Theory of Automata

Homework - 7

1) Use the pumping theorem to show that the following languages are not context-free.

a) $\{www : w \in \{a,b\}^*\}$

Ans: According to pumping lemma theorem if A is a context free language then there exists a number p where, if s is any string in A of length atleast p, then s may be divided into five pieces $w = uvxyz$ satisfying the conditions

i, for each $i \geq 0$, $uv^ixy^iz \in A$

ii, $|vy| > 0$, and

iii, $|vxy| \leq p$

Let us consider $w = a^nb \Rightarrow www = a^nba^nba^nb$

Let us prove by contradiction method. Assume given language is contextfree i.e. www is context free.

So there exists a number N such that $N > 0$

and $w \in L \Rightarrow |w| \geq N$

According to pumping lemma theorem, there exists $uvwxy \in \Sigma^*$ such that $w = uvxyz$

$|vxy| \leq N$

$|vy| > 0$

$uv^ixy^iz \in L$ for any $i \geq 0$

From the assumed language $a^nba^nba^nb$, we cannot include b on v or y because if we do so then for $i = 2, 3, \ldots$ number of b's in language will become more than 3 which doesn't satisfy the language

So lets consider $v = a^{\alpha}$ (from first a^n)

$y = a^{\beta}$ (from second a^n)

such that $\alpha + \beta \leq N$

$\Rightarrow uvxyz = a^N b a^N b a^N b$ $(i=1)$

$uv^2xy^2z = a^{N+\alpha} b a^{N+\beta} b a^N b \notin L$ $(i=2)$

So we are getting a contradiction for $i=2$ for our assumption. Therefore the given language $L = \{www : w \in \{a,b\}^*\}$ is not context free.

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b) $\{w \in \{a,b,c\}^* : w$ has equal number of a's, b's and c's $\}$

Ans: If w has equal number of a's, b's and c's then we can rewrite the language as

$$L = \{w = a^n b^n c^n : w \in \{a,b,c\}^* \; n \geq 0\}$$

Let us prove this by the contradiction method.

Assume that L is a context free language.

If L is a context free language then according to pumping lemma there exists a number N, where if w is any string in L of length atleast N, then w may be divided into five pieces $w = uvxyz$ such that $|vxy| \leq N$

$|vy| > 0$

and for $i \geq 0$, $uv^ixy^iz \in L$

Consider $a^N b^N c^N = uvxyz$. There are five cases.

i) vxy contains all a's

ii) vxy contains all b's

iii) vxy contains all c's

In all the three cases, uv^2xy^2z $(i=2) \notin L$

because in case (i) uv^2xy^2z contains $|vy|$ more a's than b's and c's,

In case (ii) uv^2xy^2z contains $|vy|$ more b's than a's and c's.

In case (iii) uv^2xy^2z contains $|vy|$ more c's than a's and b's.

Hence contradiction in all three cases.

iv) $vxy = a \ldots ab \ldots b$

v) $vxy = b \ldots bc \ldots c$

In case (iv) assume $vy = a^\alpha b^\beta$ for some $\alpha, \beta \geq 0$ & $\alpha + \beta > 0$, now $uv^2xy^2z = a^{N+\alpha} b^{N+\beta} c^N \notin L$

In case (v) assume $vy = b^\alpha c^\beta$ for some $\alpha, \beta \geq 0$ & $\alpha + \beta > 0$, now $uv^2xy^2z = a^N b^{N+\alpha} c^{N+\beta} \notin L$

Hence contradiction in (iv) and (v) case.

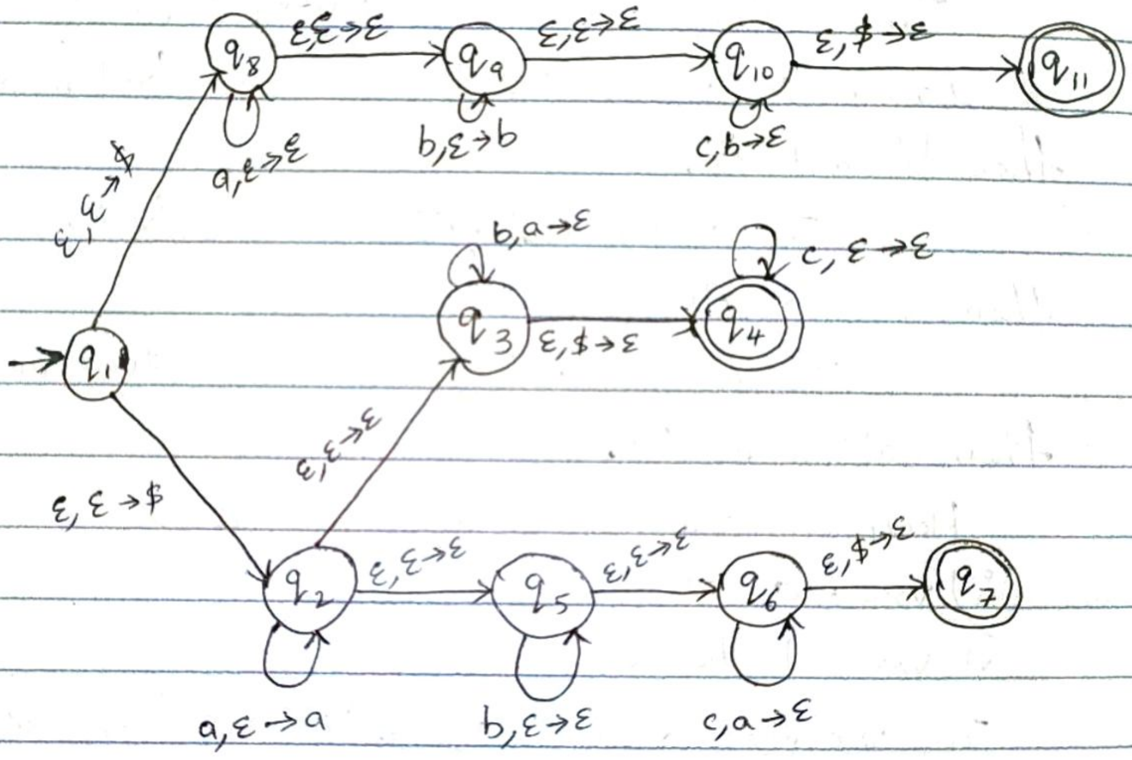
In all five cases, we got contradiction to our assumption.

So the given language L is not context free.

2) Decide whether the following language is context-free or not, and state your reason.

a) $\{a^m b^n c^p : m = n \text{ or } n = p \text{ or } m = p\}$

Ans: A language is context free if there is a push down automata that accepts it.



Since there is a pushdown automata accepting it, this language 'L' is context free.

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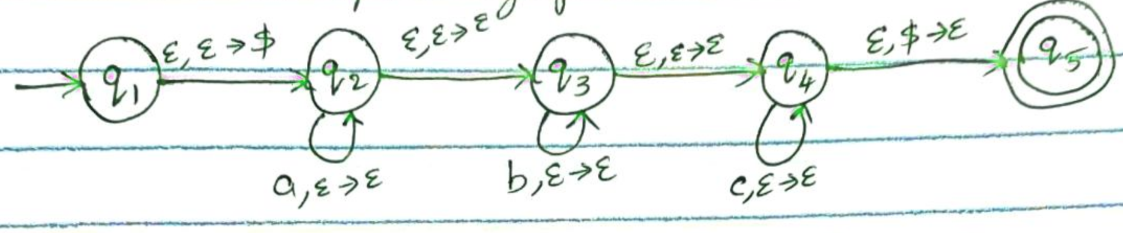
b) $\{a^m b^n c^p : m \neq n \text{ or } n \neq p \text{ or } m \neq p\}$

The given language can also be represented as

$\{a^m b^n c^p : m \neq n\} \cup \{a^m b^n c^p : n \neq p\} \cup \{a^m b^n c^p : m \neq p\}$

We Know that

$\{a^m b^n c^p : m \neq n\}$ is a context free language as it can be accepted by pushdown automata.



Similarly $\{a^m b^n c^p : n \neq p\}$ and $\{a^m b^n c^p : m \neq p\}$ are also context free

$\therefore L = \{a^m b^n c^p : m \neq n \text{ or } n \neq p \text{ or } m \neq p\}$ is a context free language. [CFG is closed under Union]

c) $\{a^m b^n c^p : m = n \text{ and } n = p \text{ and } m = p\}$

here $m = n = p = K$

$\{a^K b^K c^K : K \in N, K \geq 0\}$

So the given language is not context free because we know $\{a^n b^n c^n : n \geq 0\}$ is not a context free language [The proof for this is explained in 1 b answer]

Hence $\{a^m b^n c^p : m = n \text{ and } n = p \text{ and } m = p\}$ is not a context-free language.