CSN-212 Design and Analysis of Algorithms

[B] Mid Term Exam (Max Marks 100)

Name: **Roll Number:**

Instructions: Please read the following instructions **very carefully** before attempting the questions.

1. There are choices in questions carrying different marks. Among any set of choices, only the first UNCROSSED answer would be considered, and the rest ignored.
2. On the first left page describe the choice attempted for each question and its page numbers. Add page numbers to all used pages of your answer sheet, and clearly mark the END and Rough Work).
3. For any answer not written contiguously, clearly mention the next part "continue on page X".

Answer the following questions, carefully notice the choices amongst the subparts of a question.

1. Answer exactly **ONE** of the following **TWO** questions: **Maximum Marks: 20**

   (a) **[20 Marks] Smallest Triangle.** *Describe* and *analyze* an algorithm to compute the triangle with smallest perimeter among n points in a 2D plane.

   (b) **[10 Marks] Inversions.** *Describe* and *analyze* an algorithm to compute the number of inversions in an array $A[n]$, where an inversion is a pair of indices i, j where $i < j$ such that $A[i] > A[j]$.

2. Answer exactly **ONE** of the following **TWO** questions: **Maximum Marks: 20**

   (a) **[20 Marks] Wonderful chocolate.** Given a $a \times b$ grid where each cell can be filled with black or white such that no rectangle of both length and breadth > 1 has the same colour. *Describe* and *analyze* an algorithm to report the number of such colourings where $b \leq 2^{64}$ and $a \leq 6$.

   (b) **[10 Marks] Longest Increasing Subsequence.** Given an array of n characters, *design* and *analyze* an algorithm to report the length of its longest increasing subsequence.

3. Answer exactly **ONE** of the following **THREE** questions: **Maximum Marks: 30**

   (a) **[30 Marks] Non-dominating points in 3D.** Given a set of n points in a 3D space, *design* and *analyze* a divide and conquer algorithm to compute the set of non-dominating points, where a point (x_i, y_i, z_i) dominates (x_j, y_j, z_j) if all of $x_i \geq x_j, y_i \geq y_j$ and $z_i \geq z_j$.

   (b) **[20 Marks] Goldbach Conjecture Square.** Goldbach conjectured that every number $N > 2$ is a sum of two (possibly same) primes. If $G(N)$ is the number of such prime pairs for N, *design* and *analyze* an algorithm to compute all pairs such that $G(N) = G(a) + G(b)$ for $0 \leq a \leq b \leq N$.

   (c) **[10 Marks] Maximum and Minimum.** *Design* and *analyze* a divide an conquer algorithm to report the maximum and minimum of n numbers using $\leq \frac{3n}{2}$ comparisons.

4. Answer exactly **ONE** of the following **THREE** questions: **Maximum Marks: 30**

   (a) **[30 Marks] Triangulation problem.** A convex polygon P is a set of n points (x_i, y_i) in a 2D plane. A triangulation of P is a set of diagonals added to P such that each internal face of P becomes a triangle. The cost of a triangulation is the sum of lengths of all the added diagonals. *Design* and *analyze* and algorithm to compute the minimum weight triangulation of P.

   (b) **[20 Marks] String Matching with Wildcards.** Given a text T of length n and a pattern P of length m having wildcards $*$ which can be matched with any character of the text. Assume the character set to be of arbitrary size and the number of wildcards in P to be constant. *Design* and *analyze* an $O(m + n)$ time algorithm to find all occurrences of P in T.

   (c) **[10 Marks] Suffix Palindrome.** Given a string S of length n, *design* and *analyze* an algorithm to compute the longest palindrome of S which is also its suffix.