# PHN – 006: Quantum & Statistical Mechanics

## Assignment - 01

**Deadline for Submission: April 19, 2023, 11:59 pm.**

1. The power radiated by a spherical blackbody of radius 'R' at temperature T (= 4000K) is 100 Watts. This energy is then absorbed by another blackbody of radius '2R', predict the temperature of that black body.
2. The Planck energy density for a blackbody is plotted against the wavelength. The maxima occur at wavelength $\lambda$ = 600 nm at 5000 K. For the same blackbody at 3000 K what should be the wavelength for which the maxima occur.
3. The work function ($\Phi_0$) of a metal is 2.14 eV. Calculate the range of De Broglie wavelength of the photoelectrons in the photoelectric experiment which is carried out with the visible light source.
4. The stopping potential in the photoelectric effect is 2.012 V, where the light source is of wavelength 2000 Å. Calculate the work function of the metal used in the experiment.
5. The energy of a relativistic particle is given by $\mathbf{E = (p^2c^2 + m^2c^4)^{1/2}}$, where **m** is the mass of the particle, **p** is its momentum, and **c** is the speed of light. Calculate the ratio of group velocity **($v_g$)** to phase velocity **($v_p$).**
6. An electron is accelerated by an electric field **E = 10 V/m** within a region **0 < x < 5 (in** Å), find the wavelength of the associated wave. Also find the velocity of the electron and compare it with the speed of light.
7. It is seen in an experiment related to neutron scattering that de Broglie wavelength of a neutron is 1Å. Find the energy of the neutron.
8. A photon of energy 14.88 eV collides with an electron at rest in a head-to-head elastic collision. The wavelength of the electron scattered is 300 nm. Then find the angle with which the photon deviates from its original path.
9. Franck-Hertz experiment uses an electron of energy 4.96 eV to pass through mercury vapor. It is found that the energy of the electron decreases by 70% of initial value. Find the smallest possible wavelength of the emitted photon in the experiment.
10. The waves on the surface of water travel with a phase velocity $v_p = \frac{\sqrt{g\lambda}}{\sqrt{2\pi}}$, where $g$ is the acceleration due to gravity, and $\lambda$ is the wavelength of the wave. Show that the group velocity of a wave packet made up of these waves is $v_p/2$.
11. The speedometer of a 180 kg vehicle reads 72 km/h at a particular instant. The readings are known to have an accuracy of ± 2 km/h. What would be the minimum uncertainty in the position of the vehicle at that instant according to the Heisenberg Uncertainty Principle?
12. Find the eigenvalues and eigenfunctions of the operator d/dx.
13. Find the Hamiltonian operator of a charged particle in an electromagnetic field described by the vector potential **A** and the scalar potential $\varphi$.
14. Show that the linear momentum is not quantized.
15. The Hamiltonian operator of a system is $H = -(d^2/dx^2) + x^2$. Show that $Nx \exp(-x^2/2)$ is an eigenfunction of H and determine the eigenvalue. Also, evaluate N by normalization of the function.
16. The de Broglie wavelength $\lambda$ and the kinetic energy E for a highly relativistic electron are related as $\lambda = \alpha/E$. If $\lambda$ and E represent the numerical values in angstrom and kiloelectron volt, find the numerical value of $\alpha$.
17. When an electron is accelerated through a potential difference V, its de Broglie wavelength is given by $\lambda = \alpha/\sqrt{V}$ for nonrelativistic speeds. If $\lambda$ and V represent numerical values in angstrom and volt, find the numerical value of $\alpha$.
18. The normalized wave function of a particle is $\Psi(x) = A \exp(iax - ibt)$, where A, a and b are constants. Evaluate the uncertainty in its momentum.
19. The ground state wave function of a particle of mass m is given by $y(x) = \exp(-a^2x^4/4)$, with energy eigenvalue $\hbar^2a^2/m$. What is the potential in which the particle moves?
20. Consider a particle of mass m moving in a one-dimensional potential specified by

$$V(x) = \begin{cases} 0, & -2a < x < 2a \\ \infty, & otherwise \end{cases}$$

Find the energy eigenvalues and eigenfunctions.