# 機械学習特論

~理論とアルゴリズム~

(Basics in probabilities)

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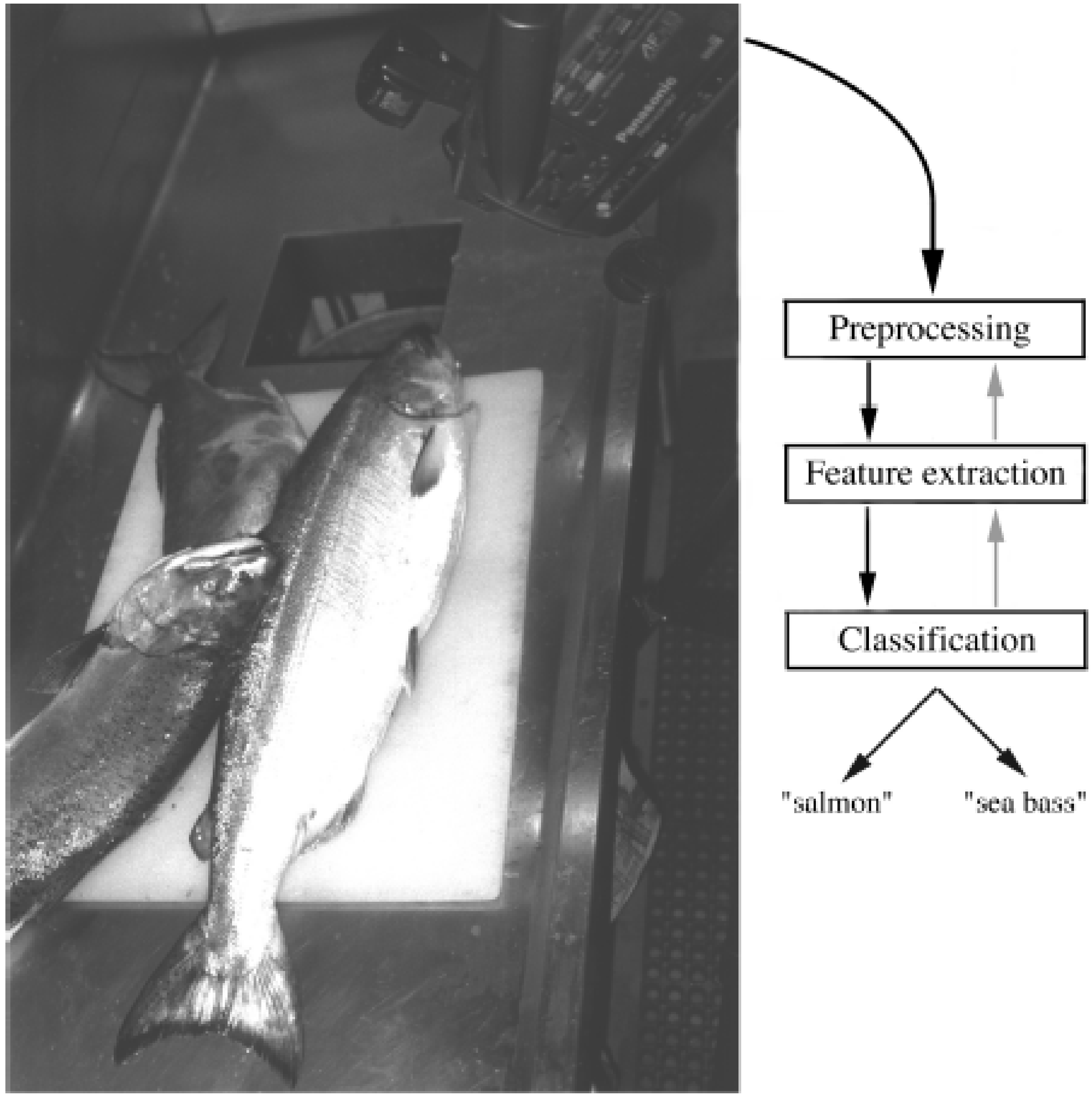
# Goal of pattern recognition: Parametric (rule-based) approach

# Process of Learning

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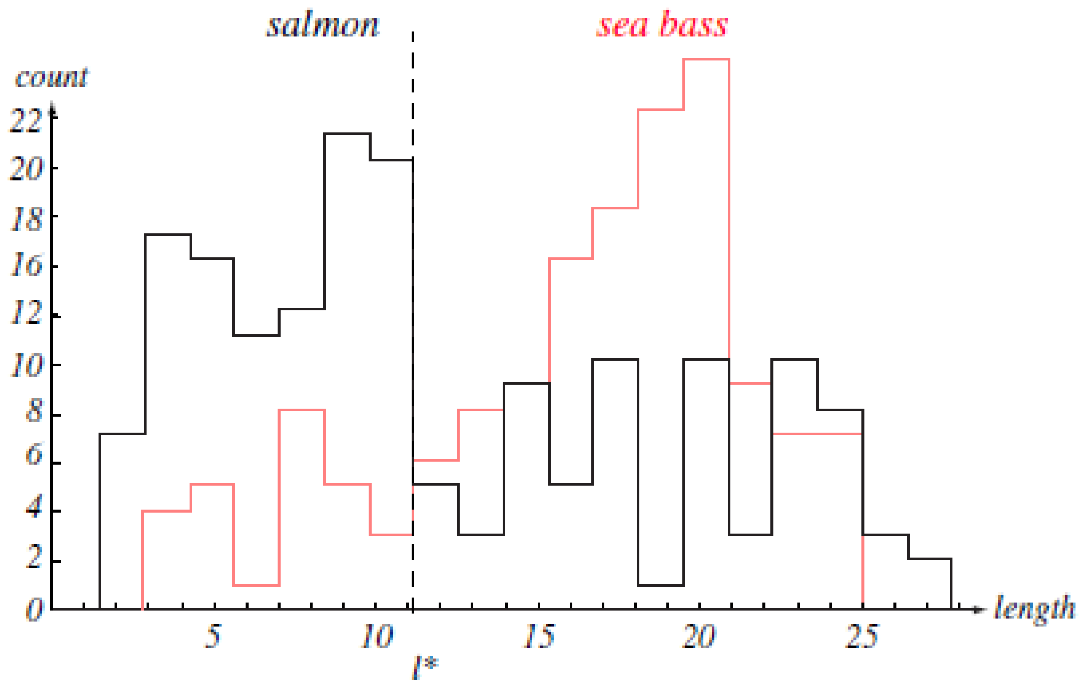
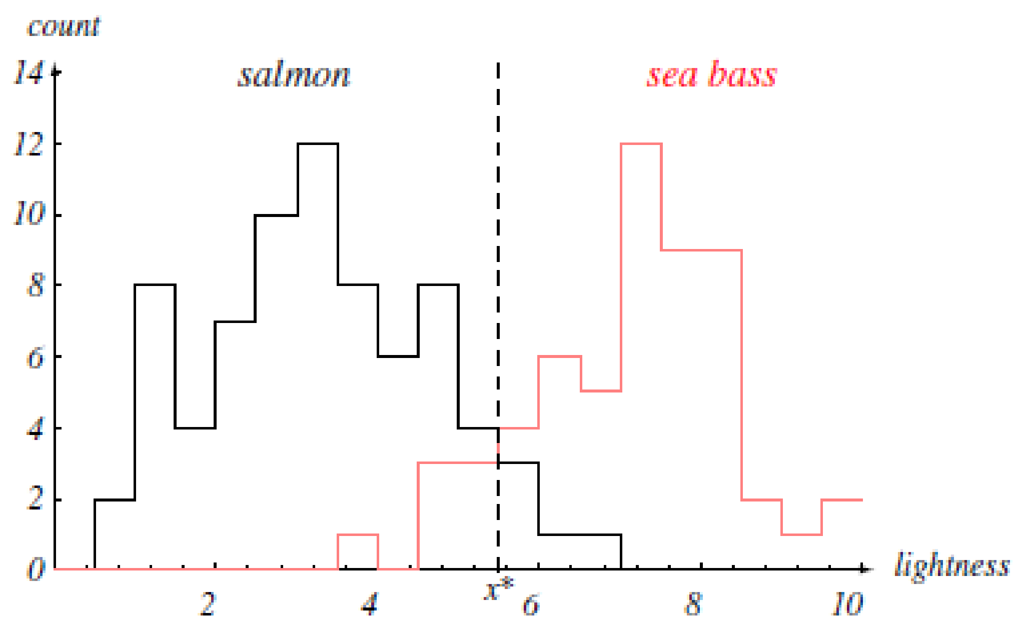
# Classification by the fish length



# Classification by the brightness

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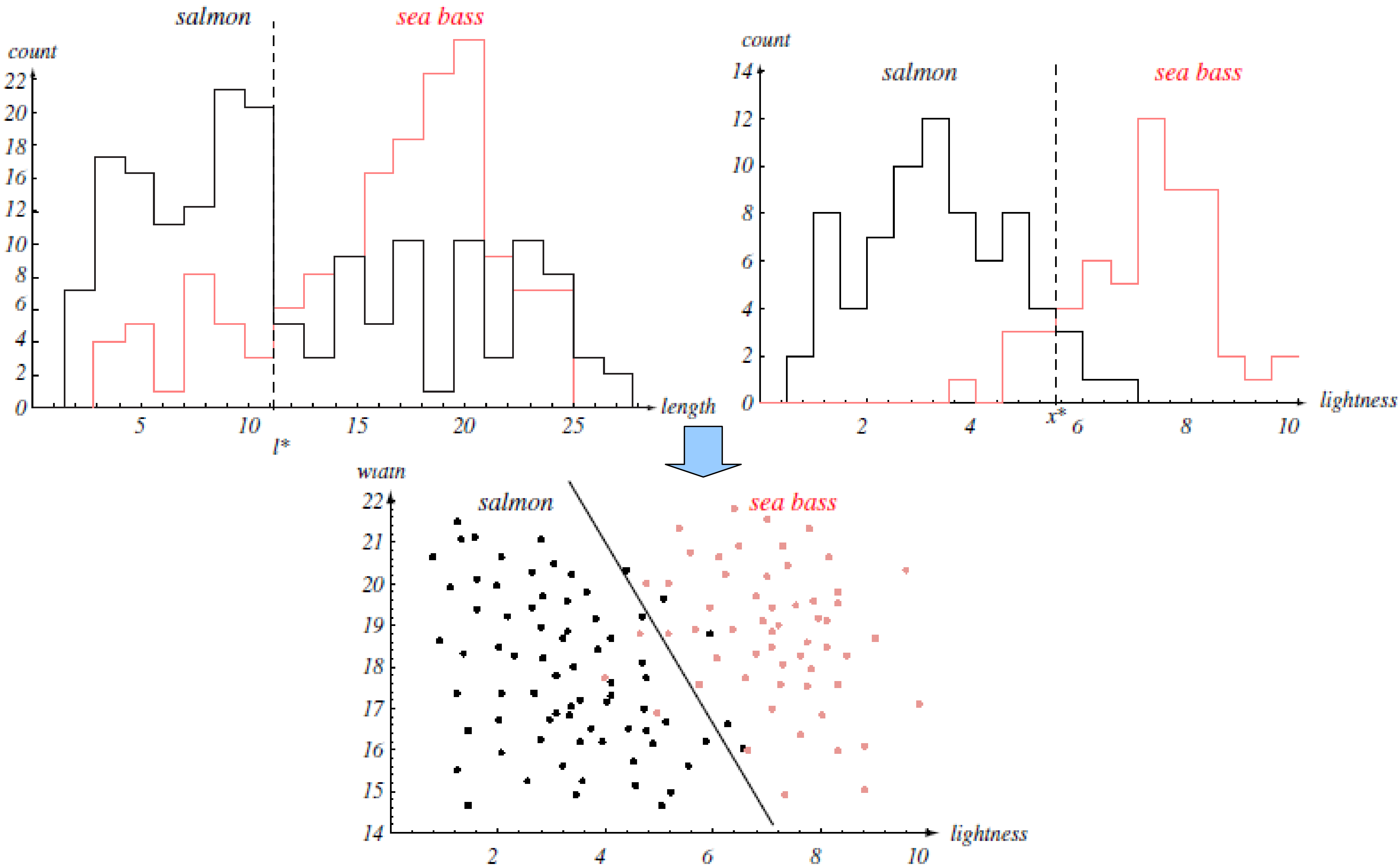
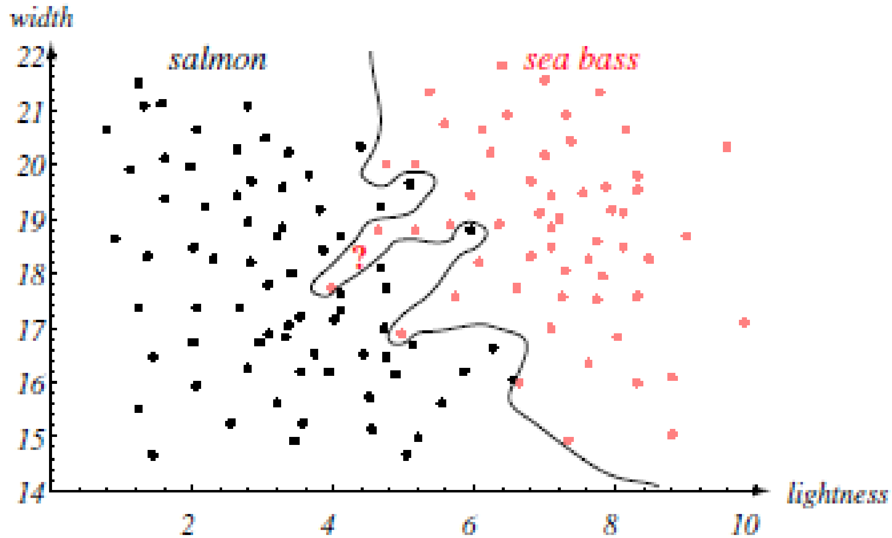
# Classification by the combined rule.



# Can we just increase the number of rules (features) ?

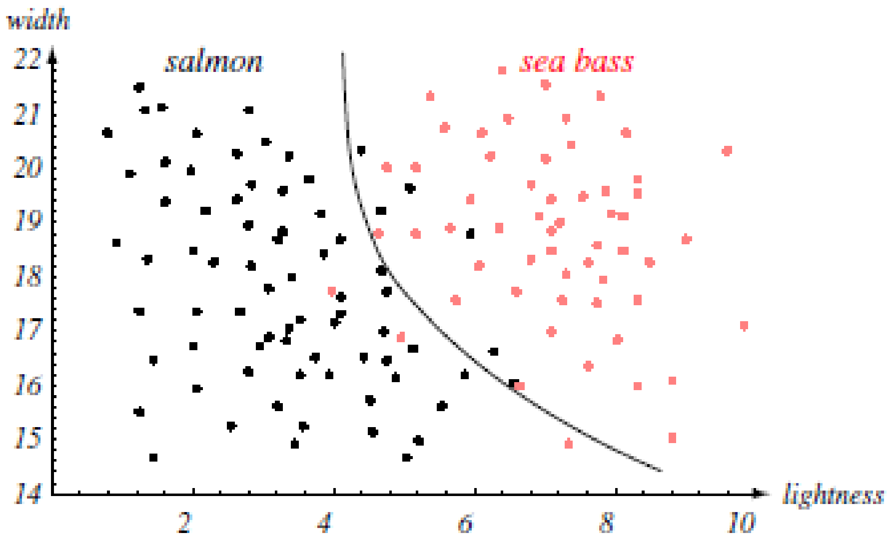
# No

- Too many rules complicate the classification boundary (below), which can be prone to a newly available data point. (overfitting)



# Good rule

- should be simple and robust to a newly available data point.



# Learning rules

Consider extracting rules for discriminating salmon and sea bass, such as size, color, etc) out of limited amount of data.

Our data can be represented in the following way. x represents rules (features), y represents salmon(1) or sea bass(0).

$$\{\{\boldsymbol{x}_{\mathbf{1}}, y_1\}, \{\boldsymbol{x}_{\mathbf{2}}, y_2\} \ldots, \{\boldsymbol{x}_{\boldsymbol{n}}, y_n\}\}$$

# Learning rules continued.

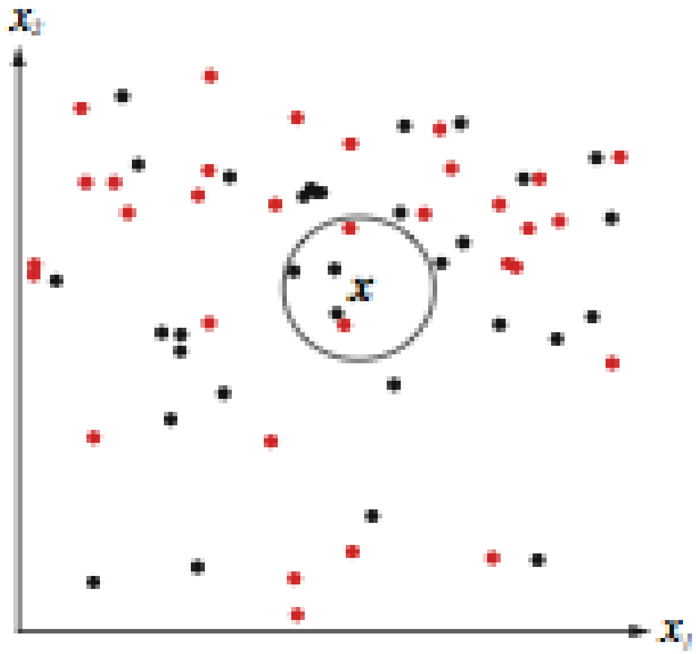
We will see different formulation and motivation for this objective. Examples covered in this lecture series are;

1) LDA
2) Logistic Regression
3) Neural Networks
4) (SVM)

We will see them later.

# Non-parametric (sample-based) approach

# k-nearest neighbor




- Procedure
  - Choose k training data points closest to the test point, then perform majority vote.
  - k should be an odd number such as 1, 3, 5, … etc.

# Review on basic probabilities

# random variable (確率変数)

Response variable y is a discrete random variable, and satisfies the following condition.

$$\sum_{i=1}^{k} p(y_i) = 1, \quad p(y_i) \geq 0$$

Explanatory variable (feature) x is generally a continuous variable, and satisfies the following condition.

$$\int p(x)\,dx = 1, \quad p(x) \geq 0$$

# Joint and conditional probability

Joint probability (同時確率) between x and y

$$p(x, y)$$

Conditional probability(条件付き確率) of y given x

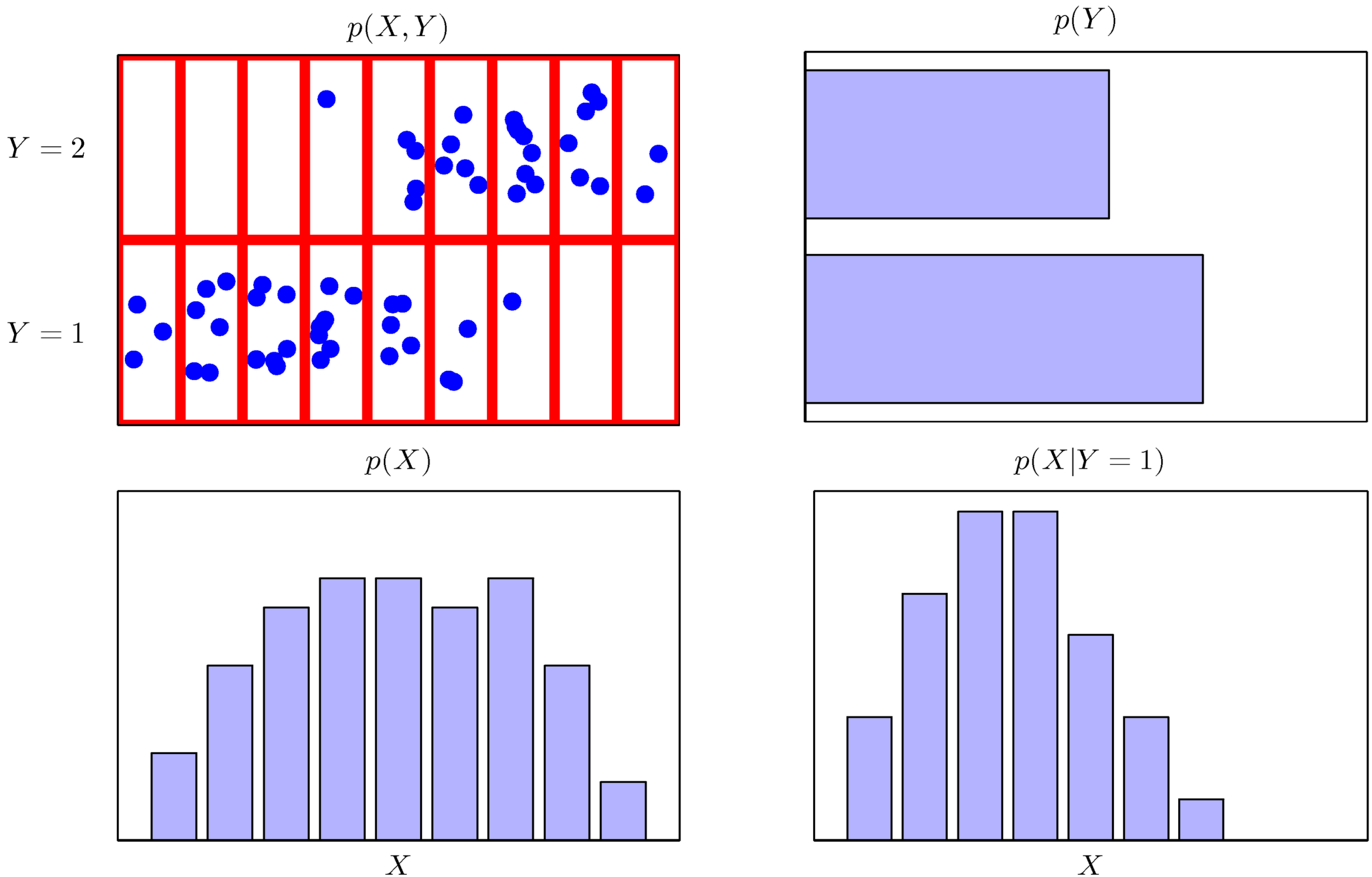
$$p(y|x)$$

# The Rules of Probability

**Sum Rule** $$p(X) = \sum_Y p(X, Y)$$

**Product Rule** $$p(X, Y) = p(Y|X)\, p(X)$$

# Example: joint, marginal and conditional probability



# Bayes' theorem

$$\boxed{p(Y|X) = \frac{p(X|Y)\,p(Y)}{p(X)} = \frac{p(X|Y)\,p(Y)}{\sum_Y p(X|Y)\,p(Y)}}$$

p(Y|X): posterior probability (事後確率)
p(X|Y): likelihood (尤度)
p(Y): prior probability ( 事前確率 )
p(X): データの確率

The denominator is often skipped, and described as

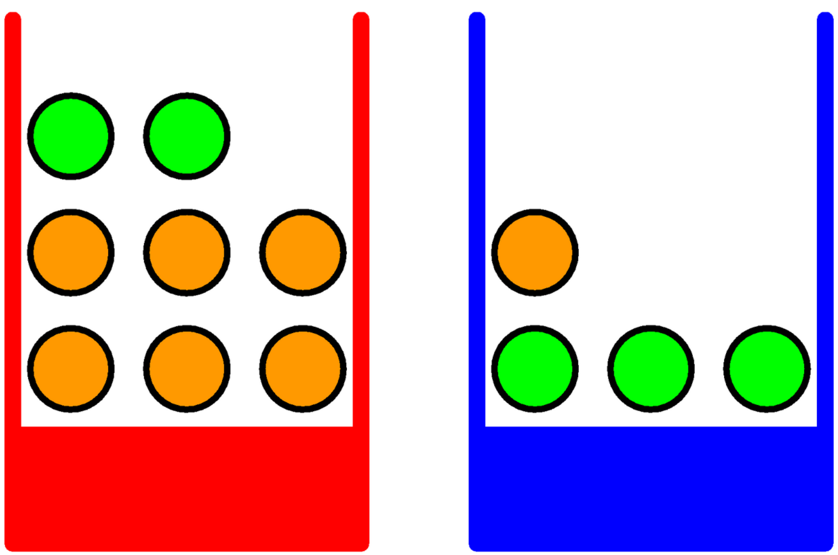
$$\boxed{p(Y|X) \approx p(X|Y)\,p(Y)}$$

# Example Bayes' theorem

Red box: 6 oranges and 2 apples

Blue box: 1 orange and 3 apples

Probability to choose red box: 40%, blue box: 60%



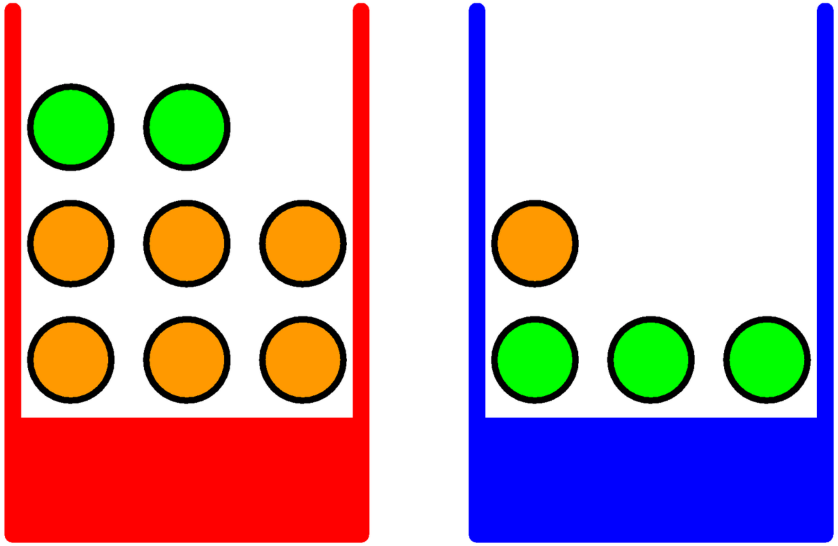
# Example: Bayes' theorem

$p(B=r)=4/10$ $p(B=b)=6/10$

$p(F=a|B=r)=1/4$ $p(F=o|B=r)=3/4$

$p(F=a|B=b)=3/4$ $p(F=o|B=b)=1/4$

What's the probability of $B=r$ when $F=o$ ?



## What's the probability of B=r when F=o ?

p(B=r)=4/10 p(B=b)=6/10

p(F=a|B=r)=1/4 p(F=o|B=r)=3/4

p(F=a|B=b)=3/4 p(F=o|B=b)=1/4

$$
\begin{aligned}
&p(B=r|F=o)\\
&=\frac{p(F=o|B=r)\,p(B=r)}{p(F=o)}\\
&=\frac{p(F=o|B=r)\,p(B=r)}{p(F=o|B=r)\,p(B=r)+p(F=o|B=b)\,p(B=b)}\\
&=\frac{3/4\times 4/10}{3/4\times 4/10+1/4\times 6/10}=\frac{2}{3}
\end{aligned}
$$

# Other statistics

- 平均(mean) $$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$$
- 分散(variance) $$var(\boldsymbol{x})=\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2$$
- 標準偏差 $$sd(\boldsymbol{x})=\sqrt{var(\boldsymbol{x})}$$

(standard deviation)

- 共分散 $$cov(\boldsymbol{x},\boldsymbol{y})=\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)$$

(covariance)

- 相関 $$cor(\boldsymbol{x},\boldsymbol{y})=\frac{cov(\boldsymbol{x},\boldsymbol{y})}{sd(\boldsymbol{x})\,sd(\boldsymbol{y})}=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2}}$$

(correlation)

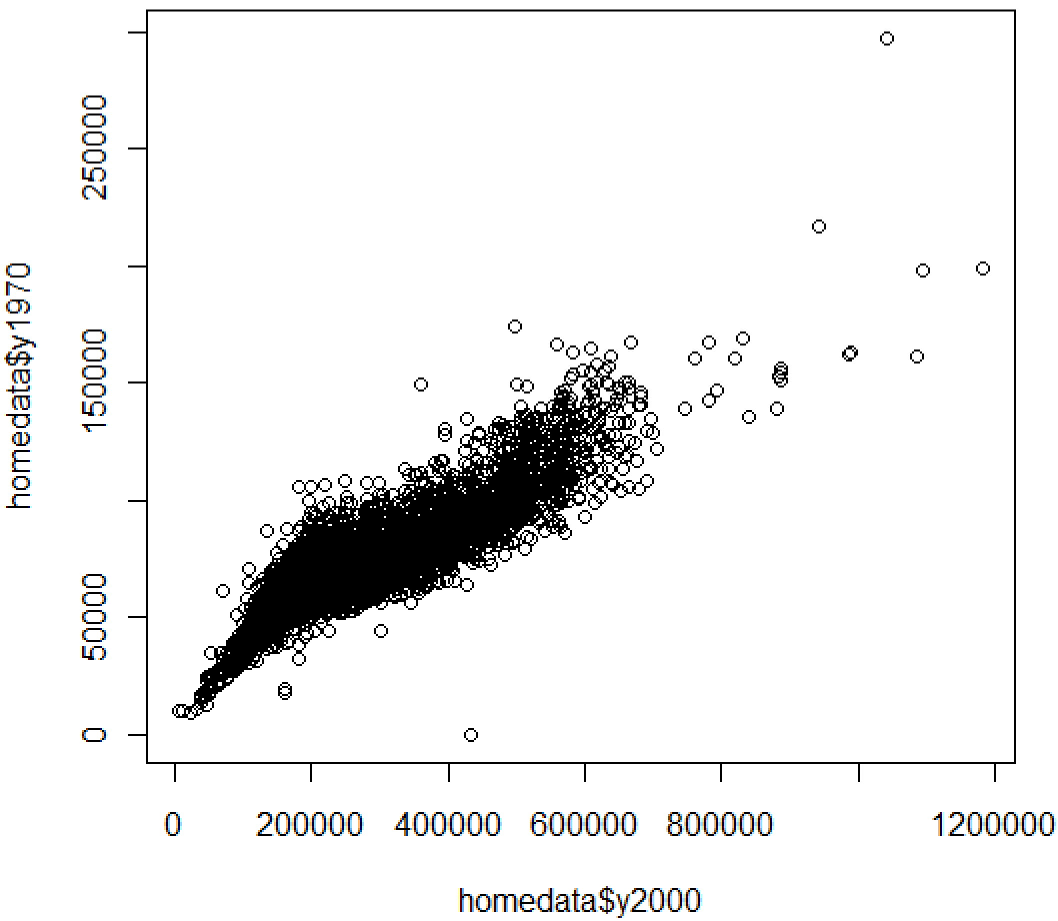
- 期待値 $$E(\boldsymbol{x})=\frac{1}{n}\sum_{i=1}^{n} x_i\, p\left(x_i\right)$$

(expectation)

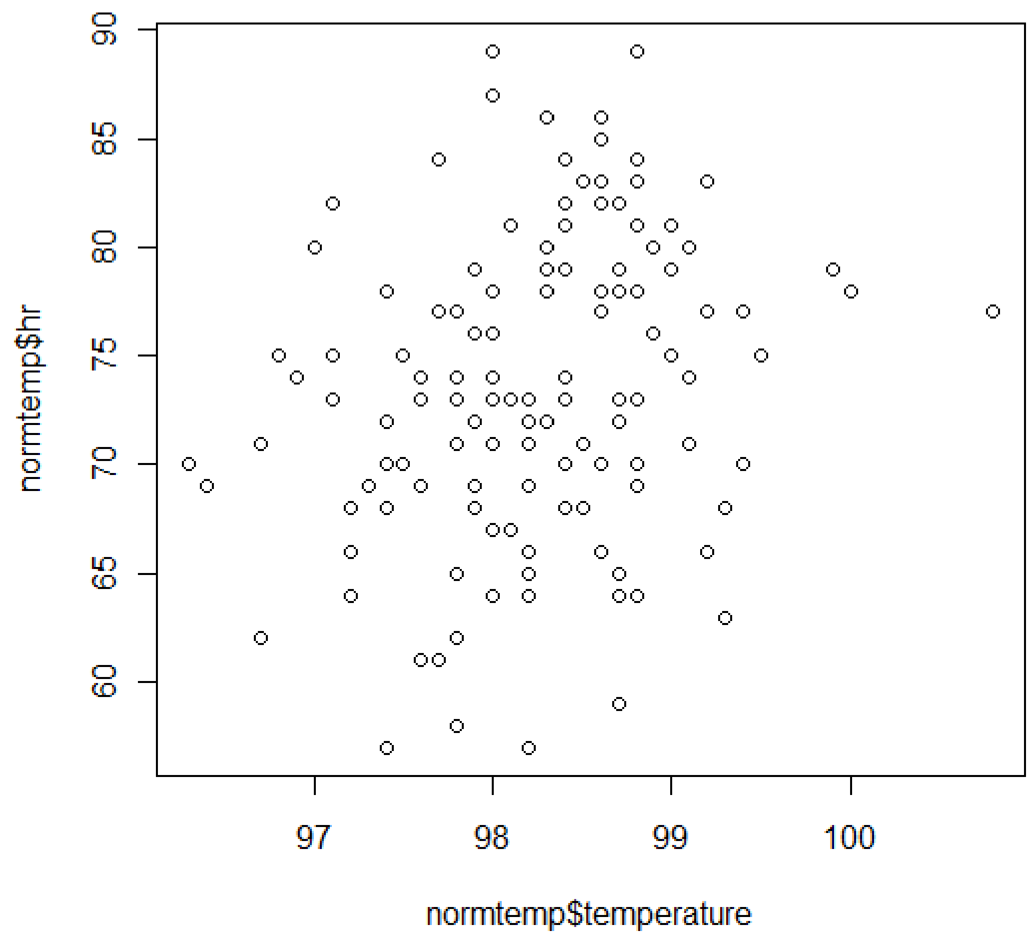
# 共分散(covariance)と相関(correlation)

correlation(相関係数) is a normalized covariance
-1 <= cor <= 1, -∞ < cov < ∞

House price 2000 vs 1970
Cov = 2610880277
Cor = 0.8926155 ( high )




temp vs heart rate
Cov = 1.313381
Cor = 0.2536564 ( low )



# Ex. 1

Suppose that we have three colourd boxes *r* (red), *b*(blue) and *g* (green). Box *r* contains 3 apples, 4 oranges and 3 limes, box *b* contains 1 apple, 1 orange and 0 limes, and box *g* contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $p(r)=0.2$, $p(b)=0.2$, $p(g)=0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any items in the box), then what is the probability of selecting an apple ? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box ?

# Ex. 2

A rare genetic disease is found. Only one in a million people has the causing gene. The genetic diagnosis for this gene is very accurate, and has 100% sensitive (no false negatives) and 99.99% specific (only 0.01% false positives). Based on Bayes' theorem, decide whether if you would take the diagnosis or not, and explain the reason.

False positive: an error telling healthy person as sick

False negative: an error telling sick person as healthy

# Monty Hall Problem