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Hypothesis Test

Consider the observations $X_1, ..., X_n \overset{\text{iid}}{\sim} D_1(\mu_1, \sigma_1)$, $Y_1, ..., Y_n \overset{\text{iid}}{\sim} D_2(\mu_2, \sigma_2)$.

With X_i independent of $Y_j \quad \forall \ (i \in \{1, ..., n\}, j \in \{1, ..., m\})$.

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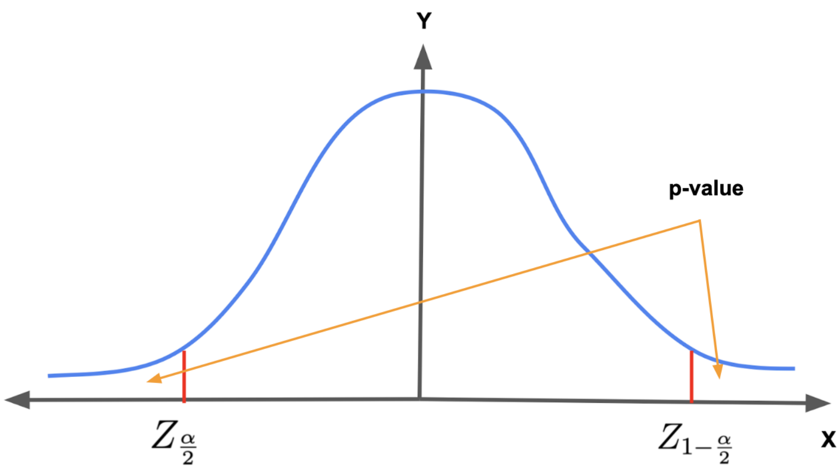
i. Equality:

$$H_0 : \mu_1 = \mu_2 \Leftrightarrow \mu_1 - \mu_2 = 0 \qquad \text{vs} \qquad H_a : \mu_1 \neq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \neq 0$$



$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If $T_{|H_0} > Z_{1-\frac{\alpha}{2}}$ **(critical value)** or $T_{|H_0} < Z_{\frac{\alpha}{2}}$ **(critical value)** then we **can reject** null hypothesis.

Else we **can't reject** null hypothesis.

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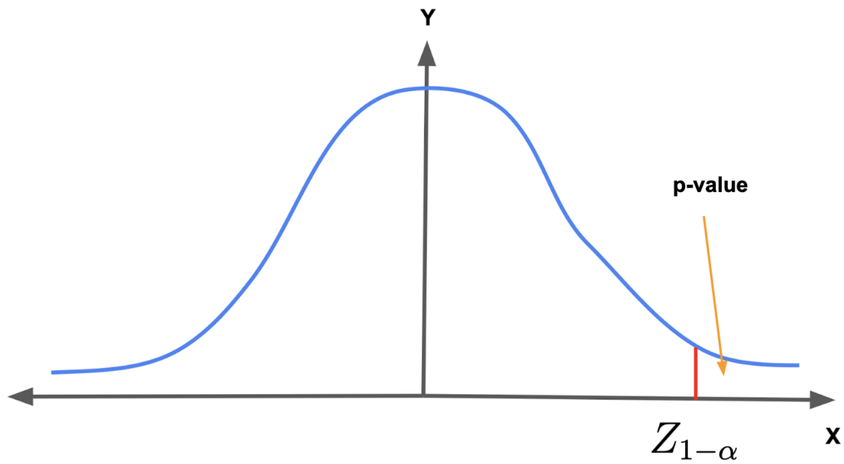
ii. Greater:

$$H_0 : \mu_1 > \mu_2 \Leftrightarrow \mu_1 - \mu_2 > 0 \qquad \text{vs} \qquad H_a : \mu_1 \leq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \leq 0$$



$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If $T_{|H_0} \leq Z_{1-\alpha}$ **(critical value)** then we **can reject** null hypothesis.

Else we **can't reject** null hypothesis.

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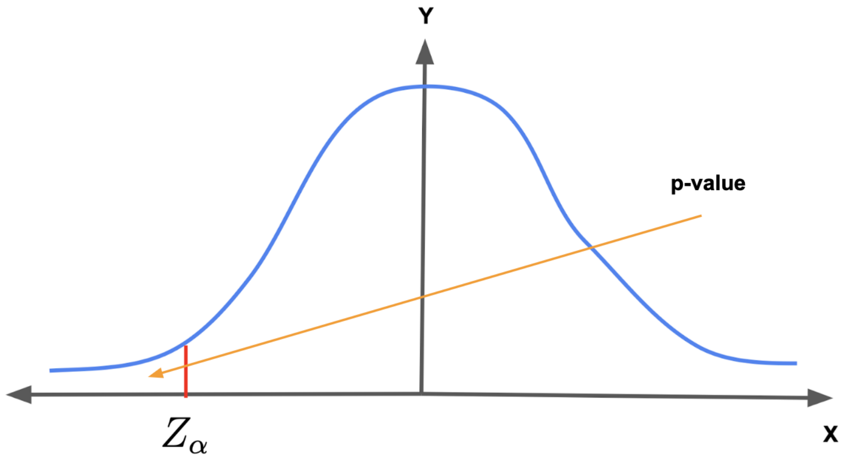
iii. Lower:

$$H_0 : \mu_1 < \mu_2 \Leftrightarrow \mu_1 - \mu_2 < 0 \qquad \text{vs} \qquad H_a : \mu_1 \geq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \geq 0$$



$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If $T_{|H_0} \geq Z_\alpha$ **(critical value)** then we **can reject** null hypothesis.

Else we **can't reject** null hypothesis.

Demostrations

$$\boxed{T_1 = \frac{1}{n} \cdot \sum_{i=1}^{n} X_i}$$

$$\boxed{T_2 = \frac{1}{m} \cdot \sum_{j=1}^{n} Y_j}$$

$$\boxed{Z = T_1 - T_2}$$

$$\mathbb{E}\left(Z\right) = \mathbb{E}\left(T_1 - T_2\right) = \mathbb{E}\left(T_1\right) - \mathbb{E}\left(T_2\right)$$

$$= \mathbb{E}\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right) - \mathbb{E}\left(\frac{1}{m} \cdot \sum_{j=1}^{m} Y_j\right) = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbb{E}\left(X_i\right) - \frac{1}{m} \cdot \sum_{j=1}^{m} \mathbb{E}\left(Y_j\right)$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \mu_1 - \frac{1}{m} \cdot \sum_{j=1}^{m} \mu_2 = \frac{1}{n} \cdot \left[n \cdot \mu_1\right] - \frac{1}{m} \cdot \left[m \cdot \mu_2\right]$$

$$= \mu_1 - \mu_2$$

$\Rightarrow$

$$\boxed{\mathbb{E}\left(Z\right) = \mu_1 - \mu_2}$$

$$\mathbb{V}\left(Z\right) = \mathbb{V}\left(T_1 - T_2\right) \overset{\text{ind}}{=} (1)^2 \cdot \mathbb{V}\left(T_1\right) + (-1)^2 \cdot \mathbb{V}\left(T_2\right) = \mathbb{V}\left(T_1\right) + \mathbb{V}\left(T_2\right)$$

$$= \mathbb{V}\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right) + \mathbb{V}\left(\frac{1}{m} \cdot \sum_{j=1}^{m} Y_j\right) \overset{\text{ind}}{=} \frac{1}{n^2} \cdot \sum_{i=1}^{n} \mathbb{V}\left(X_i\right) + \frac{1}{m^2} \cdot \sum_{j=1}^{m} \mathbb{V}\left(Y_j\right)$$

$$= \frac{1}{n^2} \cdot \sum_{i=1}^{n} (\sigma_1)^2 + \frac{1}{m^2} \cdot \sum_{j=1}^{m} (\sigma_2)^2 = \frac{1}{n^2} \cdot \left[n \cdot (\sigma_1)^2\right] + \frac{1}{m^2} \cdot \left[m \cdot (\sigma_2)^2\right]$$

$$= \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}$$

$\Rightarrow$

$$\boxed{\mathbb{V}\left(Z\right) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}$$
