L11.2 predicate Logic - the language

Chapter II - 1 (Chapter 1 of Part II)

3. Study the definitions of *sentences* and *open formulas*

3.1 sentences and open formulas

Occurrences of variables, free occurrence, free variables, bound variables

(i) $((\forall x)R(x,y))$ is a formula in which y occurs free but x does not. The formula $((\exists y)((\forall x)R(x,y)))$ has no free variables; it is a sentence.

(ii) A variable may have both a free and a bound occurrence in a single formula as do both x and y in $(((\forall x)R(x,y)) \vee ((\exists y)R(x,y)))$.

(iii) If $\varphi(x)$ is $(((\exists y)R(x,y)) \wedge ((\forall z)\neg Q(x,z)))$ and t is $f(w,u)$, then $\varphi(t) = \varphi(x/t)$ is $(((\exists y)R(f(w,u),y)) \wedge ((\forall z)\neg Q(f(w,u),z)))$. The term $g(y,s(y))$ would, however, not be substitutable for x in $\varphi(x)$.

Definition 2.6:

(i) A subformula of a formula φ is a consecutive sequence of symbols from φ which is itself a formula.

(ii) An occurrence of a variable v in a formula φ is bound if there is a subformula ψ of φ containing that occurrence of v such that ψ begins with $(\forall v)$ or $(\exists v)$. (This includes the v in $\forall v$ or $\exists v$ that are bound by this definition.) An occurrence of v in φ is free if it is not bound.

(iii) A variable v is said to occur free in φ if it has at least one free occurrence there.

(iv) A sentence of predicate logic is a formula with no free occurrences of any variable, i.e., one in which all occurrences of all variables are bound.

(v) An open formula is a formula without quantifiers.

3.2 Substitutions

Recall the application of the definition of a concept to the use of the concept, we need substitution there!

1. When working backward from statement (B1),
   - The main concept in (B1) is subset: name - subset; arguments - Σ, $Cn(\Sigma)$.
   - Definition of subset:
     A is a **subset** of B if $\forall x \;\; x \in A \Longrightarrow x \in B$.
     $\forall x$ ($\in (x, A) \rightarrow \in (x, B)$) $\rightarrow$ subset(A, B)
   - Substitution the arguments Σ, $Cn(\Sigma)$ in the use of the concept subset for the variables A and B in the definition
     A: Σ
     B: $Cn(\Sigma)$
   - Result of applying the definition to the use of the concept: the *definition instance* for the use of subset in statement (B1)
     (A1) Σ is a **subset** of $Cn(\Sigma)$ if $\forall x \;\; x \in \Sigma \Longrightarrow x \in Cn(\Sigma)$.

     $\forall x$ ($\in (x, \Sigma) \rightarrow \in (x, Cn(\Sigma))$) $\rightarrow$ subset(Σ, $Cn(\Sigma)$)

Definition 2.7: Substitution (or Instantiation) If φ is a formula and v a variable, we write $\varphi(v)$ to denote the fact that v occurs free in φ. If t is a term, then $\varphi(t)$, or $\varphi(v/t)$, is the result of substituting (or instantiating) t for all free occurrences of v in φ. We call $\varphi(t)$ an instance of φ. If $\varphi(t)$ contains no free variables, we call it a ground instance of φ.

Definition 2.8: If the term t contains an occurrence of some variable x (which is necessarily free in t) we say that t is substitutable for the free variable v in $\varphi(v)$ if all occurrences of x in t remain free in $\varphi(v/t)$.

About unique "readability" of a term or a formula

5-5-5 - syntax
--- parsing
(5-5)-5

(5-(5-5))

Study definitions. Some examination points

- Variables: bound, free,
  (p(x) /\ (((\forall x) G(x)) /\ H(x)))
  ((\forall x) (G(x) /\ H(x)))

  ((\exists x) (G(x) /\ ((\exists x) H(x))))

- Substitution
- Substitutable
  f(x, y) is not substitutable for x in formula ((\forall y)P(x))

Example of a formula and substitution

Sentence

- English: A is a subset of B if for any x, if x belongs to A then x belongs to B.
- Formula
  - Predicates
    - subset(X, Y): X is subset of Y
    - belongsTo(X, Y): X belongs to Y
  - Resulting formula
    subset(A, B) ← (
    (\forall x), (belongsTo(x, A) → belongsTo(x, B)))

(A X B) is a **subset** of (CXD)

Do substitution: A / (A X B) and B / (CXD) in the formula above
subset(A X B, CXD) ← (
(\forall x), (belongsTo(x, AXB) → belongsTo(x, CXD)))

For all x, x is an even number. (i doesn’t make sense to replace very x by a term)

Prove: if A is subset of B and B is subset of C, then A is a subset of C.

Unique readability of a term or a formula

- If t is a term, what is the form (the decomposition) of t?
  Is a constant, a variable, f(t1, …, t2) where f is a function symbol and ti a term.

Counterexample in other language: 1-1-1 ((1-1)-1) (1-(1-1))

- If \alpha is a formula, what is the form (the decomposition) of \alpha?
  Atomic formula, or (\alpha \/ \beta), …

Project

- Windows
    - Putty

Drawing for formulas (graphical form)

- Focus on intuition.

Semantics: meaning and truth - motivation