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(1.) $f(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots \infty$. [By using taylor series].

$f'(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \infty$.

$\Rightarrow$ Put value in given equation:-

$$h - \frac{h^3}{3!} + \frac{h^5}{5!} \dots \infty = 0 + h\left(1 - \frac{\theta^2 h^2}{2} + \dots \infty\right).$$

$$\Rightarrow \frac{h^2}{6} + \left(\frac{-h^4}{120}\right) \dots \infty = \frac{\theta^2 h^2}{2} - \dots \infty.$$

divide by h^2 and $h \to 0$:

$$\lim_{h \to 0} (\theta) = \sqrt{\frac{1}{6} \times 2} = \frac{1}{\sqrt{3}} \text{ Ans}.$$

(2.) (a) $e^x = 1 + x + \underbrace{\frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty}_{>0 \text{ as } x>0}$.

so: $e^x > \left(1 + x + \frac{x^2}{2}\right)$. —(i)

$\Rightarrow$ To prove (ii) part, take: $g(x) = 1 + x + \left(\frac{x^2}{2} - 1\right) e^x$

$\Rightarrow g'(x) = h(x) = 1 + \left(\frac{x^2}{2} - 1 + x\right) e^x$

$\Rightarrow h'(x) = \left(2x + \frac{x^2}{2}\right) e^x > 0 \quad \forall x > 0.$

so: $h(x) - h(0) > 0$ [as $h(x)$ is increasing]

$h(x) = g'(x) > 0$ [as $h(0) = 0$].

$g(x) - g(0) > 0$ [as $g(x)$ is increasing].

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$\Rightarrow g(x) > 0 \quad \forall x > 0 \quad (\text{as } g(0) = 0).$

$\Rightarrow 1 + x + \frac{x^2}{2} e^x > e^x$ —— (ii)

combine (i) and (ii). Ans

(2.) (b) $f(x) = x - \sin x$

$f'(x) = 1 - \cos x \geq 0$

so: $f(x) \uparrow \Rightarrow f(x) > f(0) \quad \forall (x > 0)$

$\Rightarrow \boxed{x > \sin x}$

$g(x) = \sin x - x + \frac{x^3}{6}$

$\Rightarrow g'(x) = h(x) = \cos x - 1 + \frac{x^2}{2}$

$\Rightarrow h'(x) = -\sin x + x > 0 \quad \forall (x > 0)$

$\Rightarrow h(x)$ is increasing

so $h(x) > \underbrace{h(0)}_{=0} \quad \forall (x > 0)$

$\Rightarrow g'(x) > 0 \Rightarrow g(x) \uparrow$

$\Rightarrow g(x) > g(0) \quad \forall (x > 0)$

$\Rightarrow \boxed{\sin x > \left(x - \frac{x^3}{6}\right)}$

Combine both:

we get :-

$\left(x - \frac{x^3}{3!} < \sin x < x\right)$ Ans

(2.) (C) Taylor series :-

- $\sqrt{1+x} = \left(1 + \frac{x}{2} - \frac{x^2}{8} + \underbrace{\frac{x^3}{16} - \frac{x^4}{128} \times 5}_{>0} \dots \infty\right)$

$\uparrow$ every coming pair will be > 0.

so $\sqrt{1+x} > 1 + \frac{x}{2} - \frac{x^2}{8} > 1 + \frac{x}{2} - \frac{x^3}{8}$ as $(x > 0)$

- similarly:

$\sqrt{1+x} = 1 + \frac{x}{2} + \underbrace{\left(-\frac{x^2}{8} + \frac{x^3}{16}\right)}_{<0} + \underbrace{\dots}_{<0} \infty.$

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(3.)

so: $\sqrt{1+x} < 1+\frac{x}{2}$ **Ans**

(3.) $f(x,y) = f(1,-2) + \left((x-1)\frac{\partial}{\partial x} + (y+2)\frac{\partial}{\partial y}\right) f \Big|_{(1,-2)}$

$+ \frac{1}{2}\left((x-1)\frac{\partial}{\partial x} + (y+2)\frac{\partial}{\partial y}\right)^2 f \Big|_{(1,-2)} + \ldots\ldots$

$= -10 + (-4(x-1)) + (4(y+2)) + \left(\frac{-4(x-1)^2}{2}\right)$
$+ 0 + \frac{2(x-1)(y+2)}{2}\cdot 2 + (x-1)^2(y+2).$

$= -10 + 4(y+2) - 4(x-1) - 2(x-1)^2 + 2(x-1)(y+2)$
$+ (x-1)^2(y+2)$ **Ans**

~~[after all these, will become~~ zero.].

(4.)

$f(x,y) = \frac{\pi}{4} + \left((x-1)\left(\frac{-1}{2}\right) + \frac{1}{2}(y-1)\right) + (x-1)^2\left(\frac{1}{4}\right) - \frac{1}{4}(y-1)^2.$

put $x = 1.1$ and $y = 0.9$,

so:

$f(1.1, 0.9) = \frac{\pi}{4} - (0.1) + 0 \simeq 0.685$ **Ans**

(5.) $f(x,y) = \left(f(x_0,y_0) + (x-x_0)\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)} + (y-y_0)\cdot\frac{\partial f}{\partial y}\Big|_{(x_0,y_0)}\right)$

called linear approximation.

(a) $f(x,y) = -3 - 2(x+1) - (y-1).$

$f_{xx} = 4;\ f_{xy} = -1;\ f_{yy} = 2$

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$\Rightarrow$ so $M = 4 = \max\{|f_{xx}|, |f_{xy}|, |f_{yy}|\}$.
and
$\Rightarrow$ upper bound of error term $= \frac{M}{2}\left(|x-x_0| + |y-y_0|\right)^2$

$$\leq \frac{4}{2}(0.1+0.1)^2$$
$$= 2(0.04) = 0.08$$
Ans

(5) (b) $f(x,y) = 8 + 4(x-3) - (y-2)$
$\Rightarrow E(x,y) \leq \frac{M}{2}(0.1+0.1)^2 = \frac{2}{2}(0.04) = 0.04$
Ans

where $M = \max\{|f_{xx}|, |f_{xy}|, |f_{yy}|\}$.
and $f_{xx} = 2$; $f_{yy} = 1$; $f_{xy} = -1$.

(6.) $\Rightarrow f(0,0) = 0$; $f_x = \cos x \sin y = 0$ at $(0,0)$
$f_y = \sin x \cos y = 0$ at $(0,0)$
$f_{xx} = -\sin x \sin y = 0$ at $(0,0)$
$f_{xy} = \cos x \cos y = 1$ at $(0,0)$
$f_{yy} = -\sin x \sin y = 0$ at $(0,0)$.
$\Rightarrow$ Quadratic Approximation: about origin is $\Rightarrow$
$$f(x,y) = f(0,0) + \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)f\Big|_{(0,0)}$$
$$+ \frac{1}{2}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 f\Big|_{(0,0)}$$
$$= 0 + \frac{1}{2}\left(2\cdot(xy)(1)\right) = xy.$$ Ans.

$\Rightarrow E(x,y) \leq \frac{M}{3!}\left(|x-x_0| + |y-y_0|\right)^3$

where $M = \max\{|f_{xxx}|, |f_{xxy}|, |f_{xyy}|, |f_{yyy}|\}$.
$f_{xxx} = 0;\ f_{xxy} = 0;\ f_{xyy} = 0;\ f_{yyy} = 0;$

$\Rightarrow$ but, we all know that Error can not be zero.
so looking at all funitions these include $\sin x, \cos y, \cos x, \sin y$ so: $(M)_{max} = 1$.

so $E(x,y) \leq \left(\frac{1}{6}\right).(0.008) = 0.00133\ldots$ Ans

(7.) $f(x) = \left(\frac{1}{x}\right)^x = e^{-x\ln(x)}$.

$\Rightarrow f'(x) = e^{-x\ln(x)}\left[-1-\ln(x)\right] = 0 \Rightarrow \boxed{x = \frac{1}{e}}$

when $x < \frac{1}{e} \Rightarrow f'(x) > 0$
$x > \frac{1}{e} \Rightarrow f'(x) < 0$ } so $x = \frac{1}{e}$ corresponds to maxima Ans

(8.) (a) Critical point:- $\frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$.

$\Rightarrow y - 2x - 2 = 0 = x - 2y - 2. \Rightarrow \boxed{(-2,-2)}$

$\Rightarrow$ At this point: $A = f_{xx} = -2$
$B = f_{xy} = 1$
$C = f_{yy} = -2$

so $AC - B^2 > 0$ and $A < 0 \Rightarrow$ maxima at $(-2,-2)$.
max. value $= 8$.

8.(b) $f_{xx} = 6x$; $f_{yy} = 6y$; $f_{xy} = -3a$

$\Rightarrow \frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$

$\Rightarrow x^2 - ay = 0 = y^2 - ax \Rightarrow$ $\left.\begin{matrix}(0,0)\\(a,a)\end{matrix}\right\}$ critical points. where $(a \neq 0)$.

$\Rightarrow$ For $(0,0)$:-

$f_{xx} = 0 = A$

$f_{yy} = 0 = C$

$f_{xy} = -3a = B$.

$\because AC - B^2 < 0$

$\therefore$ saddle point at $(0,0)$

$\Rightarrow$ For (a,a)

$AC - B^2 = 36a^2 - 9a^2 > 0$

and $A = 6a$ $(A > 0;\ a > 0)$ / $(A < 0;\ a < 0)$

so $a > 0 \Rightarrow$ local minima

$a < 0 \Rightarrow$ local maxima

8.(c) $f_{xx} = (2y^2 - 10)$; $f_{yy} = (2x^2 - 10)$; $f_{xy} = (4xy - 8)$

$\Rightarrow$ For critical points: $\frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$.

$\Rightarrow 2x^2y - 8x - 10y = 0 = 2y^2x - 10x - 8y$.

$\Rightarrow$ $\left.\begin{matrix}(0,0)\\(3,3)\\(-3,-3)\\(-1,1)\\(1,-1)\end{matrix}\right\}$ critical points.

$\Rightarrow$ For $(0,0)$	For $(\pm 3, \pm 3)$:-	For $(\pm 1, \mp 1)$:-
$A = C = -10$; $B = -8$.	$A = C = 8$; $B = 28$.	$A = C = -8$; $B = -12$
$\because AC - B^2 > 0$; $A < 0$	so $AC - B^2 < 0$	so $AC - B^2 < 0$
$\therefore \Rightarrow$ Local max. at $(0,0)$	$\therefore$ saddle point	$\therefore$ saddle point
Max. value $= 0$.		Ans

8.(d) $f_{xx} = 4 - 12x^2$; $f_{yy} = 4 - 12y^2$; $f_{xy} = -4$.

$\Rightarrow \frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y} \Rightarrow x - y - x^3 = 0 = x + y^3 - y$.

$\Downarrow$

$\underbrace{(0,0) \text{ and } (\pm\sqrt{2}, \mp\sqrt{2})}_{\text{critical points}}$.

$\Rightarrow$ For $(0,0)$:-

$A = C = 4$; $B = -4$

so $AC - B^2 = 0$; $A > 0$

$\therefore$ saddle point.

$\Rightarrow$ For $(\pm\sqrt{2}, \mp\sqrt{2})$:-

$A = C = -20$; $B = -4$

so : $AC - B^2 > 0$; $A < 0$

$\therefore$ Local maxima

Max. value $= 8$. Ans

8 (e) $\frac{\partial f}{\partial x} = y\cos x = 0$; $\frac{\partial f}{\partial y} = \sin x = 0$

$\Rightarrow$ so $x = n\pi$, $y = 0 \Rightarrow \left\{ \begin{array}{c} f_{xx} = 0 = f_{yy} \\ f_{xy} = 1, -1. \end{array} \right\}$

$\Rightarrow$ so $AC - B^2 < 0 \Rightarrow$ saddle point at $(n\pi, 0)$.
for $n \in I$.

9.(a) $g(x,y) = (x + y - 9)$.

$\Rightarrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \lambda\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$ [For extreema of $f(x,y)$]

$\Rightarrow \left.\begin{array}{l} 2 - 2x = \lambda \\ 2 - 2y = \lambda \end{array}\right\}$ $x = y$ $\left\{\begin{array}{c} \text{condition of} \\ \text{extreema} \\ \text{of } f(x,y) \end{array}\right\}$.

$\Rightarrow$ $f(x,y) = f(x,x) = 2+4x-2x^2 = 2(1+2x-x^2)$.

Now: $f'(x) = 2(2-2x) = 0 \Rightarrow (x=1)$

$f''(x) = -4 < 0$

So $x=1=y \Rightarrow$ will be max. correspondence and minima will be found from boundary conditions.

$\Rightarrow$ Max. value $= 4$ at $(1,1)$

Min. value $= -61$ at $(9,0)$; $(0,9)$. Ans

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9. (b) $g(x,y) = 2x^2+y^2-1$.

$\Rightarrow$ $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \lambda\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$.

$\Rightarrow$ $\left.\begin{aligned} 6x-1 &= \lambda(4x) \\ 2y &= \lambda(2y) \end{aligned}\right\}$ I. $\lambda = 1$; $x = \frac{1}{2}$; $y^2 \leq \frac{1}{2}$

II. $y = 0$; $x^2 \leq \frac{1}{2}$.

I. $f(x,y) = f = \left(\frac{1}{4}+y^2\right)$ where $y^2 \leq \frac{1}{2}$.

So $f_{min} = \frac{1}{4}$; $f_{max} = 3/4$.

II. $f(x,0) = f = (3x^2-x)$ when $x^2 \leq \frac{1}{2}$.

$f' = (6x-1) = 0 \Rightarrow \left(x = \frac{1}{6}\right) \Rightarrow$ minima.

$f_{min} = \frac{-1}{12}$ at $\left(\frac{1}{6}, 0\right)$.

$f_{max} = \left(\frac{3}{2}+\frac{1}{\sqrt{2}}\right) = \left(\frac{3+\sqrt{2}}{2}\right)$ at $\left(\frac{-1}{\sqrt{2}}, 0\right)$.

overall,

max. value $= \frac{3+\sqrt{3}}{2}$ at $\left(\frac{-1}{\sqrt{2}}, 0\right)$.

min. value $= \frac{1}{\sqrt{2}}$ at $\left(\frac{1}{6}, 0\right)$. Ans.

(10.) (a) $3 = \lambda(2x) \Rightarrow x = 3/2\lambda$
$4 = \lambda(2y) \Rightarrow y = 4/2\lambda$ } put in eqn: $\lambda = \pm 5/2$

For $\lambda = 5/2$; $(x, y) \equiv (3/5, 4/5)$ and $f_{max} = 5$.

For $\lambda = -5/2$; $(x, y) = \left(\frac{-3}{5}, \frac{-4}{5}\right)$ and $f_{min} = -5$.

Ans

(10.) (b) $m \cdot x^{m-1} y^n z^p = \lambda = n y^{n-1} x^m z^p = p z^{p-1} x^m y^n$

$\Rightarrow$ solving we get: $y = \left(\frac{n}{m}\right) x$ and $z = \left(\frac{p}{m}\right) x$

put in $x + y + z = a \Rightarrow x = \left(\frac{ma}{m+n+p}\right)$

and $y = \frac{na}{m+n+p}$, $z = \frac{pa}{m+n+p}$.

$\Rightarrow$ Value will be: $\frac{m^m n^n p^p a^{m+n+p}}{(m+n+p)^{m+n+p}}$. Ans

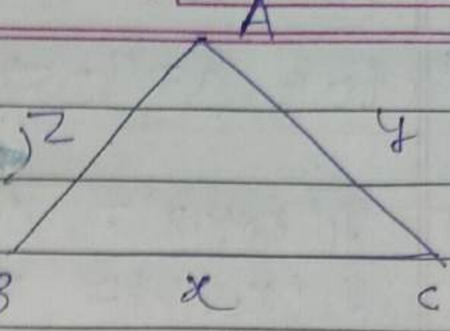
(10.) (c) $y = \frac{x}{4}\lambda$; $x = \lambda y \Rightarrow \frac{x}{y} = \frac{4y}{x} \Rightarrow \left\{ \begin{array}{l} (x = \pm 2y). \\ (\lambda = \pm 2). \end{array} \right\}$

$\Rightarrow$ Points are $(2,1)$; $(-2,-1)$; $(-2,1)$; $(2,-1)$.

Max. value 2 at $(\pm 2, \pm 1)$
Min. value -2 at $(\mp 2, \pm 1)$ } Ans

(11) $\Rightarrow$ Given: $x+y+z=p=\text{const}.$

$\Rightarrow$ Let: $g(x,y,z)=(x+y+z-p)^2$



A.t. Heron's formula:

$$\Delta = \text{area} = \sqrt{\frac{p}{2}\left(\frac{p}{2}-x\right)\left(\frac{p}{2}-y\right)\left(\frac{p}{2}-z\right)} = f(x,y,z)$$

$\Rightarrow$ $\frac{\partial f}{\partial x} = -\frac{\sqrt{\frac{p}{2}\cdot\left(\frac{p}{2}-y\right)\left(\frac{p}{2}-z\right)}}{2\sqrt{\frac{p}{2}-x}}$ and similarly f_y and f_z too.

$\Rightarrow$ $\frac{\partial g}{\partial x} = 1 = \frac{\partial g}{\partial y} = \frac{\partial g}{\partial z}.$

so: $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \lambda(1,1,1)$ [for extrema of 'f'].

$\Rightarrow$ It means: $f_x = f_y = f_z$

$\Rightarrow$ $\boxed{x=y=z}$ $\Rightarrow$ Equilateral

Ans

(12) $g(x,y,z) = x^2+y^2+z^2-24$

$f(x,y,z) = (x-1)^2+(y-2)^2+(z+1)^2 = (\text{dis})^2.$

$\Rightarrow$ $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \lambda\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right).$

$\Rightarrow$ $2(x-1) = 2\lambda x \Rightarrow x = \frac{1}{1-\lambda}.$

$\Rightarrow$ $2(y-2) = 2\lambda y \Rightarrow y = 2/1-\lambda$

$\Rightarrow$ $2(z+1) = 2\lambda z \Rightarrow z = \left(\frac{-1}{1-\lambda}\right)$

put in sphere $\Rightarrow$ $1-\lambda = \pm\frac{1}{2}.$

$\Rightarrow \lambda = 3/2, 1/2$

For $\lambda = 1/2 \Rightarrow$ distance(min) $= \sqrt{6}$ Ans

(13.) $g_1(x,y,z) = x+y+z-1.$
$g_2(x,y,z) = x^2+y^2-1.$
$f(x,y,z) = x^2+y^2+z^2.$

$$\Rightarrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial z}\right) = \lambda_1\left(\frac{\partial g_1}{\partial x}, \frac{\partial g_1}{\partial y}, \frac{\partial g_1}{\partial z}\right) + \lambda_2\left(\frac{\partial g_2}{\partial x}, \frac{\partial g_2}{\partial y}, \frac{\partial g_2}{\partial z}\right)$$

$$\Rightarrow \left.\begin{aligned} 2x &= \lambda_1 + 2\lambda_2 x \\ 2y &= \lambda_1 + 2\lambda_2 y \\ 2z &= \lambda_1 + 0 \end{aligned}\right\} \quad x = \frac{\lambda_1}{2(1-\lambda_2)};\ y = \frac{\lambda_1}{2(1-\lambda_2)} \quad z = \lambda_1/2. \quad [\lambda_2 \neq 1].$$

$\Rightarrow$ So put in plane and cylinder equation:-

$$\left.\begin{aligned} &\frac{\lambda_1}{1-\lambda_2} + \frac{\lambda_1}{2} = 1 \Rightarrow \\ &\text{and } x^2+y^2=1 \end{aligned}\right\} \quad \begin{aligned} &x = y = \pm\frac{1}{\sqrt{2}}. \\ &z = (1+\sqrt{2}), (1-\sqrt{2}). \end{aligned}$$

$\Rightarrow$ So, Farthest point on ellipse will be: $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 1+\sqrt{2}\right)$. Ans.

This whole calculation, above, done for $\lambda_2 \neq 1$, now $\lambda_2 = 1$, $\lambda_1 = 0$, $z = 0$.

$$\text{So: } \left.\begin{aligned} x+y &= 1 \\ x^2+y^2 &= 1 \end{aligned}\right\} \left.\begin{aligned} (1,0) \\ (0,1) \end{aligned}\right\}.$$

Hence, nearest point will be $(1,0,0)$; $(0,0,1)$ not $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, (1-\sqrt{2})\right)$ Ans

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(14) Quadratic approximation:- about (x_0, y_0).

$$\Rightarrow f(x,y) = f(x_0,y_0) + \sum_{k=1}^{2} \frac{1}{k!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^k f \Big|_{(x_0,y_0)}$$

$\Rightarrow$ Here, at $(1,2)$: $\begin{Bmatrix} x_0 = 1 \\ y_0 = 2 \end{Bmatrix}$ $\quad\begin{Bmatrix} h = x - x_0 \\ k = y - y_0 \end{Bmatrix}$.

$\Rightarrow f_{xy} = \frac{-2}{27}$; $f_x = \frac{4}{3}$; $f_y = \frac{2}{3}$; $f_{xx} = \frac{20}{27}$; $f_{yy} = \frac{5}{27}$;

(a) $\Rightarrow$ $$f(x,y) = 3 + \left(\frac{4}{3}(x-1) + \frac{2}{3}(y-2)\right) + \frac{1}{2}\left(\frac{20}{27}(x-1)^2 + \frac{2(x-1)(y-2)(-2/27)}{2} + \frac{5}{27}(y-2)^2\right).$$

(b) $f(1.1, 2.05) \Rightarrow$ put value of $x = 1.1$ and $y = 2.05$ in Quadratic approximation.

We get: $f(1.1, 2.05) \approx 3.170$ Ans.

(15.) $g(x,y) = x^2+y^2-1$. ; $f(x,y) = (x+y)\, e^{-(x^2+y^2)}$.

$\Rightarrow \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x}$ and $\frac{\partial f}{\partial y} = \lambda\frac{\partial g}{\partial y}$.

$$\Rightarrow \lambda = \frac{e^{-(x^2+y^2)}(1-2x(x+y))}{2x} = \frac{e^{-(x^2+y^2)}(1-2y(x+y))}{2y}.$$

$\Rightarrow \boxed{(x=y) \text{ or } (0,0)}$.

$\Rightarrow$ so $x=y \Rightarrow$ {is the condition of extrema}.

then $f = 2xe^{-2x^2}$

$f' = 0 \Rightarrow x = 1/2, -1/2 = y$

Hence,

at $\left(\frac{1}{2}, \frac{1}{2}\right) \Rightarrow \frac{1}{\sqrt{e}}$ is maximum

at $\left(\frac{-1}{2}, \frac{-1}{2}\right) \Rightarrow \frac{-1}{\sqrt{e}}$ is minimum. Ans

(16) $f(x,y) = \begin{cases} e^{-1/(x^2+y^2)} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$\Rightarrow f(x,y) = f(0,0) + \sum_{k=1}^{n} \frac{1}{k!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^k f\Big|_{(0,0)}$

where $k = y$ and $h = x$ for $(0,0)$.

(a) $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{e^{-1/h^2}}{h} = \lim_{h\to 0} \frac{(1/h)}{(e^{1/h^2})}$

$= \lim_{h\to 0} \frac{-1/h^2}{e^{1/h^2}(-2/h^3)} = \lim_{h\to 0} \left[\frac{h}{2e^{1/h^2}}\right]$

$= 0.$

similarly $f_y(0,0) = 0.$

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(b) $f_{xx}(0,0) = \lim_{h\to 0} \frac{f_x(h,0) - f_x(0,0)}{h}$

$= \lim_{h\to 0} \left[\frac{2e^{-1/h^2}\cdot(1/h^3)}{h}\right]$

$= \lim_{h\to 0} \frac{(2)\cdot(1/h^4)}{(e^{+1/h^2})} = \lim_{h\to 0} \frac{(4)(1/h^2)}{e^{1/h^2}}$

$= \lim_{h\to 0} (4) \frac{(-2/h^3)}{e^{1/h^2}\cdot(-2/h^3)} = 0.$

Similarly, $f_{xy} = 0 = f_{yy}$. at $(0,0)$.

(c) due to this, all will become zero, as there is some pattern in calculating limit so:

$f(x,y) \approx 0$ ~~approximate to a zero polynomial~~ about ~~origin~~. ~~Ans~~