M.M. 100 | **END SEMESTER EXAMINATION** | **Time: 3 Hrs**

Optimization Techniques (MAN-010)

Q1 Solve the following LPP using Two phase method:

Maximize $z = -2x_1 - x_2$

Subject to $3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 3,\ x_1, x_2 \geq 0.$ **(10)**

Q2. Show analytically that the set $S = \{(x, y) \in R^2 : x^2 + 4y^2 \leq 16\} \subset R^2$ is a convex set. **(6)**

Q3. Consider the LPP: Max z = $3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2,\ x_1 + 4x_2 + 2x_3 \geq 4,\ x_3 \leq 1,\ x_1, x_2, x_3 \geq 0$

The optimal table for the above LPP (using Big M) is obtained as:

B.V.	x_1	x_2	x_3	s_1	s_2	s_3	Solution b
$z_j - c_j$	1	0	0	2	0	1	5
s_2	7	0	0	4	1	−2	2
x_2	2	1	0	1	0	−1	1
x_3	0	0	1	0	0	1	1

where s_1 and s_3 are the slack variables corresponding to first and third constraint respectively and s_2 is the surplus variable corresponding to second constraint. Find

(i) the range of c_2 (cost coefficient of x_2) so that the current solution remains optimal.

(ii) the new optimal solution (if any) if R.H.S. of third constraint is changed to 2 from 1.

(iii) the new optimal solution if an additional constraint $4x_2 - x_3 = 2$ is added to original problem. **(14)**

Q4. Use dominance rule to reduce the size of the following payoff matrix of player A. Hence, find the optimal strategy and value of the game:

Player A \ *Player B*	B_1	B_2	B_3	B_4
A_1	1	3	11	4
A_2	8	5	2	7
A_3	4	3	−1	9

(12)

Q5. Using complementary slackness theorem, verify that *(0, 0,…, 0, 10)* is an optimal solution of the LPP:

Minimize $z = \sum_{j=1}^{10} jx_j$, Subject to $\sum_{j=i}^{10} x_j \geq i, (i = 1, 2, ..., 10)$ and $x_j \geq 0, (j = 1, 2, ..., 10)$. **(8)**

Q6. Use branch and bound to find all integer optimal solutions of the following LPP:

$$\begin{aligned}
&\text{Max } z = x_1 + x_2 \\
&\text{s/t } 2x_1 + 5x_2 \leq 16, \\
&\quad 6x_1 + 5x_2 \leq 30, \\
&\quad x_1, x_2 \geq 0 \text{ and are integers}
\end{aligned}$$

(10)

Q7 Consider the following cost **MAXIMIZING** transportation problem (TP):

Table1

	D1	D2	D3	D4	D5	a_i
S1	x				x	10
S2			x			15
S3	x		x	x		12
S4		x			x	15
b_j	5	8	20	4	15	

Table 2

	D1	D2	D3	D4	D5
S1	4	15	8	10	18
S2	16	16	24	12	7
S3	8	16	24	9	5
S4	3	10	12	17	4

Start with the cells marked with "x" (in Table 1) as an initial BFS, find the optimal solution of (TP). Table 2 is the cost matrix of the problem.

(12)

Q8. A college is having a degree course which in its present semester has five subjects. The college has considered 6 existing faculty members to teach these courses. The objective is to assign the best five teachers out of these six faculty members to teach five different subjects so that the total numbers of class hours required is minimum. Number of hours required by each faculty to teach every subject is mentioned in the table given below.

		Subjects				
		1	**2**	**3**	**4**	**5**
	1	30	39	31	38	40
	2	43	37	32	35	38
Faculty	**3**	34	41	33	41	34
	4	39	36	43	32	36
	5	32	49	35	40	37
	6	36	42	35	44	42

Find the optimal assignment of the problem if

(i) No restriction

(ii) Faculty 6 is not allowed to take Subject 3.

(10)

Q9. Use **graphical method** to minimize the time needed to process the following jobs on the machines shown below. Calculate the total time needed to complete both the jobs and the idle time for both jobs.

Job 1	Sequence of Machines: Time (in hours)	A 2	B 3	C 4	D 6	E 2
Job 2	Sequence of Machines: Time (in hours)	C 4	A 5	D 3	E 2	B 6

(8)

Q10. Consider the data of a project as shown in the following table

Activity (in weeks)	Normal time (in weeks)	Normal cost (in Rs)	Crash time (in weeks)	Crash cost (in Rs)
(1, 2)	20	200	15	300
(1, 3)	10	150	7	240
(2, 5)	15	100	10	150
(3, 4)	16	300	12	400
(3, 5)	22	450	16	570
(4, 5)	14	150	10	210

(a) Find the critical path and normal cost of the project.

(b) Using the crashing technique, find the most economical schedule for 37 weeks. **(10)**