- Name: Anvit Gupta
- Enrollment No: 22114009
- Batch: O-1 (CSE)
- Subject: MAN-001 (Assignment #01).

PAGE No.

DATE

(1) (a) let $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$

$\Rightarrow E_2(1/6)\, E_{12}(-2) \cdot E_{14}(-3)\, A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 5 & 8 \\ 0 & 0 & 3 & 3 \end{bmatrix} = M$

$\Rightarrow E_3(1/3) \cdot E_{23}(-5)\, E_{24}(-3)\, M = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{rank} = 3}$.

Ans

(1) (b) $\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 4 & 6 \\ 2 & 8 & 7 \end{bmatrix} = A$ (let.)

$\Rightarrow E_2(1/3) \cdot E_{12}(1) \cdot E_{23}(1) \cdot E_{34}(-2) \cdot E_{31}(-1)\, A = M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2/3 \\ 0 & 7 & 8 \\ 0 & 0 & -5 \end{bmatrix}$

$\Rightarrow E_{34}(5) \cdot E_3\left(\frac{3}{10}\right) \cdot E_{23}(-7)\, M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Ans

$\boxed{\text{Rank} = 3}$.

Ans

1(c) Let $A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & -3 & -5 \end{bmatrix}$

$\Rightarrow E_{13}(-3)\cdot E_{12}(-2)\,A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 14 & -18 & 4 \end{bmatrix}$.

$\Rightarrow E_2(1/7)\cdot E_{23}(-2)\,A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 1 & -9/7 & 2/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{Rank} = 2}$

Ans

1.(d) Let

$A = \begin{bmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$

$\Rightarrow E_{13}(1)\, E_{23}(-2)\, E_{24}(-3)\cdot A = \begin{bmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & -2 & 3 \end{bmatrix} = M$

$\Rightarrow E_3(1/3)\cdot E_{23}(5)\cdot E_2\left(\frac{1}{2}\right)\cdot E_{12}\cdot E_{34}\, M = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{Rank} = 3}$

Ans

(2) (a) $a(1,0,0)+b(0,1,0)+c(1,1,1)+d(-1,1,-1)=0$

$\Rightarrow a+c-d=0$
$\Rightarrow b+c+d=0$
$\Rightarrow c-d=0$

$a=0$
$c=d$
$b+2c=0$

So non zero solutions exist, it is linearly dependent.

(b) $a+2b+8c=0$
$-a+b+c=0$
$a+b+5c=0$

$\begin{pmatrix} a=-2c \\ b=-3c \end{pmatrix}$

Non zero triads exists, so linearly dependent.

(c) $a+2b-c=0$
$-a-b+3c=0$
$2a+5b+c=0$
$4a+7b-2c=0$

$a=b=c=0$
$\Downarrow$
linearly independent.

(d) $a+2b+c=0$
$2a+b-c=0$
$a+2c=0$

$a=-b$
$b=2c$
$-b=c$
} contradiction

So linearly independent.

Ans

PAGE No.
DATE

3.(a)(i) $A = \begin{pmatrix} \alpha & 1 & 2 \\ 0 & 2 & \beta \\ 1 & 3 & 6 \end{pmatrix} \xrightarrow{E_{13}\left(-\frac{1}{\alpha}\right)A} \begin{bmatrix} \alpha & 1 & 2 \\ 0 & 2 & \beta \\ 0 & 3-\frac{1}{\alpha} & 6-2/\alpha \end{bmatrix}$

Here, to have rank = 1, we have to make two rows zero which is not possible.
So No α, β exist for rank = 1.

(ii) $3-\frac{1}{\alpha} = 0 \Rightarrow \boxed{\alpha = 1/3}$
or $\frac{\beta}{2} - 2 = 0 \Rightarrow \boxed{\beta = 4}$
For rank = 2.
Ans

row echelon form $= \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & \beta/2 \\ 0 & 0 & (3\alpha-1)\left(\frac{\beta}{2}-2\right) \end{bmatrix}$.

(iii) Rank = 3 $\Rightarrow \alpha \neq \frac{1}{3}$ and $\beta \neq 4$
Ans.

(b) For consistency:
$\begin{vmatrix} 2 & 4 & (\alpha+3) \\ 1 & 3 & 1 \\ (\alpha-2) & 2 & 3 \end{vmatrix} \neq 0 \Rightarrow \boxed{\alpha \neq 3, -2}$.

- But when $\alpha = 3 \Rightarrow B = 1$ will have infinite solutions.

- $$\left[\begin{array}{ccc|c} 2 & 4 & (\alpha+3) & 3 \\ 1 & 3 & 1 & 2 \\ (\alpha-2) & 2 & 3 & B \end{array}\right] \xrightarrow[\text{form}]{\text{row echelon}} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -2 & (\alpha+1) & -2 \\ 0 & 0 & -\frac{3}{2}(\alpha-3)(\alpha+2) & B+\alpha-4 \end{array}\right]$$

$\therefore$ For consistency:

$$\text{Rank}(A) = \text{Rank}(A|B)$$

(a) For rank$(A) = 2$ (i) $\boxed{\alpha = 3}$

$B + \alpha - 4 = 0 \Rightarrow \boxed{B = 1}$

(ii) $\boxed{\alpha = -2} \Rightarrow B + \alpha - 4 = 0 \Rightarrow B = 6.$

(b) For rank$(A) = 3$: $\left.\begin{array}{l} \alpha \neq 3, -2 \\ B \neq 4 - \alpha \end{array}\right\}$ Ans

Ans: $\begin{pmatrix} \alpha = 3 \\ B = 1 \end{pmatrix}$; $\begin{pmatrix} \alpha = -2 \\ B = 6 \end{pmatrix}$; $\begin{pmatrix} \alpha \neq 3, -2 \\ B \neq 4 - \alpha \end{pmatrix}$ Ans

4. (a) $$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 1 & 1 & -6 & -4 \\ 3 & -1 & -1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc} 1 & 4 & -1 & 4 \\ 0 & 1 & 5/3 & 8/3 \\ 0 & 0 & \frac{71}{3} & \frac{71}{3} \end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} \cdot\ x + 4y - z = 4 \\ \cdot\ y + 5/3 z = 8/3 \\ \cdot\ \frac{71}{3} z = \frac{71}{3} \end{array}\right\} x = 1 = y = z$$

Ans

PAGE No.
DATE

4. (b) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 13 \end{bmatrix}$

$\Rightarrow$ Third eq^n: $0 = 13$ [Not possible]

So: no solution.

4. (c) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 3 & 4 & 2 & 4 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\left.\begin{array}{l} \Rightarrow y + z = 1 \\ \Rightarrow x + 2y + z = 2 \end{array}\right\}$ infinite solution

Ans

5. (a)(i) $D = 0 = \begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & r \\ 7 & 3 & -2 \end{vmatrix} \Rightarrow \boxed{r = 5}$. Ans

$D_x \neq 0 \Rightarrow \begin{vmatrix} 9 & 3 & 5 \\ 5 & 3 & 5 \\ 8 & 3 & -2 \end{vmatrix} \neq 0 \Rightarrow$ $s \neq 9$ Ans

5. (a) (ii) $D \neq 0 \Rightarrow r \neq 5;\ s \in R$. Ans

(iii) $D = 0,\ D_x = 0 \Rightarrow r = 5,\ s = 9$ Ans

(b) (i) $D = 0 \Rightarrow \boxed{\lambda = -3}$ Ans Here:

$D = (\lambda + 3)(\lambda - 2)$

(ii) $D \neq 0 \Rightarrow \boxed{\lambda \neq -3, 2}$

(iii) $D = 0 \Rightarrow \boxed{\lambda = 2}$ Ans

(c) (i) $D = 0 = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)^2(\lambda+2)$.

As here we have to find conditions on p, q, r also use gauss elimination method on augmented matrix:-

$$\Rightarrow \left[\begin{array}{ccc|c} \lambda & 1 & 1 & p \\ 1 & \lambda & 1 & q \\ 1 & 1 & \lambda & r \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & \lambda & r \\ 0 & (\lambda-1) & (1-\lambda) & (q-r) \\ 0 & (1-\lambda) & (1-\lambda^2) & (p-\lambda r) \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & \lambda & r \\ 0 & (\lambda-1) & (1-\lambda) & (q-r) \\ 0 & 0 & (\lambda+2)(1-\lambda) & (p+q-(1+\lambda) r) \end{array}\right]$$

(i) No solution:-

rank$(A) \neq$ rank$(A|b)$.

$\boxed{\lambda = 1}$ $\qquad$ $\boxed{\lambda = -2}$

$q \neq r$ $\qquad$ $p+q-2r \neq 0$ $\qquad$ $p+q+r \neq 0$, $q \neq r$.

(ii) Unique solution:-

rank$(A) = 3 =$ rank$(A|b)$.

$\Rightarrow$ $\lambda \neq 1, -2$

(iii) Infinite solution:-
rank(A) = rank(A|b) < 3.

$\lambda = 1$ $\quad$ $\lambda = -2$

$\begin{pmatrix} p+q = 2r \\ q = r \end{pmatrix}$ $\quad$ $p+q+r = 0$. **Ans**

6. (a) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 4 & 2 & -2 & 10 \\ 6 & -3 & 1 & 9 \end{array}\right] \longrightarrow \left[\begin{array}{cccc} 2 & -1 & 3 & 3 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -8 & 0 \end{array}\right]$

$\Rightarrow$ $2\sin x - \cos y + 3\tan z = 3$
$4\cos y - 8\tan z = 4$
$-8\tan z = 0 \Rightarrow$

$\tan z = 0$
$\cos y = 1$
$\sin x = 2$

not possible

so: no solution: proved.

(b) $\left(\begin{array}{ccccc|c} 0 & 2 & 2 & 3 & 0 & b_1 \\ 2 & 4 & 6 & 7 & 0 & b_2 \\ 1 & 1 & 2 & 2 & 0 & b_3 \end{array}\right) \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_3 \\ 0 & 2 & 2 & 3 & b_2 - 2b_3 \\ 0 & 0 & 0 & 0 & b_1 - b_2 + 2b_3 \end{array}\right]$

$\Rightarrow$ $x_1 + x_2 + 2x_3 + 2x_4 = b_3$
$2x_2 + 2x_3 + 3x_4 = b_2 - 2b_3 = b_1$

$b_1 + 2b_3 = b_2$

$\Rightarrow$ ~~[illegible]~~ infinite solutions would exist.

PAGE No.
DATE

7. Let $P(x) = ax^2 + bx + c$

$$\Rightarrow \left\{ \begin{array}{l} c = 1 \\ a + b + c = 2 \\ a - b + c = 6 \end{array} \right\} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 6 \end{array} \right]$$

we have

~~three eqn~~ :- $\begin{pmatrix} c = 1 \\ a = 3 \\ b = -2 \end{pmatrix} \Rightarrow \boxed{P(x) = 3x^2 - 2x + 1}$

Ans

8. (a) (i) $\begin{vmatrix} (3k-8) & 3 & 3 \\ 3 & (3k-8) & 3 \\ 3 & 3 & (3k-8) \end{vmatrix} \neq 0. \Rightarrow k \neq \frac{2}{3}, \frac{11}{3}$

(ii) $\begin{vmatrix} 3k-8 & 3 & 3 \\ 3 & 3k-8 & 3 \\ 3 & 3 & 3k-8 \end{vmatrix} = 0 \Rightarrow k = \frac{2}{3}, \frac{11}{3}$

(b) (i) $\begin{vmatrix} (k-1) & (3k+1) & (2k) \\ (k-1) & (4k-2) & (k+3) \\ 2 & (3k+1) & 3(k-1) \end{vmatrix} \neq 0 \Rightarrow k \neq 0, 3.$

(ii) $\begin{vmatrix} (k-1) & (3k+1) & (2k) \\ (k-1) & (4k-2) & (k+3) \\ 2 & (3k+1) & 3(k-1) \end{vmatrix} = 0 \Rightarrow k = 0, 3$

Ans

(9.) (a) Let $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

⇒ So we define: $[A|I] = \left[\begin{array}{ccc|ccc} +1 & 2 & -2 & 1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$

⇒

$$\xrightarrow{E_{12}(1)} \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{E_{23}(2/5)} \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1/5 & \frac{2}{5} & \frac{2}{5} & 1 \end{array}\right)$$

$$\xrightarrow{E_{31}(10),\ E_{32}(10)} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 5 & 4 & 10 \\ 0 & 5 & 0 & 5 & 5 & 10 \\ 0 & 0 & 1/5 & 2/5 & 2/5 & 1 \end{array}\right)$$

$$\xrightarrow{E_{21}(-2/5)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 6 \\ 0 & 5 & 0 & 5 & 5 & 10 \\ 0 & 0 & 1/5 & 2/5 & 2/5 & 1 \end{array}\right)$$

$$\xrightarrow{E_2(1/5);\ E_3(5)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 6 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{array}\right)$$

A^{-1} Ans

PAGE No. DATE

9.(b) let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 2 & 1 & 1 & 3 \end{bmatrix} \Rightarrow [A|I] = \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right)$

$$\xrightarrow{E_{12}(-2) \cdot E_{13}(-3) \cdot E_{14}(-2)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & -3 & -1 & 2 & -3 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{E_{24}(-1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & -3 & -1 & 2 & -3 & 0 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{E_{23}(-3)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 0 & -4 & 8 & 3 & -3 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[E_3(-1/4)]{E_{34}(-1/2)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & \frac{-3}{4} & \frac{3}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 0 & -1 & \frac{-3}{2} & \frac{1}{2} & \frac{-1}{2} & 1 \end{array}\right)$$

$$\xrightarrow{E_2(-1)\,;\,E_4(-1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -3/4 & 3/4 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & \frac{-1}{2} & \frac{1}{2} & -1 \end{array}\right)$$

$$\xrightarrow{E_{32}(1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & -2 & \frac{-3}{4} & \frac{3}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 \end{array}\right)$$

$$\downarrow (R_1 \to R_1 - R_2 - R_4)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & -7/4 & 3/4 & -1/4 & 1 \\ 0 & 1 & 0 & 0 & 5/4 & -1/4 & -1/4 & 0 \\ 0 & 0 & 1 & -2 & -3/4 & 3/4 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & 3/2 & -1/2 & +1/2 & -1 \end{array}\right) \xrightarrow{E_{43}(2)}$$

$$(I|A^{-1}) \xleftarrow{E_{31}(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & -7/4 & 3/4 & -1/4 & 1 \\ 0 & 1 & 0 & 0 & 5/4 & -1/4 & -1/4 & 0 \\ 0 & 0 & 1 & 0 & 9/4 & -1/4 & 3/4 & -2 \\ 0 & 0 & 0 & 1 & 3/2 & -1/2 & +1/2 & -1 \end{array}\right)$$

$\Downarrow$

where: $A^{-1} = \frac{1}{4}\begin{pmatrix} -16 & 4 & -4 & 12 \\ 5 & -1 & -1 & 0 \\ 9 & -1 & 3 & -8 \\ 6 & -2 & 2 & -4 \end{pmatrix}$ Ans

(10.) Let $X = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$; $Y = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

$\Rightarrow$ let $A = XY^T = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} (b_1\ b_2 \dots b_n)$

$\Rightarrow A = XY^T = \begin{pmatrix} a_1b_1 & a_1b_2 & \cdots \\ \vdots & & \\ a_nb_1 & \cdots & a_nb_n \end{pmatrix}$

$|A| = (a_1 a_2 \dots a_n)(b_1 b_2 \dots b_n) \left|\begin{pmatrix} 1 & 1 & 1 & \cdots \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & - & 1 \end{pmatrix}\right| = 0$ Ans

$\therefore$ So A = non-invertible

PAGE No.
DATE

(11) Obviously $|A| \neq 0 \Rightarrow$ nonsingular and invertible.

$$\Rightarrow A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix} \Rightarrow |A| = 6 \neq 0.$$

$$\Rightarrow \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix} = A \xrightarrow{E_{12}} \begin{pmatrix} 3 & 3 & 0 \\ 2 & 0 & 1 \\ 6 & 2 & 3 \end{pmatrix} \xrightarrow{E_{23}(-3)} \begin{pmatrix} 3 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\downarrow E_{31}\left(-\frac{3}{2}\right)$$

$$\Rightarrow I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xleftarrow[E_{23}]{E_1(1/3),\ E_3(1/2)} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix} \xleftarrow{E_{12}(-2/3)} \begin{pmatrix} 3 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\Rightarrow \text{so: } E_{23} E_3\left(\frac{1}{2}\right) E_1\left(\frac{1}{3}\right) E_{12}\left(-\frac{2}{3}\right) E_{31}\left(-\frac{3}{2}\right) E_{23}(-3) \ldots$$

$$\ldots E_{12} A = I$$

$$\Rightarrow \boxed{A = E_{12}^{-1} E_{23}(-3)^{-1} \ldots\ldots E_{23}^{-1}} \quad \underline{\text{Ans}}$$

(12)(a) $|A| = 0 \Rightarrow$ It means one of the eigen value of A is 0.

$\Rightarrow$ So $AX = 0$ [For $\lambda = 0$]

B can be equal to: $B = [X \mid 0 \mid \ldots \mid 0]$.

such that: $AB = [AX \mid 0 \mid \ldots \mid 0] = 0$

<u>Ans</u>

<u>proved</u>

$|A| = 0 \Rightarrow$ It means one $(\lambda = 0)$.

(12)(b) $Ax = 0 \Rightarrow$ we have to find eigen vector w.r.t $\lambda = 0 = \begin{pmatrix}\text{eigen}\\ \text{value}\end{pmatrix}$.

$$\Rightarrow \begin{pmatrix} 1 & 2 & -1 & 3 \\ -2 & 1 & 0 & 1 \\ 0 & 5 & -2 & 7 \\ -1 & 3 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0.$$

$\Rightarrow$ solving:- one solution is:
$x_1 = x_2 = x_4 = 1;\ x_3 = 6.$

so: $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ Ans

(13.) (a) Let $A = \begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix}$

$\big\downarrow E_{12}(-3)\ E_{13}(-2)\ E_{14}(-6)$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix}$$

$M = \begin{pmatrix} 1 & 7 & 0 & +3 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\big\downarrow E_2(1/3)$

$E_{21}(1) \cdot E_{23}(-2)$
$E_{24}(-5)$
$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix}$$

PAGE No.
DATE

$$\Rightarrow E_{32}(-1)\cdot E_{34}(-1)\,M = \begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

= row-reduced echelon form of A. **Ans**

13.(b) Actually, if we see (a) part solution, then:-

$$\underbrace{E_{12}(-3)\,E_{13}(-2)\,E_{14}(-6)}_{P}\,A = \begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix}$$

$$\Rightarrow P = E_{12}(-3)\cdot E_{13}(-2)\cdot E_{14}(-6)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{pmatrix} \quad \textbf{Ans}$$

13.(c) For solution, rank(A) = 3 = rank(A|Y).

$$\Rightarrow (A|Y) = \left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & X \\ 3 & 21 & 0 & 9 & 0 & Y \\ 2 & 14 & 0 & 6 & 1 & Z \\ 6 & 42 & -1 & 13 & 0 & 5 \end{array}\right)$$

$$\downarrow R_4 \to (R_4 - R_1 - R_2 - R_3).$$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 & X \\ 3 & 21 & 0 & 9 & 0 & Y \\ 2 & 14 & 0 & 6 & 1 & Z \\ 0 & 0 & 0 & 6 & 0 & 5-X-Y-Z \end{pmatrix}$$

For rank = 3 , $\boxed{x+y+z=5}$ Ans

(13) (a) $AX=0 \Rightarrow$ convert 'A' to row reduced echelon form as in (a) part:

$$\begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} x_5 &= 0 \\ x_3 + 5x_4 &= 0 \\ x_1 + 7x_2 + 3x_4 &= 0 \end{aligned}\right\} \text{Ans}$$