CS 5383

Theory of Automata

1. Select one correct answer out of 4 choices (1.5 point * 10).

1.1 Consider the following Turing machine M, where the alphabet is $\{0,1\}$:

M:= " On non-empty input string w :

1. Sweep the head from left to right across the tape, change every other 0 to 1 (i.e., the 2nd, 4th, 6th, etc., 0's, if exists, will be changed to 1)
2. Move the head to the last symbol on the tape which is **not** $\sqcup$. If this symbol is 0, accept. If the symbol is 1, reject.

Which of the following is the language decided by

M? (B)

a). $L=\{0^{2^n} \vee n \geq 0\}$

b). $L=\{w||w| is\, odd\}$

c). $L=\{w||w| is\, even\}$

d). $\overset{w}{L=\{¿w|w \in \{0,1\}^{¿}\}}$

1.2 Which of the following problem can**not** be solved by an algorithm? (C)

a). Given any deterministic finite automata and a string, determine whether this string can be accepted by the deterministic finite automata

b). Given any pushdown automata and a string, determine whether this string can be accepted by the pushdown automata

c). Given any Turing machine and a string, determine whether this string can be accepted by the Turing machine

d). None of the above

1.3 Consider the following statements, how many of them is **correct**? (C)

S1: If a language consists of all strings that can be accepted by a fixed deterministic finite automata, then it is regular

S2: If a language consists of all strings that can be accepted by a fixed pushdown automata, then it is context-free

S3: If a language consists of all strings that can be accepted by a fixed Turing machine, then it is Turing decidable

a). 0

b). 1

c). 2

d). 3

1.4 Consider two languages L_1 and L_2, which of the following statement is **wrong** on $L=\left(L_1 \cap L_2\right)$¿ (B)

a). If L_1 and L_2 are both regular, then L is also regular

b). If L_1 and L_2 are both context-free, then L is also context-free c). If L_1 and L_2 are both Turing decidable, then L is also Turing decidable

d). If L_1 and L_2 are both Turing semi-decidable, then L is also Turing semi-deciable

1.5. Consider the language $L = \{a^m b^{2n} c^{3l} : 3m + 3n + l \geq 2020\}$. Which of the followings is **correct**? (A)

a). This is a regular language

b). This is not a regular language, but is a context-free language

c). This is not a context-free language

d). All the above statements are wrong.

Hint: $L = \{a^m b^{2n} c^{3l} : 3m + 3n + l < 2020\}$ is finite and regular.

1.6. Let L be some language. How many of the following statements on $\acute{L}$ (i.e., the complement of L) are **correct**? (D)

S1: If L is regular, then $\acute{L}$ is context-free

S2: If L is context-free, then $\acute{L}$ is Turing decidable

S3: If L is Turing decidable, then $\acute{L}$ is

Turing semi-decidable

a). 0

b). 1

c). 2

d). 3

Hint: For S2, context-free means Turing decidable, then by closure property of decidable.

1.7. Let L be some language over the English alphabet that consists of 26 letters $\{a,b,c,\cdots,z\}$. Which of the following statement is **wrong**? (B)

a). If L is regular, then all the strings in L which end up with xyz also form a regular language

b). If L is context-free, then all the strings in L which contains the same number of x's and y's also form a context-free language

c). If L is Turing decidable, then all the strings in L that **cannot** be written as ww (where $w \in \{a,b,c,\cdots,z\}^{¿}$) also form a Turing decidable language

d). If L is Turing semi-decidable, then all the strings in L that **can** be written as ww (where $w \in \{a, b, c, \cdots, z\}$¿) also form a Turing semi-decidable language

Hint: Take the intersection of L and a suitable set, then use closure property. For c and d, remember that $L' = \{ww\}$ and $\acute{L}'$ are both decidable. For b, use $L = \{x^m y^n z^n : m, n \geq 0\} \wedge the that! \{x^n y^n z^n : n \geq 0\}$ is not context-free.

1.8. Let A and B be two languages. If we know $A \cup B$ and $A \cap B$ are both Turing-decidable, what can we say about A and B ? (D)

a). A and B must also be Turing decidable

b). It is possible that both A and B are **not** Turing decidable, however, they must both be Turing semi-decidable

c). It is possible that one of A and B is **not** Turing decidable; if A is **not** Turing decidable but is Turing semi-decidable, then B must be Turing decidable

d). It is possible that one of A and B is **not** Turing semi-decidable; if A is **not** Turing decidable but is Turing semi-decidable, then B **cannot** be Turing decidable

Hint: $A=A_{TM}, B=A'_{TM}$. For D, if B is Turing decidable, then by $A=((A\cup B)\cap B')\cup(A\cap B)$, A is also decidable.

1.9 Let U, V and W be three languages such that $U\subseteq V\subseteq W$. Which of the following statement is correct? (D)

a). If U and W are both regular, then V is also regular

b). If U and W are both context-free, then V is also context-free

c). If U and W are both Turing decidable, then V is also Turing decidable

d). None of the above

Hint: set $U=\varnothing, W=ground\ set$, V can be any set.

1.10 Let A,B be two disjoint languages. Which of the following statements is **correct**? (D)

a). If A and $A\circ B$ are both context-free languages,

then B is also a context-free language

$A=a^{¿}, B=\{a^p : p\ prime\}$

b). If A and $A \circ B$ are both regular languages, then B is also a regular language

$A=a^{¿}, B=\{a^p : p\ prime\}$

c). If A and $A \cup B$ are both Turing semi-decidable languages, then B is also a Turing semi-decidable language

$A=A_{TM}, B=A_{TM}^{'}$

d). If A and $A \cup B$ are both Turing decidable languages, then B is also a Turing decidable language

$B=(A \cup B) \cap \acute{A}$

2. Let L be the language that consists of all strings over the alphabet $\{a,b\}$ with an even length. (3)

2.1 Write a regular expression for L.

$(aa \cup ab \cup ba \cup bb)¿$

2.2 Write a context-free grammar that generates L.

$R = \{S \to aaS \vee |¿|baS \vee bbS \vee e\}$

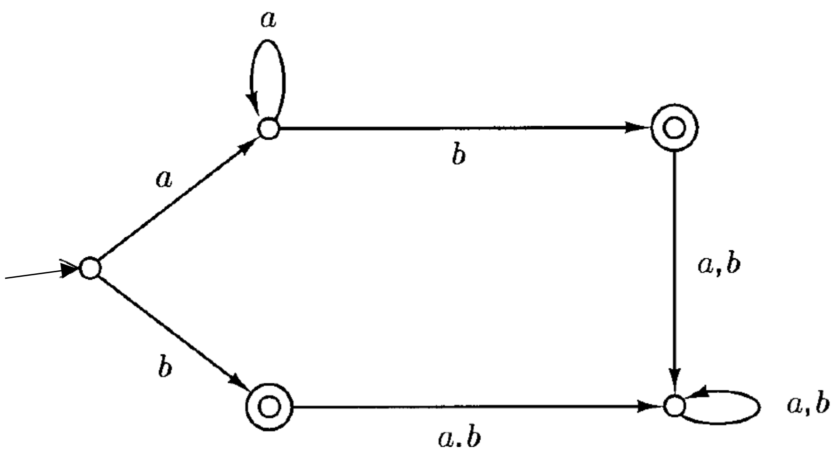
2.3 Describe a Turing machine that decides L. You can use an informal description just as Q 1.1

M:= " On non-empty input string w :

1. Sweep the head from left to right across the tape, change every other symbol (a or b) to x (i.e., the 2nd, 4th, 6th, etc., symbol a or b, if exists, will be changed to x)
2. Move the head to the last symbol on the tape which is **not** $\sqcup$. If this symbol is x, accept. If the symbol is a or b, reject.

3. Write a regular expression for the followings. (3)

3.1



$a¿b$

3.2 All strings over $\{a,b\}$ that does **not** end with

aa

$(a \cup b)$
$(¿¿¿(ab \cup bb \cup ba))$
$e \cup (a \cup b) \cup ¿$

3.3 Language generated by context-free grammar,

where $\Sigma=\{a,b\}, V=\{S,A\}, R=\{S \rightarrow aS \vee B, B \rightarrow bB \vee e\}$

$a^{¿} b^{¿}$

4. Construct a context-free grammar for the followings: (3)

4.1 $L_1=\{a^{2n}b^{3n}: w \in \{a,b\}^{¿}\}$

$\{S \rightarrow aaSbbb \vee e\}$

4.2 $L_2=\{wcw^R d: w \in \{a,b\}^{¿}\}$

$\{S \rightarrow Ad, A \rightarrow aAa \vee bAb \vee c\}$

5. Determine whether the following language is regular or not. State regular or not regular. If it is

regular, give the **regular expression**; if it is not regular, prove it using **pumping lemma**. (3)

Given a string $w \in \{a,b\}^{¿}$, we let $n_w(a)$ denote the number of a's in w , $n_w(b)$ denote the number of b's in w .

5.1 $L = \{ w \in \{a,b\}^{¿} : |n_a(w) - n_b(w)| \, is\, an\, odd\, number \}$

Hint: if the number of a's and b's differ by an odd number, then the length of string w is odd, this is essentially all strings of odd length

$(a \cup b)(aa \cup ab \cup bb \cup ba)^{¿}$

5.2 $L = \{ w \in \{a,b\}^{¿} : |n_a(w) - n_b(w)| = 0 \}$

One can directly use the proof for $\{a^n b^n : n \geq 0\}$

6. Prove that $\{(ab)^n (cd)^m (ef)^m : n \geq m \geq 1\} \cap \{(ab)^n (cd)^n (ef)^m : n \geq m \geq 1\}$ is not context-free using pumping lemma (3).

Proof the same as $\{(ab)^n (cd)^n (ef)^n : n \geq 0\}$