Tutorial -Queueing Model

Q1. Customers arrive at a milk parlour being manned by a single Individual at rate of 25 per hour. The time required to serve a customer has exponential distribution with a mean of 30 per hour. Discuss the various characteristics of the queuing system, assuming that there is only one server.

Sol1.

Arrival rate (λ) = 25 per hour, Service rate (μ) = 30 per hour

$$Traffic\ intensity\ (Utilization\ factor)(\rho) = \frac{mean\ arrival\ rate}{mean\ service\ rate} = \frac{\lambda}{\mu} = \frac{25}{30} = \frac{5}{6}$$

Expected number of units in system (L_s) = $\frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\rho}{1-\rho} = \frac{5/6}{1-5/6} = 5\ cutomers$

Expected queue length (L_q) = $\frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho} = \frac{(5/6)^2}{1-5/6} = \frac{25}{6}\ cutomers$

Expected waiting time in the queue $(Wq) = \frac{\lambda}{\mu(\mu\ \lambda)} = \frac{25}{30(30-25)} = \frac{1}{6} hour$

Q2. In a service department manned by one server, on an average 8 customers arrive every 5 minutes while the server can serve 10 customers in the same time assuming Poisson distribution for arrival and exponential distribution for service rate. Determine:

a) Average number of customers in the system.

b) Average number of customers in the queue.

c) Average time a customer spends in the system.

d) Average time a customer waits before being served.

Sol 2.

Arrival rate (λ) = 8/5 =1.6 customers per minute.

Service rate (μ) = 10/5 = 2 customers per minute.

$$Traffic\ intensity\ (\rho) = \frac{mean\ arrival\ rate}{mean\ service\ rate} = \frac{\lambda}{\mu} = \frac{1.6}{2} = 0.8$$

a) Average number of customer in the system.

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho} = \frac{0.8}{1 - 0.8} = 4\ customers$$

b) Average number of customer in the queue.

$$(L_q) = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1 - \rho} = \frac{0.8^2}{1 - 0.8} = 3.2 \text{ customers}$$

c) Average time a customer spends in the system.

$$(W_s) = \frac{1}{\mu \quad \lambda} = \frac{1}{2-1.6} = \frac{1}{0.4} hr = 2.5\ hrs.$$

d) Average time a customer waits before being served

$$W_q = \frac{\lambda}{\mu(\mu \quad \lambda)} = \frac{1.6}{2(2 \quad 1.6)} = 2\text{hrs.}$$

Q3. Consider a single-server queue with infinite buffer space

(a) Consider the situation • The inter-arrival time is a constant and is given by 1 sec • The service time required by each customer is always 0.5 sec 2 What is the mean waiting time per customer?

(b) Consider the situation • The inter-arrival time is exponentially distributed with mean 1 sec • The service time required by each customer is exponentially distributed with mean 0.5 sec What is the mean waiting time per customer?

as service time < inter-arrival time, and inter-arrival time is also constant, there is no queue formed.

Sol 3.

(a) There is no queueing at all in this case. The mean waiting time W is 0 sec

(b) The mean arrival rate is $\lambda = 1$ customers per sec. The mean service rate is $\mu = 2$ customers per sec.

we have $T = 1/(\mu - \lambda) = 1/(2 - 1) = 1$ sec

Therefore, the mean waiting time W is given by $W = T - (1/\mu) = 1 - 0.5 = 0.5$ sec

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Q.4 Arrivals at a telephone both are considered to be Poisson at an average time of 8 min between our arrival and the next. The length of the phone call is distributed exponentially, with a mean of 4 min

Determine

(a) Expected fraction of the day that the phone will be in use.

(b) Expected number of units in the queue Expected waiting time in the queue.

(c) Expected number of units in the system.

(e) Expected waiting time in the system

(f) Expected number of units in queue that from time to time.

(g) What is the probability that an arrival will have to wait in queue for service?

(h) What is the probability that exactly 3 units are in system

(i) What is the probability that an arrival will not have to wait in queue for service?

(j) What is the probability that there are 3 or more units in the system?

(k) What is the probability that an arrival will have to wait more than 6 min in queue for service?

(l) What is the probability that more than 5 units in system

(m) What is the probability that an arrival will have to wait more than 8 min in system?

(n) Telephone company will install a second booth when convinced that an arrival would have to wait for attest 6 min in queue for phone. By how much the flow of arrival is increased in order to justify a second booth.

Sol 4

The mean arrival rate = λ = 1/8 x 60 = 7.5 / hour.
The mean service = μ = x 60 = 15 / hour.

a) Fraction of the day that the phone will be in use

$$\rho = \frac{\lambda}{\mu} = \frac{7.5}{15} = 0.5$$

(b) The expected number pf units in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{7.5^2}{15(15-7.5)}$$
$$L_q = 0.5\,(units)\,person$$

(c) Expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$
$$= \frac{0.5}{7.5} = 0.066\ hrs$$

(d) Expected number of units in the system:-

$L = L_q + \lambda / \mu$
$= 0.5+0.5$
$L = 1$ person

(e) Expected waiting time in the system

$$W = Wq + \frac{1}{\mu}$$
$$= 0.066 + \frac{1}{15} = 0.133$$

(f) Expected number of units in the queue that form from time to time:-

$$D = \frac{\mu}{\mu - \lambda}$$

$$= \frac{15}{15 - 7.5} = 2 \ \ persons$$

(g) Probability that an arrival will have to wait in the system:-

$$P_{ro} = 1 - P_o$$

$$P_o = 1 - \frac{\lambda}{\mu}$$

$$= 1 - \left(1 - \frac{\lambda}{\mu}\right)$$

$$P_{ro} = \frac{\lambda}{\mu} = \frac{7.5}{15} = 0.5$$

(h) The Probability that exactly zero waits in the system:-

$$P_o = 1 - \frac{\lambda}{\mu}$$

$$= 1 - 0.5 \ = \ 0.5$$

(i) The probability that exactly 3 units in the system:-

$$P_n = P_o - \left(\frac{\lambda}{\mu}\right)^n \qquad\qquad n = 3$$

$$P_3 = 0.5(0.5)^3 = 0.0625$$

(j) Probability that an arrival will not have to wait for service:-

$$P_o = 1 - \frac{\lambda}{\mu}$$

$$= 0.5$$

(k) Probability that 3 or more units in the system:-

$$P_{n\ or\ more} = \left(\frac{\lambda}{\mu}\right)^n \qquad n = 3$$

$$P_{n\ or\ more} = 0.5^3 = 0.125$$

(l) Probability that an arrival will have to wait more than 6mins in queue for service

$$P_{ro} = \left(\frac{\lambda}{\mu}\right) e^{(\lambda-\mu)\omega}$$

$$\omega = 6\min = \frac{6}{60}\, hrs$$

$$P_{ro} = 0.5\, e^{(7.5-15)\frac{6}{60}}$$

$$P_{ro} = 0.236$$

(m) Probability that more than 5 units in the system

$$P_{ro} = \left(\frac{\lambda}{\mu}\right)^n \qquad n = 6$$

$$P_{ro} = 0.5^6 = 0.015$$

(n) Probability that an arrival will directly enter for service

P_o= 0.5

(O) Probability that arrival will have to wait more than 8mins in the system.

V = 8 / 60 hrs

$$P_{ro} = e^{(\lambda-\mu)v}$$

$$= e^{(7.5-15)\frac{8}{60}}$$

$$= 0.367$$

(p)

$$W_q = \frac{6}{60}\, hrs \quad = 0.1\, hr$$

$$\therefore \quad W_q = \frac{L_q}{\lambda} = \frac{\lambda^2}{\mu(\mu-\lambda)\lambda}$$

$$0.1 = \frac{\lambda}{15(15-\lambda)}$$

$$\therefore \quad \lambda = 9\, per\ hour.$$

To justify a second booth should be increased from 7.5 to 9 per hour

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Q.5 In a self service store with one cashier, 8 customers arrive on an average of every 5 mins. and the cashier can serve 10 in 5 mins. If both arrival and service time are exponentially distributed, then determine

(a) Average number of customer waiting in the queue for average.

(b) Expected waiting time in the queue

(c) What is the probability of having more than 6 customers In the system

Sol 5

Mean arrival rate =λ= =1.6 x 60

= 96 / hour

Mean service rate = μ = x 60

= 120 / hour.

(a) Average number of customers waiting in queue for service

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{96^2}{120(120-96)}$$

$$L_q = 3.2\ customers$$

(b) Expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{3.2}{96} = 0.033$$

(c) Probability of having more than 6 customers in the system

$$P_{6\ or\ more} = \left(\frac{\lambda}{\mu}\right)^n \qquad where \quad n = 7$$

$$= \left(\frac{96}{120}\right)^7 = 0.209$$

Q.6 Consider a box office ticket window being manned by a single server. Customer arrives to purchase ticket according to Poisson input process with a mean rate of 30/hr. the time required to serve a customer has an ED with a mean of 90 seconds determine:

(a) Mean queue length.

(b) Mean waiting time in the system.

(c) The probability of the customer waiting in the queue for more than 10min.

(d) The fraction of the time for which the server is busy.

Sol 6

The mean arrival rate = $\lambda = \quad 30/hr$

The mean service rate

$$\mu = \frac{1}{90} \times 60 \times 60$$
$$= 40/hr$$

(a) Mean queue length

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \quad \frac{30^2}{40(40-30)} \quad = 2.25\ customers$$

(b) Mean waiting time in the system

$$W = W_q + \frac{1}{\mu}$$
$$= \frac{L_q}{\lambda} + \frac{1}{\mu}$$
$$= \frac{2.25}{30} + \frac{1}{40}$$
$$= 0.1hr$$

(c) Probability of the customer waiting in queue for more than 10min.

$$W\frac{10}{60} = 1/6 hour$$

$$P_{ro} = \left(\frac{\lambda}{\mu}\right)^{e^{(\lambda-\mu)w}}$$

$$= \left(\frac{30}{40}\right)^{e^{(30-40)1/6}}$$

$$P_{ro} = 0.1416$$

(d) Fraction of time the serve is busy

$$\rho = \frac{\lambda}{\mu}$$

$$= \frac{40}{30}$$

$$= 0.75 hr$$

Q.7 A T.V repairman repair the sets in the order in which they arrive and expects that the time required to repair a set has an ED with mean 30mins. The sets arrive in a Poisson fashion at an average rate of 10/8 hrs a day.

(a) What is the expected idle time / day for the repairman?

(b) How many TV sets will be there awaiting for the repair?

Sol 7

Mean arrival rate = λ = $\frac{10}{8}$ hours

Mean service rate = μ = $\frac{1}{30}$ x 60 = 2 hours

(a) Expected idle time / day of the repair

Busy Period = $\frac{\lambda}{\mu}$ = $\frac{1.25}{2}$ = 0.625 hour

$$\therefore idle\ time = P_o = 1 - \frac{\lambda}{\mu} = 1 - 0.625 = 0.375$$

$$\therefore idle\ time / day = 0.375 \times 8 = 3\ hrs / day$$

(b) Number of T.V sets awaiting for the repair:-

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{1.25^2}{2(2-1.25)} = 1.04$$

Q.8 In a bank there is only on window. A solitary employee performs all the service required and the window remains continuously open from 7am to 1pm. It has discovered that an average number of clients is 54 during the day and the average service time is 5mins / person. Find

(a) Average number of clients in the system

(b) Average waiting time

(c) The probability that a client has to spend more than 10mins in a system.

by default will be in queue.

Sol 8

The mean arrival rate = $\lambda = \dfrac{54}{6}$

$= 9\ clients/hour$

The mean service rate $= \dfrac{1}{5} \times 60$

$= 12\ clients/hour$

(a) Average number of customer in the system

$$L = \quad L_q + \frac{\lambda}{\mu}$$

$$= \frac{\lambda}{\mu(\mu - \lambda)} + \frac{\lambda}{\mu}$$

$$= \frac{9^2}{12\,(12-9)} + \frac{9}{12}$$

$$L = 3\ clients$$

(b) Average waiting time:-

$$W_q = \frac{L_q}{\lambda} \quad = \frac{\lambda^2}{\mu(\mu-\lambda)\lambda} \quad = \quad \frac{9}{12(12-9)} \quad = 0.25$$

(c) Probability that a customer has to spend more than 10min in a system.

$$\vartheta = \frac{10}{60} \qquad = 1/6hr$$

$$P_{ro} = e^{(\lambda-\mu)v} \qquad = e^{(9-12)1/6} \qquad = 0.606$$

Q.9 A departmental Secretary receive an average of 8 job / hr. many are short jobs, while other are quiet long. Assume however, that the time to perform a job has an ED mean of 6mins determine

(a) The average elapsed time from the time the secretary receives a job, until it is completed.

(b) Average number of jobs in a system

(c) The probability that the time in the system is greater than ½hr.

(d) Probability of more than 5 jobs in the system.

Sol 9

Mean arrival rate = λ = 8 jobs / hrs

Mean service rate = μ = x 60

= 10 jobs / hrs.

(a) Average elapsed time from the time the secretary receives a job on till it is completed

$$W = Wq + \frac{1}{\mu}$$

$$= \frac{Lq}{\lambda} + \frac{1}{\mu}$$

$$= \frac{\lambda^2}{\mu(\mu - \lambda)\lambda} + \frac{1}{\mu}$$

$$= \frac{8}{10(10-8)} + \frac{1}{10} = 0.5$$

(b) Average number of jobs in the system:-

$$L = L_q + \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2}{\mu(\mu - \lambda)} + \frac{\lambda}{\mu}$$

$$= \frac{8^2}{10(10-8)} + \frac{8}{10}$$

$$L = 4 \ jobs$$

(c) Probability that the customer spends time in the system is greater than ½ hr.

$$v = 0.5 \ hr$$

$$P_{ro} = e^{(\lambda - \mu)v}$$

$$= e^{(8-10)0.5}$$

$$= 0.367$$

(d) Probability of more than 5 jobs in the system:-

$$P_{ro} = \left(\frac{\lambda}{\mu}\right)^n \qquad n = 6$$

$$P_{ro} = \frac{8}{10}^6 = 0.262$$

Q.10 At a one-man barber shop customers arrive according to P.D with a mean arrival rate of 5/hr. The hair cutting time is ED with a haircut taking 10 min on an average assuming that the customers are always willing to wait find:

(a) Average number of customers in the shop

(b) Average waiting time of a customer

doesn't include the service time.

(c) The percent of time an arrival Can walk right without having to wait

(d) The probability of a customer waiting more than 5mins

Sol 10

$$\lambda = 5/hr$$

Mean service rate= $\mu = \frac{1}{10} \times 60$

$$= 6/hr$$

(a) Average number of customer's in the shop.

$$L = L_q + \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}$$

$$= \frac{5^2}{6(6-5)} + \frac{5}{6}$$

L= 5 customers

(b) Average waiting time of a customer.

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)\lambda}$$

$$= \frac{5}{6(6-5)} \qquad = 0.833hr$$

(c) Percent of time arrival can walk right without having to wait.

$$p_o = \left(1 - \frac{\lambda}{q}\right) \times 100$$

$$= 1 - \frac{5}{6} \times 100$$

$$= 16.66\%$$

d) Probability of a customer waiting more than 5mins.

$$W = \frac{5}{60} \quad = 1/12$$

$$\therefore P_{ro} = \left(\frac{\lambda}{\mu}\right)^{e^{(\lambda-\mu)w}}$$

$$= \left(\frac{5}{6}\right)^{e^{(5-6)1/2}}$$

$$\therefore P_{ro} = 0.766$$