Closure under regular operation

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Regular operations

- **Union**: $A \cup B = \{x | x \in A \text{ or } x \in B\}$.
- **Concatenation**: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$.
- **Star**: $A^* = \{x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A\}$.

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$A = \{\text{good}, \text{bad}\}, B = \{\text{boy}, \text{girl}\}$

$A \cup B = \{\texttt{good}, \texttt{bad}, \texttt{boy}, \texttt{girl}\}$,

$A \circ B = \{\texttt{goodboy}, \texttt{goodgirl}, \texttt{badboy}, \texttt{badgirl}\}$, and

$$A^* = \{\varepsilon, \text{good, bad, goodgood, goodbad, badgood, badbad, goodgoodgood, goodgoodbad, goodbadgood, goodbadbad, } \ldots\}.$$

Regular operations

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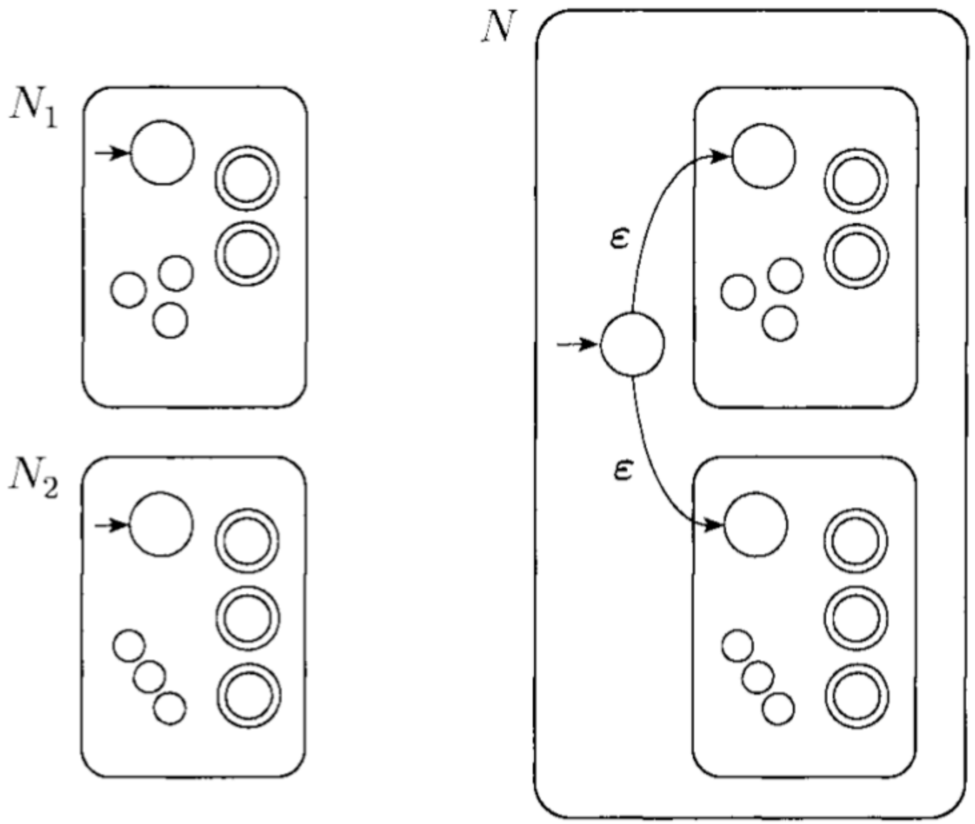

Regular operations

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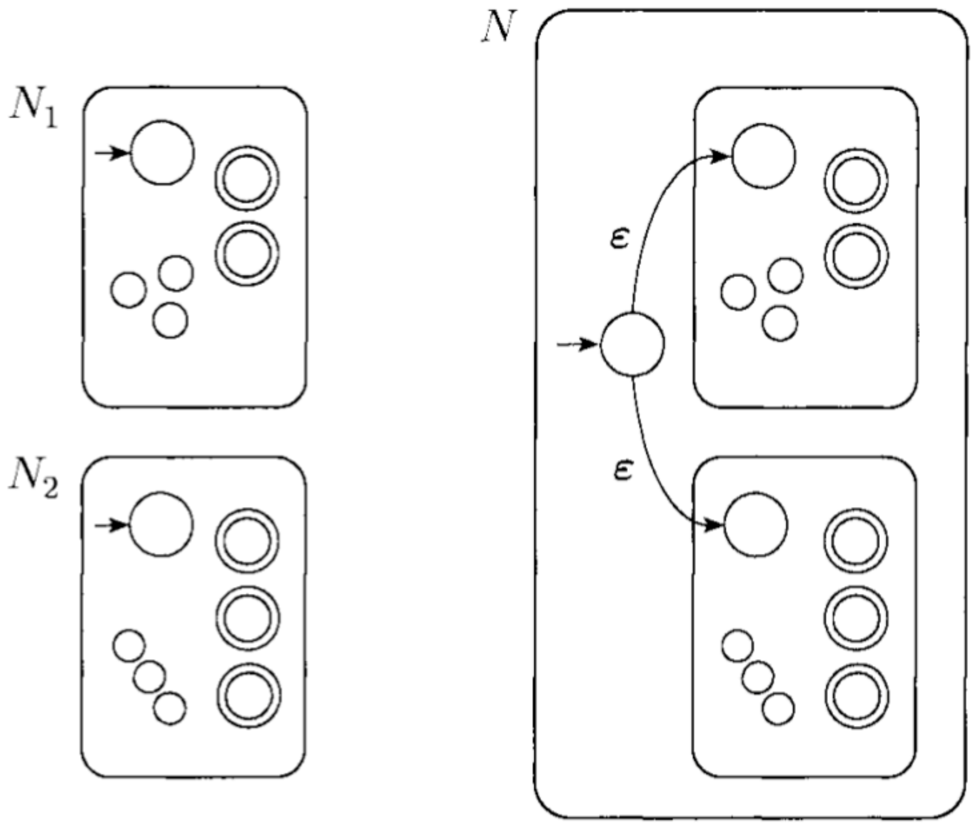
Is every string accepted by N_1 or N_2 accepted by N?

Is every string accepted by N accepted by N_1 or N_2?



Regular operations

- The class of regular languages is closed under: Union

Formal proof: state the construction of N using the 5-tuple definition.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

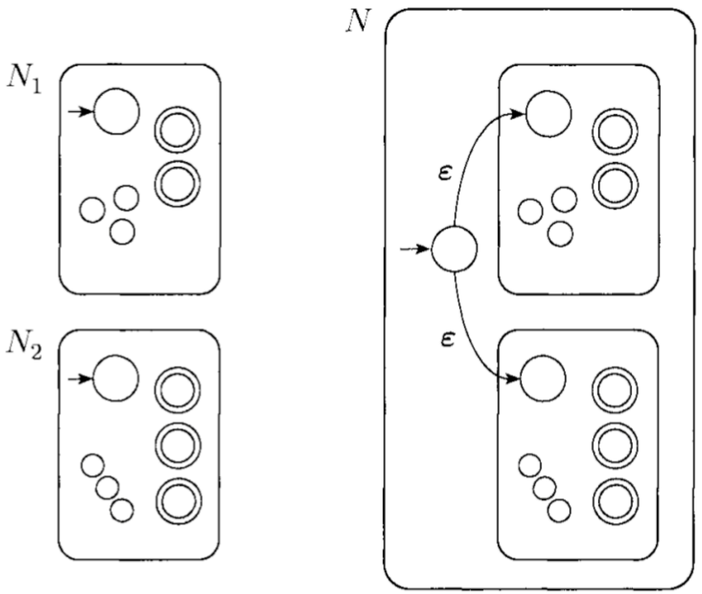
1. $Q = \{q_0\} \cup Q_1 \cup Q_2$.
The states of N are all the states of N_1 and N_2, with the addition of a new start state q_0.

2. The state q_0 is the start state of N.

3. The accept states $F = F_1 \cup F_2$.
The accept states of N are all the accept states of N_1 and N_2. That way N accepts if either N_1 accepts or N_2 accepts.

4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$



Regular operations

- The class of regular languages is closed under: Concatenation

  What does the statement mean? If languages A and B are both regular, then $A \circ B$ is also regular.

  What does regular language mean? A language is regular if and only if a DFA (NFA) recognizes it

  How to prove? If I have automata for A and B, construct automaton for $A \circ B$

Regular operations

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Is every string accepted by N_1 or N_2 accepted by N?

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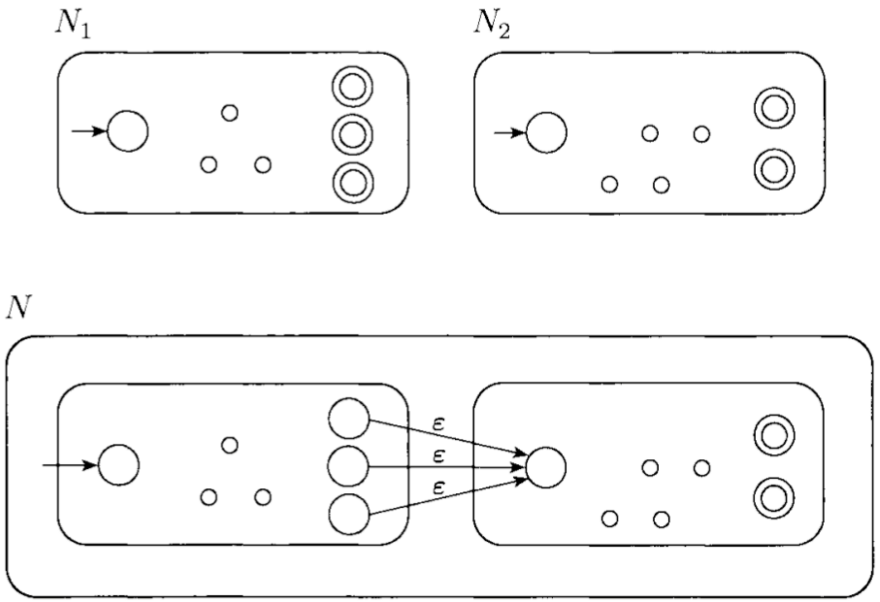
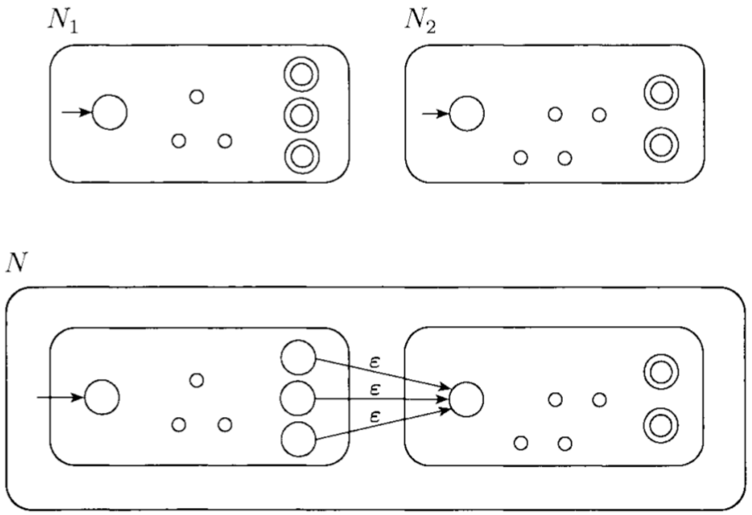


Regular operations

- The class of regular languages is closed under: Concatenation

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize A_2.



Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$.

1. $Q = Q_1 \cup Q_2$.
   The states of N are all the states of N_1 and N_2.
2. The state q_1 is the same as the start state of N_1.
3. The accept states F_2 are the same as the accept states of N_2.
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \text{ and } a = \varepsilon \\ \delta_2(q,a) & q \in Q_2. \end{cases}$$

Regular operations

- The class of regular languages is closed under: (Kleene-)star

  What does the statement mean? If language A is regular, then A^* is also regular.

  What does regular language mean? A language is regular if and only if a DFA (NFA) recognizes it

  How to prove? If I have automata for A, construct automaton for A^*

$$A^* = \{w_1 w_2 \cdots w_k : k \geq 0, w_i \in A\}$$

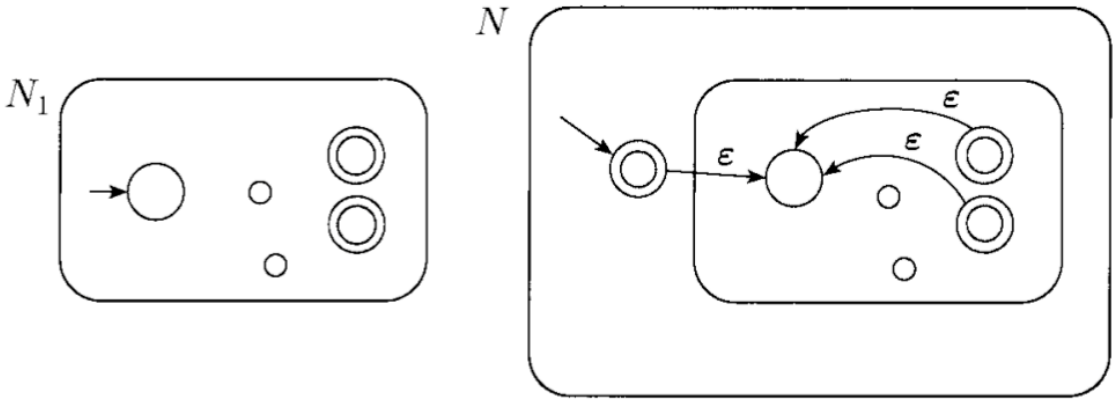
Regular operations

- The class of regular languages is closed under: Star

If I have automata for A, construct automaton for A^*

If I concatenate any k strings accepted by N_1 will it be accepted by N?

If a string is accepted by N, is it a concatenation of k strings acceptable by N_1 for some k?



Regular operations

- The class of regular languages is closed under: Star

PROOF Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.
Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*.

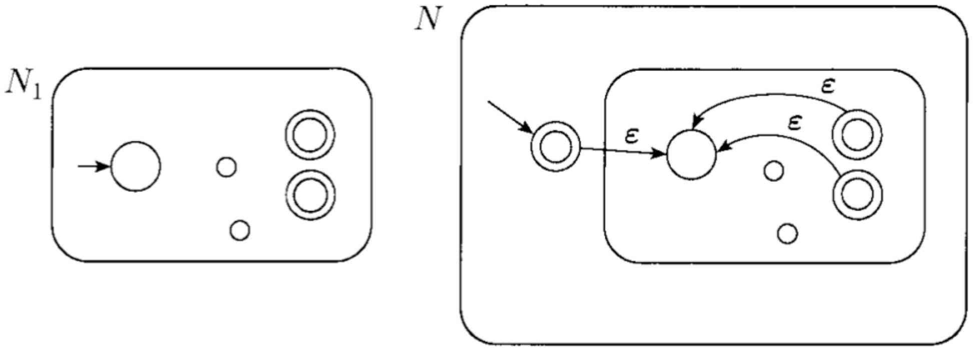
1. $Q = \{q_0\} \cup Q_1$.
The states of N are the states of N_1 plus a new start state.

2. The state q_0 is the new start state.

3. $F = \{q_0\} \cup F_1$.
The accept states are the old accept states plus the new start state.

4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q, a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$



Regular operations

- The class of regular languages is closed under: Complement and intersection (why?)