Homework 1. Basics on definitions, proofs and propositional logic

Submit your home work to blackboard by **11:59pm Tue Sept 13.** Use latex to typeset your answers to the questions (use overleaf.com to edit your latex file and get its pdf file). Please submit both your latex file and PDF file. I also attach the latex file (hw1.tex) of this homework. Here is a link to a sample Latex file on overleaf https://www.overleaf.com/read/czwmmdvqzpcj which is read only. You need to create an overleaf account, if not done yet, and then create a new project and then copy the content of the main.tex (left most pane) file in the link and paste it to the main file of your project. For the picture file in the link, click the 3 dots to the right of the file name, then click download (to your local computer), and then upload to your project. Now you can play with the latex file. Note hw1.tex (and hwHeader.tex) is also available on blackboard. In your overleaf project for hw1, you upload both of them (and delete the default main.tex file). Your "current" file should always be hw1.tex when you click the "recompile" button.

1. (10 points) Everyone is required to sign this form:
   https://forms.gle/EZRNcWq2TWXaJkZDA

2. (10) 1) Which of the following strings are official propositions (according to our definition of propositions).
   (a) $((\neg(A \vee B)) \wedge C)$
   (b) $(A \wedge B) \vee C$
   (c) $(A \wedge (B \wedge C)))$

   2) Draw the formation tree of $((\neg C) \leftrightarrow (A \vee C))$.

3. (20) Following our proof methodology prove

   $$((A \rightarrow B) \leftrightarrow C) \text{ is a proposition.}$$

   Goals for this questions:

   - Understand well the working backward proof method. Write steps for working backward on a scratch paper. During working backward, also practice application of definitions and decomposition of the current statement to prove into the "main concept" or logical connective and the rest (similar to the formation tree of a proposition).

- From your working backward steps, write the proof steps in a "forward" way. See the format of the "Final proof" in L3-ProofExamples.pdf available on blackboard.

4. (10) Although we follow the content in the book, but our class covers much more (e.g., identifying concepts and their parameters, precise definitions and etc.) than what is printed on the textbook. Also the materials in this class are so special that one unlikely can answer the questions in homework or tests without understanding what is discussed during class and studying the notes and textbook. After you complete this homework, do you strongly agree, agree, keep neutral, disagree or strongly disagree with the statements above? Explain why you answer so.

5. (50) (Read and write definitions)

   - The definition of "Definition 3.2" is not as explicit and complete as we would like. Write a precise and complete definition. (Recall the discussion of how to define valuation during class. Also recall the definition of propositions to master the definition methods there.)
   - Write the precise definition of the notation a_i (two lines below Figure 6 in Page 19 of the text book) in the proof for Theorem 2.8 (also see appendix). For any notations in your definition, find its meaning in the proof and write its precise definition here. Repeat this process until all notations are defined in this proof or outside the proof (e.g., you don't need to define $\wedge$).
   - Write the following information for each of the concepts: the a_i above, and support in "Definition 2.5 (ii)" in the text book.
     - The name of the concept:
     - The parameter(s) of the concept:
     - Meta variables in the definition of the concept:
     - The concepts used in the definition (and defined before) (including names and their parameters):

Appendix

Theorem 2.8 (Adequacy): $\{\neg, \wedge, \vee\}$ *is adequate.*

Proof: Let $A_1, \ldots, A_k$ be distinct propositional letters and let a_{ij} denote the entry (T or F) corresponding to the i^{th} row and j^{th} column of the truth table for $\sigma(A_1, \ldots, A_k)$ as in Figure 6. Suppose that at least one T appears in the last column.

A_1	$\cdots$	A_j	$\cdots$	A_k	$\cdots$	$\sigma(A_1, \ldots, A_k)$
						b_1
						b_2
						.
						.
		a_{ij}				b_i

FIGURE 6.

For any proposition α, let α^T be α and α^F be $(\neg\alpha)$. For the i^{th} row denote the *conjunction* $(A_1^{a_{i1}} \wedge \ldots \wedge A_k^{a_{ik}})$ by a_i. Let $i_1, \ldots, i_m$ be the rows with a T in the last column. The desired proposition is the *disjunction* $(a_{i_1} \vee \ldots \vee a_{i_m})$. The proof that this proposition has the given truth table is left as Exercise 14. (Note that we abused our notation by leaving out a lot of parentheses in the interest of readability. The convention is that of *right associativity*, that is, $A \wedge B \wedge C$ is an abbreviation for $(A \wedge (B \wedge C))$.) We also indicate a disjunction over a set of propositions with the usual set–theoretic terminology. Thus, the disjunction just constructed would be written as $\bigvee\{a_i | \ b_i = T\}$. $\square$