NOTE that in notes SS0 = 1 ; It is actually S (S0) and not (S S) 0.

Lecture 11 **17.2.2025**

Today's agenda:

Boolean operations:

true = $\lambda x. \lambda y. x$

false = $\lambda x. \lambda y. y$

OR = $\lambda x. \lambda y.$ x true y [version 1]

OR = $\lambda x. \lambda y.$ x x y [version 2] does not involve any other function; thus better

AND = $\lambda x. \lambda y.$ x y false [version 1]

AND = $\lambda x. \lambda y.$ x y x [version 2] does not involve any other function; thus better

NOT = $\lambda x.$ x false true

XOR = $\lambda x. \lambda y.$ x (NOT y) y

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Idea: use the functions true and false appropriately.

consider OR: x OR y [version 1]

$\lambda x. \lambda y.$ x ___ y

When we pass the actual parameter say x=true, we get true ____ y

By the property, "_____" would be returned. Since we know the answer as true so "___" is true.

What if x=false? Then the answer is whatever is the value of y. so the second argument should be y and our choice is correct.

So we get OR: $\lambda x. \lambda y.$ x true y

Take a closer look. What if x is true? It is true which is x. So we have

OR: $\lambda x. \lambda y.$ x x y now the body of OR does not contain any constant (true/false) [version 2]

Now we can design AND easily

What if x=false? Then the answer is false irrespective of the value of y. Otherwise, it is the value of y. [version 1]

Take a closer look. What if x is false? It is false which is x. So we have

AND: $\lambda x. \lambda y.$ x y x [version 2]

z can be true (first) or false (second).
It will supplied by fst or snd to get either the first or second element of the pair supplied

whenever we call first on a pair, it is like: fst (pair x y)
We have formed a pair of two values x and y and want first element of it.
lambda x. lambda y. x == First or true

Pair: (a, b)

we don't know what z can be and hence, fst and snd have to be designed separately.

The function for Pair should specify the two components and the projection function.

pair = λx. λy. λz. (z x y) x,y are variables for the actual parameters e.g., a, b

fst = λp. p (λx. λy. x) first component of the pair [NB: *fst* is not the same as *first*]

snd = λp. p (λx. λy. y) second component of the pair [*snd* is not the same as *second*]

let us compute fst (pair a b) and snd (pair a b):

pair a b = (λx. λy. λz. (z x y)) a b

= (λy. λz. (z a y) b "=" means beta-reduction

= λz. (z a b)

= N (say)

fst N = (λp. p (λx. λy. x)) N

= (λp. p (λx. λy. x)) (λz. (z a b)) see the role of *p* and *z*.

= (λz. (z a b)) (λx. λy. x) *z* may be replaced with either *first* or *second*

= (λx. λy. x) a b *first a b*

= (λy. a) b

= a

snd N = λp. p (λx. λy. y) N

= λp. p (λx. λy. y) (λz. (z a b))

= (λz. (z a b)) (λx. λy. y)

= (λx. λy. y) a b *second a b*

= (λy. y) b

= b

End of lecture