PHN-005 (Physics) B.Tech. I Year (CSE+ECE) Tutorial Sheet–II
Autumn Semester 2022

Electrostatics and Magnetostatics

1. Suppose that, instead of the Coulomb force law, one found experimentally that the force between any two charges q_1 and q_2 was given by: $\boldsymbol{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{(1-\sqrt{\alpha}\, r_{12})}{r_{12}^2}\,\boldsymbol{a}_r$, α being a constant. (a) Calculate $\oint \boldsymbol{E}.d\boldsymbol{l}$ along a contour not passing through a charge *q* where ***E*** is the electric field produced by q. (b) Obtain the modified Gauss law for a spherical Gaussian surface of radius *r* with the same charge as the center. (c) Calculate $\oiint \boldsymbol{E}\,.\,d\boldsymbol{S}$ for a sphere of radius $r+\ \delta\, r$, $\delta\, r$ being infinitesimal, and by applying the Gauss's divergence theorem to the spherical shell of inner radius *r* and outer radius $r+\ \delta\, r$, calculate $\mathbf{\nabla}.\mathbf{E}$.

2. A static charge distribution produces a radial electric field: $\boldsymbol{E} = A\frac{e^{-br}}{r^2}\,\hat{\boldsymbol{a}}_{\boldsymbol{r}}$**,** where *A* and *b* are constants. Calculate (a) the charge density and (b) the total charge.

3. Consider two electrodes, one grounded and the other, a distance *d* apart, at a constant potential V_0. Disregarding electronic collisions, calculate the constant current density between the electrodes if an unlimited supply of electrons at rest are supplied to the lower-potential electrode. [Hint: Express the charge density in terms of the current density and electron's velocity; use (1-D) Poisson's equation]

4. A sphere of radius *a* has charge *q* distributed uniformly over its surface. The sphere is surrounded by a dielectric fluid of permittivity ϵ; the fluid also contains a free charge density given by $\rho(\boldsymbol{r}) = k\,\Phi(\boldsymbol{r})$, $\Phi(\boldsymbol{r})$ being the electrostatic potential. Calculate $\Phi(\boldsymbol{r})$ assuming the same vanishes asymptotically.

5. A long, straight, cylindrical conductor of radius *a* and carrying a current *I*, has a cylindrical hole of radius *b* displaced by a distance *d* from the axis of the conductor. Calculate the magnetic field inside the hole. [Hint: Use the Maxwell's equations corresponding to the magnetostatics Gauss's law and the Ampere's law.]

6. Starting with the Biot-Savart law show explicitly that for a closed loop carrying a current I the magnetic induction at a point P away from the loop is given by: $\boldsymbol{B} = \frac{\mu_0}{4\pi} I\mathbf{\nabla}\Omega$, where Ω is the solid angle subtended by the loop at the point P. This corresponds to a magnetic scalar potential $\Phi_M = -\frac{\mu_0}{4\pi} I\Omega$. [Hint: Use Stokes law.]

7. Suppose the magnetic field on the axis of a right circular cylinder is given by: $\boldsymbol{B} = B_0(1 + \alpha\, z^2)\boldsymbol{a_z}$, α being a constant. Suppose $\boldsymbol{B}_\phi = 0$. Calculate $\boldsymbol{B}_\rho$for points very close to the axis. [Hint: Use the magnetostatic Gauss's law.]

8. Consider two current carrying loops 1 and 2. Using the Biot-Savart law, show that the force exerted on 1 because of current flowing in 2 is equal and opposite of the force exerted on 2 because of current flowing in 1.

9. Calculate the vector potential and magnetic field at a point outside and inside a thin spherical shell of radius R with a uniform surface charge density σ rotating at constant angular velocity ω.

10. Two coaxial, parallel, circular conductors of the same radius R, carrying the same current I, are separated by a distance *a* (Helmholtz galvanometer). What should be the value of *a* so that the magnetic field at the center of the line segment joining their centers, is approximately uniform?