Homework 4. Resolution. Resolution Refutation.

Submit your solution in PDF file (and Latex source file if you use Latex) to blackboard by **11:59pm Oct 31**.

1. State the soundness and completeness results about resolution, T-resolution, $\mathcal{A}$-resolution, SLD-resolution. (Make sure what "formula" your result is talking about.)

2. Let $S = \{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}$. What is S^B? What is $S^{\neg B}$?

3. Follow our proof methods, prove the following.

   If P is a PROLOG program and $G = \{\neg q_1, \ldots, \neg q_n\}$ a goal clause, then every q_i ($i \in 1..n$) is a consequence of P if and only if $P \cup \{G\}$ is unsatisfiable.

   You may use the following definition of consequence in your proof: a literal l is a *consequence of a formula S* if for every assignment $\mathcal{A}$ that satisfies S, $\mathcal{A}$ satisfies l, i.e., $l \in \mathcal{A}$.

4. Consider a PROLOG program P and a conjunction of propositional letters $p_1, ..., p_n$. To prove that $p_1, ..., p_n$ is a consequence of P, what is the set of clauses that the resolution refutations (discussed in class) are based on? Explain why these methods works.

5. Consider a PROLOG program P:

   $\{\texttt{p} :- \texttt{s}, \texttt{t}.\ \ \texttt{p} :- \texttt{q}.\ \ \texttt{q}.\ \ \texttt{q} :- \texttt{r}.\ \ \texttt{r}.\ \ \texttt{r} :- \texttt{w}.\ \ \texttt{r}.\ \ \texttt{s}.\ \ \texttt{t} :- \texttt{w}.\}$

   1) Draw an SLD-tree for goal $\neg\texttt{p}$ in terms of P.

   2) What is an SLD resolution refutation of $P \cup \{\{\neg\texttt{p}\}\}$?