Parse Tree

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TEXAS TECH
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  - Example:

$$S \rightarrow AB$$
$$A \rightarrow aA|e$$
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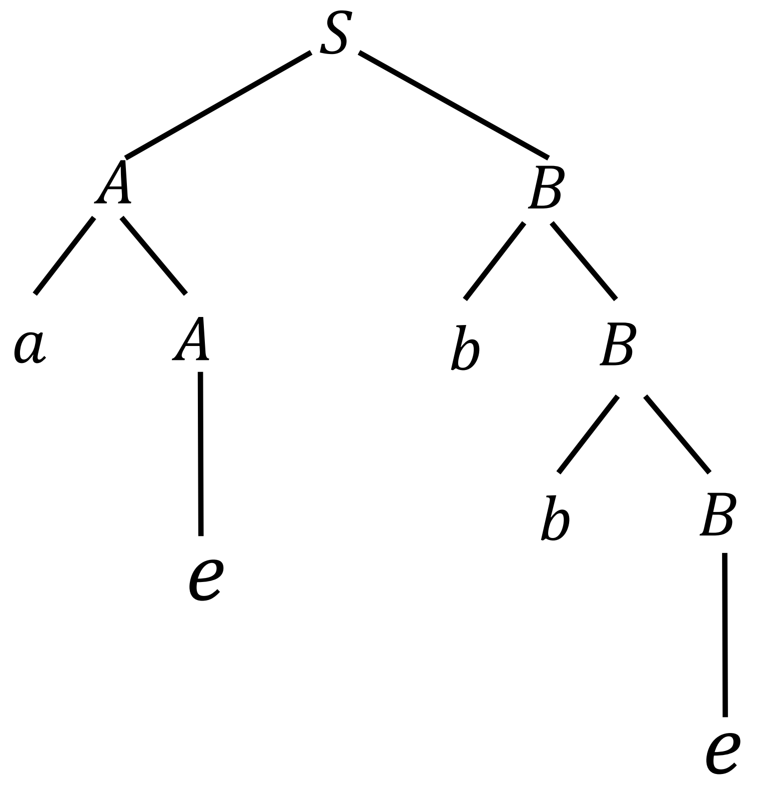

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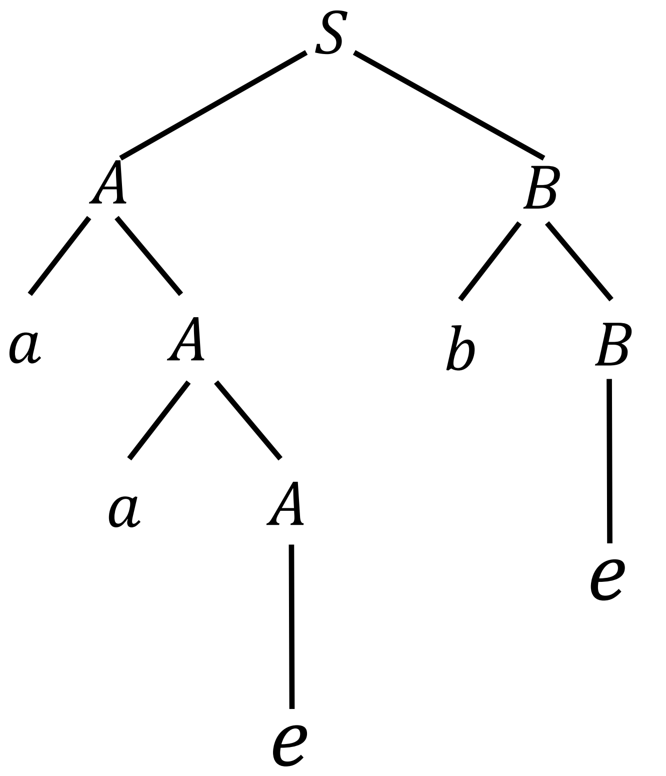

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  - A single node for $a \in \Sigma$ is a parse tree

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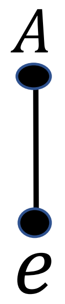

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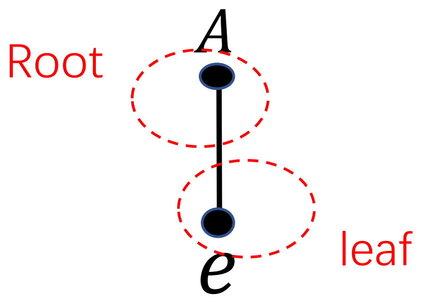

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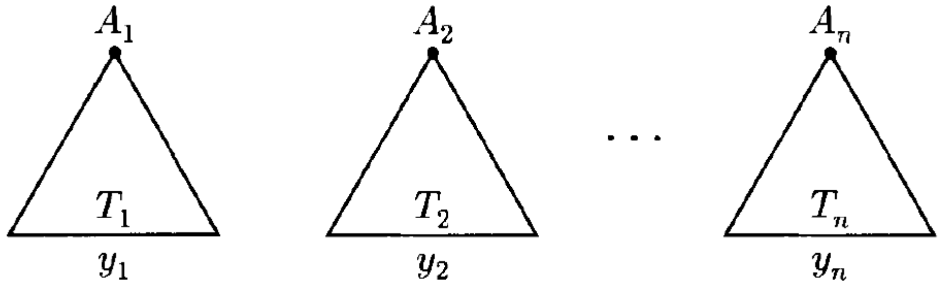

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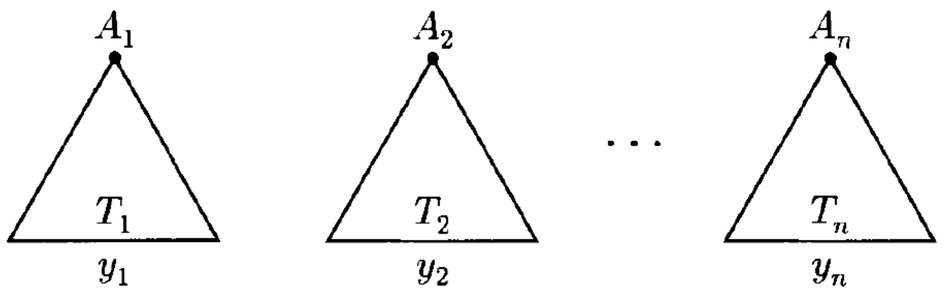

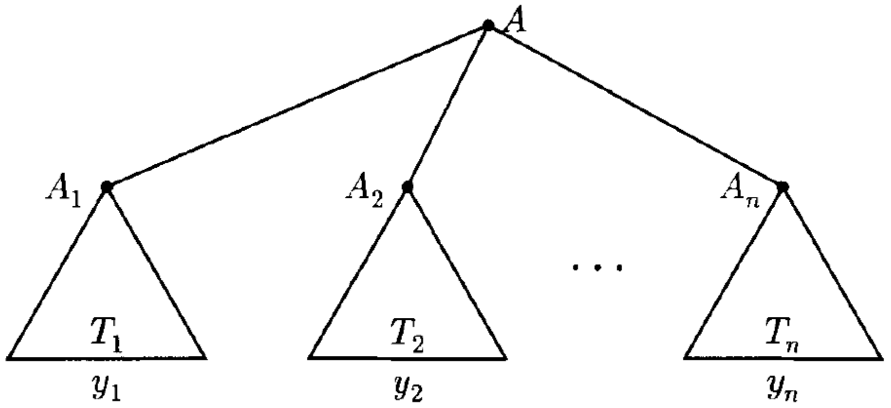

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  - Nothing else is a parse tree

Parse tree

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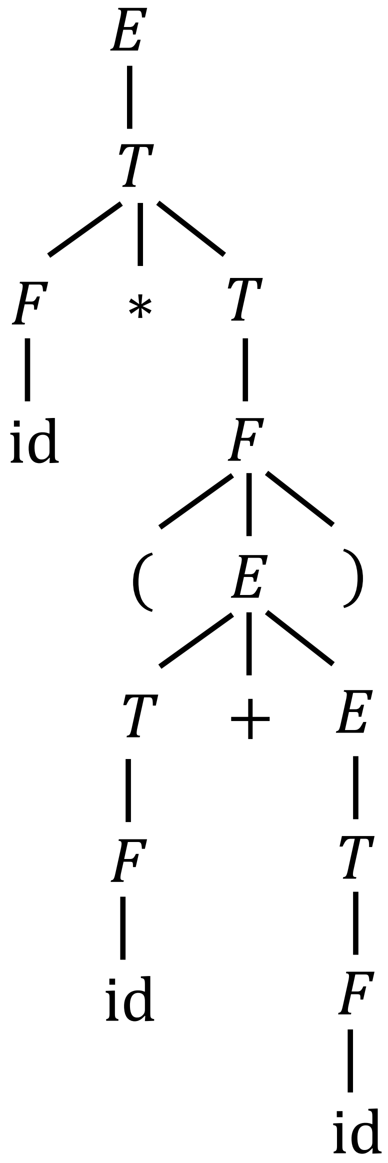
  - More examples:

$$V = \{+, *, (,), \mathrm{id}, T, F, E\}$$

$$\Sigma = \{+, *, (,), \mathrm{id}\}$$

$$E \to T + E$$
$$E \to T$$
$$T \to F * T$$
$$T \to F$$
$$F \to (E)$$
$$F \to \mathrm{id}$$



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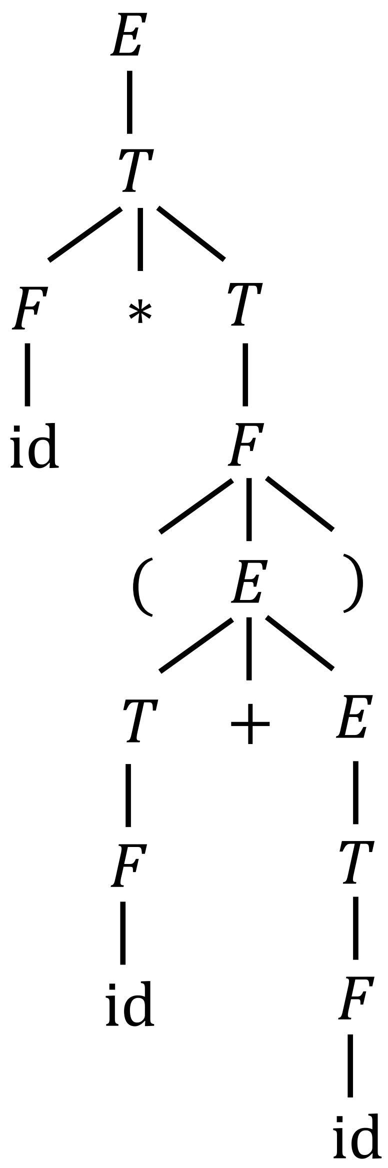
$\mathrm{id} * (\mathrm{id} + \mathrm{id})$



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$$V = \{S, (,)\},$$
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$$R = \{S \to e,$$
$$S \to SS,$$
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$$S \Rightarrow SS \Rightarrow (S)S \Rightarrow ()S \Rightarrow ()(S) \Rightarrow ()()$$
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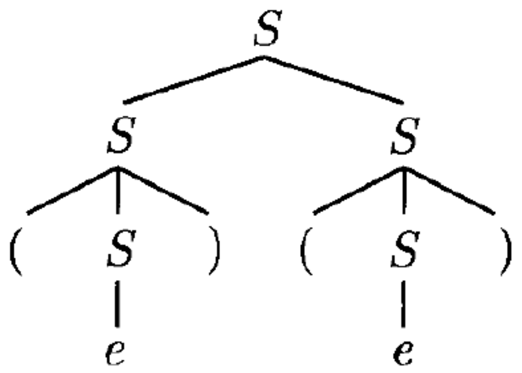

Parse tree

- Let $G = (V, \Sigma, R, S)$ be a context-free grammar, and let

$$D \Rightarrow x_1 \Rightarrow x_2 \Rightarrow \cdots \Rightarrow x_n$$

$$D' \Rightarrow x_1{}' \Rightarrow x_2{}' \Rightarrow \cdots \Rightarrow x_n{}'$$

$\in V - \Sigma$ $\in \Sigma^*$

We say D precedes D', $D \prec D'$, if $n > 2$ and there is some integer $1 < k < n$ such that

- 1. $x_i = x_i{}'$ for $i \neq k$
- 2. $x_{k-1} = x_{k-1} = uAvBw$, $u, v, w \in V^*$, $A, B \in V - \Sigma$
- 3. $x_k = uyvBw$, where $A \rightarrow y \in R$
- 4. $x_k' = uAvzw$, where $B \rightarrow z \in R$
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Differs in the order of only one step

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$$V = \{S, (,)\},$$
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$$R = \{S \to e,$$
$$S \to SS,$$
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$$D_1 = S \Rightarrow SS \Rightarrow (S)S \Rightarrow ((S))S \Rightarrow (())S \Rightarrow (())(S) \Rightarrow (())()$$
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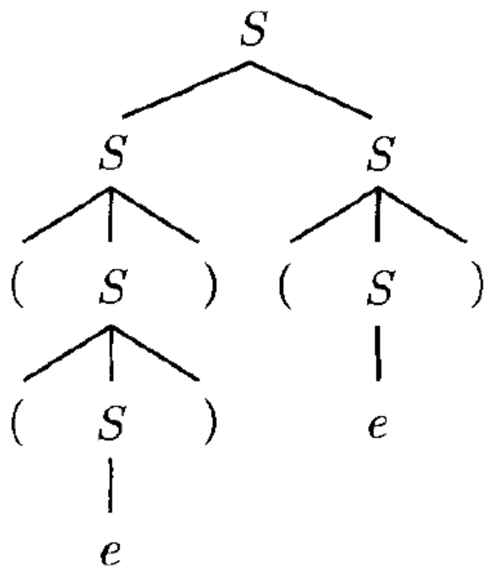

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$$D_1 \prec D_2$$

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$$D_1 \prec D_2$$
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D, D' are similar if $D \prec \cdots \prec D'$ or $D' \prec \cdots \prec D$

- D_1, D_2, D_3 are similar

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\begin{tikzcd}
 & & D_3 \arrow[dr, phantom, "\prec" sloped] & & & & \\
D_1 \arrow[r, phantom, "\prec"] & D_2 \arrow[ur, phantom, "\prec" sloped] \arrow[dr, phantom, "\prec" sloped] & & D_5 \arrow[r, phantom, "\prec"] & D_6 \arrow[dr, phantom, "\prec" sloped] & & \\
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- Similar derivations have the same parse tree

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- Similar derivations generate the same string at last
  - Is the opposite true?

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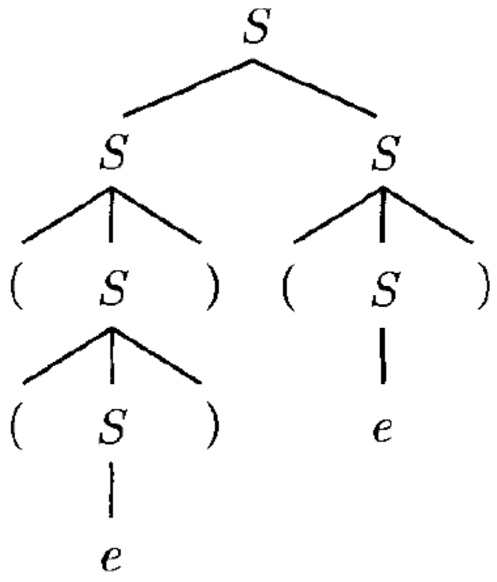

$$D' = S \Rightarrow SS \Rightarrow SSS \Rightarrow S(S)S \Rightarrow S((S))S$$
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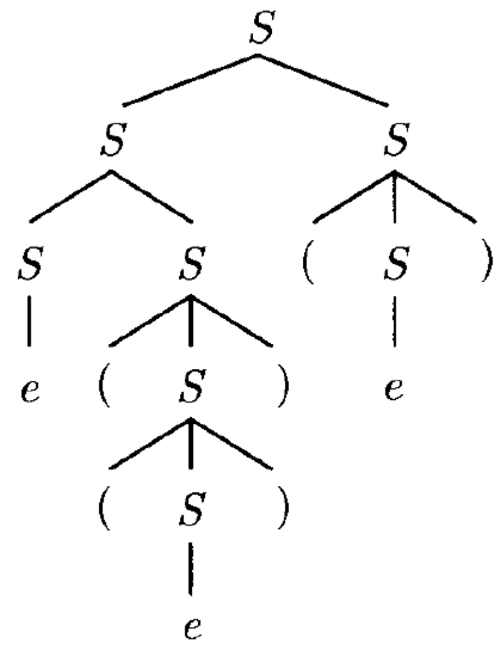

Ambiguity

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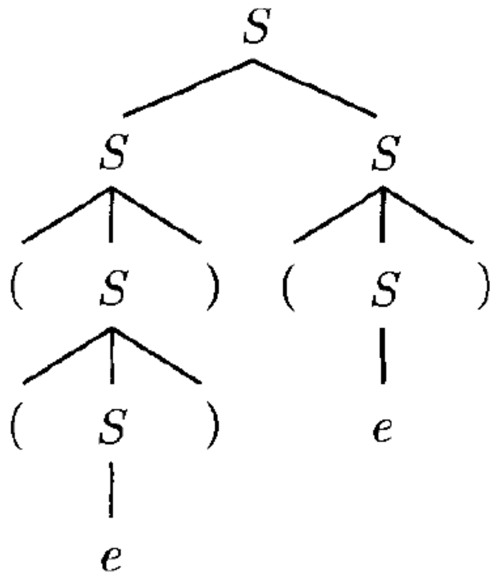

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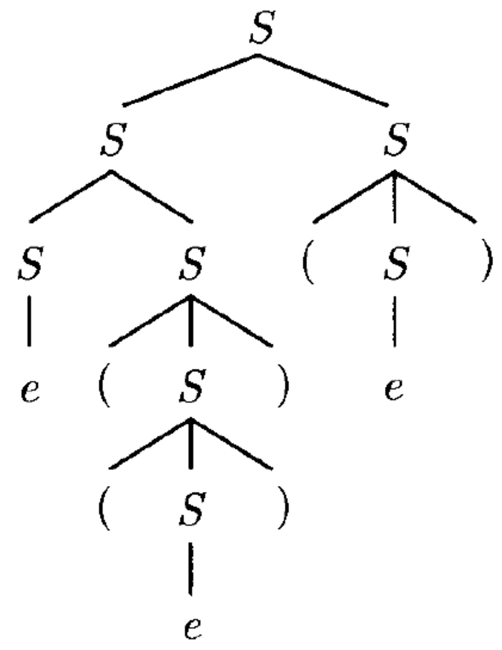

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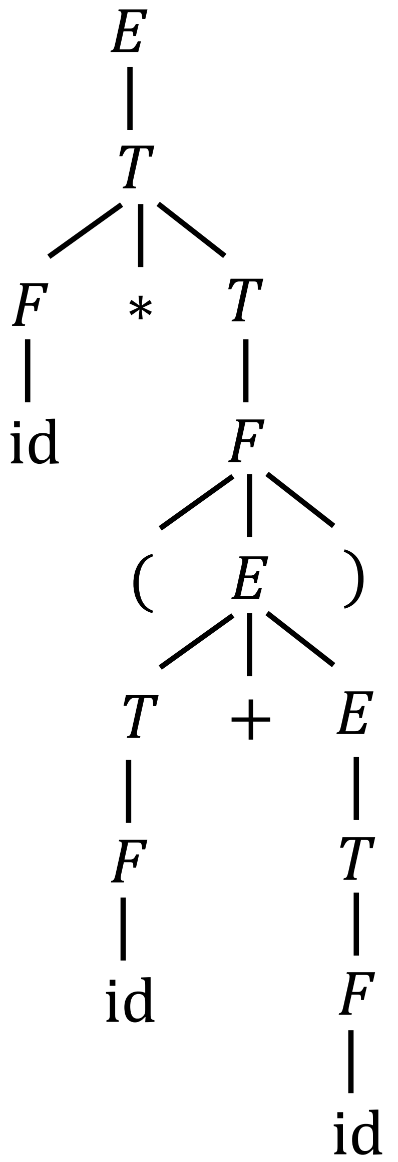
$V = \{+, *, (,), \mathrm{id}, T, F, E\}$

$\Sigma = \{+, *, (,), \mathrm{id}\}$

$E \to T + E$
$E \to T$
$T \to F * T$
$T \to F$
$F \to (E)$
$F \to \mathrm{id}$

$\mathrm{id} * (\mathrm{id} + \mathrm{id})$



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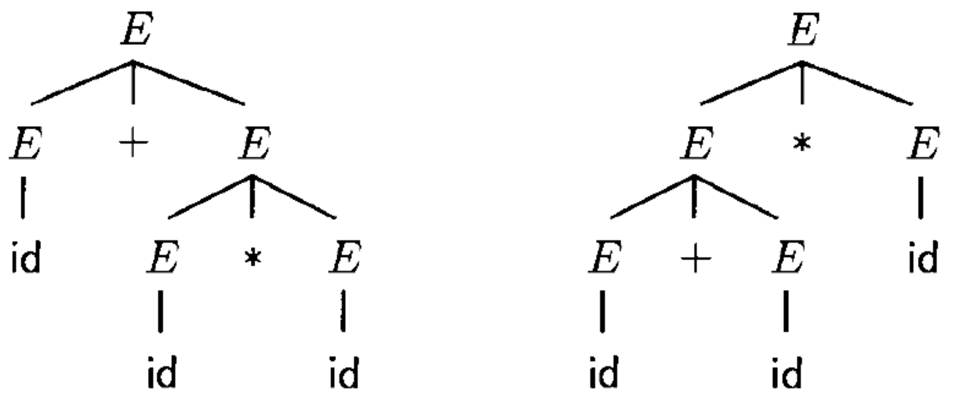
$\cancel{E \rightarrow T}$	
$T \rightarrow F * T$	$E \rightarrow E * E$
$\cancel{T \rightarrow F}$	
$F \rightarrow (E)$	$E \rightarrow (E)$
$F \rightarrow \mathrm{id}$	$E \rightarrow \mathrm{id}$

We can generate $\mathbf{id} + \mathbf{id} * \mathbf{id}$, but⋯



(a) (b)

Ambiguity

English ambiguity: I saw someone on the hill with a telescope.

- A Grammar with a string that has two or more distinct parse trees is called ambiguous

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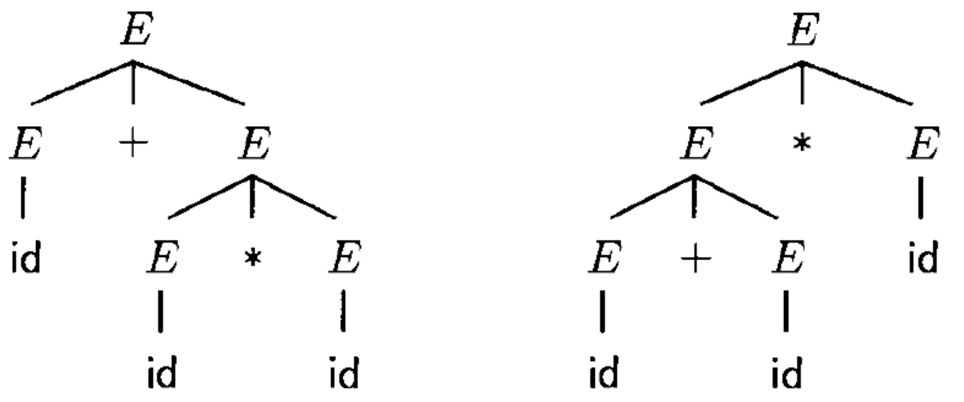
$T \rightarrow F * T$	$E \rightarrow E * E$
$\cancel{T \rightarrow F}$	
$F \rightarrow (E)$	$E \rightarrow (E)$
$F \rightarrow \mathrm{id}$	$E \rightarrow \mathrm{id}$

We can still generate $\mathbf{id} * (\mathbf{id} + \mathbf{id})$, but⋯



(a) (b)

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- A Grammar with a string that has two or more distinct parse trees is called ambiguous
- Can we construct an unambiguous grammar?

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&\qquad S \to SS,\\
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&\quad S \rightarrow AS, \\
&\quad A \rightarrow (S)\}.
\end{aligned}
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Ambiguity

- A Grammar with a string that has two or more distinct parse trees is called ambiguous
- There exist context-free languages such that all context-free grammars that generate them must be ambiguous (inherently ambiguous)
- A programming language should not be ambiguous