Theory of Automata – Homework 5

Name – Akshay Kumar Singh

R11603620

1. Consider the grammar $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\},$$
$$\Sigma = \{a, b\},$$
$$R = \{S \to AA,$$
$$A \to AAA, .$$
$$A \to a,$$
$$A \to bA,$$
$$A \to Ab\}.$$

a). Give a string of $L(G)$ that can be produced by applying the rules at most 4 times
b). Same string can be derived in different ways, e.g., $S \Rightarrow AA \Rightarrow aA \Rightarrow aa$, $S \Rightarrow AA \Rightarrow Aa \Rightarrow aa$. Give at least 2 distinct derivations for the string $babbab$
c). For any $m, n, p > 0$, describe a derivation in G of the string $b^m ab^n ab^p$

Sol: **(a)** Length no more than 4:
S => AA=> aA=>aa
S=> AA=>aA=> abA=> aba
S=>AA=> aA =>aAb =>aab
S=> AA =>bAA =>baA=> baa
S =>AA =>bAA =>bAa=> baa
S => AA =>AbA => abA =>aba
S =>AA => AbA => Aba =>aba
S =>AA =>Aa=> aa
S =>AA =>Aa =>bAa => baa
S =>AA=> Aa => Aba => aba
S => AA =>AbA => abA => aba
S => AA => AbA => Aba =>aba
S => AA => AAb => aAb =>aab
S =>AA =>AAb=> Aab => aab
Applying the rules in different orders. strings that can be generated are: aa, aab, aba, baa.

(b) Note that A =>bA =>bAb => bab, and also that A =>Ab => bAb => bab. (8 distinct derivations):

S =>AA => AbA => AbAb => Abab =>* babbab
S => AA => AAb => AbAb => Abab =>* babbab
S => AA => bAA => bAbA =>babA =>* babbab
S =>AA => AbA => bAbA => babA =>* babbab

Where each of these four has 2 ways to reach babbab in the last steps.

(c) Producing a sequence in terms of m, n, p that will produce the string $b^m ab^n ab^p$.

S => /* by rule S → AA */
AA =>* /* by m applications of rule A → bA */

b^mAA => /* by rule A → a */
b^maA =>* /* by n applications of rule A → bA */
b^mab^nA =>* by p applications of rule A → Ab */
$b^mab^nAb^p$ => /* by rule A → a */
$b^mab^nab^p$
Clearly this produces $b^mab^nab^p$ for each m, n, p.

2. Consider the grammar $G = (V, \Sigma, R, S)$, where

$$\begin{aligned} V &= \{a, b, S, A\}, \\ \Sigma &= \{a, b\}, \\ R &= \{S \to aAa, \\ &\quad S \to bAb, \\ &\quad S \to e, \\ &\quad A \to SS\}. \end{aligned}$$

Give a derivation of the string $baabbb$ in G.

Sol: Derivation of baabbb

S => bAb
=>bSSb
=>baAaSb
=>baSSaSb
=>baSaSb
=>baaSb
=>baabAbb
=>baabSSbb
=>baabSbb
=>baabbb

3. Show that the following languages are context-free by exhibiting context-free grammars generating each.
a). $\{a^m b^n : m \geq n \geq 0\}$
b). $\{a^m b^n c^{2m+n} : m, n \geq 0\}$

Sol: **(a)** :
S -> aAB / AB
A -> aA / a
B -> bB / b / e

(b) : S -> ABC / e
A -> aA / a / e
B -> bB / b / e
C -> cC / c / e