Project Write-up
PHN-006: Quantum Mechanics and Statistical Mechanics

MacColl-Hartman Effect: Is Quantum Tunneling Superluminal?

-Sarthak Shaurya
22116082
O7,ECE

1 Why I chose this topic and How is it relevant?

In the mesmerizing realms of Quantum Mechanics, reality takes on a new form. Challenging classical notions, it offers unparalleled accuracy and insights into the fundamental nature of the universe. As we explore the world of probabilities and uncertainties in Quantum Mechanics, we unravel its elegance and confront the unanswered questions that lie at its core. Quantum Mechanics beckons us to embark on a journey of wonder and discovery.

PHN-006 introduces us to many beautiful concepts in the Quantum world which appear intriguing and captivating. One such topic that fascinated me was Quantum Tunneling. Despite it appearing to be a weird phenomenon at first, I occurred to learn how important it is because of its applications in the day-to-day world- from flash memories to transistors, diodes, and satellite TV. As an ECE student, it was too exciting to learn the importance of tunneling in transistors, diodes, etc.

As I delved more into the topic, I learned about various paradoxes like the Hartman effect which questioned our understanding of the relativity theory. In the course of Quantum Mechanics and Statistical Mechanics, exploring intriguing phenomena and their implications is essential for gaining a deeper understanding of the quantum world. So, I decided to choose this topic which would allow me to learn more about the two very interesting topics of physics- Quantum tunneling and relativity, and uncover whether tunneling is really superluminal(faster than light) and to explore its implications on our understanding of spacetime and causality.

2 Introduction

Hartman effect challenges our conventional understanding of quantum tunneling by raising the question: Can quantum particles tunnel through a barrier faster than light, defying the limitations imposed by relativity?

The MacColl-Hartman effect, named after physicists E. C. G. Stueckelberg and J. G. MacColl, and later studied by Thomas F. Hartman, examines the behavior of quantum particles when confronted with a potential barrier. Quantum tunneling, a cornerstone of quantum mechanics, allows particles to pass through barriers that would be classically impenetrable. However, this effect is typically understood to occur with tunneling times that are on the order of the time it takes light to traverse the barrier.

Through this write-up, I aim to provide a comprehensive analysis of the Hartman effect, examining both the theoretical foundations and experimental observations associated with this intriguing phenomenon without going into rigorous mathematical analysis which is beyond the scope of our syllabus.

3 The Tunnel Effect

3.1 Classical Description

Imagine a minuscule entity approaching a barrier from the left such that its energy is less than the potential of the barrier. We would concur that the particle lacks the necessary vigor to surmount the barrier and would inevitably rebound upon collision.

Consider a particle with energy E approaching a barrier without any potential energy ($V = 0$). The total energy of the particle, which includes its kinetic energy (K) and potential energy (V)), can be expressed as

$$E = K + V$$

In terms of classical mechanics, we can relate the kinetic energy to the particle's momentum (p) using the equations

$$K = \frac{p^2}{2m}$$

where m represents the particle's mass and v denotes its velocity. Substituting these expressions into the total energy equation, we find

$$E = \frac{1}{2}mv^2 + V$$

To determine the velocity of the particle, we can solve for v, yielding $v = \sqrt{\frac{2}{m}(E - V)}$ Similarly, the momentum of the particle is given by $p = \sqrt{2m(E - V)}$ We can divide the scene into two regions: the left region outside the barrier, where the particle is reflected, and the right region outside the barrier, where the particle can pass through. Two boundary conditions arise: $E > V$ and $V > E$.

Let's first focus on the situation where $E > V$. In this case, the particle freely scatters through the barrier from the left side to the right side. There is no tunneling effect, and we can describe the particle's behavior in both the left and right regions. The momentum of the particle is always positive, and there are no restrictions on its velocity, which remains real and positive outside the barrier.

However, if $V > E$, the particle's behavior differs according to classical mechanics. In the realm of classical mechanics, the motion of a particle with energy E encountering a step potential with a height $V > E$ is deemed impossible. This is due to the fact that such a scenario would result in an imaginary velocity, $v = \sqrt{\frac{2}{m}(E - V)} < 0$, which is not feasible within the confines of the potential barrier.

Quantum mechanics, to our amazement, reveals a remarkable aspect: despite its minute magnitude, there exists a finite probability for the particle to be detected on the right side of the barrier. This intriguing phenomenon, known as tunneling, represents a profound departure from classical expectations. Indeed, tunneling holds significant importance in various scenarios, permeating through the fabric of quantum mechanics.

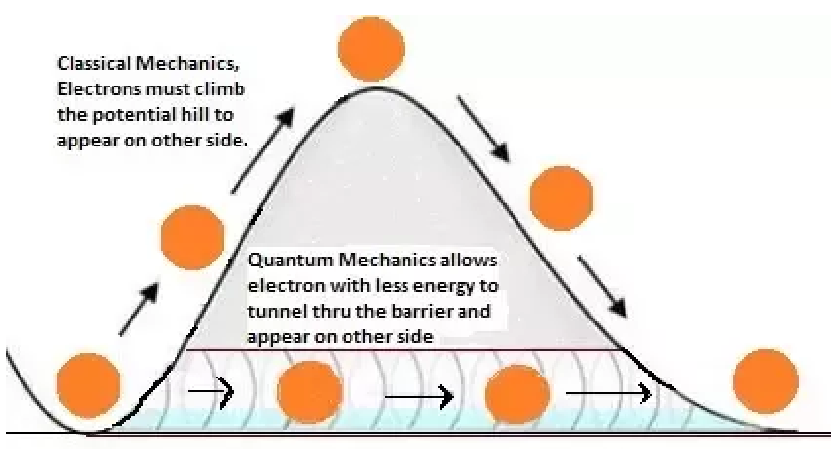


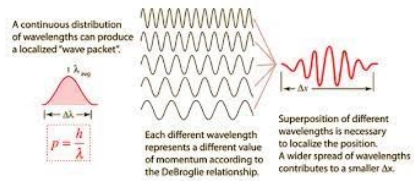
Figure 1: Comparison of Classical and Quantum Mechanics.

3.2 Wave-Particle Duality and The Uncertainty Principle

Quantum tunneling challenges classical rejection using the uncertainty principle and wave-particle duality. Particles, like electrons, lack well-defined trajectories and possess both particle and wave characteristics. This wave nature enables them to extend through space, surpassing energy barriers that classical energy alone cannot overcome. The wavefunction extends into forbidden regions, offering a non-zero probability of finding the particle on the other side. These principles defy classical expectations, allowing particles to pass through barriers despite insufficient classical energy. Quantum mechanics provides a comprehensive description, encompassing phenomena like quantum tunneling beyond classical mechanics' framework.

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

(a)



(b)

Figure 2: Uncertainty Principle

3.3 Tunneling: Explained

The history of quantum tunneling traces back to the discovery of natural radioactivity in 1896. It was not until 1928 that George Gamow, Ronald Gurney, and Edward Condon independently solved the theory of alpha decay via tunneling, showcasing particles escaping from a nucleus against high energy barriers.In the 1950s, attempts to understand tunneling in metal-semiconductor systems began, but it was in 1957 when Esaki discovered the tunnel diode, providing conclusive evidence of electron tunneling in solids. Giaever's experiment in 1960 on superconductors added to the understanding of tunneling, including measuring energy gaps in these materials.

To derive the transmission probability in tunneling, we start with the time-independent Schrödinger equation for a particle incident on a potential barrier:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0$$

where ψ is the wave function, m is the mass of the particle, E is the total energy of the particle, and $V(x)$ is the potential energy of the barrier.

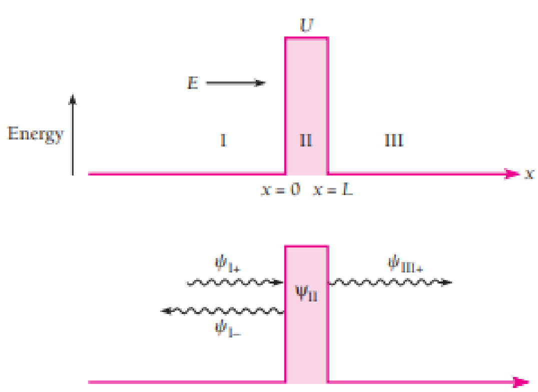


Figure 3: Partcle encountering a Barrier

Assuming the potential barrier has a finite width and the potential is constant inside the barrier, we can write the wave function in different regions as follows:

Region I (Left side of the barrier, $x < 0$):

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

Region II (Inside the barrier, $0 \leq x \leq d$):

$$\psi_{II} = Ce^{qx} + De^{-qx}$$

Region III (Right side of the barrier, $x > d$):

$$\psi_{III} = Fe^{ikx}$$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $q = \sqrt{\frac{2m(E-V)}{\hbar^2}}$ are the wave vectors in the respective regions.

To determine the transmission probability, we need to find the coefficients T and R for transmission and reflection, respectively.

Applying the boundary conditions at $x = 0$ and $x = d$, we obtain the following equations:

1. Continuity of the wave function:

$$\psi_I(0) = \psi_{II}(0) \quad (1)$$

$$\psi_{II}(d) = \psi_{III}(d) \quad (2)$$

2. Continuity of the derivative of the wave function:

$$\left.\frac{d\psi_I}{dx}\right|_{x=0} = \left.\frac{d\psi_{II}}{dx}\right|_{x=0} \quad (3)$$

$$\left.\frac{d\psi_{II}}{dx}\right|_{x=d} = \left.\frac{d\psi_{III}}{dx}\right|_{x=d} \quad (4)$$

By substituting the respective wave functions and their derivatives into the boundary condition equations (1)-(4), we can solve for the coefficients A, B, C, D, E, and F.

After solving the system of equations, we find the transmission probability T as the square of the magnitude of the coefficient F:

$$T = |F|^2$$

Approximately transmission probability is given as: $T = e^{-2qL}$.

The transmission probability gives the likelihood of the particle passing through the potential barrier.

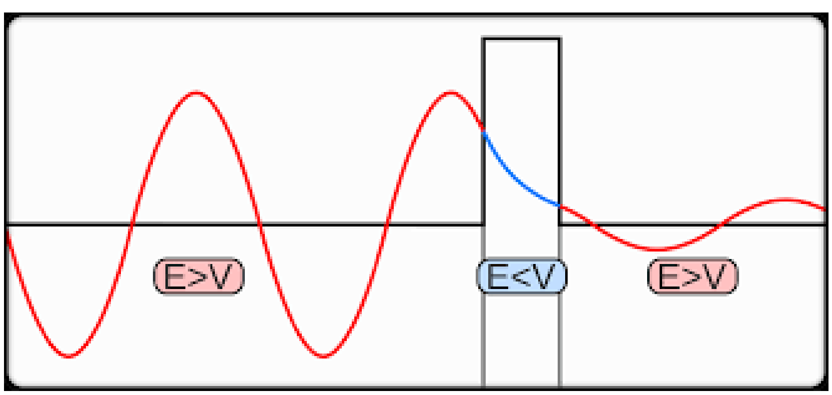
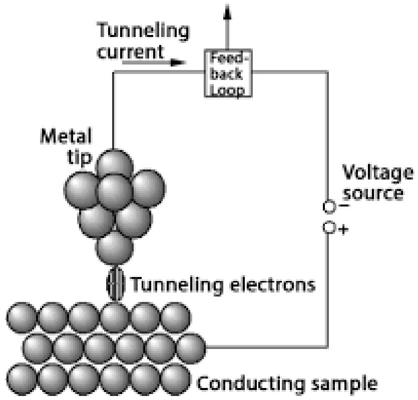
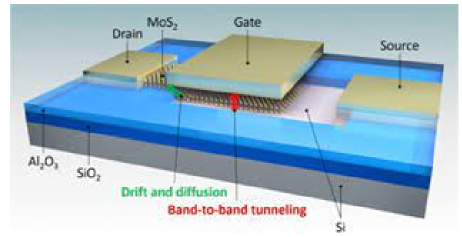


Figure 4: Wave functions in each region.

Quantum tunneling has diverse applications in cosmology, semiconductor physics, and superconductor physics. It explains phenomena such as field emission, powers electron-tunneling microscopes, and influences enzymatic reactions to enhance reaction rates. It's historical development and broad applicability make quantum tunneling a fascinating and important concept.



(a) Electron-Tunneling Microscopy



(b) Tunnel Transistors

Figure 5: Applications of tunneling

Table 1: Symbols, Physical Quantities, Units, and Nature

Symbol	**Physical Quantity**	**Units**	**Nature**
E	Energy	joules (J)	Scalar
V	Potential Energy	joules (J)	Scalar
v	Velocity	meters per second (m/s)	Vector
h	Planck's Constant	joule-seconds (J·s)	Scalar
ψ	Wave Function	unitless	Scalar
x	Displacement	meters (m)	Vector
m	Mass	kilograms (kg)	Scalar
p	Momentum	kilogram-meters per second (kg·m/s)	Vector

4 Hartman Effect

4.1 Introduction

The Hartman effect refers to the phenomenon in quantum mechanics where the tunneling time of a particle through a potential barrier appears to be independent of the barrier width.

In classical physics, one would expect that the time taken for a particle to tunnel through a barrier would increase as the barrier width increases. However, in quantum mechanics, the situation is different. The Hartman effect suggests that for barrier widths larger than the particle's de Broglie wavelength, the tunneling time remains relatively constant.

The Hartman effect has attracted significant attention because it challenges our classical understanding of time and the relationship between barrier width and tunneling time and invites us to explore the underlying principles of quantum mechanics. It has spurred theoretical and experimental investigations to better understand the factors influencing tunneling time. The phenomenon has implications in various areas, including quantum information processing, where precise control over tunneling times is crucial.

Studying the Hartman effect provides insights into the fundamental limits and possibilities of particle behavior. It serves as a reminder that the quantum world operates by its own set of rules, often defying our classical intuitions. The Hartman effect has sparked extensive theoretical and experimental investigations, and it continues to be an active area of research. Understanding the factors influencing tunneling time and reconciling it with classical expectations remains an intriguing and thought-provoking topic in quantum physics.

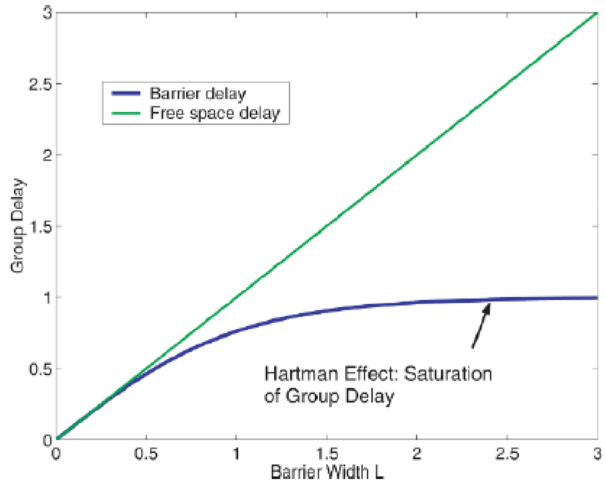


Figure 6: Tunneling time of wave packet saturates with barrier length.

4.2 History

The Hartman effect is named after Thomas E. Hartman, an American physicist who first introduced and studied this phenomenon in the context of quantum tunneling.

In 1962, Hartman published a seminal paper titled "Tunneling of a Wave Packet" in the Journal of Applied Physics. In this paper, he investigated the transmission of a wave packet through a potential barrier and found that the tunneling time appeared to be independent of the barrier width for certain conditions.

Hartman's work built upon earlier studies on tunneling, including the theoretical framework developed by George Gamow in the late 1920s. Gamow's work laid the foundation for understanding tunneling through potential barriers, but he did not specifically investigate the tunneling time aspect that came to be known as the Hartman effect.

Following Hartman's publication, the Hartman effect garnered significant attention from physicists interested in the fundamental aspects of quantum tunneling. Researchers delved deeper into the phenomenon and explored its implications in various areas of physics, including quantum optics, solid-state physics, and quantum information theory.

Since Hartman's initial work, there have been theoretical and experimental investigations to further understand and characterize the Hartman effect. Researchers have explored different potential barrier profiles, variations in particle properties, and the role of wave packet dynamics to gain deeper insights into this intriguing phenomenon.

5 Superluminal Tunneling

The Hartman effect suggests the possibility of superluminal tunneling based on experimental observations and theoretical interpretations. In certain systems, such as electromagnetic or acoustic waves propagating through specific structures, the transmission time across a barrier appears to be independent of the barrier length. This phenomenon implies that the tunneling time does not increase as the barrier thickness increases, contrary to classical expectations.

The significance of this observation lies in the comparison between the tunneling time and the time it would take for a wave to traverse the same distance in free space at the speed of light. In the Hartman effect, the measured tunneling time is often found to be shorter than or equal to the transit time in vacuum, suggesting a superluminal behavior.

One interpretation of this phenomenon is based on the behavior of evanescent waves. Evanescent waves, which are exponentially decaying in amplitude, exist near the barrier interface. These waves are not expected to propagate beyond the barrier due to their rapid decay. However, the Hartman effect suggests that there is a finite probability for these evanescent waves to tunnel through the barrier, resulting in a faster transmission time than would be expected classically.

6 Tunneling time

6.1 Defining Tunneling time

Tunneling time refers to the time it takes for a particle or a wave to traverse a barrier or potential energy barrier through the process of quantum tunneling. It is the time interval between the particle's initial interaction with the barrier and its emergence on the other side. However, defining and measuring such extremely small time is a complex task and subject to various interpretations and debates within the field of quantum mechanics.

The question of how long it takes a particle to tunnel through a potential barrier is one that has occupied physicists since the early days of quantum mechanics. The search for a general answer to that question has turned up a large number of tunneling time definitions, some of which suggest that the tunneling process is superluminal or faster than the speed of light . Let's have a look on what group delay or phase time means- concepts that have been used to assign speed of barrier traversal for decades- Phase time or group delay is the time delay experienced by different frequency components of a wave as they propagate through a medium or encounter a barrier. It represents the average time it takes for a wave packet or a collection of frequencies to pass through a system. The group delay is defined as the derivative of the phase shift with respect to frequency. It characterizes the dispersion properties of waves and helps describe how different frequencies propagate at different speeds.

6.2 Attempts to measure tunneling time

Larmor Clock- The Larmor clock is a theoretical concept used to estimate the time it takes for a particle to tunnel through a barrier. It is based on the idea that the spin of a particle, like a tiny clock hand, precesses or rotates as it moves. By observing the angle of the spin before and after tunneling, one can infer the time the particle spent inside the barrier.

Imagine the particle's spin pointing in a specific direction as it approaches the barrier. Inside the barrier, the spin undergoes a rotational motion due to the influence of the magnetic field. When the particle emerges on the other side, its spin will have changed its direction compared to the initial state. By measuring the angle of this change, one can estimate the time the particle spent within the barrier.

The Larmor clock provides a way to indirectly determine tunneling times, offering insights into the dynamics of particles during the tunneling process.

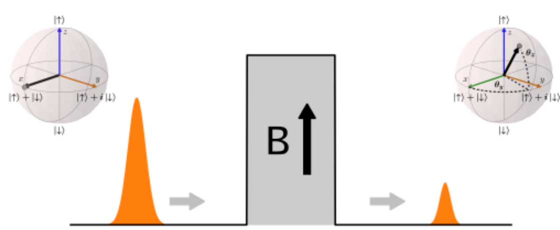


Figure 7: Larmor Clock

Atto clock- The attoclock is an experimental technique used to measure tunneling times on extremely short timescales. It involves using an intense laser pulse to ionize an atom or molecule and create an electron wave packet. As this wave packet tunnels through a potential barrier, its behavior is influenced by the tunneling time.

The attoclock works by introducing a second weaker laser pulse that interacts with the electron wave packet as it tunnels. By precisely timing the arrival of this second laser pulse, researchers can probe the phase and momentum of the electron at different stages of the tunneling process. Analyzing this information provides insights into the tunneling time.

The attoclock technique allows scientists to study tunneling dynamics at the timescale of attoseconds (billionths of a billionth of a second). It provides valuable experimental data that helps refine our understanding of the tunneling phenomenon and its time-related aspects.

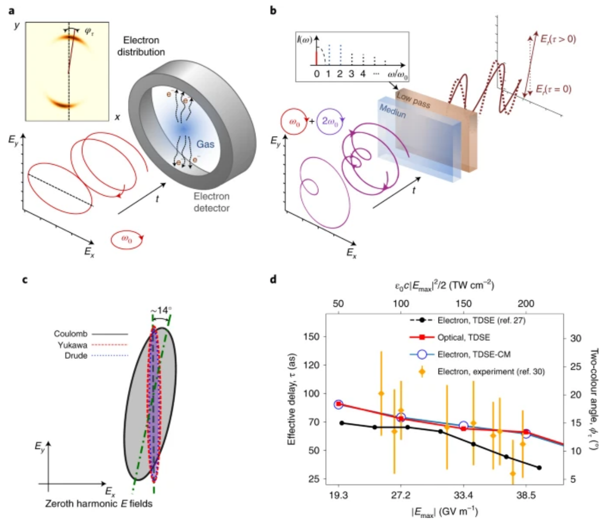


Figure 8: Atto Clock

7 Experiments Supporting Hartman Effect- A Theoretical Analysis

Directly observing the Hartman effect with quantum particles like electrons is challenging due to small timescales and environmental interference. Scientists have employed electromagnetic analogs in experiments to overcome these challenges. For instance, Enders and Nimtz used a microwave waveguide with a narrowed region as a barrier, confirming the Hartman effect by observing a constant phase shift and a shorter group delay than expected. Similar experiments at optical frequencies using photonic structures and frustrated total internal reflection validated the Hartman effect for light waves. Balcou and Dutriaux observed a saturation point in the group delay when measuring light transport between prisms, further supporting the Hartman effect. Acoustic wave experiments with phononic crystals and waveguides exhibited a consistent group delay, reinforcing the Hartman effect. These analog experiments offer valuable insights into the Hartman effect's nature, demonstrating independent group delay and the potential for superluminal tunneling.

8 Problems that arise

The proposal of superluminal tunneling, as suggested by the Hartman effect, raises several intriguing problems and challenges in our understanding of fundamental physics. Here are some of the key issues:

Causality: Superluminal tunneling implies that information or signals can seemingly travel faster than the speed of light. This challenges the principle of causality, which states that cause and effect must occur in a specific temporal order. If tunneling truly allows for superluminal transmission, it could potentially lead to paradoxes and violations of causality, questioning our understanding of cause-and-effect relationships.

Relativity: The theory of relativity, specifically special relativity, is built upon the assumption that the speed of light is an absolute speed limit. Superluminal tunneling, if confirmed, would challenge this fundamental tenet of relativity, potentially requiring modifications or extensions to the theory.

Energy and Information: Superluminal tunneling could raise questions about the conservation of energy and the transfer of information. If particles or waves can tunnel through barriers faster than light, it may be possible to transmit information or energy instantaneously, which conflicts with our current understanding of energy conservation and the limitations of information transfer.

Experimental Verification: While the Hartman effect has been observed in electromagnetic and acoustic systems, the interpretation of superluminal tunneling remains controversial. Some argue that the apparent superluminal behavior is a result of phase shifts, group delays, or other effects, rather than true faster-than-light propagation. Resolving these conflicting interpretations and obtaining clear experimental evidence for superluminal tunneling remains a challenge.

Theoretical Framework: Superluminal tunneling challenges the theoretical framework of quantum mechanics and classical physics. It prompts the need for a deeper understanding of wave-particle duality, quantum tunneling, and the interaction of waves with barriers. Exploring and reconciling these concepts within a consistent theoretical framework is an ongoing endeavor.

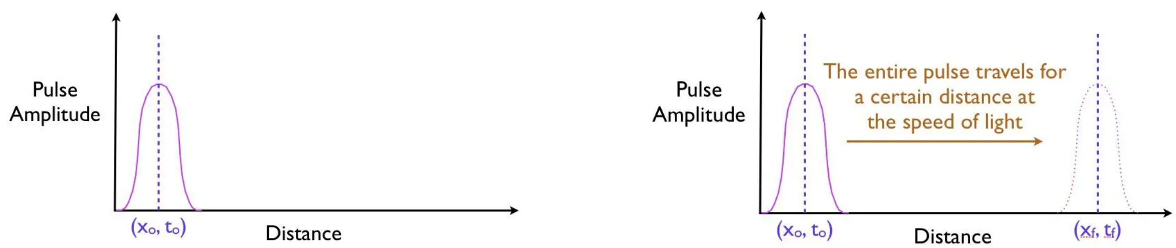
9 Theories against Superluminal Tunneling

As per many physicists, nothing in nature could break Einstein's law of relativity. So, it's not possible for Quantum Tunneling to break the speed of light.

According to many, group delay should not be used to assign a speed of barrier traversal.

Here's a basic sum-up of the viewpoints of scientists opposing Hartman Effect without going into rigorous mathematical or theoretical treatment.

We exploit the fact that we are capable of imaging pulses that move close to speed of light using novel technologies and devices. We can measure the location of a pulse before it encounters a barrier. We may also predict the position of pulse at a later instant if it were to move at speed of light.



(a) a pulse of particles- distribution in space and time inherent to those particles

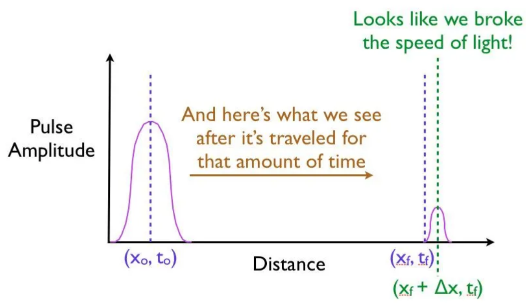
(b) Predicted position after a certain amount of time

Figure 9

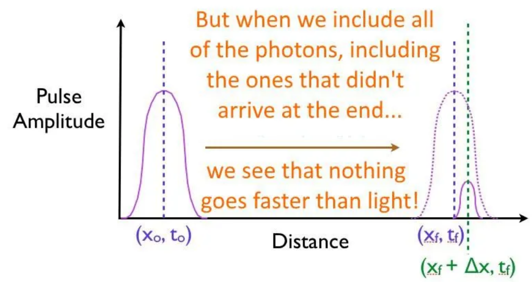
Now, we compare the location of particle after it has tunneled from what we had expected.

When tracking the individual motion of particles during tunneling, it becomes clear that the particles that successfully tunnel are part of the initial pulse's leading edge. None of these particles actually travel faster than the speed of light. The process of tunneling itself, where particles transition from a bound state on one side of a quantum barrier to an unbound state on the other side, does not involve any additional time beyond other physical effects. The overall movement of particles over a certain distance within a given time is still subject to the limitations of Einstein's relativity, which applies to every particle in all circumstances. It is remarkable that scientists have directly measured tunneling time for a single particle and demonstrated that there is no inherent delay in the tunneling process.

The group delay in tunneling is the lifetime of stored energy escaping through both ends of the barrier. It is not the transit time from input to output, as has been assumed for decades.



(a) Observed location



(b) No actual particles were traveling faster than light itself.

Figure 10

According to other arguments, even if tunneling occurs at superluminal speed, very few particles which encounter the barrier would actually be able to cross the barrier as is evident from the solution of Schrodinger's equation and the transmission probability that we obtain as its consequence. So, superluminal signaling would still not be possible.

10 Conclusion

This write-up has allowed me to delve into the fascinating realm of Quantum Mechanics and acquire new insights. It has deepened my appreciation for the fundamental principles that govern the universe.

During my study on Hartman Effect, I went through various books and research papers including the one by Hartman himself. Many experiments concluded that tunneling indeed occurs faster than light and tunneling time does not depend on the width of the barrier. However, there are serious concerns related to the understanding of the tunneling time and the way adopted to measure this time.Also, most previous work on tunneling time relied on Schrodinger's Equation which doesn't incorporate Einstien's special theory of relativity and so had no speed limit baked into it. However, to analyze FTL motion seriously, we need to consider the Dirac Equations. Whether or not, superluminal tunneling occurs, superluminal signaling isn't possible, for sure. According to a research paper published in Nature in 2020, newer methods to measure tunneling time has been devised. Sooner or later, we will get to know what is actually happening.

The process of writing this project has allowed me to explore the fascinating world of quantum tunneling and its real-life applications. The discoveries made during my research, particularly the concept of particles traveling faster than light in tunneling, have ignited a deep interest in physics and its future developments. The experience has been eye-opening, leaving me with a strong curiosity to further explore the wonders of quantum mechanics and its potential for shaping the world around us.

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