Homework – 6

1. Consider the pushdown automata $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$
\begin{aligned}
K &= \{s, f\}, \\
F &= \{f\}, \\
\Sigma &= \{a, b\}, \\
\Gamma &= \{a\}, \\
\Delta &= \{((s, a, e), (s, a)), ((s, b, e), (s, a)), ((s, a, e), (f, e)), \\
&\quad ((f, a, a), (f, e)), ((f, b, a), (f, e))\}.
\end{aligned}
$$

a). Trace all possible sequence of transitions of M on input aba

The possible computations on input aba are:

$$
\begin{aligned}
(s, aba, e) &\vdash_M (f, ba, e) \\
(s, aba, e) &\vdash_M (s, ba, a) \vdash_M (s, a, aa) \vdash_M (s, e, aaa) \\
(s, aba, e) &\vdash_M (s, ba, a) \vdash_M (s, a, aa) \vdash_M (f, e, aa)
\end{aligned}
$$

b). Show that $aba, aa, abb \notin L(M)$, but $baa, bab, baaaa \in L(M)$

The possible computations on input aa are:

$$
\begin{aligned}
(s, aa, e) &\vdash_M (s, a, a) \vdash_M (s, e, aa) \\
(s, aa, e) &\vdash_M (s, a, a) \vdash_M (f, e, a) \\
(s, aa, e) &\vdash_M (f, a, e)
\end{aligned}
$$

The possible computations on input abb are:

$$
\begin{aligned}
(s, abb) &\vdash_M (f, bb, e) \\
(s, abb) &\vdash_M (s, bb, a) \vdash_M (s, b, aa) \vdash_M (s, e, aaa)
\end{aligned}
$$

None of these computations are accepting, so M does not accept any of the strings aba, aa, abb. However, we have:

$$
\begin{aligned}
(s, baa, e) &\vdash_M (s, aa, a) \vdash_M (f, a, a) \vdash_M (f, e, e) \\
(s, bab, e) &\vdash_M (s, ab, a) \vdash_M (f, b, a) \vdash_M (f, e, e) \\
(s, baaaa, e) &\vdash_M (s, aaaa, a) \vdash_M (s, aaa, aa) \vdash_M (f, aa, aa) \vdash_M (f, a, a) \vdash_M (f, e, e)
\end{aligned}
$$

so that baa, bab and $baaaa$ are all in $L(M)$.

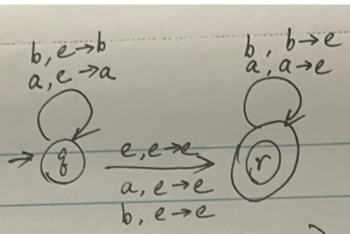
c). Describe $L(M)$ in English

$L = \{xay : x, y \in \{a, b\}^*, |x| = |y|\}$.

2. Construct a Pushdown automata that accept each of the followings:

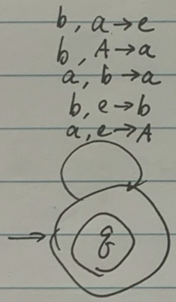
b). The language $\{w \in \{a, b\}^* : w = w^R\}$

c). The language $\{w \in \{a, b\}^* : w \text{ has the same number of } a's \text{ and } b's\}$



(3 cases, $w = uu^R$
$w = uau^R$
$w = ubu^R$)

$\{w : w \in \{a, b\}^*$, w contains twice b's as a's $\}$



"a" in string becomes "A" in stack
2 "b"s are needed to consume one "A".

$\{w : w \in \{a, b\}^*$, w contains same b's as a's $\}$