Nondeterminisitic Finite Automata

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TEXAS TECH UNIVERSITY

NFA vs. DFA

- What is the relationship between NFA and DFA languages?

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  -The language recognized by M is the set of all strings accepted by M, and denoted as $L(M)$

  - NFA (DFA) languages is the set of all languages accepted by some NFA (DFA)

NFA vs. DFA

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- NFA (DFA) languages is the set of all languages accepted by some NFA (DFA)

$L_{DFA} \subseteq L_{NFA}$, i.e., any DFA language is also an NFA language

Recall we gave a name to L_{DFA} -- regular language!

NFA and DFA definition

- NFA: A quintuple $M = (K, \Sigma, \Delta, s, F)$ where
  - K is a finite set of states
  - Σ is an alphabet
  - $s \in K$ is the initial state
  - $F \subseteq K$ is the set of final states (can be multiple)
  - Δ, the transition relation, is a subset of $K \times (\{\Sigma \cup \epsilon\}) \times K$

- DFA: A quintuple $M = (K, \Sigma, \Delta, s, F)$ where
  - …
  - δ: the transition function, a function from $K \times \Sigma$ to K

NFA vs. DFA

- A function is a special relation

  -> A DFA is also an NFA

  -> Any language recognized by a DFA is also recognized by an NFA

  -> $L_{DFA} \subseteq L_{NFA}$

NFA vs. DFA

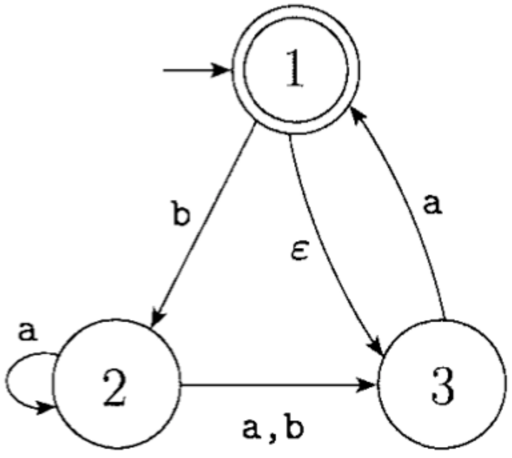
- An NFA is not necessarily DFA, but it may be equivalent to some DFA

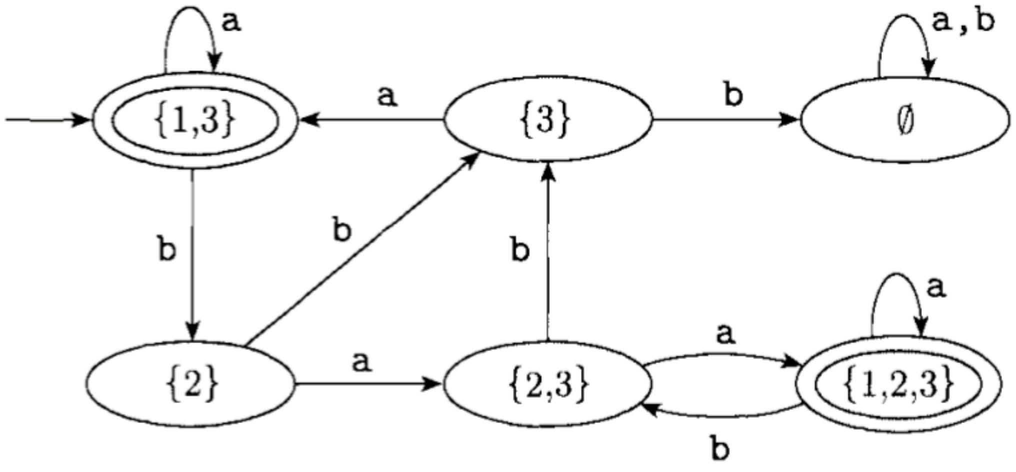
NFA vs. DFA

- We say two automata M_1 and M_2 are equivalent if $L(M_1) = L(M_2)$
- If any NFA admits an equivalent DFA, then $L_{NFA} \subseteq L_{DFA}$

An NFA is equivalent to some DFA

- Example





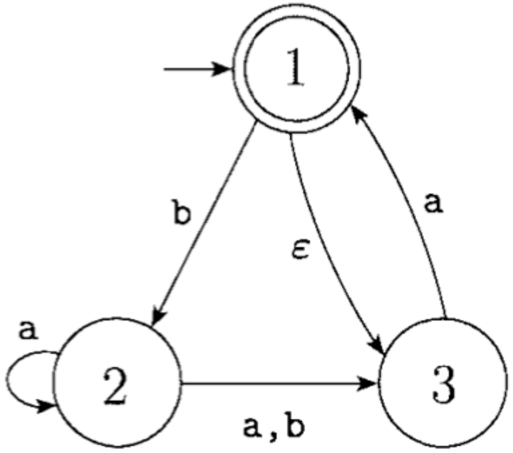
An NFA is equivalent to some DFA

- NFA: A quintuple $M = (K, \Sigma, \Delta, s, F)$
  - K: set of states
  - Σ: alphabet
  - $s \in K$: initial state
  - $F \subseteq K$: final states
  - Δ, the transition relation, is a subset of $K \times (\{\Sigma \cup e\}) \times K$

- DFA: A quintuple $M' = (K', \Sigma, \delta, s', F')$
  - $K' = 2^K$: set of states
  - Σ: alphabet
  - $s' = E(s)$: initial state
  - $F' = \{Q \subseteq K: Q \cap F \neq \emptyset\}$: final states
  - δ: $\delta(Q, a) = \cup\{E(p): p \in K \text{ and } (q, a, p) \in \Delta \text{ for some } q \in Q\}$

$$E(q) = \{p \in K: (q, e) \vdash_M^* (p, e)\}$$

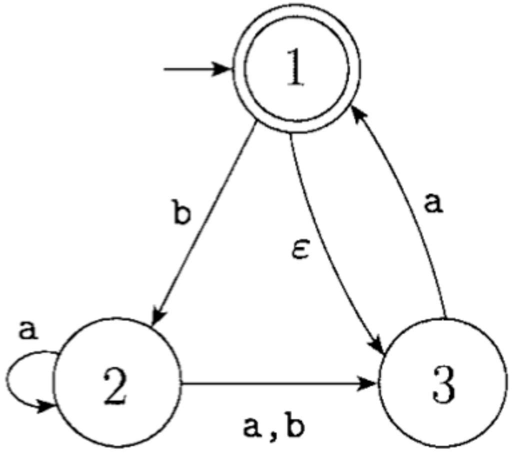
Example



All new states: power set

$$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

Example



All new states: power set

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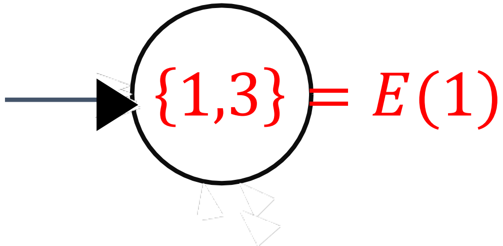
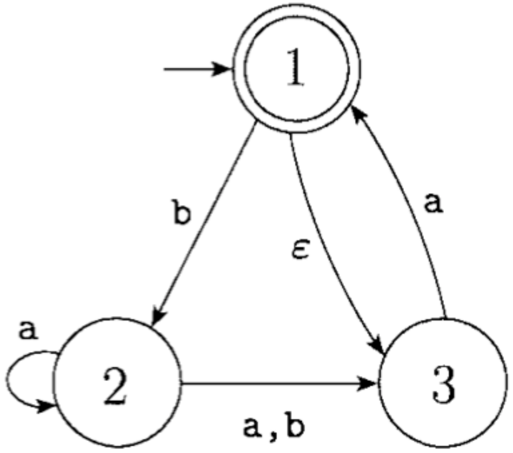
New start state: All "equivalent" states as "1"



Example



New final states: All states containing original final states

Example

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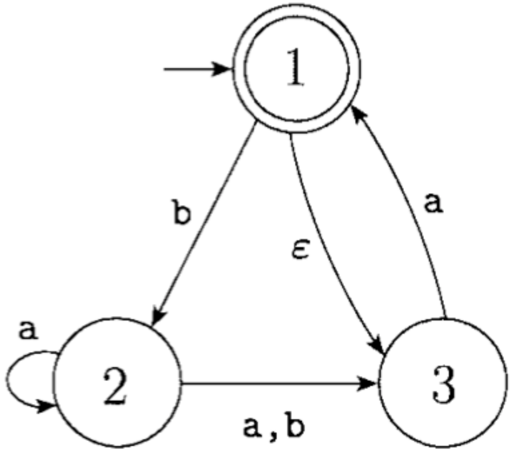
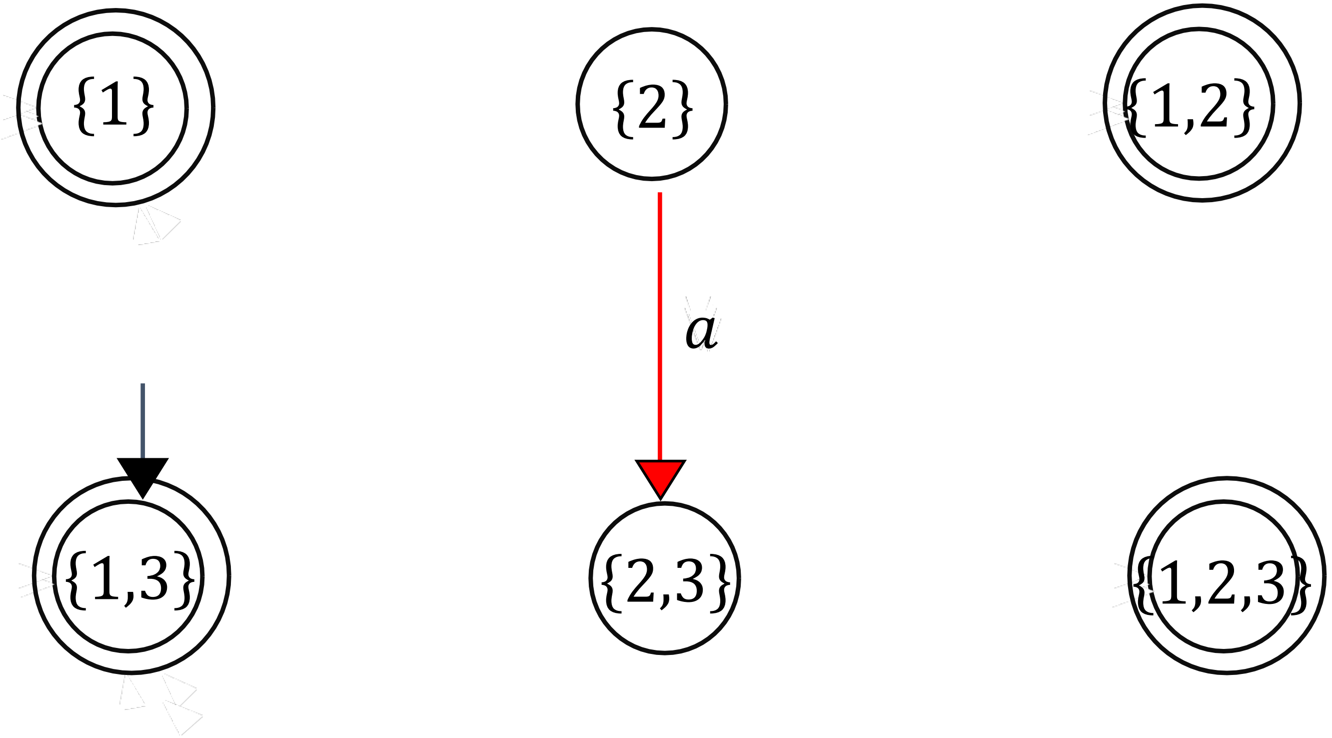
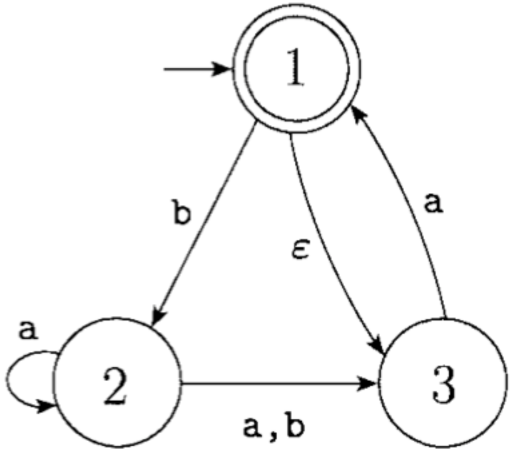


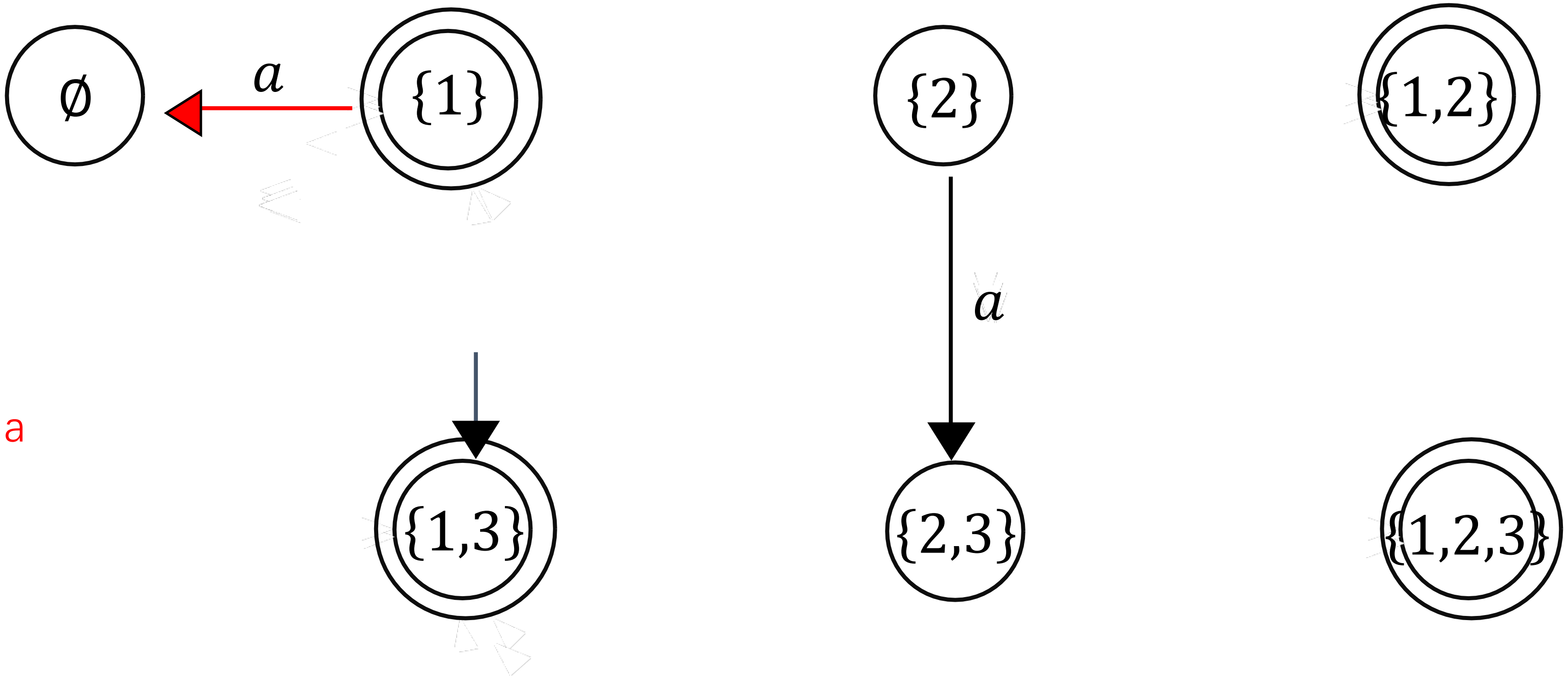


State 2 goes to both 2 and 3 on input a
And state 2 can only go to 2 and 3 on input a

Example

New final states: All states containing original final states

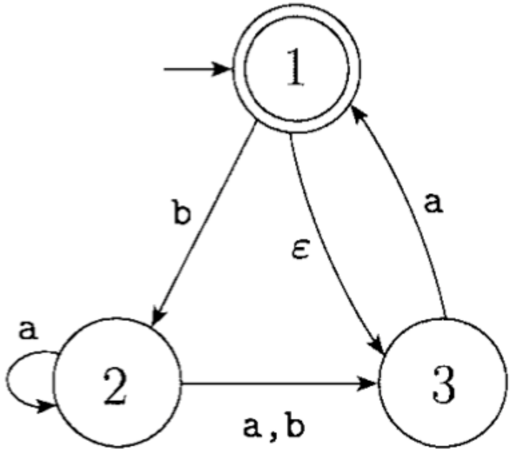


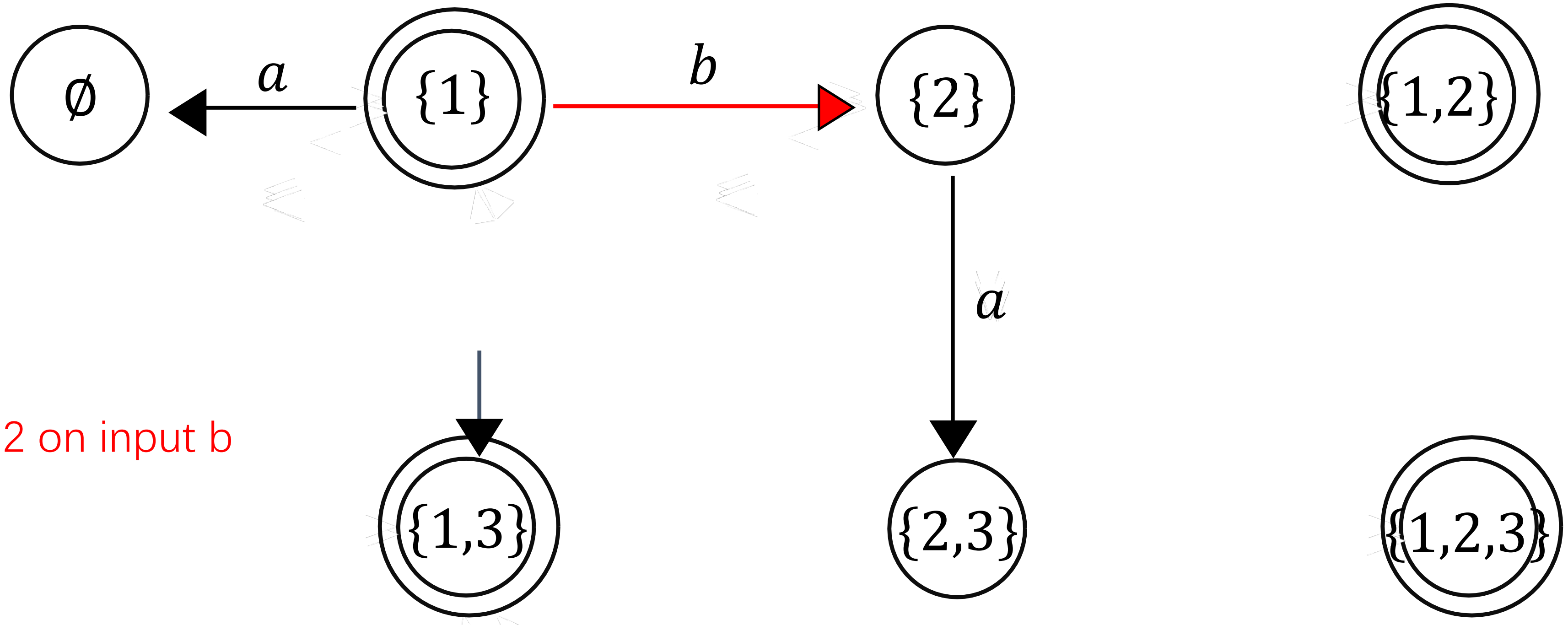


State 1 goes to no state on input a

Example

New final states: All states containing original final states

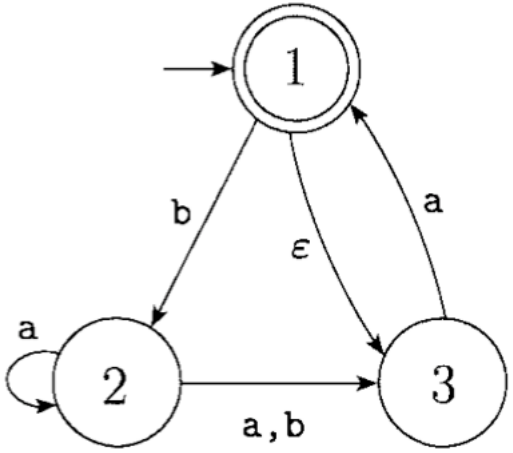


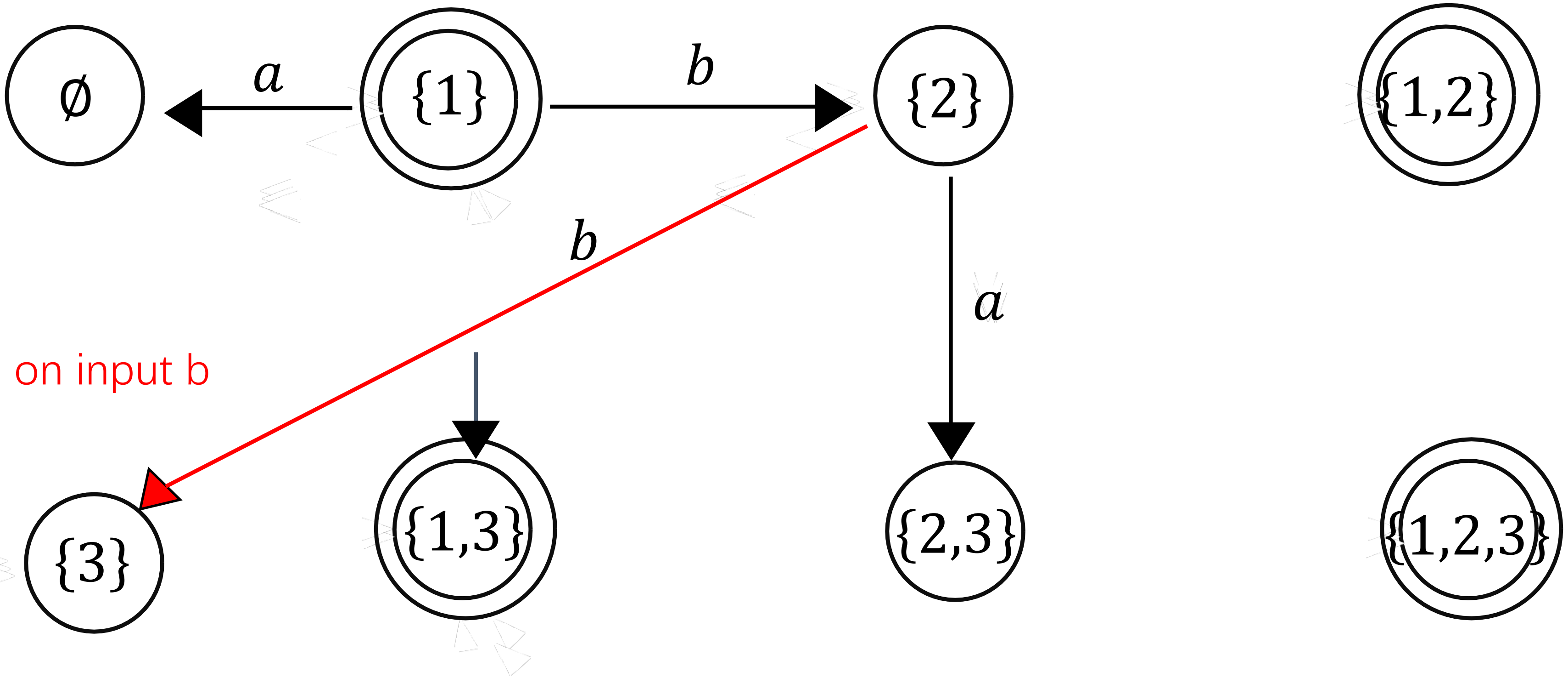


State 1 goes and only goes to state 2 on input b

Example

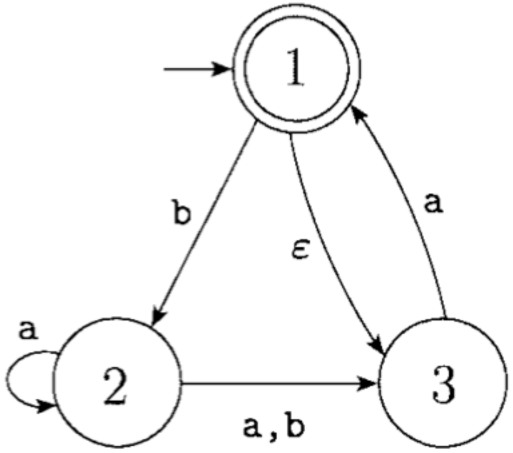
New final states: All states containing original final states



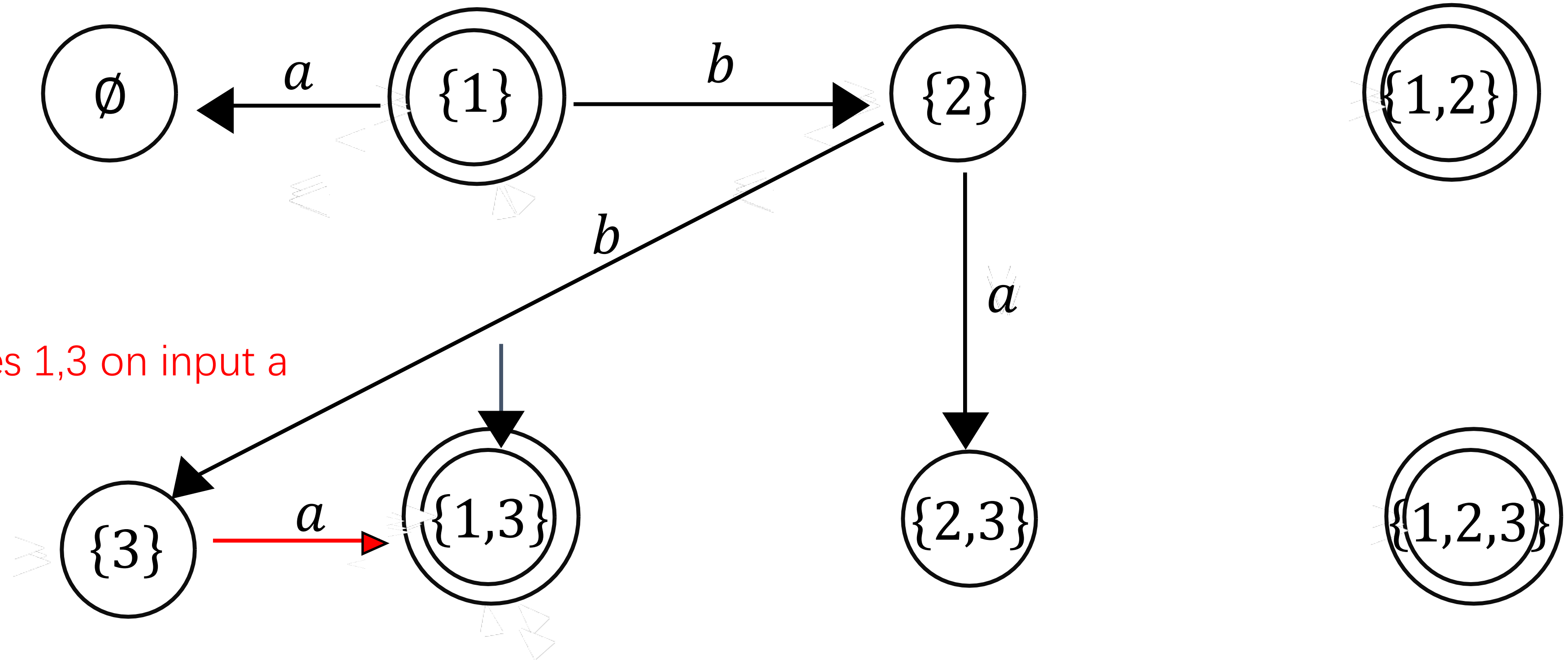


State 2 goes and only goes to state 3 on input b

Example

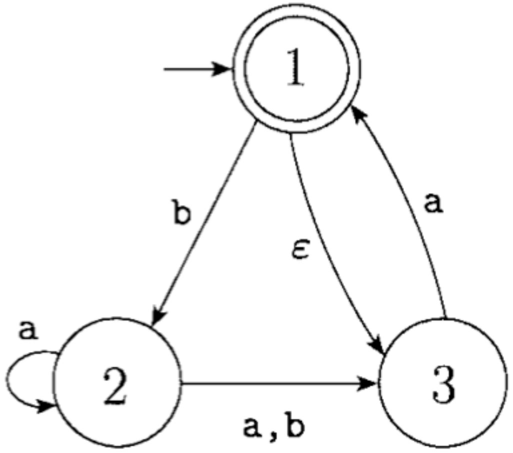


New final states: All states containing original final states



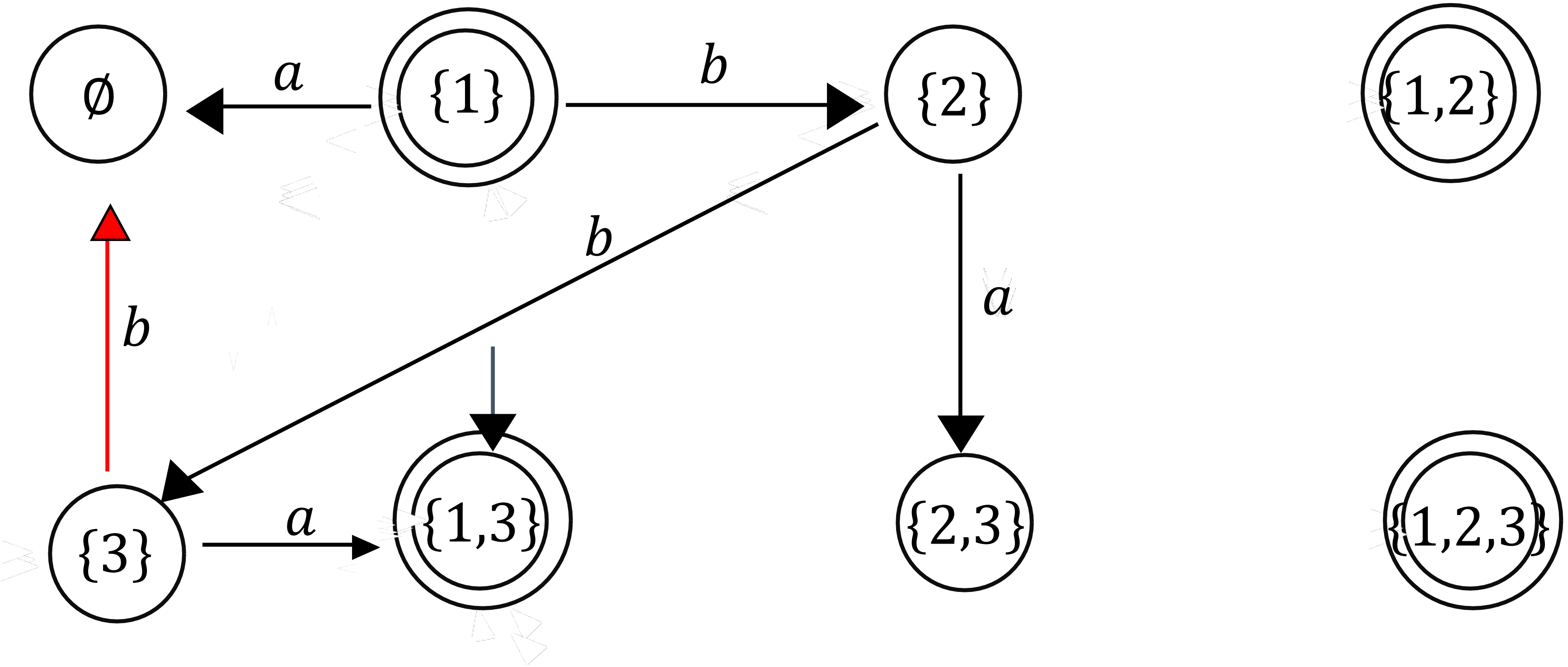
State 3 goes and only goes to states 1,3 on input a

Example



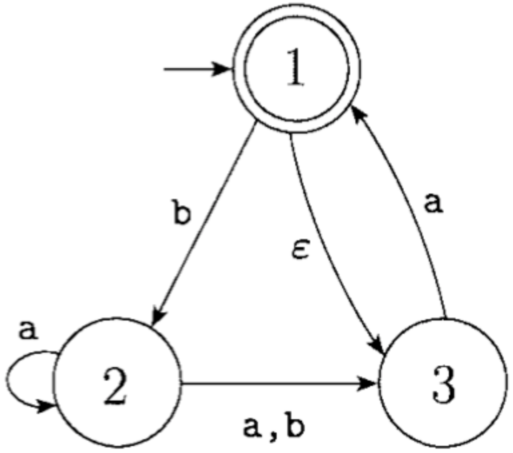
State 3 goes to no state on input b

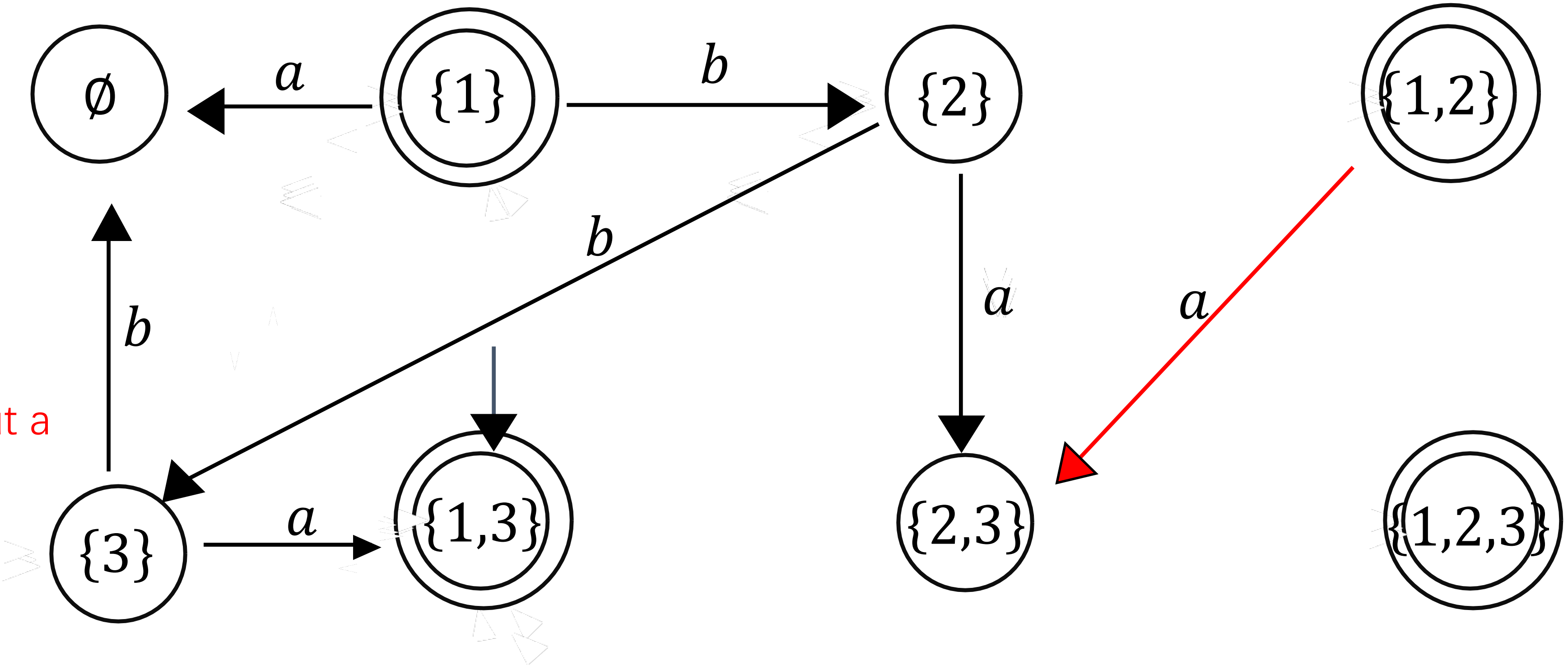
New final states: All states containing original final states



Example

New final states: All states containing original final states



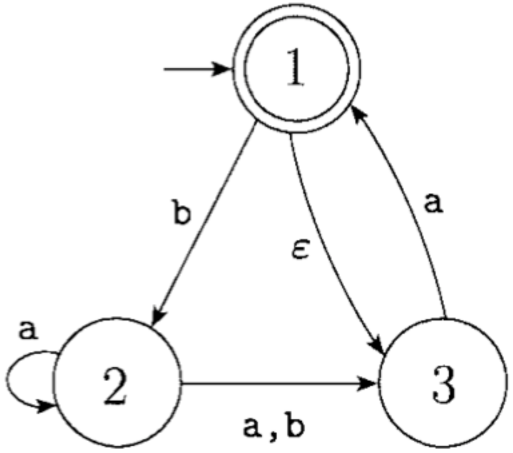


State 1 goes to no state on input a

State 2 goes and only goes
to both 2 and 3 on input a

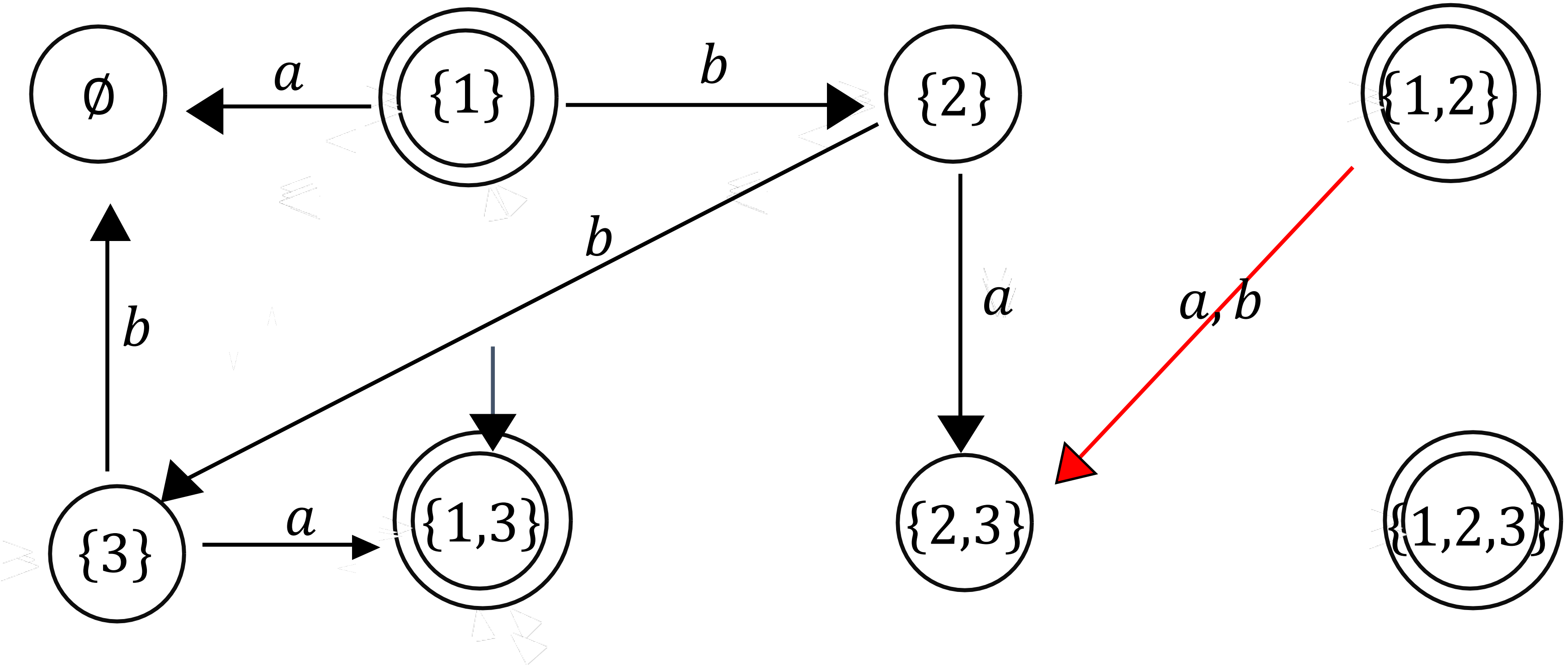
Example

New final states: All states containing original final states



State 1 goes and only goes to state 2 on input b

State 2 goes and only goes to state 3 on input b



Example

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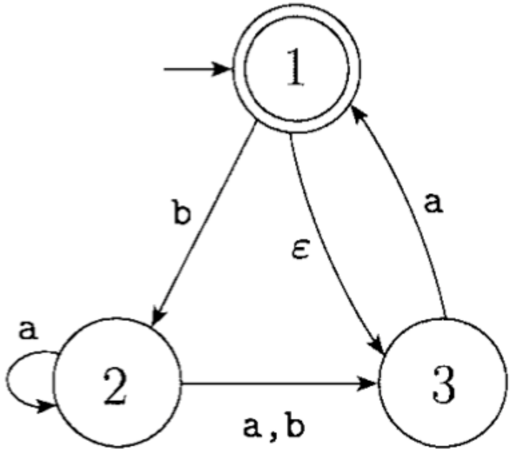
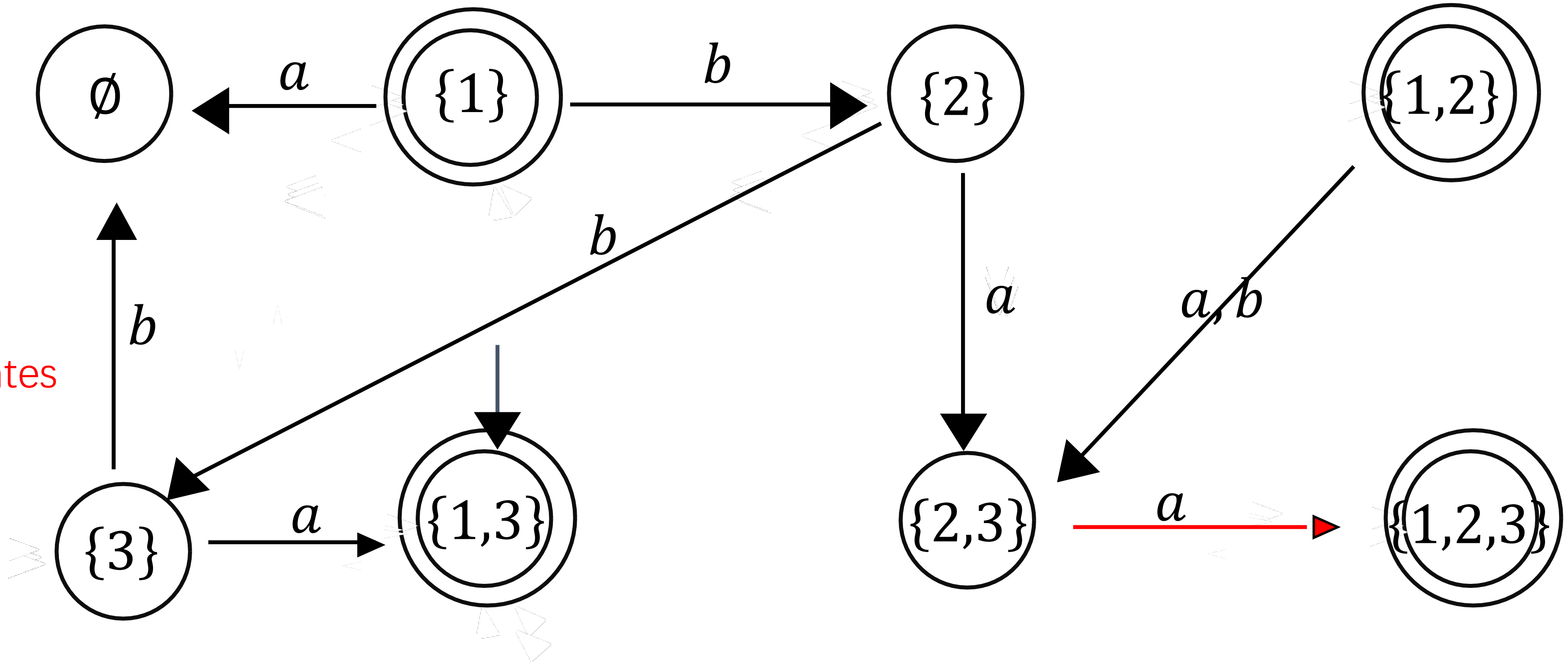


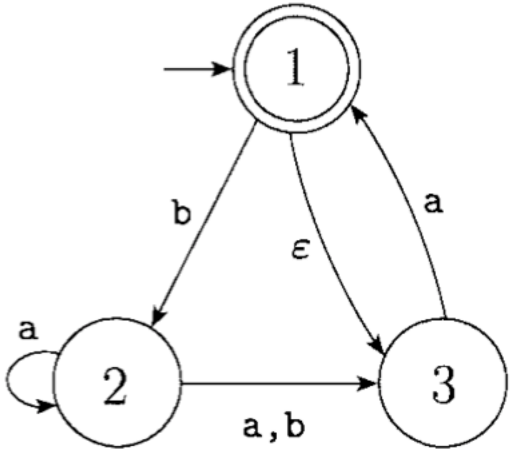


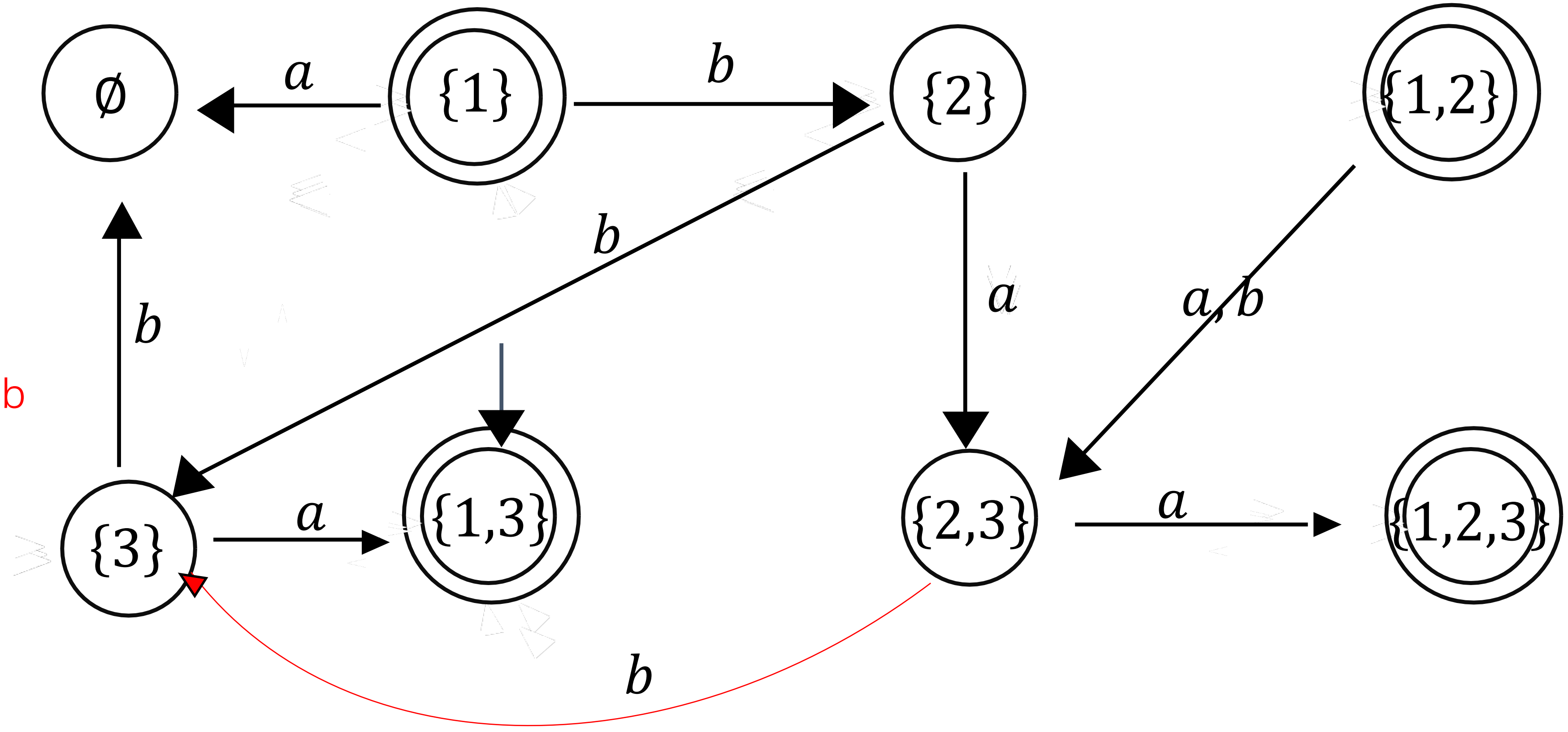
State 3 goes and only goes to states 1,3 on input a

State 2 goes and only goes to states 2, 3 on input a

Example

New final states: All states containing original final states





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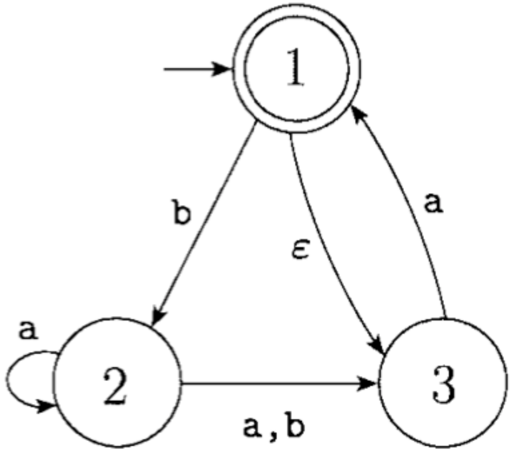

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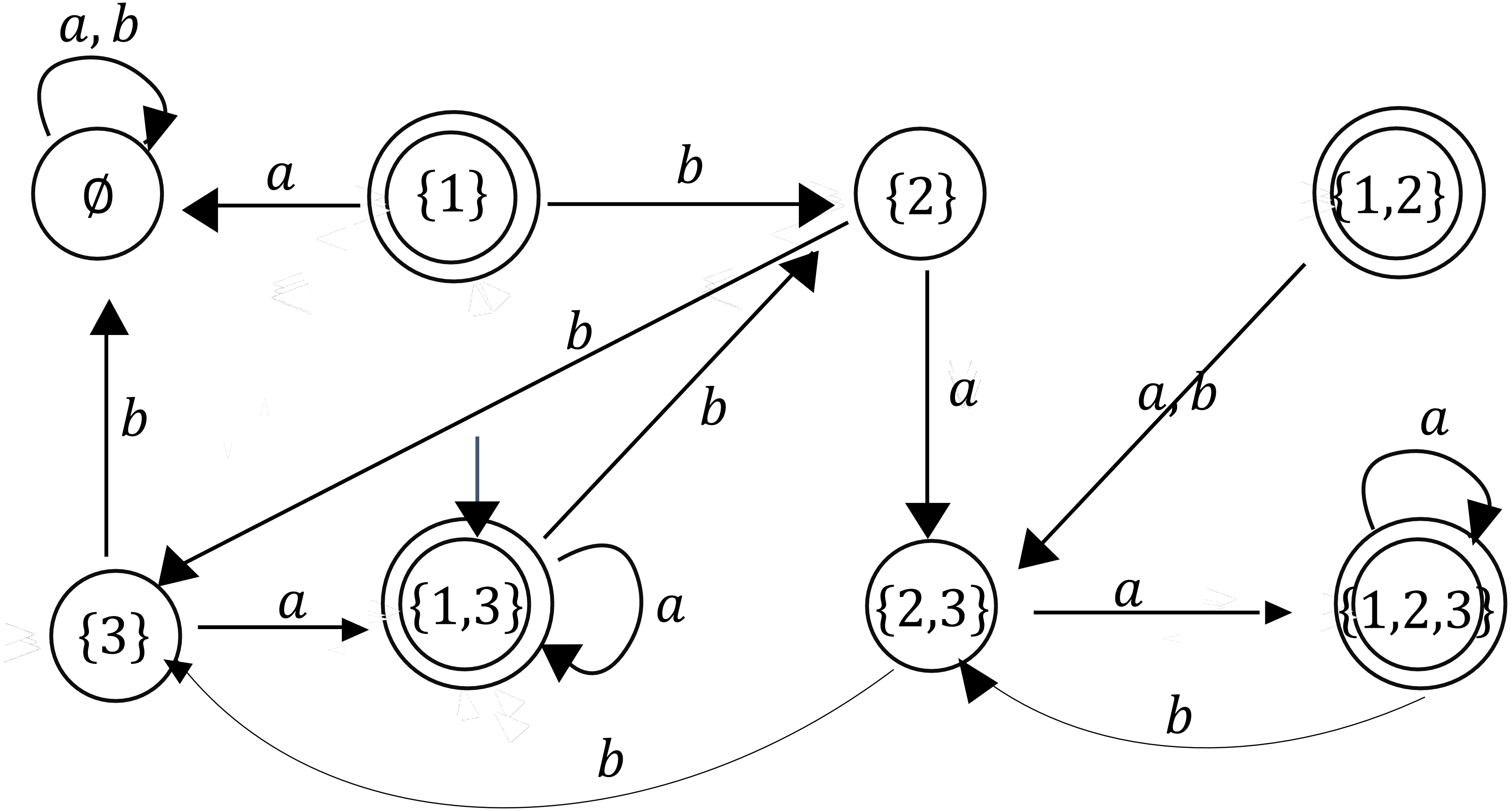
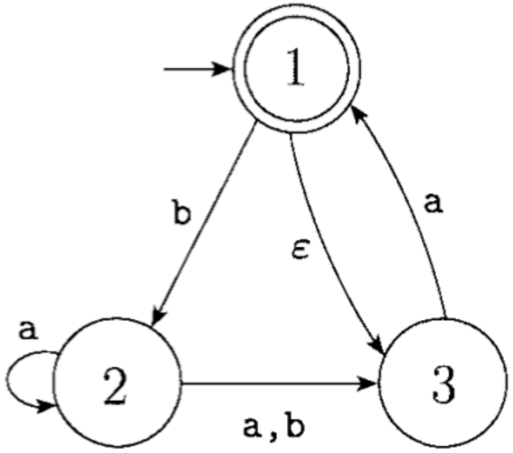
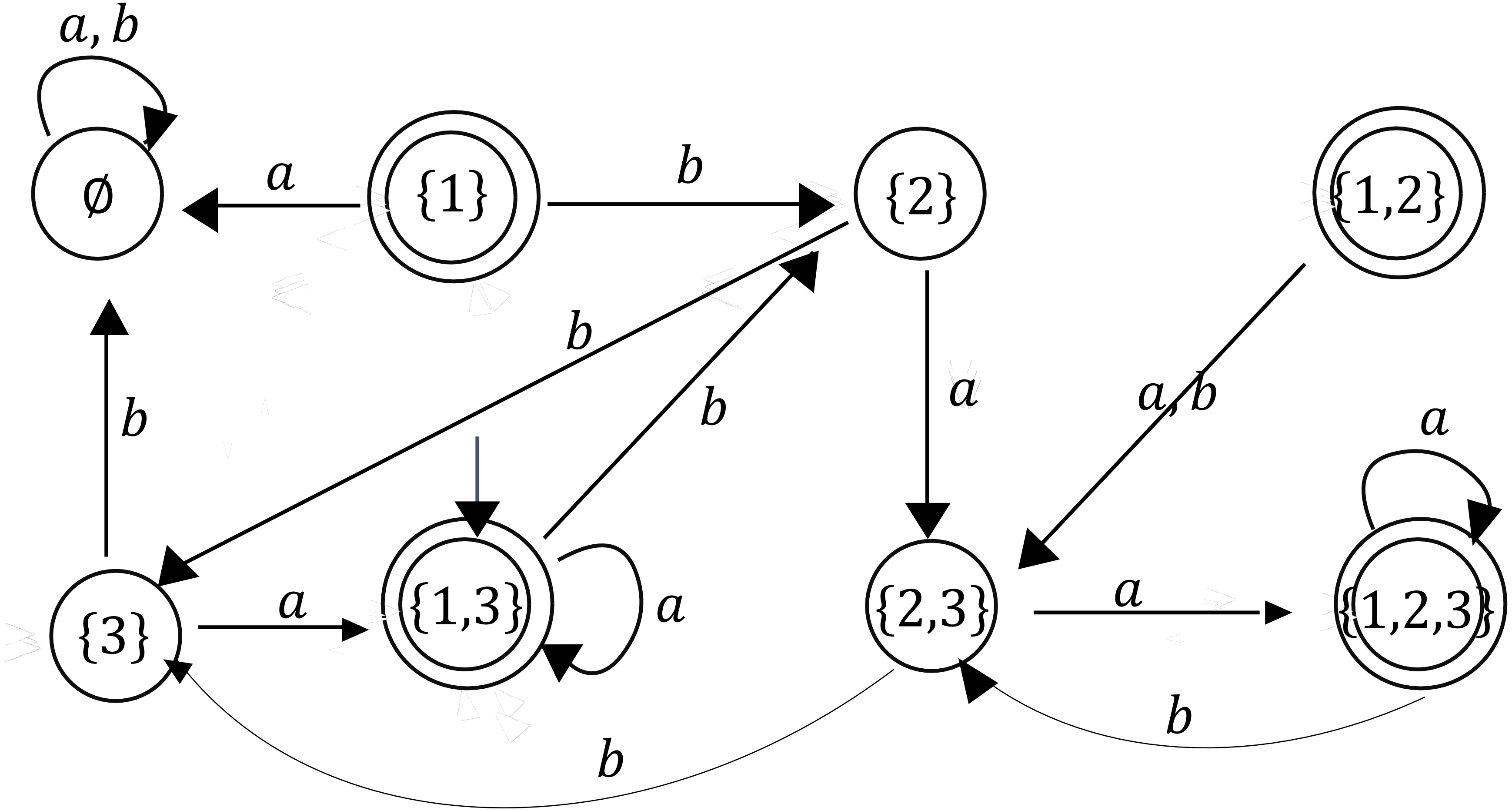


Example

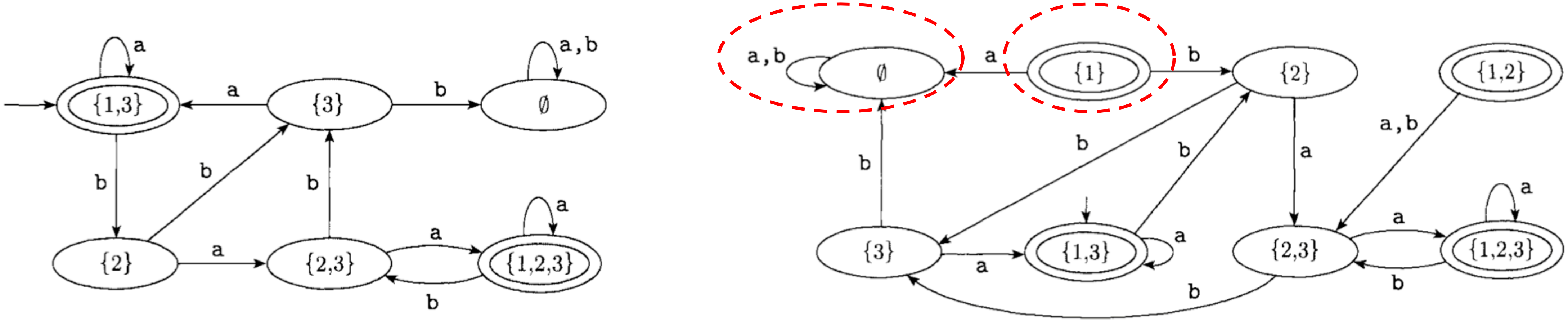


Wait, isn’ t it different from what we showed before…



Example

They are equivalent



An NFA is equivalent to some DFA

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  - δ: $\delta(Q, a) = \cup\{E(p) : p \in K \text{ and } (q, a, p) \in \Delta \text{ for some } q \in Q\}$

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

An NFA is equivalent to some DFA

- A language is regular if and only if some DFA recognizes it
- For any DFA, there is an equivalent NFA, and vice versa (i.e., $L_{DFA} = L_{NFA}$)
- $\Rightarrow$ A language is regular if and only if some NFA recognizes it.

An NFA is equivalent to some DFA

- Prove by induction:

  - Suppose the claim is true for all w such that $|w| \leq k$, we show the claim is true for $w = va$ where $|v| = k$

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

An NFA is equivalent to some DFA

- Prove by induction:

  - Suppose the claim is true for all w such that $|w| \leq k$, we show the claim is true for $w = va$ where $|v| = k$

  - $\rightarrow$: given that $(q, w) \vdash_M^* (p, e)$, there exist states r_1, r_2 such that

$$(q, va) \vdash_M^* (r_1, a) \vdash_M (r_2, e) \vdash_M^* (p, e)$$

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

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According to hypothesis, $(E(q), v) \vdash_{M'}^* (R_1, e)$ for some $R_1 \ni r_1$

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

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    According to hypothesis, $(E(q), v) \vdash_{M'}^* (R_1, e)$ for some $R_1 \ni r_1$

    $(r_1, a) \vdash_M (r_2, e)$ means $(r_1, a, r_2) \in \Delta$

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    $E(r_2) \subseteq \delta(R_1, a)$ by our construction of M'

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

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  $(r_2, e) \vdash_M^* (p, e)$ means $p \in E(r_2)$, whereas $p \in \delta(R_1, a)$, $(R_1, a) \vdash_{M'} (P, e)$

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

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 - $\leftarrow$: given that $(E(q), w) \vdash_{M'}^{*} (R_1, a) \vdash_{M'} (P, e)$, for some $P \ni p$ and some R_1 such that $\delta(R_1, a) = P$

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    $\delta(R_1, a)$ is the union of all $E(r_2)$ where $(r_1, a, r_2) \in \Delta$ for some $r_1 \in R_1$

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    $p \in P = \delta(R_1, a)$ implies there exist some $\widehat{r_1}, \widehat{r_2}$, $p \in E(\widehat{r_2})$, and $(\widehat{r_1}, a, \widehat{r_2}) \in \Delta$

Prove that: $(q, w) \vdash_{M}^{*} (p, e)$ if and only if $(E(q), w) \vdash_{M'}^{*} (P, e)$ for some set $P \ni p$

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    $p \in E(\widehat{r_2})$ means $(\widehat{r_2}, e) \vdash_{M}^{*} (p, e)$

Prove that: $(q, w) \vdash_{M}^{*} (p, e)$ if and only if $(E(q), w) \vdash_{M'}^{*} (P, e)$ for some set $P \ni p$

An NFA is equivalent to some DFA

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  - Suppose the claim is true for all w such that $|w| \leq k$, we show the claim is true for $w = va$ where $|v| = k$

  - $\leftarrow$: given that $(E(q), w) \vdash_{M'}^* (R_1, a) \vdash_{M'} (P, e)$, for some $P \ni p$ and some R_1 such that $\delta(R_1, a) = P$

    $\delta(R_1, a)$ is the union of all $E(r_2)$ where $(r_1, a, r_2) \in \Delta$ for some $r_1 \in R_1$

    $p \in P = \delta(R_1, a)$ implies there exist some $\widehat{r_1}, \widehat{r_2}$, $p \in E(\widehat{r_2})$, and $(\widehat{r_1}, a, \widehat{r_2}) \in \Delta$

    $p \in E(\widehat{r_2})$ means $(\widehat{r_2}, e) \vdash_M^* (p, e)$

    $(q, v) \vdash (\widehat{r_1}, e)$ by induction hypothesis

Prove that: $(q, w) \vdash_M^* (p, e)$ if and only if $(E(q), w) \vdash_{M'}^* (P, e)$ for some set $P \ni p$

An NFA is equivalent to some DFA

- Prove by induction:

  - Suppose the claim is true for all w such that $|w| \leq k$, we show the claim is true for $w = va$ where $|v| = k$

  - $\leftarrow$: given that $(E(q), w) \vdash_{M'}^{*} (R_1, a) \vdash_{M'} (P, e)$, for some $P \ni p$ and some R_1 such that $\delta(R_1, a) = P$

    $\delta(R_1, a)$ is the union of all $E(r_2)$ where $(r_1, a, r_2) \in \Delta$ for some $r_1 \in R_1$

    $p \in P = \delta(R_1, a)$ implies there exist some $\hat{r}_1, \hat{r}_2$, $p \in E(\hat{r}_2)$, and $(\hat{r}_1, a, \hat{r}_2) \in \Delta$

    $p \in E(\hat{r}_2)$ means $(\hat{r}_2, e) \vdash_{M}^{*} (p, e)$

    $(q, v) \vdash (r_1, e)$ by induction hypothesis

    $(q, va) \vdash_{M}^{*} (\hat{r}_1, a) \vdash_{M} (\hat{r}_2, e) \vdash_{M}^{*} (p, e)$

Prove that: $(q, w) \vdash_{M}^{*} (p, e)$ if and only if $(E(q), w) \vdash_{M'}^{*} (P, e)$ for some set $P \ni p$