Alberto Andrés Valdés González.
Degree: Mathematical Engineer.
Work position: Data Scientist.
Mail: anvaldes@uc.cl/alberto.valdes.gonzalez.96@gmail.com
Location: Santiago, Chile.

MAE relation MSE

Consider the points $(\vec{x_1}, y_1), (\vec{x_2}, y_2), ..., (\vec{x_n}, y_n)$ and its prediction $\hat{y}_1, \hat{y}_2, ..., \hat{y}_n$.

We want to prove some relation between the two measures. For this we have to remind the Cauchy-Schwarz inequality.

$$\langle \vec{u}, \vec{v} \rangle \leq ||\vec{u}|| \cdot ||\vec{v}||$$

We going to define:

$$\vec{u} = \begin{pmatrix} |y_1 - \hat{y}_1| \\ |y_2 - \hat{y}_2| \\ \vdots \\ |y_n - \hat{y}_n| \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

We can see that:

$$\boxed{\langle \vec{u}, \vec{v} \rangle = \sum_{i=1}^{n} |y_i - \hat{y}_i| \quad (1)}$$

$$\boxed{||\vec{u}|| = \sqrt{\sum_{i=1}^{n} |y_i - \hat{y}_i|^2} \quad (2)}$$

$$\boxed{||\vec{v}|| = \sqrt{n} \quad (3)}$$

Using this and the previous inequality we have that:

$$\sum_{i=1}^{n} |y_i - \hat{y}_i| \leq \sqrt{\sum_{i=1}^{n} |y_i - \hat{y}_i|^2} \cdot \sqrt{n}$$

$\Rightarrow$

$$\frac{1}{n} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i| \leq \frac{1}{n} \cdot \sqrt{\sum_{i=1}^{n} |y_i - \hat{y}_i|^2} \cdot \sqrt{n}$$

$\Rightarrow$

$$\frac{1}{n} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i| \leq \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i|^2} \leq \sqrt{\frac{1}{(n-1)} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i|^2}$$

$\Rightarrow$

$$\frac{1}{n} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i| \leq \sqrt{\frac{1}{(n-1)} \cdot \sum_{i=1}^{n} |y_i - \hat{y}_i|^2}$$

$\Rightarrow$

$$\boxed{MAE \leq \sqrt{MSE}}$$

And that's the relation between the two measures.
