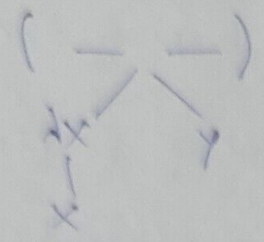
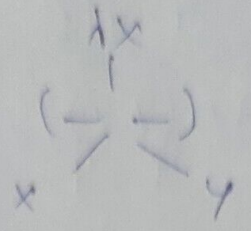
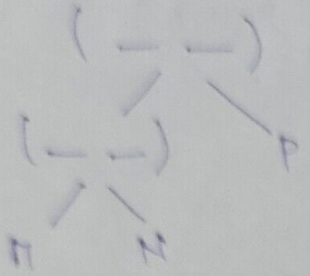
Name: Anvit Gupta  Enroll: 22114009  Batch: Btech-CSE-D1

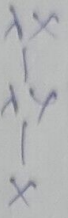
Subject: Principles of Programming languages (CSM-312)  Tutorial-1

① 1) 

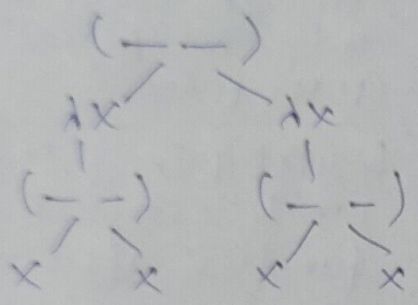
2) 

3) 

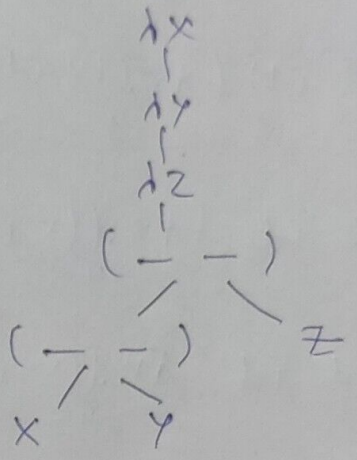
4) first = $\lambda x.\lambda y.x$



5) $SQ \equiv \lambda x.xx$



6) $\lambda x.\lambda y.\lambda z.xyz$



② 1) $\lambda x.x\ \lambda y.yx$

$\Rightarrow (\lambda x.x\ \lambda y.yx)$

$\Rightarrow (\lambda x.(x\ \lambda y.yx))$

$\Rightarrow (\lambda x.(x\ (\lambda y.yx)))$

$\Rightarrow (\lambda x.(x\ (\lambda y.(yx))))$

Ans

2) $id\ id\ \lambda x.x\ id\ z$

$\Rightarrow (id\ id\ \lambda x.x\ id\ z)$

$\Rightarrow ((id\ id)\ \lambda x.x\ id\ z)$

$\Rightarrow ((id\ id)\ (\lambda x.x\ id\ z))$

$\Rightarrow ((id\ id)\ (\lambda x.(x\ id)\ z))$

$\Rightarrow ((id\ id)(\lambda x.((x\ id)\ z)))$

Ans

(3.) 1) $\underset{1}{(}\lambda x.\ \underset{2}{(}x\ \underset{3}{(}\lambda y.\ \underset{4}{(}y\ x\underset{4}{)}\underset{3}{)}\underset{2}{)}\underset{1}{)}$ { Bracket pair-1 can be removed as only a func. abstraction of form $(\lambda x.M)$ present }

$\Rightarrow \lambda x.\ \underset{2}{(}x\ \underset{3}{(}\lambda y.\ \underset{4}{(}y\ x\underset{4}{)}\underset{3}{)}\underset{2}{)}$ { Remove pair-2 as lambda term extends as much as to the right

$\Rightarrow \lambda x.\ x\ \underset{3}{(}\lambda y.\ \underset{4}{(}y\ x\underset{4}{)}\underset{3}{)}$ { Remove pair-3 as only one abstraction of form $(\lambda x.M)$ }

$\Rightarrow \lambda x.\ x\ \lambda y.\ \underset{4}{(}y\ x\underset{4}{)}$ { Remove pair-4 as lambda terms extends as much to the right as possible }.

$\Rightarrow \lambda x.\ x\ \lambda y.\ y\ x$

Ans

2) $\underset{1}{(}\underset{2}{(}id\ id\underset{2}{)}\ \underset{3}{(}\lambda x.\ \underset{4}{(}\underset{5}{(}x\ id\underset{5}{)}\ z\underset{4}{)}\underset{3}{)}\underset{1}{)}$ { Bracket pair-1, can be removed as it has only one application of $(M\,N)$ form }

$\Rightarrow (id\ id)\ (\lambda x.((x\ id)\ z))$ { Bracket pair-2 and 5 can be removed as function application is left associative and has highest precedence }

$\Rightarrow id\ id\ (\lambda x.((x\ id)\ z))$

$\Rightarrow id\ id\ \lambda x.((x\ id)\ z)$

$\Rightarrow id\ id\ \lambda x.(x\ id)\ z$ { Bracket pair-3 can be removed as only single abstraction is present of $(\lambda x.M)$ form }

$\Rightarrow id\ id\ \lambda x.\ x\ id\ z$ { Bracket pair-4 can be removed as lambda terms extend to the right as much possible }

Ans

$\lambda x.\ x\ y\ \lambda x.\ y\ x \equiv (\lambda x_1.((x_1\ y)\ (\lambda x_2.(y\ x_2))))$

- Free variable : $y$
- Bound variable : $x_1, x_2$ [scope of $x_1$ till $\lambda x_1$ and that of $x_2$ till $\lambda x_2$]
  - $x_1$ is captured by $\lambda x_1$ (outer lambda)
  - $x_2$ is captured by $\lambda x_2$ (inner lambda)

Ans

2) $\lambda x.\ (\lambda y.\ \lambda x.\ x\ y)\ x\ y \equiv (\lambda x_1.(((\lambda y_1.(\lambda x_2.(x_2\ y_1)))\ x_1)\ y_2))$

- Free variable = $y_2$ (second 'y')
- Bound variables = $x_1, x_2, y_1$
  - $x_1$ is captured by $\lambda x_1$ (outer $\lambda x$)
  - $x_2$ is captured by $\lambda x_2$ (inner $\lambda x$)
  - $y_1$ is captured by $\lambda y_1$

{scope of $x_1, x_2, y_1$ are till $\lambda x_1, \lambda x_2$ and $\lambda y_1$ respectively}

Ans

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(5.) 1) $(\lambda g.\ g\ 5)\ (\lambda x.\ x+3)$

- Using CBN :-
  - $(\lambda g.\ g\ 5)\ (\lambda x.\ x+3)$
  - $=_B (\lambda x.\ x+3)\ 5$
  - $=_B 5+3 = 8$ Ans

- Using CBV :-
  - $(\lambda g.\ g\ 5)\ (\lambda x.\ x+3)$
  - $=_B (\lambda x.\ x+3)\ 5$
  - $=_B 5+3 = 8$ Ans

Here, the sequence of CBN and CBV are both same as $(\lambda x.\ x+3)$ can't be evaluated further, in function application

2) $(\lambda x.\ x\ x\ x)\ (\lambda x.\ x\ x\ x)$

- Using CBN :-
  - $(\lambda x.\ x\ x\ x)\ (\lambda x.\ x\ x\ x)$
  - $=_B (\lambda x.\ x\ x\ x)\ (\lambda x.\ x\ x\ x)\ (\lambda x.\ x\ x\ x) =_B \ldots\ldots$

It leads to infinite computation. Ans

⇒ Using CBV:-

⇒ $(\lambda x.xxx)(\lambda x.xxx)$

$=_\beta (\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx) =_\beta \ldots$

It leads to infinite computation. ∴