CS 5383

Theory of Automata

1. Select one correct answer out of 4 choices (1.5 point * 10).

1.1 Which of the following language is context-free but is **not** regular (B)

a).$\{a^n bcd: n \geq 0\}$ b).$\{a^n b^n cd: n \geq 0\}$

c).$\{a^n b^n c^n d: n \geq 0\}$ d).$\{a^n b^n c^n d^n: n \geq 0\}$

1.2 Which of the following statements is **wrong**? (C)

a). If a language is accepted by a deterministic finite automata, then it is context-free

b). If a language is accepted by a pushdown automata, then it is context-free

c). If a language is **not** accepted by a deterministic finite automata, then it is not context-free

d). If a language is **not** accepted by a pushdown automata, then it is not context-free

1.3. Which of the following statements is **wrong**? (D)

a). A string may be generated through different derivations

b). Different derivations may correspond to the same parse tree

c). A string may be generated through different parse tree

d). Different parse tree may correspond to the same derivation

1.4 Consider the language $(\Sigma\Sigma)^*$ where $\Sigma = \{a, b\}$, and let

$$R_1 = \{S \to aaS \mid bbS \mid abS \mid baS \mid e\}$$

$$R_2 = \{S \to aSa \mid bSb \mid aSb \mid bSa \mid e\}$$

$$R_3 = \{S \to Saa \mid Sbb \mid abS \mid baS \mid e\}$$

be three rules. Which of them can be used to generate the language? (B,C)

a). Only one of them

b). Two of them

c). All of them

d). None of them

Comment: There is a typo when I type R_3, it actually should be $R_3 = \{S \to Saa \mid Sbb \mid Sab \mid Sba \mid e\}$, in that case C is correct. But because of the typo, both B and C are acceptable.

1.5. Consider the language $L = \{a^m b^n c^l : m + n + l \geq 2020\}$. Which of the followings is **correct**? (A)

a). This is a regular language

b). This is not a regular language, but is a context-free language

c). This is not a context-free language

d). All the above statements are wrong.

1.6 Which of the following statements is **correct**? (D)

a). If A and $A \circ B$ are both context-free languages, $A \cap B = \emptyset$, then B is also a context-free language

$A = ba^*, B = \{a^p : p\ is\ prime\}$

b). If A and $A \cap B$ are both context-free languages, then B is also a regular language

c). If A and $(A \circ B)^*$ are both context-free languages, $A \cap B = \emptyset$, then B is also context-free

d). None of the above

1.7 Let L be a context-free language over alphabet Σ. Which of the followings is **correct**? (B,D)

a). It is possible that any subset of L is not context-free b). It is possible that any subset of L is not regular

c). It is possible that for any $A \subseteq \Sigma^*$, $L \subseteq A$, A is not context-free

d). It is possible that L does **not** contain a subset which is context-free but is not regular

$L = \emptyset$

Comment: In this class (and also other CS or Math courses), any means "every" or "all", so b) should be interpreted as It is possible that every subset of L is not regular. This is wrong since emptyset is a subset and is regular. But I see some students understands it in a way that It is possible that some subset of L is not regular. I want to emphasize that this is not a correct way of understanding a mathematical/CS statement, "any" and "some" mean different things. But since I did not emphasize this in class, I also include B as a correct answer.

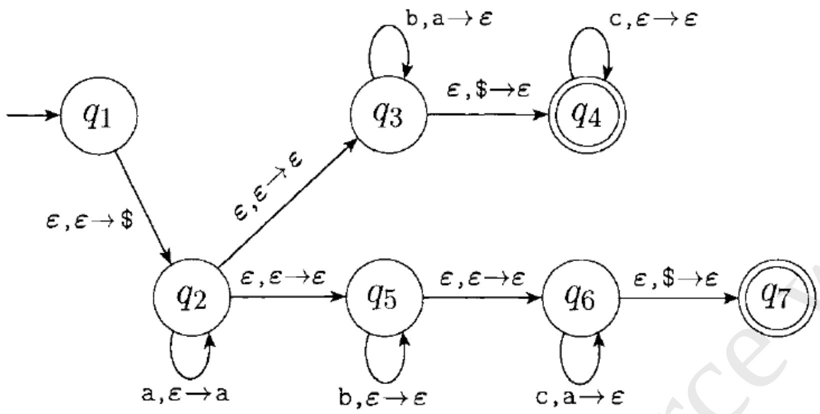
1.8 Which of the followings is **wrong**? (D)

a). There exists a context-free language whose complement is also a context-free language

b). There exists a context-free language whose complement is **not** a context-free language c). There exists a regular language whose complement is also a regular language

d). There exists a regular language whose complement is **not** a regular language

1.9 Consider the following PDA



Which of the following strings is **not** accepted by it? (A)

a). $aaaaa$

b). $bbbbb$

c). $ccccc$

d). $aabbcc$

1.10 Which of the following statement is wrong? (C)

a). The intersection of two context-free languages can be regular.

$$\{a^i b^i : i \geq 1\} \cap \{b^i a^i : i \geq 1\}$$

b). The intersection of two non-regular languages can be context-free

$$\{a^i b^i : i \geq 1\} \cup \left(\Sigma^* - \{a^i b^i : i \geq 1\}\right)$$

c). The complement of a non-context-free language can be regular

d). The complement of a non-regular language can be context-free.

complement of $\{a^i b^i : i \geq 1\} \circ b^*$

2. Let L be the language that consists of all strings over the alphabet $\{a, b\}$.

2.1 Write a regular expression for L.

$$(a \cup b)^*$$

2.2 Write a context-free grammar that generates L.

$$R = \{S \rightarrow aS|bS|e\}$$

3. Construct a context-free grammar for the followings:

3.1 $L_1 = \{a^n b^{2n} : w \in \{a, b\}^*\}$

$$\{S \rightarrow aSbb|e\}$$

3.2 $L_2 = \{wcw^R : w \in \{a, b\}^*\}$

$$\{S \rightarrow aSa|bSb|c\}$$

4. Consider the following Grammar:

1. <SENTENCE> →<NOUN-PHRASE> <VERB-PHRASE>
2. <NOUN-PHRASE> → <CMPLX-NOUN> | <CMPLX-NOUN><PREP-PHRASE>
3. <VERB-PHRASE> → <CMPLX-VERB> | <CMPLX-VERB><PREP-PHRASE>
4. <PREP-PHRASE> → <PREP><CMPLX-NOUN>
5. <CMPLX-NOUN> → <ARTICLE> <NOUN>
6. <CMPLX-VERB> → <VERB> | <VERB><NOUN-PHRASE>
7. <ARTICLE> → a | the
8. <NOUN> → boy | girl | flower
9. < VERB> → touches | likes | sees
10. <PREP> → with

Give a derivation for the sentence "The boy likes the girl with the flower".

(replace touches with likes in the following derivations)

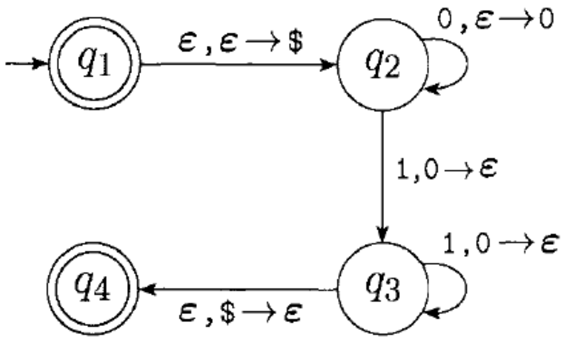
Here is one derivation:

⟨SENTENCE⟩ ⇒ ⟨NOUN-PHRASE⟩⟨VERB-PHRASE⟩ ⇒
⟨CMPLX-NOUN⟩⟨VERB-PHRASE⟩ ⇒
⟨CMPLX-NOUN⟩⟨CMPLX-VERB⟩⟨PREP-PHRASE⟩ ⇒
⟨ARTICLE⟩⟨NOUN⟩⟨CMPLX-VERB⟩⟨PREP-PHRASE⟩ ⇒
The boy ⟨VERB⟩⟨NOUN-PHRASE⟩⟨PREP-PHRASE⟩ ⇒
The boy ⟨VERB⟩⟨NOUN-PHRASE⟩⟨PREP⟩⟨CMPLX-NOUN⟩ ⇒
The boy touches ⟨NOUN-PHRASE⟩⟨PREP⟩⟨CMPLX-NOUN⟩ ⇒
The boy touches ⟨CMPLX-NOUN⟩⟨PREP⟩⟨CMPLX-NOUN⟩ ⇒
The boy touches ⟨ARTICLE⟩⟨NOUN⟩⟨PREP⟩⟨CMPLX-NOUN⟩ ⇒
The boy touches the girl with ⟨CMPLX-NOUN⟩ ⇒
The boy touches the girl with ⟨ARTICLE⟩⟨NOUN⟩ ⇒
The boy touches the girl with the flower

Here is another derivation:

⟨SENTENCE⟩ ⇒ ⟨NOUN-PHRASE⟩⟨VERB-PHRASE⟩ ⇒
⟨CMPLX-NOUN⟩⟨VERB-PHRASE⟩ ⇒ ⟨ARTICLE⟩⟨NOUN⟩⟨VERB-PHRASE⟩ ⇒
The boy ⟨VERB-PHRASE⟩ ⇒ The boy ⟨CMPLX-VERB⟩ ⇒
The boy ⟨VERB⟩⟨NOUN-PHRASE⟩ ⇒
The boy touches ⟨NOUN-PHRASE⟩ ⇒
The boy touches ⟨CMPLX-NOUN⟩⟨PREP-PHRASE⟩ ⇒
The boy touches ⟨ARTICLE⟩⟨NOUN⟩⟨PREP-PHRASE⟩ ⇒
The boy touches the girl ⟨PREP-PHRASE⟩ ⇒
The boy touches the girl ⟨PREP⟩⟨CMPLX-NOUN⟩ ⇒
The boy touches the girl with ⟨CMPLX-NOUN⟩ ⇒
The boy touches the girl with ⟨ARTICLE⟩⟨NOUN⟩ ⇒
The boy touches the girl with the flower

5. Below is the pushdown automata for $\{0^n 1^n : n \geq 0\}$



Based on this, design a pushdown automata for $\{0^n 1^n : n \geq 0\} \cup \{1^n 0^n : n \geq 0\}$

6. Prove that $\{a^n b^{2n} c^{3n} : n \geq 1\}$ is not context-free pumping lemma (4).

a). Suppose on the contrary that $L = \{a^n b^{2n} c^{3n} : n \geq 0\}$ is CFG, then there exists some sufficiently large number N, for any $n \geq N$, we have $a^n b^{2n} c^{3n} = uvxyz$ such that $|vy| > 0$, $|vxy| \leq N$, and $uv^i xy^i z \in L$ for any $i \geq 0$.
Pick $n = N$ and consider $a^N b^{2N} c^{3N} = uvxyz$. $|vxy| \leq N$, so there are 5 different possibilities.
i). $vxy = a \cdots a,\ or\ b \cdots b, or\ c \cdots c$, i.e., it only consists one symbol
We show the case of $vxy = a \cdots a,$ the other two cases are the same. Since $|vy| > 0$, we know $v^2 xy^2$ contains exactly $|vy|$ more a’s than vxy. That is, $uv^2 xy^2 z$ will contain $N + |vy| > N$ copies of a, i.e., $uv^2 xy^2 z = a^{N+|vy|} b^{2N} c^{3N} \notin L,$ contradicting that $uv^i xy^i z \in L$ for any $i \geq 0$.
(The other two cases are optional: $vxy = b \cdots b$. Since $|vy| > 0$, we know $v^2 xy^2$ contains exactly $|vy|$ more b’s than vxy. That

is, $uv^2xy^2z = a^N b^{2N+|vy|} c^{3N} \notin L$, contradicting that $uv^ixy^iz \in L$ for any $i \geq 0$.)

ii). $vxy = a \cdots ab \cdots b$ or $vxy = b \cdots bc \cdots c$, i.e., vxy contains both a, b or b, c. We show that case of $vxy = a \cdots ab \cdots b$, the other case is the same. Since $|vy| > 0$, we assume $vy = a^\alpha b^\beta$ for some $\alpha, \beta \geq 0$ and $\alpha + \beta > 0$. Now we have $uv^2xy^2z = a^{N+\alpha} b^{2N+\beta} c^N \notin L$, contradicting that $uv^ixy^iz \in L$ for any $i \geq 0$

Note that since $|vxy| \leq N$, it is impossible for vxy to contain all a, b, c. Thus we have exhausted all the possibilities.

7. Construct a context-free grammar for the following language:

$$L = \{w^R \overline{w}: w \in \{a, b\}^*\}$$

Where w^R is the reverse of w, and $\overline{w}$ is the opposite of w, that is, $\overline{w}$ is obtained by replacing every occurrence of a in w by b, and replacing every occurrence of b in w by a. For example, if $w = aab$, then $w^R = baa$, $\overline{w} = bba$.

$$R = \{S \to aSb |\ bSa | e\}$$