3.4.2025

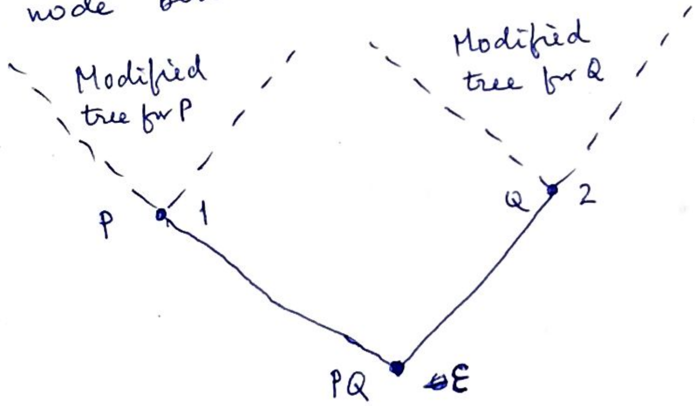
Definition (The construction-tree of a term)

Each node in the tree has two labels: a position ad a subterm.
The tree is defined for an arbitrary term M as follows:

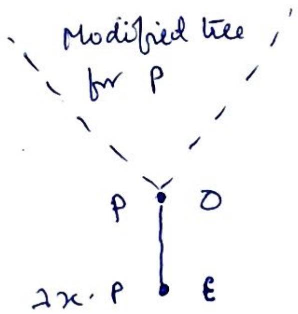
(i) If $M \equiv x$, its tree is a single node labelled with x and the empty position (denoted by ε - epsilon).

$$x \bullet \varepsilon$$

(ii) If $M \equiv PQ$, its tree is obtained by first concatenating "1" onto the left end of each position-label in the tree for P, then concatenating "2" onto the left end of each position-label in the tree for Q, and then placing an extra node beneath the two modified trees, as shown below:



(iii) If $M \equiv \lambda x.P$, its tree is obtained by first concatenating "0" onto the left end of each position-label in the tree for P, and then placing an extra node beneath the modified tree, as shown below



Exercise:

Construct the construction-tree for $M \equiv (\lambda x . yx)(\lambda z . x(yx))$

$M \equiv PQ$ where $P \equiv \lambda x . yx$ $\quad Q \equiv \lambda z . x(yx)$

$P \equiv \lambda x . P'$ where $P' = yx$

$P' = P'' Q''$ where $P'' \equiv y$ $\quad Q'' \equiv x$

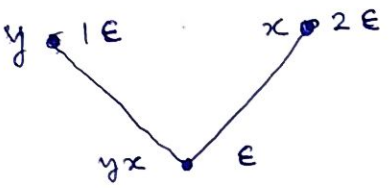
Now P'' and Q'' cannot be split any further.

1. Draw the trees for P'' and Q''

$y \bullet \varepsilon$ $\qquad$ $x \bullet \varepsilon$

(P'') tree $\qquad$ (Q'') tree

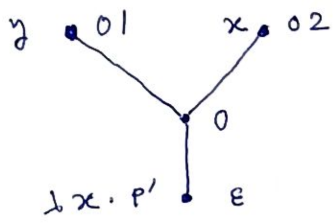
2. The tree for P' (obtained from step '1' and the rule for PQ)



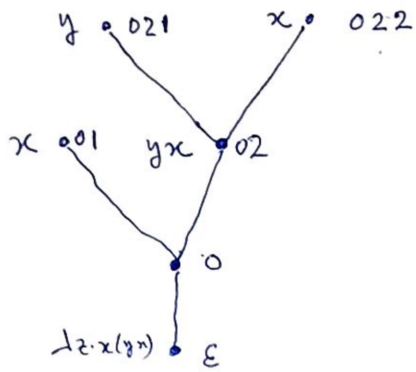
since $\alpha . \varepsilon = \varepsilon . \alpha = \alpha$ for any α
so we can label
1ε as 1 and 2ε as 2

(P'-tree)

3. The tree for P
(obtained from "2" and the rule for $\lambda x . M$)



4. The tree for Q



5. The tree for M:
(obtained from "3" and "4" and the rule for PQ)