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Confidence Interval for Parameters

We have mainly 3 ways of give a confidence interval for parameters.

1. Fisher Information: If we have the estimator $\hat{\theta_{MLE}}$ we can use that:

$$(\hat{\theta} - \theta) \sim N(0, I(\theta)^{-1}), \qquad n \to +\infty$$

Deduction:

$$z_{\frac{\alpha}{2}} \leq \frac{\hat{\theta} - \theta}{\sqrt{I(\theta)^{-1}}} \leq z_{1-\frac{\alpha}{2}}$$

$\Rightarrow$

$$\frac{z_{\frac{\alpha}{2}}}{\sqrt{I(\theta)}} \leq \hat{\theta} - \theta \leq \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{I(\theta)}}$$

$\Rightarrow$

$$\frac{z_{\frac{\alpha}{2}}}{\sqrt{I(\theta)}} - \hat{\theta} \leq -\theta \leq \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{I(\theta)}} - \hat{\theta}$$

$\Rightarrow$

$$\hat{\theta} - \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{I(\theta)}} \leq \theta \leq \hat{\theta} - \frac{z_{\frac{\alpha}{2}}}{\sqrt{I(\theta)}}$$

$\Rightarrow$

$$\boxed{\hat{\theta} + \frac{z_{\frac{\alpha}{2}}}{\sqrt{I(\theta)}} \leq \theta \leq \hat{\theta} + \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{I(\theta)}}}$$

Regulary we use $I(\theta)$ evaluated on $\theta = \hat{\theta}$.

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2. Notable case: For example when we have the estimator:

$$S^2 = \frac{1}{(n-1)} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

We now that:

$$T = \frac{(n-1) \cdot S^2}{\sigma^2} \sim \chi^2_{n-1}$$

Deduction:

$$t_{\frac{\alpha}{2},(n-1)} \leq T \leq t_{\left(1-\frac{\alpha}{2}\right),(n-1)}$$

$\Rightarrow$

$$t_{\frac{\alpha}{2},(n-1)} \leq \frac{(n-1) \cdot S^2}{\sigma^2} \leq t_{\left(1-\frac{\alpha}{2}\right),(n-1)}$$

$\Rightarrow$

$$\frac{1}{t_{\left(1-\frac{\alpha}{2}\right),(n-1)}} \leq \frac{\sigma^2}{(n-1) \cdot S^2} \leq \frac{1}{t_{\frac{\alpha}{2},(n-1)}}$$

$\Rightarrow$

$$\frac{(n-1)}{t_{\left(1-\frac{\alpha}{2}\right),(n-1)}} \leq \frac{\sigma^2}{S^2} \leq \frac{(n-1)}{t_{\frac{\alpha}{2},(n-1)}}$$

$\Rightarrow$

$$\boxed{\frac{(n-1)}{t_{\left(1-\frac{\alpha}{2}\right),(n-1)}} \cdot S^2 \leq \sigma^2 \leq \frac{(n-1)}{t_{\frac{\alpha}{2},(n-1)}} \cdot S^2}$$

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3. Analyze particular case: Given a estimator $\hat{\theta}$ we have to analyze the distribution of the estimator and then we create a confidence interval.
