Working Backwards/Problem Decomposition for "Finding" Proofs

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Outline

1. On working backwards/problem decomposition
2. Proof Example 1
3. Proof Example 2

Note. *These slides are best used in the full screen mode so that when scoll up or down, you have the experience of the intended animation.*

On working backwards/problem decomposition

Working backwards/problem decomposition

- Work backward / problem decomposition. To prove a statement, we work backward from the statement (or decompose the statement). We need to understand the structure of (or parse) the statement. Then we find the main concept, decompose the main concept using *definitions* or *logic*.
- We also need to know how to work forward.

Proof Example 1

Prove that A is a proposition.

Proof.

QED

Prove that A is a proposition.

Proof.

(B1) A is a proposition.

QED

Prove that A is a proposition.

Proof.

(B1) A is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement

QED

Prove that A is a proposition.

Proof.

(B2) A proposition letter is a proposition. Definition of proposition.

(B1) A is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement

QED

Prove that A is a proposition.

Proof.

(B2) A proposition letter is a proposition. Definition of proposition.

(B1) A is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement

QED

Prove that A is a proposition.

Proof.

(B3) A is a proposition letter.

(B2) A proposition letter is a proposition. Definition of proposition.

(B1) A is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement

QED

Prove that A is a proposition.

Proof.

(B3) A is a proposition letter. This is true by definition of proposition letter.

(B2) A proposition letter is a proposition. Definition of proposition.

(B1) A is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement

QED

Final proof

Prove that A is a proposition.

Proof.

(1) A is a proposition letter. By definition of *proposition letter*.

(2) A proposition letter is a proposition. Definition of *proposition*.

(3) A is a proposition. By (1) and (2) [We replace "a proposition letter" by A].

QED

Proof Example 2

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B1) $(\neg(A \to B))$ is a proposition.

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B2) If $(A \to B)$ is a proposition, $(\neg(A \to B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \to B)$

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B3) $(A \to B)$ is a proposition.

(B2) If $(A \to B)$ is a proposition, $(\neg(A \to B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \to B)$

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B4) If A and B are propositions, $(A \to B)$ is a proposition.

Definition of *proposition* with $\alpha = A$ and $\beta = B$

(B3) $(A \to B)$ is a proposition.Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = A$ and $\beta = B$.

(B2) If $(A \to B)$ is a proposition, $(\neg(A \to B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \to B)$

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B5) A and B are propositions

(B4) If A and B are propositions, $(A \to B)$ is a proposition.

Definition of *proposition* with $\alpha = A$ and $\beta = B$

(B3) $(A \to B)$ is a proposition.Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = A$ and $\beta = B$.

(B2) If $(A \to B)$ is a proposition, $(\neg(A \to B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \to B)$

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

......

(B5) A and B are propositions

(B4) If A and B are propositions, $(A \to B)$ is a proposition.

Definition of *proposition* with $\alpha = A$ and $\beta = B$

(B3) $(A \to B)$ is a proposition.Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = A$ and $\beta = B$.

(B2) If $(A \to B)$ is a proposition, $(\neg(A \to B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \to B)$

(B1) $(\neg(A \to B))$ is a proposition. Main concept: proposition. Use definition of *proposition* to decompose the statement. We need substitution of meta variables: $\alpha = (A \to B)$

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B5) A and B are propositions.

(B1) to (B4): see the previous page

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

(B5) A and B are propositions. Main concept: **and** – a logical connective. Use understanding of logical connective, We need to show each one separately.

(B1) to (B4): see the previous page

QED

Prove that $(\neg(A \to B))$ is a proposition.

Proof.

......

(B7) A is a proposition

......

(B6) B is a proposition

(B5) A and B are propositions. Main concept: **and** – a logical connective. Use understanding of logical connective, We need to show each one separately.

(B1) to (B4): see the previous page

QED

Final proof

Prove that A is a proposition.

Proof.

(1) A is a proposition letter. By definition of *proposition letter*.

(2) A is a proposition. By (1) and definition of *proposition*.

(3) B is a proposition letter. By definition of *proposition letter*.

(4) B is a proposition. By (3) and definition of *proposition*.

(5) A and B are propositions. By (2), (3) and logical connective **and**.

(6) If A and B are propositions, $(A \to B)$ is a proposition.

Definition of *proposition* with $\alpha = A, \beta = B$

(... to be continued in the next page ...)

Final proof

(... continue from the previous page ...)

(7) $(A \rightarrow B)$ is a proposition. By (5) and (6).

(8) If $(A \rightarrow B)$ is a proposition, $(\neg(A \rightarrow B))$ is a proposition.

Definition of *proposition* with $\alpha = (A \rightarrow B)$

(9) $(\neg(A \rightarrow B))$ is a proposition. By (7) and (8).

QED