Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)

Exercise-6

1. Consider the LPP

   Max $z = 3x_1 + 2x_2 + 5x_3$

   s/t $x_1 + 2x_2 + x_3 \le 430,\ 3x_1 + 2x_3 \le 460,\ x_1 + 4x_2 \le 420,\ x_1, x_2, x_3 \ge 0.$

   Given that x_2, x_3, x_6 (slack variable corresponding to constraint 3) form the optimal basis and inverse of the optimal basis is, row-wise; ½, -1/4, 0; 0, ½ 0; -2, 1, 1. Form the optimal table based on this information.

2. In problem 1, find the optimal solution if the objective function is changed to
   (i) $z = 4x_1 + 2x_2 + x_3$ (ii) $z = 3x_2 + x_3$

3. In problem 1, a fourth variable is added with the technological (constraint) coefficients as 3, 2 and 4. Determine the optimal solution if the profit per unit of the new variable is given as 5 and 10.

4. Solve this problem using big M-method.
   Max $z = 5x_1 + 2x_2 + 3x_3$ s/t $x_1 + 5x_2 + 3x_3 = 30,\ x_1 - 5x_2 - 6x_3 \le 40,$ all vari $\ge 0.$

5. In problem 4, find the optimal solution, using sensitivity analysis if the objective function is changed to
   (i) Max $z = 12x_1 + 5x_2 + 2x_3$ (ii) Min $z = 2x_2 - 5x_3$

6. In problem 4, suppose that the technological coefficients of x_2 are $(5-a, -5+a)$ instead of $(5, -5)$, where a is a nonnegative parameter. Find the value of a so that the solution remains optimal.

7. In problem 4, suppose that the right hand side of the constraint becomes $(30 + a, 40 - a)$, a is nonnegative parameter. Determine the values of a so that the solution of the problem remain optimal.

8. Solve the LPP: *Minimize* $z = -x_1 + x_2 + x_3$
   Subject to $-2x_1 + x_2 + x_3 \ge 2,\ x_1 - 2x_2 + 2x_3 = 2,\ x_1, x_2, x_3 \ge 0$
   by Big M method. Find the optimal solution of the changed LPP obtained from the above LPP by employing the following (using the concepts of sensitivity analysis):

   (a) Changing the RHS of second constraint to 8.
   (b) Add the constraint $x_1 + x_2 + x_3 \le 1$.
   (c) The cost c_1 of x_1 is changed from -1 to -3.
   (d) Add the constraint $x_1 + x_2 + x_3 \ge 4$.
   (f) Add the variable x_4 with cost -2 and column $(2, -1)^T$

10. (i) Consider the problem $Max\, z = -x_1 + 2x_2 - x_3$ subject to $x_1 + 2x_2 - 2x_3 \leq 4$, $x_1 - x_3 \leq 3$, $2x_1 - x_2 + 2x_3 \leq 2$, $x_1, x_2, x_3 \geq 0$.

The optimal table of the above LPP is:

B.V	x_1	x_2	x_3	s_1	s_2	s_3	Solution
Z	9/2	0	0	3/2	0	1	8
x_2	3	1	0	1	0	1	6
s_2	7/2	0	0	½	1	1	7
x_3	5/2	0	1	½	0	1	4

(a) Find the range of the cost coefficient c_2 of variable x_2 such that present solution remains optimal.

(b) If the RHS of the original problem is changed to (5,4,1) then find the optimal solution.

(c) Find the range of b_2 (RHS of second constraint) so that the present solution remains optimal.

(d) Find the optimal solution after adding a new constraint $3x_1 - x_2 \geq 1$.