First of all we have to prove the solution for:

$$x^2+px+q=0$$

$$x_{1,2}=\frac{-p\pm\sqrt{p^2-4q}}{2}$$

Demonstration: We want to rid of the "x" term.

Let $x=y+k$

$x^2+px+q=0$

$\Rightarrow (y+k)^2+p\cdot(y+k)+q=0$

$\Rightarrow y^2+2yk+k^2+p\cdot y+k\cdot p+q=0$

$[2k+p]\cdot y=0 \Rightarrow 2k+p=0 \Rightarrow k=-\frac{p}{2}$

$\Rightarrow y^2+\frac{p^2}{4}-\frac{p^2}{2}+q=0$

$\Rightarrow y^2-\frac{p^2}{4}+q=0 \Rightarrow y^2=\frac{p^2}{4}-q$

$\Rightarrow y^2=\frac{p^2-4q}{4} \Rightarrow y=\pm\frac{\sqrt{p^2-4q}}{2}$

But how $x=y+k=y-\frac{p}{2} \Rightarrow x=\frac{-p\pm\sqrt{p^2-4q}}{2}$