

Closure of a Set of Functional Dependencies

The set of all dependencies that include F (the set of functional dependencies that are specified on relation schema R) as well as all dependencies that can be inferred from F is called the closure of F ; it is denoted by F^+ .

We shall use the notation F^+ to denote the closure of the set F , that is, the set of all functional dependencies that can be inferred given the set F . F^+ contains all of the functional dependencies in F .

$R(ABC)$

$$F = \{A \rightarrow B, B \rightarrow C\}$$

$$F^+ = ?$$

ABC

$$2^3 = 8$$

ϕ	$\phi^+ = \phi$	$\phi \rightarrow \phi$	$\phi \rightarrow A$	$\phi \rightarrow B$	$\phi \rightarrow C$	$\phi \rightarrow ABC$
A	$A^+ = ABC$ $2^3 = 8$	$A \rightarrow A$ $A \rightarrow B$ $A \rightarrow C$	$A \rightarrow BC$ $A \rightarrow AB$ $A \rightarrow AC$	$A \rightarrow \phi$	$A \rightarrow ABC$	
B	$B^+ = BC$	$B \rightarrow C$		$8 - 1 = 7$		
C	$C^+ = C$	$2^1 = 2$	$C \rightarrow \phi$, $C \rightarrow C$	$2^2 = 4$	$4 - 1 = 3$	
AB	$(AB)^+ = ABC$			$8 - 1 = 7$	$2 - 1 = 1$	
BC	$(BC)^+ = BC$			$4 - 1 = 3$		
AC	$(AC)^+ = ABC$			$8 - 1 = 7$		
ABC	$(ABC)^+ = ABC$			$8 - 1 = 7$		

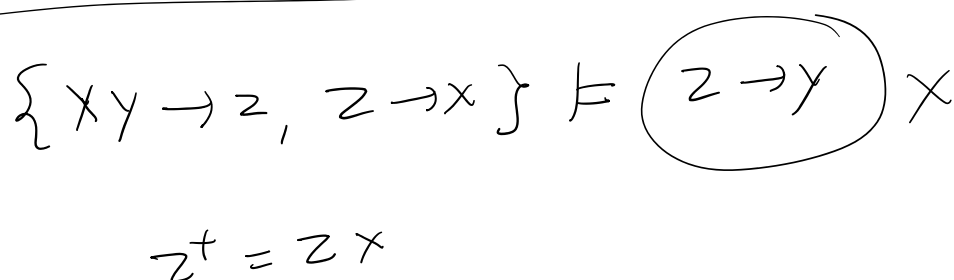
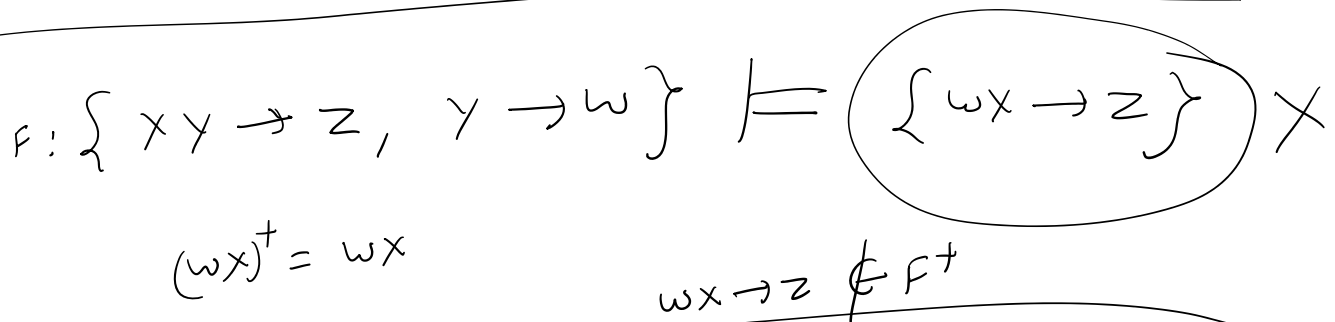
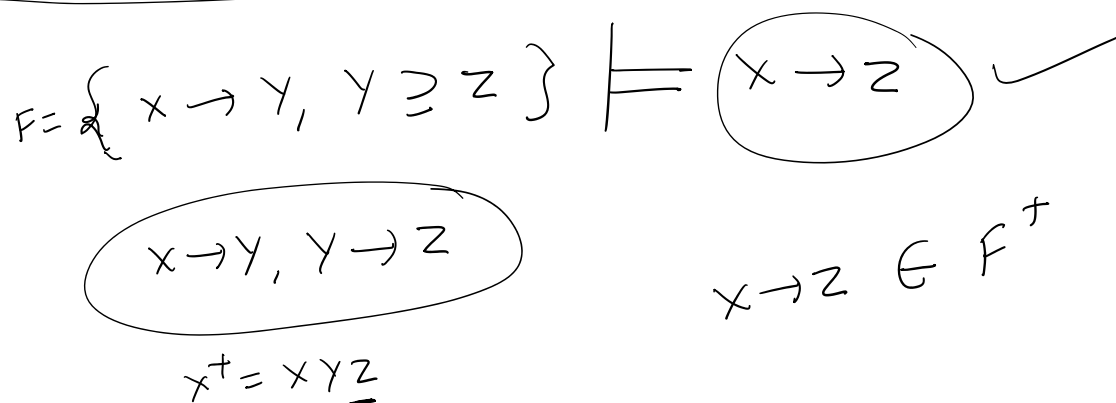
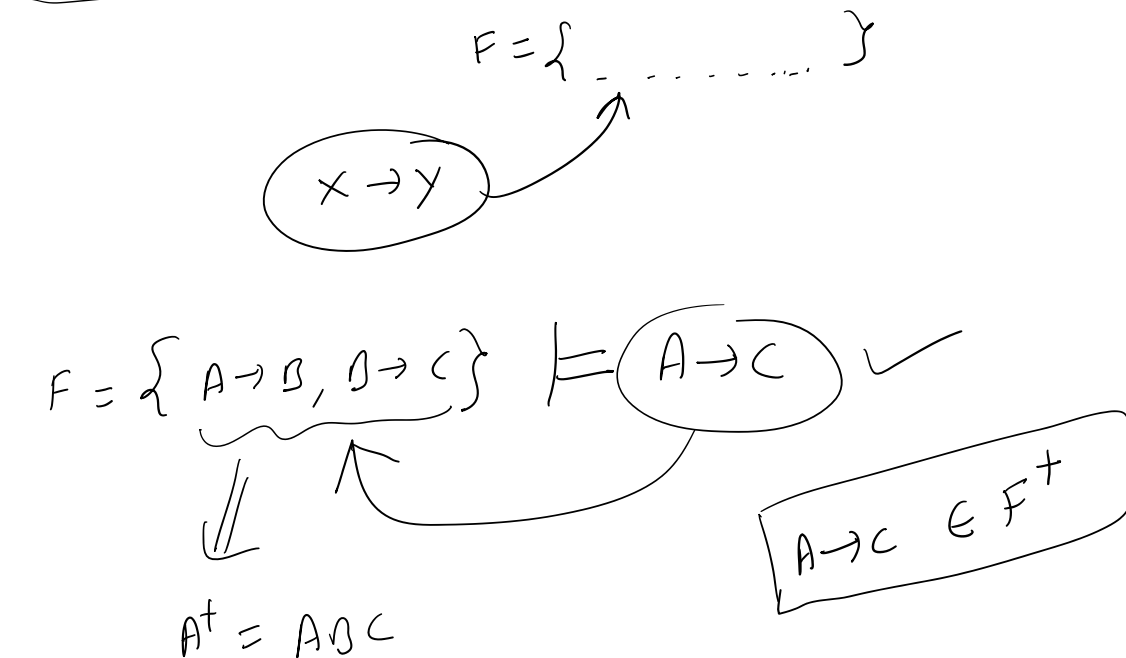
$$F^+ = 43 - 8 = 35$$

$$F^+ = \left\{ \begin{array}{l} 43 \text{ FDS} \\ 35 \end{array} \right\}$$

Membership Test

To check whether the given FD $X \rightarrow Y$ is a member of F^+ (closure of FD Set F) or not

We use the notation $F \models X \rightarrow Y$ to denote that the functional dependency $X \rightarrow Y$ is inferred from the set of functional dependencies F .



$$\{x \rightarrow y, y \rightarrow z\} \models x \rightarrow yz \quad \checkmark$$

$$x^+ = xyz$$

$$\{xy \rightarrow z, z \rightarrow w\} \models \{x \rightarrow w\} \quad x$$
$$x^+ = x$$

Cover

A set of functional dependencies F is said to cover another set of functional dependencies G if every FD in G is also in F^+ ; that is, if every dependency in G can be inferred from F ; alternatively, we can say that G is covered by F .

Equivalence

Two sets of functional dependencies F and G are equivalent if $F^+ = G^+$. Therefore, equivalence means that every FD in F can be inferred from G , and every FD in G can be inferred from F ; that is, F is equivalent to G if both the conditions (F covers G) and (G covers F) hold.

$$(F \equiv G) \Leftrightarrow ((1) \wedge (2)) \Leftrightarrow (F^+ = G^+)$$

①

$$(F \equiv G) \Leftrightarrow (F^+ = G^+)$$

- ① F covers G : $F^+ \supseteq G$ All F.D. of G member of F set
- ② G covers F : $G^+ \supseteq F$

$$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$$

$$G = \{A \rightarrow BC, D \rightarrow AE\}$$

F covers G :-

$$F^+ \supseteq G$$

$$F^+ \equiv F$$

$$G^+ \equiv G$$

$$F: \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\} \models \begin{array}{l} A \rightarrow BC \checkmark \\ D \rightarrow AE \checkmark \end{array}$$

G covers F :- $G: \{A \rightarrow BC, D \rightarrow AE\} \models \begin{array}{l} A \rightarrow B \checkmark \\ AB \rightarrow C \checkmark \\ D \rightarrow AC \checkmark \\ D \rightarrow E \checkmark \end{array}$

$$G^+ \supseteq F$$

$$(F \equiv G)$$

$$F^+ = G^+ \checkmark$$

$$(F^+)^+ = F^+$$

$$(F^+ \equiv F) \quad (F^+)^+ = F^+$$

Canonical cover or Minimal Cover

A canonical cover of F is a **Minimal** set of functional dependencies equivalent to F , having no redundant dependencies or redundant parts of dependencies

$$F := \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$

$$\equiv \{A \rightarrow B, B \rightarrow C\} : F_1$$

$$F \equiv F_1$$

$$F^+ = F_1^+$$

$$F_1^+ = F_2^+$$

$$F_1: \{A \rightarrow B, A \rightarrow B\}$$

$$\{A \rightarrow B\} F_2$$

$$A \rightarrow B$$

$$AD \rightarrow BD$$

$$AD \rightarrow B, AD \rightarrow D$$

Extraneous Attributes:

Consider F , and a functional dependency, $A \rightarrow B$

"Extraneous": Are there any attributes in A or B that can be safely removed?

- Without changing the constraints implied by F

An attribute of a functional dependency is said to be extraneous if we can remove it without changing the closure of the set of functional dependencies.

$$(I) \quad F = \{AB \rightarrow C, A \rightarrow B\}$$

$$A \rightarrow B$$

$$AB \rightarrow C$$

$$\{AB \rightarrow C, AB \rightarrow C\}$$

$$\{AB \rightarrow C, AB \rightarrow C\} \equiv \{AB \rightarrow C\}$$

$$A \rightarrow B$$

$$A \rightarrow C$$

$$A^+ = ABC$$

$$(AB)^+ = ABC$$

$$AB \rightarrow C$$

$$(II) \quad \{AB \rightarrow C, A \rightarrow C\} \equiv \{A \rightarrow C\}$$

$$(III) \quad \{A \rightarrow BC, B \rightarrow C\} \equiv \{A \rightarrow B, B \rightarrow C\}$$

$$X \rightarrow BCD$$

$$BC \rightarrow D$$

$$\equiv$$

$$X \rightarrow BC$$

$$BC \rightarrow D$$

$$(IV) \quad \{AB \rightarrow CD, BC \rightarrow D\} \equiv \{AB \rightarrow C, BC \rightarrow D\}$$

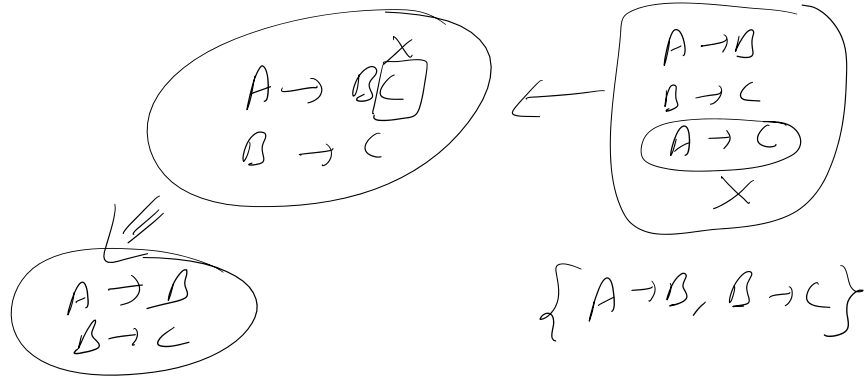
$$(ABD)^+ = ABCD$$

Q.1 F: $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C, A \rightarrow B\}$

$A \rightarrow BC, B \rightarrow C, AB \rightarrow C, \cancel{A \rightarrow B}$

$A \rightarrow BC, B \rightarrow C, AB \rightarrow C$
 \uparrow

$A \rightarrow B, B \rightarrow C, AB \rightarrow C$
 \uparrow



F: $\{ABCD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

$\cancel{A} \cancel{B} \cancel{C} \cancel{D} \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D$

$AC \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D$

$AC \rightarrow \cancel{D}E, E \rightarrow D, A \rightarrow B$

$\{AC \rightarrow E, E \rightarrow D, A \rightarrow B\}$

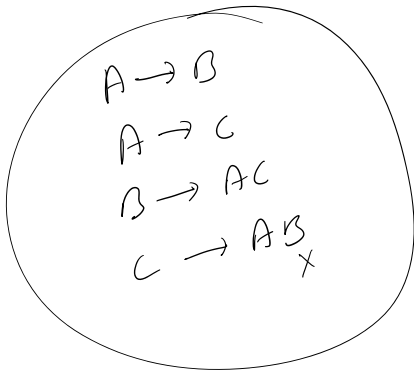
F: $\{x \rightarrow yz, y \rightarrow xz, z \rightarrow x\}$

$x \rightarrow yz \quad y \rightarrow xz \quad z \rightarrow x$

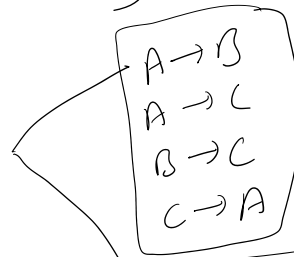
$F^+ = F_1^+$

$x \rightarrow y, y \rightarrow z, z \rightarrow x$ F_1

$F: \{A \rightarrow BC, B \rightarrow AC, C \rightarrow AB\}$



$A \rightarrow B$
 $A \rightarrow C$
 $B \rightarrow AC$
 \times
 $C \rightarrow A$



$A \rightarrow B$
 $B \rightarrow C$
 $C \rightarrow A$

