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Prediction Variance

Its depends of each model. For example for the autoregressive model we have that:

$$\boxed{X_t = \phi \cdot X_{t-1} + Z_t}$$

What is the variance if we want to predict n steps forward?

First of all we have $\hat{X}_0$ (observed value).

$$\hat{X}_1 = \phi \cdot \hat{X}_0 + Z_1$$

$$\hat{X}_2 = \phi \cdot \hat{X}_1 + Z_2 = \phi \cdot [\phi \cdot \hat{X}_0 + Z_1] + Z_2 = \phi^2 \cdot \hat{X}_0 + \phi \cdot Z_1 + Z_2$$

$\Rightarrow$

$$\boxed{\hat{X}_n = \phi^n \cdot \hat{X}_0 + \sum_{i=0}^{n-1} \phi^i \cdot Z_{(n-i)}}$$

$\Rightarrow$

$$\begin{aligned}
\mathbb{V}\left(\hat{X}_n\right) &= \mathbb{V}\left(\phi^n \cdot \hat{X}_0 + \sum_{i=0}^{n-1} \phi^i \cdot Z_{(n-i)}\right) = \mathbb{V}\left(\sum_{i=0}^{n-1} \phi^i \cdot Z_{(n-i)}\right) \\
&= \sum_{i=0}^{n-1} \mathbb{V}\left(\phi^i \cdot Z_{(n-i)}\right) = \sum_{i=0}^{n-1} (\phi^2)^i \cdot \mathbb{V}\left(Z_{(n-i)}\right) = \sum_{i=0}^{n-1} (\phi^2)^i \cdot \sigma^2 \\
&= \sigma^2 \cdot \sum_{i=0}^{n-1} (\phi^2)^i = \sigma^2 \cdot \left(\frac{1-(\phi^2)^n}{1-\phi^2}\right)
\end{aligned}$$

$\Rightarrow$

$$\boxed{\mathbb{V}\left(\hat{X}_n\right) = \sigma^2 \cdot \left(\frac{1-(\phi^2)^n}{1-\phi^2}\right)}$$

How $|\phi| < 1$ then $\mathbb{V}\left(\hat{X}_n\right)$ is an increasing function in terms of n.
