Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)

Exercise – 4

1. Find an optimal solution to the following linear program:
   Min $z = x_1 + 2x_2 + 3x_3$, s/t $x_1 + 3x_2 + 6x_3 = 6$, $x_1\ x_2, x_3 \ge 0$.

2. Use the Simplex method to solve
   Max $z = x_1 + x_2$, s/t $x_1 + 2x_2 \le 3$, $2x_1 + x_2 \le 3$, $x_1, x_2 \ge 0$,
   Plot the feasible region using x_1 and x_2 as coordinates. Follow the solution steps of the simplex method graphically by interpreting the shift from one b.f.s. to the next in the feasible region. (1, 1)

3. Use the simplex method to solve:
   Min $z = -x_1 - x_2$, s/t $x_1 + 2x_2 \le 3$, $x_1 + x_2 \le 2$, $3x_1 + 2x_2 \le 6$, $x_1, x_2 \ge 0$.
   Find an alternate optimal solution if one exists. (1, 1) ; (2, 0)

4. Use the simplex method to solve:
   Max $z = 4x_1 + 3x_2 + 2x_3 + x_4$
   Subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \le 12$, $2x_1 + x_2 + 3x_3 + 2x_4 \le 12$
   $x_1, x_2, x_3, x_4 \ge 0$ ($x_1 = 4 = x_2$)
   Is the optimal solution unique? Why or why not?

5. Solve by the simplex method:
   (i) Min $z = -5x_1 - 3x_2$
   Subject to $x_1 + x_2 + x_3 = 2$, $5x_1 + 2x_2 + x_4 = 10$
   $3x_1 + 8x_2 + x_5 = 12$, $x_1, x_2, x_3, x_4, x_5 \ge 0$
   Verify your result graphically.
   (ii) Max $z = 3x_1 + x_2 + 2x_3$
   Subject to $12x_1 + 3x_2 + 6x_3 + 3x_4 = 9$, $8x_1 + x_2 - 4x_3 + 2x_5 = 10$
   $3x_1 - x_6 = 0$, $x_1, \ldots, x_6 \ge 0$. (3 iteration; (0, 0, 3/2, 0, 8, 0))

6. Use the simplex method to show that the following problem has unbounded solution:
   Max $z = x_1 + x_2$, s/t $3x_1 - 4x_2 \ge -3$, $x_1 - x_2 - x_3 = 0$, $x_1, x_2, x_3 \ge 0$.

7. Solve the following systems of equations using simplex method:
   (i) $2x_1 + x_2 - x_3 = 1$, $-2x_1 + 2x_2 - x_3 = -2$, $x_1 + x_3 = 3$, $x_i \ge 0$. (1, 1, 2)
   (ii) $x_1 - x_2 + x_3 = 1$, $x_1 + x_3 = 2$, $2x_1 + x_2 + 2x_3 = 3$. (Inconsistent)

8. Describe the big M and the two phase methods. Which should be preferred and why?
   Solve the following problems by both these methods:
   (i) Min $z = 3x_1 + 5x_2$, s/t $x_1 + 3x_2 \ge 3$, $x_1 + x_2 \ge 2$, $x_1, x_2 \ge 0$. (3/2 , 3/2)
   (ii) Max $z = 4x_2 - 3x_1$, s/t $x_1 - x_2 \ge 0$, $2x_1 - x_2 \ge 2$, $x_1, x_2 \ge 0$. (2, 2)
   (iii) Max $z = 5x_1 - 3x_2 + 4x_3$, s/t $x_1 - x_2 \le 1$, $-3x_1 + 2x_2 + 2x_3 \le 1$, $4x_1 - x_3 = 1$,
   $x_2 \ge 0_1$ x_1 unrestricted. (3/5 , 0, 7/5)
   (iv) Max $z = 3x_1 + 2x_2$, s/t $2x_1 + x_2 \le 2$, $3x_1 + 4x_2 \ge 12$, $x_1, x_2 \ge 0$.
   (infeasible)

Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)

Exercise-5

1. Write the duals of the following problems:

   (i) Max $z = x_1 - 2x_2 + 4x_3 - 3x_4$, s/t $x_1 + x_2 - 3x_3 + x_4 = 9$,
   $3x_1 + 5x_2 + 2x_3 - 7x_4 \le 5$, $x_1 - 3x_2 + 5x_4 \ge 8$, $x_1, x_2, x_3, x_4 \ge 0$

   (ii) Min $z = 2x_1 + x_2 + x_3$, s/t $x_1 + x_2 - x_3 \ge 1$, $-2x_1 + x_3 \le 0$, $x_1 - x_2 + x_3 = 2$,
   $x_1 \ge 0, x_2 \le 0$

2. Write the dual of problem 1 (ii) in the form in which
   (a) all the dual variables are nonnegative (b) all the dual constraints are of $\le$ type.
3. Show that dual of the dual is the primal problem.
4. If both the primal and the dual problems are feasible, then show that both have optimal solutions.
5. Show that the following problem and its dual are infeasible.
   Max $z = 8x_1 + 6x_2$, s/t $2x_1 - x_2 \ge 2, -4x_1 + 2x_2 \ge 1$, $x_1, x_2 \ge 0$
6. Write the dual of the problem: Max $z = x_1 + 2x_2 + x_3$,
   s/t $x_1 + x_2 - x_3 \le 2$, $x_1 - x_2 + x_3 = 1$, $2x_1 + x_2 + x_3 \ge 2$, $x_1 \ge 0$, $x_2 \le 0$
   and using the duality theory show that maximum of z can not exceed one.
7. It is given that the LPP: $\max z = p^T x$, s/t $Ax \le b$, $x \ge 0$ has on optimal solution. Suppose the requirement vector b is changed to another vector d. If the problem so obtained is feasible, then prove that it has an optimal solution.
8. Sow by inspection that the dual of the problem:
   Max $z = -2x_1 + 3x_2 + 5x_3$, s/t $x_1 - x_2 + x_3 \le 15$, $x_1, x_2, x_3 \ge 0$
   is infeasible. What can you say about the solution of the primal?
9. (i) Solve the following problem graphically. Write its dual. Then using the complementary slackness theorem obtains the solution of the dual problem.

   Maximize $z = 2x_1 + 3x_2$
   subject to $x_1 + x_2 \le 3$, $2x_1 + 3x_2 \ge 3$, $-x_1 + x_2 \le 0$, $x_1 \le 2$, $x_1, x_2 \ge 0$.

   (ii) Write the dual of the problem:

   Minimize $z = x_1 + 2x_2 + 3x_3 + 4x_4$,
   subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \ge 30, 2x_2 + 3x_3 + 2x_4 \ge 40$ $x_1, x_2, x_3, x_4 \ge 0$.
   Solve the dual graphically. Then using the complementary slackness theorem, obtain the solution of the above problem.

10. Describe the dual simplex method. Using it, solve the following problems:

    (i) *Min* $z = 2x_1 + x_2$, s/t $3x_1 + x_2 \ge 3$, $4x_1 + 3x_2 \ge 6, x_1 + 2x_2 \le 3$,
    $x_1, x_1, x_2 \ge 0$. (3/5, 6/5)

    (ii) Min $z = x_1 + 4x_2 + 3x_4$, s/t $x_1 + 2x_2 - x_3 + x_4 \ge 3$ (0, 14/9, 1/9, 0)
    s/t $x_1 + 2x_2 - x_3 + x_4 \ge 3, -2x_1 + x_2 + 4x_3 + x_4 \ge 2$, $x_1, x_2, x_3, x_4 \ge 0$.

11. Write the duals of the problems 8 (i) & (iii) of Ex. 4 and from the optimal tables of the primal problems obtained by the big M method, write the optimal solution of the dual problems.