Homework 3. Tableau Proof. Resolution.

Submit your solution in PDF file and Latex source file to blackboard by **05:00pm Mon, Oct 10**.

1. (15) For Definition 6.2, write the following information in the order they occur in the definition

   - For each concept defined by this definition, write its name and parameters (if there is any),

     Defined Concepts:
     1) Tableau Proof (Parameters are α and Σ)
     2) Provable (Parameter is Σ)
     3) $\Sigma \vdash \alpha$ (Parameters are α and Σ)

   - For each concept used in this definition, write its name and arguments (if there is any), and

     Used Concepts:
     1) Proposition (Argument is α)
     2) Or
     3) Tableau (Argument is Σ)
     4) Root Entry (Argument is $F\alpha$)
     5) Contradictory (Argument is Tableau)
     6) Contradictory (Argument is path)
     7) If
     8) And
     9) Proof(tableau proof) (Arguments are α and Σ)

   - Write meta variables in the definition.

     Meta Variables:
     α and Σ

2. (15) For Definition 8.4, write the following information in the order they occur in the definition

- For each concept defined by this definition, write its name and parameters (if there is any),

  Defined Concepts:
  1) Deduction (Parameters are C and S)
  2) Proof (Parameters are C and S)
  3) Provable (Parameters are C and S)
  4) $S \vdash_R C$ (Parameters are C and S)
  5) Refutation (Parameter is S)
  6) Refutable (Parameter is S)
  7) $S \vdash_R \Box$ (Parameters are S and $\Box$)

- For each concept used in this definition, write its name and arguments (if there is any), and

  Used Concepts:
  1) Formula (Argument is S)
  2) Clauses (Argument is C)
  3) Or
  4) Resolvent (Arguments are C_i, C_j, C_k)
  5) If
  6) There is
  7) Deduction
  8) Deduction (Arguments are $\Box$ and S)
  9) If
  10 There is
  11) Deduction

- Write meta variables in the definition.

  Meta Variables:
  C_i, C_j, C_k, C, S

3. (15) i) Find the definition of *assignment* from Chapter 8. Write the definition below.

   Answer:

   An assignment $\mathcal{A}$ is a consistent set of literals, i.e., one not containing both p and $\neg p$ for any propositional letter p.

   ii) Write the definition of another concept, whose name contains "assignment", that was defined before (see L04).

   Answer:

   A truth assignment $\mathcal{A}$ is a function that assigns to each propositional letter A a unique truth value $\mathcal{A}$(A) $\in \{T, F\}$

iii) Is it precise for us to understand *truth assignment* as the combination of the English meaning of truth and the definition of *assignment* in i)? Why?

Answer:

NO
Reason: According to the definition of assignment in i) and the English meaning of truth, it means that the consistent set of literals should be assigned with a truth value T. Whereas truth assignment means assigning T of F to the literals.

4. (15) i) Write the result of applying the definition of *satisfiable* (see Section 2.3 of L04).

$$\{\{\neg A\}, \{A, \neg B\}, \{B\}\}.$$

Answer:

1) The given formula is $\{\{\neg A\}, \{A, \neg B\}, \{B\}\}$.
2) We now need to check if the given formula is satisfiable using the definition of satisfiable.
3) The satisfiablity rule states that the assignment $\mathcal{A}$ satisfies S $\mathcal{A} \models S$ iff $\forall C \in S$, $C \cap \mathcal{A} \neq \emptyset$
4) The given formula has the following clauses.
$C_1 = \{\neg A\}$
$C_2 = \{A, \neg B\}$
$C_3 = \{B\}$
5) We now check the given formula with different assignments available.
6) Case i:
Consider the assignment $\mathcal{A} = \{A, B\}$
$C_1 \cap A = \{\neg A\} \cap \{A, B\} = \emptyset$
$\mathcal{A} = \{A, B\}$ cannot be the assignment as the satisfiability criteria for C_1 is not satisfied.
7) Case ii:
Consider the assignment $\mathcal{A} = \{\neg A, B\}$
$C_1 \cap A = \{\neg A\} \cap \{\neg A, B\} = \{\neg A\}$
$C_2 \cap A = \{A, \neg B\} \cap \{\neg A, B\} = \emptyset$
$\mathcal{A} = \{\neg A, B\}$ cannot be the assignment as the satisfiability criteria for C_2 is not satisfied.
8) Case iii:
Consider the assignment $\mathcal{A} = \{A, \neg B\}$
$C_1 \cap A = \{\neg A\} \cap \{A, \neg B\} = \emptyset$
$\mathcal{A} = \{A, \neg B\}$ cannot be the assignment as the satisfiability criteria for C_1 is not satisfied.
9) Case iv:
Consider the assignment $\mathcal{A} = \{\neg A, \neg B\}$
$C_1 \cap A = \{\neg A\} \cap \{\neg A, \neg B\} = \{\neg A\}$
$C_2 \cap A = \{A, \neg B\} \cap \{\neg A, \neg B\} = \{\neg B\}$
$C_3 \cap A = \{B\} \cap \{\neg A, \neg B\} = \emptyset$

$\mathcal{A} = \{\neg A, \neg B\}$ cannot be the assignment as the satisfiability criteria for C_3 is not satisfied.
10) We have tried with all possible cases of assignment but the given formula $S = \{\{\neg A\}, \{A, \neg B\}, \{B\}\}$. is not satisfiable.

ii) According to the result above, is the formula satisfiable? If yes, give an assignment satisfying it.

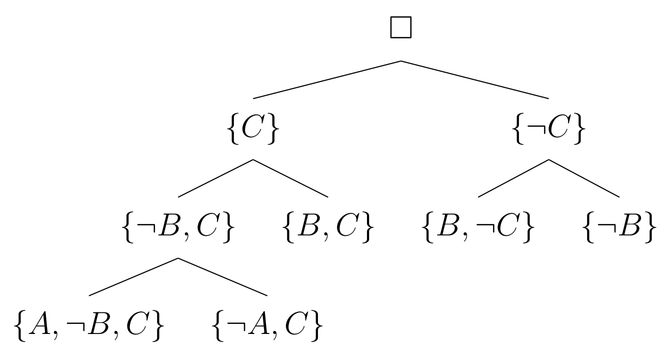
Answer:

1) NO. With the above result, we can confirm that the formula $\{\{\neg A\}, \{A, \neg B\}, \{B\}\}$. is not satisfiable.
2) As the formula is not satisfiable, there will be no assignment satisfying it.

5. (20) Find a resolution tree refutation of the following formula:

$$\{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}.$$

Answer:



6. (20) Prove that if the formula $S = \{C_1, C_2\}$ is satisfiable and C is the resolvent of C_1 and C_2, then C is satisfiable. Use our proof methodologies and format. Do not copy the proof in the book.

Answer:

1) Let C_1 and C_2 be two clauses.
2) C be the resolvent of C_1 and C_2
3) Since C is the resolvent of C_1 and C_2, we can write $C_1 = \{l\} \cup C_1'$ and $C_2 = \{\bar{l}\} \cup C_2'$ and C= $C_1' \cup C_2'$.
4) Since C_1 and C_2 are satisfiable, there exists an assignment $\mathcal{A}$ such that $\mathcal{A} \cap C_1 \neq \emptyset$ and $\mathcal{A} \cap C_2 \neq \emptyset$. [According to the definiton of satisfiable]

5) Since $\mathcal{A}$ is an assignment, it should either contain l or $\bar{l}$ but not both. In this scenario, two cases occur.[According to the definition of Assignment]
6) Case i: $\mathcal{A}$ contains l.
Since $l \in \mathcal{A}$, $\bar{l} \notin \mathcal{A}$
Thus we can say that $\mathcal{A} \models C_2^{'}$. [Because $\mathcal{A}$ and $C_2^{'}$ doesn't contain $\bar{l}$ and $C_2^{'} \cap \mathcal{A} \neq \emptyset$].
Now we can say that $\mathcal{A} \models C$. [Because $\mathcal{A} \models C_2^{'}$ and C= $C_1^{'} \cup C_2^{'}$ which implies $C \cap \mathcal{A} \neq \emptyset$].
7) Case ii: $\mathcal{A}$ contains $\bar{l}$.
Since $\bar{l} \in \mathcal{A}$, $l \notin \mathcal{A}$
Thus we can say that $\mathcal{A} \models C_1^{'}$. [Because $\mathcal{A}$ and $C_1^{'}$ doesn't contain l and $C_1^{'} \cap \mathcal{A} \neq \emptyset$].
Now we can say that $\mathcal{A} \models C$. [Because $\mathcal{A} \models C_1^{'}$ and C= $C_1^{'} \cup C_2^{'}$ which implies $C \cap \mathcal{A} \neq \emptyset$].
8) From cases i and ii, we can say that C is satisfiable from S which has an assignment $\mathcal{A}$. Therefore f the formula $S = \{C_1, C_2\}$ is satisfiable and C is the resolvent of C_1 and C_2, then C is satisfiable.