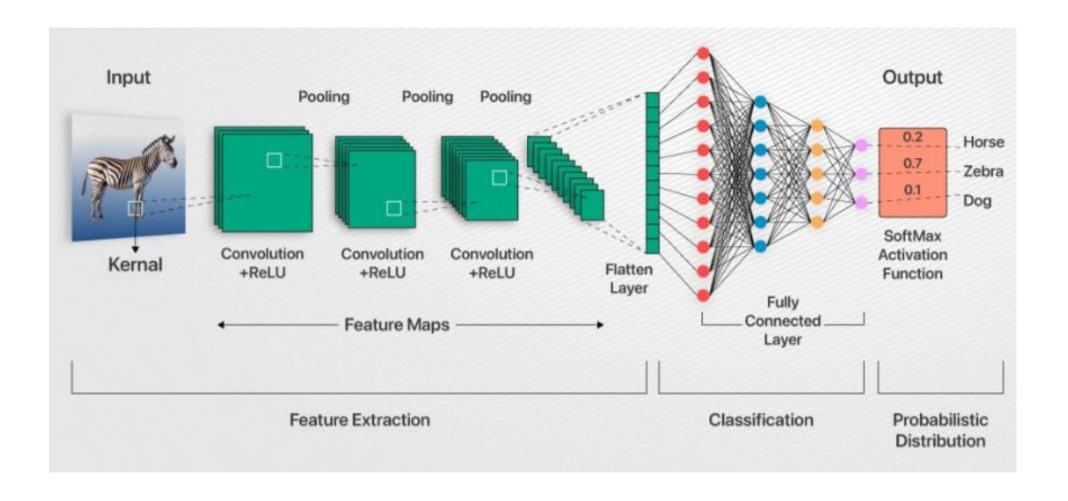
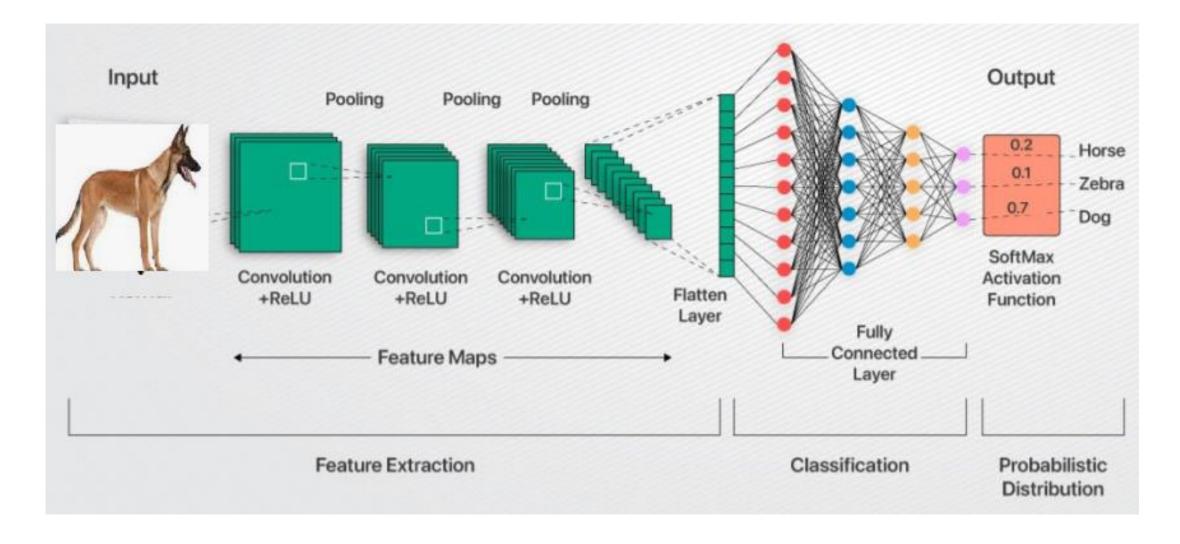
Sequence Learning

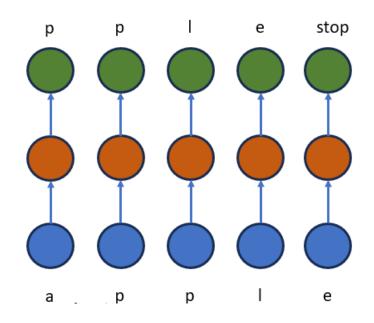


In CNNs and feed forward network size of input was always fixed. (For the whole dataset, the size of the input image is fixed.)

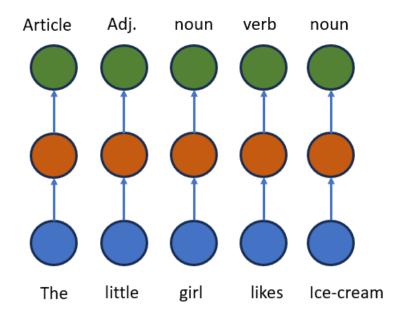


Also every input is independent of previous or future inputs.

- But there are some applications where input size is not fixed
- Also inputs can be dependent

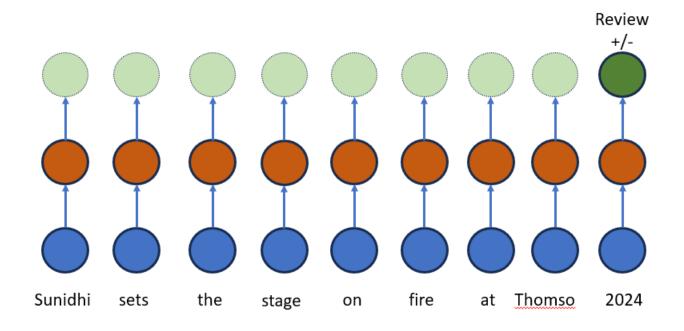


- Example of Auto completion (Predict the next character or word = Language Modelling)
- A sequence learning problem considering each input as one time step



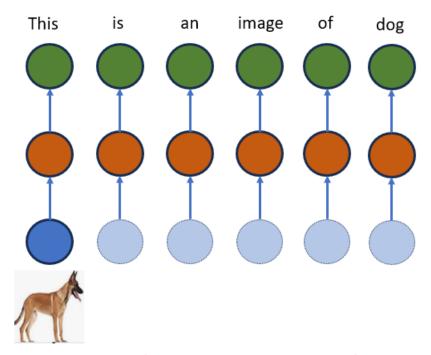
(Example of Parts Of Speech Tagging)

- Task of tagging parts of sentence
- Here input and output are available at every time step



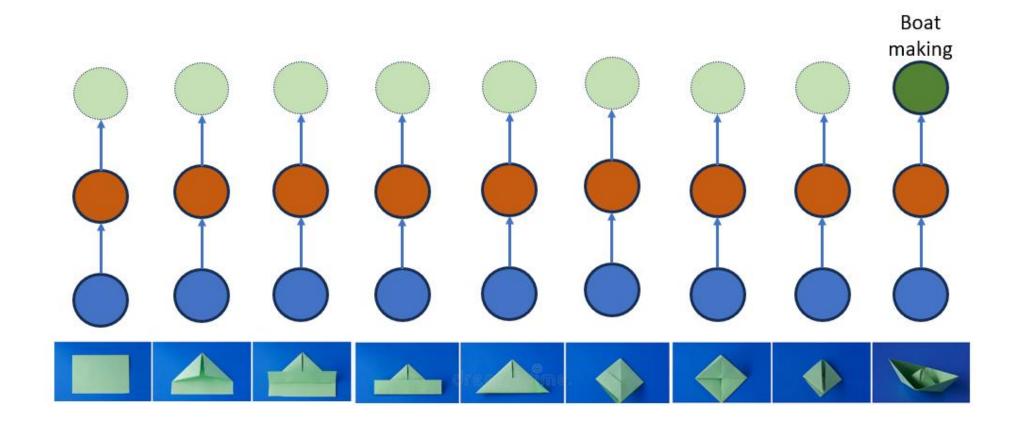
(many to one RNN)

• Sometimes there is no need of output at every time step



(one to Many RNN)

• Image to caption

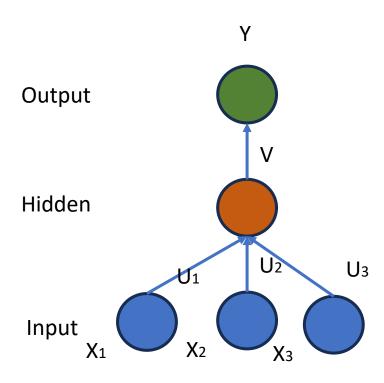


Video activity prediction example

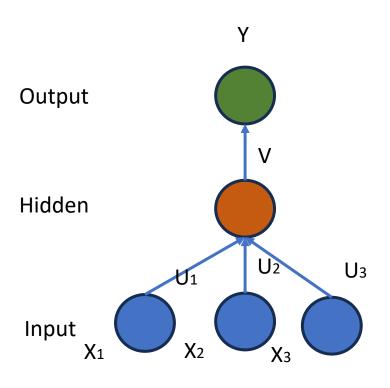
So, there is need of network which works on sequences which can

- Handle variable number of inputs
- Check dependency among inputs
- Ensure same function execution at each time step (same parameter)

Dependency among inputs



Dependency among inputs



Problem:

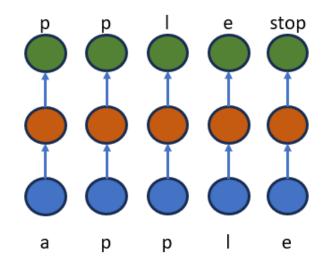
Every time calculated function is different

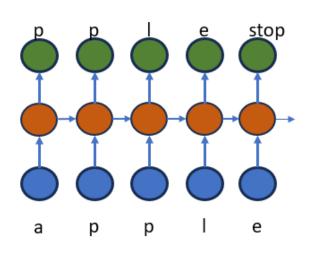
Y=
$$f_1(X_1)$$

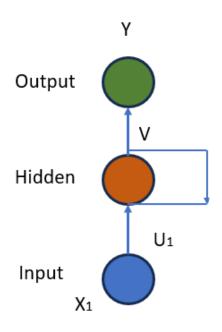
Y= $f_2(X_1, X_2)$
Y= $f_3(X_1, X_2, X_3)$

Trying to solve a sequence learning problem through ANN. X1 will go first and we want that the function estimated will be of x1 only. Then in the next timestep, x2 will go and function estimated will be of x1 and x2...and so on.

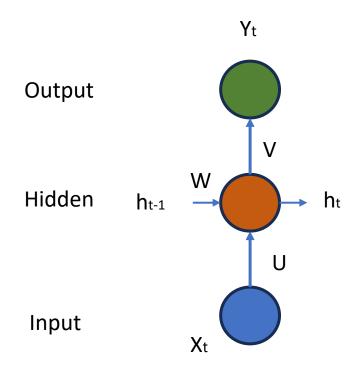
Solution: Recurrent connection







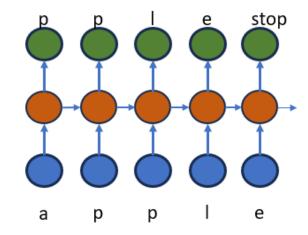
Function

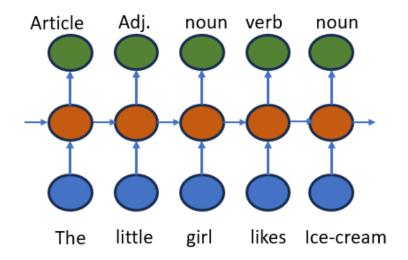


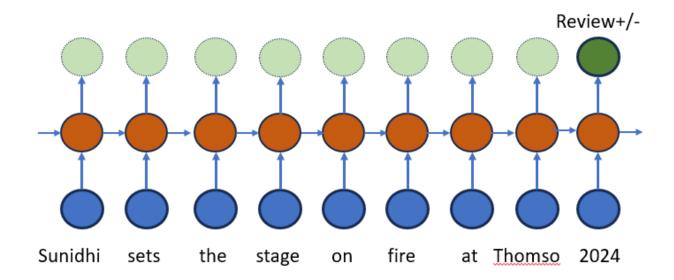
$$h_t = f_{UW}(x_t, h_{t-1})$$

new state / input vector at old state some time step some function with parameters U_&W

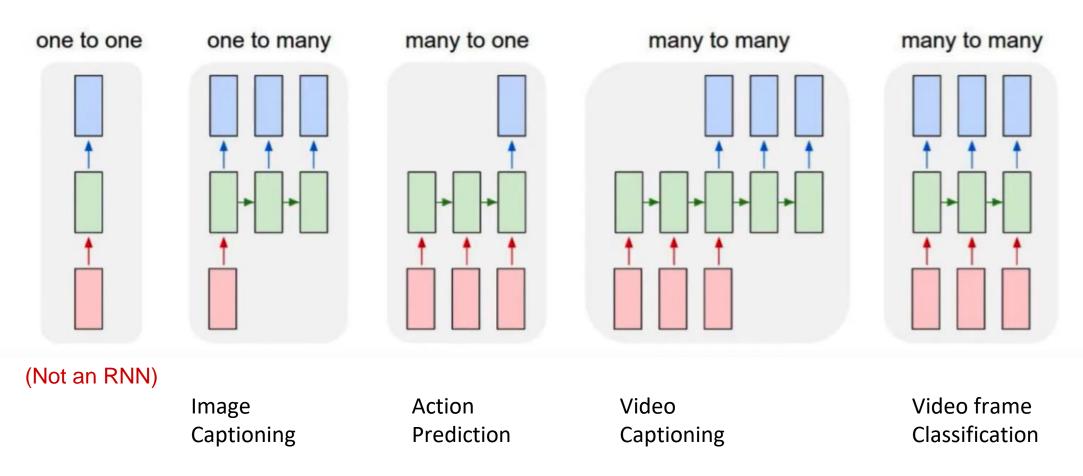
 $h_t = act_fn(UX_t + Wh_{t-1} + b)$
 $h_t = tanh(UX_t + Wh_{t-1} + b)$
 $h_t = out_fn(Vh_t + b)$
 $h_t = softmax(Vh_t + b)$
 $t = time-step$



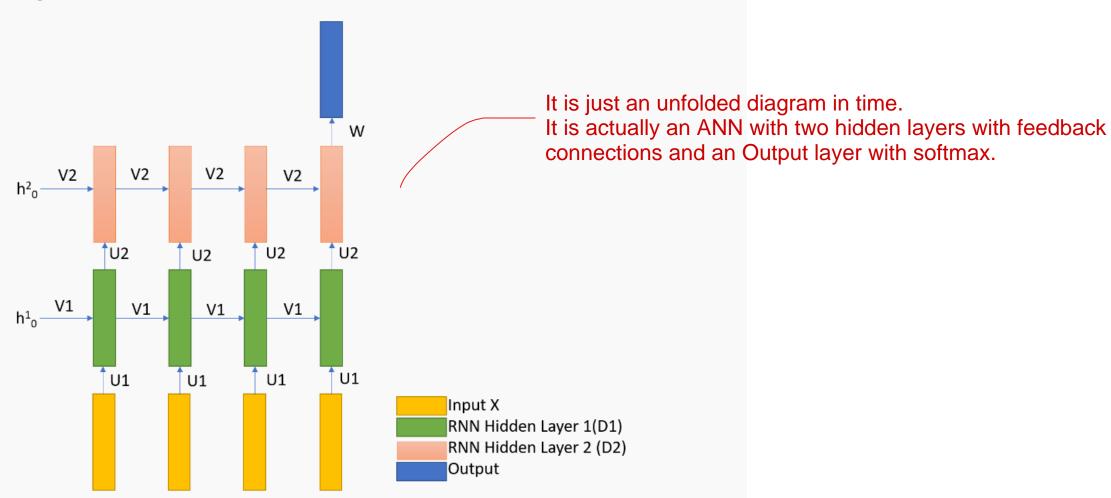




Variants



The RNN given below is used for classification:



The dimensions of the layers of RNN are as follows:

Input X $\in \mathbb{R}^{234}$

Hidden Layer 1 D1 $\in \mathbb{R}^{1024}$

Hidden Layer 2 D2 $\in \mathbb{R}^{512}$

Number of classes:15'

Note: Do not consider the bias term'

Solution

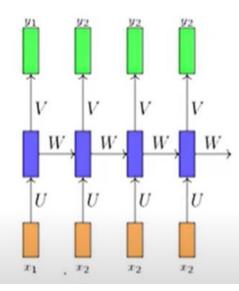
U1= 234*1024 =239616

V1=1024*1024=1048576

U2= 1024*512=524288

V2= 512*512=262144

W=512*15=7680



$$x_i \in \mathbb{R}^n$$
 (n-dimensional input)

$$s_i \in \mathbb{R}^d$$
 (d-dimensional state)

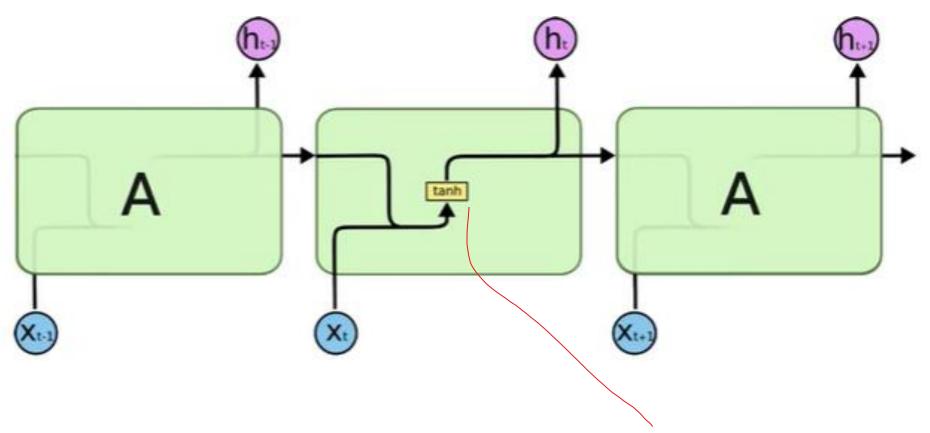
$$y_i \in \mathbb{R}^k$$
 (say k classes)

$$U \in \mathbb{R}^{n \times d}$$

$$V \in \mathbb{R}^{d \times k}$$

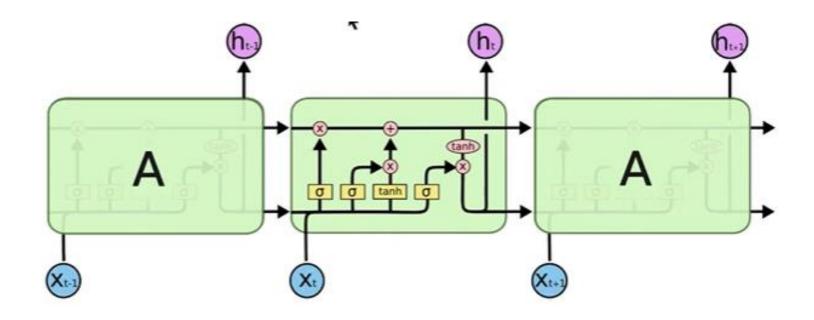
$$W \in \mathbb{R}^{d \times d}$$

RNN

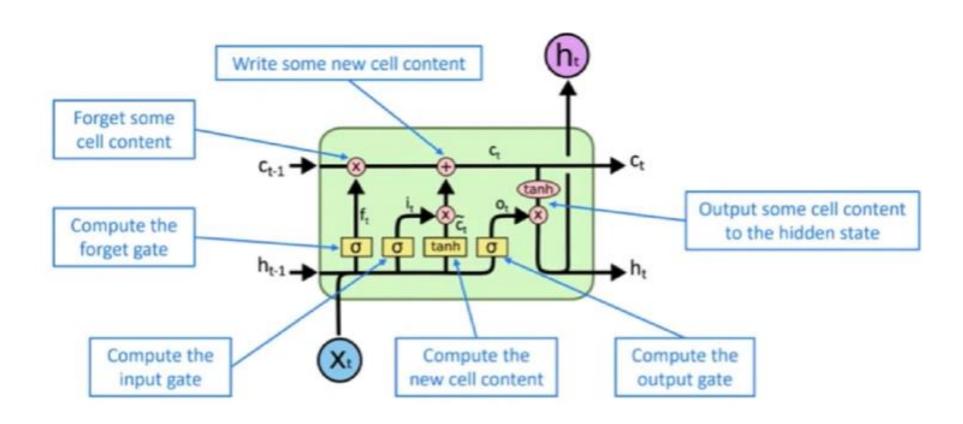


Default activation used in an RNN layer will be tanh.

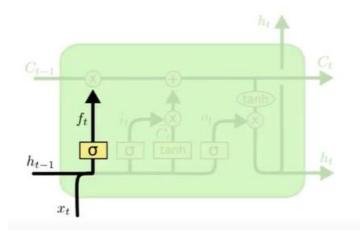
LSTM



There are three gates in LSTM: Forget, Input and output.

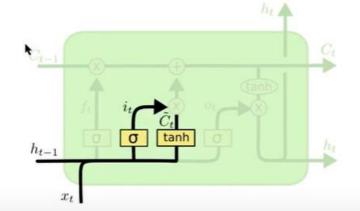


Forget gate: controls what is kept vs what is forgotten, from previous cell state



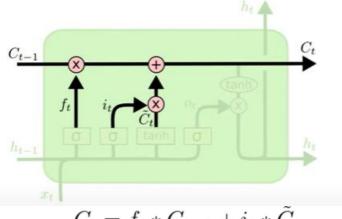
$$f_t = \sigma \left(W_f \cdot [h_{t-1}, x_t] + b_f \right)$$

Input gate: decides what information to throw away from the cell state Cell content: new content to be written to cell



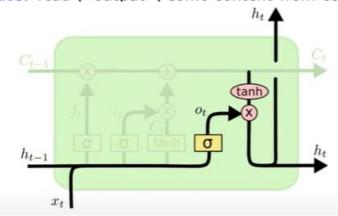
$$i_t = \sigma \left(W_i \cdot [h_{t-1}, x_t] + b_i \right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Output gate: controls what parts of cell are output to hidden state Hidden state: read ("output") some content from cell



$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

Forget gate: controls what is kept vs forgotten, from previous cell state

Input gate: controls what parts of the new cell content are written to cell

Output gate: controls what parts of cell are output to hidden state

New cell content: this is the new content to be written to the cell

<u>Cell state</u>: erase ("forget") some content from last cell state, and write ("input") some new cell content

Hidden state: read ("output") some content from the cell

Sigmoid function: all gate values are between 0 and 1

$$f_{t} = \sigma (W_{f} \cdot [h_{t-1}, x_{t}] + b_{f})$$

$$i_{t} = \sigma (W_{i} \cdot [h_{t-1}, x_{t}] + b_{i})$$

$$o_{t} = \sigma (W_{o} [h_{t-1}, x_{t}] + b_{o})$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$h_t = o_t * \tanh(C_t)$$

Recurrent Neural Network

- In this chapter we will learn about
 - Fundamental concepts in RNNs
 - The main problem RNNs face
 - And the solution to the problems
 - How to implement RNNs
 - Variations of the RNNs
 - Convolution with RNN

Recurrent Neural Network

- Predicting the future is what we do all the time
 - Finishing a friend's sentence
 - Anticipating the smell of coffee at the breakfast or
 - Catching the ball in the field

Recurrent Neural Network

- Unlike all the nets we have discussed so far
 - RNN can work on sequences of arbitrary lengths
 - Rather than on fixed-sized inputs

Recurrent Neural Network - Applications

- RNN can analyze time series data
 - Such as stock prices, and
 - Tell you when to buy or sell



Recurrent Neural Network - Applications

- In autonomous driving systems, RNN can
 - Anticipate car trajectories and
 - Help avoid accidents



Recurrent Neural Network - Applications

- RNN can take sentences, documents, or audio samples as input and
 - Make them extremely useful
 - o For natural language processing (NLP) systems such as
 - Automatic translation
 - Speech-to-text or
 - Sentiment analysis







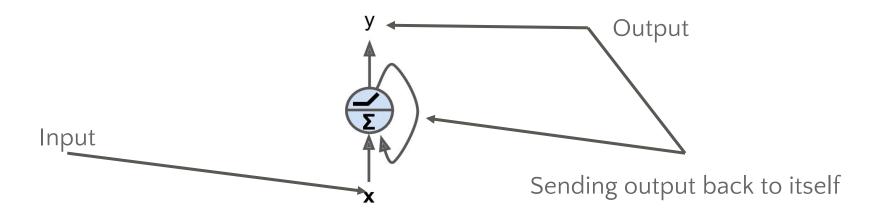
Neutra



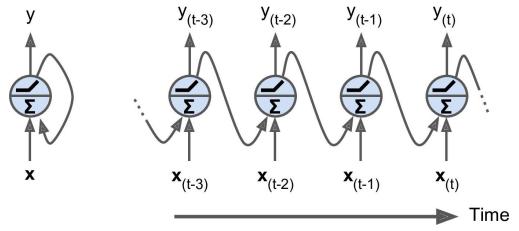
Positive

- RNN looks much like a feedforward neural network
 - Except it has connections pointing backward

The simplest possible RNN

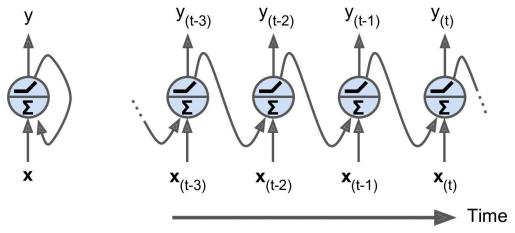


- At each time step t (also called a frame)
 - o This recurrent neuron receives the inputs $\mathbf{x}_{(t)}$
 - o As well as its own output from the previous time step $_{y(t-1)}$



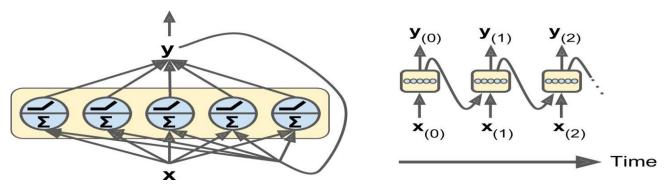
A recurrent neuron (left), unrolled through time (right)

- We can represent this tiny network against the time axis (See below figure)
- This is called *unrolling the network through time*



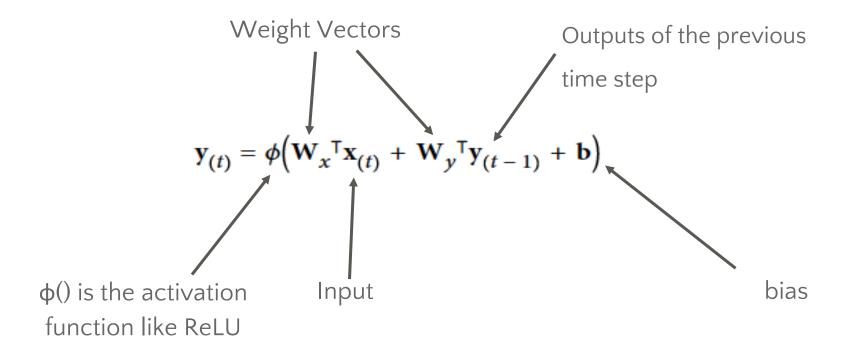
A recurrent neuron (left), unrolled through time (right)

- We can easily create a layer of recurrent neurons
- At each time step t, every neuron receives both the
 - \circ Input vector $\mathbf{x}_{(t)}$ and
 - Output vector from the previous time step $y_{(t-1)}$



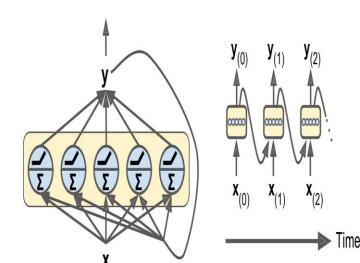
A layer of recurrent neurons (left), unrolled through time(right)

Output of a single recurrent neuron for a single instance



Vectorized form of previous equation

$$\begin{aligned} \mathbf{Y}_{(t)} &= \phi \Big(\mathbf{X}_{(t)} \mathbf{W}_x + \mathbf{Y}_{(t-1)} \mathbf{W}_y + \mathbf{b} \Big) \\ &= \phi \Big(\Big[\mathbf{X}_{(t)} \quad \mathbf{Y}_{(t-1)} \Big] \mathbf{W} + \mathbf{b} \Big) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \end{bmatrix} \end{aligned}$$



Used to compute a whole layer's output in one-shot for a whole mini-batch

$$\mathbf{Y}_{(t)} = \phi \left(\mathbf{X}_{(t)} \mathbf{W}_{x} + \mathbf{Y}_{(t-1)} \mathbf{W}_{y} + \mathbf{b} \right)$$

$$= \phi \left(\begin{bmatrix} \mathbf{X}_{(t)} & \mathbf{Y}_{(t-1)} \end{bmatrix} \mathbf{W} + \mathbf{b} \right) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_{x} \\ \mathbf{W}_{y} \end{bmatrix}$$

- Y_(t) is an m x n_{neurons} matrix containing the
 - Layer's outputs at time step t for each instance in the minibatch
 - o m is the number of instances in the mini-batch
 - n_{neurons} is the number of neurons

$$\mathbf{Y}_{(t)} = \phi \left(\mathbf{X}_{(t)} \mathbf{W}_{x} + \mathbf{Y}_{(t-1)} \mathbf{W}_{y} + \mathbf{b} \right)$$

$$= \phi \left(\begin{bmatrix} \mathbf{X}_{(t)} & \mathbf{Y}_{(t-1)} \end{bmatrix} \mathbf{W} + \mathbf{b} \right) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_{x} \\ \mathbf{W}_{y} \end{bmatrix}$$

- $X_{(t)}$ is an m × n_{inputs} matrix containing the inputs for all instances
 - o n_{inputs} is the number of input features

$$\mathbf{Y}_{(t)} = \phi \left(\mathbf{X}_{(t)} \mathbf{W}_{x} + \mathbf{Y}_{(t-1)} \mathbf{W}_{y} + \mathbf{b} \right)$$

$$= \phi \left(\begin{bmatrix} \mathbf{X}_{(t)} & \mathbf{Y}_{(t-1)} \end{bmatrix} \mathbf{W} + \mathbf{b} \right) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_{x} \\ \mathbf{W}_{y} \end{bmatrix}$$

- W_x is an $n_{inputs} \times n_{neurons}$ matrix containing the connection weights for the inputs of the current time step
- W_y is an $n_{neurons} \times n_{neurons}$ matrix containing the connection weights for the outputs of the previous time step

$$\mathbf{Y}_{(t)} = \phi \left(\mathbf{X}_{(t)} \mathbf{W}_{x} + \mathbf{Y}_{(t-1)} \mathbf{W}_{y} + \mathbf{b} \right)$$

$$= \phi \left(\begin{bmatrix} \mathbf{X}_{(t)} & \mathbf{Y}_{(t-1)} \end{bmatrix} \mathbf{W} + \mathbf{b} \right) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_{x} \\ \mathbf{W}_{y} \end{bmatrix}$$

- The weight matrices W_x and W_y are often concatenated into a single weight matrix W of shape $(n_{inputs} + n_{neurons}) \times n_{neurons}$
- **b** is a vector of size n_{neurons} containing each neuron's bias term

Memory Cells

- Since the output of a recurrent neuron at time step t is a

- We can say that it has a form of *memory*
- A part of a neural network that
 - o Preserves some state across time steps is called a memory cell

Memory Cells

- In general a cell's state at time step t, denoted $h_{(t)}$ is a
 - o Function of some inputs at that time step and
 - o Its state at the previous time step $h_{(t)} = f(h_{(t-1)}, x_{(t)})$
- Its output at time step t, denoted $y_{(t)}$ is also a
 - Function of the previous state and the current inputs

Memory Cells

- In the case of basic cells we have discussed so far
 - The output is simply equal to the state
 - o But in more complex cells this is not always the case

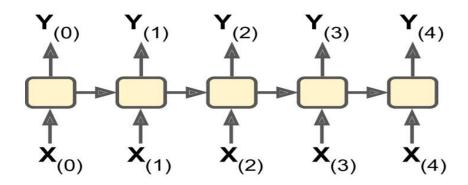
y $y_{(0)}$ $y_{(1)}$ $y_{(2)}$ h $h_{(0)}$ x $x_{(0)}$ x $x_{(1)}$ x xTime

The output of an RNN, LSTM or GRU cell may be different from the state of the cell at that time step.

A cell's hidden state and its output may be different

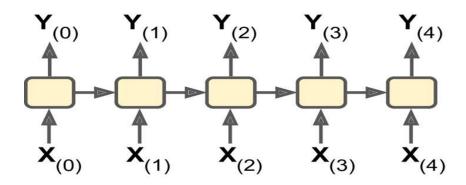
Sequence-to-sequence Network

- An RNN can simultaneously take a
 - Sequence of inputs and
 - Produce a sequence of outputs



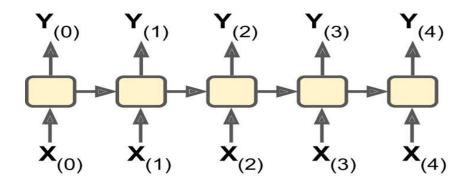
Sequence-to-sequence Network

- This type of network is useful for predicting time series
 - Such as stock prices



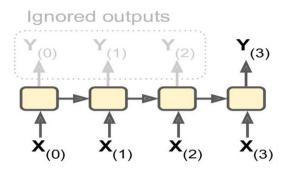
Sequence-to-sequence Network

- We feed it the prices over the last N days and
 - o It must output the prices shifted by one day into the future
 - i.e., from N 1 days ago to tomorrow



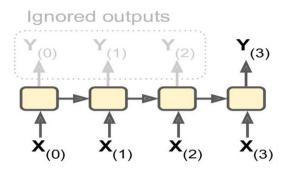
Sequence-to-vector Network many to one

- Alternatively we could feed the network a sequence of inputs and
 - Ignore all outputs except for the last one



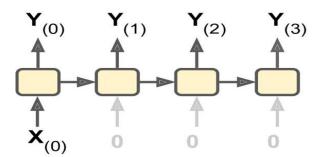
Sequence-to-vector Network

- We can feed this network a sequence of words
 - Corresponding to a movie review and
 - The network would output a sentiment score
 - e.g., from -1 [hate] to +1 [love]

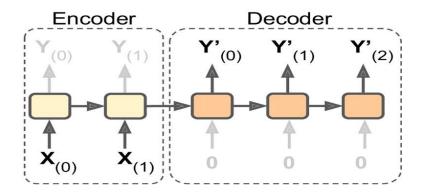


Vector-to-sequence Network one to many

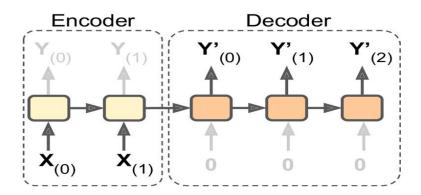
- We could feed the network a single input at the first time step and
 - Zeros for all other time steps and
 - Let it output a sequence
- For example, the input could be an image and the
 - Output could be a caption for the image



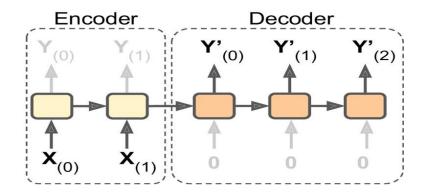
- In this network, we have
 - o sequence-to-vector network, called an encoder followed by
 - o vector-to-sequence network, called a decoder



- This can be used for translating a sentence
 - From one language to another



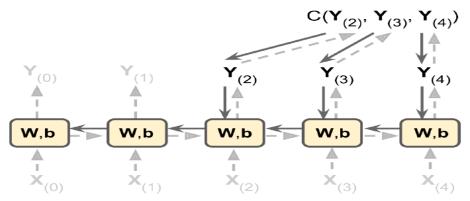
- We feed the network sentence in one language
 - The encoder converts this sentence into single vector representation
 - Then the decoder decodes this vector into a sentence in another language



- This two step model works much better than
 - Trying to translate on the fly with a
 - Single sequence-to-sequence RNN
- Since the last words of a sentence can affect the
 - First words of the translation
 - So we need to wait until we know the whole sentence

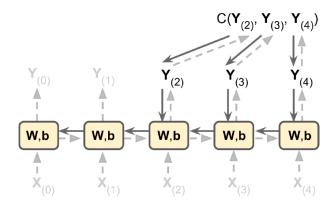


• To train an RNN, the trick is to unroll it through time and then simply use regular backpropagation. This strategy is called **backpropagation through time (BPTT)**.



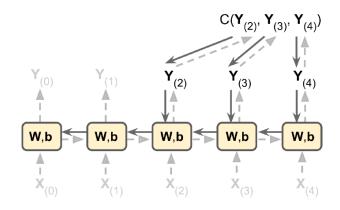
- Dotted line shows forward pass.
- Solid line shows backward pass.
 //or propagation of cost in the backward pass

- There is a first forward pass through the unrolled network (represented by the dashed arrows)
- Then the output sequence is evaluated using a cost function C(Y(0), Y(1), ...Y(T)) (where T is the max time step)



- The gradients of that cost function are then propagated backward through the unrolled network
- Finally the model parameters are updated using the gradients computed during BPTT

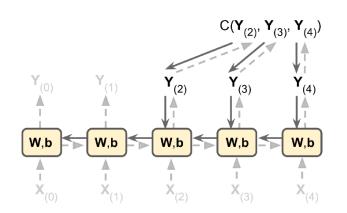
3rd type of backward arrows: from neuron-3 to nuron-2, neuron-2 to nuron-1



Ist type of backward arrows: C to Y2, Y3, Y4

2nd type of backward arrows: Y2, Y3, Y4 to the respective neuron

- Note that the gradients flow backward through all the outputs used by the cost function, not just through the final output
- Since the same parameters W and b are used at each time step, backpropagation will do the right thing and sum over all time steps



if you derive, then it will come to the sum over all timesteps.

Basic RNNs

- Suppose you are studying
 - o the number of active users per hour on your website,
 - or the daily temperature in your city,
 - or a company's financial health, measured quarterly using multiple metrics

- Suppose you are studying
 - o the number of active users per hour on your website,
 - or the daily temperature in your city,
 - or a company's financial health, measured quarterly using multiple metrics
- Here, the data is a sequence of one or more values per time step
- This is called a **time series**

Forecasting a Time Series – Types



Forecasting a Time Series - Univariate

The term "univariate time series" refers to a time series that consists of single (scalar) observations recorded sequentially over equal time increments

Examples:

- The number of active users per hour on your website,
- Or the daily temperature in your city

Forecasting a Time Series - Multivariate

Multivariate time series model is an extension of the univariate case and involves two or more input variables

Examples:

 A company's financial health, measured quarterly using multiple metrics

What is Forecasting? Predicting future values.



Another common task is to fill in the blanks: to predict missing values from the past.

This is called **imputation**

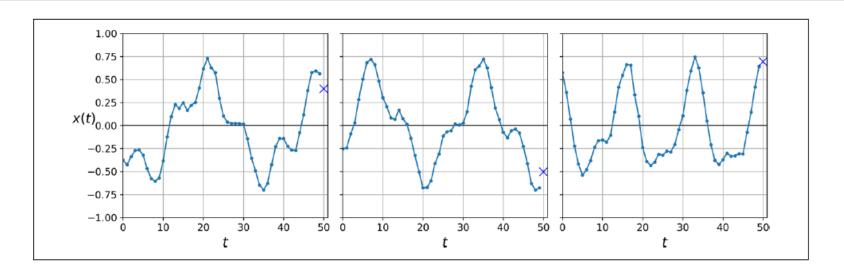
Trend is a general systematic linear or (most often) nonlinear component that changes over time and does not repeat

For example, if you are studying the number of active users on your website,
 and it is growing by 10% every month

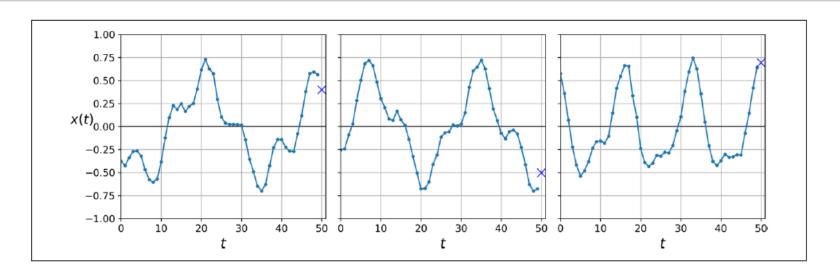
Seasonality is a general systematic linear or (most often) nonlinear component that changes over time and does repeat

 For example, if you are trying to predict the amount of sunscreen lotion sold every month, you will probably observe strong seasonality: since it sells well every summer, a similar pattern will be repeated every year

When using traditional models, we have to remove the trend and seasonality but with RNN we don't need to worry about that.



The goal is to forecast value at next time step (represented by X) for each of these 50 steps long ____?___ time series. Is it Univariate or multivariate?



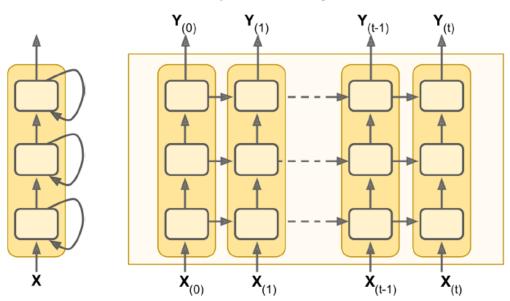
The goal is to forecast value at next time step (represented by X) for each of these 50 steps long **univariate** time series.

Deep RNNs

Deep RNNs

It is quite common to stack multiple layers of cells, as shown below.

This gives you a deep RNN



- Some information is lost at each time step in the RNN
- So after a while, the RNN's state contains virtually no trace of the first inputs

This can be a showstopper





- To tackle this problem, various types of cells with long-term memory have been introduced
- They have proven so successful that the basic cells are not used much anymore
- LSTM is one of these cells

LSTM

LSTM Cell

The Long Short-Term Memory (LSTM) cell was proposed in 1997 by Sepp
 Hochreiter and Jürgen Schmidhuber and was improved by Alex Graves, Haşim
 Sak, Wojciech Zaremba, and many more



Sepp Hochreiter



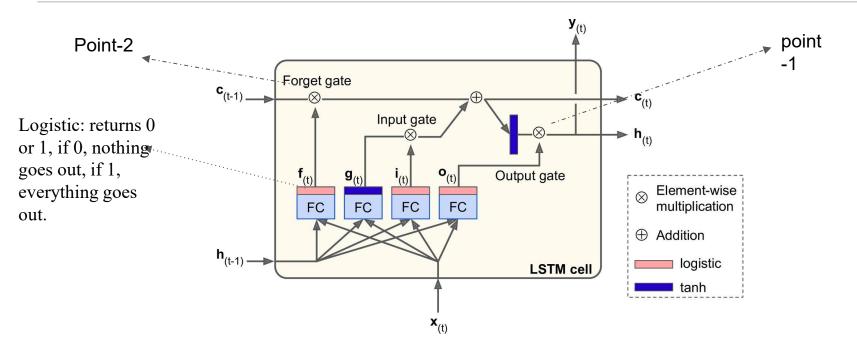
Jürgen Schmidhuber

LSTM Cell

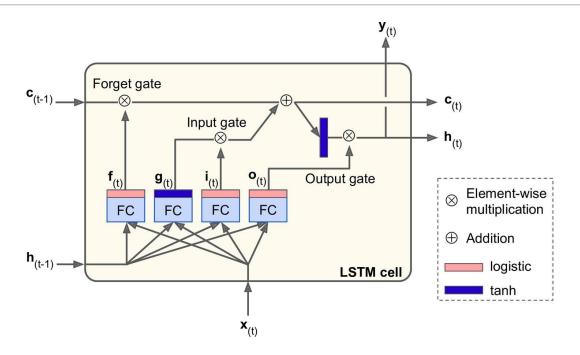
"Can be used as a black box - behaves like basic cell"

Except

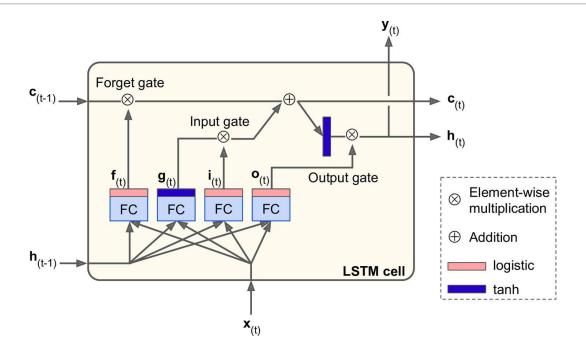
- It will perform much better
- Training will converge faster
- And it will detect long-term dependencies in the data



The architecture of a basic LSTM cell



• The **LSTM cell** looks exactly like a regular cell, except that its state is split in two vectors: $h_{(t)}$ and $c_{(t)}$, here "c" stands for "cell"

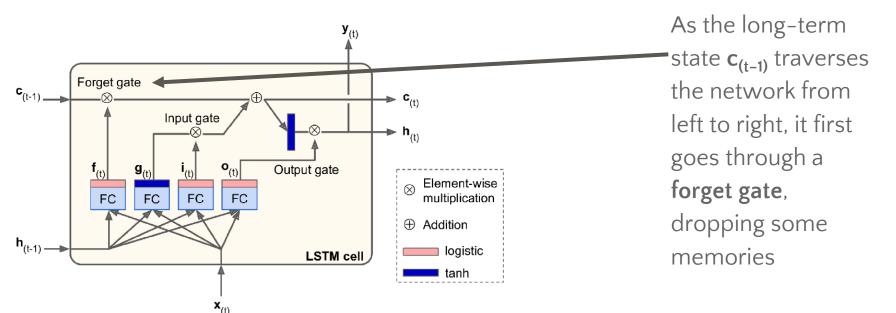


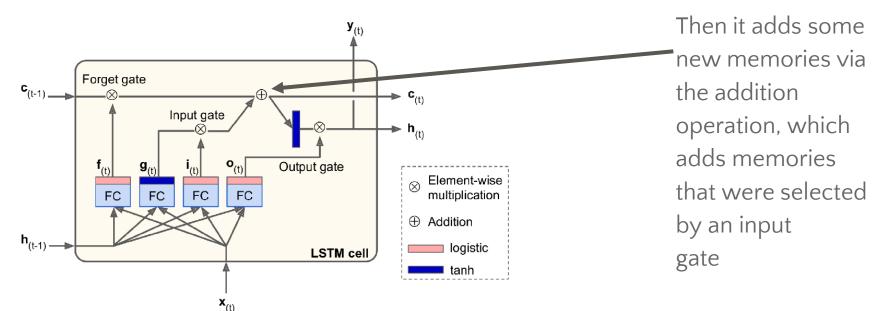
• We can think of $h_{(t)}$ as the short-term state and $c_{(t)}$ as the long-term state

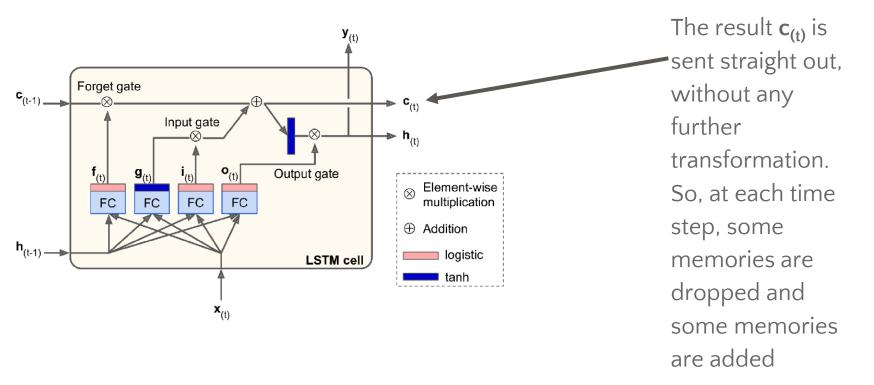
Understanding the LSTM cell structure

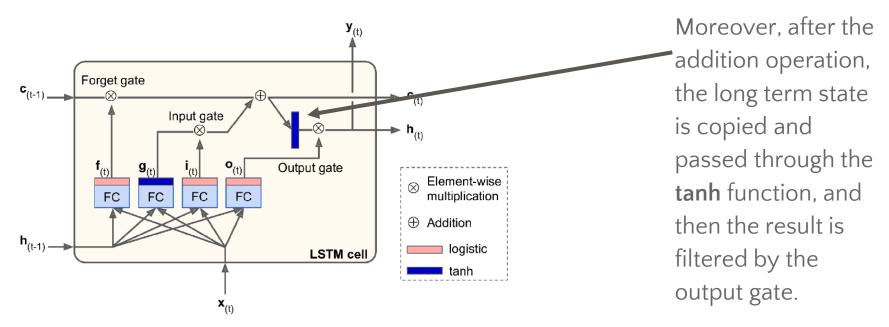
- The key idea is that the network can learn
 - What to store in the long-term state,
 - What to throw away,
 - And what to read from it

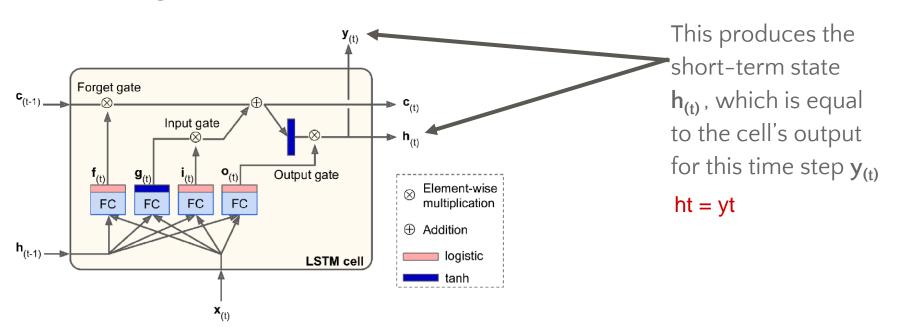
CRUX of LSTM



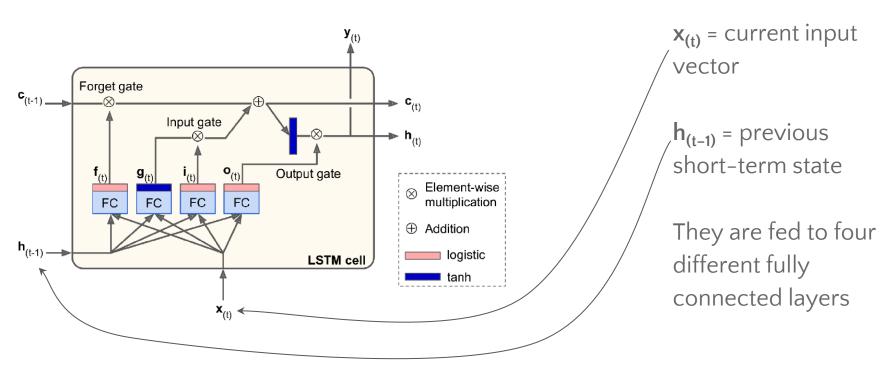




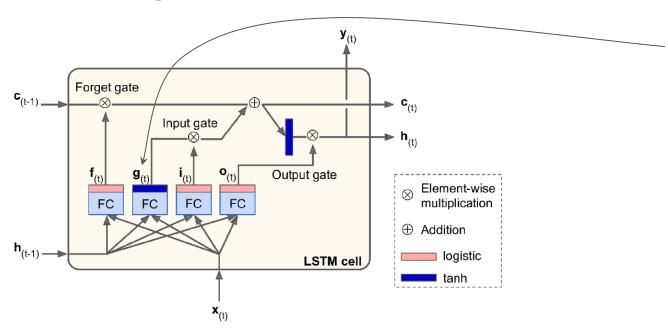




Now let's look at where new memories come from and how the gates work

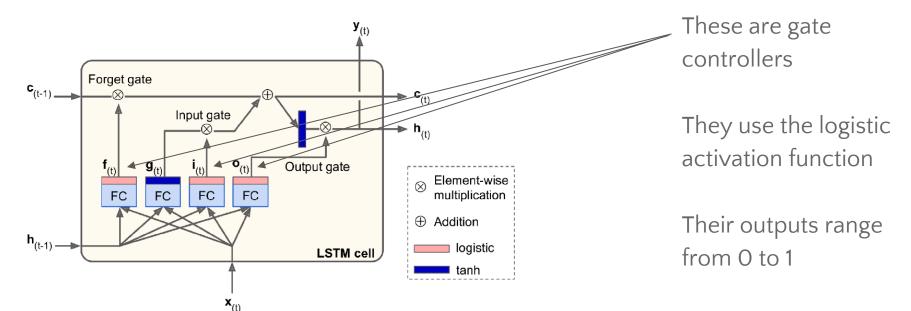


Understanding the LSTM cell structure

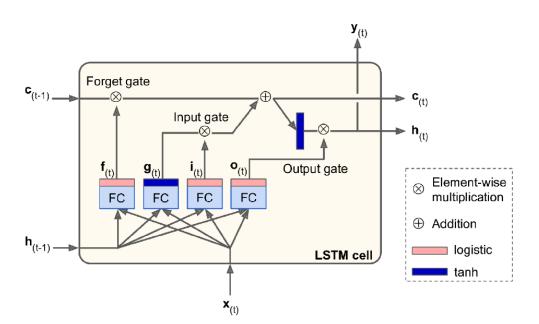


This is the main layer which analyzes $\mathbf{x}_{(t)}$ and $\mathbf{h}_{(t-1)}$

This layer's output is partially stored in the long-term state

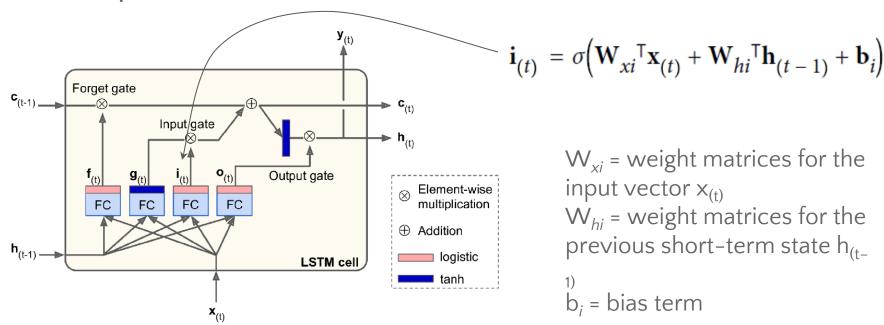


LSTM computations

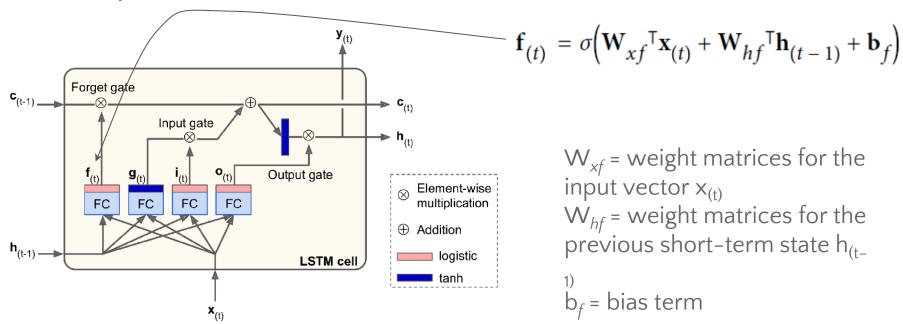


- The following slides summarizes how to compute the cell's long-term state, its short-term state, and its output at each time step for a single instance
- The equations for a whole mini-batch are very similar

LSTM computations

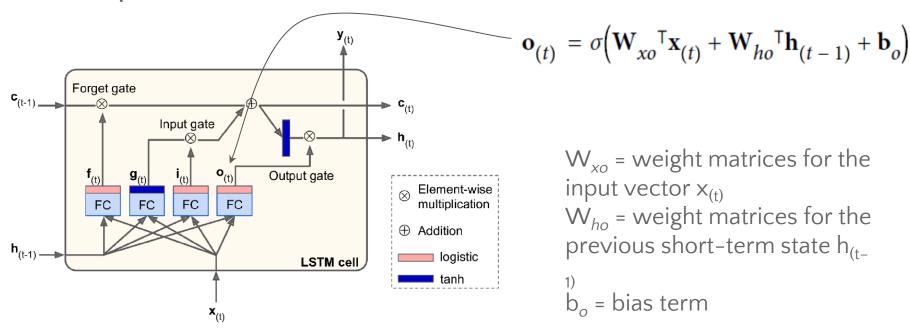


LSTM computations

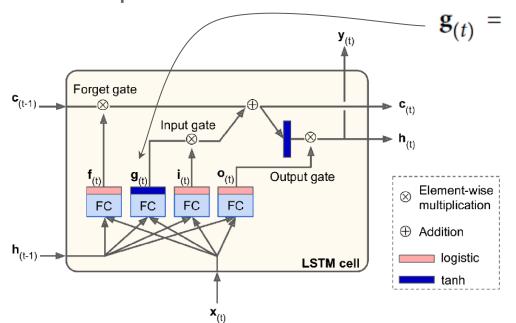


Note that TensorFlow initializes b_f to a vector full of 1s instead of 0s. This prevents forgetting everything at the beginning of training

LSTM computations



LSTM computations



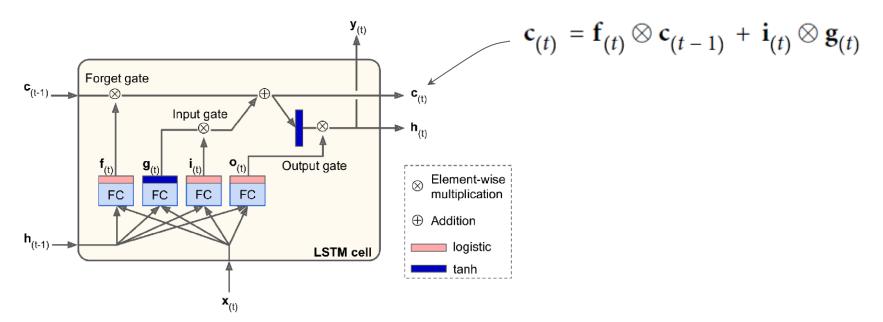
$$\mathbf{g}_{(t)} = \tanh \left(\mathbf{W}_{xg}^{\mathsf{T}} \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{\mathsf{T}} \mathbf{h}_{(t-1)} + \mathbf{b}_{g} \right)$$

 W_{xg} = weight matrices for the input vector $x_{(t)}$

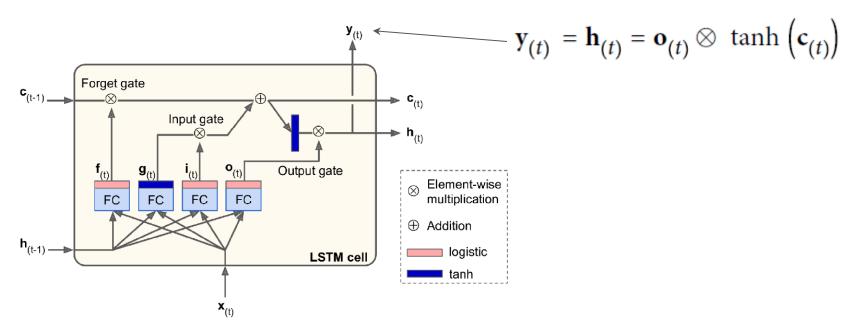
 W_{hg} = weight matrices for the previous short-term state $h_{(t-)}$

$$b_q = bias term$$

LSTM computations



LSTM computations



LSTM Cell

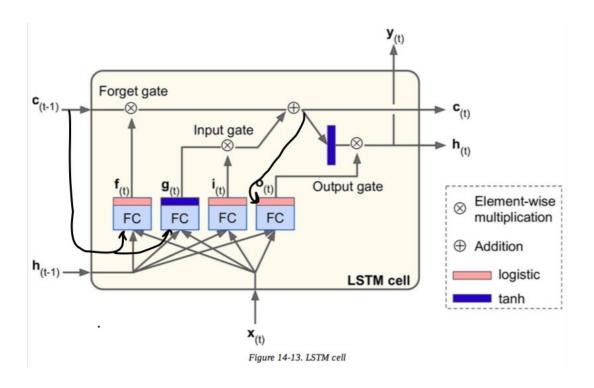
Conclusion

- A LSTM cell can learn to
 - Recognize an important input, that's the role of the input gate,
 - Store it in the long-term state,
 - Learn to preserve it for as long as it is needed, that's the role of the forget gate,
 - And learn to extract it whenever it is needed

This explains why they have been amazingly successful at capturing long-term patterns in time series, long texts, audio recordings, and more.

- In a **basic LSTM cell**, the gate controllers can look only at the input $\mathbf{x}_{(t)}$ and the previous short-term state $\mathbf{h}_{(t-1)}$
- It may be a good idea to give them a bit more context by letting them peek at the long-term state as well
- This idea was proposed by Felix Gers and Jürgen Schmidhuber in 2000

- They proposed an LSTM variant with extra connections called peephole connections:
 - o The previous long-term state $\mathbf{c}_{(t-1)}$ is added as an input to the controllers of the forget gate and the input gate,
 - o And the current long-term state $\mathbf{c}_{(t)}$ is added as input to the controller of the output gate.



GRU Cell

GRU Cell

The Gated Recurrent Unit (GRU) cell was proposed by **Kyunghyun Cho** et al. in a 2014 paper that also introduced the Encoder–Decoder network we discussed earlier



Kyunghyun Cho

GRU Cell

- LSTM or GRU cells are one of the main reasons behind the success of **RNNs** in recent years
- In particular for applications in **natural language processing (NLP)**