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Confidence Interval for Parameters

We have mainly 3 ways of give a confidence interval for parameters.

1. Fisher Information: If we have the estimator $\hat{\theta}_{MLE}$ we can use that:

$$(\hat{\theta} - \theta) \sim N(0, I(\theta)^{-1}), \quad n \rightarrow +\infty$$

Deduction:

$$z_{\frac{\alpha}{2}} \leq \frac{\hat{\theta} - \theta}{I(\theta)^{-1}} \leq z_{1-\frac{\alpha}{2}}$$

\Rightarrow

$$\frac{z_{\frac{\alpha}{2}}}{I(\theta)} \leq \hat{\theta} - \theta \leq \frac{z_{1-\frac{\alpha}{2}}}{I(\theta)}$$

\Rightarrow

$$\frac{z_{\frac{\alpha}{2}}}{I(\theta)} - \hat{\theta} \leq -\theta \leq \frac{z_{1-\frac{\alpha}{2}}}{I(\theta)} - \hat{\theta}$$

\Rightarrow

$$\hat{\theta} - \frac{z_{1-\frac{\alpha}{2}}}{I(\theta)} \leq \theta \leq \hat{\theta} - \frac{z_{\frac{\alpha}{2}}}{I(\theta)}$$

\Rightarrow

$$\boxed{\hat{\theta} + \frac{z_{\frac{\alpha}{2}}}{I(\theta)} \leq \theta \leq \hat{\theta} + \frac{z_{1-\frac{\alpha}{2}}}{I(\theta)}}$$

Regularly we use $I(\theta)$ evaluated on $\theta = \hat{\theta}$.

2. Notable case: For example when we have the estimator:

$$S^2 = \frac{1}{(n-1)} \cdot \sum_{i=1}^n (x_i - \bar{x})^2$$

We now that:

$$T = \frac{(n-1) \cdot S^2}{\sigma^2} \sim \chi_{n-1}^2$$

Deduction:

$$t_{\frac{\alpha}{2},(n-1)} \leq T \leq t_{(1-\frac{\alpha}{2}), (n-1)}$$

\Rightarrow

$$t_{\frac{\alpha}{2},(n-1)} \leq \frac{(n-1) \cdot S^2}{\sigma^2} \leq t_{(1-\frac{\alpha}{2}), (n-1)}$$

\Rightarrow

$$\frac{1}{t_{(1-\frac{\alpha}{2}), (n-1)}} \leq \frac{\sigma^2}{(n-1) \cdot S^2} \leq \frac{1}{t_{\frac{\alpha}{2}, (n-1)}}$$

\Rightarrow

$$\frac{(n-1)}{t_{(1-\frac{\alpha}{2}), (n-1)}} \leq \frac{\sigma^2}{S^2} \leq \frac{(n-1)}{t_{\frac{\alpha}{2}, (n-1)}}$$

\Rightarrow

$$\boxed{\frac{(n-1)}{t_{(1-\frac{\alpha}{2}), (n-1)}} \cdot S^2 \leq \sigma^2 \leq \frac{(n-1)}{t_{\frac{\alpha}{2}, (n-1)}} \cdot S^2}$$

3. Analyze particular case: Given a estimator $\hat{\theta}$ we have to analyze the distribution of the estimator and then we create a confidence interval.
