1. T=(((w (λx. (λy. (λz. ((x z)(y z)))))) u) v)
=(1 (2 (3 w (4 λx. (5 λy. (6 λz. (7 (8 x z 8)(9 y z 9) 7) 6) 5) 4) 3) u 2) v 1)
Let P = (4 λx. (5 λy. (6 λz. (7 (8 x z8)(9 y z 9) 7) 6) 5) 4)
T = (1 (2 (3 w P 3) u 2) v 1)
= (1 (2 (3 w P 3) u 2) v 1)
Let Q = (3 w P 3)
T = (1 (2 Q u 2) v 1)
Let R = (2 Q u 2)
T = (1 R v 1)
= R v no ambiguity since (M N)≡ M N
= (2 Q u 2)v
= Q u v by left associativity of application f g h ≡ (f g) h
= (3 w P 3) u v
= w P u v by left associativity of application (1)
P = (4 λx. (5 λy. (6 λz. (7 (8 x z8)(9 y z 9) 7) 6) 5) 4)
Now P is of the form (4 λx. P′ 4)
From (1): (4 cannot be removed, otherwise meaning would be changed, since now (1) becomes w λx. P′ u v ≡ w λx.(P′ u v)
Since (5, (6 are within the scope of (4, so it can be removed without changing the meaning
Since (7 is within the scope of (6, so it can be removed without changing the meaning
If (8, (9 are removed then we get x z y z ≡ ((x z) y) z whose meaning is different from that given. So both (8, (9 cannot be removed. But removing (8 only does not change the meaning since by left associativity(x z)(y z) ≡ x z(y z)
So we get
P = (λx. λy. λz. x z(y z)) (2)
Substituting (2) into (1) gives:
T = w (λx. λy. λz. x z(y z)) u v

2. (a) M = (λx. x y) λz. w λw. w z y x
FV(x) = {x}   FV(M N) = FV(M) U FV(N)   FV(λx.M) = FV(M)\{x}

M = M1 M3
M1 = (λx. x y)   M3 = λz. w M2   M2 = λw. w z y x
FV(M1) = FV(x y)\{x} by rule of abstraction
= (FV(x) U FV(y)) \{x} by rule of application
= {x} U {y} \ {x} by rule FV(x) = {x}
= {y}
FV(M2) = ({z} U {y} U {x} U {w}) \{w} = {x,y,z} by rule of application, abstraction, and FV(x) = {x}

$FV(M3) = (\{w\} \cup \{x,y,z\}) \setminus \{z\} = \{w,x,y\}$ by rule of application, abstraction
$FV(M) = FV(M1) \cup FV(M3) = \{y\} \cup \{w,x,y\} = \{x,y,w\}$

(b) $M = x\ \lambda z.\ x\ \lambda w.\ w\ z\ y$
$= x\ \lambda z.\ x\ M1 \qquad M1 = \lambda w.\ w\ z\ y$
$= x\ M2 \qquad M2 = \lambda z.\ x\ M1$
$FV(M1) = FV(w\ z\ y) \setminus \{w\} =$
$FV(w) \cup FV(z) \cup FV(y) \setminus \{w\} =$
$\{w,z,y\} \setminus \{w\} = \{y,z\}$ by rule of application, abstraction, and $FV(x) = \{x\}$
$FV(M2) = FV(x\ M1) \setminus \{z\} =$
$FV(x) \cup FV(M1) \setminus \{z\} =$
$\{x\} \cup \{y, z\} \setminus \{z\}$
$= \{x,y\}$ by rule of application, abstraction, and $FV(x) = \{x\}$
$FV(M) = FV(x) \cup FV(M2) = \{x\} \cup \{x,y\} = \{x,y\}$ by rule of application and $FV(x) = \{x\}$

3. T=(1 (2 λf. (3 (4 λg. (5 (6 f f 6) g 5) 4) (7 λh. (8 k h 8) 7) 3) 2) (9 λx. (10 λy. y 10) 9) 1)
T= (2 λf...... 2) N where N = (λx. (λy. y))
T=β ((λg. ((N N) g)) (λh. (k h)))) [N/f]
Let Q= (λh. (k h))
T=β (N N) Q [Q/g]
NN =β (λy. y) since x does not occur in the body
(λx. P1) P2 =β P1 if x does not occur in P1
T = (λy. y) Q =β (λh. (k h)) [Q/y]

[there are other reductions possible, but some intermediate steps would be same as above]

4. λx.λy.λz. x y z z
Case2: M1 = λy.λz. x y z z
Case2: M2 = λz. x y z z
Case3: M3 = x y z z
= ((x y) z) z by left associativity
Case3: M4: (x y)
Case1: let x : a->b y : a
M4 : b Now (M4 z) cannot be unified, so x: a-> b->c
now M4 : b->c z:b Now (M4 z) : c which cannot be unified with z:b So we modify x:a-> b->b->c, now M4 z: b->c, M3 : c
so the PT of the given term becomes (a->b->b->c)->a->b->c