HSS-01: Economics

Lesson: 04

Production

Production Decisions, Variable Inputs, Returns to Scale, Costs in Short & Long Run, Economies of Scope

6.1. PRODUCTION DECISIONS

The Real Role of a Firm

You and me, if we have enough time, can also build a house, isn't it? At least a very small one…

We can convert inputs into output, isn't it? Why do we need a firm/company?

- *Benefits of a firm*
  - Organizational efficiency – machines, workers, managers
  - Large scale process automation – factories (More than sum of the parts.)

A natural extension of you and me working together… As we grow, we become a small scale firm, then a medium scale firm, and finally a large enterprise.

Factors of production

Production (in a firm) is done in a way to minimize the costs of production.

- Land
- Capital
- Labour
- Inputs (e.g., natural resources)

- Technology – efficiency of transforming inputs to output

Since a firm wants to minimize the costs of production,

why do you think they would pay you more for your job?

You would ideally expect that, isn't it?

Production Function of a Firm

The highest output q that a firm can produce for every specified combination of inputs.

$$q = F(K, L)$$

- K = capital
- L = labour

Imagine that the function F() is applied on a given set of inputs/materials to produce output.

In above equation, technology is assumed to be a constant.

Short Run vs. Long Run

You take time to learn how to write an essay. A firm takes time to learn to minimize costs.

- ***Short run:*** Period of time in which quantities of one or more production factors cannot be changed. At least one factor (fixed input) that cannot be varied.
- ***Long run:*** Amount of time needed to make all production inputs variable.

In the short run, firms vary the intensity with which they utilize a given plant and machinery; in the long run, they vary the size of the plant.

All fixed inputs in the short run represent the outcomes of previous long-run decisions based on estimates of what a firm could profitably produce and sell.

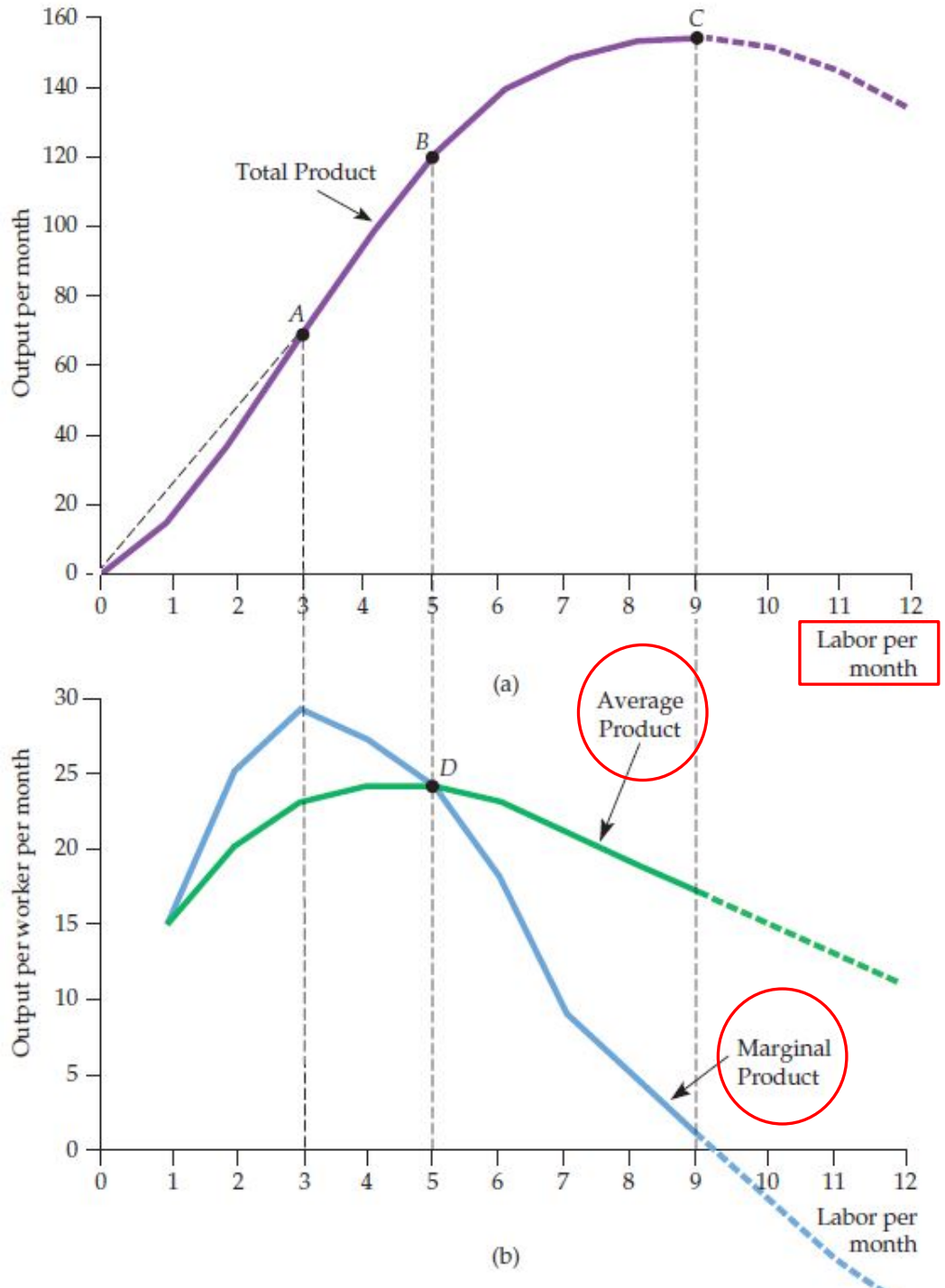
6.2. PRODUCTION WITH ONE VARIABLE INPUT (LABOR)

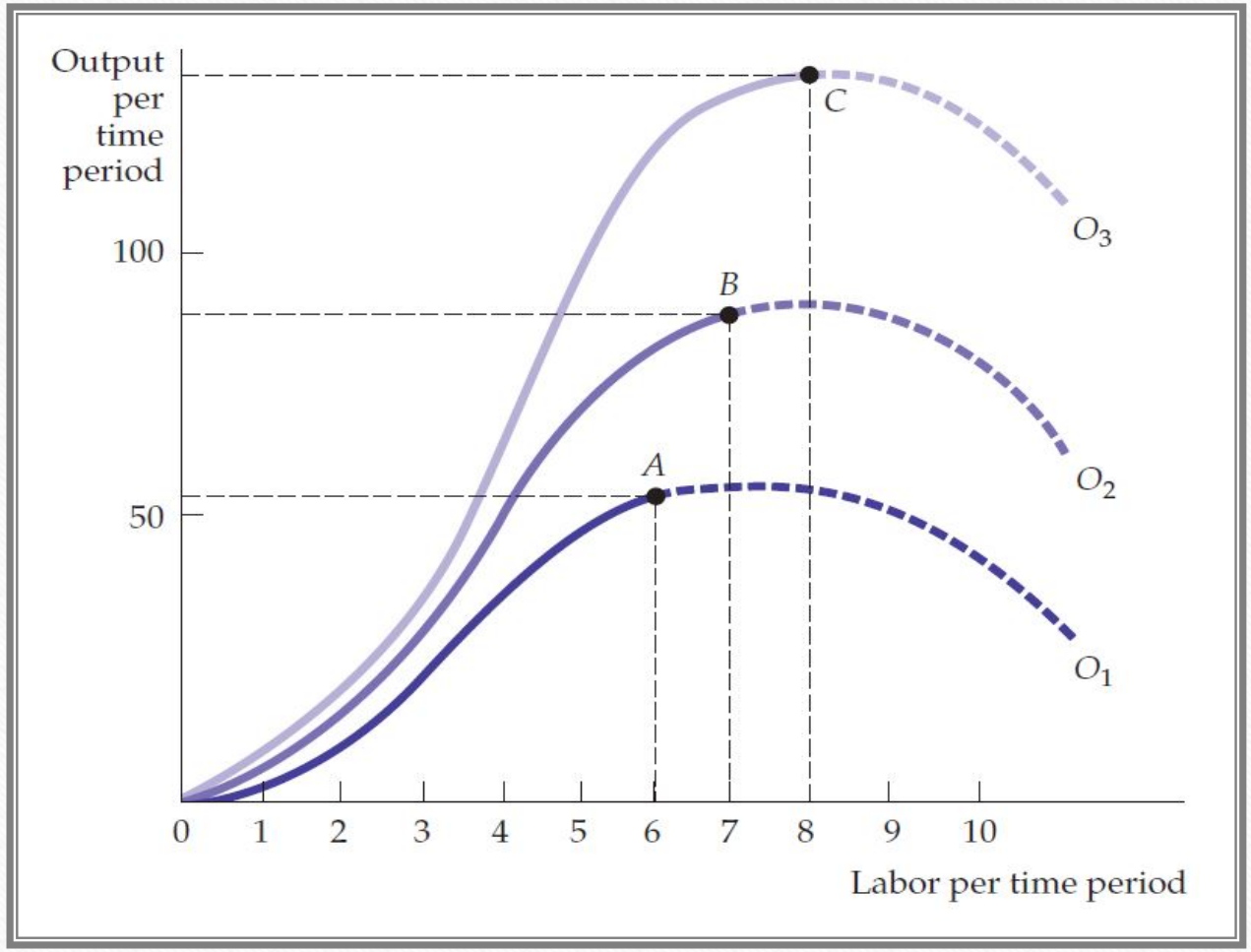
One Variable Input -- Labor

Average output = q/L
Marginal output = Δq/ΔL

Once the labor input exceeds 9 units (point C), the marginal product becomes negative, so that total output falls as more labor is added.

But why?





Law of Diminishing Marginal Returns

As the use of an input increases with other inputs fixed, the resulting additions to output will eventually decrease.

This holds irrespective of the quality/productivity of labor.

Labor Productivity

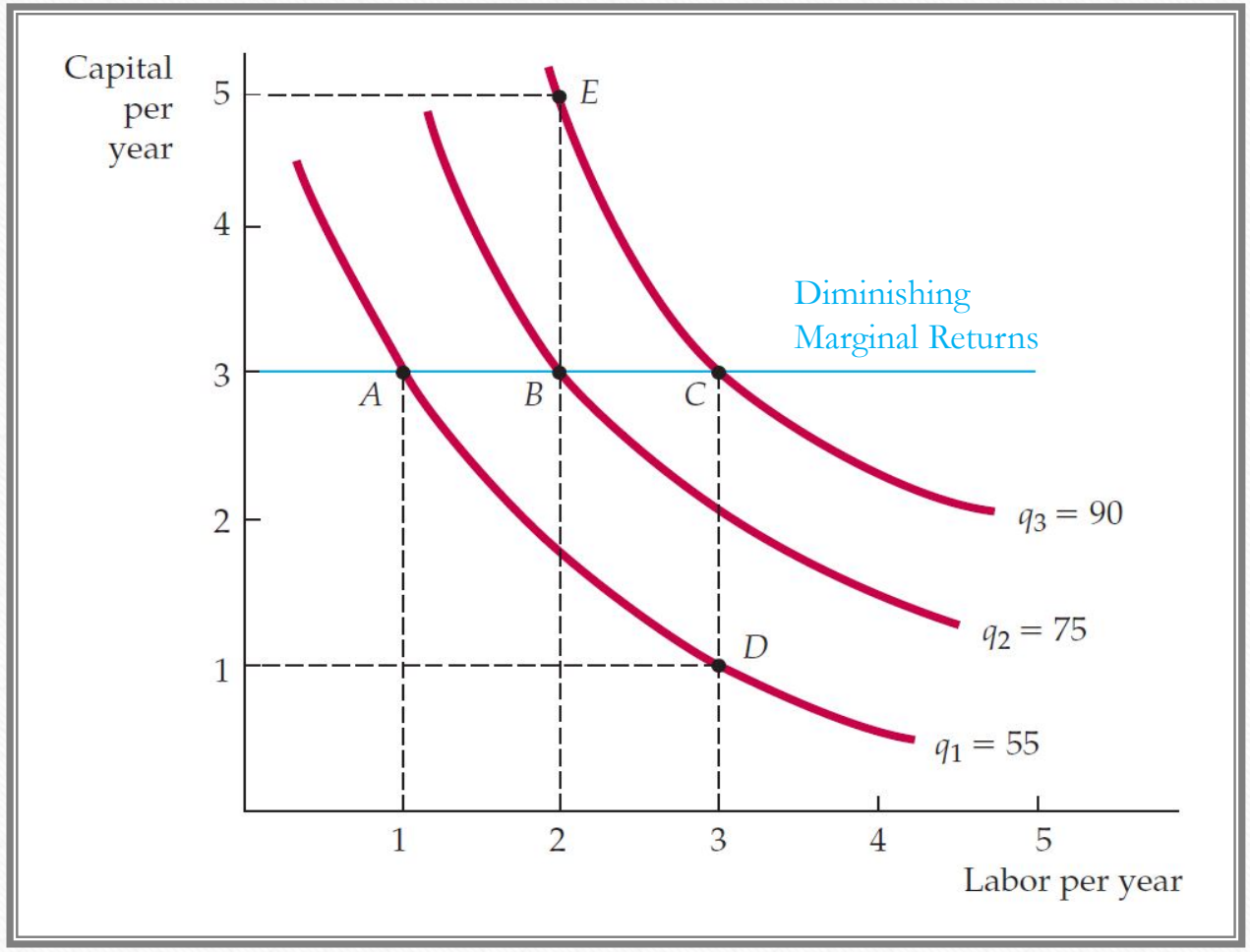
Average product of labor for an entire industry or for the economy as a whole.

- Determines the real *standard of living* in an economy (e.g., a country).
  - Income as a producer is spent to purchase items, as a consumer.
  - Consumers, in aggregate economy, can increase consumption by increasing production.
- Sources of growth in labor productivity
  - Stock of capital – amount of capital available for use in production.
  - Technological change -- development of new technologies that allow labor (and other factors of production) to be used more effectively

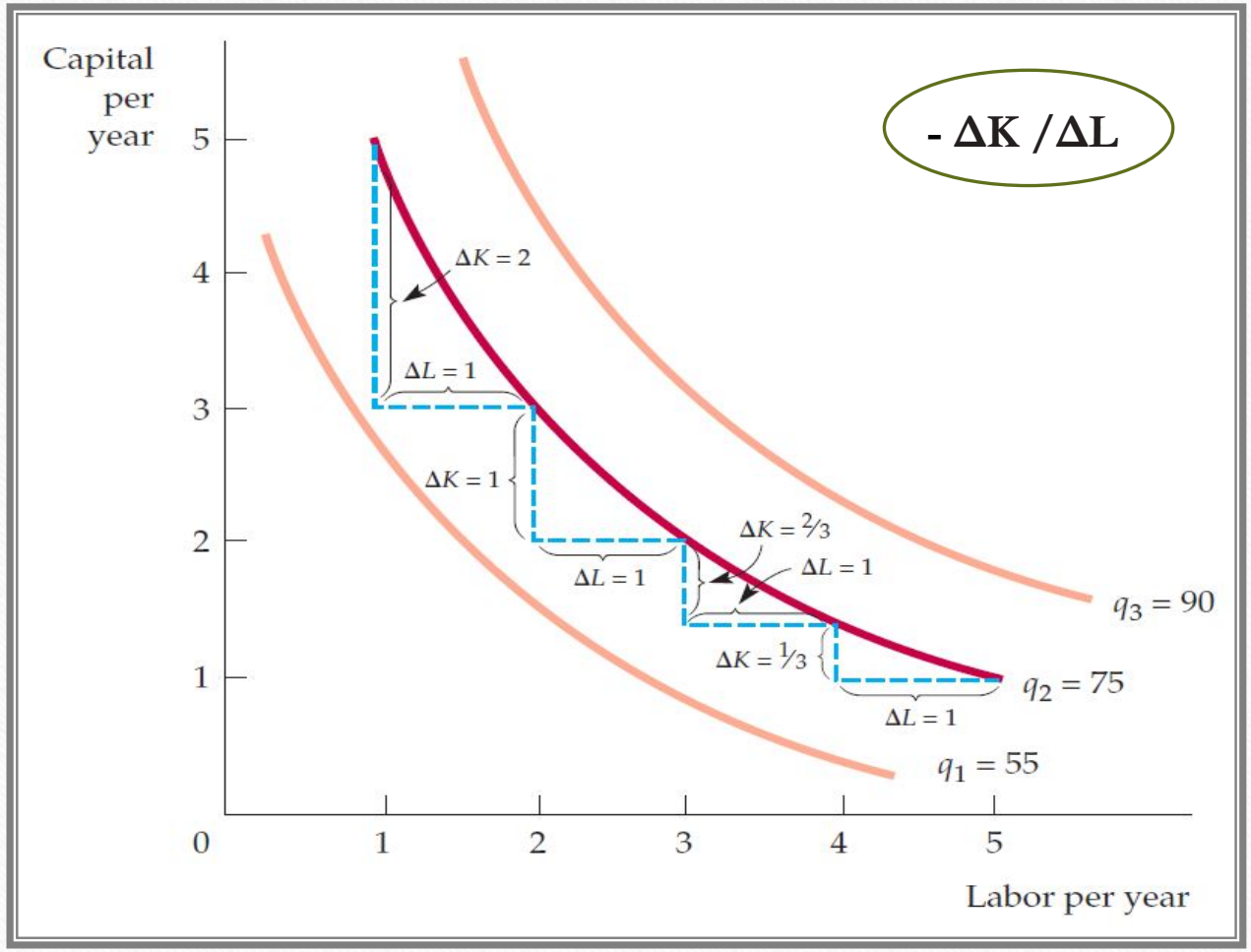
What about AI?

6.3. PRODUCTION WITH TWO VARIABLE INPUTS (LABOR)

Production Isoquants



Production **isoquants** show the various combinations of inputs necessary for the firm to produce a given output. A set of isoquants, or **isoquant map**, describes the firm's production function. Output increases as we move from isoquant $q1$ (at which 55 units per year are produced at points such as *A* and *D*), to isoquant $q2$ (75 units per year at points such as *B*), and to isoquant $q3$ (90 units per year).



Marginal Rate of Technical Substitution

Isoquants are downward sloping and convex.

The slope of the isoquant at any point measures the marginal rate of technical substitution (MTRS)—the ability of the firm to replace capital with labor while keeping the same level of output.

On isoquant q_2, the MRTS falls from 2 to 1 to 2/3 to 1/3.

Substitution among Inputs

The marginal rate of technical substitution between two inputs is equal to the ratio of the marginal products of the inputs.

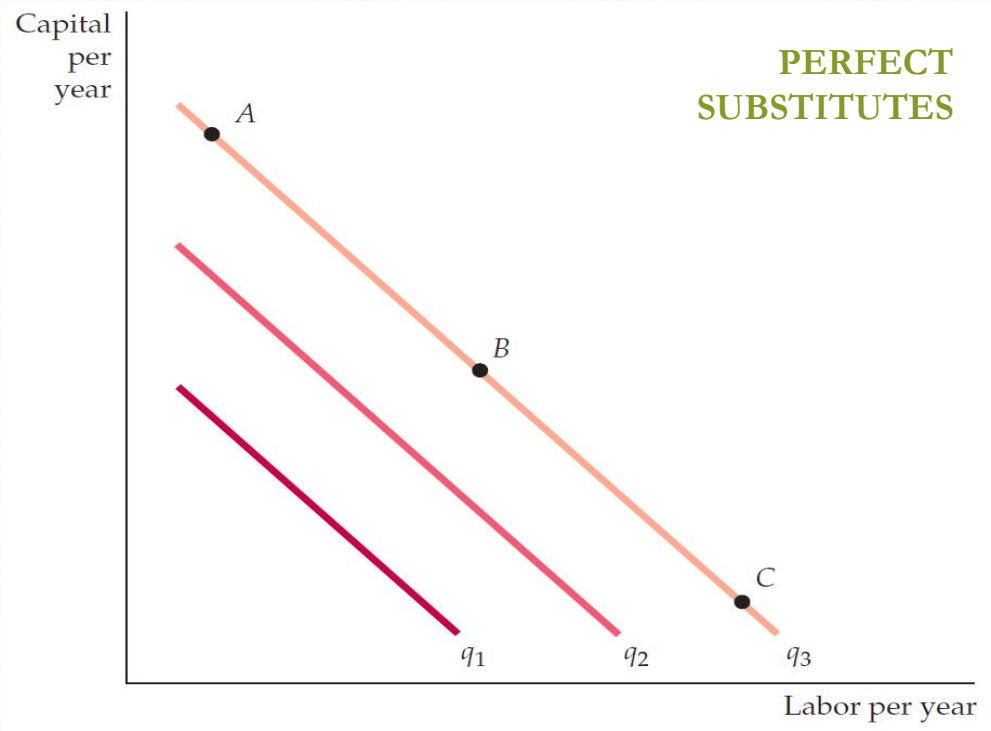
- *Along an isoquant, we have:*

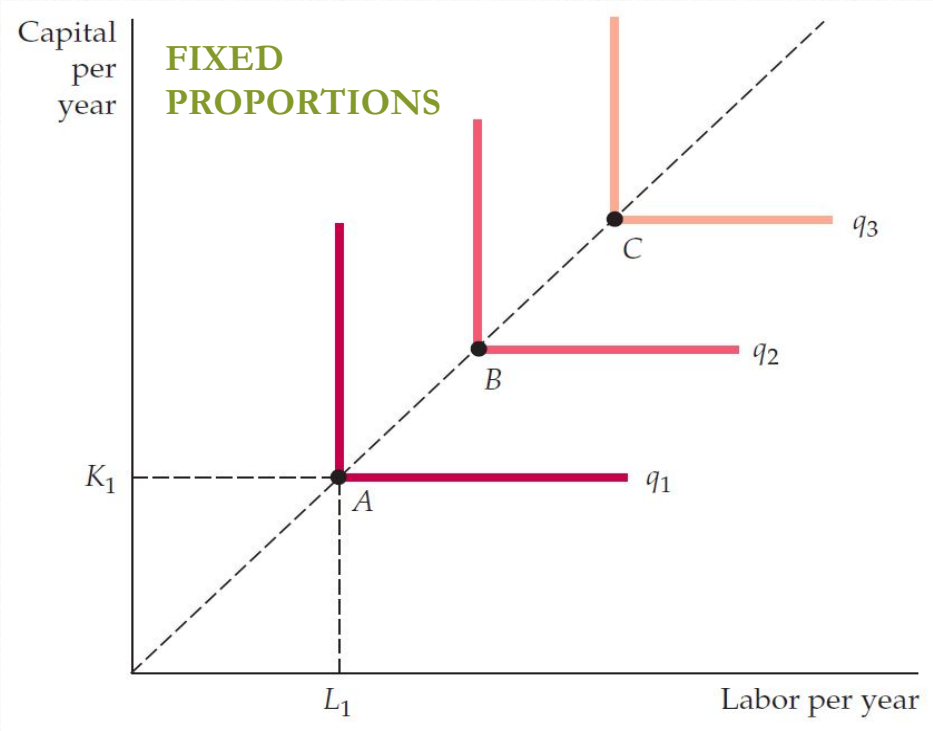
$$(\mathrm{MP}_L)(\Delta L) + (\mathrm{MP}_K)(\Delta K) = 0$$

- *and on rearranging, we get:*

$$\mathbf{MRTS} = \mathbf{MP}_L \,/\, \mathbf{MP}_K$$

Special Cases of Production Functions



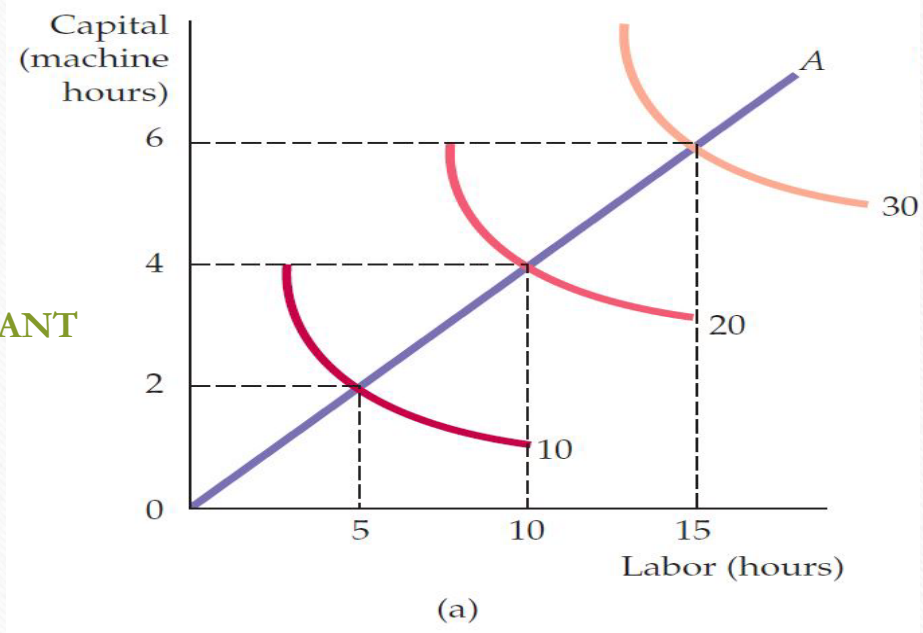


6.4. RETURNS TO SCALE

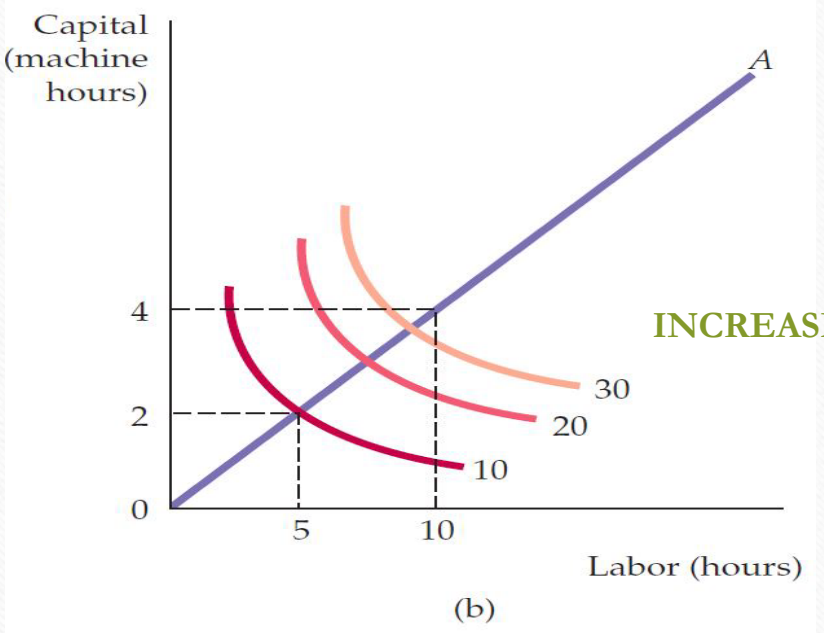
Returns to Scale – Increasing/Decreasing/Constant

Rate at which output increases as inputs are increased proportionately.

CONSTANT



INCREASING



7.1. MEASURING COST

Various Types of Costs

- Accounting cost = just note down the transactions of assets and liabilities (upto now)
- Economic cost = overall costs (upto now + future)
- Opportunity cost = cost of best alternative opportunity
- Sunk cost = cost/expenditure that can not be recovered in future (no alternative use)
- Fixed cost FC = cost that does not vary with the level of production output
- Variable cost VC = cost that varies with level of output
- Total cost TC = fixed cost + variable cost

Cost Calculations

- Marginal cost MC =
  - Increase in cost resulting from the production of one extra unit of output
  - $\Delta TC / \Delta q = \Delta VC / \Delta q$
- Average total cost ATC = TC / q
- Average fixed cost AFC = FC / q
- Average variable cost AVC = VC / q

7.2. COST IN THE SHORT RUN

Nature of Marginal Cost

- When there are diminishing marginal returns, marginal cost will increase as output increases.
- If the marginal product of labor decreases only slightly as the amount of labor is increased, costs will not rise so quickly when the rate of output is increased.
- Suppose labor is hired at a fixed wage w (in a competitive market). Then

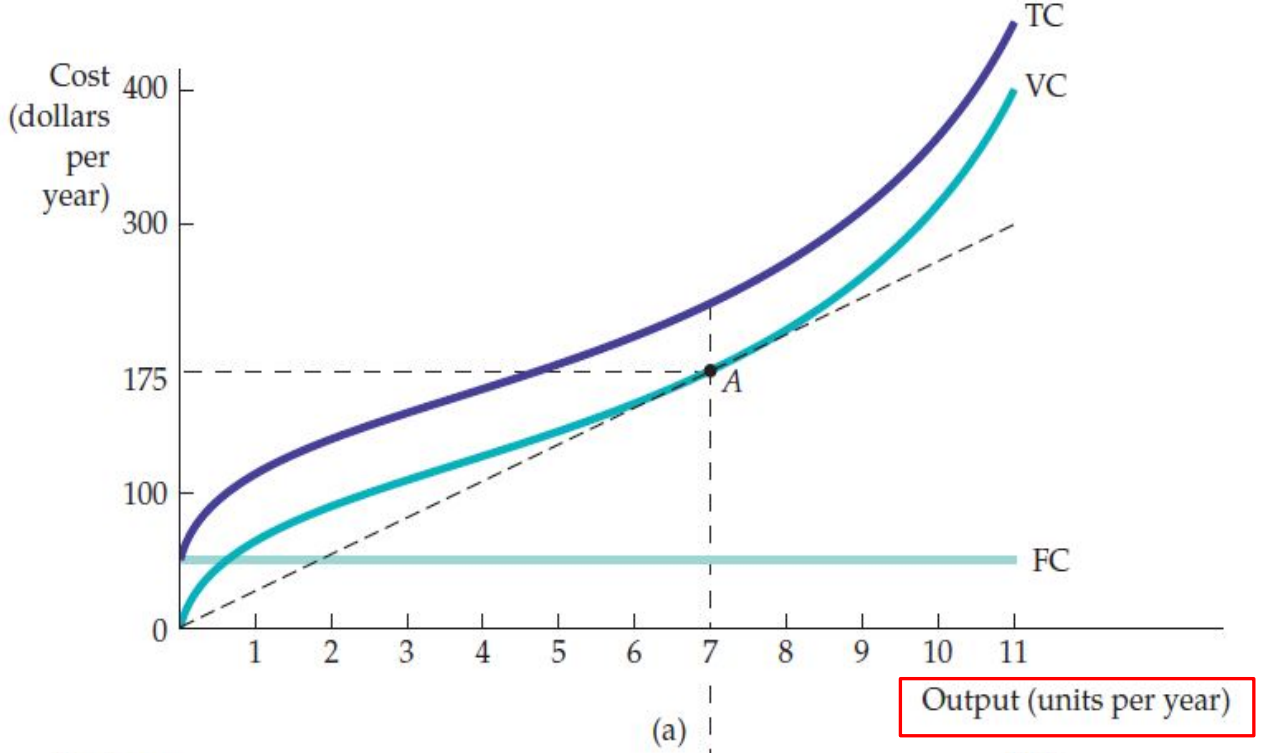
$$MC = w\,\Delta L / \Delta q = w / MP_L$$

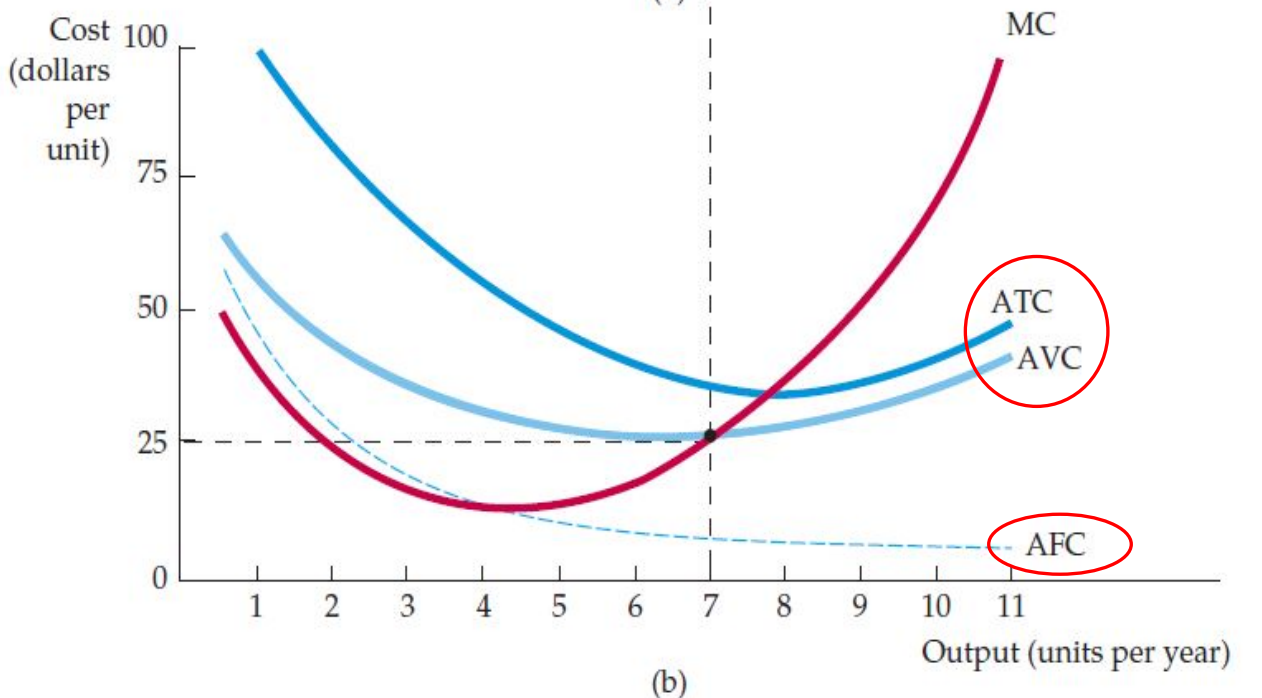
Cost Curves for a Firm

In (a) total cost TC is the vertical sum of fixed cost FC and variable cost VC.

In (b) average total cost ATC is the sum of average variable cost AVC and average fixed cost AFC.

Marginal cost MC crosses the average variable cost and average total cost curves at their minimum points.





7.3. COST IN THE LONG RUN

Cost Minimizing Input Choice

- Price of Capital
  - Capital expenditure per year (a flow measure), also called user cost of capital

$$r = \text{Depreciation rate} + \text{Interest rate.}$$

- Rental Rate of Capital
  - Cost per year of renting one unit of capital
  - Capital that is purchased can be treated as though it were rented at a rental rate equal to the user cost of capital

Isocost Line

Graph showing all possible combinations of labor and capital that can be purchased for a given total cost

- $C = wL + rK \quad => \quad K = C/r - (w/r)L$

$$\Delta K / \Delta L = -(w/r)$$

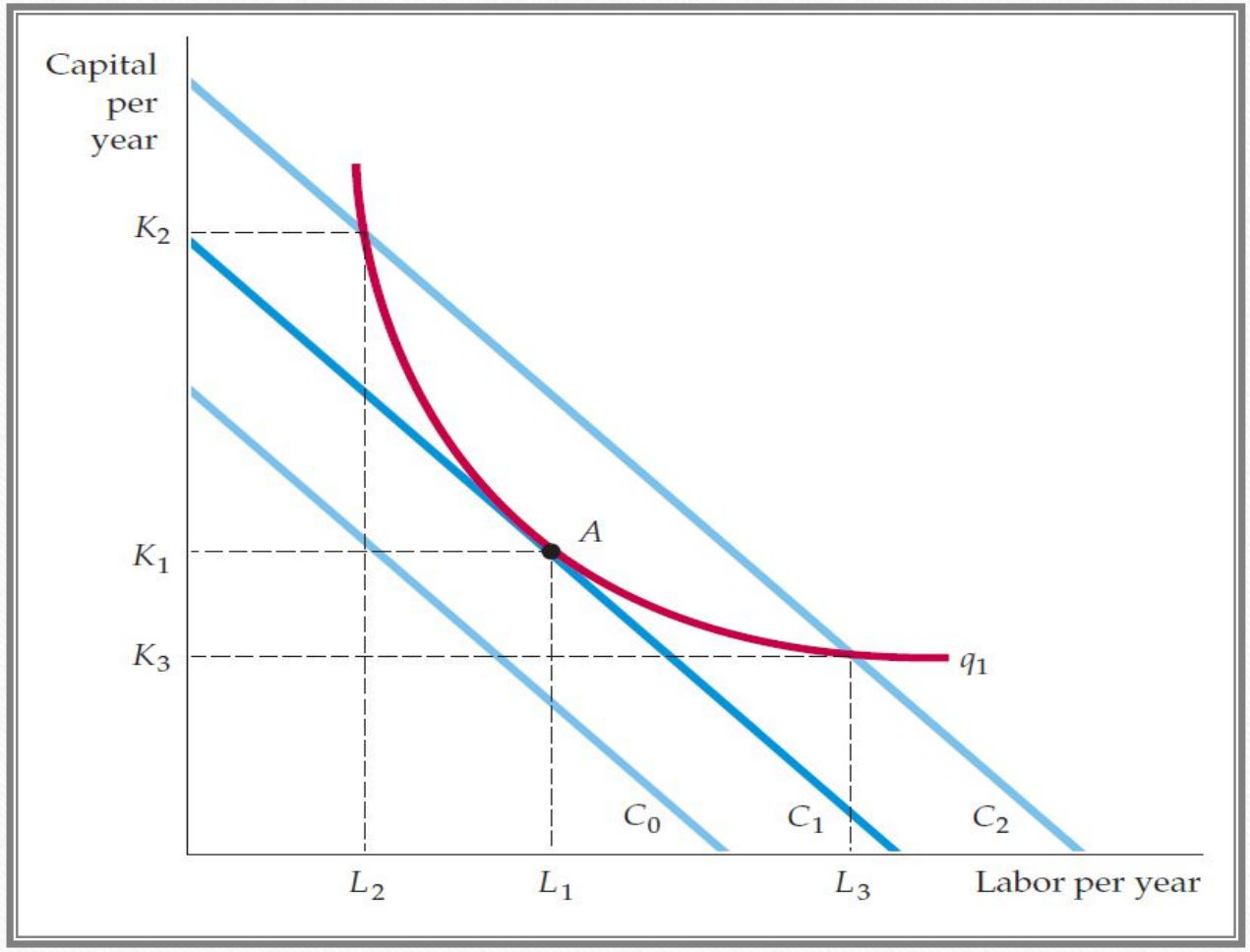
- If the firm gave up a unit of labor (and recovered w dollars in cost) to buy w/r units of capital at a cost of r dollars per unit, its total cost of production would remain the same.

Isocost Line & Production

- Since **MRTS** = **$-\Delta K / \Delta L$** = **MP_L / MP_K** we have :

$$MP_L / w = MP_K / r$$

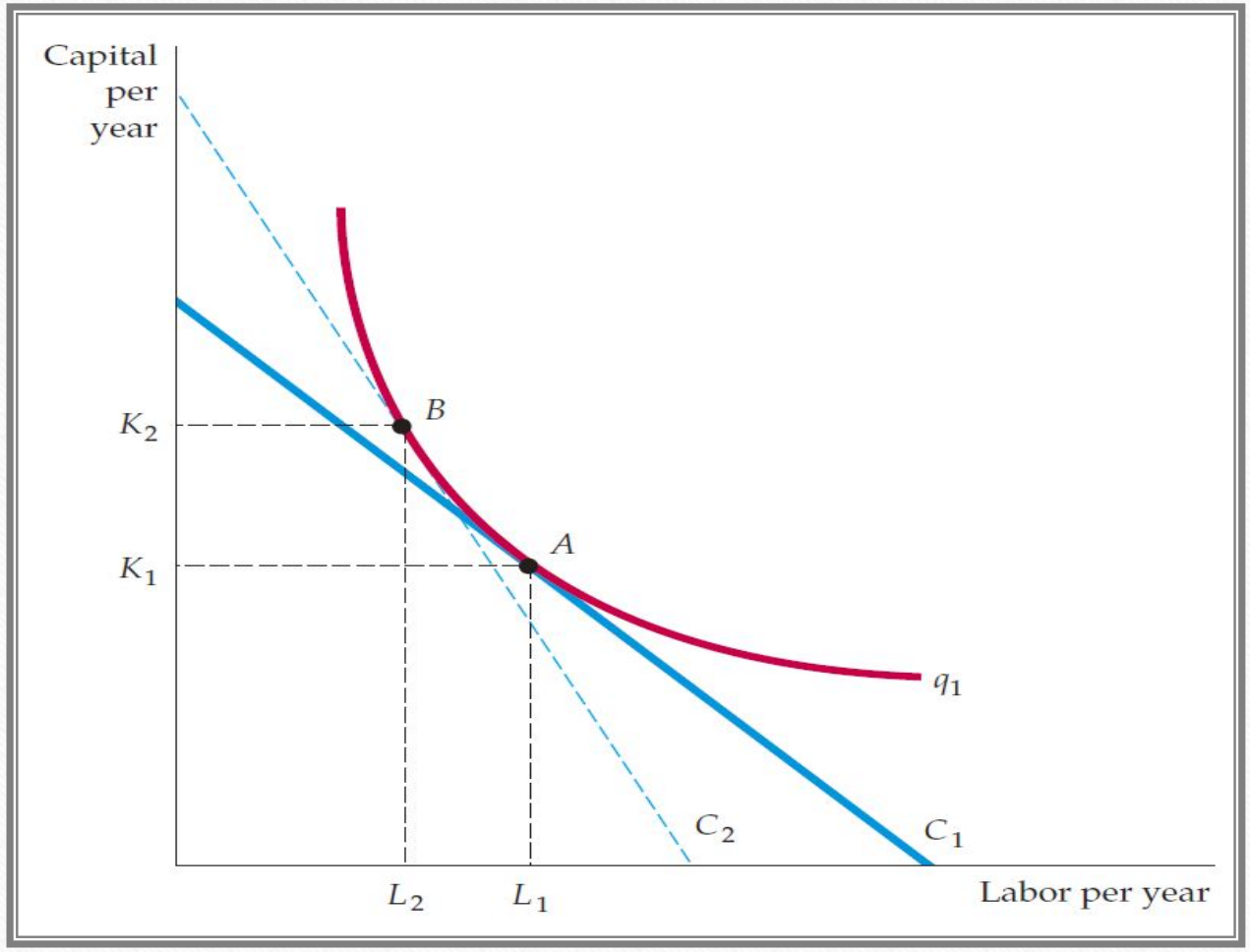
- For a a cost-minimizing firm, additional output that results from spending an additional dollar for labor is the same as that of spending for capital.



Producing Output at Minimum Cost

Isocost curve *C*1 is tangent to isoquant *q*1 at *A* and shows that output *q*1 can be produced at minimum cost with labor input *L*1 and capital input *K*1.

Other input combinations—*L*2, *K*2 and *L*3, *K*3 — yield the same output but at higher cost.



Input Substitution as Input Price Changes

Facing an isocost curve $C1$, the firm produces output $q1$ at point A using $L1$ units of labor and $K1$ units of capital.

When the price of labor increases, the isocost curves become steeper.

Output $q1$ is now produced at point B on isocost curve $C2$ by using $L2$ units of labor and $K2$ units of capital.

Expansion Path

Combinations of labor and capital that the firm will choose to minimize costs at each output level.

- ✓ The curve passing through the points of tangency between
  - the firm's isocost lines, and
  - its isoquants.

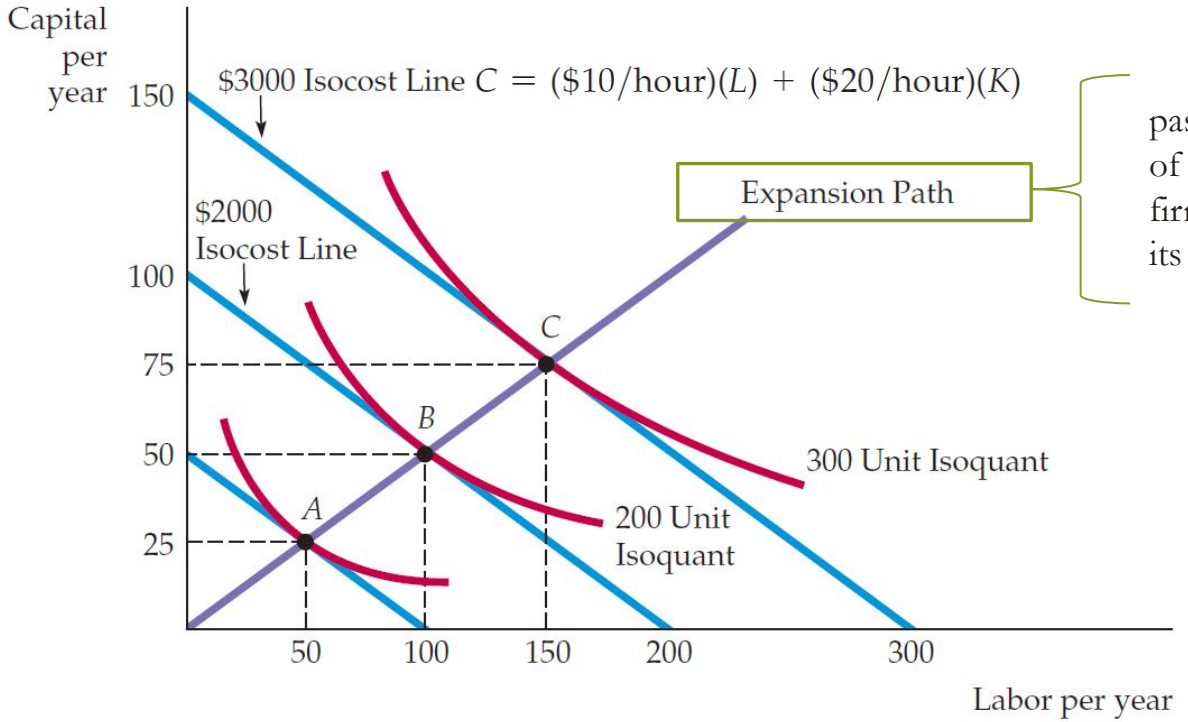
- ✓ As long as the use of both labor and capital increases with output, the curve will be upward sloping.

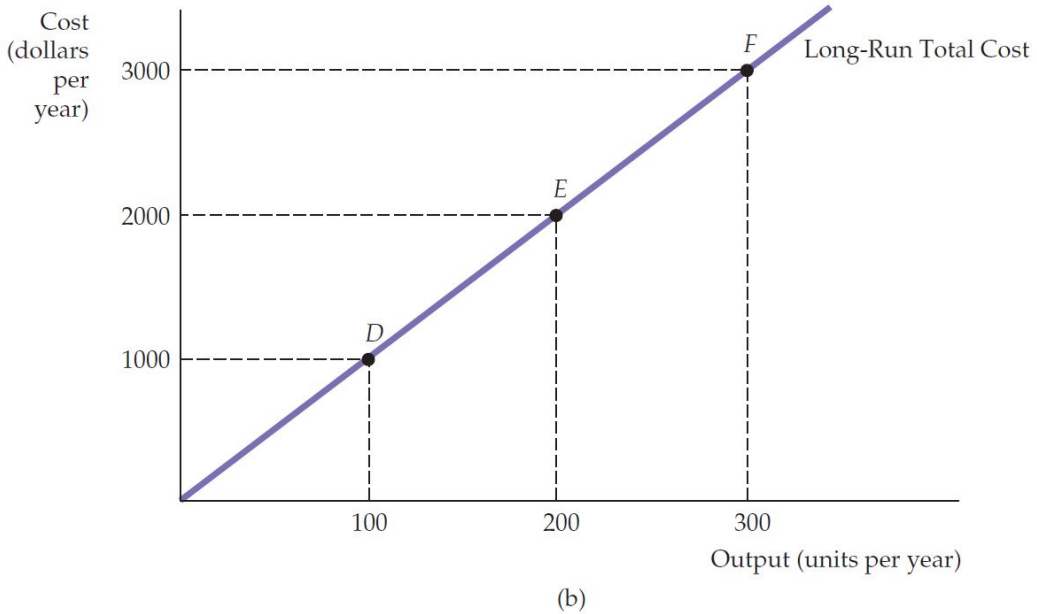
.

Cost Minimization with Varying Output

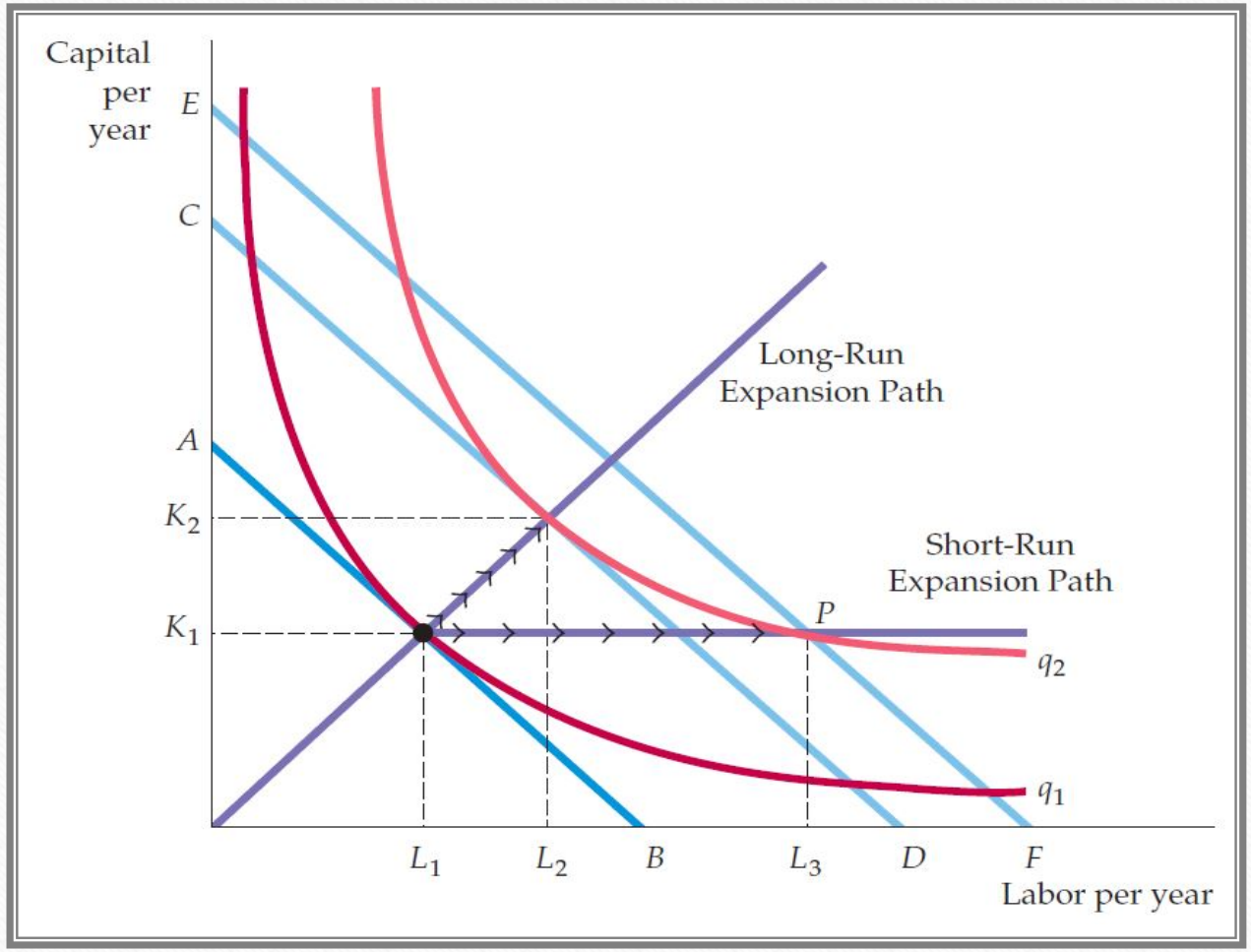
In **(a)**, the **expansion path** (from the origin through points *A, B*, and *C*) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output in the long run—i.e., when both inputs to production can be varied.

In **(b)**, the corresponding long-run total cost curve (from the origin through points *D, E*, and *F*) measures the least cost of producing each level of output.





7.4. LONG RUN vs. SHORT RUN

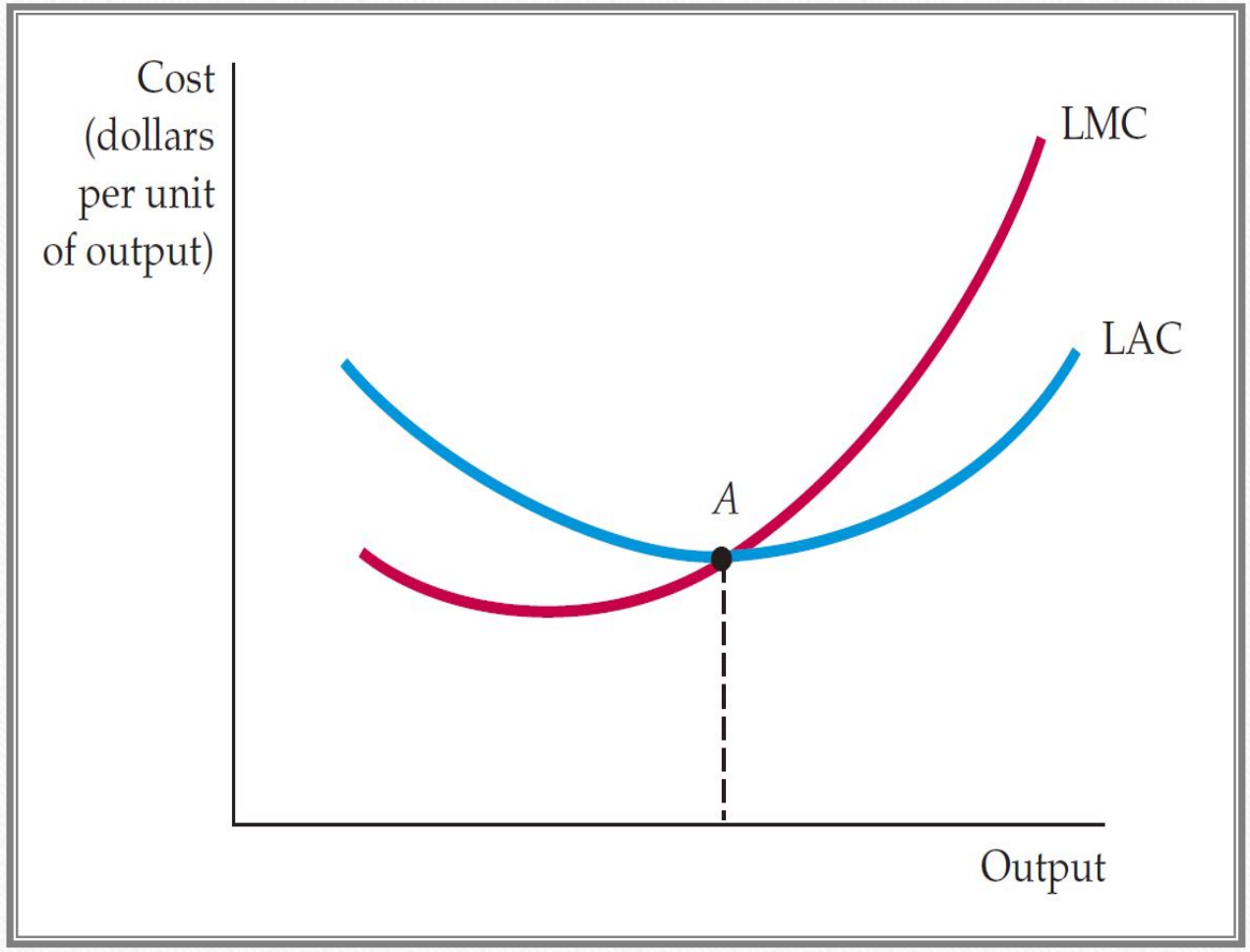


Inflexibility in Short Run

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

Output is initially at level q_1. **In the short run**, output q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1.

In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2.



Long Run Costs – Average, Marginal

When a firm is producing at an output at which the **long-run average cost LAC** is falling, the **long-run marginal cost LMC** is less than LAC.

Conversely, when LAC is increasing, LMC is greater than LAC.

The two curves intersect at *A*, where the LAC curve achieves its minimum.

Economies of **Scale**

Situation in which **output can be doubled** for **less than a doubling of cost**.

- Some cases in which costs decreases:
  - If the firm operates on a larger scale, workers can specialize in the activities at which they are most productive.
  - The firm may be able to acquire some production inputs at lower cost because it is buying them in large quantities and can therefore negotiate better prices.
- Some cases in which costs increases:
  - At least in the short run, factory space and machinery may make it more difficult for workers to do their jobs effectively.
  - Managing a larger firm may become more complex and inefficient as the number of tasks increases.

Economies of **Scale**

Situation in which **output can be doubled** for **less than a doubling of cost**.

- **vs. Returns to Scale**
  - Increasing returns to scale => Output more than doubles when the quantities of all **inputs** are doubled
  - Economies of Scale => Output more than doubles even with less than a doubling of **cost**
    - Basically, **additional inputs** obtained during doubling the total output **comes at less cost**.
      - For e.g., technology can be used only when the input size is large (machines to milk cows).

Economies of **Scale**

Measured using cost-output elasticity

$$E_C = (\Delta C/C) / (\Delta q/q) = MC / AC$$

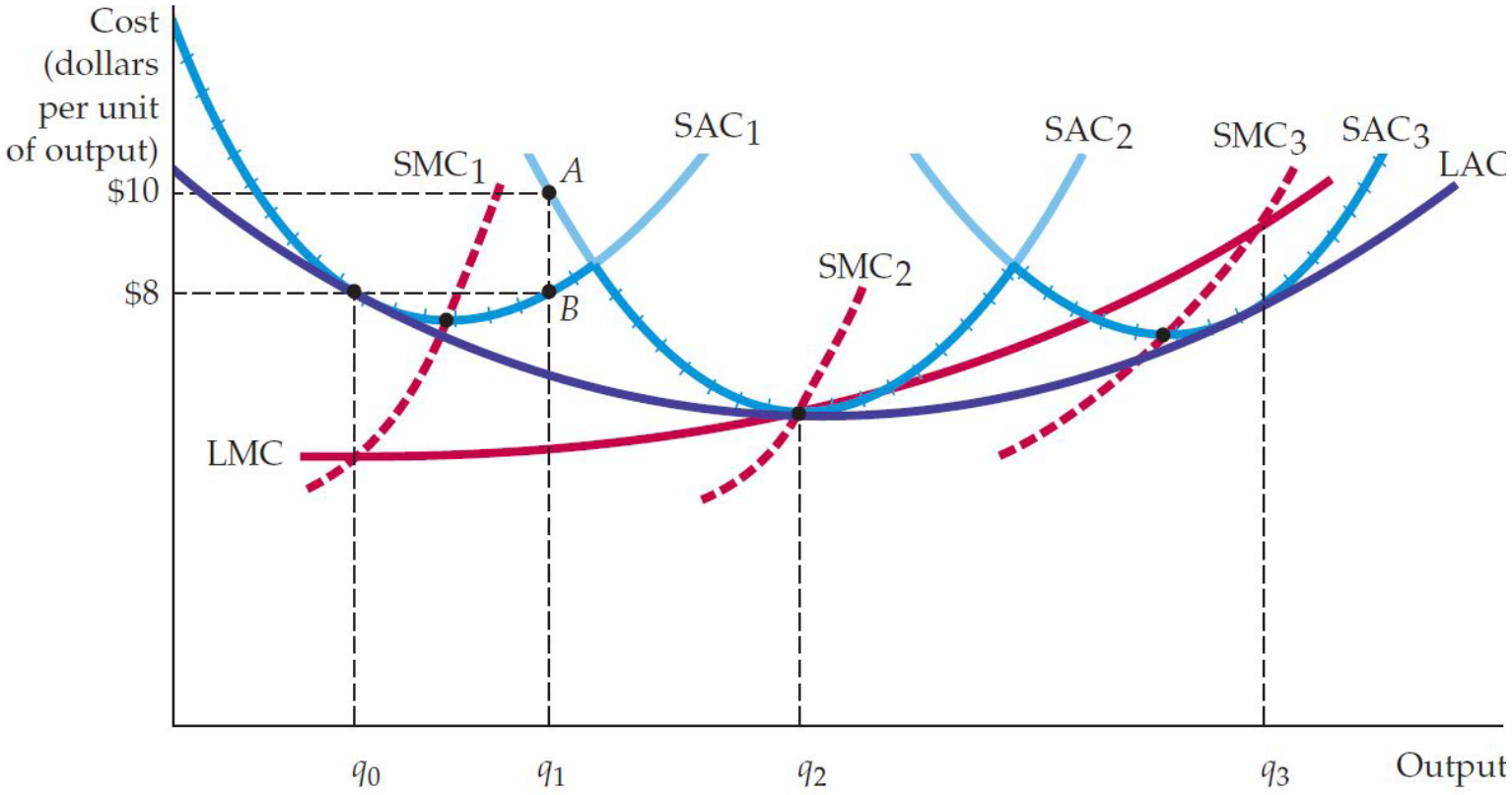
- $E_C = 1$ => Neither economies nor diseconomies of scale
  - Costs increase proportionately with output
  - Constant returns to scale would apply if input proportions were fixed
- $E_C < 1$ => Economies of scale
  - Costs increase less than proportionately with output
  - Marginal cost is less than average cost (both are declining)

Relationship btw. Long & Short Run Costs

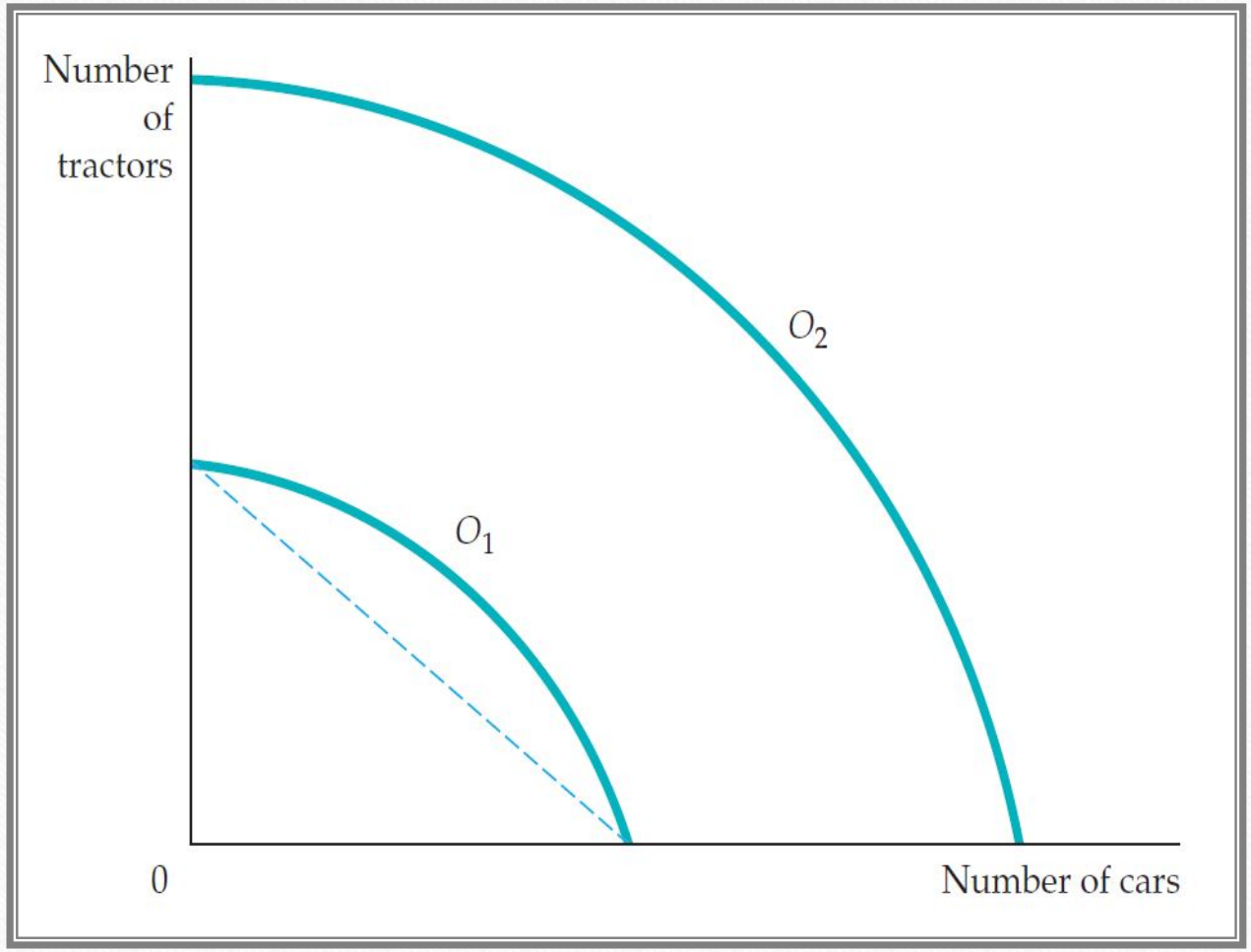
Long-run Cost with Economies and Diseconomies of Scale

The long-run average cost curve LAC is the envelope of the short-run average cost curves SAC_1, SAC_2, and SAC_3.

With economies and diseconomies of scale, the minimum points of the short-run average cost curves do not lie on the long-run average cost curve.



7.5. PRODUCTION WITH TWO OUTPUTS



Product Trans-formation Curves

The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of production inputs.

The product transformation curves O_1 and O_2 are bowed out (or concave) because there are economies of scope in production.

Economies of **Scope**

Situation in which **joint output of a single firm is greater** than output that could be achieved by two different firms when each produces a single product.

- Degree of economies of scope
  - Percentage of cost savings resulting when two or more products are produced jointly rather than individually.

$$\frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}$$

COBB-DOUGLAS COST AND PRODUCTION FUNCTIONS

We have already seen this. Please recall.

Inputs optimizing condition

$$MP_L / w = MP_K / r$$

Cobb-Douglas Production Function

- q is the rate of output
- K is the quantity of capital
- L is the quantity of labor

$$q = F(K, L) = A K^{\alpha} L^{\beta}$$

- A, a, β are positive constants
- Assume $a < 1$, $\beta < 1$ so that MP_L and MP_K are decreasing
- $a + \beta = 1$ => Firm has constant returns to scale

Cobb-Douglas Production Function

- $MP_L = \beta A K^{\alpha} L^{\beta-1}$
- $MP_K = \alpha A K^{\alpha-1} L^{\beta}$

<=

$$q = F(K, L) = A K^{\alpha} L^{\beta}$$

- $\mathbf{MP_L / w = MP_K / r}$

=>

$$L = \frac{\beta r}{\alpha w} K$$

Cobb-Douglas Production Function

$$K = \left(\frac{\alpha\, w}{\beta\, r}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$$q = F(K, L) = A\, K^{\alpha} L^{\beta}$$

and similarly,

$$L = \left(\frac{\beta\, r}{\alpha\, w}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$$L = \frac{\beta\, r}{\alpha\, w} K$$

Cobb-Douglas Production Function

$$K = \left(\frac{\alpha w}{\beta r}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$$L = \left(\frac{\beta r}{\alpha w}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

What do these equations say?

Factor demands, given output!

How do *K* and *L* change when:

w increases?
r increases?
A increases?

Cobb-Douglas Production Function

$$K = \left(\frac{\alpha w}{\beta r}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$$L = \left(\frac{\beta r}{\alpha w}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$C(q, w, r) = wL + rK$

The cost function.

Can you see that when $a + \beta = 1$,
the firm has has constant returns to scale?

What happens when $a + \beta < 1$?