

Forward and Inverse Kinematics

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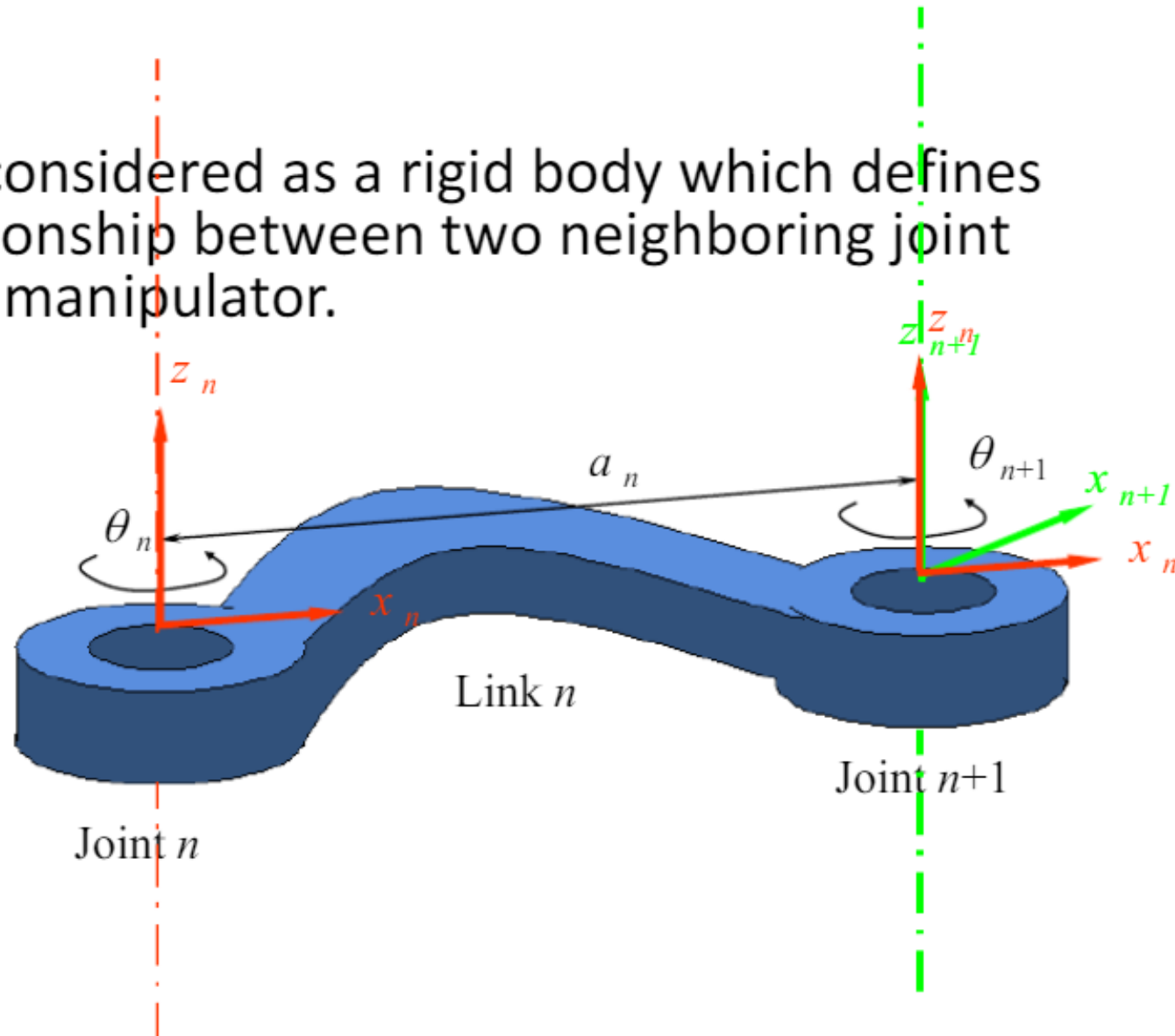
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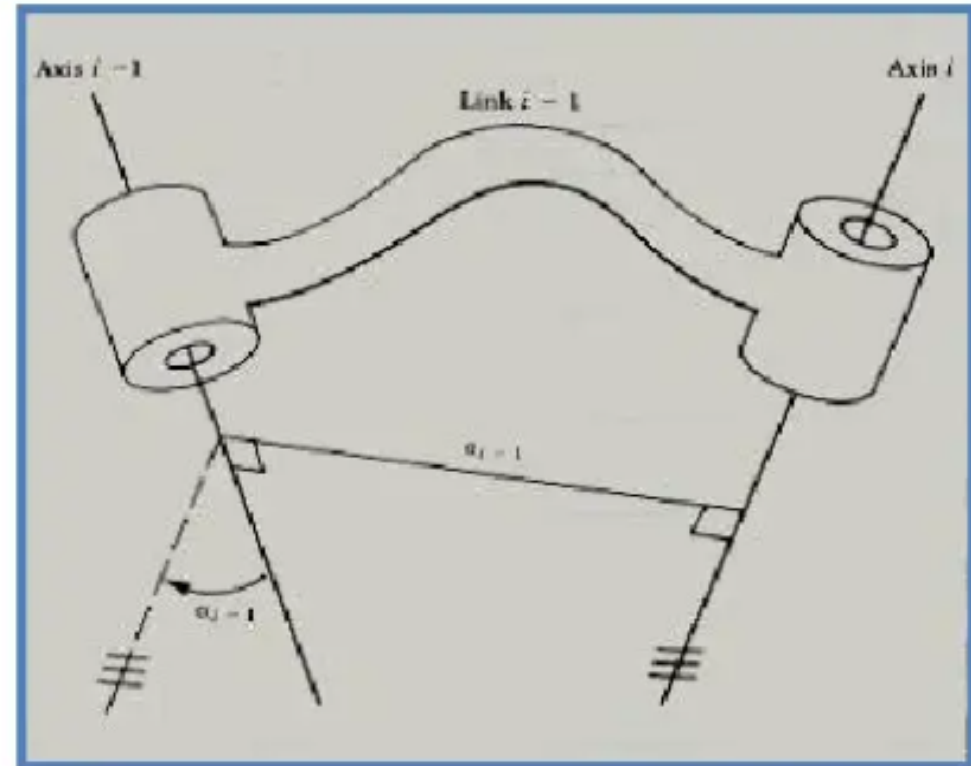
Link

- A link is considered as a rigid body which defines the relationship between two neighboring joint axes of a manipulator.



The Kinematics Function of a Link

- ◆ The kinematics function of a link is to maintain a fixed relationship between the two joint axes it supports.
- ◆ This relationship can be described with two parameters: the link length a , the link twist α



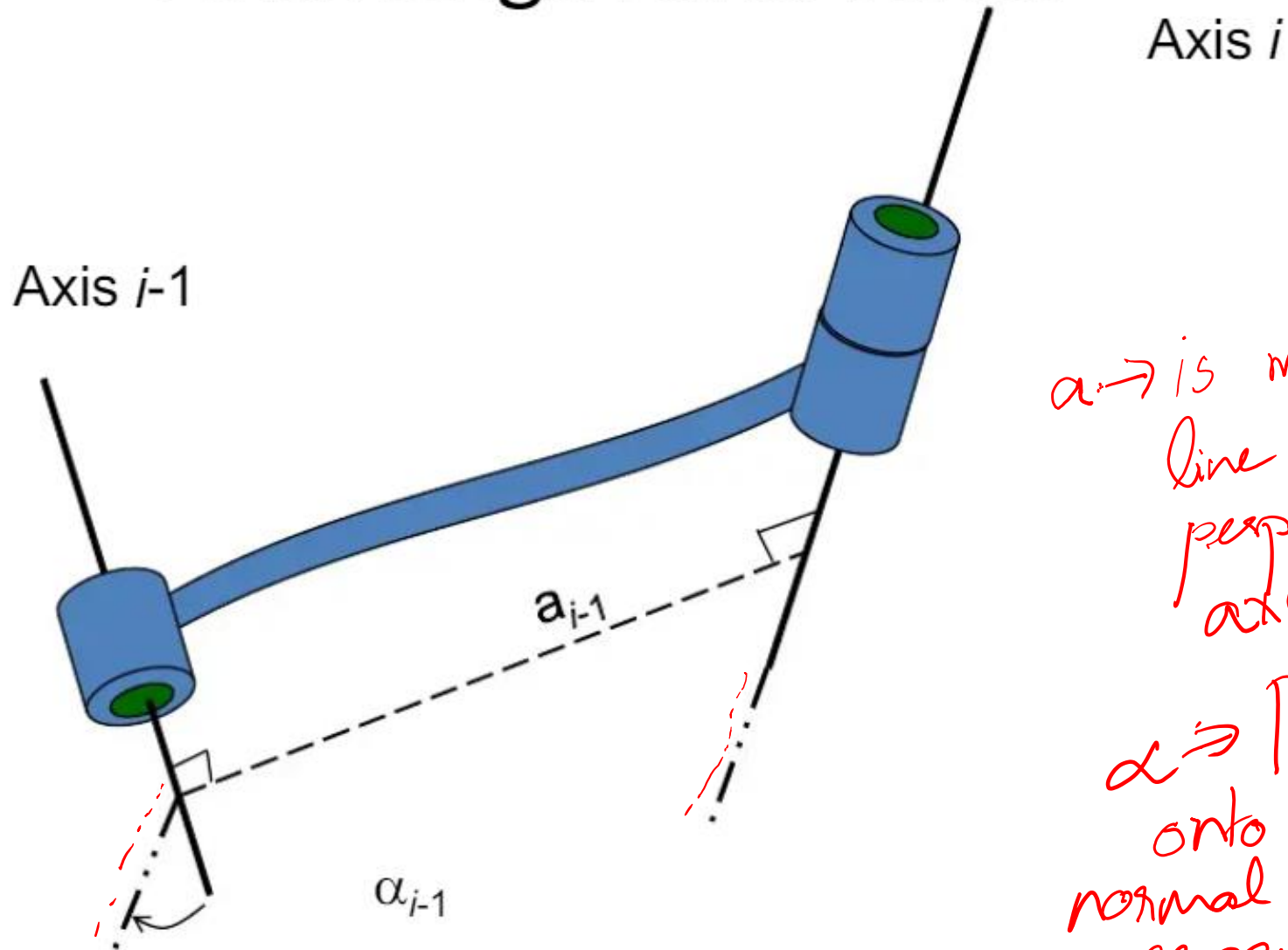
Link Length

- ◆ Is measured along a line which is mutually perpendicular to both axes.
- ◆ The mutually perpendicular always exists and is unique except when both axes are parallel.

Link twist

- ◆ Project both axes $i-1$ and i onto *the plane whose normal is the mutually perpendicular line*, and measure the angle between them
- ◆ Right-hand sense

Link Length and Twist



$a \rightarrow$ is measured along the line mutually perpendicular to both axes.

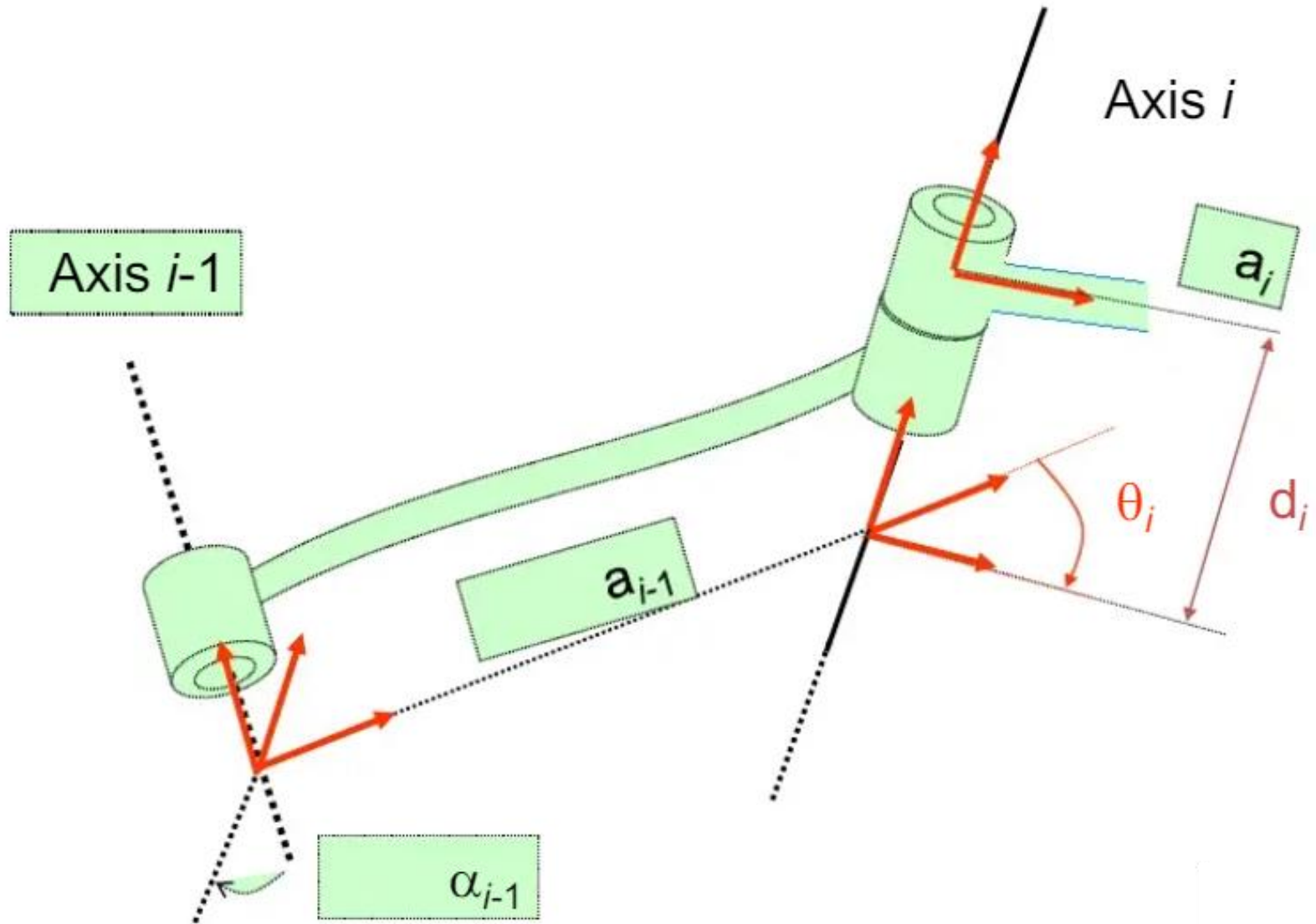
$\alpha \rightarrow$ Project both axes onto the plane which normal is mutually perpendicular line and measure angle.

Joint Parameters

A joint axis is established at the connection of two links. This joint will have two normals connected to it one for each of the links.

- The relative position of two links is called link offset d_n which is the distance between the links (the displacement, along the joint axes between the links).
- The joint angle θ_n between the normals is measured in a plane normal to the joint axis.

Link and Joint Parameters



Link and Joint Parameters

4 parameters are associated with each link. You can align the two axis using these parameters.

- Link parameters:

a_n the length of the link.

α_n the twist angle between the joint axes.

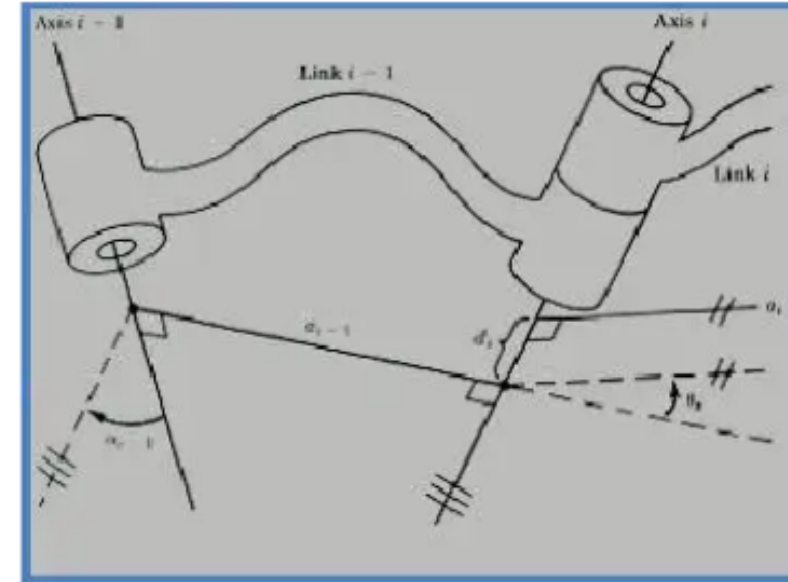
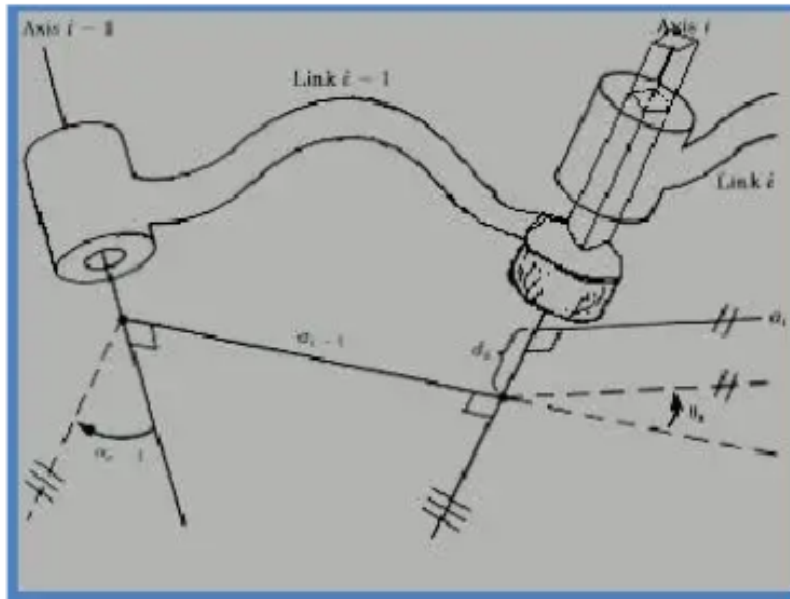
- Joint parameters:

θ_n the angle between the links.

d_n the distance between the links

Link Connection Description:

For Revolute Joints: a , α , and d are all fixed, then " θ_i " is the Joint Variable.



For Prismatic Joints: a , α , and θ are all fixed, then " d_i " is the Joint Variable.

These four parameters: (Link-Length a_{i-1}), (Link-Twist α_{i-1}), (Link-Offset d_i), (Joint-Angle θ_i) are known as the Denavit-Hartenberg Link Parameters.

Links Numbering Convention

Base of the arm: Link-0

1st moving link: Link-1

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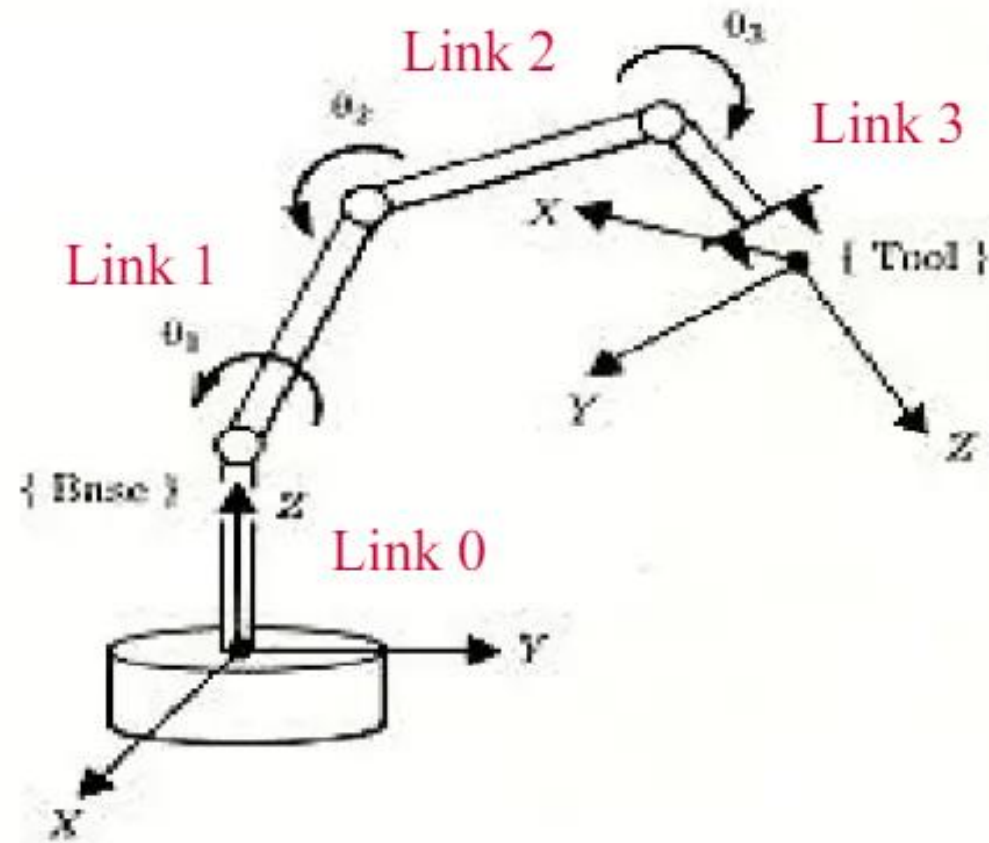
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Last moving link Link-n



A 3-DOF Manipulator Arm

First and Last Links in the Chain

- $a_0 = a_{n+1}$
- $\alpha_0 = \alpha_{n+1}$
- *If joint 1 is revolute: $d_0 = 0$ and θ_1 is arbitrary*
- *If joint 1 is prismatic: d_0 is arbitrary and $\theta_1 = 0$*

We are interested in **two** kinematics topics

Forward Kinematics (angles to position)

What you are given: The length of each link
The angle of each joint

What you can find: The position of any point
(i.e. it's (x, y, z) coordinates)

*position of end
effector/gripper*

Inverse Kinematics (position to angles)

What you are given: The length of each link
The position of some point on the robot

What you can find: The angles of each joint needed to obtain
that position

*position of end
effector/
gripper*

Animation of Forward and Inverse Kinematics

- <https://www.youtube.com/watch?v=rlurtdGVQCK>