State Equations/Table/Diagram

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State equation

The behavior of a clocked sequential circuit can be described algebraically by means of state equations.

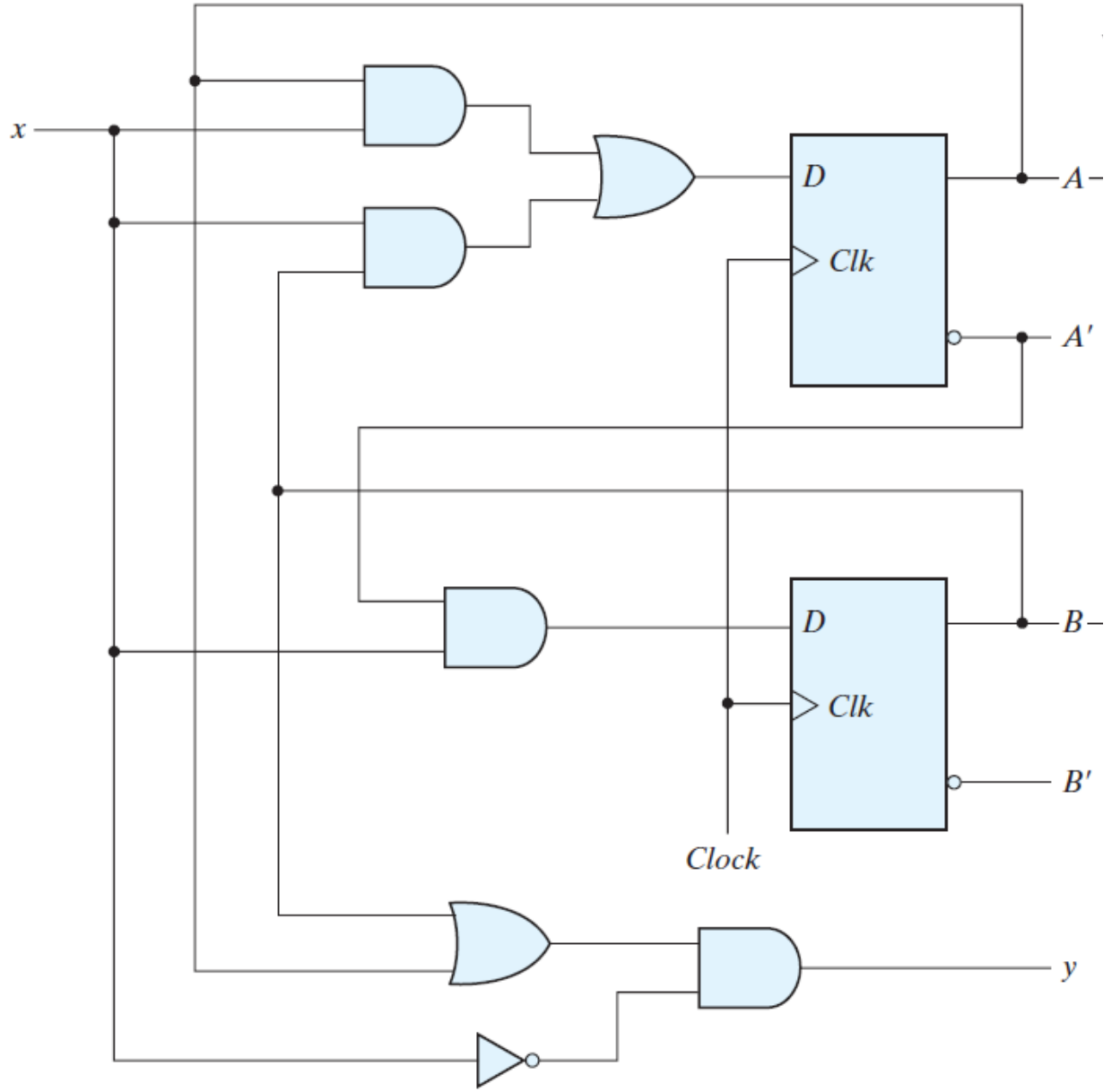
A *state equation* (also called a *transition equation*) specifies the next state as a function of the present state and inputs.

Consider this circuit.

Show equations for A(t+1), B(t+1) and y

$$A(t+1) = A(t)x(t) + B(t)x(t)$$
$$B(t+1) = A'(t)x(t)$$

$$y(t) = [A(t) + B(t)]x'(t)$$



State Table

The time sequence of inputs, outputs, and flip-flop states can be enumerated in a *state table*

The state table for the circuit shown on previous slide is shown next.

The table consists of four sections labeled *present state*, *input*, *next state*, and *output* .

The present-state section shows states of flip-flops A and B at any given time t .

The input section gives a value of x for each possible present state.

The next-state section shows the states of the flip-flops one clock cycle later, at time $t + 1$.

The output section gives the value of y at time t for each present state and input condition.

State table

$$A(t+1) = A(t)x(t) + B(t)x(t)$$
$$B(t+1) = A'(t)x(t)$$

$$y(t) = [A(t) + B(t)]x'(t)$$

Table 5.2
State Table for the Circuit of Fig. 5.15

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Table 5.2
State Table for the Circuit of Fig. 5.15

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Table 5.3
Second Form of the State Table

Present State		Next State				Output	
		x = 0		*x* = 1		*x* = 0	*x* = 1
A	*B*	*A*	*B*	*A*	*B*	*y*	*y*
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

State diagram

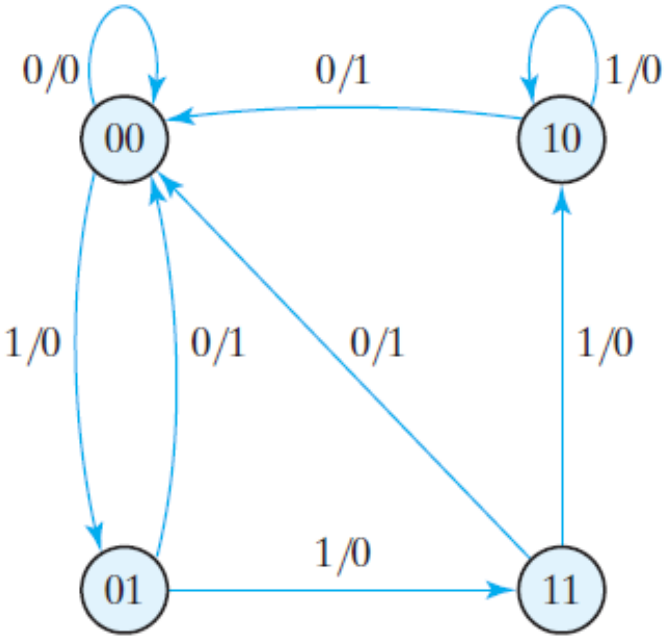
The information available in a state table can be represented graphically in the form of a state diagram.

In this type of diagram, a state is represented by a circle, and the (clock-triggered) transitions between states are indicated by directed lines connecting the circles.

State diagram

Table 5.3
Second Form of the State Table

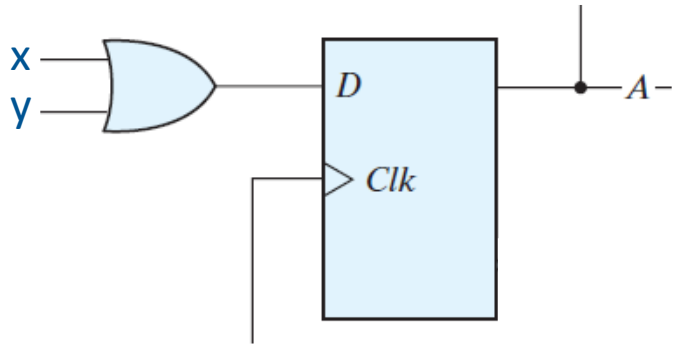
Present State		Next State				Output	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0



Flip-flop input equations

We adopt the convention of using the flip-flop input symbol to denote the input equation variable and a subscript to designate the name of the flip-flop output.

The following input equation specifies an OR gate with inputs x and y connected to the D input of a flip-flop whose output is labeled with the symbol Q :

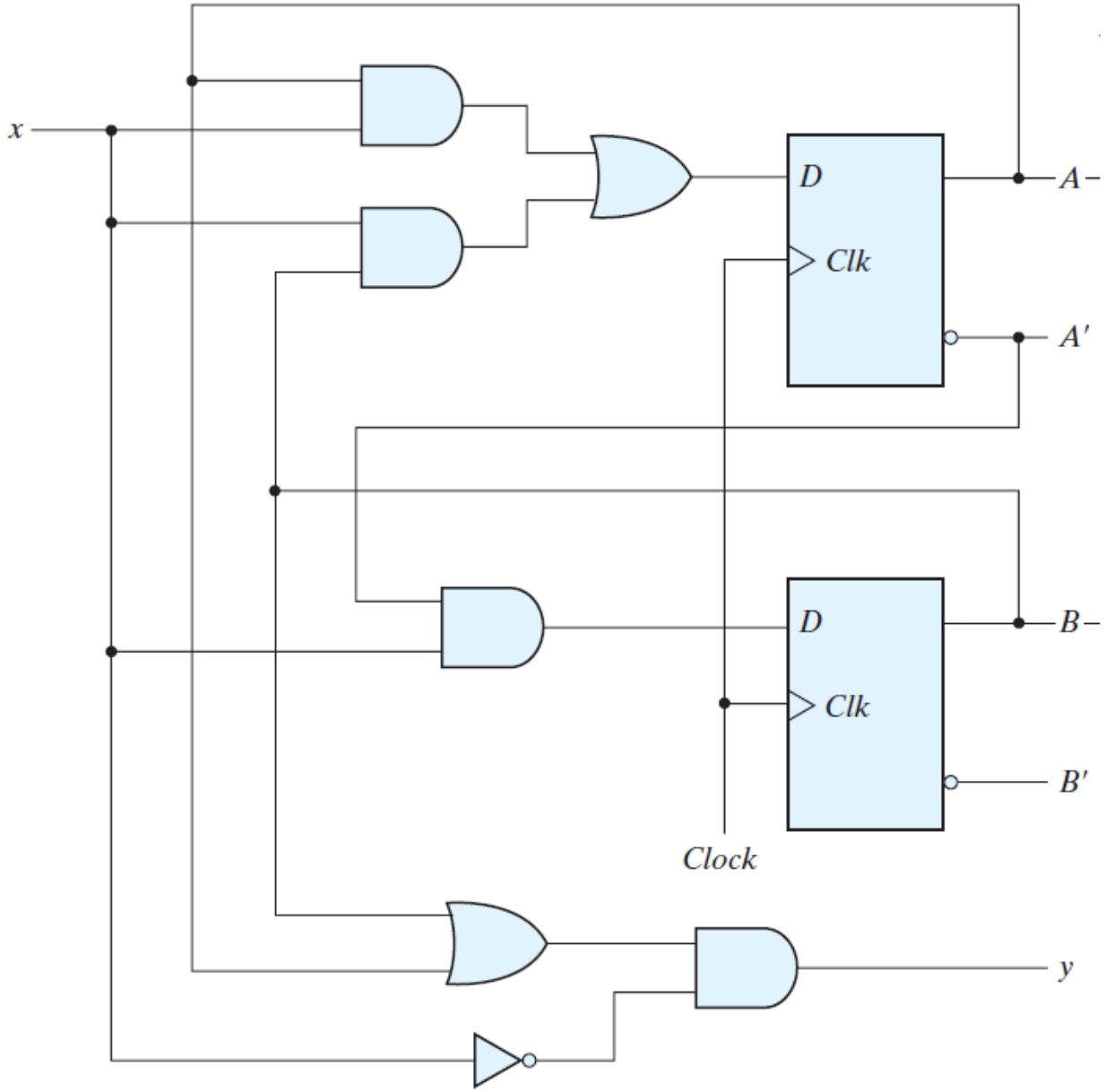


$$D_Q = x + y$$

Flip-flop input equations

Equations of gates shown in previous diagram

$$D_A = Ax + Bx$$
$$D_B = A'x$$
$$y = (A + B)x'$$



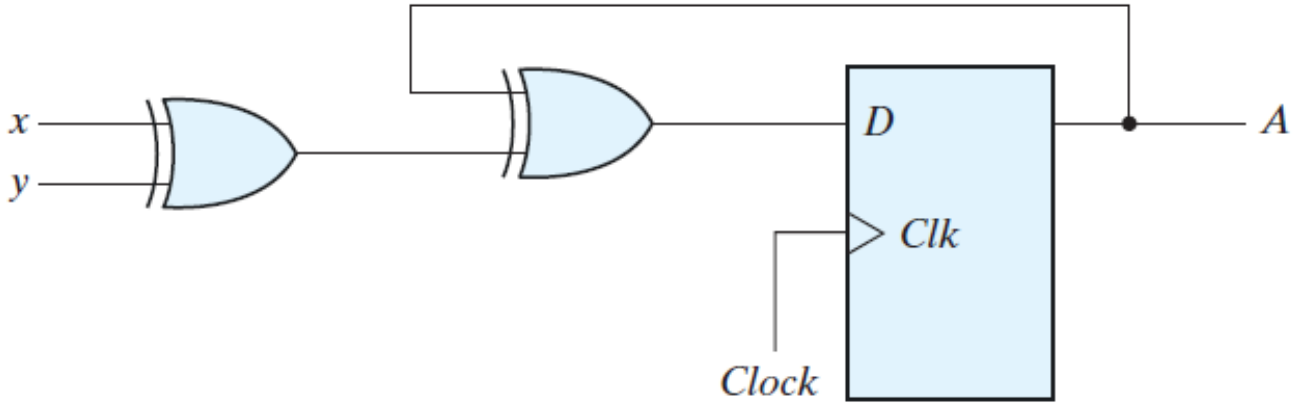
Analyzing clocked sequential circuit with D flip-flop

- Consider this circuit: $D_A = A \oplus x \oplus y$

- The next-state values are obtained from the state equation

$$A(t+1) = A \oplus x \oplus y$$

- The expression specifies an odd function and is equal to 1 when
  - only one variable is 1
  - or when all three variables are 1.

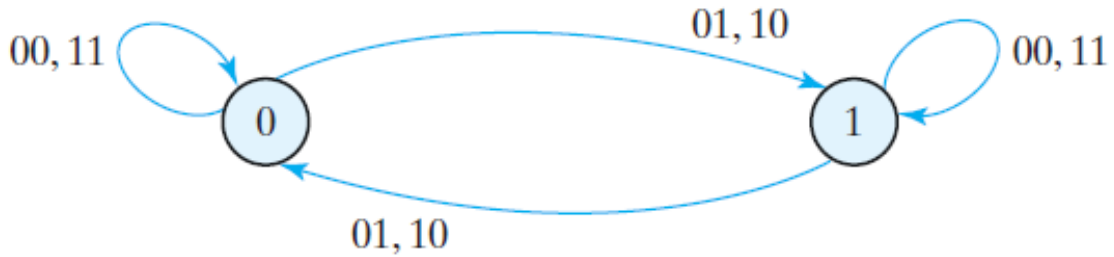


(a) Circuit diagram

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table

The circuit has one flip-flop and two states.



(c) State diagram

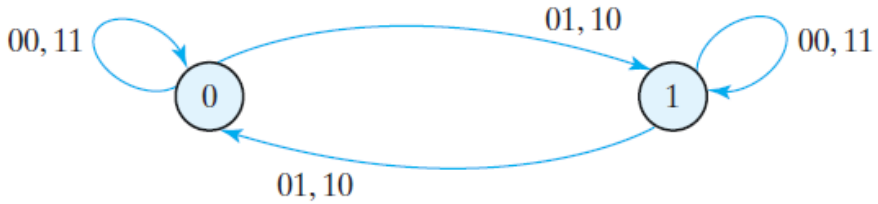
Discussion

The circuit has one flip-flop and two states. The state diagram consists of two circles, one for each state.

The present state and the output can be either 0 or 1, as indicated by the number inside the circles.

A slash on the directed lines is not needed, because there is no output from a combinational circuit.

The two inputs can have four possible combinations for each state. Two input combinations during each state transition are separated by a comma to simplify the notation.



(c) State diagram