Closure of Attribute Sets

Attribute Closure

Given a set of attributes α, define the ***closure*** of α **under** *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*

α^+

$R(ABCD)$

F.D. $F = \{A \to B,\ B \to C,\ C \to D\}$

$A \to A$

$A^+ = ABCD$

$\boxed{A^+ = ABCD}$

$A \to A$
$A \to B$
$A \to C$
$A \to D$
u_{c_1}

$A \to BC$
$A \to CD$
u_{c_2}

$A \to ABC$
u_{c_3}

$A \to ABCD$
or
R
u_{c_4}

$A \to R$

$B^+ = BCD$ $\quad (AB)^+ = ABCD$

$C^+ = CD$ $\quad D^+ = D$

$F: \{AB \to CD,\ AF \to D,\ DE \to F,\ C \to G,\ F \to E,\ G \to A\}$

$(CF)^+ = CFGADE$

$(BG)^+ = BGACD$

$(AF)^+ = AFDE$

$(AB)^+ = ABCDG$

Inference Rules

IR1 (reflexive rule): If $X \supseteq Y$, then $X \to Y$.

IR2 (augmentation rule): $\{X \to Y\} \models XZ \to YZ$.

IR3 (transitive rule): $\{X \to Y, Y \to Z\} \models X \to Z$.

IR4 (decomposition or projective rule): $\{X \to YZ\} \models X \to Y$. and $X \to Z$

IR5 (union or additive rule): $\{X \to Y, X \to Z\} \models X \to YZ$.

IR6 (pseudotransitive rule): $\{X \to Y, WY \to Z\} \models WX \to Z$.

$X \to Y$ $\quad X^+ = XY \quad (XZ)^+ = YZX$

$XZ \to YZ$

$X \to Y, Y \to Z$

$X^+ = XYZ$

$X \to Z$

$X \to YZ$ $\quad X^+ = XYZ$

$X \to Y$ and $X \to Z$

$X \to Y$ and $X \to Z$

$X^+ = XYZ$

$X \to YZ$

$X \to Y, \quad WY \to Z$

$(WX)^+ = WXYZ$

$WX \to Z$

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute α^+, and check if α^+ contains all attributes of R.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \to \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \to S$.

Candidate key and Super key using Attribute Closure

K → R means K+ determines all the attributes of the relation R

K is a superkey for relation schema *R* if and only if $K \rightarrow R$

K is a candidate key for *R* if and only if

- $K \rightarrow R$, and
- for no $\alpha \subset K$, $\alpha \rightarrow R$

R(ABCD)

$K \rightarrow R$

$K \rightarrow ABCD$

$\boxed{(K : SK) \Leftrightarrow (K \rightarrow R)}$

$\boxed{(K : CK) \Leftrightarrow ((K \rightarrow R) \cap (\text{minimal Cond}^n))}$

$\boxed{((K : CK)') \Leftrightarrow ((K \not\rightarrow R) \vee (\text{fail minimal Cond}^n))}$

$p \Leftrightarrow q$

T	T
F	F

✓

T	F
F	T

×

$q' \Leftrightarrow p'$

$p' \Leftrightarrow q'$

R(ABCD) F :- {A → B, B → C, C → D}

CK : A $\qquad A^+ = ABCD$ or R

SK : A $\qquad A \rightarrow R$

R(ABCDE) {AB → C, C → D, B → E}

$(AB)^+ = ABCDE = R$ $\qquad \boxed{AB \rightarrow R}$

S.K. :- AB

C.K :- AB

$A^+ = A$

$B^+ = BE$

$A \not\rightarrow R$

R(ABCDE)

$R(ABCDE)$

$\{AB \to C,\ C \to D,\ B \to EA\}$

$(AB)^+ = R$

$A \not\to R$

$B \not\to R$

AB: SK ✓

: CK ✗

C.K: B

S.K: B

$A^+ = A$

$B^+ = R$

$B \to R$

$R(ABCDEF)$ $\{A \to BCDEF,\ \boxed{BC \to ADEF}$ ✓, $B \to F,\ D \to E\}$

$A^+ = R$

$A \to R$

CK: A, BC

BC
|
A

BC
|
A

CK: A, BC

Prime attribute $= \{A, B, C\}$

$(BC)^+ = R$

$(BC)^+ = BCADEF$

$BC \to R$

$(C)^+ = C$

$(B)^+ = BF$

$R(ABCDEF)$

$\{AB \to C,\ C \to D,\ D \to E,\ E \to F,\ F \to A\}$

$(AB)^+ = R$

AB, CB, DB, FB, EB

$R(ABCDE)$

CK: ABCDE

$R(ABCDE)$

$F: \{ \quad \}$

$CK: ABCDE$

$ABCDE \rightarrow R$

$R(ABCDE)$ $\{A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A\}$

$CK:- AE, BE, CE, DE$

$R(ABCDEH)$

$\{A \rightarrow B, BC \rightarrow D, E \rightarrow C, D \rightarrow A\}$

$\boxed{BEH, AEH, DEH}$

BEH

AEH

$P.A = \{B, E, H, A, D$

$BCEH \rightarrow BEH$

$D\boxed{EH}$

$R(ABCD)$

$\{AB \rightarrow C, C \rightarrow D, CE \rightarrow F, F \rightarrow G\}$

$(AB)^+ = R$

$C.K. = AB$