[CSN212] Assignment 5
Maximum Flow and Complexity Theory
[Maximum Marks 100]

NOTE: Feel free to use any algorithm or proof covered in class as a black box.

1 Cycle Cover [20 Marks]

A cycle cover of a directed graph $G = (V, E)$ is a set of vertex-disjoint simple cycles that cover all the vertices, i.e., every vertex appears exactly once in the cycle cover. Design and analyze an algorithm to compute whether a graph contains a cycle cover or not. Also, prove the correctness of the algorithm.

2 Exact Path Length [20 Marks]

Given a weighted directed graph $G = (V, E)$ having positive edge weights and two vertices $s, t \in V$. Show that computing whether an $s - t$ path of a given length d exists is NP-Hard by reducing it from the Subset sum problem.

3 Cardinality DAG problem [20 Marks]

Given a directed acyclic graph (DAG) $G = (V, E)$ with a unique source s and a unique sink t, where each edge is labelled a character from set L, and an integer k. The Cardinality DAG problem is to decide if there is an $s - t$ path such that the set of labels of the edges of the path has cardinality exactly k. Say for $k = 1$ there must be a path having edges of the same label. Prove that this problem is NP-complete by reducing from 3-CNF-SAT.

4 Harder Scheduling Problem [20 Marks]

Given a single machine and n tasks $a_1, \cdots, a_n$ where each task a_i requires t_i time to complete and gives a profit p_i if completed before the deadline d_i. The schedule for performing the tasks on the machine is such that no two tasks overlap. Show that computing whether a schedule exists which gives a profit P is NP-Complete.

5 Problems from Tutorials [20 Marks]

Formally solve the following problems from tutorials:

1. Design and analyze an algorithm for Hunter survival problem.
2. Design and analyze an algorithm for tiling a chessboard problem.
3. Prove that Maximum independent set (function version) is NP-Hard
4. Prove that Subgraph-Isomorphism is NP Complete.