END TERM EXAMINATION
(Autumn 2022-2023)
Course- PHN-005 (Electrodynamics and Optics)

Time-**3 hours**

Total Marks- **100** Weight: **50%**

Instructions:

(1) **All questions carry 20 marks**. Bold-faced letters denote vectors or position vector.
(2) **Answer any 10 of 11 parts in Question no. 1**. Short reasons for all parts attempted in Question no. 1, must be given.
(3) Attempt **any four of the five questions numbered 2 to 6**.
(4) Solutions to all parts of a question should be written without inclusion of solutions of parts of some other question.

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1. **Answer any ten parts of the following. Each part carries 2 marks.**

(i) Which one of the following is correct for Newton's ring experiment and its concentric circular fringes-
a) Division of amplitude and fringes of equal inclination
b) Division of wave-front and fringes of equal inclination
c) Division of amplitude and fringes of equal thickness
d) Division of wave-front and fringes of equal thickness

(ii) A light wave travelling in a medium of refractive index 1.33 suffers reflection at the surface of a medium of refractive index 1.42. The additional path difference due to reflection is
(a) $\lambda/4$ **(b)** $\lambda/2$ **(c)** λ **(d)** 0

(iii) In Young's double -hole experiment, interference fringes are formed using sodium light, which predominantly comprises two lines at 5890 $\overset{o}{\mathrm{A}}$ and 5896 $\overset{o}{\mathrm{A}}$. The distances between the two sources and source plane to screen distance are 5 mm and 20 cm, respectively. The regions on the screen (y) where the fringe will disappear-
(a) $y \approx 1.1$ cm **(b)** $y \approx 2.2$ cm **(c)** $y \approx 0.5$ cm **(d)** None

(iv) Which of the following is/are correct for the missing order in double slit diffraction pattern-
(a) An interference maximum and a diffraction minimum correspond to the same angular position
(b) An interference minimum and a diffraction maximum correspond to the same angular position
(c) An interference minimum and a diffraction minimum correspond to the same angular position
(d) An interference maximum and a diffraction maximum correspond to the same angular position

(v) Consider two diffraction gratings G1 and G2. G1 has 10 lines spaced 1 cm apart and G2 has 100,000 lines spaced 1μm apart. The two gratings are illuminated normally with an incident wavelength of 500 nm. Which of the following is/are correct for G1 and G2 gratings-
(a) The maximum resolving power of G1 is larger than that of G2
(b) The maximum resolving power of G2 is larger than that of G1
(c) G1 and G2 have same resolving power
(d) Resolving power of the gratings can not be calculated with the given parameters

(vi) Calculate explicitly the divergence of the curl of a vector in a three-dimensional orthogonal curvilinear system with coordinates (u_1, u_2, u_3) and scale factors (h_1, h_2, h_3).

(vii) Prove that the area (pseudo-)vector for a non-self-intersecting planar closed curve is given by $\frac{1}{2}\oint \boldsymbol{dr} \times \boldsymbol{r}$.

(viii) A spherical conductor has a uniform conductivity σ and a uniform volume charge density ρ. Given the differential form of the Ohm's law: $\boldsymbol{J} = \sigma\boldsymbol{E}$, obtain $\boldsymbol{E} = \boldsymbol{E}(\boldsymbol{r}, t)$.

(ix) For a constant magnetic field intensity along the z axis, assuming the x and y components of the electric field to be the same, find the electric field for which the Poynting vector will be irrotational.

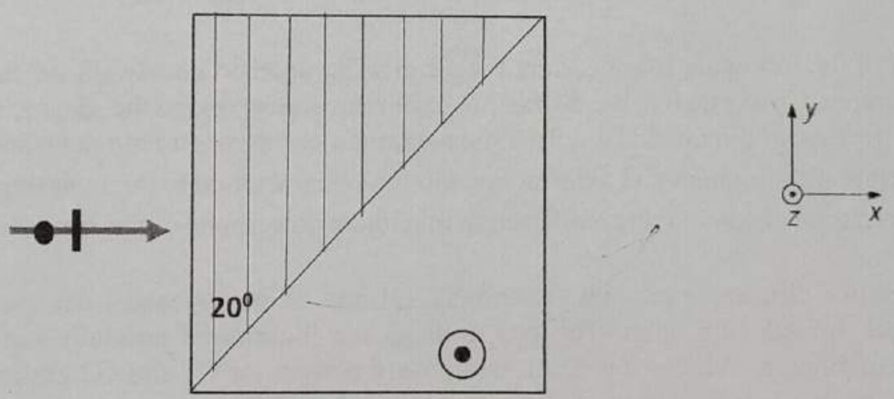
(x) From the Maxwell's equations, what happens to $\boldsymbol{E}(\boldsymbol{r}, t), \boldsymbol{B}(\boldsymbol{r}, t)$ if (a) the signs of all the source charges are reversed ("charge conjugation"), (b) under the transformation $\boldsymbol{r} \to -\boldsymbol{r}, t \to t$ ("parity"), (c) under the transformation $\boldsymbol{r} \to \boldsymbol{r}, t \to -t$ ("time reversal")?

(xi) The four sets of Maxwell's equations are a total of eight (2 scalar and 2 vector) equations in 3+3=6 components of the electric and magnetic fields. Doesn't this overdetermine the components? Why?

2. **(a)** Consider two diffraction gratings one with 10 lines spaced 1 cm apart and the other with 100,000 lines spaced 1 μm apart. Compare the two gratings in terms of ***(i)*** maximum resolving power, ***(ii)*** number of observable orders and ***(iii)*** intensities of the principal maxima for normal incidence of light of wavelength 500 nm. **[3+4+3]** [4+6+4]

(b) Show that the first order and second order spectra from a diffraction grating will never overlap when the grating is used for studying a light beam containing wavelength components from 400 to 700 nm. Assume the incidence of the light to be normal to the grating surface. **[5]** [6]

3. Consider a combination of two similar Calcite prisms as shown schematically in the figure below. The optic axis of the first prism is along the y axis, and the optic axis of the second prism is along the z axis. Such a combination of the prism is called Wollaston prism. Assume the angle of the prism to be 20° and $n_o \sim 1.658$ & $n_e \sim 1.486$. If an unpolarized beam is incident on the Wollaston prism, calculate the angular separation between the two orthogonally polarized output beams. **[15]** [20]



4. A quarter wave plate is rotated between two crossed Polaroids. If an unpolarized beam is incident on the first Polaroid, discuss the variation of intensity of the emergent beam as the quarter wave plate is rotated. What will happen if we have a half wave instead of a quarter wave plate? **[10+5]** [[5+5]

5. (a) Given a three-dimensional orthogonal coordinate system with coordinates (u_1, u_2, u_3) and scale factors (h_1, h_2, h_3), with unit tangent vectors $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$, calculate and write down $\frac{\partial \hat{a}_1}{\partial u_2}$ in terms of $\hat{a}_{1,2,3}$ and (derivative(s) of) $h_{1,2,3}$.

[8]

(b) The current density inside a slab of thickness 2d along the z axis (with the slab of infinite extent along the x and y axes) is given by $\boldsymbol{H} = -\lambda \nabla \times J, \frac{\partial J}{\partial t} = \lambda' \boldsymbol{E}$ ("London equations") with $\boldsymbol{E} = \boldsymbol{D} = 0$ everywhere and there being no surface currents and surface charges. The magnetic field is $H_1 \boldsymbol{a}_y$ for $z \leq -d$ and $H_2 \boldsymbol{a}_y$ for $z \geq d$ for constant H_1 and H_2. Find $\boldsymbol{H}$ inside the slab assuming $\boldsymbol{H} = H_y(z)\, \hat{\boldsymbol{a}}_y$.

[12]

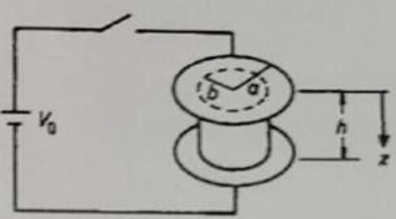
6.

(a) A spherical wave is given by:

$$\boldsymbol{E}(r, \theta, \phi, t) = A \frac{\sin\theta}{r}\left(\cos(kr - \omega t) - \frac{\sin(kr - \omega t)}{kr}\right)\hat{\boldsymbol{a}}_\phi, \qquad \frac{\omega}{k} = c.$$

Show that ***E*(*r*,*t*)** obeys Maxwell's equations and find out the corresponding ***B*(*r*,*t*)**. [10]

(b) Two conducting discs of radius a are separated by a distance $h \ll a$. A solid cylinder of radius b, length h and volume resistivity ρ fills symmetrically the gap between the disks. The discs can be assumed to have been connected to a battery for a very long time. [10]



(i) Neglecting edge effects and inductance, calculate ***E*** and ***B*** and hence ***S*** in the gap as functions of time, after the battery has been disconnected from the capacitor.

(ii) Can you reconcile the ***E*** and ***B*** obtained in (i) with Maxwell's equations corresponding to the differential form of Faraday's law?

[Hints: 1. $\boldsymbol{B} = \nabla \times \boldsymbol{A},\ \boldsymbol{E} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t}$

2. (For determining the vector potential) Use: the solution of $x\, y''(x) + y'(x) + a\, x\, y(x) = b\, x$, is given by $y(x) = b/a + c_1 J_0(x\, a^{1/2}) + c_2 Y_0(x\, a^{1/2})$, where $J_0(x)$ and $Y_0(x)$ are Bessel functions of the first and second kinds of order 0.

3. $J_0(x) = \sum_{n=0}^{\infty} \frac{(-)^n}{n!^2}\left(\frac{x}{2}\right)^{2n}$; $Y_0(x \sim 0) \sim \frac{2}{\pi}\log x$]