times d_3$

  $i \; k \quad k+1 \; j \quad d_{i-1} \; d_k \; d_j$

∴ General form:

$$c[i,j] = \min \{ c[i,k] + c[k+1,j] + d_{i-1} \times d_k \times d_j \}$$

Q. How many set of calculations are present?

↳ $|j-i|$ ~~or $i-j$~~.

So to find cost[i, j], there are $|j-i|$ many multiplication methods.

eg: cost[1, 3] = 3 − 1 = 2 calculations

That is exactly what we have done:

$(A_1 \times A_2) A_3$ and $A_1 (A_2 \times A_3)$

Q. What about the k values?

↳ k = i to j (j not included)

eg: matrix 1 to 3

then k = 1 and k = 2.

matrix 1 to 4 multiplications

then k = 1, k = 2, k = 3.

matrix 4 to 6:

then k = 4, 5.

$$i \leq k < j$$

Q. $A_1 \times A_2 \times A_3 \times A_4$

d_0 3 2 d_1 2 4 d_2 4 2 d_3 2 5 d_4

A: i = 1 so k = 1, 2, 3.

j = 4

So:

$$c[1,4] = \min \begin{cases} k=1 & c[1,1] + c[2,4] + d_0 \times d_1 \times d_4 \\ k=2 & c[1,2] + c[3,4] + d_0 \times d_2 \times d_4 \\ k=3 & c[1,3] + c[4,4] + d_0 \times d_3 \times d_4 \end{cases}$$

$i-j=0$
$i-j=1$
$i-j=2$
⋮

we find $c[i,j]$ in this order. (From min to max).

cost table.

	1	2	3	4
1	0	24	28	58
2		0	16	36
3	Not needed		0	40
4				0

k table.

	1	2	3	4
1	0	1	1	3
2		0	2	3
3			0	3
4				0

Step 1: We can say diagonal values = 0.
($i-j=0$, cost of a matrix multiplying with itself).

~~in $c[1,4]$ when $k=1$~~
~~$c[1,1] + c[2,4] + d_0 \times d_1 \times d_4$~~
$0 + ? + d_0 \times d_1 \times d_4$

~~$c[2,4]$ = min~~
~~Similarly $c[1,2]$ is also~~

$d_0 = 3$ $d_4 = 5$
$d_1 = 2$
$d_2 = 4$
$d_3 = 2$

Step 2: ($i-j=1$ pairs.)
In cost table, $i-j=0$ → is filled
Now find cost of $i-j=1$ pairs,

$$c[1,2] = \min_{k=1} \{ c[1,1] + c[2,2] + d_0 \times d_1 \times d_2$$

Fill corresponding values of k in k table.

$= 0 + 0 + (3 \times 2 \times 4)$
$= 24$ (Fill in table)

$$c[2,3] = \min_{k=2} \{ c[2,2] + c[3,3] + d_1 \times d_2 \times d_3$$

$= 0 + 0 + 2 \times 4 \times 2$
$= 16$

$$c[3,4] = \min_{k=3} \{ c[3,3] + c[4,4] + d_2 \times d_3 \times d_4$$

$= 0 + 0 + 4 \times 2 \times 5$
$= 40$

Step 3:

i-j = 2

$c[1,3] = \min \begin{cases} k=1 & c[1,1] + c[2,3] + d_0 \times d_1 \times d_3 \\ & = 0 + 16 + (3 \times 2 \times 2) = 28 \checkmark \\ k=2 & c[1,2] + c[3,3] + d_0 \times d_2 \times d_3 \\ & = 24 + 0 + (3 \times 4 \times 2) \\ & = 24 + 24 = 48 \end{cases}$

Min @ k=1 is 28.

↳ write in cost & k table.

$c[2,4] = \min \begin{cases} k=2 & c[2,2] + c[3,4] + d_1 \times d_2 \times d_4 \\ & = 0 + 40 + (2 \times 4 \times 5) \\ & = 80 \\ k=3 & c[2,3] + c[4,4] + d_1 \times d_3 \times d_4 \\ & = 16 + 0 + (2 \times 2 \times 5) \\ & = 36 \checkmark \end{cases}$

min @ k=3 & cost = 36

Step 4:

i-j = 3

~~cost~~

$c[1,4] = \min \begin{cases} k=1 & c[1,1] + c[2,4] + d_0 \times d_1 \times d_4 \\ & = 0 + 36 + (3 \times 2 \times 5) = 36 + 30 = 66. \\ k=2 & c[1,2] + c[3,4] + (d_0 \times d_2 \times d_4) \\ & = 24 + 40 + (3 \times 4 \times 5) = 64 + 60 = 124 \\ k=3 & c[1,3] + c[4,4] + (d_0 \times d_3 \times d_4) \\ & = 28 + 0 + (3 \times 2 \times 5) = 28 + 30 = 58 \end{cases}$

min @ k=3 is 58.

Parenthesis possibilities:

$A_1 \times A_2 \times A_3 \times A_4$

1. $(A_1 \times A_2) \times (A_3 \times A_4)$
2. $A_1 ((A_2 \times A_3) \times A_4)$
3. $((A_1 \times A_2) A_3) A_4$
4. $A_1 (A_2 (A_3 \times A_4))$
5. $(A_1 (A_2 \times A_3)) A_4$

Gen →

To find the no: of parenthesis possibilities ↓

$$\frac{2(n-1)C_{(n-1)}}{n}$$

✓ $\frac{2(4-1)C_{4-1}}{4}$

$= \frac{6C_3}{4} = \frac{6 \times 5 \times 4}{1 \times 2 \times 3 \times 4}$

$= 5$

Use k table for optimal matrix multiplication.

$A_1 \quad A_2 \quad A_3 \quad A_4$

k value of $[1, 4] = 3$.

So, $(A_1 \; A_2 \; A_3) \; A_4$.

k value of $[1, 3] = 1$

So, $((A_1) \; A_2 \; A_3) \; A_4$

↓ this is the optimal matrix multiplication.

Q. Using DP find the paranthesisation for multiplying the chain of matrics

$A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \quad A_6$

$\langle 30 \times 35 \rangle \; \langle 35 \times 15 \rangle \; \langle 15 \times 5 \rangle \; \langle 5 \times 10 \rangle \; \langle 10 \times 20 \rangle \; \langle 20 \times 25 \rangle$

A: No: of paranthesis possibilities:

$$\frac{2(n-1)C_{n-1}}{n} = \frac{2(5)C_5}{6} = \frac{10C_5}{6}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5 \times 6}$$

$$= 6 \times 7 = 42$$

$d_0 = 30 \quad d_2 = 15 \quad d_4 = 10 \quad d_6 = 25$

$d_1 = 35 \quad d_3 = 5 \quad d_5 = 20$

cost table:

	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	7125	9625
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

k table:

	1	2	3	4	5	6
1	0	1	1	3	3	3
2		0	2	3	3	3
3			0	3	3	3
4				0	4	5
5					0	5
6						0

$c[1, 2] = k=1 \{ c[1,1] + c[2,2] + d_0 \times d_1 \times d_2$

$= 0 + 0 + 30 \times 35 \times 15$

$= 15750$

$c[2,3] = k=2 \{ c[2,2] + c[3,3] + d_1 \times d_2 \times d_3$

$= 0 + 0 + (35 \times 15 \times 5)$

$= 2625$

$c[3,4] = k=3 \{ c[3,3] + c[4,4] + d_2 \times d_3 \times d_4$

$= 0 + 0 + (15 \times 5 \times 10)$

$= 750$

$c[4,5] = k=4 \{ c[4,4] + c[5,5] + d_3 \times d_4 \times d_5$

$= 0 + 0 + (5 \times 10 \times 20)$

$= 1000$

$c[5,6] = k=5 \{ c[5,5] + c[6,6] + d_4 \times d_5 \times d_6$

$= 10 \times 20 \times 25$

$= 5000$

$c[1,3] = \min$

$k=1$: $c[1,1] + c[2,3] + d_0 \times d_1 \times d_3$

$= 0 + 2625 + 30 \times 35 \times 5$

$= 2625 + 5250 = 7875$ ✓

$k=2$: $c[1,2] + c[3,3] + d_0 \times d_2 \times d_3$

$= 15750 + 0 + (30 \times 15 \times 5)$

$= 18000$

$c[2,4] = \min$

$k=2$: $c[2,2] + c[3,4] + d_1 \times d_2 \times d_4$

$= 0 + 750 + (35 \times 15 \times 10)$

$= 750 + 5250 = 6000$

$k=3$: $c[2,3] + c[4,4] + (d_1 \times d_3 \times d_4)$

$= 2625 + 0 + (35 \times 5 \times 10)$

$= 2625 + 1750$

$= 4375$ ✓

$c[3,5] = \min$

$k=3$: $c[3,3] + c[4,5] + (d_2 \times d_3 \times d_5)$

$= 0 + 1000 + (15 \times 5 \times 20)$

$= 2500$ ✓

$k=4$: $c[3,4] + c[5,5] + (d_2 \times d_4 \times d_5)$

$= 750 + 0 + (15 \times 10 \times 20)$

$= 3750$

$c[4,6] = \min$

$k=4$: $c[4,4] + c[5,6] + d_3 \times d_4 \times d_6$

$0 + 5000 + (5 \times 10 \times 25)$

$= 6250$

$k=5$: $c[4,5] + c[6,6] + d_3 \times d_5 \times d_6$

$1000 + 0 + (5 \times 20 \times 25)$

$= 3500$ ✓

$d_0 = 30$ $d_1 = 35$ $d_2 = 15$ $d_3 = 5$
$d_4 = 10$ $d_5 = 20$ $d_6 = 25$

$c[1,4] = \min$
- $k=1$: $c[1,1] + c[2,4] + d_0 \times d_1 \times d_4$ $= 0 + 4375 + (30 \times 35 \times 10) = 14875$
- $k=2$: $c[1,2] + c[3,4] + d_0 \times d_2 \times d_4$ $= 15750 + 750 + (30 \times 15 \times 10) = 21000$
- $k=3$: $c[1,3] + c[4,4] + d_0 \times d_3 \times d_4$ $= 7875 + 0 + (30 \times 5 \times 10) = 9375$ ✓

$c[2,5] = \min$
- $k=2$: $c[2,2] + c[3,5] + d_1 \times d_2 \times d_5$ $= 0 + 2500 + (35 \times 15 \times 20) = 13000$
- $k=3$: $c[2,3] + c[4,5] + d_1 \times d_3 \times d_5$ $= 2625 + 1000 + (35 \times 5 \times 20) = 7125$ ✓
- $k=4$: $c[2,4] + c[5,5] + d_1 \times d_4 \times d_5$ $= 4375 + 0 + (35 \times 10 \times 20) = 11375$

$c[3,6] = \min$
- $k=3$: $c[3,3] + c[4,6] + d_2 \times d_3 \times d_6$ $= 0 + 3500 + (15 \times 5 \times 25)$ $= 5375$ ✓
- $k=4$: $c[3,4] + c[5,6] + d_2 \times d_4 \times d_6$ $750 + 5000 + (15 \times 10 \times 25)$ $= 9500$
- $k=5$: $c[3,5] + c[6,6] + d_2 \times d_5 \times d_6$ $2500 + 0 + (15 \times 20 \times 25) = 10000$

$c[1,5] = \min$
- $k=1$: $c[1,1] + c[2,5] + d_0 \times d_1 \times d_5$ $= 0 + 7125 + (30 \times 35 \times 20) = 28125$
- $k=2$: $c[1,2] + c[3,5] + d_0 \times d_2 \times d_5$ $15750 + 2500 + (30 \times 15 \times 20) = 27250$
- $k=3$: $c[1,3] + c[4,5] + d_0 \times d_3 \times d_5$ $= 7875 + 1000 + 30 \times 5 \times 20 = 11875$ ✓
- $k=4$: $c[1,4] + c[5,5] + d_0 \times d_4 \times d_5$ $9375 + 0 + (30 \times 10 \times 20) = 15375$

$c[2,6] = \min$
- $k=2$: $c[2,2] + c[3,6] + d_1 \times d_2 \times d_6 = 0 + 5375 + d_1 d_2 d_6$ $= 5375 + (35 \times 15 \times 25) = 18500$
- $k=3$: $c[2,3] + c[4,6] + d_1 \times d_3 \times d_6 = 2625 + 3500 + d_1 d_3 d_6$ $= 6125 + (35 \times 5 \times 20) = 9625$ ✓
- $k=4$: $c[2,4] + c[5,6] + d_1 \times d_4 \times d_6 = 4375 + 5000 + d_1 d_4 d_6$ $= 9375 + (35 \times 10 \times 25) = 18125$
- $k=5$: $c[2,5] + c[6,6] + d_1 d_5 d_6 = 7125 + 0 + d_1 d_5 d_6$ $= 7125 + (35 \times 20 \times 25) = 24625$

$c[1,6] = \min$

$k=1$: $c[1,1] + c[2,6] + d_0 d_1 d_6$
$= 0 + 9625 + (30 \times 35 \times 25)$
$= 35875$

$k=2$: $c[1,2] + c[3,6] + d_0 d_2 d_6$
$= 15750 + 5375 + (30 \times 15 \times 25)$
$= 32375$

$k=3$: $c[1,3] + c[4,6] + d_0 d_3 d_6$
$= 7875 + 3500 + (30 \times 5 \times 25)$
$= 15125$ ✓

$k=4$: $c[1,4] + c[5,6] + d_0 d_4 d_6$
$= 9375 + 5000 + (30 \times 10 \times 25)$
$= 21875$

$k=5$: $c[1,5] + c[6,6] + d_0 d_5 d_6$
$= 11875 + 0 + (30 \times 20 \times 25)$
$= 26875$

Among the 42 possible matrix multiplication chains, the optimal one is:

$k[1,6] = 3$ $\quad k[4,6] = 5$

$((A_1)(A_2 \times A_3)) \times ((A_4 \times A_5)(A_6))$

$k[1,3] = 1$

Verification:

A_1 A_2 A_3 A_4 A_5 A_6

30×35 $\quad 35\times15$ $\quad 15\times5$ $\quad 5\times10$ $\quad 10\times20$ $\quad 20\times25$

$35\times15\times5 \rightarrow 2625$

$5\times10\times20 \rightarrow 1000$

A_1 A_{23} A_{45} A_6

30×35 $\quad 35\times5$ $\quad 5\times20$ $\quad 20\times25$

$30\times35\times5 \rightarrow 5250$

$5\times20\times25 \rightarrow 2500$

$A_{123} \times A_{456}$

30×5 $\quad 5\times25$

$30\times5\times25 \rightarrow 3750$

Multiplications $= 2625 + 1000 + 5250 + 2500 + 3750$
$= 15,125$
↳ The optimal multiplications (cost in cost table for $c[1,6]$).

∴ Parathesis pattern is:

$$(A_1 \times (A_2 \times A_3))((A_4 \times A_5) \times A_6)$$

27/06/24/Thursday.

- Fractional knap sack - greedy approach.
- 0/1 knap sack pblm - cannot use greedy.
  - use DP.

0/1 Knap Sack Problem:

- Dynamic Pgmming
- We take either the whole thing or we do not take it.
- No: of rows - no: of items + 1
  - name the rows from 0.
- No: of cols - max capacity of container + 1

eg: weight = {3, 4, 6, 5}
profit = {2, 3, 1, 4} W = 8kg.

i ↓

P	W	0	1	2	3	4	5	6	7	8
		0	0	0	0	0	0	0	0	0
2	3*	1	0	0	0	2*	2	2	2	2
3	4*	2	0	0	0	2	3*	3	3	5
4	5*	3	0	0	0	2	3	4*	4	5
1	6*	4	0	0	0	2	3	4	4*	5

6-5=1, 4+0, 3
7-5=2, 4+0, 5
8-5=3, 4+2, 5
1,4
1,5

Step 1: Write items in ↑order of weight.
Step 2: Write their profits alongside the weights

P	W
2	3
3	4
4	5
1	6

Write this to the left of table.

Step 3: for zeroth row & column : value is zero.
Step 4: until min: weight, value is zero.
Here min weight = 3 so in 1st row upto column 3, value = 0
Step 5: Once you put the value of profit at the [row][column], put the same along the row.
eg: here min wt 3 has profit 2.
So, 2nd row from 3rd column, value is 2.

Step 6: Now the nxt weight is 4 kg, upto 4th column look at how it is made according to previous row.

max(3+0, 2) = 3
↓ value in [previous row] [col no: 4−4] ← wt
↳ previous row value

max(3+0, 2)
↳ value in [previous row] [col no: 5−4] ← wt = [1][1]

max(3+0, 2)
max(3+2, 2) value in [1][2−4] = [1][3] = 2

Step 7. Continue on to nxt iteration.

Step 8. Now after completing the table, put wts in ↑ order & write 1 if it is included else put 0.

↳ We put zero if value of wt col is same as previous row.

3	4	5	6
			0

(last col) → col 8 of row 4 (which corresponds to 4th item with wt 6) ~~is same as~~ has value same as previous row.

Now go to previous row.

3	4	5	6
		1	0

→ The last col value = 6 previous row last col value is different so 5 is included.

Now profit = 6 − 4 (↳ profit of 5) = 2.

3	4	5	6
	0	1	0

→ Remaining profit = 2. Check for 2. It appears in col 3, but it repeats in previous row so value of 4 = 0

3	4	5	6
1	0	1	0

→ Remaining profit = 2. Present in col 3 of row 1. This value does not repeat in previous row (no more previous rows) so it is included.

Remaining profit = 2 − 2 = 0.

Thus we included item 1 and item 3 with weights 3 & and 5 units respectively.

Q. ~~Sol~~ Solve for 0/1 knap sack pblm.

Wt = { 2, 3, 4, 5 } Max capacity = 8kg

profit = { 1, 2, 5, 6 }

p	w	0	1	2	3	4	5	6	7	8
		0	0	0	0	0	0	0	0	0
1	2	0	0	1*	1	1	1	1	1	1
2	3	2	0	0	1	2	2	3	3	3
5	4	3	0	0	1	2	5*	5	6	7
6	5	4	0	0	1	2	5	6*	6	7

2	3	4	5
0	1	0	1

5th obj: 8 − 6 = 2

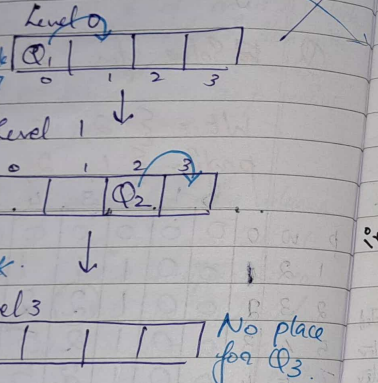
2nd obj: 2 − 2 = 0

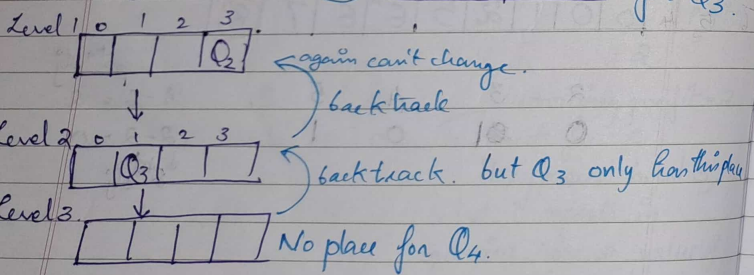
$$m[i,w] = \max\{m(i-1, w),\ m(i-1, w-w(i)+p(i))\}$$

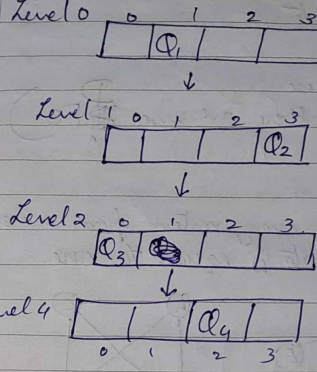
N-Queen Pblm:

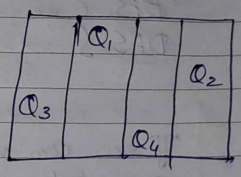
- using backtracking.

4-Queen









for implementation

Generalising : positions other queens can't be in.

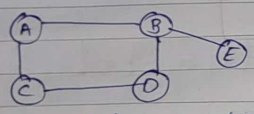
eg: Q in (1, 2) [→] (top left to bottom right) [↙]

row	col	diag 1	diag 2
(1,0)	(0,2)	(0,1)	(0,3)
(1,1)	(2,2)	(2,3)	~~(1,2)~~ (2,1)
(1,3)	(3,2)	↓	(3,0)
		diff = 3-2 = 1	↓
		2-1 = 1	sum = 1+2 = 3
		(same diff.)	(same sum).
		↓	↓
i same		~~i same~~	i+j → same.
	j same.	j-1 → same.	

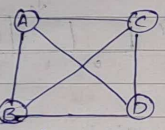
Connected Components

BFS } we use these
DFS } (Here we use DFS).

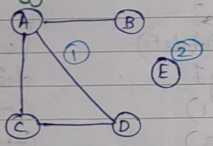
If we can visit every vertex from a vertex then connected components is equal to one.



connected component = 1
Strongly connected component.



#CC = 1
Strongly connected component



#CC = 2

case ↓ edge.

↳ ~~from~~ every vertex ~~we can go to all~~ is connected to every other vertex (through another vertex at least)

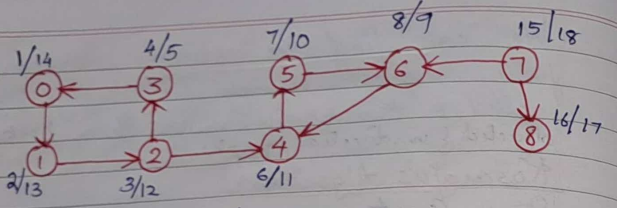
such a graph is called strongly connected graph.

We can find the strongly connected components from a graph using DFS. Algorithm is called Kosaraju's Algorithm. Works in both directed & undirected graphs.

Kosaraju's Algorithm:

Step 1: Create an empty stack.
Step 2: Do DFS traversal of the graph.
Step 3: During DFS traversal mark starting and finishing time.
Step 4: When the finishing time of a vertex is marked, push the vertex into the stack.
Step 5: Reverse the directions of all arcs to obtain the transpose graph.
Step 6: Pop the vertex one by one from stack while stack is not empty. While popping, let the popped vertex be V. Then take V as a source and do DFS call on V. This DFS call will give one strongly connected component or SCC.
Step 7: Repeat until stack is empty.

Q. Find the SCC for the following graph?

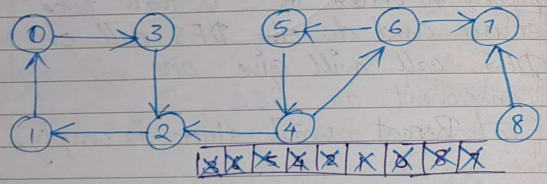


A: → DFS traversal

- While outo the stack in the order of finishing time.

3	6	5	4	2	1	0	8	7

- Now stack is complete.
- Now draw transpose of graph by reversing edges.

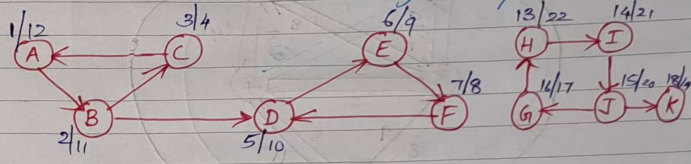


Now in this do popping.

1. 7 (it is not connected anywhere).
2. 8 (it is connected to 7 but it is already popped)
3. 0 → 3 → 2 → 1 (path in transpose graph). 1, 2 & 3 are already visited thanks to this so when they come up in stack, just pop them out. No need to do DFS.
4. 4 → 6 → 5 (The others: 2 was already been popped out).

★ Strongly Connected Graph → the whole graph is a single component.

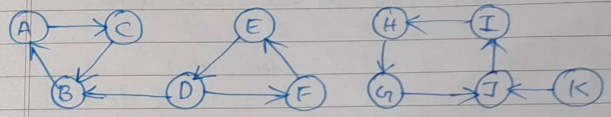
Q.



A: Stack:

~~G~~	~~K~~	~~E~~	~~D~~	~~B~~	~~A~~	~~C~~	~~K~~	~~J~~	~~I~~	~~H~~

transpose.



1. H → G → J → I
2. K
3. A → C → B
4. D → F → E

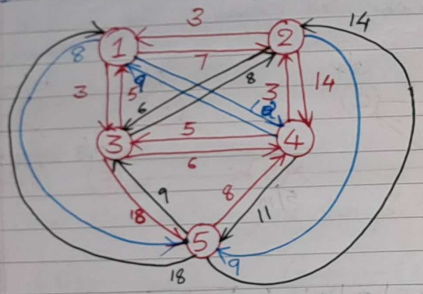
28/06/24 Friday.

Travelling Sales Man Problem (TSP):

- **Branch & Bound** algorithm/strategy.
- Start from one vertex and visit all other vertices exactly once and then return back to the starting vertex.

Prestep: Convert graph to adjacency matrix.
Step1: Have a state space tree.

Q.



A: * Mark self loops as infinity if they are not present.

	1	2	3	4	5
1	∞	7	(3)	12	8
2	(3)	∞	6	14	9
3	(5)	8	∞	6	18
4	9	(3)	5	∞	11
5	18	14	9	(8)	∞

→ Base Matrix.

Step1: Find the reduced cost matrix (Reduce the rows and then columns... that is, subtract min: cost from corresponding rows first and then from columns.)
↓ all rows should have one zero atleast and every columns should have atleast one zero as well.

Step2: Write the min: value for each row. (I have circled it in the base matrix) Now subtract each value in the row with the min value.

After subtracting.

∞	4	0	9	(5)
0	∞	3	11	6
0	3	∞	1	13
6	0	2	∞	8
10	6	1	0	∞

~~cost 1~~
→ col: wise not reduced. (only last col). (so do for only last col)

Now find total cost for ~~each row~~ min value
= 3+3+5+3+8 = 22// (sum of min values selected).

$$\begin{bmatrix} \infty & 4 & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & 3 & \infty & 1 & 8 \\ 6 & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix}$$

Reduced cost of col = 5 (we used 5)
↳ the min value of col selected

∴ Total reduced cost = 22+5
= 27

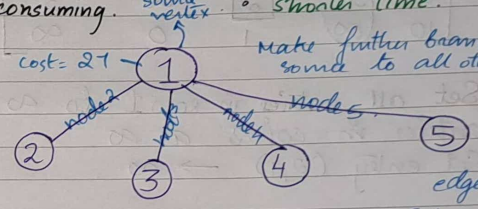
Step3: Check whether the resultant matrix is reduced or not (We checked & found out it was not column wise). That is, check all rows and cols have atleast one zero value or not. Here all rows had zeroes but the 5th column did not have zero. So, we subtract the min: value, 5, from the 5th column.

Step4: Find the cost of the reduced matrix.
Cost = total cost reduced from the original matrix = 3+3+5+8+5 (rows, last col)
= 27,

Step 5: Make the space tree matrix which is generated by least cost branch and bound. Mark the root node values as 27 (the reduced cost). Starting from the root node generate nodes 2, 3, 4 & 5.

★ Full cost BfB:
- make branches from all nodes.
- time consuming.

Least Cost BfB:
- we take branch with least cost.
- shorter time.



in first iteration both edge & circle value are all same.

Step 5 continuation: Find out the reduced matrix for each node (1,2), (1,3), (1,4) and (1,5).

cost (1,2):

In the matrix made, mark value in 1st and 2nd col ~~and~~ as ∞ as well as cell (2,1) as ∞.

↓ Matrix becomes

∞	∞	∞	∞	∞	
∞	∞	3	11	1	1
0	∞	∞	1	8	2 +
6	∞	2	∞	3	3
10	∞	1	0	∞	

Set all entries in row 1 to ∞ & set all entries in col 2 as ∞.
Set entry (2,1) → ∞.

Now reduce the resulting matrix & find the cost. (No need to consider all ∞ row/col)

↳

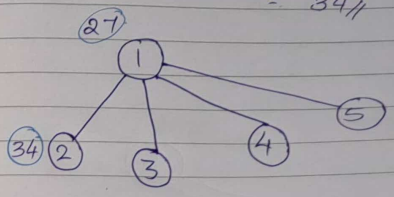
∞	∞	∞	∞	∞	→ all cols have atleast one zero. So completely reduced.
∞	∞	2	10	0	
0	∞	∞	1	8	
4	∞	0	∞	1	
10	∞	1	0	∞	

Total reduction cost = 2 + 1 = 3
(value in ~~original~~ base matrix for this step)

∴ Cost of (1,2) = cost(1) + A(1,2) + reductionCost In This Iteration

= 27 + 4 + 3

= 34 //



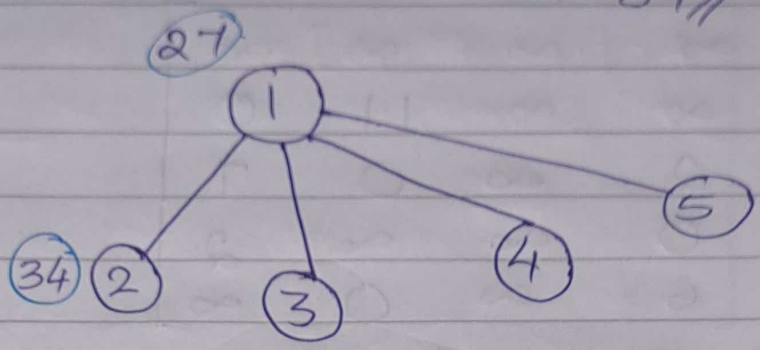
Total reduction cost = 2 + 1 = 3

value in original base matrix for this step

∴ Cost of (1,2) = cost(1) + A(1,2) + reductionCostInThisIteration

= 27 + 4 + 3

= 34 //



29/6/24 Saturday

cost (1,3):

Make values of row 1, column 3 and cell (3,1) into ∞ in the base matrix (matrix at the end of finding cost of 1).

∞	∞	∞	∞	∞	
0	∞	∞	1	1	
∞	3	∞	1	8	1
6	0	∞	∞	3	
10	6	∞	0	∞	

↓ now reduce...

∞	∞	∞	∞	∞
0	∞	∞	11	(1)
∞	2	∞	0	18
6	0	∞	∞	3
10	6	∞	0	∞

+ 1

↓ column reduce.

∞	∞	∞	∞	∞
0	∞	∞	11	0
∞	2	∞	0	7
6	0	∞	∞	2
10	6	∞	0	∞

reduced cost = 1+1 = 2

c(1,3) = c(1) + A(1,3) + reduced Cost
= 27 + 0 + 2 = 29

cost (1,4):
A[1,4] = 9 (in base matrix).
1st row, 4th col & A[4,1] = ∞

∞	∞	∞	∞	∞
0	∞	3	∞	1
0	3	∞	∞	8
∞	0	2	∞	3
10	6	(1)	∞	∞

1 +

row reduce.

∞	∞	∞	∞	(∞)
0	∞	3	∞	(1)
0	3	∞	∞	8
∞	0	2	∞	3
9	5	0	∞	∞

1 +

↓ column reduce

∞	∞	∞	∞	∞
0	∞	3	∞	0
0	3	∞	∞	7
∞	0	2	∞	2
9	5	0	∞	∞

Reduced cost = 1+1 = 2.

c[1,4] = A[1,4] + c[1] + reduced Cost
= 9 + 27 + 2 = 38

cost [1,5]:
1st row, 5th col & cell [5,1] = ∞

∞	∞	(∞)	∞	∞
0	∞	3	11	∞
0	3	∞	1	∞
6	0	2	∞	∞
∞	6	(1)	0	∞

already row reduced so just col: reduce

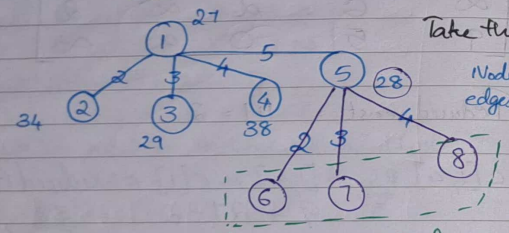
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 1 & \infty & \infty \\ \infty & 6 & 0 & 0 & \infty \end{bmatrix}$$

reducedCost = 1

c[1,5] = c[1] + A[1,5] + reducedCost

= 27 + 0 + 1

= 28



The base matrix for these would be the matrix you got at the end of step 5 ...

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 1 & \infty & \infty \\ \infty & 6 & 0 & 0 & \infty \end{bmatrix}$$ → The BASE MATRIX

node 5 to node 2..

cost(5,6) → (5,2) → This is 1→5→2.

So make (2,1) = ∞.

5th row, 2nd col of cell [2,1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 11 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 6 & \infty & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$ 2 + 1

↓ now reduce

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & 10 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$ 1 + 4

↓ col reduce

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & 9 & \infty \\ 0 & \infty & \infty & 0 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

cost[5,2] = c[5] + A[5,2] + reducedCost

= 28 + 6 + 4 = 38 //

cost(5,7): → cost(5,3)

5th row, 3rd col of [3,1] = ∞

∞	∞	∞	∞	∞
0	∞	∞	11	∞
∞	3	∞	1	∞
6	0	∞	∞	∞
∞	∞	∞	∞	∞

now reduce

1+

∞	∞	∞	∞	∞
0	∞	∞	11	∞
∞	2	∞	0	∞
6	0	∞	∞	∞
∞	∞	∞	∞	∞

c[5,3] = c[5] + A[5,3] + reducedCost

= 28 + 0 + 1 = 29//

cost(5,8) → cost(5,4]: 1→5→4

5th row, 4th col of [4,1] = ∞

∞	∞	∞	∞	∞
0	∞	2	∞	∞
0	3	∞	∞	∞
∞	0	1	∞	∞
∞	∞	∞	∞	∞

1+

∞	∞	∞	∞	∞
0	∞	1	∞	∞
0	3	∞	∞	∞
∞	0	0	∞	∞
∞	∞	∞	∞	∞

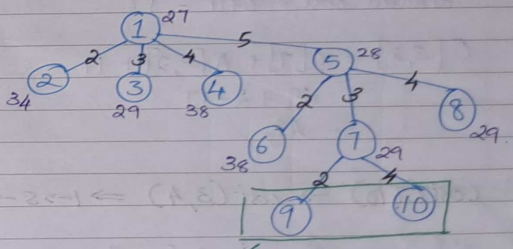
c[5,4] = c[5] + A[5,4] + reducedCost

= 28 + 0 + 1 = 29//

c[5,3] = 29 ✓

c[5,4] = 29

} You can any one of them...



~~cost~~

Base Matrix → Matrix of c[5,7]

∞	∞	∞	∞	∞
0	∞	∞	11	∞
∞	2	∞	0	∞
6	0	∞	∞	∞
∞	∞	∞	∞	∞

cost(7,9) → cost(3,2) ⇒ 1→5→3→2

3rd row, 2nd col of [2,1] = ∞

∞	∞	∞	∞	∞
∞	∞	∞	11	∞
∞	∞	∞	∞	∞
6	∞	∞	∞	∞
∞	∞	∞	∞	∞

6+11

reduced cost = 6+11 = 17

C[3,2] = C[7] + A[3,2] + 17
= 29 + 2 + 17
= 48

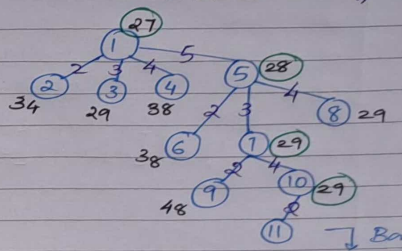
cost(7,10) → cost(3,4) ⇒ 1→5→3→4

3rd row, 4th col of [4,1] = ∞

∞	∞	∞	∞	∞
0	∞	∞	∞	∞
∞	∞	∞	∞	∞
∞	0	∞	∞	∞
∞	∞	∞	∞	∞

already → row & col reduced.

C[3,4] = C[7] + A[3,4] + 0
= 29 + 0 = 29



↓ Base Matrix

∞	∞	∞	∞	∞
0	∞	∞	∞	∞
∞	∞	∞	∞	∞
∞	0	∞	∞	∞
∞	∞	∞	∞	∞

~~Make~~ cost(10,11) → cost(4,2) ⇒ 1→5→3→4→2.

row 4, col 2 of [2,1] = ∞

∞

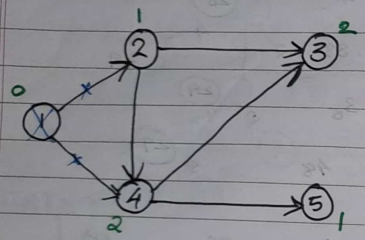
Thus cost(4,2) = 29+0+0
= 29

That is: The path: 1→5→3→4→2 & cost=29

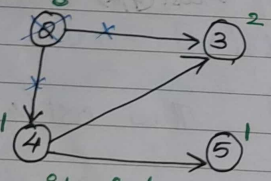
3/07/24/Wednesday.

Topological Sorting / Topological Ordering:

Ordering of vertices of a graph in linear order.

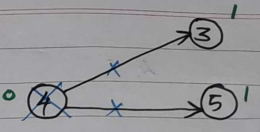


Step1: Write in degree of each vertex
Step2: Now we start with any vertex with indegree=0. here it is vertex 1.
Step3: Display that vertex & remove the edges coming out from that vertex & the vertex

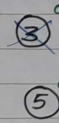


1,

Step4: Again write indegrees to the resultant graph...
Step5: Repeat...



1, 2



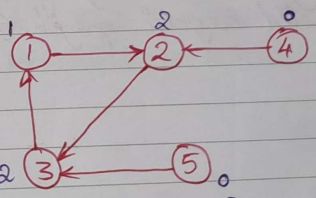
1, 2, 4,

Take any one from these.

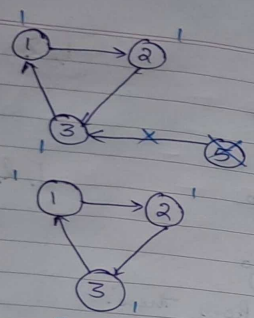
1, 2, 4, 3, 5.
or
1, 2, 4, 5, 3
if we take 5 before 3.

Thus the linear orders are: 1, 2, 4, 3, 5
1, 2, 4, 5, 3

Q.



A: Start from 4,

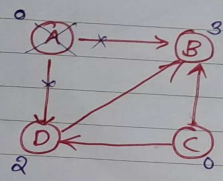


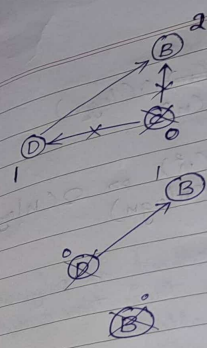
4, 5

Not possible. The same condition occurs even if we start from 5...
It becomes 5, 4, and then this condition.
So we cannot display it in topological order.
This is because there is a cycle.

Topological ordering occurs only if the graph is a directed acyclic graph.

Q.





A,

A, C

A, C, D

A, C, D, B

the topological ordering ← or

C, A, D, B

P, NP, NP Hand, NP- Complete Problems:

NP- Hand → NP-H
NP - complete → NP-C

Solutions

P	NP
can solve in polynomial time problems.	cannot ~~veri~~ solve in polynomial time but can verify in polynomial time problems.

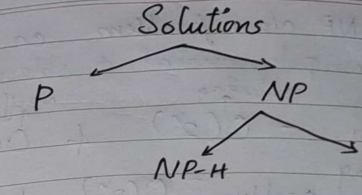
eg: Binary Search → ~~O(n)~~ $O(\log_2 n)$
Linear Search → $O(n)$

Quick Sort → $O(n^2)$ or $\Omega(n \log n)$
Merge Sort → $O(n \log n)$

All the above can be solved in polynomial time as shown using their time complexities. Hence they are P problems.

Sudoku → can be solved → max is 2^n exponential
→ But verification → n unit of time ∴ polynomial time.

→ Sudoku is an NP problem. It takes exponential time for solving & takes polynomial time for verification once it is filled. Hence this is an NP problem.



The solutions of an algorithm can be divided into polynomial time - P & non-polynomial time - NP.

P-class problems:

A problem which can be solved on polynomial time is known as P-class problems.
eg: all sorting & searching problems.
sum of n integers. etc.

Linear search → $O(n)$
Binary search → $O(\log_2 n)$
Sum of integers → $O(n)$

NP-class problems:

A problem which cannot be solved on polynomial time but is verified in polynomial time is

known as NP class problem or non-deterministic polynomial problem.

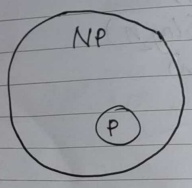
eg: Sudoku - $O(2^n)$ (solving) + $O(n)$ (verification)

Travelling Sales Man problem - $O(2^n)$ (solving) + $O(n)$ (verification)

In NP class problem, we have a problem X, and S is the solution to the problem and A is the algorithm used to solve X.

NP problem means we cannot solve X in polynomial time but can verify the solution of X is correct or not in polynomial time.

P is a subset of NP.



NP class	P class
∘ hard to solve.	∘ easy to solve.
∘ easy to verify.	∘ easy to verify.
∘ time complexity = exponential time.	∘ time complexity = polynomial time.
∘ Intractable pblms.	∘ Tractable pblms

A problem is called tractable if & only if (iff) there is an efficient algorithm that solves it in polynomial time. That is; it can be solved in (n^k) time where k is a constant.

eg: $O(n)$, $O(n^2)$, $O(n^3)$, ... etc where n is the size of input.

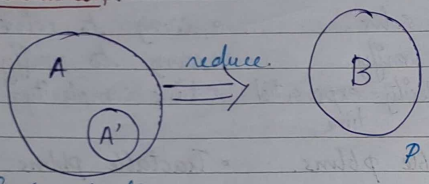
Intractable pblms means there is no efficient algorithm that solves it in polynomial time

eg: $O(2^n)$, $O(3^n)$, $O(n^n)$ (n!) etc.

eg: Travelling Sales man pblm, Knap Sack pblms, Decision pblms that can be solved by guess etc.

6/7/24/Saturday.

Reduction:



NP class problem

P class problem

A'- ~~so~~ subset of A which is also NOT a P class problem.
- NP-H problem.

A - NP-C problem

NP-C: both NP-H and NP problem. Such problems are called NP-C.

Travelling Sales ~~Person~~ Man Problem is NP-C.

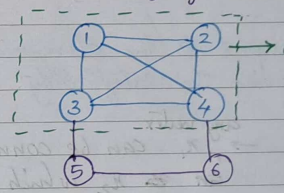
~~Quick~~ Clique Decision Problem: (CDP)

→ Decision Problem - Yes/No problems
- NP-C

→ Optimization - NP-H

In a graph, if subgraph of that graph is a complete graph then that complete graph is a clique. The no: of vertices in the complete graph part is called size of the clique.

eg: K=4 → size of the clique



To prove a problem is NP-C:
Step 1) prove it is NP
Step 2) prove it is NP-H
if ~~2~~ both are proven then it is NP-C.

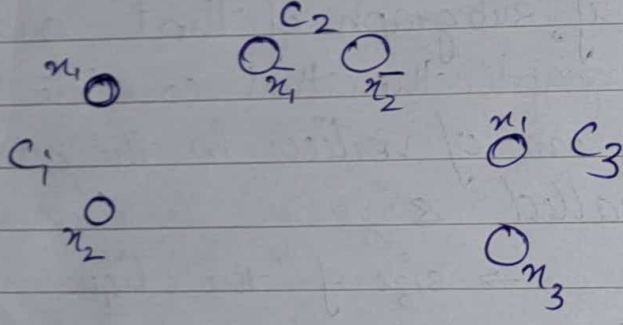
Q. Prove that CDP is NP-C and NP-H?

A. Satisfiability Rule:

1. $F = \underbrace{(x_1 \vee x_2)}_{C_1} \wedge \underbrace{(\bar{x}_1 \vee \bar{x}_2)}_{C_2} \wedge \underbrace{(x_1 \vee x_3)}_{C_3}$

$x_1, x_2, x_3 \Rightarrow$ boolean variables.

Now we represent these clauses (C_1, C_2, C_3) as vertices. Now represent the boolean variables as vertices



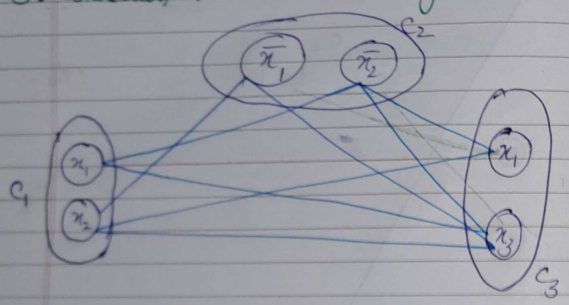
Now with respect to the above vertices, we draw a graph following the satisfiability.

Rule. $x_1 \vee x_2$ → any vertex x_1 can be connected to ~~or~~ x_2 which is in same clause.

In CDP we connect only to diff: clauses.

Rules:

1. Connection b/w same vertices → not possible
2. Connections within same clause → not possible.
3. Connection b/w same vertices but negations → not possible



$(x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_1 \vee x_3)$

Now identify the cliques.

K=3. boolean

So take 3 variables

x_1	x_2	x_3
0	0	0

Apply in $F = (x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_1 \vee x_3)$

= 0

= False.

So this is false.

Coin Change using DP:

Using DP, find the diff: no: of ways with which you get the change/weight using the coins denominations.

eg: coins = {2, 3, 5, 10}

W = 15

↳

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
3	1	0	1	1	1	1	2	1	2	2	2	2	3	2	3	3
5	1	0	1	1	1	2	2	2	3	3	4	4	5	5	6	7
10	1	0	1	1	1	2	2	2	3	3	5	4	6	6	7	9

This is the no: of combinations possible with these coins

Steps: 1. First column = 1.

2. Before 3rd column copy values from first row.

3. cell[3,3] = [previous row value] + [3-3][row][3-3]
= 0 + cell[0,row][0] = 0 + 1 = 1

4. cell[3,4] = 1 + 0 = 1

Continue...