Theory of Automata

Homework – 6

1) Consider the pushdown automata $M=(K, \Sigma, \Gamma, \Delta, s, F)$, where

$K = \{s, f\}$,

$F = \{f\}$,

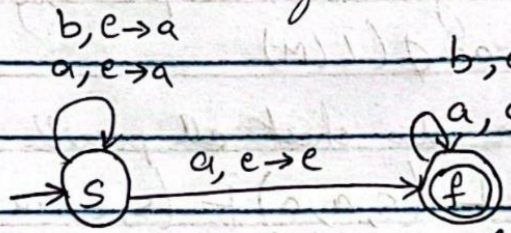
$\Sigma = \{a, b\}$,

$\Gamma = \{a\}$,

$\Delta = \{((s,a,e),(s,a)), ((s,b,e),(s,a)), ((s,a,e),(f,e)), ((f,a,a),(f,e)), ((f,b,a),(f,e))\}$

a) Trace all the possible sequence of transitions of M on input aba

Ans: The statechart diagram for above machine is



The possible sequence of transitions are

i) $(s, aba, e) \vdash (s, ba, a) \vdash (s, a, aa) \vdash (s, e, aaa)$

ii) $(s, aba, e) \vdash (s, ba, a) \vdash (s, a, aa) \vdash (f, e, aa)$

iii) $(s, aba, e) \vdash (f, ba, e)$

All the three cases cannot be accepted.

b) Show that aba, aa, abb $\notin L(M)$, but baa, bab, baaaa $\in L(M)$

Ans: A string is accepted by the machine M, if the middle symbol in a string which is odd in length is 'a'.

Consider the string aba:

The string is not accepted by M as it doesn't contain 'a' in the middle. aba $\notin L(M)$

We can also check all possible transitions.

- $(s, aba, e) \vdash (s, ba, a) \vdash (s, a, aa) \vdash (s, e, aaa)$ X
- $(s, aba, e) \vdash (s, ba, a) \vdash (s, a, aa) \vdash (\not{s}, e, aa)$ X
- $(s, aba, e) \vdash (\not{s}, ba, e)$ X

None of the transitions is accepted.

Consider the string aa:

The string is not accepted by M as it is even in length. aa $\notin L(M)$

We can also check all possible transitions.

- $(s, aa, e) \vdash (s, a, a) \vdash (s, e, aa)$ X
- $(s, aa, e) \vdash (s, a, a) \vdash (\not{s}, e, a)$ X
- $(s, aa, e) \vdash (\not{s}, a, a)$ X

None of the transitions is accepted.

Consider the string abb:

The string is not accepted by M as it doesn't contain 'a' in the middle of string. abb $\notin L(M)$

We can also confirm with all possible transitions.

- $(s, abb, e) \vdash (s, bb, a) \vdash (s, b, aa) \vdash (s, e, aaa)$ X
- $(s, abb, e) \vdash (\not{s}, bb, e)$ X

None of the transitions is accepted.

Consider the string baa:

The string is odd in length and contains 'a' in the middle. So it is accepted by M.

$(s, baa, e) \vdash (s, aa, a) \vdash (f, a, a) \vdash (f, e, e)$ ✓

baa $\in L(M)$ as it is accepted by M.

Consider the string bab:

The string is odd in length and contains 'a' in the middle. So it is accepted by M.

$(s, bab, e) \vdash (s, ab, a) \vdash (f, b, a) \vdash (f, e, e)$ ✓

bab $\in L(M)$ as it is accepted by M.

Consider the string baaaa:

The string is odd in length and contains 'a' in the middle. So it is accepted by M.

$(s, baaaa, e) \vdash (s, aaaa, a) \vdash (s, aaa, aa) \vdash (f, aa, aa) \vdash$
$(f, a, a) \vdash (f, e, e)$ ✓

Since the string is accepted by M, baaaa $\in L(M)$

∴ The strings baa, bab, baaaa are accepted by M and belongs to language (M).

baa, bab, baaaa $\in L(M)$

c) Describe L(M) in English.

Ans: The language of M is the set of strings whose middle symbol is 'a' and string is odd in length.

$L(M) = \{xay \in \{a,b\}^* : |x| = |y|\}$

$x = \{a,b\}^*$

$y = \{a,b\}^*$

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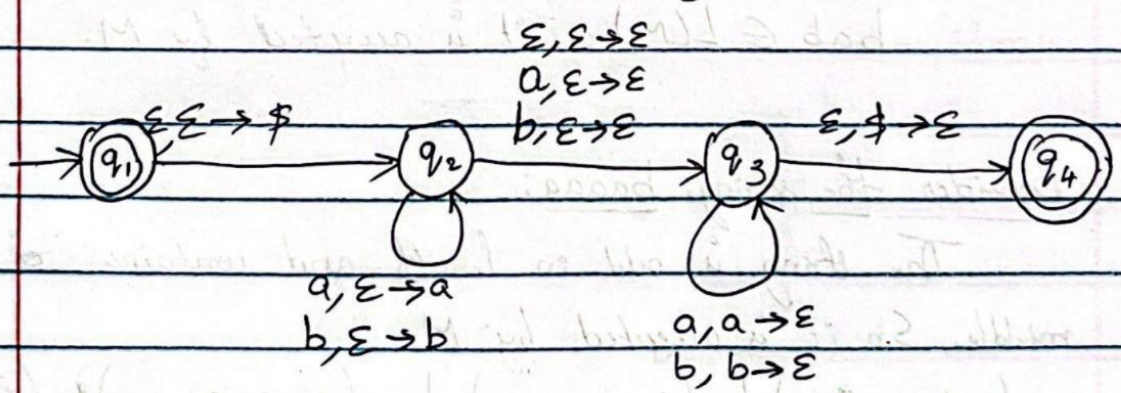
2) Construct a Pushdown automata that accept each of the followings:

a) The language $\{w \in \{a,b\}^* : w = w^R\}$

Ans: w = string

w^R = reverse of a string



Pushdown automata for the language M:

$M = \{Q, \Sigma, \Gamma, \Delta, s, F\}$

$Q = \{q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, \$\}$

$$\Delta = Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$$

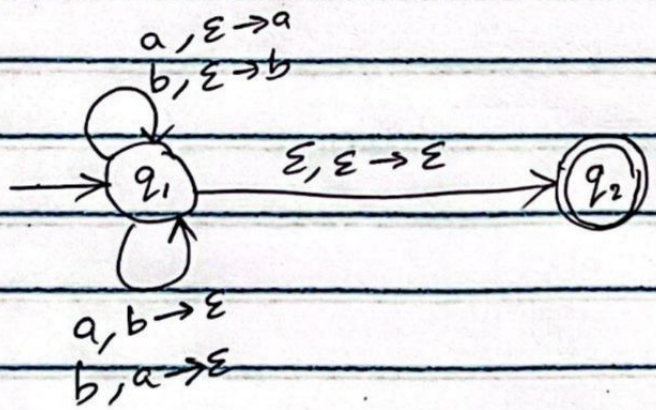
Input	a				b				ε			
Stack	a	b	$	ε	a	b	$	ε	a	b	$	ε
q_1	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	$\{(q_2, \$)\}$
q_2	∅	∅	∅	$\{(q_2, a), (q_3, \varepsilon)\}$	∅	∅	∅	$\{(q_2, b), (q_3, \varepsilon)\}$	∅	∅	∅	$\{(q_3, \varepsilon)\}$
q_3	$\{(q_3, \varepsilon)\}$	∅	∅	∅	∅	$\{(q_3, \varepsilon)\}$	∅	∅	∅	∅	$\{(q_4, \varepsilon)\}$	∅
q_4	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅

$s = \{q_1\}$

$F = \{q_1, q_4\}$

b) The language $\{w \in \{a,b\}^* : w$ has the same numbers of a's and b's.

Ans:



Pushdown automata for the language M:

$M = \{Q, \Sigma, \Gamma, \Delta, s, F\}$

$Q = \{q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b\}$

$$\Delta = \{((q_1, a, c), (q_1, a)), ((q_1, a, a), (q_1, a)), ((q_1, b, c), (q_1, b)),$$
$$((q_1, b, b), (q_1, b)), ((q_1, a, b), (q_1, c)),$$
$$((q_1, b, a), (q_1, c)), ((q_1, c, c), (q_2, c))\}$$

$$S = \{q_1\}$$

$$F = \{q_2\}$$