Department of Computer Science and Engineering IIT Roorkee

CSN-373: Probability Theory for Computer Engineering

Mid-Term Examination

Time: 1 hour 30 minutes Date: September 10, 2024 Full Marks: 50

There are 5 questions. Answer all the questions

Q1. (a) If the density function of X is

$$f(x) = 1, \quad 0 < x < 1$$

determine $E[e^{tX}]$. Find the nth moment of X.

(b) Let X_i denote the percentage of votes cast in a given election that are for candidate i, and suppose that X_1 and X_2 have a joint density function

$$f_{X_1,X_2}(x,y) = \begin{cases} 3(x+y) & \text{if } x \geq 0, y \geq 0, 0 \leq x+y \leq 1 \\ 0 & \text{if otherwise} \end{cases}$$

(i) Find the marginal densities of X_1 and X_2;

(ii) Find $E[X_i]$ and $Var(X_i)$ for $i = 1, 2$. (5+5)

Q2. At least one-half of an airplane's engines are required to function in order for it to operate. If each engine independently function with probability p, for what values of p is a 4-engine plane more likely to operate than a 2-engine plane? (10)

Q3. A random variable is said to be normally distributed with parameters μ and σ^2, and we write $X \sim N(\mu, \sigma^2)$, if its density function is $f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < \infty$. Find the maximum likelihood estimators for μ and σ^2. Are the estimators so obtained unbiased? If not find related unbiased estimators. (10)

Q4. Let X be a random variable having the probability mass function $P\{X = 1\} = p$ and $P\{X = 0\} = 1 - p$. The entropy of X is denoted by $H(p)$. Determine the value of p for which $H(p)$ is maximum. Show all the relevant derivations. (10)

Q5. Suppose E and F are two events in a sample space. Prove or disprove the following statement

$$P(E \cap F) \geq P(E) + P(F) - 1.$$

Justify your answer. (10)

The End