

# CS 5381 Analysis of Algorithms

## Homework 3

Fall 2022

**Total 180 points**

### Graph Algorithms

1. (30 points) The **incidence matrix** of a directed graph  $G = (V, E)$  with no self-loop is a  $|V| \times |E|$  matrix  $B = (b_{ij})$  such that

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i \\ 1 & \text{if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise.} \end{cases}$$

Describe what the entries of the matrix product  $BB^T$  represent, where  $B^T$  is the transpose of  $B$ .

2. (30 points) Show that using a single bit to store each vertex color suffices by arguing that the BFS algorithm on page 152 of the Lecture Notes would produce the same result if its line 18 were removed.
3. (30 points) Use the example on page 156 of the Lecture Notes to argue that in a breadth-first search, the value  $u.d$  assigned to a vertex  $u$  is independent of the order in which the vertices appear in each adjacency list.

4. (30 points) Prove that in a depth-first search of an undirected graph  $G$ , every edge of  $G$  is either a tree edge or a back edge.
5. (30 points) Show that if an edge  $(u, v)$  is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.
6. (30 points) Give an example of a connected graph such that the set of edges  $\{(u, v) : \text{there exists a cut } (S, V - S) \text{ such that } (u, v) \text{ is a light edge crossing } (S, V - S)\}$  does not form a minimum spanning tree.