3

$R(A, B, C, D)$

r

$r_1 = \pi_{ABC}(R)$, $r_2 = \pi_{AD}(R)$

$R(ABCD)$ → ABC, AD

$S = r_1 \bowtie r_2$

lossy

$A \not\rightarrow ABC$

$A \not\rightarrow AD$

$\boxed{S = r_1 \bowtie r_2}$ $\boxed{ABCD}$

$\boxed{S \subset r}$ ✗ $\quad$ $\boxed{r \cup S = r}$ ✗ $\quad$ $\boxed{r \bowtie S = r}$ ✓

✓ $\boxed{S \supset r}$ true $\quad$ $\boxed{r \cup S = S}$ ✓ $\quad$ $\boxed{r \bowtie S = S}$ ✗

R

A	B	C
1	2	1
2	2	3
3	3	3

3

S

A	B	C
1	2	1
2	2	3
3	3	3
1	2	3
2	2	1

5

$R \subset S$

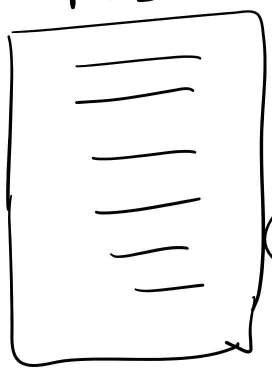
$$R \bowtie S = \pi_{ABC}\left(\sigma_{\boxed{(R.A = S.A) \wedge (R.B = S.B) \wedge (R.C = S.C)}} (R \times S) \right) = R$$

$R \times S$

121

223

333



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Dependency Preservation

Let R be a relation schema that is decomposed into two schemas with attribute sets X and Y, and let F be a set of FDs over R. The **projection of F on** X is the set of FDs in the closure F^+ (not just F!) that involve only attributes in X. We will denote the projection of F on attributes X as F_X. Note that a dependency $U \rightarrow V$ in F^+ is in F_X only if *all* the attributes in U and V are in X.

The decomposition of relation schema R with FDs F into schemas with attribute sets X and Y is **dependency-preserving** if $(F_X \cup F_Y)^+ = F^+$. That is, if we take the dependencies in F_X and F_Y and compute the closure of their union, we get back all dependencies in the closure of F. Therefore, we need to enforce only the dependencies in F_X and F_Y; all FDs in F^+ are then sure to be satisfied. To enforce F_X, we need to examine only relation X (on inserts to that relation). To enforce F_Y, we need to examine only relation Y.

Definition. Given a set of dependencies F on R, the **projection** of F on R_i, denoted by $\pi_{R_i}(F)$ where R_i is a subset of R, is the set of dependencies $X \rightarrow Y$ in F^+ such that the attributes in $X \cup Y$ are all contained in R_i. Hence, the projection of F on each relation schema R_i in the decomposition D is the set of functional dependencies in F^+, the closure of F, such that all the left- and right-hand-side attributes of those dependencies are in R_i. We say that a decomposition $D = \{R_1, R_2, \ldots, R_m\}$ of R is **dependency-preserving** with respect to F if the union of the projections of F on each R_i in D is equivalent to F; that is, $((\pi_{R_1}(F)) \cup \text{K} \cup (\pi_{R_m}(F)))^+ = F^+$.

If a decomposition is not dependency-preserving, some dependency is lost in the decomposition.

R F → R_1, R_2

F_1 F_2

$\boxed{(F_1 \cup F_2)^+ = F^+}$ ✓

$\boxed{(F_1 \cup F_2)^+ \subset F^+}$

$R(ABCD)$

$F: \{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow D\}$

$D: \{AB,\ BC,\ CD\}$

$\boxed{(F_1 \cup F_2 \cup F_3)^+ = F^+}$ ✓

$(F_1 \cup F_2 \cup F_3)^+ \subset F^+$

$R_1(AB)$	$R_2(BC)$	$R_3(CD)$
$A^+ = AB$	$B^+ = BC$	$C^+ = CD$
$\boxed{A \rightarrow B}$ F_1	$\boxed{B \rightarrow C}$ F_2	$\boxed{C \rightarrow D}$ F_3
$B^+ = B$	$C^+ = C$	$D^+ = D$

$\boxed{(F_1 \cup F_2 \cup F_3)^+ = F^+}$ ✓

D.P. ✓

$R(ABCD)$ $F: \{A \to B,\ B \to C,\ C \to D,\ D \to A\}$

$D: \{AD, BC, CD\}$ (R_1, R_2, R_3)

$\boxed{AD}$

$A^+ = AD$

$D^+ = AD$

F_1: $\boxed{A \to D,\ D \to A}$

$\boxed{BC}$

$B^+ = BC$

$C^+ = BC$

F_2: $\boxed{B \to C,\ C \to B}$

$\boxed{CD}$

$C^+ = CD$

$D^+ = CD$

F_3: $\boxed{C \to D,\ D \to C}$

$(F_1 \cup F_2 \cup F_3)^+$

$(F')^+ \subseteq F^+$

$F_1 \cup F_2 \cup F_3 = \{A \to D,\ D \to A,\ B \to C,\ C \to B,\ C \to D,\ D \to C\} = F'$

$F = \{A \to B,\ B \to C,\ C \to D,\ D \to A\}$

$A^+ = ADCB$

$(F_1 \cup F_2 \cup F_3)^+ \subseteq F^+$

$(F')^+ \subseteq F^+$ ✓

$(F')^+ \supseteq F^+$

$F^+ \subseteq (F')^+$ ✓

$X \subseteq Y$ and $Y \subseteq X$

$X = Y$

$F^+ = (F')^+$

D.P. ✓

$R(ABCDE)$

$F: \{AB \to CD,\ C \to D,\ D \to E\}$

$D: \{ABC, CD, DE\}$ (R_1, R_2, R_3)

$\boxed{ABC}$ R_1

$A^+ = A$

$B^+ = B$

$C^+ = C\cancel{DE}$

$C^+ = C$

$(AB)^+ = ABC\cancel{DE}$

$(AC)^+ = AC\cancel{DE}$

$(BC)^+ = BC\cancel{DE}$

F_1: $\boxed{AB \to C}$

$\boxed{CD}$ R_2

$C^+ = CD$

$D^+ = D$

F_2: $\boxed{C \to D}$

$\boxed{DE}$ R_3

F_3: $\boxed{D \to E}$

$F' = F_1 \cup F_2 \cup F_3 = \{AB \to C,\ C \to D,\ D \to E\}$

$AB^+ = ABCDE$

$AB \to CD$

$F^+ = (F')^+$

DP ✓

$R(ABCDEF)$

$F: \{AB \to CD,\ C \to D,\ D \to E,\ E \to F\}$

$D: \{AB,\ CDE,\ EF\}$

$R_1(AB)$

$\boxed{}$

F_1

$R_2(CDE)$

$C^+ = CDE$
$D^+ = DE$
$E^+ = E$
$(CD)^+ = CDEF$
$(DE)^+ = DE$
$(CE)^+ = CED$

$C \to DE$
$D \to E$
$CD \to E$
$CE \to D$

F_2

$C \to D$
$D \to E$

F_2

$R_3(EF)$

$\boxed{E \to F}$

F_3

$(F')^+ \subseteq F^+$ ✓
$F^+ \nsubseteq (F')^+$ ✓

$(F')^+ \subset F^+$

$F_1 \cup F_2 \cup F_3 = \{C \to D, D \to E, E \to F\} = F'$

$F = \{AB \to CD,\ C \to D,\ D \to E,\ E \to F\}$

lost

$(F')^+ \subset F^+$ ✓

not D.P.

$\boxed{A \to BCD}$

$AB \to CD,\ AC \to BD,\ AD \to BC$
$ABC \to D,\ ACD \to B,\ ABD \to C$

$A \to BCD$