

Lecture 10

Syntax Analysis

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February 11, 2025



• Example of Shift-Reduce parsing



- Example of Shift-Reduce parsing
- Issuses in bottom-up parsing



- Example of Shift-Reduce parsing
- Issuses in bottom-up parsing
- Handle



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- Issuses in bottom-up parsing
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- Configuration of LR parser





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- ullet Summarize all possible stack prefixes lpha as a parser state
- ullet Parser state is defined by a DFA state that reads in the stack lpha
- Accept states of DFA are unique reductions





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 - ► The set of viable prefixes is a regular language
 - Construct an automaton that accepts viable prefixes





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 - Symbols on the left of "." are already on the stacks
 - Symbols on the right of "." are expected in the input



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 - Added items have "." located at the beginning
 - ▶ No symbol of these items is on the stack as yet





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- Intuitively $A \to \alpha.B\beta$ indicates that we might see a string derivable from $B\beta$ as input



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 - ▶ If $A \to \alpha.B\beta$ is in closure(I) and $B \to \gamma$ is a production then $B \to .\gamma$ is in closure(I)
- ullet Intuitively A o lpha.Beta indicates that we might see a string derivable from Beta as input
- ullet If input $B o\gamma$ is a production then we might see a string derivable from γ at this point



• Consider the grammar

$$E' \to E$$

$$E \to E + T|T$$

$$T \to T * F|F$$

$$F \to (E)|id$$



• Consider the grammar

$$E' \rightarrow E$$

 $E \rightarrow E + T | T$
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• Grammar:

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• I_0 : $closure(E' \rightarrow .E)$



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• $I_1 : goto(I_0, E)$



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•
$$I_1$$
: $goto(I_0, E)$
 $E \rightarrow E$.
 $F \rightarrow F_1 + T$



• Grammar:

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- $l_1 : goto(l_0, E)$ $E \rightarrow E$. $E \rightarrow E + T$
- I_2 : $goto(I_0, T)$



 $F \rightarrow .id$

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- $l_1 : goto(l_0, E)$ $E \rightarrow E$. $F \rightarrow F_1 + T$
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- I_1 : $goto(I_0, E)$
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$$E \rightarrow E. + T$$

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 - $F \rightarrow T$.
 - $T \rightarrow T. * F$
- I_3 : $goto(I_0, F)$
 - $T \rightarrow F$





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• I_4 : $goto(I_0, ())$ $F \rightarrow (.E)$ $E \rightarrow .E + T$ $E \rightarrow .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$ $F \rightarrow .id$



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- I_5 : $goto(I_0, id)$



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- I_5 : $goto(I_0, id)$ $F \rightarrow id$.



• I_6 : $goto(I_1, +)$



• $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$ $F \rightarrow .id$



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- I_7 : $goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
- I_8 : $goto(I_4, E)$



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• $goto(I_4, T)$ is I_2

- I_6 : $goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow F$
 - $F \rightarrow .(E)$
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- I_7 : $goto(I_2, *)$ $T \rightarrow T * F$

 - $F \rightarrow .(E)$
 - $F \rightarrow id$
- I_8 : $goto(I_4, E)$ $\mathsf{F} \to (\mathsf{E}.)$
 - $E \rightarrow E. + T$



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 - $T \rightarrow T * F$
 - $F \rightarrow .(E)$

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$$E \rightarrow E. + T$$

175 IT ROCKELL

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- $goto(I_4, ()is I_4)$
- $goto(I_4, id)$ is I_5

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- $goto(I_4, id)$ is I_5
- \bullet I_9 : $goto(I_6, T)$



- I_6 : $goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow F$ $F \rightarrow .(E)$ $F \rightarrow id$
- I_7 : $goto(I_2, *)$ $T \rightarrow T * F$ $F \rightarrow .(E)$ $F \rightarrow id$
- I_8 : $goto(I_4, E)$ $\mathsf{F} \to (\mathsf{E}.)$

$$F \rightarrow (E.)$$

 $E \rightarrow E. + T$

- $goto(I_4, T)$ is I_2 • $goto(I_4, F)$ is I_3
- $goto(I_4, ()is I_4)$
- $goto(I_4, id)$ is I_5
- \bullet I_9 : $goto(I_6, T)$ $E \rightarrow E + T$. $T \rightarrow T * F$



- $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
- I_8 : $goto(I_4, E)$ $F \rightarrow (E.)$ $E \rightarrow E. + T$

- $goto(I_4, T)is$ I_2
- $goto(I_4, F)$ is I_3
- goto(I₄, () is I₄
- $goto(I_4, id)$ is I_5
- $I_9: goto(I_6, T)$ $E \rightarrow E + T.$ $T \rightarrow T. * F$
- $goto(I_6, F)$ is I_3



• $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$

 $F \rightarrow id$

- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
- I_8 : $goto(I_4, E)$ F \rightarrow (E.)

$$E \rightarrow E. + T$$

 $E \rightarrow E. +$

- $goto(I_4, T)$ is I_2
- $goto(I_4, F)$ is I_3
- goto(I₄, () is I₄
- $goto(I_4, id)$ is I_5
- I_9 : $goto(I_6, T)$ $E \rightarrow E + T$.
 - $T \rightarrow T. * F$
- $goto(I_6, F)$ is I_3
- $goto(I_6, ()is I_4)$

- I_6 : $goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$

 $F \rightarrow .id$

- F
 ightarrow .id
- I_8 : $goto(I_4, E)$ $F \rightarrow (E.)$ $E \rightarrow E. + T$

- $goto(I_4, T)is$ I_2
- $goto(I_4, F)$ is I_3
- goto(I₄, () is I₄
- $goto(I_4, id)$ is I_5
- I_9 : $goto(I_6, T)$ $E \rightarrow E + T$. $T \rightarrow T * F$
- $goto(I_6, F)$ is I_3
- $goto(I_6, ()is I_4)$
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- $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
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- goto(I₄,()is I₄
- $goto(I_4, id)$ is I_5
- I_9 : $goto(I_6, T)$ $E \rightarrow E + T$. $T \rightarrow T_* * F$
- $goto(I_6, F)$ is I_3
- goto(I_6 ,() is I_4
- $goto(I_6, id)$ is I_5

• I_{10} : $goto(I_7, F)$ $T \rightarrow T * F$.



- $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$

 $F \rightarrow id$

• I_8 : $goto(I_4, E)$ $F \rightarrow (E.)$ $E \rightarrow E. + T$

- $goto(I_4, T)is$ I_2
- $goto(I_4, F)$ is I_3
- goto(I₄, () is I₄
- $goto(I_4, id)$ is I_5
- $I_9: goto(I_6, T)$ $E \rightarrow E + T$. $T \rightarrow T * F$
- $goto(I_6, F)$ is I_3
- $goto(I_6, ()is I_4)$
- $goto(I_6, id)$ is I_5

- I_{10} : $goto(I_7, F)$ $T \rightarrow T * F$.
- goto(I₇, ()is I₄
 goto(I₇, id)is I₅



- I_6 : $goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$
 - $F \rightarrow .(E)$
 - extstyle F
 ightarrow .id
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
- I_8 : $goto(I_4, E)$ $F \rightarrow (E.)$ $E \rightarrow E. + T$

- $goto(I_4, T)is$ I_2
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- $goto(I_4, id)$ is I_5
- I_9 : $goto(I_6, T)$ $E \rightarrow E + T$. $T \rightarrow T. * F$
- $goto(I_6, F)$ is I_3
- $goto(I_6, ()is I_4)$
- $goto(I_6, id)$ is I_5

- I_{10} : $goto(I_7, F)$ $T \rightarrow T * F$.
- goto(I₇, ()is I₄
 goto(I₇, id)is I₅
- $I_11 : goto(I8,))$ $F \to (E).$



- $I_6: goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$
 - $F \rightarrow .(E)$
 - F
 ightarrow .id
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$ $F \rightarrow .(E)$ $F \rightarrow .id$
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- I_{10} : $goto(I_7, F)$ $T \rightarrow T * F$.
- goto(I₇, ()is I₄
 goto(I₇, id)is I₅
- $I_11 : goto(I8,))$ $F \to (E).$
- $goto(I_8, +)$ is I_6



- I_6 : $goto(I_1, +)$ $E \rightarrow E + .T$ $T \rightarrow .T * F$ $T \rightarrow .F$ $F \rightarrow .(E)$
- $I_7 : goto(I_2, *)$ $T \rightarrow T * .F$

 $F \rightarrow id$

- $F \rightarrow .(E)$
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- goto(I₇, ()is I₄
 goto(I₇, id)is I₅
- $I_11 : goto(I8,))$ $F \to (E).$
- $goto(I_8, +)is$ I_6
- goto(l₉,*)is l₇









