Quaternion

Dr. Divya Udayan J, Ph.D.(Konkuk University, S.Korea)

Department of CSE
Amrita School of Engineering, Amritapuri Campus,
Amrita Vishwa Vidyapeetham
Email: divyaudayanj@am.amrita.edu
Mobile: 9550797705

Quaternion

- Way to represent complex numbers in higher dimensions
- Using 4D notation – named as 'Quaternion'
- Complex number is a combination of real and imaginary parts

  a+ib
- 3D equivalent is a+ib+jc where $i^2 = j^2 = -1$
- When forming product of two such numbers, cannot resolve ij and ji
- Extension -> a+ib+jc+kd where $i^2 = j^2 = k^2 = -1$
- When forming product of such numbers, again created problems in solving ij, jk,ki and their mirrors ji,kj,ik

Hamilton rules

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k, \quad jk = i, \quad ki = j$$

$$ji = -k, \quad kj = -i, \quad ik = -j$$

Extra notes

7.8.2 Quaternions

As mentioned earlier, quaternions were invented by Sir William Rowan Hamilton in the mid-nineteenth century. Sir William was looking for a way to represent complex numbers in higher dimensions, and it took 15 years of toil before he stumbled upon the idea of using a 4D notation – hence the name 'quaternion'.

Knowing that a complex number is the combination of a real and imaginary quantity: $a+ib$, it is tempting to assume that its 3D equivalent is $a+ib+jc$ where $i^2=j^2=-1$. Unfortunately, when Hamilton formed the product of two such objects, he could not resolve the dyads ij and ji, and went on to explore an extension $a+ib+jc+kd$ where $i^2=j^2=k^2=-1$. This too, presented problems with the dyads ij, jk, ki and their mirrors ji, kj and ik. But after many years of thought Hamilton stumbled across the rules:

$$
\begin{aligned}
&i^2=j^2=k^2=ijk=-1\\
&ij=k, \quad jk=i, \quad ki=j\\
&ji=-k, \quad kj=-i, \quad ik=-j.
\end{aligned}
$$

Problems

- Refer note book