Assignment: (4)

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(1.)

(1.) (i) $u = \cos^{-1}\left(\frac{x^2+y^2}{\sqrt{x}-\sqrt{y}}\right) \Rightarrow \cos u = \frac{x^2+y^2}{\sqrt{x}-\sqrt{y}} = f(x,y).$

$\Rightarrow$ 'cos u' is homogenous function with degree = $3/2$;

So:

$$x \cdot \frac{\partial(\cos u)}{\partial x} + y \cdot \frac{\partial(\cos u)}{\partial y} = \frac{3}{2} \cdot \cos(u).$$

$$\Rightarrow \quad x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = -\frac{3}{2}\frac{\cos(u)}{\sin(u)} = -\frac{3}{2}\cot(u).$$

Hence, proved.

(2) $\Rightarrow z = z_1 + z_2$

where $z_1 = x^m f(y/x)$; homogenous function of degree 'm'.

$z_2 = x^n g(y/x)$; homogenous function of degree 'n'.

So, we have:

$$x\frac{\partial z_1}{\partial x} + y\frac{\partial z_1}{\partial y} = m z_1 \quad \text{—(i)}$$

$$x\frac{\partial z_2}{\partial x} + y\frac{\partial z_2}{\partial y} = n z_2 \quad \text{—(ii)}.$$

$\Rightarrow$ Add (i) and (ii) :-

$$\boxed{x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = (m z_1 + n z_2)} \quad \text{—(iii)}$$

$\Rightarrow$ take $\partial/\partial x$:- of (iii) :-

$$\frac{\partial z}{\partial x} + x\frac{\partial^2 z}{\partial x^2} + y\cdot\frac{\partial^2 z}{\partial x \cdot \partial y} = m\frac{\partial z_1}{\partial x} + n\frac{\partial z_2}{\partial x} \quad \text{—(iv)}$$

$$\frac{\partial z}{\partial y} + y\frac{\partial^2 z}{\partial y^2} + x\cdot\frac{\partial^2 z}{\partial y \cdot \partial x} = m\cdot\frac{\partial z_1}{\partial y} + n\cdot\frac{\partial z_2}{\partial y} \quad \text{—(v)}.$$

- as partials are continous,

$$\frac{\partial^2 z}{\partial x \cdot \partial y} = \frac{\partial^2 z}{\partial y \cdot \partial x}.$$

- Add (iv) and (v) ; respectively multiplied by x and y first :-

$$\left(x^2\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} + 2xy\frac{\partial^2 z}{\partial x \cdot \partial y}\right) + \left(x\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right)$$
$$= m\left(x\cdot\frac{\partial z_1}{\partial x} + y\frac{\partial z_1}{\partial y}\right) + n\left(x\cdot\frac{\partial z_2}{\partial x} + y\cdot\frac{\partial z_2}{\partial y}\right).$$

$$\Rightarrow (L.H.S - mnz) + \left(x\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right) = m(mz_1) + n(nz_2)$$

[By using (i), (ii)].

$$\Rightarrow (L.H.S) = \left[m^2 z_1 + n^2 z_2 + mn(z_1 + z_2)\right] - \left(x\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right)$$

by using $z = (z_1 + z_2)$;.

$\Rightarrow$ L.H.S $= (m+n)(m z_1 + n z_2) - \left(x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y}\right)$.

$= (m+n-1)\left(\frac{\partial z}{\partial x} \cdot x + y \cdot \frac{\partial z}{\partial y}\right)$. $=$ R.H.S

Hence, proved.

(3.) (f) is homogenous with degree (-2);

so:

$x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} = -2f$. Hence, proved.

(4.) $\frac{du}{dt} = \frac{\partial u}{\partial x}\left(\frac{dx}{dt}\right) + \frac{\partial u}{\partial y}\left(\frac{dy}{dt}\right) + \frac{\partial u}{\partial z} \cdot \left(\frac{dz}{dt}\right)$ — (1)

(i) $\frac{du}{dt} = \left[\cos(x^2+y^2) \cdot 2x\right] \cdot (2t) + \left[\cos(x^2+y^2) \cdot 2y\right](3t^2)$

$= 2\cos(x^2+y^2) \cdot [2xt + 3yt^2]$. Ans

(ii) $\frac{du}{dt} = \left[\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{-y}{x^2}\right)\right] \cdot (e^t + e^{-t}) + \left[\frac{1}{\left(1+\left(\frac{y}{x}\right)^2\right)} \cdot \left(\frac{1}{x}\right)\right](e^t - e^{-t})$

$= \frac{1}{x\left(1+\frac{y^2}{x^2}\right)}\left\{\underbrace{(e^t - e^{-t})}_{= x} - \left(\frac{y}{x}\right) \cdot \underbrace{(e^t + e^{-t})}_{= y}\right\}$.

$= \frac{x^2 - y^2}{(x^2+y^2)} = \frac{-2}{(e^{2t} + e^{-2t})}$. Ans

(4.) (iii) $\frac{du}{dt} = (2x)(2e^{2t}) + (2y)\left[2e^{2t}\cos(3t) - 3e^{2t}\sin(3t)\right]$

$+ (2z)\left[2e^{2t}\sin(3t) + 3e^{2t}\cos(3t)\right]$.

Put value of x, y, z.

$\frac{du}{dt} = 8e^{4t}$. Ans

(5.) • $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$

$= -3\sin(x^3+y^3)\left[x^2(t) + y^2(2s)\right]$.

• $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$

$= -3\sin(x^3+y^3)\left[x^2(s) + y^2(2t)\right]$.

(6.) • $z = \frac{1-xy}{(x+y)}$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{(x+y)(-y) - (1-xy)(1)}{(x+y)^2} = \frac{-(1+y^2)}{(x+y)^2}$ Ans

$\Rightarrow \frac{\partial z}{\partial y} = \frac{(x+y)(-x) - (1-xy)(1)}{(x+y)^2} = \frac{-(1+x^2)}{(x+y)^2}$ Ans

To match the answers:
substitute 1 over $(xy+yz+zx)$ or vice-versa

$\Rightarrow \dfrac{\partial z}{\partial x} = -\dfrac{(xy+yz+zx+y^2)}{(x+y)^2} = -\dfrac{(y+z)}{(x+y)}$ Ans.

similarly for $(\partial z/\partial y)$.

(7). • at $t=0 \Rightarrow (x=0, y=0)$.

• $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt}$ —— (i)

$\Rightarrow \dfrac{dx}{dt}(3x^2+1) = -(e^t+2t+1)$ —(ii)

$\Rightarrow \dfrac{dy}{dt}(1+t^3+3ty^2) = -(1+y^3+3yt^2)$ —(iii)

$\Rightarrow$ calculate for $x=0, y=0, t=0$,

we have : $\dfrac{dx}{dt} = -2$ and $\dfrac{dy}{dt} = -1$.

$\Rightarrow \dfrac{\partial z}{\partial x} = 0$ and $\dfrac{\partial z}{\partial y} = 2$.

so: $\dfrac{dz}{dt} = 0 + 2(-1) = -2$ Ans

(8). $V = r^n(3\cos^2\theta - 1)$.

• $\dfrac{\partial V}{\partial r} = n\cdot r^{n-1}(3\cos^2\theta - 1)$; $\dfrac{\partial V}{\partial \theta} = -r^n(3\sin(2\theta))$.

$\Rightarrow$ substitute in eq^n:

$$\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial V}{\partial r}\right) + \frac{\frac{\partial}{\partial \theta}\left(\sin\theta\cdot\frac{\partial V}{\partial \theta}\right)}{\sin\theta} = 0.$$

$\Rightarrow n(3\cos^2\theta - 1)(n+1)\cdot r^n + \dfrac{1}{\sin\theta}(r^n)\cdot(3)\left[\sin(2\theta) + \cos(2\theta)\sin\theta\right]$.

$$\Rightarrow r^n \left[n(n+1)(3\cos^2\theta - 1) - \frac{(1\times 3)}{\sin\theta} \left[\sin(3\theta) + \sin\theta \cdot \cos(2\theta) \right] \right] = 0$$

$$\Rightarrow n(n+1) = \frac{3\left[\frac{\sin(3\theta)}{\sin\theta} + \cos(2\theta) \right]}{(3\cos^2\theta - 1)}$$

$$= \frac{3\left[3 - 4\sin^2\theta + 2\cos^2\theta - 1 \right]}{(3\cos^2\theta - 1)} = 6.$$

So: $n^2 + n - 6 = 0 \Rightarrow \boxed{n = 2, -3}$. Ans

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9. $$\frac{\partial r}{\partial x_i} = \left(\frac{x_i}{r}\right); \quad \frac{\partial V}{\partial x_i} = \left(\frac{\partial V}{\partial r}\right)\cdot\left(\frac{\partial r}{\partial x_i}\right)$$

$\Rightarrow$ take: $$\frac{\partial}{\partial x_i}\left(\frac{\partial V}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(\frac{\partial V}{\partial r}\cdot\frac{\partial r}{\partial x_i}\right)$$

$$= \frac{\partial V}{\partial r}\cdot\frac{\partial}{\partial x_i}\left(\frac{\partial r}{\partial x_i}\right) + \frac{\partial r}{\partial x_i}\,\frac{\partial}{\partial x_i}\left(\frac{\partial V}{\partial r}\right)$$

$$= \frac{\partial V}{\partial r}\cdot\frac{\partial}{\partial x_i}\left(\frac{x_i}{r}\right) + \left(\frac{x_i}{r}\right)\left(\frac{\partial}{\partial r}\cdot\left(\frac{\partial V}{\partial r}\right)\frac{\partial r}{\partial x_i}\right)$$

$$= \frac{\partial V}{\partial r}\left(\frac{r\cdot(1) - (x_i)\left(\frac{x_i}{r}\right)}{r^2}\right) + \frac{\partial^2 V}{\partial r^2}\cdot\left(\frac{x_i^2}{r^2}\right)$$

$$= \left(\frac{\partial^2 V}{\partial r^2}\right)\cdot\left(\frac{x_i^2}{r^2}\right) + \left(\frac{\partial V}{\partial r}\right)\cdot\left(\frac{r^2 - x_i^2}{r^3}\right)$$

$\Rightarrow$ Now, take summation: (1).

$$\sum_{i=1}^{n} \frac{\partial^2 V}{\partial x_i^2} = \frac{\partial^2 V}{\partial r^2} \cdot \frac{\cancel{(x_1^2 + x_2^2 \cdots x_n^2)}}{r^2} + \frac{\partial V}{\partial r}\left(\frac{nr^2 - r^2}{r^3}\right)$$

$$= \frac{\partial^2 V}{\partial r^2} + \left(\frac{n-1}{r}\right) \cdot \left(\frac{\partial V}{\partial r}\right) \cdot \text{Ans}$$

(10.) (a) $w_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial x}\right)$.

$$\Rightarrow w_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial u}\right) \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial u} \cdot \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial v}\right) \cdot \frac{\partial v}{\partial x} + \frac{\partial w}{\partial v} \cdot \cancel{\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)}^{0}.$$

$$= \frac{\partial^2 w}{\partial u^2} \cdot (x^2) + \frac{\partial w}{\partial u}(1) + y^2 \frac{\partial^2 w}{\partial v^2} + 0.$$

(b) $w_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial y}\right)$.

$$\Rightarrow w_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial u}\right) \cdot \frac{\partial u}{\partial y} + \frac{\partial w}{\partial u} \cdot \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial v}\right) \cdot \frac{\partial v}{\partial y} + \frac{\partial w}{\partial v} \cdot \cancel{\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right)}^{0}.$$

$$= \frac{\partial^2 w}{\partial u^2}(y^2) + \frac{\partial w}{\partial u}(-1) + \frac{\partial^2 w}{\partial v^2} x^2 + 0.$$

Add both: $w_{xx} + w_{yy} = (x^2 + y^2) \cdot (w_{uu} + w_{vv})$

$$= 0$$

Hence, proved.

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(11.) Jacobian $= \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \end{vmatrix}$

$= \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \rho\cos\theta\cos\phi & \rho\cos\theta\sin\phi & -\rho\sin\theta \\ -\rho\sin\theta\sin\phi & \rho\sin\theta\cos\phi & 0 \end{vmatrix}$

$= \rho^2\sin\theta$ Ans

(12.) $\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \sqrt{2} & -\sqrt{\frac{2}{3}} \\ \sqrt{2} & \sqrt{\frac{2}{3}} \end{vmatrix} = \frac{4}{\sqrt{3}}$ Ans

(13.) (a) $x/y = e^{f(x,y)}$.

$\Rightarrow g(x) = \frac{1}{2}\left(\frac{x}{y} + 3\cdot\left(\frac{y}{x}\right)\right)$.

$\boxed{g = \frac{1}{2}\left(e^{f} + 3\cdot e^{-f}\right)}$ Ans

so both are functionally dependent.

(b) $g = \frac{1-y/x}{1+y/x} = \frac{1-f}{1+f}$ Ans

both are functionally dependent.

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(14.) $w = u\left(v - \frac{(u^2 - v)}{2}\right) = \frac{3v - u^2}{2} \cdot u.$

So all functions are functionally dependent, so they are linearly dependent too and jacobian is equal to zero.

(15.) Jacobian $= \begin{vmatrix} e^x \cos(y) & 1 \\ -e^x \sin(y) & -\tan(y) \end{vmatrix} = 0.$

$\Rightarrow \ln(f) = g$ [observation].

which makes clear that f and g are functionally dependent.

(16.) $\begin{Bmatrix} u = 3st + 2s^2 + 2t^2 \\ v = st - 2s^2 - 2t^2 \end{Bmatrix}$

$\Rightarrow$ Jacobian $= \frac{\partial(u,v)}{\partial(s,t)} = \begin{vmatrix} \frac{\partial u}{\partial s} & \frac{\partial v}{\partial s} \\ \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} \end{vmatrix} = J$

$\Rightarrow J = \begin{vmatrix} (3t + 4s) & (t - 4s) \\ (3s + 4t) & (s - 4t) \end{vmatrix} = 16(s^2 - t^2)$ Ans

(17.) Jacobian $= \begin{vmatrix} (1-u_2) & u_2(1-u_3) & u_2u_3(1-u_4) & u_2u_3u_4 \\ -u_1 & u_1(1-u_3) & u_1u_3(1-u_4) & u_1u_3u_4 \\ 0 & -u_1u_2 & u_2u_1(1-u_4) & u_1u_2u_4 \\ 0 & 0 & -u_1u_2u_3 & u_1u_2u_3 \end{vmatrix}$

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$$\Rightarrow \text{Jacobian} = (u_1 u_2)(u_1 u_2 u_3)(u_1)\begin{vmatrix} -1 & (1-u_3) & u_3(1-u_4) & u_3 u_4 \\ 0 & -1 & (1-u_4) & u_4 \\ 0 & 0 & -1 & 1 \end{vmatrix} \quad 1$$

$$= u_1^3 u_2^2 u_3 \quad \underline{\text{Ans}}$$

(18.) $3t^3 - 3(x+y+z)t^2 + 3(x^2+y^2+z^2)t - (x^3+y^3+z^3) = 0.$

$\Rightarrow$ $u+v+w = x+y+z$

$uv+vw+wu = x^2+y^2+z^2$

$uvw = \dfrac{x^3+y^3+z^3}{3}.$

$\Rightarrow$ take $(\partial/\partial x)$ of all:

$u_x + v_x + w_x = 1$ —— (1)

$(v+w)u_x + (w+u)v_x + (u+v)w_x = 2x$ —— (2)

$(vw)u_x + (wu)v_x + (uv)w_x = x^2$ —— (3)

$\Rightarrow$ (2) $-(v+w)$(1) :-

$(u-v)v_x + (u-w)w_x = 2x - v - w$ —— (4)

$\Rightarrow$ (3) $-(vw)$(1) :-

$w(u-v)v_x + v(u-w)w_x = x^2 - vw$ —— (5)

Solve (4) and (5) : $v_x = \dfrac{(x-v)^2}{(u-v)(w-v)}.$

Similarly, by symmetry, others can be also be calculated.

$\Rightarrow$ so Jacobian $= J = \dfrac{\partial(u,v,w)}{\partial(x,y,z)}$

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$$\Rightarrow J = \begin{vmatrix} \frac{(x-u)^2}{(v-u)(w-u)} & \frac{(x-v)^2}{(u-v)(w-v)} & \frac{(x-w)^2}{(u-w)(v-w)} \\ \frac{(y-u)^2}{(v-u)(w-u)} & \frac{(y-v)^2}{(u-v)(w-v)} & \frac{(y-w)^2}{(u-w)(v-w)} \\ \frac{(z-u)^2}{(v-u)(w-u)} & \frac{(z-v)^2}{(u-v)(w-v)} & \frac{(z-w)^2}{(u-w)(v-w)} \end{vmatrix}$$

- let $d = (u-v)(w-v).(w-u)(v-u).(u-w)(v-w).$

$$\Rightarrow J = \frac{1}{d} \begin{vmatrix} (x-u)^2 & (x-v)^2 & (x-w)^2 \\ (y-u)^2 & (y-v)^2 & (y-w)^2 \\ (z-u)^2 & (z-v)^2 & (z-w)^2 \end{vmatrix}$$

$\{R_1 \to R_1 - R_2\}$
$\{R_2 \to R_2 - R_3\}$

$$= \frac{(x-y)(y-z)}{d} \begin{vmatrix} x+y-2u & x+y-2v & x+y-2w \\ y+z-2u & y+z-2v & y+z-2w \\ (z-u)^2 & (z-v)^2 & (z-w)^2 \end{vmatrix}$$

$\{R_1 \to R_1 - R_2\}$

$$= \frac{(x-y)(y-z)(x-z)}{d} \begin{vmatrix} 1 & 1 & 1 \\ y+z-2u & y+z-2v & y+z-2w \\ (z-u)^2 & (z-v)^2 & (z-w)^2 \end{vmatrix}$$

$\{C_3 \to C_3 - C_2\}$
$\{C_2 \to C_2 - C_1\}$

$$= \frac{(x-y)(y-z)(x-z)(u-v)(v-w)(u-w) \times 2}{d}$$

$$= \frac{-2(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)} \text{. Ans}$$

(19) $r = \sqrt{x^2+y^2}$

$\Rightarrow \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$; $\frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$

$\Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial r}{\partial x}\right) = \frac{\partial^2 r}{\partial x^2} = \frac{\sqrt{x^2+y^2}\cdot(1) - \frac{x^2}{\sqrt{x^2+y^2}}}{(x^2+y^2)} = \frac{y^2}{(x^2+y^2)^{3/2}}$

$\Rightarrow \frac{\partial^2 r}{\partial y^2} = \frac{x^2}{(x^2+y^2)^{3/2}}$.

$\Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial r}{\partial x}\right) = \frac{\partial^2 r}{\partial y \cdot \partial x} = \frac{-xy}{(x^2+y^2)^{3/2}}$.

put in (i), (ii), (iii) $\Rightarrow$ Hence, proved.

(20) $\Rightarrow \frac{\partial u}{\partial x} = \frac{3x^2-3yz}{x^3+y^3+z^3-3xyz}$; $\frac{\partial u}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}$.

$\frac{\partial u}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$;

$\Rightarrow E = L.H.S = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$.

$= \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{3}{x+y+z}\right)$

$= (3)\left(\frac{-3}{(x+y+z)^2}\right)$

$= \frac{-9}{(x+y+z)^2}$ Ans

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1.(ii) $x\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y} + \frac{3\cot(u)}{2} = 0$

take $\partial/\partial x$ and $\partial/\partial y$:

$$x\cdot\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\cdot\frac{\partial^2 u}{\partial x\cdot\partial y} + -\frac{3}{2}\operatorname{cosec}^2(u)\cdot\frac{\partial u}{\partial x} = 0 \quad \text{(1)}$$

$$y\cdot\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + x\cdot\frac{\partial^2 u}{\partial x\cdot\partial y} + -\frac{3}{2}\operatorname{cosec}^2(u)\cdot\frac{\partial u}{\partial y} = 0 \quad \text{(2)}$$

$\Rightarrow (x)\times(1) + (y)\times(2)$:-

$$\text{L.H.S} = \frac{3}{2}\operatorname{cosec}^2(u)\left(x\frac{\partial u}{\partial x} + y\cdot\frac{\partial u}{\partial y}\right) + \frac{3}{2}\cot(u).$$

$$= \frac{3}{2}\cot(u)\left(1 - \frac{3}{2}\operatorname{cosec}^2(u)\right).$$