Dynamical systems

ME 416 - Prof. Tron

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Dynamical systems are defined by three elements:

State: A vector with all the quantities of interest of a system.

Inputs: What we can control to affect how the state evolves.

Model: Differential equation indicating how the state changes over time given the current state and inputs.

Example (differential drive robot) As we derived in class, the linear and angular velocity of a differential drive robot are governed by the equation:

$$\dot{z} = A(z)u, \tag{1}$$

where the three elements of the dynamical system are:

State: The vector $z = \begin{bmatrix} {}^{\mathcal{W}}T_{\mathcal{B}} \\ \theta \end{bmatrix}$.

Inputs: The vector $u = \begin{bmatrix} u_{LW} \\ u_{RW} \end{bmatrix}$.

Model: Equation (1), where $A(z)$ is the 3×2 matrix derived in another part of these notes.

★★TODO: talk about outputs.

1 Dynamical systems with open-loop control

Open loop control refers to the fact that the inputs as a function of time $u(t)$ are decided beforehand. By substituting $u(t)$ in the dynamical system equation, we obtain a (generally non-autonomous) ODE which we can integrate to see what is the resulting trajectory of the system $z(t)$. The integration of the ODE can be done either analytically (if possible) or approximately using Euler's method.

Since dynamical models are often idealization of real systems, open loop control can approximately maneuver the system on a desired trajectory, but it is not robust (i.e., the actual trajectory of the system will not be exactly the same as the desired one).

Example (differential drive robot) In one of the questions in Homework 2 you are asked to derive the trajectory $z(t)$ for inputs $u(t)$ which are constant. The odometry nodes in the other questions of Homework 2 compute $z(t)$ when $u(t)$ is piecewise constant.

2 Dynamical systems with feedback control

A *state feedback controller* is given by a mapping $u(z)$ (or, more generally $u(z, t)$), which decides inputs depending on the current state of the system (z, and possibly also the time t). Substituting $u(z)$ in the dynamical system model, one gets the *closed-loop system* model. Feedback control is much more robust than open-loop control, since it can compare the actual state with the desired one, and apply corrections to the inputs accordingly.