

**Indian Institute of Technology Roorkee**  
**MAN-001(Mathematics-1)**  
**Autumn Semester: 2022-23**  
**Assignment-8: (Application of Multiple Integrals)**

- (1) Find the area of the region included between the cardioids  $r = a(1 + \cos\theta)$  and  $r = a(1 - \cos\theta)$  .
- (2) Find the volume of a wedge intercepted between the cylinder  $x^2 + y^2 = 2ax$  and the planes  $z = mx$  and  $z = nx$  ( $m > n$ ).
- (3) Find the volume of a tetrahedron with vertices at  $(0, 0, 0)$ ,  $(a, a, 0)$ ,  $(0, a, 0)$  and  $(0, a, a)$  .
- (4) Use cylindrical coordinates to compute the integral  $\iiint_D z(x^2 + y^2)^{-\frac{1}{2}} dx dy dz$ , where  $D$  is the solid bounded above by the plane  $z = 2$  and below by the surface  $2z = x^2 + y^2$  .
- (5) The average value of a function  $f$  over a solid region  $D$  is defined as  $\frac{1}{\text{vol. of } D} \iiint_D f(x, y, z) dx dy dz$  . Find the average value of  $f(x, y, z) = x + y + z$  over the sphere  $x^2 + y^2 + z^2 = 4$ .
- (6) Find the volume bounded above by the sphere  $x^2 + y^2 + z^2 = 32$  and below by the paraboloid  $x^2 + y^2 = 4z$  .
- (7) Find the volume of the torus generated by revolving the circle  $x^2 + y^2 = 4$  about the line  $x = 3$  .
- (8) Find the volume bounded by the surfaces  $z = 4 - x^2 - \frac{1}{4}y^2$  and  $z = 3x^2 + \frac{y^2}{4}$ .
- (9) Evaluate  $\iiint z^2 dx dy dz$  over the region common to the sphere  $x^2 + y^2 + z^2 = a^2$  and the cylinder  $x^2 + y^2 = ax$  .
- (10) Find the centre of gravity of the area bounded by the parabola  $y^2 = x$  and the line  $x + y = 2$ , treating the density as constant.
- (11) Find the mass of a plate in the shape of the curve  $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$  , the density being given by  $\rho = \mu xy$ .
- (12) A solid body of constant density  $\rho$  is obtained by revolving the cardioid  $r = a(1 + \cos\theta)$  about the initial line. Find its M.I. about a straight line through the pole and perpendicular to the initial line.

**Answers.**

- (1)  $\frac{a^2}{2}(3\pi - 8)$  (2)  $(m - n)a^3\pi$  (3)  $\frac{a^3}{6}$  (4)  $\frac{32\pi}{5}$  (5) 0 (6)  $64\pi(4\sqrt{2} - \frac{7}{6})$  (7)  $24\pi^2$   
(8)  $4\sqrt{2}\pi$  (9)  $\frac{2}{15}a^5\pi$  (10)  $(\frac{8}{5}, -\frac{31}{54})$  (11)  $\frac{\mu a^2 b^2}{20}$  (12)  $\frac{352}{105}\rho a^5\pi$ .