CS 5381 Analysis of Algorithms
Homework 2

Fall 2022

Total 240 points

Dynamic Programming

1. (30 points) Verify the optimal revenue r_i, $i = 1, \ldots, 10$ on page 70 of the Lecture Notes.

2. (30 points) Consider a modification of the rod-cutting problem in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Provide a pseudocode of a dynamic-programming algorithm to solve this modified problem.

3. (30 points) Compute the optimal value of $m[2, 5]$ of the example of a chain of 6 matrices on page 100 of the Lecture Notes.

4. (30 points) Let $R(i, j)$ be the number of times that table entry $m[i, j]$ is referenced while computing other table entries in a call of MATRIX-CHAIN-ORDER algorithm on page 98 of the Lecture Notes. Show that the total number of references for the entire table is

$$\sum_{i=1}^{n} \sum_{j=i}^{n} R(i, j) = \frac{n^3 - n}{3}.$$

5. (30 points) Suppose that in the rod-cutting problem, we also had limit l_i on the number of pieces of length i that we are allowed to produce, for $i = 1, 2, \ldots, n$. Show that the optimal-substructure property no longer holds.

Greedy Algorithms

6. (30 points) Using the procedure on page 125 of the Lecture Notes, show that the set $\{a_2, a_4, a_9, a_{11}\}$ is also an optimal solution.

7. Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the i-th element of set A, and let b_i be the i-th element of set B. You then receive a payoff of $\prod_{i=1}^{n} a_i^{b_i}$.

– (30 points) Give an algorithm that will maximize your payoff.

– (30 points) Show that your algorithm maximizes the payoff.