# CS 5383

# Theory of Automata

1. Select one correct answer out of 4 choices (1.5 point * 10).

1.1 Which of the followings belong to $2^{\{a,b\}} \times \{a, b\}$? (C)

a).$\{\{a\}, a\}$ b).$\{a, \{a\}\}$

c).$(\{a\}, a)$ d).$(a, \{a\})$

1.2 Which of the following statements is **wrong**? (C)

a). A set is a subset of itself

b). Emptyset is a subset of any set

c). A set is a subset of its powerset

d). The cardinality of emptyset is 0

1.3. Which of the following statements is **correct**? (B)

a). A string is a set of symbols from an alphabet

b). The length of the concatenation of two strings can be the same as one of the them

c). The length of a string is at least 1

d). The concatenation of prefix and suffix of a string $w$ is $w$ itself

1.4 Which of the followings describe the regular expression $(\Sigma\Sigma)^*$ (B)

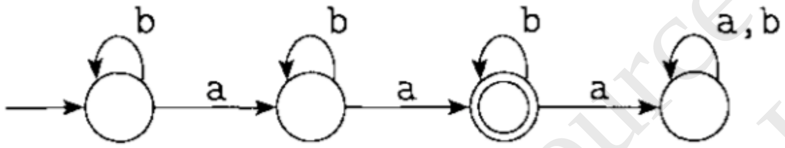
a). Any strings over alphabet $\Sigma$

b). Any strings of even length over alphabet $\Sigma$

c). $\{aa: a \in \Sigma\}$, i.e., any strings consisting of two identical symbols

c). $\{(aa)^*: a \in \Sigma\}$, i.e., any strings consisting of an even number of identical symbols

1.5. Consider the following DFA




Which of the followings describes its language over $\{a, b\}$? (C)

a). All string that contain at most two a's

b). All string that contain at least two a's

c). All string that contain exactly two a's

d). All strings that does not contain two a's

1.6 Which of the following statements is **correct**? (D)

a). If $A$ and $A \circ B$ are both regular languages, $A \cap B = \emptyset$, then $B$ is also a regular language

$A = a^*, B = \{a^i b^j: i \neq j, i, j \geq 1\}$

b). If $A$ and $A \cap B$ are both regular languages, then

$B$ is also a regular language

c). If $A$ and $(A \circ B)^*$ are both regular languages, $A \cap B = \emptyset$, then $B$ is also a regular language

d). If $A$ and $A \cup B$ are both regular languages, $A \cap B = \emptyset$, then $B$ is also a regular language

$$B = (A \cup B) \cap \bar{A}$$

1.7 Let $L$ be a regular language over alphabet $\Sigma$. Which of the followings is **correct**? (B)

a). It is possible that any subset of $L$ is not regular

b). All strings of $L$ that has an even length is regular

c). It is possible that for any $A \subseteq \Sigma^*$, $L \subseteq A$, $A$ is not regular

d). The set of all strings that is formed by the concatenation of strings in $L$ may be nonregular

1.8 Which of the followings is **wrong**? (A)

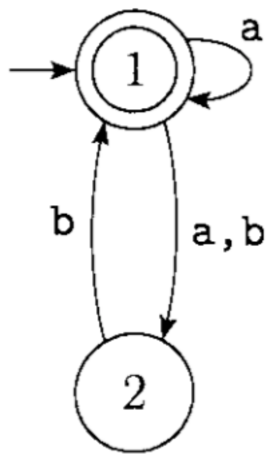
a). A nondeterministic finite automaton is also a deterministic finite automaton.

b). A deterministic finite automaton is also a nondeterministic finite automaton

c). If a language $L$ is regular, then there exists a deterministic finite automaton such that the set of strings it accepts is exactly $L$.

d). If a language $L$ is regular, then there exists a deterministic finite automaton such that the set of strings it does not accept is exactly $L$.

1.9 Consider the following NFA




Which of the following strings is **not** accepted by it? (B)

a). $aaaaba$

b). $bbbbb$

c). $ababab$

d). $aabba$

1.10 Which of the following statement is wrong? (D)

a). The intersection of two non-regular languages can be regular.

$$\{a^i b^i : i \geq 1\} \cap \{b^i a^i : i \geq 1\}$$

b). The union of two non-regular languages can be regular.

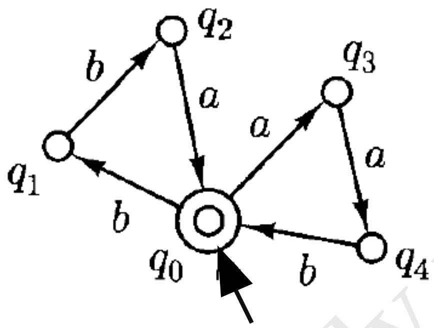
$$\{a^i b^i : i \geq 1\} \cup \left(\Sigma^* - \{a^i b^i : i \geq 1\}\right)$$

c). The concatenation of two non-regular languages can be regular.

$$\{a^i b^i : i \geq 1\} \circ b^*$$

d). The complement of a non-regular language can be regular.

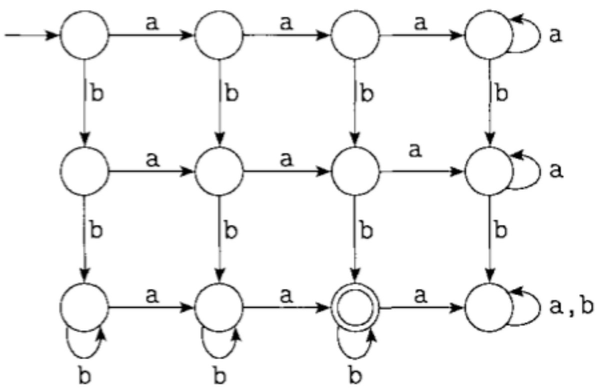
1. Write the regular expression for the language accepted by the following NFA. (1)




$(bba \cup aab)^*$

It is wrong to write $(bba)^*(aab)^*$

2. Consider the following DFA (2):




(The DFA is for $w$ contains exactly two a's and at least two b's)

2.1 Give 1 string that is accepted by the above DFA

2.2 Give 1 string that is not accepted by the above DFA

3. Write the regular expression for the following sets (3)

4.1 All strings over $\{a, b\}$ that are odd in length

$$(\mathrm{a} \cup \mathrm{b})\big((a \cup b)(a \cup b)\big)^*$$

4.2 All strings over $\{a, b\}$ whose length is **not** a multiple of 3

$$\Big((a \cup b) \cup \big((a \cup b)(a \cup b)\big)\Big)\big((a \cup b)(a \cup b)(a \cup b)\big)^*$$

4.3 All strings over $\{a, b\}$ that start with $aa$ and end with $bb$

$$aa(a \cup b)^*bb$$

4. Construct DFA or NFA that recognizes the following language (either draw a state diagram or write down the 5-tuple description). (1)

$\{\mathrm{w} \in \{\mathrm{a, b}\}^*: w \text{ has } abab \text{ as a substring}\}$

5. Prove that $\{a^n ba^m ba^{2m+n}: n, m \geq 1\}$ is not regular using pumping lemma (4).

   Proof: Suppose $L = \{a^n ba^m ba^{m+n}: n, m \geq 1\}$ is regular. According to the pumping theorem, there exists some constant $N_0$ such that if $|a^n ba^m ba^{m+n}| \geq n_0$, then there exist some $x, y, z$ such that $a^n ba^m ba^{m+n} = xyz$ such that $|xy| \leq n_0, |y| \geq 1,$ and $xy^i z \in L$ for any $i \in N$.

   Take $n = n_0, m = 1$, then we have $a^{n_0} baba^{1+n_0} = xyz$ such that $|xy| \leq n_0, |y| \geq 1,$ and $xy^i z \in L$ for any $i \in N$. Since $|xy| \leq n_0, |y| \geq 1,$ we know $x = a^\alpha, y = a^\beta$ for some $\beta \geq 1$. Hence $xy^i z = a^{\alpha+i\beta} a^{n_0-\alpha-\beta} baba^{1+n_0} \in L$ for any $i \geq 0$ by pumping theorem. However, taking $i = 2$, $a^{\alpha+2\beta} a^{n_0-\alpha-\beta} baba^{1+n_0} = a^{n_0+\beta} baba^{1+n_0}$. It is easy to see that $n_0 + \beta + 1 > n_0 + 1$ as $\beta \geq 1$, whereas $a^{n_0+\beta} baba^{1+n_0} \notin L$, contradicting that $xy^i z = a^{\alpha+i\beta} a^{n_0-\alpha-\beta} baba^{1+n_0} \in L$ for any $i \geq 0$. Hence, $L$ is not regular.

6. (4) Answer the following questions and state your reason (The alphabet is $\{a, b\}$ for all following questions)

6.1 Is $\{ab\}$ a regular language? For any fixed integer $i$, is $\{a^i b^i\}$

regular? (1)

Yes, write DFA or regular expression

6.2 Is $\{ab\} \cup \{a^2b^2\}$ regular? For any fixed integer $i$, is $\cup_{h=1}^{i} \{a^h b^h\}$ regular? (1)

Yes, write DFA or regular expression

6.3 Is $\lim_{i\to\infty} \cup_{h=1}^{i} \{a^h b^h\}$ regular? (2)

No. Proof using pumping lemma.