We have:

$$\boxed{ax^3+bx^2+cx+d=0}$$

We want to get rid of the "x^2" term.

Develop: Let $\boxed{x=y+k}$

$$\Rightarrow x^3=(y+k)^3=y^3+3y^2.k+3k^2.y+k3$$

$$x^2=(y+k)^2=y^2+2yk+k^2$$

$$a\cdot[y^3+3y^2k+3k^2.y+k^3]+b\cdot[y^2+2yk+k^2]$$

$$+c\cdot[y+k]+d=0$$

$$\Rightarrow$$

$$a\cdot y^3+\underline{3ak\cdot y^2}+3k^2.a\cdot y+k^3.a+\underline{b\cdot y^2}+2bk\cdot y$$

$$+b\cdot k^2+c\cdot y+c\cdot k+d=0$$

$$[3ak+b]\cdot y^2=0$$

$$\Rightarrow 3ak+b=0$$

$$3ak=-b$$

$$\boxed{k=\frac{-b}{3a}}$$

$$\Rightarrow$$

$$a\cdot y^3+3\cdot\left(\frac{-b}{3a}\right)\cdot\left(\frac{-b}{3a}\right).a\cdot y+\left(\frac{-b}{3a}\right)^3\cdot a+2b\cdot\left(\frac{-b}{3a}\right)\cdot y$$

$$+b\cdot\left(-\frac{b}{3a}\right)^2+c\cdot y+c\cdot\left(\frac{-b}{3a}\right)+d=0$$

$$\Rightarrow$$

$$a\cdot y^3+\left(\frac{b^2}{3a}\right)\cdot y-\frac{b^3}{27a^2}-\left(\frac{2b^2}{3a}\right)\cdot y+\frac{b^3}{9a^2}+c\cdot y$$

$$-\left(\frac{b\cdot c}{3a}\right)+d=0$$

$$\Rightarrow$$

$$a\cdot y^3+\left[-\left(\frac{b^2}{3a}\right)+c\right]\cdot y+\left[\left(\frac{2b^3}{27a^2}\right)-\left(\frac{b\cdot c}{3a}\right)+d\right]=0$$

$$y^3+\left[-\left(\frac{b^2}{3a^2}\right)+c\right]\cdot y\cdot\left[\left(\frac{2b^3}{27a^3}\right)-\left(\frac{b\cdot c}{3a^2}\right)+\left(\frac{d}{a}\right)\right]=0$$

$$\boxed{p=-\left(\frac{b^2}{3a^2}\right)+c} \quad \boxed{q=\left(\frac{2b^3}{27a^3}\right)-\left(\frac{b\cdot c}{3a^2}\right)+\left(\frac{d}{a}\right)}$$

$$\Rightarrow \boxed{y^3+p\cdot y+q=0}$$