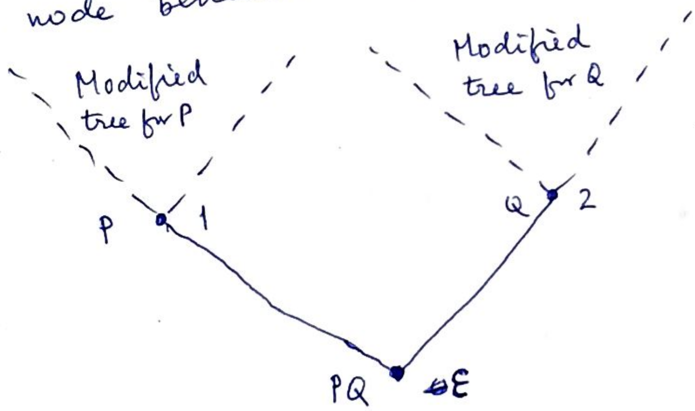
3.4.2025

Definition (The Construction-tree of a term)

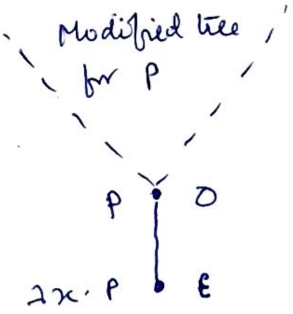
Each node in the tree has two labels: a position ad a subterm.
The tree is defined for an arbitrary term M as follows:

(i) If $M \equiv x$, its tree is a single node labelled with x and the empty position (denoted by ε - epsilon).

(ii) If $M \equiv PQ$, its tree is obtained by first concatenating "1" onto the left end of each position-label in the tree for P, then concatenating "2" onto the left end of each position-label in the tree for Q, and then placing an extra node beneath the two modified trees, as shown below:



(iii) If $M \equiv \lambda x . P$, its tree is obtained by first concatenating "0" onto the left end of each position-label in the tree for P, and then placing an extra node beneath the modified tree, as shown below



if we throw away the type/delta rules (discussed in proof tree or deduction tree), then the structure of proof or deduction tree is isomorphic to the construction tree.

Exercise:

Construct the construction-tree for $M \equiv (\lambda x . y x)(\lambda z . x(y x))$

$M \equiv P\,Q$ where $P \equiv \lambda x . y x$ $\quad Q \equiv \lambda z . x(y x)$

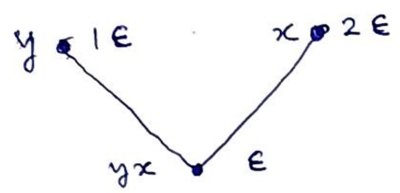
$P \equiv \lambda x . P'$ where $P' = y x$

$P' = P'' Q''$ where $P'' \equiv y$ $\quad Q'' \equiv x$

Now P'' and Q'' cannot be split any further.

1. Draw the trees for P'' and Q''

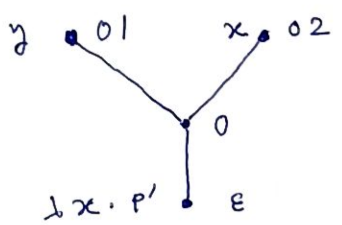
2. The tree for P' (obtained from step '1' and the rule for PQ)



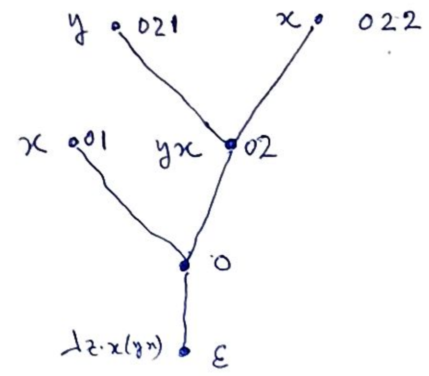
(P'-tree)

since $\alpha . \varepsilon = \varepsilon . \alpha = \alpha$ for any α
so we can label
1ε as 1 and 2ε as 2

3. The tree for P
(obtained from "2" and the rule for $\lambda x . M$)



4. The tree for Q



5. The tree for M:
(obtained from "3" and "4" and the rule for PQ)