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# Method of Moments

In statistics, the method of moments is a method of estimation of population parameters. The same principle is used to derive higher moments like skewness and kurtosis.

If we have to estimate $n$ parameters, the method consists on present the next equations:

$$\boxed{\mathbb{E}[X^j] = \hat{\mu_j}} \qquad \boxed{\hat{\mu_j} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i^j}$$

$$\boxed{\mathbb{E}[X^1] = \hat{\mu_1}}$$

$$\vdots$$

$$\boxed{\mathbb{E}[X^n] = \hat{\mu_n}}$$

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**Example:** $X_i \overset{\text{iid}}{\sim} Exp(\lambda)$. Compute $\hat{\lambda}$.

We have to estimate 1 parameter.

$$\mathbb{E}[X] = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$\boxed{\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

**Example:** $X_i \overset{\text{iid}}{\sim} Poisson(\lambda)$. Compute $\hat{\lambda}$.

We have to estimate 1 parameter.

$$\mathbb{E}[X] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$\lambda = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$\boxed{\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

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**Example:** $X_i \overset{\text{iid}}{\sim} Bernoulli(p)$. Compute $\hat{p}$.

We have to estimate 1 parameter.

$$\mathbb{E}[X] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$p = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\Rightarrow$

$$\boxed{\hat{p} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

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**Example:** $X_i \overset{\text{iid}}{\sim} Normal(\mu, \sigma)$. Compute $\hat{\mu}$ and $\hat{\sigma^2}$.

We have to estimate 2 parameters.

$$\mathbb{E}[X] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\mathbb{E}[X^2] = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$\Rightarrow$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\mu^2 + \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

$\Rightarrow$

$$\boxed{\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

$\Rightarrow$

$$\hat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \hat{\mu}^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}\cdot\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 + \bar{x}\cdot\bar{x} - 2\cdot\bar{x}\cdot\bar{x}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 + \frac{1}{n}\sum_{i=1}^{n} \bar{x}\cdot\bar{x} - 2\cdot\bar{x}\cdot\frac{1}{n}\sum_{i=1}^{n} x_i$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 + \frac{1}{n}\sum_{i=1}^{n} \bar{x}\cdot\bar{x} + \frac{1}{n}\sum_{i=1}^{n}(-2\cdot\bar{x}\cdot x_i)$$

$$= \frac{1}{n}\cdot\sum_{i=1}^{n}[x_i^2 - 2\cdot\bar{x}\cdot x_i + \bar{x}^2] = \frac{1}{n}\cdot\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$\Rightarrow$

$$\boxed{\hat{\sigma^2} = \frac{1}{n}\cdot\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

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