Theory of Automata

Home work-8

1) Let $M = (K, \Sigma, \delta, s, \{h\})$, where

$$K = \{q_0, q_1, h\},$$
$$\Sigma = \{a, b, \sqcup, \triangleright\},$$
$$s = q_0,$$

and δ is given by the following table.

q	σ	$\delta(q,\sigma)$
q_0	a	(q_1, b)
q_0	b	(q_1, a)
q_0	$\sqcup$	$(h, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	a	$(q_0, \rightarrow)$
q_1	b	$(q_0, \rightarrow)$
q_1	$\sqcup$	$(q_0, \rightarrow)$
q_1	$\triangleright$	$(q_1, \rightarrow)$

a) Trace the computation of M starting from the configuration $(q_0, \triangleright aabbba)$.

Ans: $(q_0, \triangleright \underline{a}abbba) \vdash_M (q_1, \triangleright b\underline{a}bbba)$

$\vdash_M (q_0, \triangleright ba\underline{b}bba)$

$\vdash_M (q_1, \triangleright bb\underline{b}bba)$

$\vdash_M (q_0, \triangleright bbb\underline{b}ba)$

$\vdash_M (q_1, \triangleright bba\underline{b}ba)$

$\vdash_M (q_0, \triangleright bba\underline{b}ba)$

$\vdash_M (q_1, \triangleright bba\underline{a}ba)$

$$\vdash_M (q_0, \triangleright bbaa\underline{b}a)$$
$$\vdash_M (q_1, \triangleright bbaa\underline{a}a)$$
$$\vdash_M (q_0, \triangleright bbaaa\underline{a})$$
$$\vdash_M (q_1, \triangleright bbaaa\underline{b})$$
$$\vdash_M (q_0, \triangleright bbaaab\underline{\sqcup})$$
$$\vdash_M (h, \triangleright bbaaab\underline{\sqcup})$$

b) Describe informally what M does when started in q_0 on any square of a tape.

Ans: i) Swap 'a' with 'b' and 'b' with 'a'

ii) M moves to the right side

iii) M swaps 'a' with 'b' and 'b' with 'a'.

iiii) When there is a blank input, it stops.

2) Let $M = (K, \Sigma, \delta, s, \{h\})$, where
$$K = \{q_0, q_1, q_2, h\},$$
$$\Sigma = \{a, b, \sqcup, \triangleright\},$$
$$s = q_0,$$

and δ is given by the following table (the transitions on $\triangleright$ are $\delta(q, \triangleright) = (q, \triangleright)$, and are omitted).

q	a	$\delta(q, a)$
q_0	a	$(q_1, \leftarrow)$
q_0	b	$(q_0, \rightarrow)$
q_0	$\sqcup$	$(q_0, \rightarrow)$
q_1	a	$(q_1, \leftarrow)$
q_1	b	$(q_2, \rightarrow)$
q_1	$\sqcup$	$(q_1, \leftarrow)$
q_2	a	$(q_2, \rightarrow)$
q_2	b	$(q_2, \rightarrow)$
q_2	$\sqcup$	$(h, \sqcup)$

Trace the computation of M starting from the
configuration $(q_0, \triangleright a\underline{b}b\sqcup bb\sqcup\sqcup\sqcup\sqcup aba)$

Ans: $(q_0, \triangleright a\underline{b}b\sqcup bb\sqcup\sqcup\sqcup\sqcup aba) \vdash_M (q_0, \triangleright ab\underline{b}\sqcup bb\sqcup\sqcup\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\underline{\sqcup} bb\sqcup\sqcup\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup \underline{b}b\sqcup\sqcup\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup b\underline{b}\sqcup\sqcup\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup bb\underline{\sqcup}\sqcup\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup bb\sqcup\underline{\sqcup}\sqcup\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup bb\sqcup\sqcup\underline{\sqcup}\sqcup aba)$

$\vdash_M (q_0, \triangleright abb\sqcup bb\sqcup\sqcup\sqcup\underline{\sqcup} aba)$

$\vdash_M (q_0, \triangleright abb\sqcup bb\sqcup\sqcup\sqcup\sqcup \underline{a}ba)$

$\vdash_M (q_1, \triangleright abb\sqcup bb\sqcup\sqcup\sqcup\underline{\sqcup} aba)$

$\vdash_M (q_1, \triangleright abb\sqcup bb\sqcup\sqcup\sqcup\underline{\sqcup} aba)$

$$\vdash_M (q_1, \triangleright abb \sqcup bb \underline{\sqcup} \sqcup \sqcup aba)$$
$$\vdash_M (q_1, \triangleright abb \sqcup b\underline{b} \sqcup \sqcup \sqcup aba)$$
$$\vdash_M (q_2, \triangleright abb \sqcup bb \underline{\sqcup} \sqcup \sqcup aba)$$
$$\vdash_M (h, \triangleright abb \sqcup bb \underline{\sqcup} \sqcup \sqcup aba)$$