PHN – 006: Quantum & Statistical Mechanics

Assignment - 2

Deadline for Submission: June 07, 2023, 11:59 pm.

1. Calculate the probability current density j(x) for the wave function.
$$\Psi(x) = u(x)\,exp[i\varphi(x)],$$
where u, φ are real.
2. Find the probability current density **j(r, t)** associated with the charged particle of charge e and mass m in a magnetic field of vector potential **A** which is real.
3. A particle of mass m trapped in the potential V(x) = 0 for $-a \leq x \leq a$ and V(x) = ∞ otherwise. Evaluate the probability of finding the trapped particle between x = 0 and x = a/n when it is in the n^{th} state.
4. A harmonic oscillator is in the ground state. (i) Where is the probability density maximum? (ii) What is the value of maximum probability density?
5. Show that the zero-point energy of (1/2) ħw of a linear harmonic oscillator is a manifestation of the uncertainty principle.
6. Electrons with energies 1 eV are incident on a barrier 5 eV high 0.4 nm wide.
(i) Evaluate the transmission probability. What would be the probability (ii) if the height is doubled, (iii) if the width is doubled?
7. At what temperature would we find about half as many hydrogen atoms in the first excited state as in the ground state.
8. Classify the following particles and atoms can or cannot exhibit Bose-Einstein Condensation, even in principle.
    a) Neutron
    b) Positron
    c) Photon
    d) α-particle
    e) Hydrogen Molecule
    f) $^{23}Na_{11}$
    g) $^{30}K_{19}$
9. Four identical particles can be in any five states. What are the number of possible ways of distributing them in various states according to Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac statistics.
10. There are 2.54×10^{22} free electrons per cm^3 in Sodium. Calculate its Fermi energy and Fermi Temperature ($h = 6.63 \times 10^{-34}$ J-sec, $m_e = 9.31 \times 10^{-31}$, $K_B = 1.38 \times 10^{-23}$ J/K and $1eV = 1.6 \times 10^{-19}$ J)
11. The total power emitted by a spherical blackbody of radius R at a temperature T is P1. Let P2 be the total power emitted by another spherical blackbody of Radius 16R and at Temperature T/8. The ratio P1/P2 is _______________.
12. The free energy for a photon gas $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant. The entropy S and pressure of photon gas are to be calculated.
13. The radial wave function for H- like atom is given by $R = [2/a_0^{3/2}]\exp[-r/a_0]$. Calculate the ground state energy of that electron.
14. For a 2p electron calculate the magnitude of the orbital angular momentum. Find the kinetic energy after calculating the most probable distance of this electron from the nucleus. Take $\Psi_{2p} = A\, r^*\exp(-r/2a_0)$, where A is a constant.
15. Calculate the energy splitting of the hyperfine splitting of the level with spectroscopic term $^2P_{3/2}$.
16. Verify the mass of the electron by the data of the experiment in which on applying magnetic field 2T splits the 400nm spectral line into Zeeman components with separation of 0.01491nm.
17. If in a chemical composition the two isotopes are present such that their atomic masses are in ratio 2:1, what can you say about spectral lines associated with them. Explain your answer.
18. If the partition function of a system is given by $Z = \exp[8\pi^5VK^3t^3/45h^3c^3]$. The specific heat of the system depends on temperature as $(T)^n$. Calculate the value of n.
19. For a canonical ensemble there are two energy levels such that the heat reservoir is at 500K. The separation between the two levels is 0.25 eV. Calculate the temperature at which the probability that the system is in higher energy level is ¼. [Take lower level to be E = 0].
20. A system, having N number of non-interacting particles which are distinguishable all having spin 1, is in thermal equilibrium. Calculate the entropy of the particle for $N = 2 \times 10^{25}$.