Midterm Review

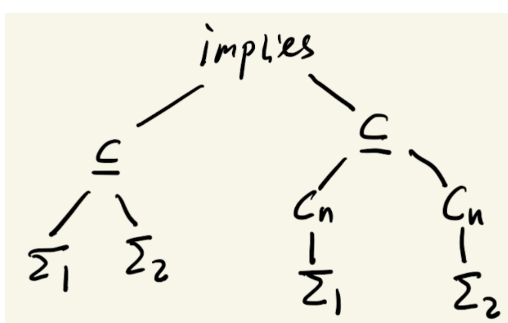
Midterm covers L02-L07 (chapters till 8)
Closed book test 50 minutes. One letter size cheat sheet is allowed. No answers of hw should be allowed in the cheat sheet. Bring ID with you.

L02 Propositional Logic - syntax

- Motivating example for propositional logic
- Learning skills
    - Concepts. For each concept, there are
        - Concept NAME and parameters, when being defined
        - Concept NAME and arguments, when being used
    - Precise statement (using only logical symbols (connectives, quantifiers), and concepts defined). Precise definition (is a precise statement except that some concepts are new and defined in the statements)
    - Given a precise statement or definition, identify/recognize
        - Concepts that are defined with names and parameters
        - Concepts (and logical symbols) that are defined before and used it with names and arguments.
        - Meta variables used in the statement/definition
    - Tree structure of a statement (and definition): similar to formation tree of proposition
        - In the tree, we put a concept name (example including logical connectives/quantifiers) as parent and its parameters/arguments as children). See formation tree examples in L02
        - Example the tree of statement

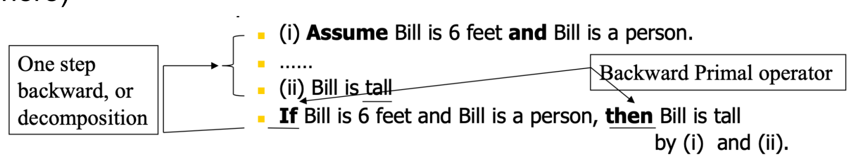
          $\Sigma_1 \subseteq \Sigma_2$ implies $Cn(\Sigma_1) \subseteq Cn(\Sigma_2)$

          Is

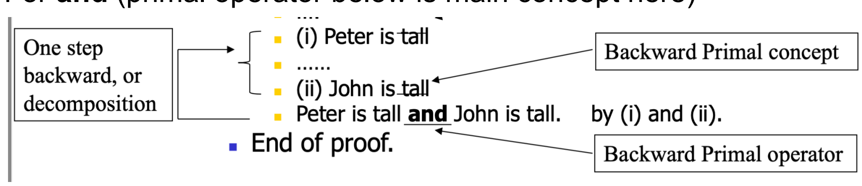
          

          The root of a tree is called "main concept." For example, "implies" is the main concept of the WHOLE statement. The left "subset" is the main concept of the sub-statement $\Sigma_1 \subseteq \Sigma_2$.
    - Key reasoning based on definitions: apply definition of a concept to a use of the concept (many examples around Page 9 of L04)
    - Prove a statement (thanks to definitions and preciseness of the statement)

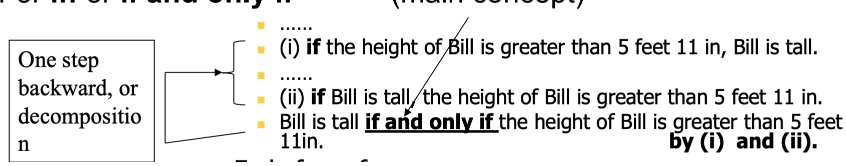
- Working backwards proof (whole L3). It heavily uses *application of the definition of a concept to a use of the concept.*
  - Need an empty paper with enough space.
  - Start at the very top by writing **Proof**.
  - At the very bottom we copy the statement to prove here, and in a new line write **QED**. This statement to prove is called goal statement.
  - Every statement written from the top of the paper is assumed to be true.
  - Work backwards one step from the current goal statement. (See around P9 of L04 for examples)
    - Draw the tree for the current goal statement. The root concept name is the main concept name.
    - Know how to do one step backwards for the main concept that is defined in the course to produce new goal statement
    - Know how to do one step backwards for a standard logical symbols to produce new goal statement
      - For **if / implies** (primal operator below is main concept here)



      - For **and** (primal operator below is main concept here)



      - For **iff** or **if and only if** (main concept)



      - For **for all** $$\forall$$

        =====

        ......

        (i) For every person x

        ......

        (ii) x is tall

        <u>For every person x,</u> x is tall. by (i) and (ii)

        ======

      - For **exists** $$\exists$$

        =====

        ......

(i) We construct a person x
……
(ii) x is tall
<u>There exists a person x</u> such that x is tall.   by (i) and (ii)
======

        - Apply the one step working backwards methods continuously until the current goal statement can be directly derived from the statement at the top (i.e., statements assumed to be true).
- Propositional logic
    - Syntax: definition of proposition.
        - Note the definition methods used.
    - Formation tree of a proposition

L03 - Proof example/method
Examples of using working backwards to prove statements. See also working backwards above.

L04 Propositional Logic - Semantics
- Motivating example
- Precise definition of semantics of proposition logic
    - The "ultimate concept about meaning of propositions": $\Sigma \models \sigma$ where $$\Sigma$$ is a set of propositions and $$\sigma$$ is a proposition.
    - The ultimate meaning definition depends on other concepts: e.g., definition of "meaning" of logical connectives, truth assignment, valuation etc.
        - Adequacy of logical connectives
            - Idea behind why {$$\neg$$, $$\land$$, $$\lor$$} is adequate.
- Learning skills: prove statement(s) using working backwards and the application of the definition of *consequence*. (Section 2.3)
- Self-learn concepts: their definition and statement (e.g., theorem) about concepts.
    - Motivation of the concept
    - Intuitive meaning of the concept
    - Write or understanding the definition of a concept
    - Create example(s) of a concept
    - Proofs (working backwards)

L05 Tableau proof (of the consequence of a set of premises (a set of propositions))
- Motivating example of tableau proof
- Atomic tableau / Tableau (definition and motivation)
- Tableau proof (definition) and and concepts it depends
- Finished tableau and concepts it depends
- Complete systematic tableau

L06 Soundness and Completeness of Tableau Proofs
- Motivation

- Able to state soundness result and completeness result of a proof (methods) such as tableau proof
- Proof of the soundness and completeness results using lemmas and proof by contradiction methods
- Proof of the lemmas (5.4, 5.2) by inductive proofs.
- Inductive proof method
  - Induction on what?
  - Base case statement and its proof
  - Inductive hypothesis. (Don't prove this. We use it)
  - Inductive proof statement and its proof.
  - Example: see detailed proof of 5.4 in Section 5 and 6.
- Tableau proof for deductions from premises. Its soundness and completeness results, and their proofs.

L07 Resolution

- Motivation
- *Resolution deduction* or *proof*, *resolution refutation* and the concepts they depend on.
- Soundness and completeness results (in terms of *unsatisfiability* and *resolution refutation*).