PHN-006 PROJECT

WAVE-PARTICLE DUALITY

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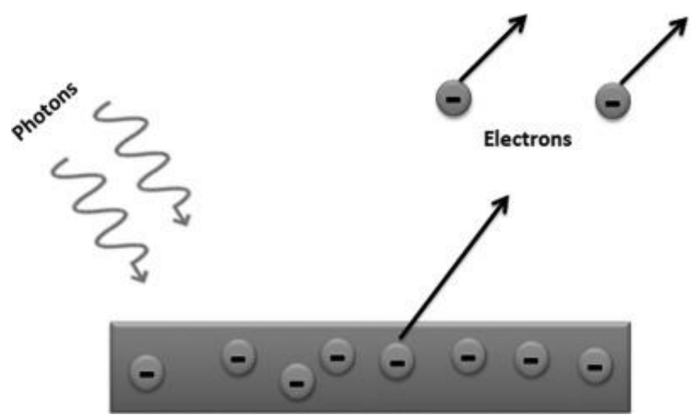
The topic chosen for the project is 'Wave-Particle Duality'. At the time I was studying for the Joint Entrance Examination (JEE), I had come across the photo electric effect while studying about atomic structure in chemistry. It was intriguing to find out how our understanding of the physics of both light and atomic scale particles such as the electron were shaped by this and other such experiments. The fact that Einstein was awarded the Nobel Prize for explaining the photo electric effect and not for the famous equation, $E=mc^2$, as I had initially believed; instilled into me a zeal to at least study this phenomenon, if not understand it completely.



The Photo electric Effect

The topic of this project relates to the subject on a foundational level, as it plays a crucial role in understanding the behavior of particles and their connection to macroscopic properties.

Newton's initial belief was that light consisted of particles, but subsequent investigations unveiled its wave-like behavior. Nevertheless, the advent of the twentieth century brought about a pivotal revelation that light, at times, exhibited particle-like characteristics. Similarly, the electron was historically perceived as a particle until evidence surfaced indicating its wave-like behavior in numerous aspects. Consequently, it becomes apparent that light and electrons do not conform exclusively to either particle or wave behavior. Thus, a consensus has emerged, acknowledging their dual nature and categorizing them as entities that resemble neither.

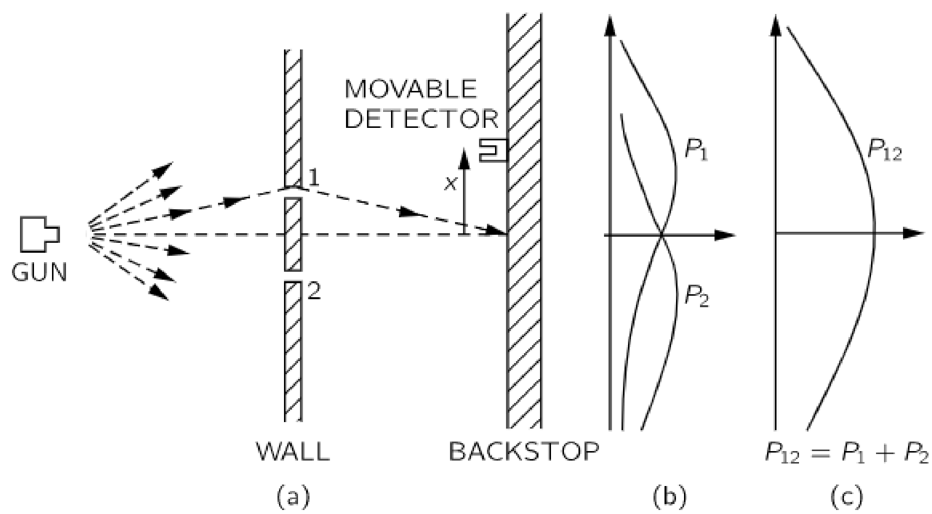
A coincidence arises in the similarity of electron behavior to that of light. The quantum properties exhibited by atomic constituents, such as electrons, protons, neutrons, and photons, share a commonality in being referred to as "particle waves". Therefore, the knowledge gained regarding electron properties is equally applicable to all "particles," including light photons.

The gradual accumulation of information about atomic and small-scale behavior during the first quarter of the 20th century produced an increasing confusion which was finally resolved in 1926 and 1927 by Schrödinger, Heisenberg, and Born. They finally obtained a consistent description of the behavior of matter on a small scale.

Let us take an experiment to understand the quantum behavior of electrons by comparing it to the behavior of familiar particles like bullets and waves such as water waves.

1. A setup involving a machine gun, a wall with two holes, a backstop, and a bullet detector is used. The objective is to determine the probability of a bullet passing through the holes and reaching a specific distance, denoted as x, from the center.

   Due to the unpredictable nature of bullet trajectories, probabilities are considered instead of definite outcomes. The probability is determined by counting the number of bullets reaching the detector within a certain time frame and comparing it to the total number of bullets hitting the backstop. Alternatively, if the firing rate remains constant, the probability can be calculated proportionally based on the number of bullets reaching the detector within a standard time interval.

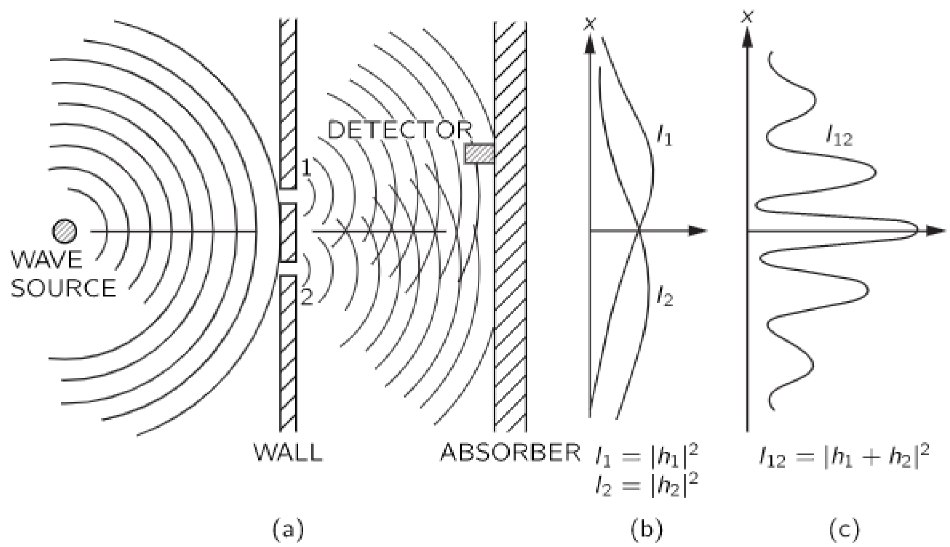
   

   In this idealized experiment, the bullets are assumed to be indestructible and always arrive in distinct groups called lumps. The detector consistently captures whole bullets, and the group size is independent of the firing rate, leading to the conclusion that "bullets always arrive in identical lumps." The detector measures the probability of a lump's arrival as a function of x. The anticipated results, depicted in part (c) of the diagram, show the probability (P_{12}) plotted against x. P_{12} is highest near the center and decreases significantly as x increases. To understand why P_{12} has its maximum value at x=0, separate experiments are conducted with one hole covered at a time, resulting in probability curves (P_1 and P_2) representing the distribution of bullets passing through each hole. Comparing parts (b) and (c), we find the important result that

$$P_{12}=P_1+P_2.$$

The result is an observation of *no interference*.

2. For the water wave experiment, a shallow trough of water is used with a wave source that generates circular waves. The setup includes a wall with two holes, an absorber wall made of sand, and a detector that measures the intensity of the wave motion. The intensity of the wave can vary and has no distinct "lumpiness." By measuring the wave intensity at different positions (x), a curve labeled I_{12} is obtained.



Wave interference plays a crucial role in understanding the intensity pattern. When the waves diffract at the holes, new circular waves emerge from each hole, leading to interference. By covering one hole at a time, intensity curves I_1 and I_2 are observed. I_1 represents the intensity of the wave from hole 1, and I_2 represents the intensity of the wave from hole 2.

The observed intensity (I_{12}) when both holes are open does not simply result from adding I_1 and I_2. Instead, the waves interfere with each other. Constructive interference occurs when the waves are in phase, resulting in a higher intensity at certain points where I_{12} reaches its maximum. This constructive interference happens when the distance from the detector to one hole is a whole number of wavelengths longer or shorter than the distance to the other hole.

In contrast, destructive interference occurs when the waves arrive at the detector out of phase, with a phase difference of π. This leads to a lower wave intensity as the amplitudes of the waves subtract. Destructive interference occurs when the distance between hole 1 and the detector differs from the distance between hole 2 and the detector by an odd number of half-wavelengths. The low values of I_{12} in the diagram correspond to these positions of destructive interference.

The relationship between the intensities I_1, I_2, and I_{12} can be described using complex numbers. The height of the water wave at the detector for the wave from hole 1 is represented as (the real part of) $h_1e^{i\omega t}$, where h_1 is the complex amplitude. The intensity is proportional to the square of the absolute value $|h_1|^2$. Similar expressions apply to hole 2. When both holes are open, the wave heights add up, resulting in the intensity $|h_1+h_2|^2$.

In the context of interfering waves, disregarding the constant of proportionality, the proper relationships can be expressed as follows:

$$I_1 = |h_1|^2$$
$$I_2 = |h_2|^2$$
$$I_{12} = |h_1 + h_2|^2$$

Comparing this result to the one obtained with bullets, it is evident that they differ significantly. Expanding $|h_1 + h_2|^2$ yields:

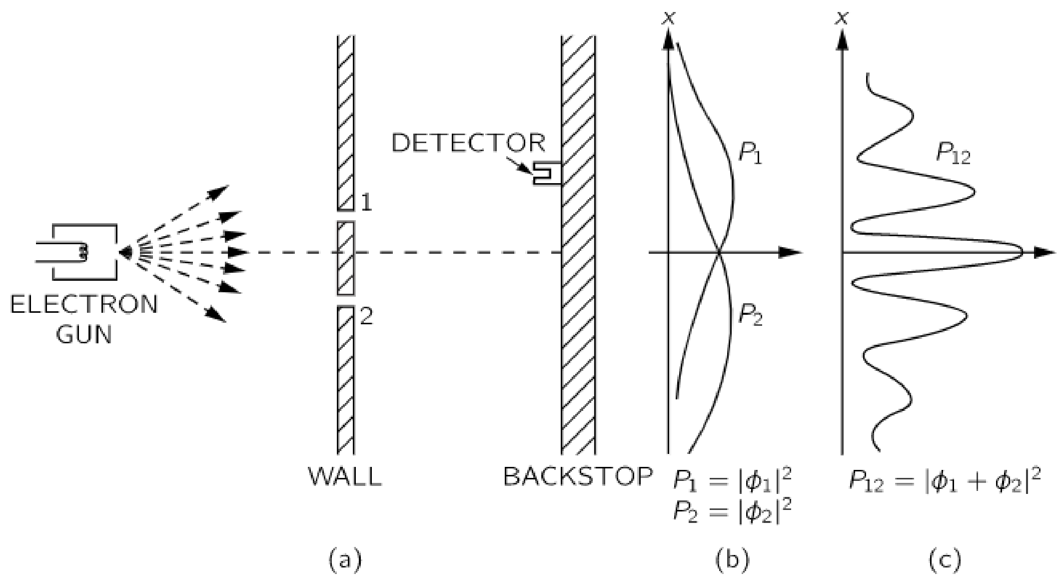
$$|h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos\delta$$

where δ represents the phase difference between h_1 and h_2. In terms of intensities, we can rewrite it as:

$$I_{12} = I_1 + I_2 + 2\sqrt{(I_1I_2)}\cos\delta$$

The final term in above equation denotes the "interference term." This summarizes the behavior of water waves, where the intensity can take any value and exhibits *interference.*

3. In a hypothetical experiment involving electrons, an electron gun with a heated tungsten wire and a surrounding metal box emits electrons accelerated towards the walls. Some electrons pass through a hole in the box, and they have similar energy. A wall with two holes and a backstop plate are positioned in front of the electron gun. A movable detector, such as a Geiger counter or an electron multiplier connected to a loudspeaker, is placed in front of the backstop.



It's important to note that this experiment is theoretical and cannot be practically executed due to the required small scale. However, previous experiments conducted under specific conditions provide insight into the expected outcomes.

In this electron experiment, the detector detects distinct and identical "clicks" without any partial clicks. These clicks occur randomly, resembling the pattern of a Geiger counter. Counting the clicks over extended periods shows that the numbers obtained in two equal time intervals are nearly the same, allowing for the determination of an average click rate (e.g., clicks per minute).

While the rate of clicks varies as the detector is moved, the intensity of each click remains constant. Lowering the wire's temperature in the electron gun reduces the click rate, but the sound of each click remains unchanged. When two separate detectors are placed at the backstop, only one detector clicks at a time, except for rare instances of closely spaced clicks. This suggests that the electrons arrive at the backstop in discrete and identical "lumps," with only one lump arriving at a time.

Similar to the bullet experiment, the relative probability of an electron lump reaching the backstop at different distances (x) from the center can be determined experimentally. This probability is derived from observing the click rate while keeping the electron gun's operation consistent. The probability of a lump arriving at a specific x is proportional to the average click rate at that position.

The result of the experiment is represented by the P_{12} curve in part (c) of the diagram, providing insight into the behavior of electrons in the experiment.

For electrons:

$$P_{12} \neq P_1 + P_2$$

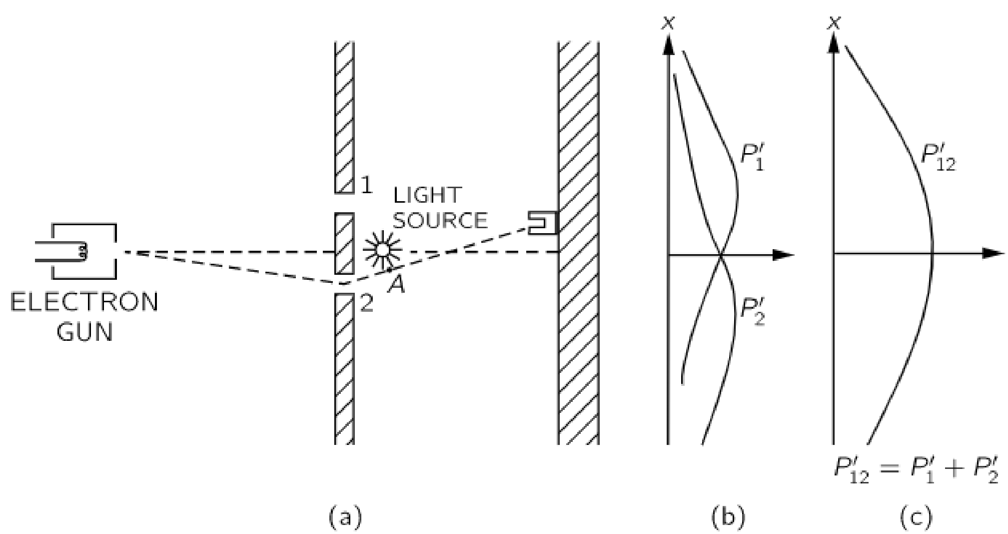
In the electron experiment, it is observed that electrons exhibit dual behavior, behaving as both particles and waves. They arrive in discrete lumps or particles, but their arrival probability follows a wave-like distribution. In classical wave analysis, intensity is calculated using the mean square of wave amplitudes, while in quantum mechanics, complex numbers are necessary to accurately represent electron amplitudes.

Although the probability of electrons arriving through both holes can be straightforwardly described, it is not equal to the sum of the probabilities of arrival through each hole separately. This discrepancy highlights that the proposition that, electrons go through either hole 1 or hole 2, is likely false. Further experiments can be conducted to test this conclusion and explore the intricacies of electron behavior.

Let us take the aforementioned proposition and call it Proposition A.

Proposition A: Each electron either goes through hole 1 or it goes through hole 2.

The next experiment involves scattering light off electrons passing through two holes. Observing the flashes of light reveals that electrons go through either hole 1 or hole 2, but not both simultaneously. This supports the notion that when observed, electrons choose a specific path, in line with Proposition A.



Analyzing the probabilities of electron paths reveals that when observed, electrons behave as expected, going through one hole or the other without interference. However, when the flashes of light are ignored and the detector clicks are combined, the interference pattern reemerges. This suggests that observing the electrons and scattering light disrupts their motion, causing interference effects to vanish.

Lowering the light intensity results in some electrons passing undetected, indicating that light behaves like discrete particles called photons. Attempts to observe electrons without disturbing them using longer wavelengths of light lead to the loss of the interference pattern. This supports Heisenberg's uncertainty principle, which states that determining the hole a particle passes through disrupts the interference pattern.

The uncertainty principle is a fundamental aspect of nature and underpins quantum mechanics, a successful theory describing matter. If a way to bypass the uncertainty principle were found, it would invalidate quantum mechanics. Therefore, the uncertainty principle is a critical element in our understanding of nature.

Regarding Proposition A, it is concluded that an apparatus capable of determining the electron's path can identify whether it goes through hole 1 or hole 2. However, without disturbing the electrons or observing their paths, making deductions about their trajectory is inappropriate to avoid errors in analysis and accurately describe nature.

First Principles of Quantum Mechanics

1. The probability of an event in an ideal experiment is given by the square of the absolute value of a complex number ϕ which is called the probability amplitude:

   P=probability,

   ϕ=probability amplitude,

   $P=|\phi|^2$.

2. When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference:

$$\phi = \phi_1 + \phi_2,$$

$$P = |\phi_1 + \phi_2|^2.$$

3. If an experiment is performed which is capable of determining whether one or another alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost:

$$P = P_1 + P_2.$$

In conclusion, the experiments exploring the quantum behavior of electrons have revealed the fascinating and puzzling nature of the microscopic world. They have demonstrated the wave-particle duality of electrons, highlighted the uncertainty principle, and emphasized the profound impact of observation on quantum systems. These experiments have laid the foundation for quantum mechanics, shaping our understanding of nature and opening doors to new frontiers of scientific exploration.

Citation

The Feynman Lectures on Physics https://www.feynmanlectures.caltech.edu/III_toc.html

Photo electric effect https://doi.org/10.1016/B978-0-12-818111-9.00003-3

Heisenberg and the wave particle duality https://doi.org/10.1016/j.shpsb.2005.08.002