Theory of Automata – Home Work 4

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1. **Prove that $\{0^n1^n2^n: n \geq 1\}$ is not a regular language.**

Sol: Suppose **A1** = $\{0^n1^n2^n: n \geq 1\}$ is regular. Let p be the pumping length given by the pumping lemma. Choose s = 0p1p2p. By the lemma, |xy| ≤ p and |y| > 0 therefore p ≥ 0 so s ∈ A1. Clearly, |s| ≥ p thus s = xyz for some x, y and z. Since |xy| ≤ p, xy cannot extend beyond the first p symbols of s, meaning xy = 0k where 1 ≤ k ≤ p. Let us write x = 0a, y = 0b , z = 0c 1p2p. The number of 0's, 1's and 2's in s are given by a + b + c = p. Let i = 0 such that s = xyi z = xz. The number of 1's in s is p whereas the number of 0's in s is a + c. For s ∈ A, the number of 0's in s must equal the number of 1's in s , namely a + c = p. Substituting for p, we have a + c = a + b + c with equality holding when b = 0. Because |y| > 0 and |y| = b, b > 0, thus s ∈/ A, a contradiction. Therefore, **A1** is non-regular.

2. **For arbitrary constant c, is $\{0^n1^n2^n: n \geq c\}$ regular or not?**

Sol: Suppose **A1** = $\{0^n1^n2^n: n \geq c\}$ is regular. Let p be the pumping length given by the pumping lemma. Choose s = 0p1p2p. By the lemma, |xy| ≤ p and |y| > 0 therefore p ≥ 0 so s ∈ A1. Clearly, |s| ≥ p thus s = xyz for some x, y and z. Since |xy| ≤ p, xy cannot extend beyond the first p symbols of s, meaning xy = 0k where 1 ≤ k ≤ p. Let us write x = 0a, y = 0b , z = 0c 1p2p. The number of 0's, 1's and 2's in s are given by a + b + c = p. Let i = 0 such that s = xyi z = xz. The number of 1's in s is p whereas the number of 0's in s is a + c. For s ∈ A, the number of 0's in s must equal the number of 1's in s , namely a + c = p. Substituting for p, we have a + c = a + b + c with equality holding when b = 0. Because |y| > 0 and |y| = b, b > 0, thus s ∈/ A, a contradiction. Therefore, **A1** is non-regular.

3. **The decimal notation for a number is the number written in the usual way, as a string over the alphabet $\{0, 1, \cdots 9\}$. For example, the decimal notation for 13 is a string of length 2. In unary notation, only the symbol "I" is used; thus 5 would be represented as IIIII in unary notation. Show that each of the following is or is not a regular language. (For regular languages, write down its regular expression or describe the automata accepting it; for languages that are not regular, prove it using pumping lemma)**

3.1 {w: w is the unary notation for a number that is a multiple of 7}

Sol : L = {w : w is the unary notation for a natural number that is a multiple of 7}. L is regular since it can be described by the regular expression (1111111)*.

3.2 $\{\mathbf{w}: \mathbf{w}$ is the unary notation for $10^n, n \geq 1\}$

Sol: L = {w : w is, for some n ≥ 1, the unary notation for 10n }. So L = {1111111111, 1100, 11000, …}. L isn't regular, since clearly any machine to accept L will have to count the 1's. We can prove this using the pumping lemma: Let w = 1P , N ≤ P and P is some power of 10. y must be some number of 1's. Clearly, it can be of length at most P. When we pump it in once, we get a string s whose maximum length is therefore 2P. But the next power of 10 is 10P. Thus s cannot be in L.