Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)
Mid Semester Examination (20-04-2023)

M.M. 50 **Time: 1.5 Hrs.**

Note: There are five question in the question paper. All the questions are compulsory.

Q1. A retired employee wants to invest Rs. 20, 00, 000/- which he received as provident fund. He was made acquainted with two schemes. In scheme A, he is ensured that for each rupee invested will earn Rs. 0.6 a year, and in scheme B, each rupee will earn Rs. 1.4 after two years. In scheme A, investments can be made annually, while in scheme B investments are allowed for periods that are multiples of two years only. Formulate this problem so that the employee invests his hard earn money to maximize the earnings at the end of three years? (assume x_{iA} = the amount invested in i^{th} year under scheme A, and x_{iB} = the amount invested in i^{th} year under scheme B, $i=1, 2, 3$).

[10]

Q2. Show that for a maximization type LPP, the maximum value of the objective function will attain at least at one of the vertex, provided the feasible region of the LPP is bounded.

[8]

Q3. Consider the following LPP:

$$\begin{aligned} \text{Max } & z = 3x_1 + 2x_2 \\ \text{Subject to } & -x_1 + 2x_2 + x_3 = 4, \\ & 3x_1 + 2x_2 + x_4 = 14, \\ & x_1 - x_2 + x_5 = 3, \\ & x_1, x_2, x_3, x_4, x_5 \geq 0. \end{aligned}$$

Let $x_B = (x_3, x_4, x_1)$ be basic variables and respective $B^{-1} = \begin{pmatrix} 1 & 0 & x \\ y & 1 & -3 \\ 0 & z & 1 \end{pmatrix}$.

(i) Without performing Simplex iterations, calculate the missing entries x, y, z of B^{-1}.

(ii) Without performing Simplex iterations, find all the entries of the table starting with basic variables $x_B = (x_3, x_4, x_1)$.

(iii) Is the table so formed optimal? If not, find the optimal table and hence the optimal solution.

(iv) Is the optimal solution obtained unique? If not, find the alternate optimal solution.

[15]

Q4. Consider the following LPP:

$$\text{Minimize } \quad Z = x_1 + x_2 + x_3$$
$$\text{Subject to } \; -x_1 + x_3 \leq 4, \quad x_1 + x_2 + 2x_3 \geq 3, \quad x_1 \geq 0,\, x_2 \leq 0,\, x_3 \geq 0.$$

Find its dual and then using the duality theory, prove that the above linear program is feasible but has no optimal solution. **[10]**

Q5. Consider the problem (P):

$$\text{Max } z = \text{Min}(3x - 10, -5x + 5)$$
$$\text{s/t } \quad 0 \leq x \leq 5.$$

(i) Solve the problem (P) graphically.

(ii) Formulate the problem (P) as a LPP.

[7]