Operation And Supply Chain Management (IBM-311)

TUTORIAL 2

INVENTORY MANAGEMENT

Q 1) A manufacturer uses Rs. 10,000 worth of an item during the year. He has estimated the ordering costs as Rs.25 per order and carrying costs as 12.5% of average inventory value. Find the optimal order size, number of orders per year, time period per order and total cost.

Solution

Here D = Rs. 10,000, C_o = Rs. 25, C_h = 12.5% of average inventory value/unit

$$TC = \frac{25D}{Q} + (0.125)\frac{Q}{2}$$ where Q is order size in rupees

Applying Eqs. (9.6) to (9.9)

$$\text{We get, } Q^* = \sqrt{\frac{2 \times 25 \times 10{,}000}{0.125}} = \text{Rs. } 2{,}000$$

$$t^* = \sqrt{\frac{2 \times 25}{(0.125)(10{,}000)}} = \frac{1}{5} \text{ yrs} = 73 \text{ days}$$

$$N = \frac{1}{t^*} = 5$$

$$TC^* \text{ (Variable Cost only)} = \sqrt{2 \times 25 \times 0.125 \times 10{,}000} = \text{Rs. } 250$$

Q 2) An item is used at a uniform rate of 50,000 units per year. No shortage is allowed and delivery is at an infinite rate. The ordering, receiving and hauling cost is Rs. 13 per order, while inspection cost is Rs. 12 per order. Interest costs Rs. 0.056 and deterioration and obsolescence cost Rs. 0.004 respectively per year for each item actually held in inventory plus Rs. 0.02 per year per unit based on the maximum number of units in inventory. Calculate the EOQ. If lead time is 20 days, frnd re-order level

Solution

Given that ordering cost $= C_o =$ Rs. 13 + Rs. 12 = Rs. 25 per order

Storage Cost (C_h) based on actual inventory (= average inventory)
= Rs. 0.056 + Rs. 0.004 = Rs. 0.060

Storage Cost (C_h) based on the Maximum inventory = Rs. 0.02 per unit/year

Demand = 50,000 units/yr.

$$\text{Total variable cost} = TC = \frac{25 \times 50{,}000}{Q} + \frac{0.060\,Q}{2} + 0.02\,Q$$

$$= \frac{1250{,}000}{Q} + 0.1\,Q/2$$

Thus,

$$Q^* = \sqrt{\frac{2 \times 25 \times 50{,}000}{0.1}} = 5{,}000 \text{ units}$$

$$\text{Re–order level (R.O.L)} = L \times D = \frac{20 \times 50{,}000}{365}$$

$$= 2{,}740 \text{ units}$$

Q 3) The demand for an item each costing Re 1, is 10,000 units per year. The ordering cost is Rs. 10. Inventory carrying charge is 20% based on the average inventory per year. Stock-out cost is Rs. 5 per unit of shortage incurred. Find various parameters

Solution

Here $D = 10{,}000,\ C_o = 10,\ C_h = 20\%$ of Re $1 = 0.2,\ C_s =$ Rs. 5

Therefore, $Q^* = EOQ = \sqrt{\left(\dfrac{2 \times 10 \times 10{,}000}{0.20}\right)\left(\dfrac{0.20+5}{5}\right)} = 1{,}020$ units

The inventory level $I^* = \sqrt{\left(\dfrac{2 \times 10 \times 10{,}000}{0.20}\right)\left(\dfrac{5}{0.20+5}\right)} = 980$ units

The shortage level $= Q^* - I^* = 1020 - 980 = 40$ units

Cycle period, $t^* = \dfrac{1020}{10{,}000} = 0.102$ yr $= 37.23$ days or 37 days

Number of orders/yr $= \dfrac{10{,}000}{1020} = 9.8$ (or 10)

Total Variable Cost $= \sqrt{\dfrac{2 \times 10 \times 0.20 \times 5 \times 10{,}000}{(0.20+5)}} =$ Rs. 196.12

Q 4) A unit is used at the rate of 100 per day and can be manufactured at a rate of 600 per day. It costs Rs. 2000 to set up the manufacturing process and Rs. 0.1 per unit per day held in inventory based on the actual inventory any time. Shortage is not allowed. Find the minimum cost and the optimum number of units per manufacturing run.

Solution

Here $D = 100$ units, $P = 600$ units, $C_o =$ Rs. 2000, $C_h =$ Rs. 0.1

Therefore, $Q^* = \sqrt{\dfrac{2 \times 2000 \times 100 \times 600}{(0.1)(600-100)}} = 2190$ units

and $TC^* = \sqrt{\dfrac{2 \times (0.1) \times 2000 \times (600-100) \times 100}{600}} =$ Rs. 182.6

Q 5) Annual demand for an item is 2400 units. Ordering cost is Rs. 350, inventory carrying charge is 24% of the purchase price per year. Purchase prices are:

P1 = Rs. 10 for purchasing Q1 < 500

P2 = Rs. 9.25 for purchasing 500 =<Q2= < 750

P3 = Rs. 8.75 for purchasing 750 =<Q3

Determine the optimum purchase quantity.

Solution

As per Step 1, $Q_3^* = \sqrt{\dfrac{2 \times 350 \times 2400}{(0.24)(8.75)}} = 894$ units

Since 894 is greater than 750, optimum purchase quantity is 894 units.

Q 6) Consider above question with thc ordering cost of Rs. 100 only

Solution

$$\text{Here } Q_3^* = \sqrt{\frac{2 \times 100 \times 2400}{(0.24)(8.75)}} = 478 \text{ units}$$

Since $478 < 750 = b_3$, we next compute,

$$Q_2^* = \left(\frac{2 \times 100 \times 2400}{(0.24)(9.25)}\right)^{1/2} = 465 \text{ units}$$

Since $465 < 500 = b_2$, we next compute,

$$Q_1^* = \left(\frac{2 \times 100 \times 2400}{(0.24)(10)}\right)^{1/2} = 447 \text{ units} > 0$$

We now compare the total costs for purchasing $Q_1^* = 447$, $b_2 = 500$ and $b_3 = 750$ units respectively.

From Eq. (9.21), we get

$$TC_1 \text{ (for purchasing 447)} = 10 \times 2400 + \frac{100 \times 2400}{447} + \frac{1}{2}(0.24)(10)\,447$$

$$= \text{Rs. } 25{,}085$$

$$TC_2 \text{ (for } Q_2 = 500) = 9.25 \times 2400 + \frac{100 \times 2400}{500} + \frac{1}{2}(0.24)(9.25)(550)$$

$$= \text{Rs. } 23{,}247$$

$$TC_3 \text{ (for } Q_3 = 750) = 8.75 \times 2400 + \frac{100 \times 2400}{750} + \frac{1}{2}(0.24)(8.75)(750)$$

$$= \text{Rs. } 22{,}119.50^*$$

Therefore, the economic purchase quantity for this problem is $Q_3^* = 750$ units.

Q 7) From the following calculate **(i)** Re-ordering Level and **(ii)** Minimum Level

Minimum usage 100 units per week Normal usage 200 units per week

Maximum usage 300 units per week Re-order period 4 to 6 weeks

Solution

(i) Re-ordering Level

*Re-ordering level= Maximum consumption * Lead Time [maximum]*

Re-ordering level= 300 * 6

Re-ordering level= 1,800 Units per week

(ii) Minimum Level

Minimum level= Reorder level – (Average consumption x lead time [Average])

Minimum level= 1,800 – (200 x 5)

Minimum level= 1,000 Units per week

Q8) Calculate Ordering Level, Minimum Level and Maximum Level from the following data:

EOQ

Re-order quantity 1,500 units Re-order period 4 to 6 weeks

Maximum consumption 400 units per week Average consumption 300 units per week

Minimum consumption 250 units per week

Solution

(i) Ordering Level

*Ordering level= Maximum consumption * Lead Time [maximum]*

Ordering level= 400 * 6

Ordering level= 2,400 Units per week

(ii) Minimum Level

Minimum level= Reorder level – (Average consumption x lead time [Average])

Minimum level= 2,400 – (300 x 5)

Minimum level= 900 Units per week

(iii) Maximum Level

*Maximum stock level= Reorder level – (Min consumption * Lead time [minimum]) + EOQ*

*Maximum stock level= 900 – (250 * 4) + 1,500*

Maximum stock level= 2,400 – (1,000) + 1,500

Maximum stock level= 2,900 Units per week

Q 9) The following information is available in respect of component DP 5:

Maximum stock level	8,400 units
Budgeted consumption- maximum	1,500 units per month
Budgeted consumption- minimum	800 units per month
Estimated delivery period	Maximum 4 months and minimum 2 months

You are required to calculate Re-order level

Solution

Ordering Level

*Ordering level= Maximum consumption * Lead Time [maximum]*

Ordering level= 1,500 * 4

Ordering level= 6,000 Units per week

Q 10) From the following date for the last twelve months, compute the Average Stock Level for a component.

Maximum usage in a month 300 units Minimum usage in a month 200 units

Average usage in a month 225 units Re-ordering quantity 750 units

Time lag procurement of material Maximum 6 months and Minimum 2 months

Solution

Average Stock Level

Average Stock Level = Minimum Stock Level + ½ of EOQ

Minimum level= Reorder level – (Average consumption x lead time [Average])

*Re-ordering level= Maximum consumption * Lead Time [maximum]*

*Re-ordering level= Maximum consumption * Lead Time [maximum]*

Re-ordering level= 300 * 6

Re-ordering level= 1,800 Units per month

Minimum level= Reorder level – (Average consumption x lead time [Average])

Minimum level= 1,800 – (225 x 4)

Minimum level= 900 Units per month

Average Stock Level = Minimum Stock Level + ½ of EOQ

Average Stock Level = 900 + ½ (750)

Average Stock Level = 1,275 Units per month

Q 11) An auto parts supplier sells Hardy-brand batteries to car dealers and auto mechanics. The annual demand is approximately 1,200 batteries. The supplier pays $28 for each battery and estimates that the annual holding cost is 30 percent of the battery's value. It costs approximately $20 to place an order (managerial and clerical costs). The supplier currently orders 100 batteries per month.

a. Determine the ordering, holding, and total inventory costs for the current order quantity.
b. Determine the economic order quantity (EOQ).
c. How many orders will be placed per year using the EOQ?
d. Determine the ordering, holding, and total inventory costs for the EOQ. How has ordering cost changed? Holding cost? Total inventory cost?

Solution We are given the following information:

annual demand: $D = 1200$ batteries per year

item cost: $c = \$28$ per battery

holding cost: $H = ic = 0.30(28) = \$8.40$ per battery per year

order cost: $S = \$20$ per order

current order quantity: $Q = 100$ batteries

a. The current ordering and holding costs are: $\frac{D}{Q}S + \frac{Q}{2}H = \frac{1200}{100}(20) + \frac{100}{2}(8.40) = 240 + 420 = \$660.$

b. The EOQ is $Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2\times1200\times20}{8.40}} = 75.6 \rightarrow 76$ batteries.

c. The company will place $\frac{D}{Q^*} = \frac{1200}{76} = 15.8$ orders per year.

d. The new ordering and holding costs are: $\frac{D}{Q^*}S + \frac{Q^*}{2}H = \frac{1200}{76}(20) + \frac{76}{2}(8.40) = 315.79 + 319.20 = \$634.99$. The company will save $25.01 by using the EOQ.

Q 12) Upon closer inspection, the supplier determines that the demand for batteries is normally distributed with mean 4 batteries per day and standard deviation 3 batteries per day. (The supplier is open 300 days per year.) It usually takes about 4 days to receive an order from the factory.
a. What is the standard deviation of usage during the lead time?

b. Determine the reorder point needed to achieve a service level of 95 percent.

c. What is the safety stock? What is the holding cost associated with this safety stock?

d. How would your analysis change if the service level changed to 98 percent?

Solution In addition to the information from the problem above, we are told:

average demand rate: $d = 4$ batteries per day

standard deviation of demand: $\sigma_d = 3$ batteries *per day*

lead time: $L = 4$ days

300 operating days per year

a. The standard deviation of demand during the lead time is $\sigma_L = \sigma_d\sqrt{L} = 3(\sqrt{4}) = 6$ batteries.

b. The reorder point is equal to the average demand expected during the lead time plus some safety stock. The service level is 95 percent. Examining the Table of Normal Demand Percentages (see the last page of this handout), we see that this corresponds to $z = 1.65$ (take the average of $z = 1.6$ and $z = 1.7$, since 95 percent is between 94.5 and 95.5 percent). In other words, we need to keep 1.65 standard deviations worth of extra inventory on hand to ensure that the probability of running out is less than 5 percent. Now we can figure out the reorder point: $R = dL + z\sigma_L = 4(4) + 1.65(6) = 16 + 9.9 = 25.9 \rightarrow 26$ batteries. We place an order for 76 batteries when the inventory level drops to 26 batteries.

c. The safety stock is the inventory in excess of the expected demand during the lead time. In other words, the safety stock is $26 - 16 = 10$ batteries. The associated holding cost is simply $10 \times H = 10 \times 8.40 = \$84.00$.

d. If the service level changes to 98 percent, then we must go back and determine a new z. Consulting the table, we see that 98 percent is between 97.7 and 98.2 percent. Averaging the two corresponding z values gives us $z = 2.05$ for a service level of 98 percent. Thus, the reorder point will be: $R = dL + z\sigma_L = 4(4) + 2.05(6) = 16 + 12.3 = 28.3 \rightarrow 29$ batteries. Place an order for 76 units when the inventory level drops to 29 units.

Q 13) Foster Drugs, Inc., handles a variety of health and beauty aid products. A particular hair conditioner product costs Foster $2.95 per unit. The annual holding cost rate is 20 percent. Using an EOQ model, they determined that an order quantity of 300 units should be used. The lead time to receive an order is one week, and the demand is normally distributed with a mean of 150 units per week and a standard deviation of 40 units per week.

a. What is the reorder point if the firm is willing to tolerate a 1-percent chance of a stockout during an order cycle?

b. What safety stock and annual safety stock cost are associated with your recommendation in part a?

c. Foster is considering making a transition to a periodic-review system in an attempt to coordinate ordering of some of its products. The review period would be two weeks and the delivery lead time would remain one week. What target inventory level would be needed to ensure the same 1-percent risk of stockout?

d. What is the safety stock associated with your answer to part c? What is the annual cost associated with holding this safety stock?

e. Compare your answers to parts b and d. If you were the manager of Foster Drugs, would you choose a continuous- or periodic-review system?

$\boxed{\textbf{Solution}}$ We are given the following information:

item cost: $c = \$2.95$ per unit

carrying "interest rate": $i = 0.20$ per unit per year

lead time: $L = 1$ week

average demand rate: $d = 150$ units per week

standard deviation of demand: $\sigma_d = 40$ units per week

current order quantity: $Q = 300$ units

a. A 1-percent stockout risk corresponds to a service level of 99 percent. Consulting the table, we see that this service level corresponds to $z = 2.3$. First, compute the mean and standard deviation of demand during the lead time: $dL = 150(1) = 150$ and $\sigma_L = \sigma_d\sqrt{L} = 40\sqrt{1} = 40$. Now we have all of the information necessary to determine the reorder point: $R = dL + z\sigma_L = 150 + 2.3(40) = 150 + 93.05 = 243.05 \rightarrow 243$ units.

b. The safety stock is 93 units, and the holding cost associated with the safety stock is $93 \times H = 93 \times ic = 93 \times 0.20(2.95) = \$54.87$.

c. We are told that the review period is two weeks so $P = 2$. Use this number, along with $L = 1$ to compute the mean and standard deviation of demand during the lead time and the review period: $d(P + L) = 150(2 + 1) = 450$ and $\sigma_{P+L} = \sigma_d\sqrt{P + L} = 40\sqrt{2 + 1} = 69.28$. The z value remains the same. The target inventory level is: $T = d(P + L) + z\sigma_{P+L} = 450 + 2.3(69.28) = 450 + 159.35 = 609.35 \rightarrow 610$ units.

d. The safety stock is 160 units (rounded up), and the holding cost associated with the safety stock is $160 \times H = 160 \times ic = 160 \times 0.20(2.95) = \$94.40$.

e. The periodic review method requires a larger safety stock, so it costs more: $94.40 - 54.87 = \$39.53$. However, using a fixed-period method facilitates ordering multiple items from a single supplier, so it is probably worth the extra cost.