As we see previously:

$$\boxed{z^3 = k_1} \qquad \boxed{z^3 = k_2}$$

$$\Rightarrow \boxed{\begin{array}{ll} z_1 = \sqrt[3]{k_1} & z_4 = \sqrt[3]{k_2} \\ z_2 = \sqrt[3]{k_1}\cdot w_1 & z_5 = \sqrt[3]{k_2}\cdot w_1 \\ z_3 = \sqrt[3]{k_1}\cdot w_2 & z_6 = \sqrt[3]{k_2}\cdot w_2 \end{array}}$$

Now we want to prove that:

$$\boxed{\frac{1}{w_2} = w_1} \quad \boxed{\frac{1}{w_1} = w_2} \quad \boxed{\frac{1}{\sqrt[3]{k_1}} = -\left(\frac{3}{p}\right)\cdot\sqrt[3]{k_2}}$$

$$\boxed{\frac{1}{\sqrt[3]{k_2}} = -\left(\frac{3}{p}\right)\cdot\sqrt[3]{k_1}}$$

Proof:

(1) $\boxed{\frac{1}{w_2} = w_1}$

As $w_2 = \frac{-\sqrt{3}i - 1}{2}$

$$\frac{1}{w_2} = -\left(\frac{2}{\sqrt{3}i+1}\right) = -\left(\frac{2}{\sqrt{3}i+1}\right)\cdot\left(\frac{\sqrt{3}i-1}{\sqrt{3}i-1}\right)$$

$$= -\frac{2}{\left([\sqrt{3}i]^2 - 1^2\right)}\cdot\left(\sqrt{3}i-1\right) = -\frac{2}{(-3-1)}\cdot\left(\sqrt{3}i-1\right)$$

$$= \frac{1}{2}\cdot\left(\sqrt{3}i-1\right) = \frac{\sqrt{3}i-1}{2} = w_1 \quad //$$

(2) $\boxed{\frac{1}{w_1} = w_2}$

As $w_1 = \frac{\sqrt{3}i - 1}{2}$

$$\frac{1}{w_1} = \frac{2}{\sqrt{3}i-1} = \frac{2}{\left(\sqrt{3}i-1\right)}\cdot\frac{\left(\sqrt{3}i+1\right)}{\left(\sqrt{3}i+1\right)}$$

$$= \frac{2\left(\sqrt{3}i+1\right)}{[\sqrt{3}i]^2 - 1^2} = \frac{2\cdot\left(\sqrt{3}i+1\right)}{-3-1}$$

$$= -\frac{\left(\sqrt{3}i+1\right)}{2} = w_2 \quad //$$

(3) $\boxed{\frac{1}{\sqrt[3]{k_1}} = -\left(\frac{3}{p}\right)\cdot\sqrt[3]{k_2}}$

$$\frac{1}{\sqrt[3]{k_1}} = \frac{\sqrt[3]{k_2}}{\sqrt[3]{k_1}\cdot\sqrt[3]{k_2}} = \frac{\sqrt[3]{k_2}}{\sqrt[3]{k_1\cdot k_2}}$$

$$k_1\cdot k_2 = \left[\left(-\frac{q}{2}\right) + \frac{1}{2}\cdot\sqrt{q^2+\frac{4p^3}{27}}\right]\left[\left(-\frac{q}{2}\right) - \frac{1}{2}\sqrt{q^2+\frac{4p^3}{27}}\right]$$

$$= \left(-\frac{q}{2}\right)^2 - \left[\frac{1}{2}\cdot\sqrt{q^2+\frac{4p^3}{27}}\right]^2$$

$$= \frac{q^2}{4} - \frac{1}{4}\cdot\left(q^2+\frac{4p^3}{27}\right) = \frac{q^2}{4} - \frac{q^2}{4} - \frac{p^3}{27}$$

$$= \frac{-p^3}{27} \Rightarrow \boxed{\sqrt[3]{k_1\cdot k_2} = -\frac{p}{3}}$$

$$\Rightarrow \boxed{\frac{1}{\sqrt[3]{k_1}} = \left(-\frac{3}{p}\right)\cdot\sqrt[3]{k_2}} \quad //$$

(4) $\boxed{\frac{1}{\sqrt[3]{k_2}} = -\left(\frac{3}{p}\right)\cdot\sqrt[3]{k_1}}$

$$\frac{1}{\sqrt[3]{k_2}} = \frac{\sqrt[3]{k_1}}{\sqrt[3]{k_1}\cdot\sqrt[3]{k_2}} = \frac{\sqrt[3]{k_1}}{\sqrt[3]{k_1\cdot k_2}}$$ — As we can see previously:

$$= \sqrt[3]{k_1}\cdot\left(-\frac{3}{p}\right)$$

$$\Rightarrow \boxed{\frac{1}{\sqrt[3]{k_2}} = \left(-\frac{3}{p}\right)\cdot\sqrt[3]{k_1}} \quad //$$