Lecture 9

Syntax Analysis

Awanish Pandey

Department of Computer Science and Engineering
Indian Institute of Technology
Roorkee

February 7, 2025

Take aways from the last class

- Calculation of *follow* set

Take aways from the last class

- Calculation of *follow* set
- Creation of parse table

Take aways from the last class

- Calculation of *follow* set
- Creation of parse table
- Error recovery technique.

Take aways from the last class

- Calculation of *follow* set
- Creation of parse table
- Error recovery technique.
- Bottom up parsing

Take aways from the last class

- Calculation of *follow* set
- Creation of parse table
- Error recovery technique.
- Bottom up parsing
- Shift-Reduce parser

Example

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$
E.+id	shift

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$
E.+id	shift
E+.id	shift

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$
E.+id	shift
E+.id	shift
E+id.	reduce $E \rightarrow id$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$
E.+id	shift
E+.id	shift
E+id.	reduce $E \rightarrow id$
E+E.	reduce $E \rightarrow E + E$

Example

- Assume grammar is $E \rightarrow E + E | E * E | id$
- Parse id*id+id

String	Action
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce $E \rightarrow id$
E*E.+id	reduce $E \rightarrow E * E$
E.+id	shift
E+.id	shift
E+id.	reduce $E \rightarrow id$
E+E.	reduce $E \rightarrow E + E$
E.	ACCEPT

Shift reduce parsing

Shift reduce parsing

- Symbols on the left of "." are kept on a stack

Shift reduce parsing

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."

Shift reduce parsing

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack

Shift reduce parsing

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack

Shift reduce parsing

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack
- The most important issue: when to shift and when to reduce

Shift reduce parsing

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack
- The most important issue: when to shift and when to reduce
- Reduce action should be taken only if the result can be reduced to the start symbol

Bottom up parsing

- A more powerful parsing technique

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible

Bottom up parsing

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible
- Allows better error recovery

Issues in bottom up parsing

Issues in bottom up parsing

- How do we know which action to take

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?
- Sometimes parser can reduce but it should not: $X \rightarrow \epsilon$ can always be reduced!

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?
- Sometimes parser can reduce but it should not: $X \rightarrow \epsilon$ can always be reduced!
- Given stack δ and input symbol 'a', should the parser

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?
- Sometimes parser can reduce but it should not: $X \rightarrow \epsilon$ can always be reduced!
- Given stack δ and input symbol 'a', should the parser
  - Shift a onto stack (making it δa)

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?
- Sometimes parser can reduce but it should not: $X \rightarrow \epsilon$ can always be reduced!
- Given stack δ and input symbol 'a', should the parser
  - Shift a onto stack (making it δa)
  - Reduce by some production $A \rightarrow \beta$ assuming that stack has form $a\beta$ (making it aA)

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - which production to use for reduction?
- Sometimes parser can reduce but it should not: $X \rightarrow \epsilon$ can always be reduced!
- Given stack δ and input symbol 'a', should the parser
  - Shift a onto stack (making it δa)
  - Reduce by some production $A \rightarrow \beta$ assuming that stack has form $a\beta$ (making it aA)
  - How to keep track of length of β?

Handle

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs
- **Handle pruning**: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs
- **Handle pruning**: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A
- A right most derivation in reverse can be obtained by handle pruning

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs
- **Handle pruning**: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A
- A right most derivation in reverse can be obtained by handle pruning
- Handles always appear at the top of the stack and never inside it. This makes stack a suitable data structure

Handle

- A string that matches right hand side of a production and whose replacement gives a step in the reverse right most derivation
- If $S \rightarrow^{rm*} aAw \rightarrow^{rm} a\beta w$ then β (corresponding to production $A \rightarrow \beta$) in the position following 'a' is a handle of $a\beta w$. The string w consists of only terminal symbols
- We only want to reduce handle and not any rhs
- **Handle pruning**: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A
- A right most derivation in reverse can be obtained by handle pruning
- Handles always appear at the top of the stack and never inside it. This makes stack a suitable data structure
- Bottom up parsing is based on recognizing handles

Conflicts

Conflicts

- The general shift-reduce technique is:

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- However, what happens when there is a choice

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- However, what happens when there is a choice
  - What action to take in case both shift and reduce are valid. **Shift-Reduce conflict**

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- However, what happens when there is a choice
  - What action to take in case both shift and reduce are valid. **Shift-Reduce conflict**
  - Which rule to use for reduction if reduction is possible by more than one rule? **Reduce-Reduce conflict**

Conflicts

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- However, what happens when there is a choice
  - What action to take in case both shift and reduce are valid. **Shift-Reduce conflict**
  - Which rule to use for reduction if reduction is possible by more than one rule? **Reduce-Reduce conflict**
- Conflicts come either because of ambiguous grammars or parsing method is not powerful enough

Shift-Reduce conflict

Shift-Reduce conflict

- Consider the grammar $E \rightarrow E + E | E * E | id$

Shift-Reduce conflict

- Consider the grammar $E \rightarrow E + E | E * E | id$
- input $id + id * id$

Shift-Reduce conflict

- Consider the grammar $E \rightarrow E + E | E * E | id$
- input $id + id * id$

stack	input	action
E+E	*id	reduce E→E+E
E	*id	shift
E*	id	shift
E*id		reduce E→id
E*E		reduce E→E*E
E		

Shift-Reduce conflict

- Consider the grammar $E \rightarrow E + E | E * E | id$
- input $id + id * id$

stack	input	action
E+E	*id	reduce E→E+E
E	*id	shift
E*	id	shift
E*id		reduce E→id
E*E		reduce E→E*E
E		

stack	input	action
E+E	*id	shift
E+E*	id	shift
E+E*id		reduce E→id
E+E*E		reduce E →E*E
E+E		reduce E→ E+E
E		

Reduce-Reduce conflict

Reduce-Reduce conflict

- Consider the grammar $M \rightarrow R + R | R + c | R$
  $R \rightarrow c$

Reduce-Reduce conflict

- Consider the grammar $M \rightarrow R + R | R + c | R$
  $R \rightarrow c$
- input $c + c$

Reduce-Reduce conflict

- Consider the grammar $M \rightarrow R + R | R + c | R$
  $R \rightarrow c$
- input $c + c$

stack	input	action
	c+c	shift
c	+c	reduce R→C
R	+c	shift
R+	c	shift
R+c		reduce R→c
R+R		reduce M→ R+R
M		

Reduce-Reduce conflict

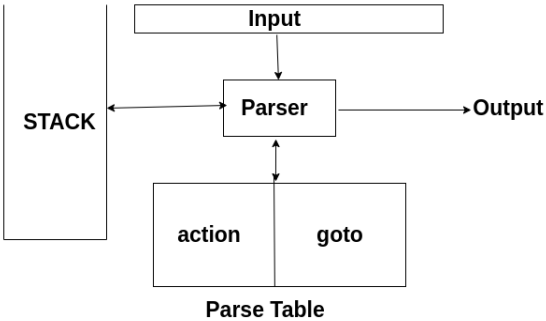
- Consider the grammar $M \rightarrow R + R | R + c | R$
  $R \rightarrow c$
- input $c + c$

stack	input	action
	c+c	shift
c	+c	reduce R→C
R	+c	shift
R+	c	shift
R+c		reduce R→c
R+R		reduce M→ R+R
M		

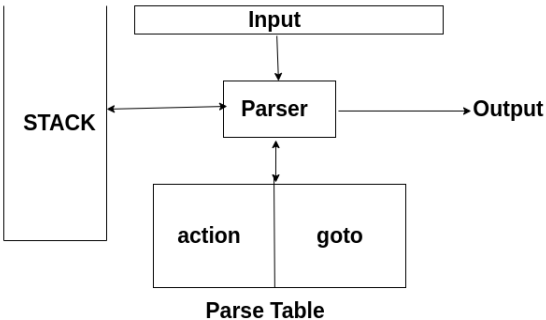
stack	input	action
	c+c	shift
c	+c	reduce R→C
R	+c	shift
R+	c	shift
R+c		reduce M→R+c
M		

LR Parsing

LR Parsing

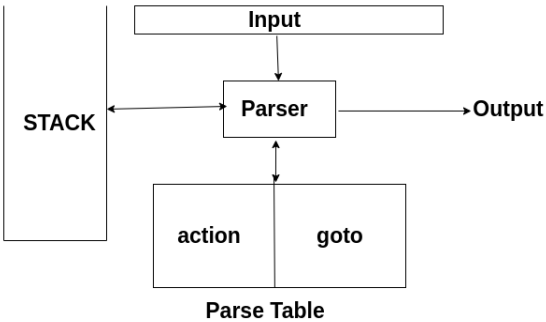


LR Parsing



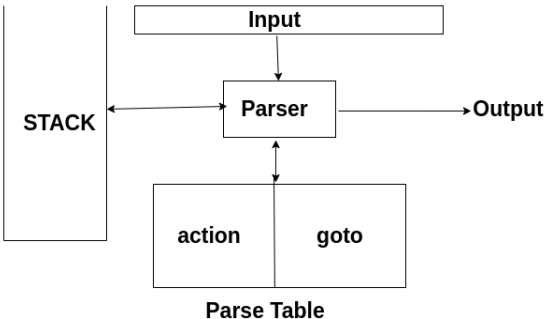
- Input contains the input string.

LR Parsing



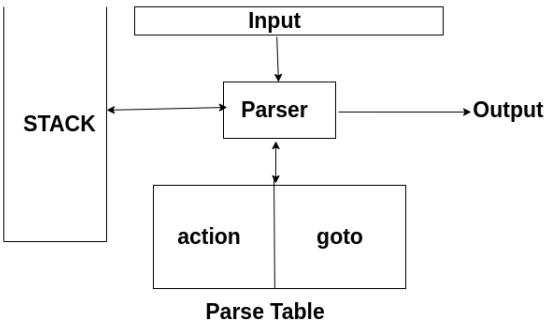
- Input contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2 \cdots X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.

LR Parsing



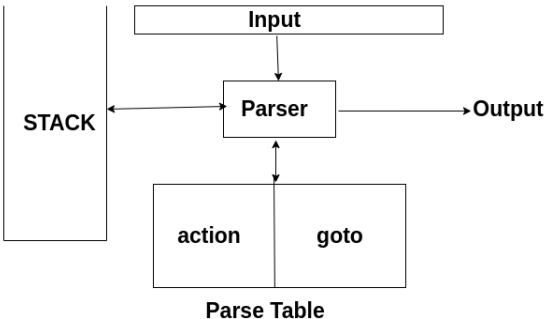
- Input contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2 \cdots X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.
- Tables contain **action** and **goto** parts.

LR Parsing



- Input contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2 \cdots X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.
- Tables contain **action** and **goto** parts.
- action table is indexed by state and terminal symbols.

LR Parsing



- Input contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2 \cdots X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.
- Tables contain **action** and **goto** parts.
- action table is indexed by state and terminal symbols.
- goto table is indexed by state and non terminal symbols.

Actions in an LR (shift reduce) parser

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol
- Action $[S_i, a_j]$ can have four values

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol
- Action $[S_i, a_j]$ can have four values
  1. shift a_j to the stack and goto state S_k

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol
- Action $[S_i, a_j]$ can have four values
  1. shift a_j to the stack and goto state S_k
  2. reduce by a rule

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol
- Action $[S_i, a_j]$ can have four values
  1. shift a_j to the stack and goto state S_k
  2. reduce by a rule
  3. accept

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_j is current input symbol
- Action $[S_i, a_j]$ can have four values
  1. shift a_j to the stack and goto state S_k
  2. reduce by a rule
  3. accept
  4. error

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$

  Then the configuration becomes

  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$

  Then the configuration becomes

  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$
- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n \$$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$

  Then the configuration becomes

  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$

  Then the configuration becomes

  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n \$$

  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$

Configurations in LR parser

$Stack : S_0X_1S_1X_2 \cdots X_mS_m \quad Input : a_ia_{i+1} \cdots a_n\$$

- If $action[S_m, a_i] = shift\ S$
  Then the configuration becomes
  $Stack : S_0X_1S_1 \cdots X_mS_ma_iS \quad Input : a_{i+1} \cdots a_n\$$
- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$
  Then the configuration becomes
  $Stack : S_0X_1S_1 \cdots S_{m-r}X_{m-r}S \quad Input : a_ia_{i+1} \cdots a_n\$$
  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$
- If $action[S_m, a_i] = accept$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$
- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n \$$
  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$
- If $action[S_m, a_i] = accept$
  Then parsing is completed. *HALT*

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$
- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n \$$
  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$
- If $action[S_m, a_i] = accept$
  Then parsing is completed. *HALT*
- If $action[S_m, a_i] = error$

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

- If $action[S_m, a_i] = shift\ S$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots X_m S_m a_i S \quad Input : a_{i+1} \cdots a_n \$$
- If $action[S_m, a_i] =$ reduce $A \rightarrow \beta$
  Then the configuration becomes
  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n \$$
  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$
- If $action[S_m, a_i] = accept$
  Then parsing is completed. *HALT*
- If $action[S_m, a_i] = error$
  Then invoke error recovery routine

LR parsing Algorithm

Algorithm $\mathrm{LR}_{parsing}{}_{Algorithm}$

1: Initial State *Stack* : S_0 *Input* : $w\$$
2: **for** *TRUE* **do**
3: **if** $action[S, a] = shift \quad S'$ **then**
4: push(a); push(S'); ip++
5: **else if** action[S,a] = reduce $A \rightarrow \beta$ **then**
6: pop $(2 * |\beta|)$ symbols;
7: push(A); push (goto[S",A]) {S" is the state after popping symbols}
8: **else if** action[S,a] = accept **then**
9: exit
10: **else**
11: error()
12: **end if**
13: **end for**

Example

- Consider the grammar

Example

- Consider the grammar $E \to E + T | T \qquad T \to T * F | F \qquad F \to (E) | id$

state	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Parse id + id * id

stack	input	action
0	id + id * id$	s5
0id5	+ id*id$	reduce F→id
0F3	+id*id$	reduce T→ F
0T2	+id*id$	reduce E→T
0E1	+id*id$	shift 6
0E1+6	id*id $	shift 5
0E1+6id5	*id $	reduce F→id
0E1+6F3	*id $	reduce T→F
0E1+6T9	*id $	shift 7
0E1+6T9*7	id $	shift 5
0E1+6T9*7id5	$	reduce F → id
0E1+6T9*7F10	$	reduce T →T*F
0E1+6T9	$	reduce E→E+T
0E1	$	acc