

Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)
Transportation and Assignment Problems

Ex- 7

1. Iron ore is to be transported from three mines to four steel mills situated in different cities. Find the minimum cost transportation schedule given the following cost matrix:

		Steel Mills				Ore Available
		A	B	C	D	
Mines	I	14	56	48	27	13
	II	82	35	21	81	19
	III	99	31	71	63	16
Ore required		7	14	21	6	

2. In a flood relief operation, there are four bases of operations B_i ($i=1,2,3,4$) from where air crafts can take relief materials to three targets T_j ($j = 1,2,3$). Because of the difference in air crafts, range to target and flying altitudes, the relief material (in tons) per aircraft from any base that can be delivered to any target differs according to following table:

	T_1	T_2	T_3
B_1	8	6	5
B_2	6	6	6
B_3	10	8	4
B_4	8	6	4

The daily sortie capacity of each of the four bases is 150 sorties per day and the daily requirement of sorties on each target is 200. Find the allocation of sorties that maximizes the total tonnage over all the targets. If the problem has alternative solutions, find one.

3. A steel company has three open hearth furnaces and five rolling mills. Shipping cost of steel from furnaces to rolling mills are shown in the following table. Find the optimal shipping schedule.

Mills Furnaces	M ₁	M ₂	M ₃	M ₄	M ₅	Capacities (in quintal)
F ₁	4	2	3	2	6	8
F ₂	5	4	5	2	1	12
F ₃	6	5	4	7	3	14
Requirement (in quintal)	4	4	6	8	8	

4. Solve the following transportation problem for minimum transportation cost.

	D ₁	D ₂	D ₃	D ₄	D ₅	Availability
O ₁	20	19	14	21	16	40
O ₂	15	20	13	19	16	60
O ₃	18	15	18	20	–	70
Requirement	30	40	50	40	60	

where – indicates that it is not possible to transport goods from origin O_3 to destination D_5 .

5. Solve the following transportation problem for minimum cost starting with the degenerate basis $x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_{42} = 60$

	D ₁	D ₂	D ₃	Availability
O ₁	4	5	2	30
O ₂	4	1	3	40
O ₃	3	6	2	20
O ₄	2	3	7	60
Demand	40	50	60	

6. A company has four plants producing the same product. The production cost differs from one plant to another as do the cost of raw materials. There are five regional warehouses. Sales price at each is different. The maximum sales, capacity, unit transportation costs etc. are given in the following table. Determine the transportation schedule which maximizes the over all profit.

		Plants					
		1	2	3	4	Sales	
Production cost (Rs.)		15	18	14	13	Price	
Sales							
Raw material cost (Rs.)		10	9	12	8	(Rs.)	
		Warehouse					
		1	3	9	5	4	34
		2	1	7	4	5	32
Transportation cost (Rs.)		3	5	8	3	6	31
		4	7	3	8	2	31
		5	4	5	6	7	31
Capacity			150	200	175	100	

7. There are five persons and five jobs. The cost of performing different jobs by different persons are given in the following table. Find an assignment to minimize total cost.

	1	2	3	4	5
1	5	3	4	7	1
2	2	3	7	6	5
3	4	1	5	2	4
4	3	8	1	2	3
5	4	2	5	7	1

8. There are four sales representatives and four territories. Due to their different abilities, the yearly sales expected to be carried out by each representative in different territories are as follows:

		Territories			
		I	II	III	IV
Salesman	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Find the optimum assignment so as to maximize sales. What happens if salesman D can not be assigned to territory IV ?

9. Five jobs may be processed on any of the five machines available. The profit (in Rs.) from any machine processing any job is given below.

		Job				
Machine	32	38	40	28	40	
	40	24	28	21	36	
	41	27	33	30	37	
	22	38	41	36	36	
	29	33	40	31	39	

What is the optimum assignment that maximizes the profits ? What happens if fourth job can not be assigned to second machine ?

10. A department head has four tasks to be performed and three subordinates. The subordinates differ in efficiency. The estimates of the time, each subordinate would take to perform each task, is given below. How should we allocate the tasks, not more than one to each person so as to minimize the total man hour? Which job is left incomplete at the expiry for this period?

	I	II	III
A	9	26	15
B	13	27	6
C	35	20	15
D	18	30	20

ANSWERS :

1. $I \rightarrow 1 = 7$, $I \rightarrow 4 = 6$, $II \rightarrow 3 = 19$, $III \rightarrow 2 = 14$, $III \rightarrow 3 = 2$, Minimum cost = 1235.
 2. $1 \rightarrow 1 = 100$, $1 \rightarrow 3 = 50$, $3 \rightarrow 1 = 100$, $3 \rightarrow 2 = 50$, $4 \rightarrow 2 = 150$, $2 \rightarrow 3 = 150$, Maximum tonnage = 4250. Alternate solution
(i) $1 \rightarrow 2 = 100$, $1 \rightarrow 3 = 50$, $2 \rightarrow 3 = 150$, $3 \rightarrow 1 = 50$, $3 \rightarrow 2 = 100$, $4 \rightarrow 1 = 150$:
(ii) $1 \rightarrow 1 = 100$, $1 \rightarrow 3 = 50$, $2 \rightarrow 3 = 150$, $3 \rightarrow 2 = 150$, $4 \rightarrow 1 = 100$, $4 \rightarrow 2 = 50$.
 3. $1 \rightarrow 1 = 4$, $1 \rightarrow 2 = 4$, $2 \rightarrow 4 = 8$, $2 \rightarrow 5 = 4$, $3 \rightarrow 1 = 0$, $3 \rightarrow 3 = 5$, $3 \rightarrow 5 = 4$.
 4. $1 \rightarrow 5 = 40$, $2 \rightarrow 3 = 50$, $2 \rightarrow 5 = 10$, $3 \rightarrow 1 = 30$, $3 \rightarrow 2 = 40$, Minimum cost = 2590.
 5. Minimum cost 300.
 6. Maximum profit 1955.
 7. (2, 1) , (5, 2) , (4, 3) , (3, 4) , (1, 5) , Minimum cost 8.
 8. (i) $A \rightarrow I$, $B \rightarrow III$, $C \rightarrow II$, $D \rightarrow IV$ or
(ii) $A \rightarrow I$, $B \rightarrow II$, $C \rightarrow III$, $D \rightarrow IV$, Maximum sales 99.
 9. Maximum profit 191.
 10. $I \rightarrow A$, $III \rightarrow B$, $II \rightarrow C$, Total time = 35, job D.
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