Theory of Automata

Homework - 4

1) Prove that $\{0^n 1^n 2^n : n \geq 1\}$ is not a regular language.

Ans: Proof by Pumping Lemma:

In this process, if the language $L = \{0^n 1^n 2^n : n \geq 1\}$ is not accepted by pumping Lemma, then it is not regular.

We consider the language L is regular.

If a language L is regular, then there exists some $p \geq 1$ such that for any string $w \in L$, $|w| \geq p$, we has $w = xyz$ such that

$$x = 0^k, \quad y = 0^{p-k}, \quad z = 1^p 2^p$$

here $\Rightarrow |xy| \leq p$

$\Rightarrow y \neq \epsilon$

$\Rightarrow xy^i z \in L$ for any $i \geq 0$

$$\Rightarrow xy^i z = 0^k (0^{p-k})^i 1^p 2^p$$
$$= 0^k 0^{ip-ik} 1^p 2^p$$

From the statement above $k + i(p-k) = p$

Here if $i = 1$ then it satisfies. But if $i=2$ then the above statement doesn't satisfy

If $i=2$ then
$$xy^2z = 0^k 0^{2p-2k} 1^p 2^p$$

$\Rightarrow xy^2z = 0^{2P-K} 1^P 2^P \notin L$

So the language $L = \{0^n 1^n 2^n : n \geq 1\}$ is not regular as it is a contradiction to the definition of Pumping Lemma.

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2) For arbitary constant c, is $\{0^n 1^n 2^n : n \geq c\}$ regular or not?

Ans: To check if a language is regular or not we can initially consider the language as regular and check it with pumping lemma.

If it satisfies pumping lemma, then it is regular, else if there is contradiction, it is non-regular.

By pumping Lemma, if L is regular, then there exists $n \geq 1$ such that if $w \in L$ $|w| \geq n$, then $w = xyz$ such that $y \notin \epsilon$, $|xy| \leq n$, $xy^iz \in L$ in any $i \geq 0$

Here $x = 0^{n-K}$

$y = 0^K$

$z = 1^n 2^n$

Consider $n = 3$, $c = 2$

$\Rightarrow xy^iz = 0^1 (0^2)^i 1^3 2^3$

For $i = 1 \Rightarrow 0^3 1^3 2^3 \in L$

For $i = 2 \Rightarrow 0^5 1^3 2^3 \notin L$

As $xy^2z \notin L$, it is a contradiction to the definition of pumping lemma.

So our assumption is wrong.

$\{0^n 1^n 2^n : n \geq c\}$ for arbitary constan c is not regular.

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3) The decimal notation for a number is number written in usual way, as a string over alphabet $\{0, 1, \ldots 9\}$. For example 13 is a String of length 2. In unary notation, only symbol "1" is used. Thus 5 is represented as 11111 in unary notation. Show each of following is or not regular language.

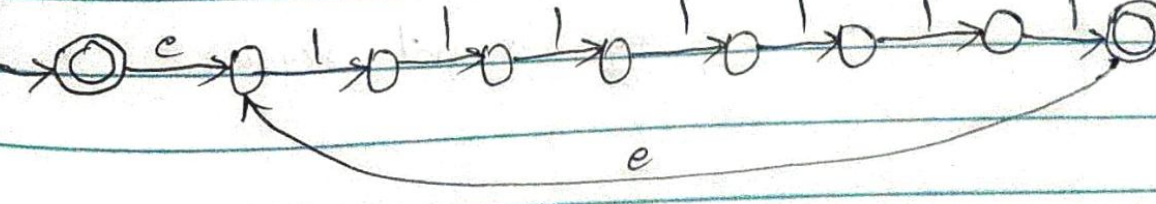
Ans: 3.1) $\{w : w$ is unary notation for a number that is a multiple of $7\}$

Multiples of 7 are 0, 7, 14, 21, ...

In Unary language, 7 is written as 1111111

The regular expression for the given language can be written as $(1111111)^*$

This regular expression can be represented as the following automata



As the language can be expressed as a regular expression and can also be represented as automata, the language is regular.

3.2) $\{w : w$ is the unary notation for $10^n, n \geq 1\}$

$10^n, n \geq 1 = \{10, 100, 1000, \ldots\}$

So the language is represented as

$10 \Rightarrow 1111111111 \Rightarrow 1^{10}$

$\Rightarrow 100 \Rightarrow 1^{100} \quad \Rightarrow 10000 = 1^{10000}$

$L = \{1^{10}, 1^{100}, 1^{1000}, 1^{10000}, \ldots\}$

Let us consider the language is regular, So by the definition of pumping lemma, there exists some n such that for $w \in L$, $|w| \geq n$, we have $w = xyz$

such that $y \neq e$

$|xy| \leq n$

$xy^iz \in L$ for $i \geq 0$

$\Rightarrow x = 1^2 \quad y = 1^3 \quad z = 1^5$

$xy^1z = 1^{10} \in L$

$xy^2z = 1^2 1^6 1^5 = 1^{13} \notin L$

Since $xy^iz \notin L$ for $i = 2$, the assumption is wrong as it is a contradiction.

The language is not regular.