Lecture 9

Syntax Analysis

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Take aways from the last class

- Calculation of *follow* set

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- Creation of parse table

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- Error recovery technique.

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- Bottom up parsing

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- Shift-Reduce parser

Example

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string successfully reduced to start symbol.

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Bottom up parsing

LR grammars accept more class of languages compared with LL grammars.

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- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible
- Allows better error recovery

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  - How to keep track of length of β?

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- Bottom up parsing is based on recognizing handles

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  - Which rule to use for reduction if reduction is possible by more than one rule? **Reduce-Reduce conflict**
- Conflicts come either because of ambiguous grammars or parsing method is not powerful enough

if there are multiple entries in bottom up parse table, then grammar can be ambiguous, or parser technique is not powerful enough.

Shift-Reduce conflict

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stack	input	action
E+E	*id	reduce E→E+E
E	*id	shift
E*	id	shift
E*id		reduce E→id
E*E		reduce E→E*E
E		

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considering the precedence of * over +, this one is correct.

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  $R \rightarrow c$

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Reduce-Reduce conflict

- Consider the grammar $M \rightarrow R + R | R + c | R$
  $R \rightarrow c$
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stack	input	action
	c+c	shift
c	+c	reduce R→C
R	+c	shift
R+	c	shift
R+c		reduce R→c
R+R		reduce M→ R+R
M		

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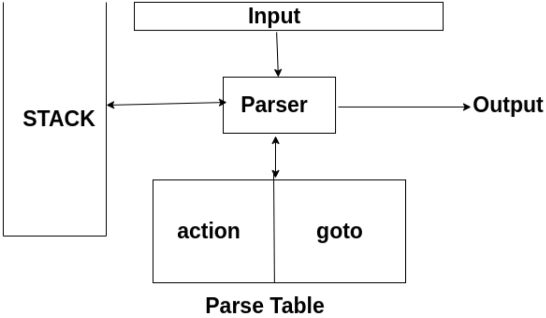

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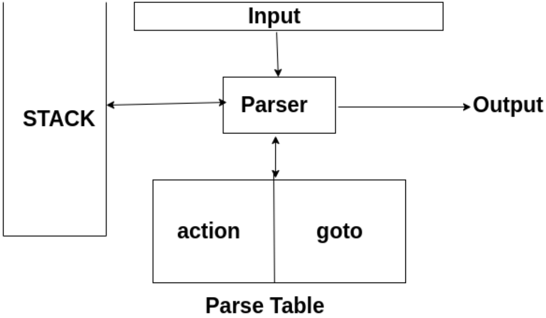

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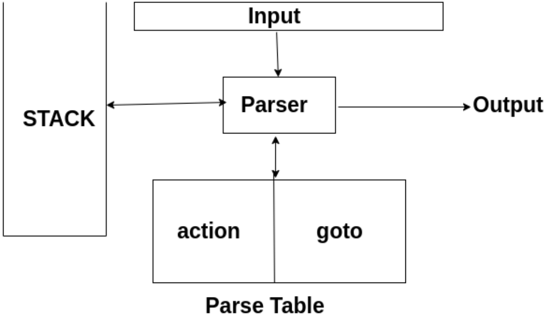

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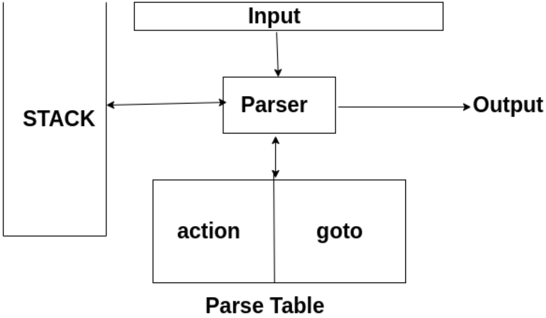

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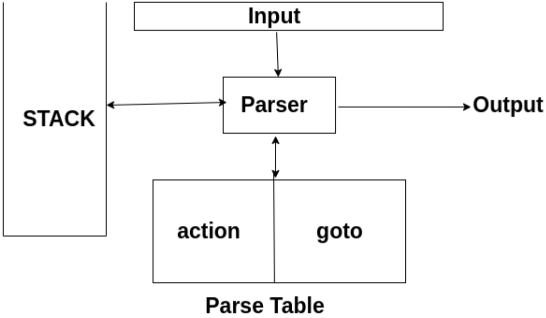

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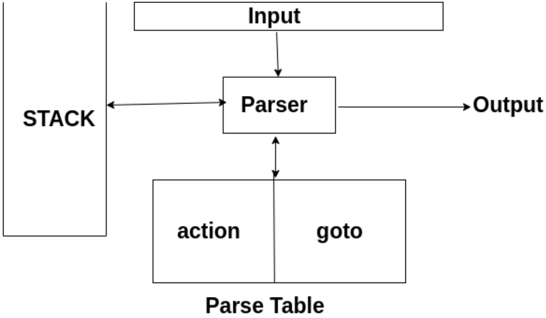

- Input contains the input string.
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- Tables contain **action** and **goto** parts.
- action table is indexed by state and terminal symbols.
- goto table is indexed by state and non terminal symbols.

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  4. error

Configurations in LR parser

$Stack : S_0 X_1 S_1 X_2 \cdots X_m S_m \quad Input : a_i a_{i+1} \cdots a_n \$$

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  Then the configuration becomes

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  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$

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- If $action[S_m, a_i] = error$

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top of stack will be always stack symbol

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  Then the configuration becomes

  $Stack : S_0 X_1 S_1 \cdots S_{m-r} X_{m-r} S \quad Input : a_i a_{i+1} \cdots a_n\$$

  $Where \quad r = |\beta| and \quad S = goto[S_{m-r}, A]$

X{m-r} will be A

- If $action[S_m, a_i] = accept$

  Then parsing is completed. *HALT*

No need to check anything when "accept" comes. Checks have been done during parse table creation.

- If $action[S_m, a_i] = error$

  Then invoke error recovery routine

LR parsing Algorithm

Algorithm LR$_p$*arsing*$_A$*lgorithm*

```
Initial State $Stack : S_0$   $Input : w\$$
for $TRUE$ do
    if $action[S, a] = shift \quad S'$ then
        push(a); push(S'); ip++
    else if  action[S,a] = reduce $A \rightarrow \beta$ then
        pop $(2 * |\beta|)$ symbols;
        push(A); push (goto[S",A]) {S"  is the state after popping symbols}
    else if action[S,a] = accept then
        exit
    else
        error()
    end if
end for
```

Example

- Consider the grammar

Example

- Consider the grammar $E \to E + T | T \qquad T \to T * F | F \qquad F \to (E) | id$

state	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Parse id + id * id

stack	input	action
0	id + id * id$	s5
0id5	+ id*id$	reduce F→id
0F3	+id*id$	reduce T→ F
0T2	+id*id$	reduce E→T
0E1	+id*id$	shift 6
0E1+6	id*id $	shift 5
0E1+6id5	*id $	reduce F→id
0E1+6F3	*id $	reduce T→F
0E1+6T9	*id $	shift 7
0E1+6T9*7	id $	shift 5
0E1+6T9*7id5	$	reduce F → id
0E1+6T9*7F10	$	reduce T →T*F
0E1+6T9	$	reduce E→E+T
0E1	$	acc