

Theory of Automata

Homework - 5

1) Consider the grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\},$$

$$\Sigma = \{a, b\}$$

$$R = \left\{ \begin{array}{l} S \rightarrow AA, \\ A \rightarrow AAA, \\ A \rightarrow a, \\ A \rightarrow bA, \\ A \rightarrow Ab \end{array} \right\}$$

a) Give a string of $L(G)$ that can be produced by applying the rules at most 4 times.

Ans:

$$S \rightarrow AA$$

$$S \rightarrow bAA (A \rightarrow bA)$$

$$S \rightarrow baA (A \rightarrow a)$$

$$S \rightarrow baa (A \rightarrow a)$$

b) Same string can be derived in different ways.

eg. $S \Rightarrow AA \Rightarrow aA \Rightarrow aa$, $S \Rightarrow AA \Rightarrow Aa \Rightarrow aa$. Give at least 2 distinct derivations for the string babbab

Ans: 1st Derivation:

$$S \rightarrow AA$$

$$S \rightarrow bAA (A \rightarrow bA)$$

$$S \rightarrow bAbA (A \rightarrow Ab)$$

$$S \rightarrow bAbbA (A \rightarrow Ab)$$

$$S \rightarrow babbaA (A \rightarrow a)$$

$$S \rightarrow ba b b A b \quad (A \rightarrow Ab)$$

$$S \rightarrow babbab \quad (A \rightarrow a)$$

2nd Derivation:

$$S \rightarrow AA$$

$$S \rightarrow AAb \quad (A \rightarrow Ab)$$

$$S \rightarrow AbAb \quad (A \rightarrow bA)$$

$$S \rightarrow AbbAb \quad (A \rightarrow bA)$$

$$S \rightarrow Abbab \quad (A \rightarrow a)$$

$$S \rightarrow bAbbab \quad (A \rightarrow bA)$$

$$S \rightarrow babbab \quad (A \rightarrow a)$$

c) For any $m, n, p > 0$, describe a derivation in G of the string $b^m a b^n a b^p$

Ans: Consider $S \rightarrow AA$

$$S \rightarrow bAA \quad (A \rightarrow bA)$$

$$S \rightarrow b \overset{\downarrow}{b} AA \quad (A \rightarrow bA)$$

(Apply $A \rightarrow bA$ for first A 'm' times)

$$S \rightarrow b^m AA$$

$$S \rightarrow b^m A b A \quad (A \rightarrow Ab)$$

$$S \rightarrow b^m A b b A \quad (A \rightarrow Ab)$$

(Apply $A \rightarrow Ab$ for first A 'n' times)

$$S \rightarrow b^m A b^n A$$

$$S \rightarrow b^m a b^n A \quad (A \rightarrow a)$$

$$S \rightarrow b^m a b^n A b \quad (A \rightarrow Ab)$$

$$S \rightarrow b^m a b^n A b b \quad (A \rightarrow Ab)$$

$$S \rightarrow b^m a b^n A b b b \quad [A \rightarrow A b]$$

(Apply $A \rightarrow A b$ for first A 'p' times)

$$S \rightarrow b^m a b^n A b^p$$

$$S \rightarrow b^m a b^n a b^p \quad (A \rightarrow a)$$

2) Consider the grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\},$$

$$\Sigma = \{a, b\},$$

$$R = \{S \rightarrow a A a,$$

$$S \rightarrow b A b$$

$$S \rightarrow e,$$

$$A \rightarrow S S\}.$$

Ans:

$$S \rightarrow b A b$$

$$S \rightarrow b S S b \quad (A \rightarrow S S)$$

$$S \rightarrow b a A a S b \quad (A \rightarrow a A a)$$

$$S \rightarrow b a A a b A b b \quad (S \rightarrow b A b)$$

$$S \rightarrow b a S S a b A b b \quad (A \rightarrow S S)$$

$$S \rightarrow b a S a b A b b \quad (S \rightarrow e)$$

$$S \rightarrow b a a b A b b \quad (S \rightarrow e)$$

$$S \rightarrow b a a b S S b b \quad (A \rightarrow S S)$$

$$S \rightarrow b a a b S b b \quad (S \rightarrow e)$$

$$S \rightarrow b a a b b b \quad (S \rightarrow e)$$

3) Show that the following languages are context free by exhibiting context free grammars generating each.

a) $\{a^m b^n : m \geq n \geq 0\}$

Ans: The Context free grammar for this language is

$$G = \{V, \Sigma, R, S\}$$

$$V = \{a, b, S\}$$

$$\Sigma = \{a, b\}$$

$$R = \{S \rightarrow aS, \\ S \rightarrow aSb, \\ S \rightarrow e\}$$

$$S \rightarrow AX$$

$$A \rightarrow Aa \mid a \mid e$$

$$X \rightarrow aXb \mid e$$

Since we can generate CFG, given language is Context free.

b) $\{a^m b^n c^{2m+n} : m, n \geq 0\}$

The context free grammar for the language is

$$G = \{V, \Sigma, R, S\}$$

$$V = \{a, b, c, A, B, S\}$$

$$\Sigma = \{a, b, c\}$$

$$R = \{S \rightarrow A, \\ A \rightarrow aAcc \mid e \mid B, \\ B \rightarrow bBc \mid e\}$$

$$S \rightarrow aScc \mid A$$

$$A \rightarrow bAc \mid e$$

Since we can generate a CFG, the given language is Context free.