

Closure of Attribute Sets

Attribute Closure

Given a set of attributes α , define the **closure** of α under F (denoted by α^+) as the set of attributes that are functionally determined by α under F

α^+

$R(ABCD)$

$F = \{A \rightarrow B, B \rightarrow C, C \rightarrow D\}$

F.D.

$A \rightarrow A$

$B^+ = BCD$

$C^+ = CD$

$D^+ = D$

$(AB)^+ = ABCD$

$A^+ = ABCD$

$A^+ = ABCD$

$A \rightarrow A$	$A \rightarrow BC$	$A \rightarrow ABC$	$A \rightarrow ABCD$
$A \rightarrow B$	$A \rightarrow CD$	\cup_{C_3}	\cup_{C_4}
$A \rightarrow C$	\cup_{C_2}		
$A \rightarrow D$	\cup_{C_1}		

$A \rightarrow R$

$$F : \{ AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A \}$$

$$(CF)^+ = CFGADE$$

$$(BG)^+ = BGACD$$

$$(AF)^+ = AFDE$$

$$(AB)^+ = ABCDG$$

Inference Rules

IR1 (reflexive rule): If $X \supseteq Y$, then $X \rightarrow Y$. ✓

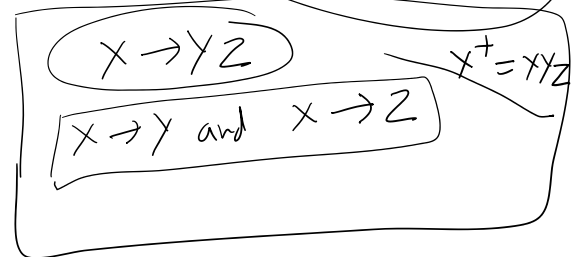
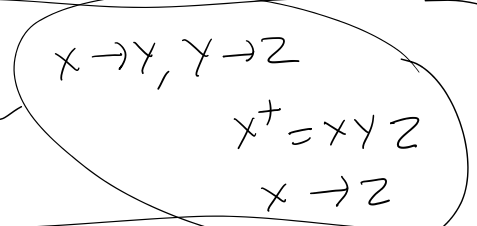
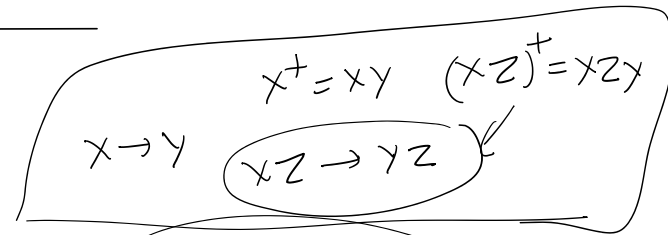
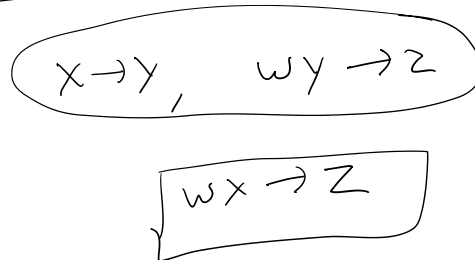
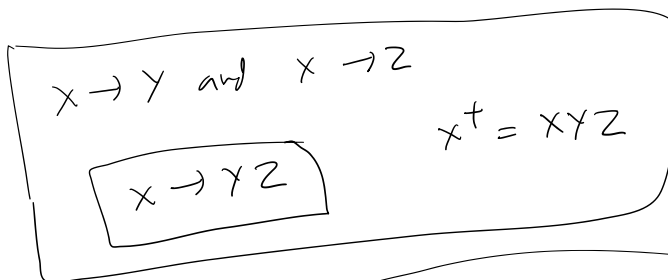
IR2 (augmentation rule): $\{X \rightarrow Y\} \models XZ \rightarrow YZ$. ✓

IR3 (transitive rule): $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$. ✓

IR4 (decomposition or projective rule): $\{X \rightarrow YZ\} \models X \rightarrow Y$ and $X \rightarrow Z$. ✓

IR5 (union or additive rule): $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$. ✓

IR6 (pseudotransitive rule): $\{X \rightarrow Y, WY \rightarrow Z\} \models WX \rightarrow Z$. ✓



Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
 - ✓ To test if α is a superkey, we compute α^+ and check if α^+ contains all attributes of R .
- Testing functional dependencies
 - ✓ To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
 - That is, we compute α^+ by using attribute closure, and then check if it contains β .
 - Is a simple and cheap test, and very useful
- Computing closure of F
 - ✓ For each $\gamma \subseteq R$, we find the closure γ^+ , and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Candidate key and Super key using Attribute Closure

$K \rightarrow R$ means K^+ determines all the attributes of the relation R

K is a superkey for relation schema R if and only if $K \rightarrow R$

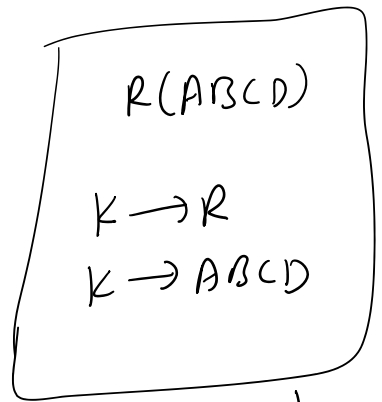
K is a candidate key for R if and only if

- $K \rightarrow R$, and
- for no $\alpha \subset K$, $\alpha \rightarrow R$

$$(K : SK) \Leftrightarrow (K \rightarrow R)$$

$$(K : CK) \Leftrightarrow ((K \rightarrow R) \wedge (\text{minimal Cond}^n))$$

$$((K : CK)) \Leftrightarrow ((K \nrightarrow R) \vee (\text{fail minimal Cond}^n))$$



$$P \Leftrightarrow Q \quad a' \Leftrightarrow P'$$

T	T
F	F

✓

T	F
F	T

✗

$$P' \Leftrightarrow Q'$$

$$R(ABCD) \quad F := \{A \rightarrow B, B \rightarrow C, C \rightarrow D\}$$

$$CK : A$$

$$A^+ = ABCD \text{ or } R$$

$$SK : A$$

$$A \rightarrow R$$

$$R(ABCDE) \quad \{AB \rightarrow C, C \rightarrow D, B \rightarrow E\}$$

$$(AB)^+ = ABCDE = R$$

$$AB \rightarrow R$$

$$S.K. :- AB$$

$$C.K. :- AB$$

$$A^+ = A$$

$$B^+ = BE$$

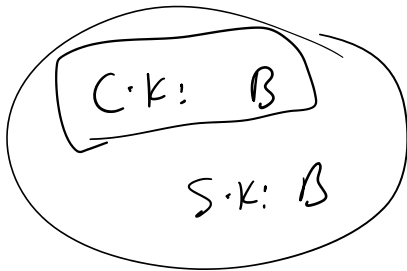
$$A \nrightarrow R$$

$$R(ABCDE)$$

$R(ABCDE)$

$\{AB \rightarrow C, C \rightarrow D, B \rightarrow EA\}$

$$(AB)^+ = R$$



$$A^+ = A$$

$$B^+ = R$$

$$B \rightarrow R$$

$$A \not\rightarrow R$$

$$B \not\rightarrow R$$

AB: SK ✓

: CK ✗

$R(AB CDEF)$

$\{A \rightarrow BCDEF, BC \rightarrow ADEF\}$

$B \rightarrow F, D \rightarrow E$

$$A^+ = R$$

$$A \rightarrow R$$

CK: A, BC

Prime attribute = $\{A, B, C\}$

CK: A, BC

BC
|
A

BC
|
A

$$(BC)^+ = R$$

$$(BC)^+ = BC ADEF$$

$$BC \rightarrow R$$

$$(C)^+ = C$$

$$(B)^+ = BF$$

$R(AB CDEF)$

$\{AB \rightarrow C, C \rightarrow D, D \rightarrow E, E \rightarrow F, F \rightarrow A\}$

$$(AB)^+ = R$$

AB, CB, DB, FB, EB

$R(AB CDE)$

CK: ABCDE

$R(AB CDE)$

$F: \{ \}$

$CK: AB CDE$

$AB CDE \rightarrow R$

$R(AB CDE) \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A \}$

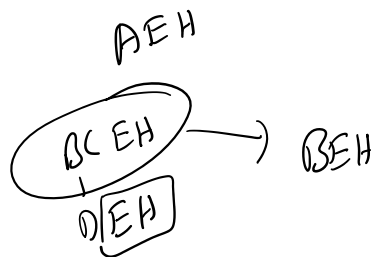
$CK: AE, BE, CE, DE$

$R(AB CDEH)$

$\{ A \rightarrow B, BC \rightarrow D, E \rightarrow C, D \rightarrow A \}$

BEH, AEH, DEH

BEH



$P.A = \{ B, E, H, A, D \}$

$R(AB CD)$

$\{ AB \rightarrow C, C \rightarrow D, \textcircled{CE \rightarrow F, F \rightarrow G} \}$

$(AB)^+ = R$

$C.K. = AB$