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# Sorting Probabilities

When we are talking about choose elements, first of all we have to do two distinctions.

**1. The order:**

(a) Doesn't matter = **Combinations**

(b) Matters = **Permutations**

**2. Is repetitions allowed?:**

(a) Yes

(b) No

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## 1. Combinations without repetition

$$\boxed{\binom{n}{i}}$$

Choose $i$ elements from a set of $n$ elements.

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## 2. Combinations with repetition

$$\boxed{\binom{i+n-1}{i}}$$

Choose $i$ elements from a set of $n$ elements.

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## 3. Permutations without repetition

$$\boxed{\frac{n!}{(n-i)!}}$$

Choose $i$ elements from a set of $n$ elements.

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## 4. Permutations with repetition

$$\boxed{n^i}$$

Choose $i$ elements from a set of $n$ elements.

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# Exercises

**Question 1.** How many subsets have a set of $n$ elements. Consider the empty subset as possible.

To create a subset we have to do a <u>combination without repetion</u>.

All the subset of $i$ elements is:

$$\binom{n}{i}$$

But we have to sum of the possible subsets:

$$\boxed{T = \sum_{i=0}^{n} \binom{n}{i}}$$

The binomial theorem says:

$$\sum_{i=0}^{n} \binom{n}{i} a^i \cdot b^{n-i} = (a+b)^n$$

For this to compute $T$ we have to use $a = 1$ and $b = 1$ then:

$$T = \sum_{i=0}^{n} \binom{n}{i} = (1+1)^n = 2^n$$

$\Rightarrow$

$$\boxed{T = 2^n}$$

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**Question 2.** If we reorder the letters of the word MATHEMATICS, Which is the probability of the vowels be together?

First of all we have to see that the probality is:

$$\boxed{P = \frac{\text{number of ways of reorder keeps the vowels together}}{\text{number of ways of reorder all the letters}}}$$

Now we going to build the next 3 tables:

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**Letters**

| Letter | Frequency |
|---|---|
| M | 2 |
| A | 2 |
| T | 2 |
| H | 1 |
| E | 1 |
| I | 1 |
| C | 1 |
| S | 1 |

**Total of letter:** 11.
**Letters with repetition:** 3.

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**Consonants**

| Consonant | Frequency |
|---|---|
| M | 2 |
| T | 2 |
| H | 1 |
| C | 1 |
| S | 1 |

**Total of consonants:** 7.
**Consonants with repetition:** 2.

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**Vowels**

| Vowel | Frequency |
|---|---|
| A | 2 |
| E | 1 |
| I | 1 |

**Total of vowels:** 4.
**Vowels with repetition:** 1.

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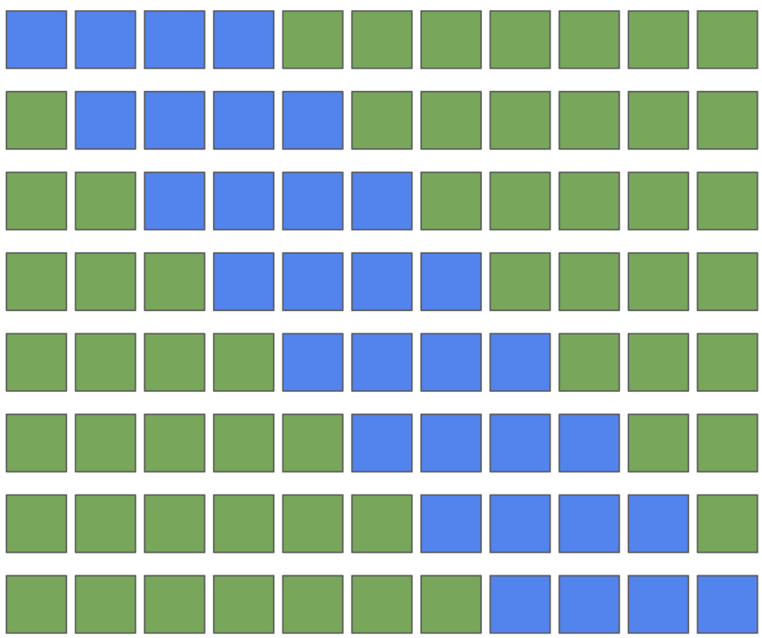
Using the table of the **Letters** we can see that:

$$\boxed{\text{number of ways of reorder all the letters} = \frac{11!}{2! \cdot 2! \cdot 2!}} \tag{1}$$

Now to compute the number of ways of reorder keeps the vowels together we going to board this problem as the vowels was an unique block (blue) and the consonants was an unique block (green) with the capacity of divide it to merge with the vowels.



You can see the combination between the two blocks are 8 (number of consonants + 1).

Now we have to compute the number of ways or order the vowels and consonants.

$$\boxed{\text{number of ways of reorder the vowels block} = \frac{4!}{2!}}$$

$$\boxed{\text{number of ways of reorder the consonants block} = \frac{7!}{2! \ \cdot 2!}}$$

Now we have that:

$$N_{VT} = \text{number of ways of reorder keeps the vowels together}$$

$$N_M = \text{number of ways of merge vowels block and consonants block}$$

$$N_V = \text{number of ways of order the vowels block}$$

$$N_C = \text{number of ways of order the consonants block}$$

$$N_{VT} = N_M \cdot N_V \cdot N_C = (8) \cdot \left(\frac{4!}{2!}\right) \cdot \left(\frac{7!}{2! \ \cdot 2!}\right) = \frac{8! \ \cdot 4!}{2! \ \cdot 2! \ \cdot 2!}$$

$\Rightarrow$

$$\boxed{N_{VT} = \frac{8! \ \cdot 4!}{2! \ \cdot 2! \ \cdot 2!}} \quad (2)$$

Combining (1) and (2) we have that:

$$P = \frac{\frac{8! \ \cdot 4!}{2! \ \cdot 2! \ \cdot 2!}}{\frac{11!}{2! \ \cdot 2! \ \cdot 2!}} = \frac{8! \ \cdot 4!}{11!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{11 \cdot 10 \cdot 9} = \frac{4 \cdot 1 \cdot 1 \cdot 1}{11 \cdot 5 \cdot 3} = \frac{4}{165}$$

$\Rightarrow$

$$\boxed{P = \frac{4}{165}}$$

As you can see all the problem is about **permutations without repititions.**

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