

HMM Practice Questions:

Assume you have three boxes, each containing a certain number of apples and oranges. At any point in time, you select a box at random, and then a fruit from that box (i.e., an apple or orange) and record your finding (**A** for apple and **O** for orange). You immediately replace the fruit so that the total number of apples and oranges stays the same over time and repeat the process. Unfortunately, in between you forgot to write down the boxes you chose and simply have an account of apples and oranges. Assume the following quantity of fruits:

- **Box 1:** 2 apples, 2 oranges
- **Box 2:** 3 apples, 1 orange
- **Box 3:** 1 apple, 3 oranges

1. Draw a Hidden Markov Model to represent this problem. Show a state diagram in addition to two-dimensional parameter arrays **a** (for transitions) and **b** (for emission probabilities).
2. Compute the probability of seeing box sequence $\pi = (1,1,3,3,2)$ and fruit sequence $\mathbf{x} = (\mathbf{A},\mathbf{A},\mathbf{O},\mathbf{O},\mathbf{A})$. Show your work.
3. Compute the optimal set of boxes corresponding to the fruit sequence given in the previous problem (π^*). That is, which box was each piece of fruit most likely to be selected from?
4. How much better is your path than that given in Problem 2? Compute this value by using a log-odds ratio given as:

$$\log \frac{P(\pi^* | \mathbf{x})}{P(\pi | \mathbf{x})}$$

where the denominator is using the path from problem 2.