CS 5381 Analysis of Algorithms
Solutions to Homework 3

Fall 2022

1.

$$BB^T(i,j) = \sum_{e \in E} b_{ie} b^T_{ej} = \sum_{e \in E} b_{ie} b_{je}$$

- If $i = j$, then $b_{ie}b_{je} = 1$ whenever e enters or leaves vertex i, and 0 otherwise.
- If $i \neq j$, then $b_{ie}b_{je} = -1$ when $e = (i,j)$ or $e = (j,i)$, and 0 otherwise.

Thus,

$$BB^T(i,j) = \begin{cases} \text{degree of } i = \text{in-degree} + \text{out-degree} & \text{if } i = j \\ -(\text{\# of degrees connecting } i \text{ and } j) & \text{if } i \neq j. \end{cases}$$

2. The BFS procedure cares only whether a vertex is white or not. A vertex v must become non-white at the same time that $v.d$ is assigned a finite value so that we do not attempt to assign to $v.d$ again, and so we need to change vertex colors in lines 5 and 14. Once we have changed a vertex's color to non-white, we do not need to change it again.

3. The BFS algorithm does not assume that the adjacency lists are in any particular order. In the example on page 156 of the Lecture Notes, if t precedes x in $Adj[w]$, we can get the breadth-first tree as shown in this example.

4. Let (u, v) be an arbitrary edge of G, and suppose without loss of generality that $u.d < v.d$. Then the search must discover and finish v before it finishes u (while u is gray), since v is on u's adjacency list. If the first time that the search explores edge (u, v), it is in the direction from u to v, then v is undiscovered (white) until that time, for otherwise the search would have explored this edge already in the direction from v to u. Thus, (u, v) becomes a tree edge. If the search explores (u, v) first in the direction from v to u, then (u, v) is a back edge, since u is still gray at the time the edge is first explored.

5. Proof by contraction: Suppose the edge (u, v) was not a light edge crossing any cut of the graph, then this edge can not be contained in a minimum spanning tree.

6. A triangle whose edge weights are all equal is a graph in which every edge is a light edge crossing some cut. But the triangle is cyclic, so it is not a minimum spanning tree.