[CSN212] Assignment 1
Complexity Analysis
Maximum Marks 100

1 Lockdown Loss [10 Marks]

In these days of Lockdown, Motu's Father bussiness is in Loss. On Zeroth day of Lockdown the loss was Rs P, then Rs Q on the First day. Motu observed loss as a function and wanted to calculate the loss on Nth day of the Lockdown. He observed that the loss is dependent on the previous days, i.e.

$F(n) = F(n-1) + F(n-2) + F(n-1) \times F(n-2)$

Design a polynomial time algorithm that computes the loss on Nth day given P, Q and N.

Note: In the matrix formulation clearly define the **variable**, **base case**, and the **size** of the matrix.

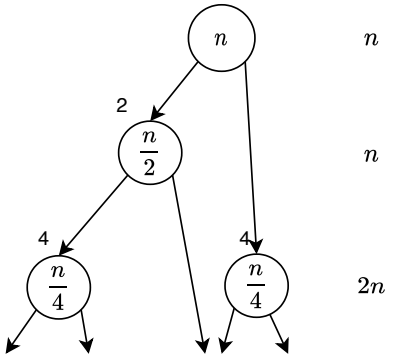
2 Recursion and analysis [15 Marks]

. Consider the following recurrence relation.

$$T(n) = 2T(n/2) + 4T(n/4) + n \text{ with } T(1) = 1$$

1. Make its recurrence tree, grouping all instances of $T(n/2^i)$ at the level i. What is the total time taken $T_i(n)$ at each level $0 \leq i \leq 4$?
2. Note that total time taken $T_i(n)$ at a level $i \geq 2$ satisfies $T_i(n) = T_{i-1}(n) + T_{i-2}(n)$. Complete the recursion tree proof to get $T(n) = O(n^{1+\log_2 \varphi})$, where φ is the golden ratio.
3. Verify the solution $T(n) = O(n^{1+\log_2 \varphi})$ using substitution method.

Hint: *Levels $0, 1, 2$ of the recursion tree are shown below. Grouping identical elements is used for avoiding extensive drawing.*



For the other cases, think of fibonacci sequence, where $f_0 = f_1 = 1$ and $f_i = f_{i-1} + f_{i-2}$ for $i \geq 2$. Use $\sum_{i=1}^{k} f_i = f_{k+2} - 1$, $f_k = O(\varphi^k)$ and $\varphi^2 = \varphi + 1$, where φ is the golden ratio.

3 Analysing Quicksort [20 Marks]

We know the quick sort performs differently based on input. The recursive formulation can be written as:

$$T(n) = T(k) + T(n - k - 1) + n$$

1. Show that the above expression is $O(n^2)$ and $\Omega(n \log n)$.
2. Substitute suitable values for k to show $\Theta(n^2)$ and $\Theta(n \log n)$ for different values of k, and describe the corresponding input accordingly.

4 Generalized Master's Method [30 Marks]

Consider the following generalized form of recursion on two terms

$$T(n) = aT(\frac{n}{b}) + cT(\frac{n}{d}) + f(n)$$

Describe and prove any tight bounds possible and the criteria for the same.

Note: *Refer to Section 4.5 of CLRS book (3rd Edition) for exhaustive proof of master's method, and follow a similar rigorous approach.*

5 Textbook Problems [25 Marks]

Solve the following problems from CLRS Textbook (3rd Edition)
3.1-8, 3.2-4, 3.2-5,4.3-7,4.5-5,4.6-3