Tutorial-2 - CSIV-312 - Principles of Programming Languages

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CSE-04

~~1. plus = dm dn df dy m f (n f y)~~

1. plus = dm. dn. df. dy. m f (n f y)

For m = 3, n = 2:

m = df. dy. f (f (f y))
n = df. dy. f (f y)

⇒ plus 3 2 = ((dm. dn. df. dy. m f. (n f. y)) 3) 2
= (dn. df. dy. (df. dy. f (f (f y))) f (n f y)) 2
= df. dy. (df. dy. f (f (f y))) f ((df. dy. f (f y)) f y)
= df. dy. (df. dy. f (f (f y))) f (f (f y))
= df. dy (dy. f (f (f y))) (f (f y))
= df. dy f (f (f (f (f y))))

∴ 5

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By interchanging the order of m n:

plus = dn.dm.df.dy. mf (nf.y)

plus 2 3

=((dn.dm.df.dy mf (nf y) 2) 3

=(dm.df.dy.mf ((df.dy. f (f y)) f y)) 3

=(dm.df.dy. mf (f (fy))) 3

= df.dy. (df.dy. f (f (fy))) f (f (fy))

= df. dy. (dy. f (f (fy))) (f (fy))

= df.dy. f (f (f (f (fy))))

= 5

---

2. multiply = dm.dn. df.dy. n (mf) y

m = 2 = df.dy. f (fy)
n = 3 = df.dy. f (f (fy))

multiply 2 3 =

= (dm. dn. df.dy. n (mf) y) 2 3

= (dm. dn. df. dy. n (mf) y) df.dy. f (fy) 3

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$= (dn \cdot df \cdot dy \; n \; (df \cdot dy \cdot f(fy) \; f) \; y) \; 3$

$= (dn \cdot df \cdot dy \; n \; (dy \cdot f(fy)) \; y) \; 3$

$= (dn \cdot df \cdot dy \; n \; (dy \cdot f(fy)) \; y) \; df \cdot dy \cdot f(f(fy))$

$= df \cdot dy \cdot df \cdot dy \cdot f(f(fy)) \; (dy \cdot f(fy)) \; y$

$= df \cdot dy \cdot dy \cdot (dy \cdot f(fy)) \; ((dy \cdot f(fy)) \; ((dy \cdot f(fy)) \; y)) \; y$

$= df \cdot dy \cdot dy \cdot f(f(f(f(f(fy))))) \; y$

$= \boxed{df \cdot dy \cdot f(f(f(f(f(fy)))))}$

$= \boxed{6}$

By changing the order of m n in multiply:

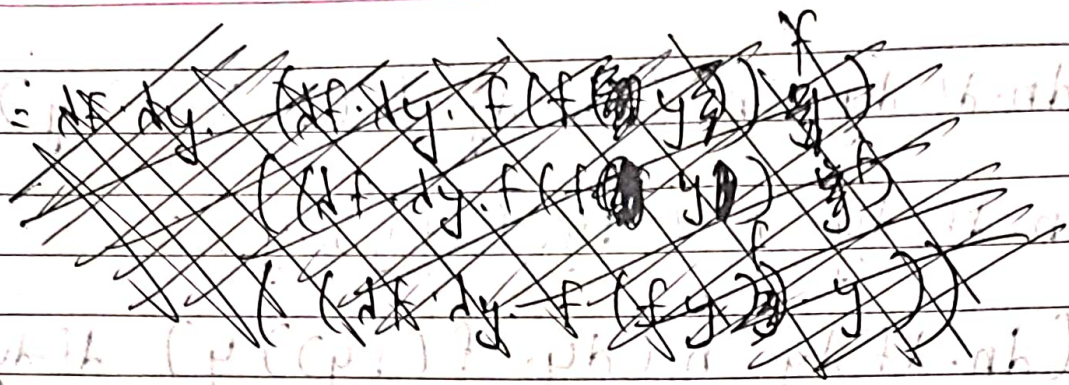
multiply = dn dm df dy n (m f) y.

multiply 3 2

$= (dn \cdot dm \cdot df \cdot dy \cdot n(m\,f)\,y) \; df \cdot dy \cdot f(f(fy)) \; 2$

$= (dm \cdot df \cdot dy \; df \cdot dy \cdot f(f(fy)) \; (m\,f) \; y) \; 2$

$= (dm \cdot df \cdot dy \; (mf) \; ((mf) \; ((mf) \; y))) \; df \cdot dy \cdot f(f$



$= df\,dy\,(df\,dy\,f\,(f\,y)\,f)$
$\quad (df\,dy\,f\,(f\,y)\,y)$

$= df\,dy\,(df\,dy\,f\,(f\,y)\,f)$
$\quad (df\,dy\,f\,(f\,y)\,f)$
$\quad (df\,dy\,f\,(f\,y)\,f)\,y))$

$= df\,dy\,(df\,dy\,f\,(f\,y)\,f)$
$\quad (df\,dy\,f\,(f\,y)\,f)$
$\quad f\,(f\,y)))$

$= df\,dy\,(df\,dy\,f\,(f\,y)\,f)$
$\quad f\,(f\,(f\,(f\,y)))$

$= \boxed{df\,dy\;f\,(f\,(f\,(f\,(f\,(f\,y)))))}$

$= 6$

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3. exponent = $\lambda m.\lambda n.\ n\ m$.

exponent 3 2

$= (\lambda m.\lambda n.\ n\ m)\ 3\ 2$

$= (\lambda n.\ n\ \ \lambda f.\lambda y.\ f\ (f\ (f\ y)))\ 2$

$= \lambda f.\lambda y.\ f\ (f\ y)\ \ \lambda F.\lambda g.\ F\ (F\ (F\ g))$

$= \lambda z.\ \lambda F.\lambda g.\ F\ (F\ (F\ g))\ (\lambda f.\lambda y.\ f\ (f\ (f\ y)))$

$= \lambda y.\ \lambda f.\lambda y.\ f\ (f\ (f\ y))\ (\lambda y.\lambda z.\ y\ (y\ (y\ z)))\ y$

$= \lambda y.\ \lambda f.\lambda y.\ f\ (f\ (f\ y))\ (\lambda z.\ y\ (y\ (y\ z)))$

$= \lambda y.\ \lambda f.\lambda w.\ f\ (f\ (f\ w))\ (\lambda z.\ y\ (y\ (y\ z)))$

$= \lambda y.\ \lambda w.\ \lambda z.\ y\ (y\ (y\ z))\ (\lambda z.\ y\ (y\ (y\ z)))$

$\quad (\lambda z.\ y\ (y\ (y\ z))\ w))$.

$= \lambda y.\lambda w.\ \lambda z.\ y\ (y\ (y\ z))\ (\lambda z.\ y\ (y\ (y\ z)))$

$\quad (\lambda z.\ y\ (y\ (y\ z))\ w))$

$= \lambda y.\ \lambda w.\ \lambda z.\ y\ (y\ (y\ z))\ (y\ (y\ (y\ (y\ (y\ (y\ w))))))$

$= \boxed{\lambda y.\ \lambda w.\ y\ (y\ (y\ (y\ (y\ (y\ (y\ (y\ (y\ w)))))))))}$

$= 9$

~~exponent = 2~~

By changing the order of m, n:

exponent = dn.dm. n m

exponent 2 3

$= (dn.dm\ n\ m)\ (df.dy.f(fy))(df.dy.(f(f(fy$

$= (dm.(df.dy.f(fy))\ m)\ (df.dy.(f(f(fy))$

$= (df.dy.f(fy))(df.dy.(f(f(fy))))$

$= dy.(df.dy.f(f(fy)))((df.dy.f(f(fy)))y$

$= dy.(df.dy.f(f(fy)))(dy.dz.y(y.(yz))$

$= dy.(df.dy.f(f(fy)))(dz.y(y(yz)))$

$= dy.(df.dw.f(f(fy^{w})))(dz.y(y.(yz)))$

$= dy.dw.(dz.y(y(yz)))((dz.y(y(yz))$

$(dz.y(y.(yz))$

$w z))$

$= dy.dw.(dz.y(y(yz))\ (dz.y(y(yz))$

$(y(y(y.w)))$

$= dy.dw.(dz.y(y(yz))\ y(y(y(y(y(yw)))$

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$= \lambda dy.\ dw.\ (y\ (y\ (y\ (y\ (y\ (y\ (y\ (y\ (y\ w))))))))))$

$= \lambda df.\ dy.\ (f\ (f\ (f\ (f\ (f\ (f\ (f\ (f\ (f\ (f\ y)))))))))))$

$= 9$

4. LHS = 2 × 3 + 4

= add (mult 2 3) 4

= 4

add m n
$= \lambda f.\ dy.\ m\ f\ (n\ f\ y)$

mult 2 3 n
$= \lambda f.\ dy.$

4. LHS = 2 × 3 + 4

= plus (multiply 2 3) 4

= plus (($\lambda m.\ dn.\ df.\ dy.\ n\ (m\ f)\ y$) 2 3) 4

= plus (($\lambda n.\ df.\ dy.\ n\ (df.\ dy.\ f\ (f\ y)\ f)\ y$) 3) 4

= plus (($\lambda n.\ df.\ dy.\ n\ (\lambda y.\ f\ (f\ y))\ y$) 3) 4

= plus ($\lambda f.\ dy.$ ($\lambda f.\ dy.\ f\ (f\ (f\ y))$) ($\lambda g.\ f\ (f\ y)$) y)) 4

= plus ($\lambda f.\ dy.$ ($\lambda y.\ (\lambda g.\ f\ (f\ y))$ (($\lambda y.\ f\ (f\ y)$) ($\lambda g.\ f\ (f\ y)$) y))) 4

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= plus (λf.λy. f(f(f(f(f(f y)))))) 4

= (λm.λn.λf.λy.((m f)(n f y))) 6 4

= (λn.λf.λy.(((λf.λy.f(f(f(f(f(f y)))))) f)(n f y))) 4

= (λn.λf.λy.((λy.f(f(f(f(f(f y))))))(n f y))) 4

= (λn.λf.λy.f(f(f(f(f(f(n f y))))))) 4

= λf.λy.f(f(f(f(f(f((λf.λy.f(f(f(f y))))) f y))))))

= λf.λy.f(f(f(f(f(f(f(f(f(f y)))))))))

= 10 = RHS.

Hence proved