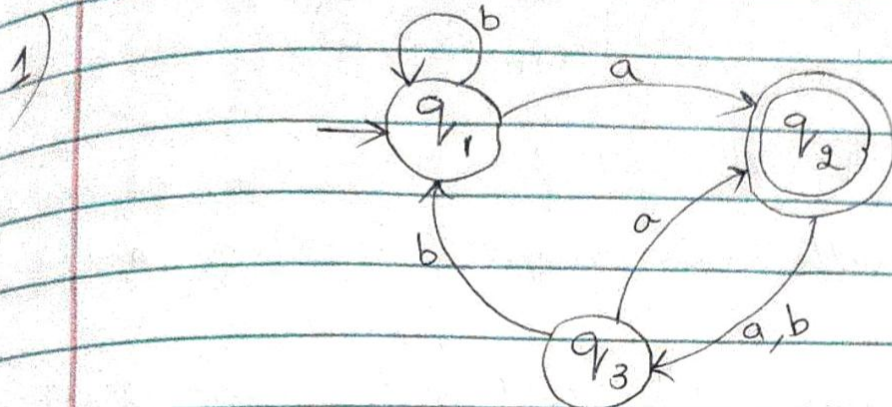


# Theory of Automata

## Homework - 2



$M_1$

a) The start state for the  $M_1$  DFA is  $q_1$

b) The set of accept states for  $M_1$  is  $\{q_2\}$

c) On giving the input "aabb", the machine goes through following states



d) The machine accepts the string  $w$  if  $(q_0, w) \vdash_{M_1}^* (q, \epsilon)$  for some  $q \in F$

Here  $F = \{q_2\}$

So  $(q_1, aabb) \vdash_{M_1} (q_2, abb)$

$\vdash_{M_1} (q_3, bb)$

$\vdash_{M_1} (q_1, b)$

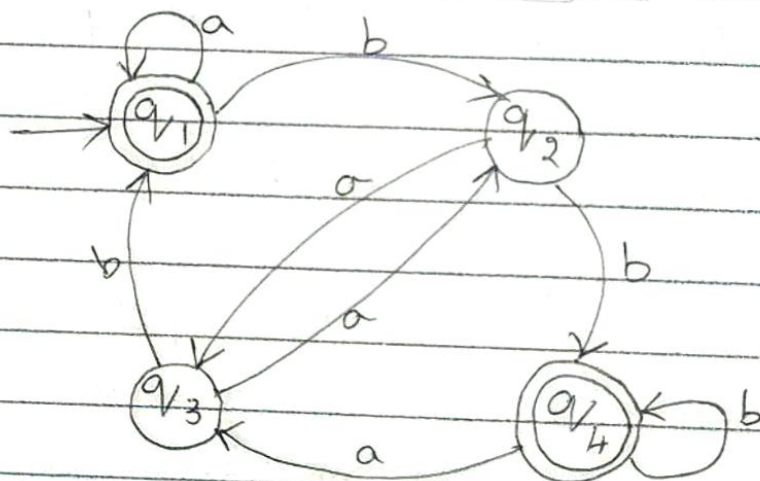
$\vdash_{M_1} (q_1, \epsilon)$

$$\Rightarrow (q_1, aabb) \vdash_{M_1}^* (q_1, \epsilon)$$

$$q_1 \notin \{q_2\}$$

$M_2$  will not accept the string "aabb"

e) The machine  $M_1$  will not accept string  $\epsilon$  as DFA does not accept  $\epsilon$ .

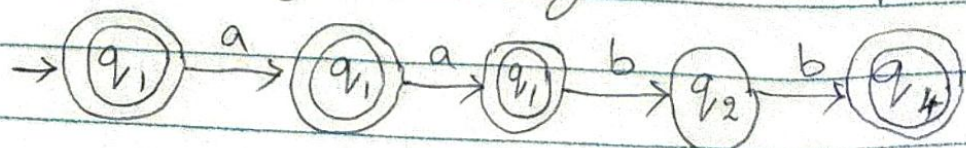


$M_2$

a) Start state  $q_1$

b) Set of accept states for  $M_2$  is  $\{q_1, q_4\}$

c)  $M_2$  goes through following states for input "aabb"





$$d) (q_1, aabb) \vdash_{M_2} (q_1, abb)$$

$$\vdash_{M_2} (q_1, bb)$$

$$\vdash_{M_2} (q_2, b)$$

$$\vdash_{M_2} (q_4, \epsilon)$$

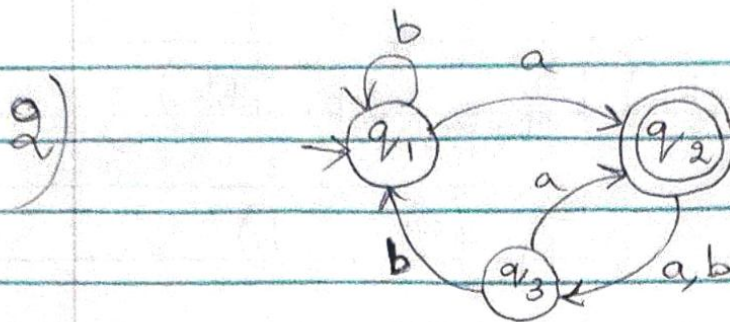
$$\Rightarrow (q_1, aabb) \vdash_{M_2}^* (q_4, \epsilon)$$

$q_4 \in \{q_1, q_4\}$  set of final states.

$M_2$  will accept the string "aabb"

e) The machine  $M_2$  will not accept the string  $\epsilon$  because

-  $\epsilon$  move is not allowed in DFA  
and  $\epsilon$  move will change it from DFA to NFA.



$M_1$

$$M_1 = (\{q_1, q_2, q_3\}, \{a, b\}, \delta, q_1, \{q_2\})$$

$Q = \{q_1, q_2, q_3\}$  are set of states

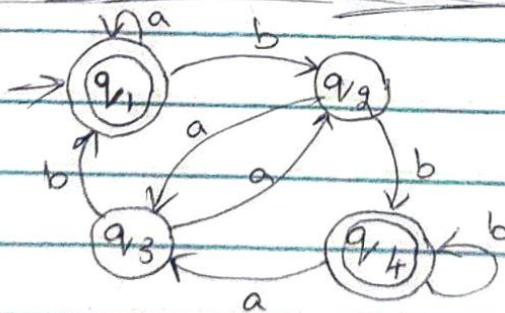
$\Sigma = \{a, b\}$  are alphabets

$$\delta =$$

	a	b
$q_1$	$q_2$	$q_1$
$q_2$	$q_3$	$q_3$
$q_3$	$q_2$	$q_1$

$q_0 =$  start state ( $q_1$ )

$F =$  set of final states  $\{q_2\}$



$= M_2$

$$M_2 = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$



$$\delta =$$

	a	b
$q_1$	$q_1$	$q_2$
$q_2$	$q_3$	$q_4$
$q_3$	$q_2$	$q_1$
$q_4$	$q_3$	$q_4$

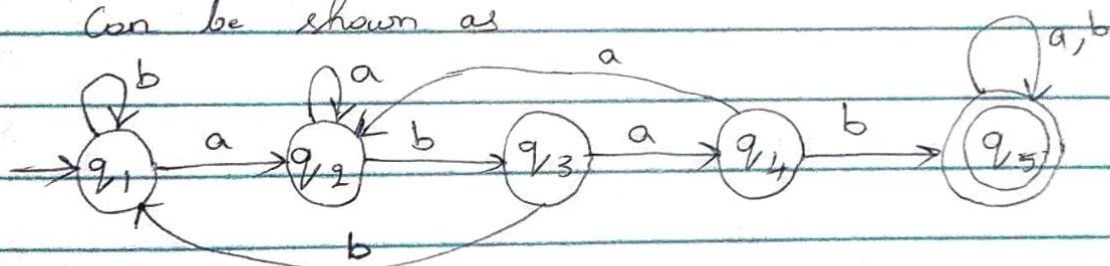
$q_0 = \text{start state} = q_1$

$F = \text{set of final states} = \{q_1, q_4\}$

$$\Rightarrow M_2 = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_1, q_4\})$$

3) DFA for

3.1)  $\{w \in \{a, b\}^* : w \text{ has } \text{abab} \text{ as substring}\}$   
 can be shown as



$$M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = \{a, b\}$$

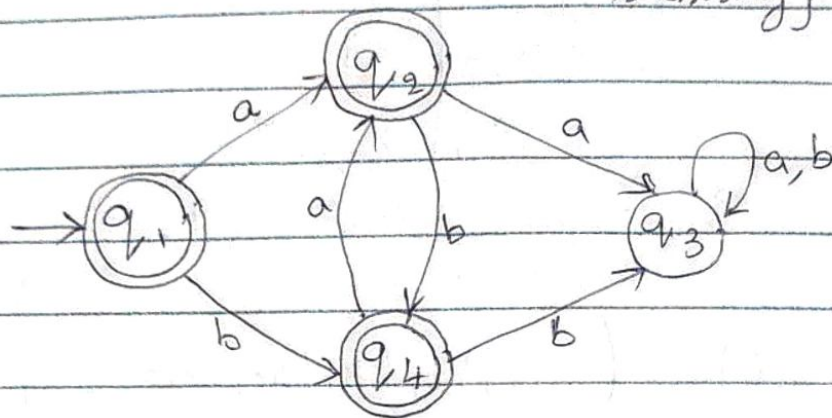
$$q_0 = q_1$$

$$F = \{q_5\}$$

$$\delta =$$

	a	b
$q_1$	$q_2$	$q_1$
$q_2$	$q_2$	$q_3$
$q_3$	$q_4$	$q_1$
$q_4$	$q_2$	$q_5$
$q_5$	$q_5$	$q_5$

3.2) DFA for  
 $\{w \in \{a, b\}^* : w \text{ has neither } aa \text{ or } bb \text{ as a substring}\}$



$$M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_1$$

$$F = \{q_1, q_2, q_4\}$$

$$\delta =$$

	a	b
$q_1$	$q_2$	$q_4$
$q_2$	$q_3$	$q_4$
$q_3$	$q_3$	$q_3$
$q_4$	$q_2$	$q_3$