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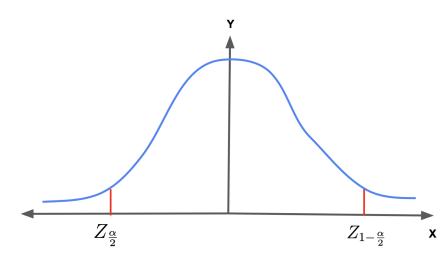
# Hypothesis Test

Consider the observations  $X_1, ..., X_n \stackrel{\text{iid}}{\sim} D_1(\mu_1, \sigma_1), Y_1, ..., Y_n \stackrel{\text{iid}}{\sim} D_2(\mu_2, \sigma_2).$ 

With  $X_i$  independent of  $Y_j \quad \forall \ (i \in \{1,...,n\}, j \in \{1,...,m\}).$ 

#### i. Equality:

$$H_0: \mu_1 = \mu_2 \Leftrightarrow \mu_1 - \mu_2 = 0$$
 vs  $H_a: \mu_1 \neq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \neq 0$ 



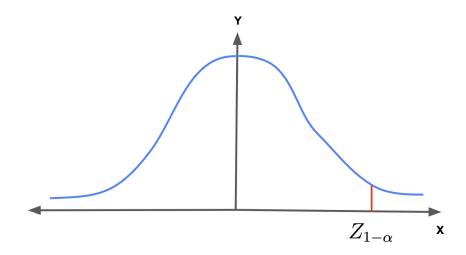
$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$
$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If  $T_{|H_0} > Z_{1-\frac{\alpha}{2}}$  (critical value) or  $T_{|H_0} < Z_{\frac{\alpha}{2}}$  (critical value) then we can reject null hypothesis.

Else we can't reject null hypothesis.

### ii. Greater:

$$H_0: \mu_1 > \mu_2 \Leftrightarrow \mu_1 - \mu_2 > 0$$
 vs  $H_a: \mu_1 \leq \mu_2 \Leftrightarrow \mu_1 - \mu_2 \leq 0$ 



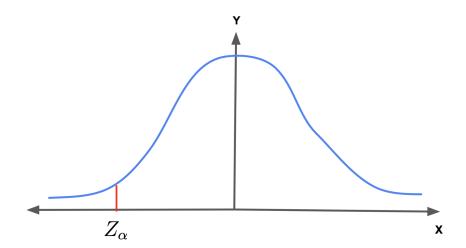
$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$
$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If  $T_{|H_0} \leq Z_{1-\alpha}$  (critical value) then we can reject null hypothesis.

Else we can't reject null hypothesis.

## iii. Lower:

$$H_0: \mu_1 < \mu_2 \Leftrightarrow \mu_1 - \mu_2 < 0$$
 vs  $H_a: \mu_1 \ge \mu_2 \Leftrightarrow \mu_1 - \mu_2 \ge 0$ 



$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

$$T_{|H_0} = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}$$

If  $T_{|H_0} \geq Z_{\alpha}$  (critical value) then we can reject null hypothesis.

Else we can't reject null hypothesis.

## **Demostrations**

$$T_1 = \frac{1}{n} \cdot \sum_{i=1}^n X_i$$

$$T_2 = \frac{1}{m} \cdot \sum_{j=1}^n Y_j$$

$$Z = T_1 - T_2$$

$$\mathbb{E}(Z) = \mathbb{E}(T_1 - T_2) = \mathbb{E}(T_1) - \mathbb{E}(T_2)$$

$$= \mathbb{E}\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_{i}\right) - \mathbb{E}\left(\frac{1}{m} \cdot \sum_{j=1}^{m} Y_{j}\right) = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbb{E}\left(X_{i}\right) - \frac{1}{m} \cdot \sum_{j=1}^{m} \mathbb{E}\left(Y_{j}\right)$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \mu_{1} - \frac{1}{m} \cdot \sum_{j=1}^{m} \mu_{2} = \frac{1}{n} \cdot [n \cdot \mu_{1}] - \frac{1}{m} \cdot [m \cdot \mu_{2}]$$

$$= \mu_{1} - \mu_{2}$$

 $\Rightarrow$ 

$$\mathbb{E}\left(Z\right) = \mu_1 - \mu_2$$

$$\mathbb{V}\left(Z\right) = \mathbb{V}\left(T_{1} - T_{2}\right) \stackrel{\text{ind}}{=} (1)^{2} \cdot \mathbb{V}\left(T_{1}\right) + (-1)^{2} \cdot \mathbb{V}\left(T_{2}\right) = \mathbb{V}\left(T_{1}\right) + \mathbb{V}\left(T_{2}\right)$$

$$= \mathbb{V}\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_{i}\right) + \mathbb{V}\left(\frac{1}{m} \cdot \sum_{j=1}^{m} Y_{j}\right) \stackrel{\text{ind}}{=} \frac{1}{n^{2}} \cdot \sum_{i=1}^{n} \mathbb{V}\left(X_{i}\right) + \frac{1}{m^{2}} \cdot \sum_{j=1}^{m} \mathbb{V}\left(Y_{j}\right)$$

$$= \frac{1}{n^{2}} \cdot \sum_{i=1}^{n} (\sigma_{1})^{2} + \frac{1}{m^{2}} \cdot \sum_{j=1}^{m} (\sigma_{2})^{2} = \frac{1}{n^{2}} \cdot \left[n \cdot (\sigma_{1})^{2}\right] + \frac{1}{m^{2}} \cdot \left[m \cdot (\sigma_{2})^{2}\right]$$

$$= \frac{\sigma_{1}^{2}}{n} + \frac{\sigma_{2}^{2}}{m}$$

 $\Rightarrow$ 

$$\mathbb{V}(Z) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}$$