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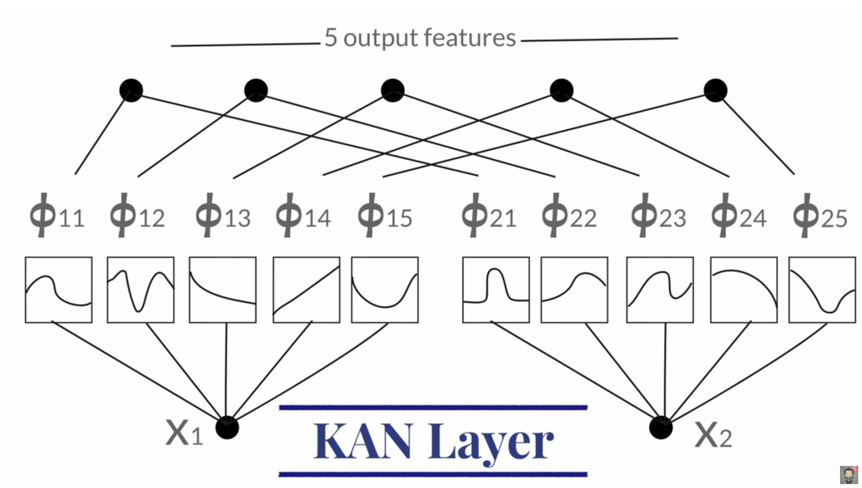
KAN

Kolmogorov Arnold Representation Theorem

Kolmogorov-Arnold representation theorem states that if f is a multivariate continuous function on a bounded domain, then it can be written as a finite composition of continuous functions of a single variable and the binary operation of addition. More specifically, for a smooth $f : [0,1]^n \Rightarrow \mathbb{R}$.

$$\boxed{f(x) = f(x_1, x_2, ..., x_n) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)}$$

KAN Layer



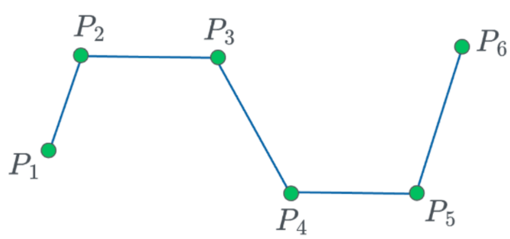
Activation functions

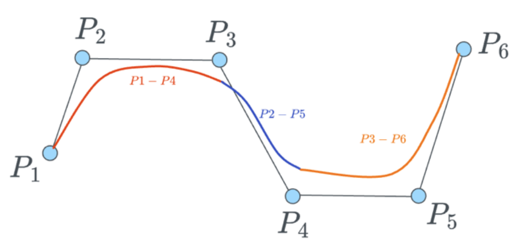
We going to use B-splines. B-Splines provide a more efficient way to represent curves, especially when dealing with a large number of data points.

Unlike high-degree polynomials, B-Splines use a series of lower-degree polynomial segments, which are connected smoothly.

In other words, instead of extending Bezier curves to tens of hundreds of data points, which leads to an equally high degree of the polynomial, we use multiple lower-degree polynomials and connect them together to form a smooth curve.

To exemplify, consider the set of data points in the image below:





Example 1:

Consider $f(x, y) = x \cdot y$

$$f(x, y) = f(x_1, x_2) = \sum_{q=1}^{5} \Phi_q \left(\phi_{q,1}(x_1) + \phi_{q,2}(x_2)\right) = \Phi_1 \left(\phi_{1,1}(x_1) + \phi_{1,2}(x_2)\right) +$$

$$\Phi_2 \left(\phi_{2,1}(x_1) + \phi_{2,2}(x_2)\right) + \Phi_3 \left(\phi_{3,1}(x_1) + \phi_{3,2}(x_2)\right) + \Phi_4 \left(\phi_{4,1}(x_1) + \phi_{4,2}(x_2)\right) + \Phi_5 \left(\phi_{5,1}(x_1) + \phi_{5,2}(x_2)\right)$$

We can define:

$$\boxed{\Phi_1(u) = \exp(u) \qquad \Phi_2(u) = \Phi_3(u) = \Phi_4(u) = \Phi_5(u) = 0}$$

$$\boxed{\phi_{1,1}(x_1) = ln(x_1) \qquad \phi_{1,2}(x_1) = ln(x_2)}$$

$\Rightarrow$

$$f(x,y) = \exp\left(ln(x) + ln(y)\right) = \exp\left(ln\left[x \cdot y\right]\right) = x \cdot y$$

Example 2:

Consider $f(x,y) = \dfrac{x}{y}$

$$f(x,y) = f(x_1,x_2) = \sum_{q=1}^{5} \Phi_q\left(\phi_{q,1}(x_1) + \phi_{q,2}(x_2)\right) = \Phi_1\left(\phi_{1,1}(x_1) + \phi_{1,2}(x_2)\right) +$$

$$\Phi_2\left(\phi_{2,1}(x_1) + \phi_{2,2}(x_2)\right) + \Phi_3\left(\phi_{3,1}(x_1) + \phi_{3,2}(x_2)\right) + \Phi_4\left(\phi_{4,1}(x_1) + \phi_{4,2}(x_2)\right) + \Phi_5\left(\phi_{5,1}(x_1) + \phi_{5,2}(x_2)\right)$$

We can define:

$$\boxed{\Phi_1(u) = \exp(u) \qquad \Phi_2(u) = \Phi_3(u) = \Phi_4(u) = \Phi_5(u) = 0}$$

$$\boxed{\phi_{1,1}(x_1) = ln(x_1) \qquad \phi_{1,2}(x_1) = -ln(x_2)}$$

$\Rightarrow$

$$f(x,y) = \exp\left(ln(x) - ln(y)\right) = \exp\left(ln\left[\frac{x}{y}\right]\right) = \frac{x}{y}$$
