Alberto Andrés Valdés González.
Degree: Mathematical Engineer.
Work position: Data Scientist.
Mail: anvaldes@uc.cl/alberto.valdes.gonzalez.96@gmail.com
Location: Santiago, Chile.

Central Limit Theorem

In probability theory, the central limit theorem (CLT) establishes that, in many situations, for independent and identically distributed random variables, the sampling distribution of the standardized sample mean tends towards the standard normal distribution even if the original variables themselves are not normally distributed.

Consider $X_i \overset{\text{iid}}{\sim} D(\mu, \sigma^2)$.

We define:

$$\boxed{Y_n = \frac{1}{n} \cdot \sum_{i=1}^{n} X_i}$$

Now:

$$\mathbb{E}\left[Y_n\right] = \mathbb{E}\left[\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right] = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbb{E}\left[X_i\right] = \frac{1}{n} \cdot \sum_{i=1}^{n} \mu = \frac{1}{n} \cdot (n \cdot \mu) = \mu$$

$\Rightarrow$

$$\boxed{\mathbb{E}\left[Y_n\right] = \mu}$$

On the other hand:

$$\mathbb{V}\left[Y_n\right] = \mathbb{V}\left[\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right] = \frac{1}{n^2} \cdot \sum_{i=1}^{n} \mathbb{V}\left[X_i\right] = \frac{1}{n^2} \cdot \sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2} \cdot (n \cdot \sigma^2) = \frac{\sigma^2}{n}$$

$\Rightarrow$

$$\boxed{\mathbb{V}\left[Y_n\right] = \frac{\sigma^2}{n}}$$

Thus:

$$\boxed{\frac{Y_n - \mathbb{E}[Y_n]}{\sqrt{\mathbb{V}[Y_n]}} \sim N(0,1)}$$
