Indian Institute of Technology Roorkee
Theory of Games
Optimization Techniques (MAN-010)

Ex-9

1. Examine the following payoff matrices for saddle points. In case the saddle point exists, find the optimal strategies and value of the game. In every case verify that

$$\max_i \min_j a_{ij} \le \min_j \max_i a_{ij}$$

(i) $\begin{bmatrix} -1 & 3 \\ -2 & 10 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -1 & -2 \\ 1 & 0 & 1 \\ -2 & -1 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 0 & 2 & -3 & 0 \\ -2 & 0 & 0 & 3 \\ 3 & 0 & 0 & -4 \\ 0 & -3 & 4 & 0 \end{bmatrix}$ (vii) $\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$

2. Solve the games with the following payoff matrices.

(i) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$

4. Solve graphically the games whose payoff matrices are the following.

(i) $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$

5. Use the notion of dominance to simplify the following payoff matrices and then solve the game.

(i) $\begin{bmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 1 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix}$

6. Write both the primal and the dual LP problems corresponding to the rectangular games with the following payoff matrices. Solve the game by solving the LP problem by simplex method.

(i) $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$