CSN373: Probability Theory for Computer Engineering
DA:431: Probability Theory
Assignment:1

Do not copy your answer from the internet/friends. Last date of submission is 22 August 2024.

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1. A fair coin is tossed until a head is observed. Describe the sample space. How probable is this to get the first head in exactly 7 tosses? If possible, let us now assume that the coin is not fair and $P(H) > P(T)$. Let $P(H) = P(T) + 0.025$. Does the probability of getting the first head in exactly 7 tosses increase from the earlier case? How do you interpret it in general? What if $P(H) \approx 1$?

2. Suppose one coin (fair or unfair) is tossed two times. If the information of getting a head in the second toss does not have any effect on getting a head in the first toss, then show that all the outcomes that one can get from the two tosses are indeed independent.

3. Suppose Bob thinks of a random integer X between 1 to 20 uniformly at random. Alice tries to guess it by asking questions only of the form, is $X = x$, with every guess being independent of her previous guesses, and uses the following probability mass function on $\{1, 2, \ldots, 20\}$:

$$P_A(x) = \begin{cases} \frac{3}{40} & \text{if } 1 \leq x \leq 10 \\ \frac{1}{60} & \text{if } 11 \leq x \leq 15 \\ \frac{1}{30} & \text{if } 15 \leq x \leq 20. \end{cases}$$

   What is the expected number of guesses needed for Alice to guess it correctly? What happens if Alice also uses the uniform distribution (like Bob) instead of P_A?

4. Suppose you arrive at your hostel canteen at 11 PM and order food. Suppose that the food will be served at some time between 11 and 11:30 following the distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{\sqrt{x}}{100} & 0 \leq x < 16 \\ \frac{3}{5} & 16 \leq x < 18 \\ \frac{1}{20} & 18 \leq x < 30 \\ 1 & 30 \leq x. \end{cases}$$

   Here 'x' denotes the time in minutes you waited for the food. What is the probability that you will have to wait longer than 9 minutes? If, at 11:09, the food has yet to be served, what is the probability that you will have to wait at least an additional 10 minutes? What is the probability that the food will be served at 11:16 PM?

5. Let X_B be the random variable that denotes the number that Bob thinks of and X_A be the random variable that denotes what Alice chooses while guessing as in Question no. 3. Assume that X_A and X_B are independent. Define $X := \max\{X_A, X_B\}$. Is X a discrete random variable? Find its pmf (/pdf) and cdf.

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