1. Determine whether the following statement is correct or wrong, and state your reason.

(). Let A, B be two languages over $\{a, b\}$. If both A and $A \cup B$ are regular, then B is regular.

False, consider $A = \{a, b\}^*$ and $B = \{a^n b^n : n \geq 0\}$.

(). Let L be a regular language, then $\{w \in L : |w|\ is\ even\}$ is also regular.

True. All even length language over the alphabet is regular, intersecting with L is also regular by closure property.

(). Every regular language can be generated by a context free grammar.

True. Every regular language is a context-free language.

e). Let L_1 be context-free language and L_2 be regular language, then $L_1^* L_2^*$ is also context-free.

True. CFG is closed under Kleene star and concatenation.

(). There exists some context-free language which cannot be decided by a Turing machine.

False. $L_{CFG} \subseteq L_{REC}$

(). Even if we take the union, intersection and concatenation of regular languages infinitely many times, we cannot generate a context-free language.

False. Consider $L_i = \{a^i b^i\}$

( ) The complement of every recursive enumerable language is recursive enumerable.

False. (Closure property)

( ) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

False. It is Semi-decidable.

( ) Every regular language is recursively enumerable.

True. Language Hierarchy.

2. (a) Prove that $\{a^{6n}b^{3n}c^{2n} : n \geq 0\}$ is not context-free using pumping theorem.

   Same proof to Exam 2 question.

   (b) The followings prove a contrary to (a), determine each of them is right or wrong.

   (i) $\{a^{6n}b^{3n}c^{2m}, n \geq 0, m \geq 0\}$ is context-free

   True

   (ii) $\{a^{6n}b^{3m}c^{2m}, n \geq 0, m \geq 0\}$ is context-free

   True

   (iii) Intersection of two context-free languages is context-free.

   False.

   (iv) $\{a^{6n}b^{3n}c^{2m}, n \geq 0, m \geq 0\} \cap \{a^{6n}b^{3m}c^{2m}, n \geq 0, m \geq 0\} = \{a^{6n}b^{3n}c^{2n} : n \geq 0\}$

True.

(This is one of most challenging problems in this class and represent the most difficult question that you might encounter in multiple choice questions. There will be one or two such questions.)

(6%) Let A and B be disjoint, recursively enumerable languages. Further let $\overline{A \cup B}$ also be recursively enumerable. What can you say about A and B?

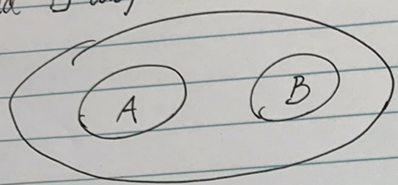
(a) It is possible that neither A nor B is decidable.

(b) At least one among A and B is decidable.

(c) Both A and B are decidable.

Recursively enumerable = Semi-decidable

A and B disjoint



ground set

$$\Rightarrow A \subseteq \overline{B}$$
$$B \subseteq \overline{A}$$

[Formal proof: $A \cap B = \emptyset \Rightarrow \overline{A} \cup \overline{B} = U$

$$\Rightarrow A \cap (\overline{A} \cup \overline{B}) = A$$
$$\parallel$$
$$(A \cap \overline{A}) \cup (A \cap \overline{B})$$
$$\parallel$$
$$A \cap \overline{B}$$]

A and $\overline{A \cup B}$ semi-decidable

closure property

$$A \cup (\overline{A \cup B}) = A \cup (\overline{A} \cap \overline{B})$$
$$= (A \cup \overline{A}) \cap (A \cup \overline{B})$$
$$= A \cup \overline{B}$$
$$= \overline{B} \quad \text{semi-decidable}$$

similarly, $B \cup (\overline{A \cup B}) = B \cup \overline{A} = \overline{A}$ semi-decidable

Since A and $\overline{A}$ both semi-decidable

$\Rightarrow$ A is decidable (slide 32)

Since B and $\overline{B}$ both semi-decidable
B is decidable