Lecture Outline

- Strategic Role of Forecasting in Supply Chain Management
- Components of Forecasting Demand
- Time Series Methods
- Forecast Accuracy
- Time Series Forecasting Using Excel
- Regression Methods

Learning Objectives

- Discuss the strategic role of forecasting in supply chain management
- Describe the forecasting process and identify the components of forecasting demand
- Forecast demand using various time series models, including exponential smoothing, and trend and seasonal adjustments
- Discuss and calculate various methods for evaluating forecast accuracy
- Use Excel to create various forecast models
- Develop forecasting models with linear and multiple regression analysis

Supply & Demand

Operations & Supply Chains	Sales & Marketing	
Supply > Demand		Wasteful Costly
Supply < Demand		Opportunity Loss Customer Dissatisfaction
Supply = Demand		Ideal

Process variation

- Varieties of offered goods or services
- Variation in demand
- Random variation
- Assignable variation

Process variation

- Varieties of offered goods or services
- *Variation in demand*
- Random variation common cause variation
- Assignable variation special cause variation

Illustration

- Apparel trends are rarely unplanned
- Looks, styles, and colors can often be traced back
  - Result of vast amounts of
    - Information
    - Sophisticated forecasting
    - Expert professional analysis
- Identify fundamental facts about past trends and forecasts
- Determine factors most likely to affect future trends
  - Economic
  - Technological
  - Fashion

Factors for denim jeans

- Fiber innovations
- Price and availability of cotton
- Advances in manufacturing processes and machinery
- Shifts in global manufacturing locations
- Shipping changes
- Shifting global markets
- Sustainability issues
- Fashion factors
  - Designs
  - Colours
  - Styles
  - Media
  - Blogs
  - Celebrity
  - Apparel trade shows

Forecast

- **Forecast** – a statement about the future value of a variable of interest
  - We make forecasts about such things as weather, demand, and resource availability
  - Forecasts are important to making informed decisions

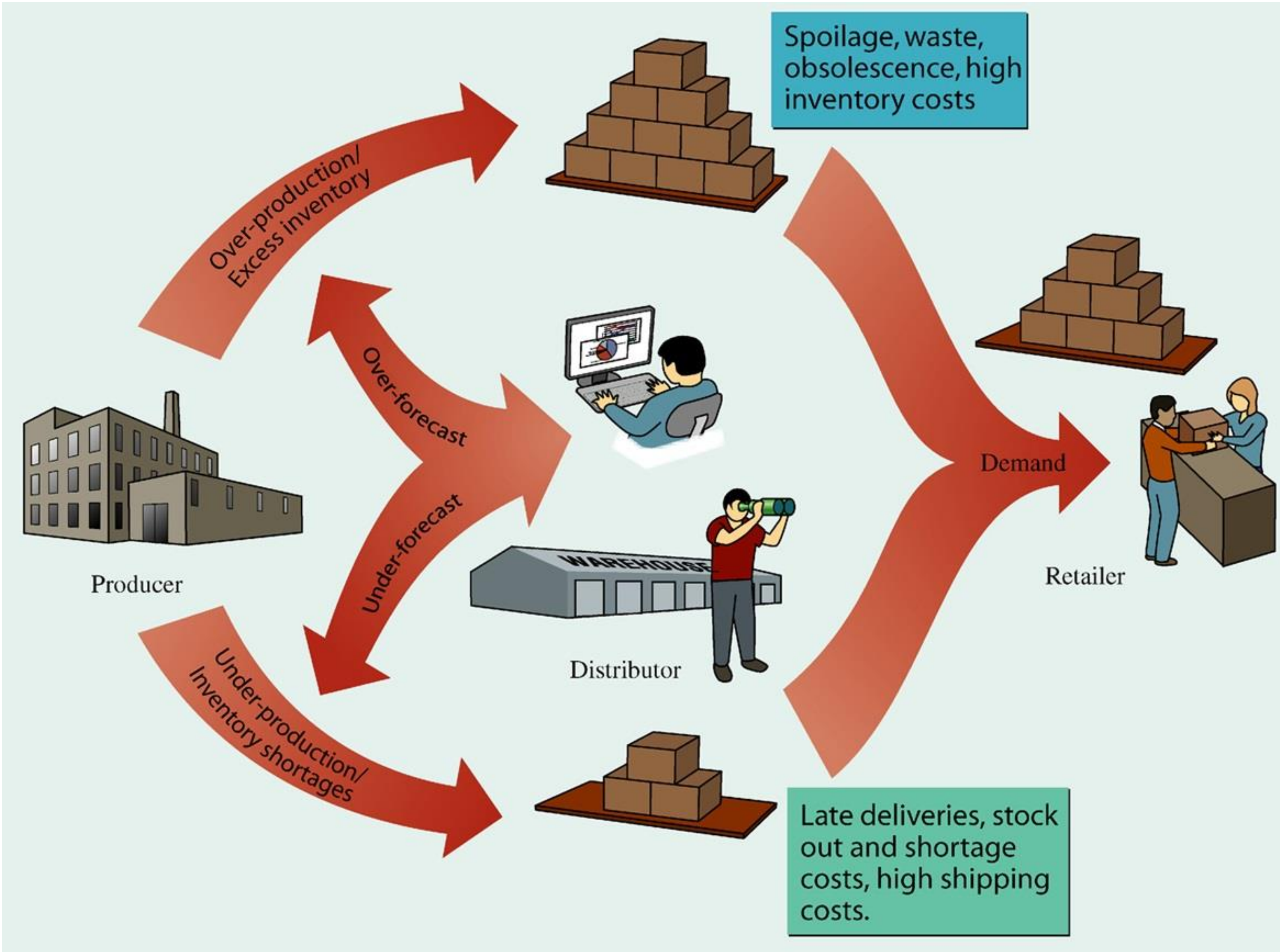
Two Important Aspects of Forecasts

- Expected level of demand
  - The level of demand may be a function of some structural variation such as trend or seasonal variation
- Accuracy
  - Related to the potential size of forecast error

Forecast Uses

- **Plan the system**
  - Generally involves long-range plans related to:
    - Types of products and services to offer
    - Facility and equipment levels
    - Facility location
- **Plan the use of the system**
  - Generally involves short- and medium-range plans related to:
    - Inventory management
    - Workforce levels
    - Purchasing
    - Production
    - Budgeting
    - Scheduling

The Effect of Inaccurate Forecasting



Features Common to All Forecasts

1. Techniques assume some underlying causal system that existed in the past will persist into the future
2. Forecasts are not perfect
3. Forecasts for groups of items are more accurate than those for individual items
4. Forecast accuracy decreases as the forecasting horizon increases

Forecasts are not Perfect

– Forecasts are not perfect:
  - Because random variation is always present, there will always be some residual error, even if all other factors have been accounted for.

Elements of a Good Forecast

The forecast

- should be *timely*
- should be *accurate*
- should be *reliable*
- should be expressed in *meaningful units*
- should be *in writing*
- technique should be *simple to understand and use*
- should be *cost-effective*

Steps in the Forecasting Process

1. Determine the purpose of the forecast
2. Establish a time horizon
3. Obtain, clean, and analyze appropriate data
4. Select a forecasting technique
5. Make the forecast
6. Monitor the forecast errors

Forecast Accuracy and Control

- Allowances should be made for forecast errors
  - It is important to provide an indication of the extent to which the forecast might deviate from the value of the variable that actually occurs
- Forecast errors should be monitored
  - Error = Actual – Forecast
  - If errors fall beyond acceptable bounds, corrective action may be necessary

Forecast Accuracy Metrics

$$\text{MAD} = \frac{\sum \left|\text{Actual}_t - \text{Forecast}_t\right|}{n}$$

- MAD weights all errors evenly

$$\text{MSE} = \frac{\sum \left(\text{Actual}_t - \text{Forecast}_t\right)^2}{n-1}$$

- MSE weights errors according to their squared values

$$\text{MAPE} = \frac{\sum \frac{\left|\text{Actual}_t - \text{Forecast}_t\right|}{\text{Actual}_t} \times 100}{n}$$

- MAPE weights errors according to relative error

Forecast Error Calculation

Period	Actual (A)	Forecast (F)	(A-F) Error	|Error|	$Error^2$	[|Error|/Actual]x100
1	107	110	-3	3	9	2.80%
2	125	121	4	4	16	3.20%
3	115	112	3	3	9	2.61%
4	118	120	-2	2	4	1.69%
5	108	109	1	1	1	0.93%
			Sum	13	39	11.23%
				n = 5	*n-1* = 4	*n* = 5
				MAD	**MSE**	**MAPE**
				= 2.6	= 9.75	= 2.25%

Forecasting Approaches

- **Qualitative Forecasting**
  - Qualitative techniques permit the inclusion of *soft* information such as:
    - Human factors
    - Personal opinions
    - Hunches
  - These factors are difficult, or impossible, to quantify
- **Quantitative Forecasting**
  - These techniques rely on *hard* data
  - Quantitative techniques involve either the projection of historical data or the development of associative methods that attempt to use *causal variables* to make a forecast
  - Diffusion Models

time series
E.S
ARIMA
MA
Regression
y (IV) = DV

Qualitative Forecasts

- Forecasts that use subjective inputs such as opinions from consumer surveys, sales staff, managers, executives, and experts
  - Executive opinions
    - a small group of upper-level managers may meet and collectively develop a forecast
  - Sales force opinions
    - members of the sales or customer service staff can be good sources of information due to their direct contact with customers and may be aware of plans customers may be considering for the future
  - Consumer surveys — Preference Survey
    - since consumers ultimately determine demand, it makes sense to solicit input from them
    - consumer surveys typically represent a *sample* of consumer opinions
  - Other approaches
    - managers may solicit 0pinions from other managers or staff people or outside experts to help with developing a forecast.
    - the Delphi method is an iterative process intended to achieve a consensus

Time-Series Forecasts

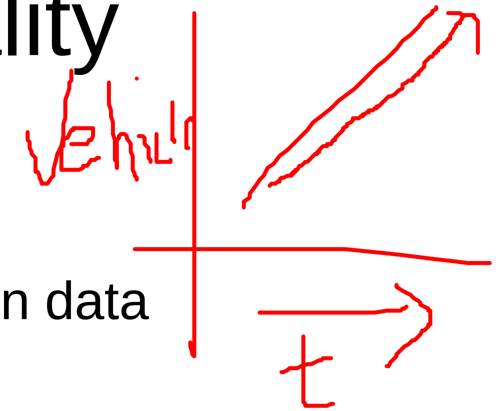
- Forecasts that project patterns identified in recent time-series observations
  - **Time-series** - a time-ordered sequence of observations taken at regular time intervals

- Assume that future values of the time-series can be estimated from past values of the time-series

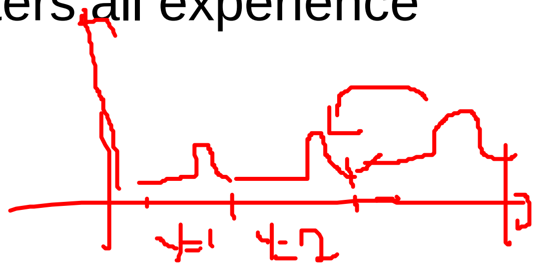
Time-Series Behaviors

- Trend
- Seasonality
- Cycles
- Irregular variations
- Random variation

Trends and Seasonality



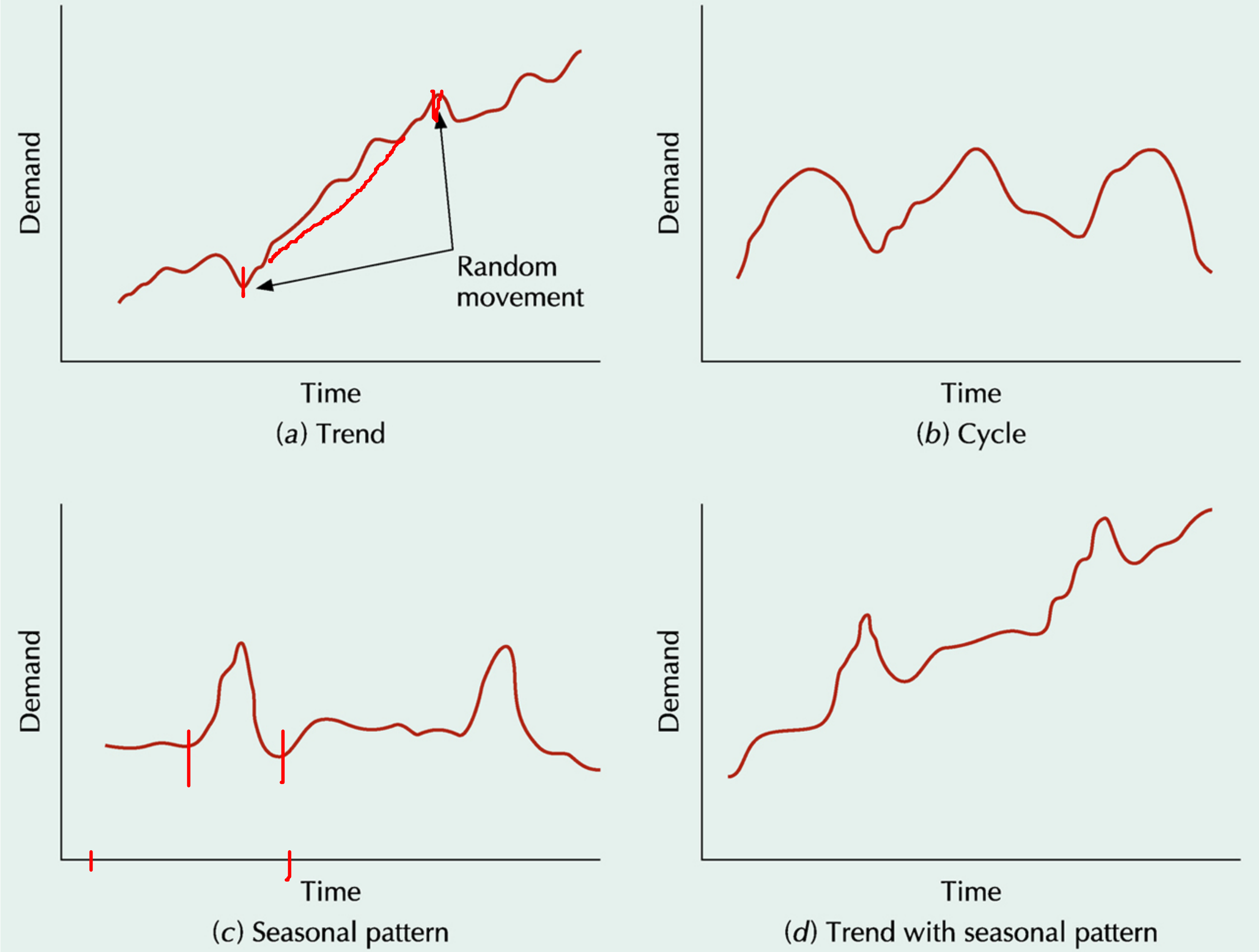
- **Trend**
  - A long-term upward or downward movement in data
    - Population shifts
    - Changing income
- **Seasonality**
  - Short-term, fairly regular variations related to the calendar or time of day
  - Restaurants, service call centers, and theaters all experience seasonal demand



Cycles and Variations

- **Cycle**
  - Wavelike variations lasting more than one year
    - These are often related to a variety of economic, political, or even agricultural conditions
- Irregular variation
  - Due to unusual circumstances that do not reflect typical behavior
    - Labor strike
    - Weather event
- **Random Variation**
  - Residual variation that remains after all other behaviors have been accounted for

Forms of Forecast Movement



(*a*) Trend

(*b*) Cycle

(*c*) Seasonal pattern

(*d*) Trend with seasonal pattern

Time-Series Forecasting - Naïve Forecast

- **Naïve Forecast**
  - Uses a single previous value of a time series as the basis for a forecast
    - The forecast for a time period is equal to the previous time period’s value
  - Can be used with
    - a stable time series
    - seasonal variations
    - trend

Time-Series Forecasting - Averaging

t=1 2 3 4

- These techniques work best when a series tends to vary about an average
  - Averaging techniques smooth variations in the data
  - They can handle step changes or gradual changes in the level of a series
  - Techniques

    1. Moving average
    2. Weighted moving average
    3. Exponential smoothing

$(x<1)$

$x - x^2 - x^3$

G.P.

Moving Average

- Technique that averages a number of the most recent actual values in generating a forecast

$$F_t = \text{MA}_n = \frac{\sum_{i=1}^{n} A_{t-i}}{n} = \frac{A_{t-n} + \ldots + A_{t-2} + A_{t-1}}{n}$$

where

F_t = Forecast for time period t

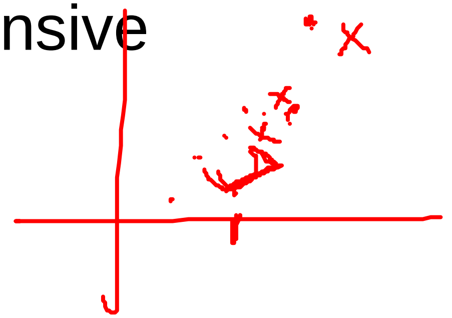
MA_n = n period moving average

A_{t-i} = Actual value in period $t-i$

n = Number of periods in the moving average

Moving Average

- As new data become available, the forecast is updated by adding the newest value and dropping the oldest and then re-computing the average
- The number of data points included in the average determines the model's sensitivity
  - Fewer data points used-- more responsive
  - More data points used-- less responsive



Weighted Moving Average

- The most recent values in a time series are given more weight in computing a forecast
  - The choice of weights, *w*, is somewhat arbitrary and involves some trial and error

$$F_t = w_t(A_t) + w_{t-1}(A_{t-1}) + \ldots + w_{t-n}(A_{t-n})$$

where

w_t = weight for period t, w_{t-1} = weight for period $t-1$, etc.

A_t = the actual value for period t, A_{t-1} = the actual value for period $t-1$, etc.

$$ES = F_{t+1} = \lambda(A_t) + (1-\lambda)F_t$$

$$= \lambda(A_t) + (1-\lambda)(\lambda A_{t-1} + (1-\lambda)F_{t-1})$$

$$= \lambda A_t + (1-\lambda)\lambda A_{t-1} + (1-\lambda)^2 F_{t-1}$$

Linear Trend

- A simple data plot can reveal the existence and nature of a trend
- Linear trend equation

$$F_t = a + bt$$

where

$$F_t = \text{Forecast for period } t$$

$$a = \text{Value of } F_t \text{ at } t = 0$$

$$b = \text{Slope of the line}$$

$$t = \text{Specified number of time periods from } t = 0$$

Estimating slope and intercept

- Slope and intercept can be estimated from historical data

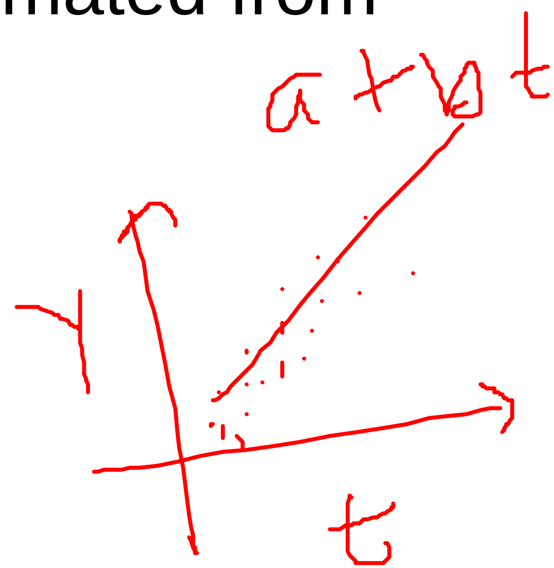
$$b = \frac{n\sum ty - \sum t \sum y}{n\sum t^2 - \left(\sum t\right)^2}$$

$$a = \frac{\sum y - b\sum t}{n} \text{ or } \bar{y} - b\bar{t}$$

where

$n =$ Number of periods

$y =$ Value of the time series



$(a + bt_i) = \hat{y}_i$

$D_i^2 = (y_i - \hat{y}_i)^2$

Loss function $= T.D = \sum (y_i - \hat{y}_i)^2$

Moving Average: Naïve Approach

MONTH	ORDERS PER MONTH	FORECAST
Jan	120	
Feb	90	
Mar	100	
Apr	75	
May	110	
June	50	
July	75	
Aug	130	
Sept	110	
Oct	90	
Nov	-	

Moving Average: Naïve Approach

MONTH	ORDERS PER MONTH	FORECAST
Jan	120	-
Feb	90	120
Mar	100	90
Apr	75	100
May	110	75
June	50	110
July	75	50
Aug	130	75
Sept	110	130
Oct	90	110
Nov	-	90

Simple Moving Average

$$MA_n = \frac{\sum_{i=1}^{n} D_i}{n}$$

where

n = number of periods in the moving average
D_i = demand in period i

3-month Simple Moving Average

MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	
Feb	90	
Mar	100	
Apr	75	
May	110	
June	50	
July	75	
Aug	130	
Sept	110	
Oct	90	
Nov	-	

$$MA_3 = \frac{\sum_{i=1}^{3} D_i}{3}$$

3-month Simple Moving Average

MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	–
Feb	90	–
Mar	100	–
Apr	75	103.3
May	110	88.3
June	50	95.0
July	75	78.3
Aug	130	78.3
Sept	110	85.0
Oct	90	105.0
Nov	-	110.0

$t = n+1$

$$MA_3 = \frac{\sum_{i=1}^{3} D_i}{3}$$

$$= \frac{90 + 110 + 130}{3}$$

$= 110$ orders for Nov

5-month Simple Moving Average

MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	
Feb	90	
Mar	100	
Apr	75	
May	110	
June	50	
July	75	
Aug	130	
Sept	110	
Oct	90	
Nov	-	

$$MA_5 = \frac{\sum_{i=1}^{5} D_i}{5}$$

5-month Simple Moving Average

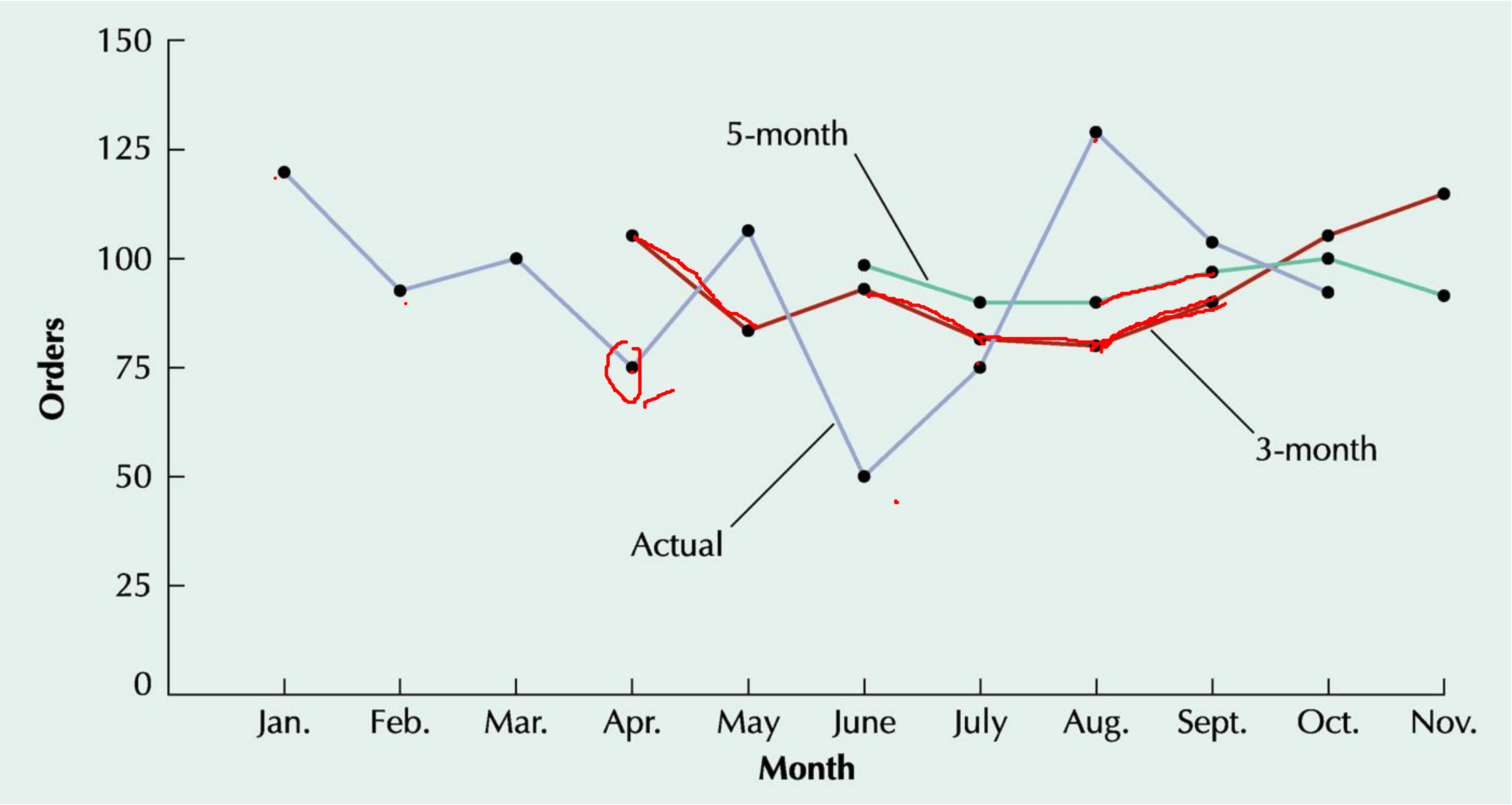
MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	–
Feb	90	–
Mar	100	–
Apr	75	–
May	110	–
June	50	99.0
July	75	85.0
Aug	130	82.0
Sept	110	88.0
Oct	90	95.0
Nov	-	91.0

$$MA_5 = \frac{\sum_{i=1}^{5} D_i}{5}$$

$$= \frac{90 + 110 + 130+75+50}{5}$$

= 91 orders for Nov

Smoothing Effects



Weighted Moving Average

- Adjusts moving average method to more closely reflect data fluctuations

$$WMA_n = \sum_{i=1}^{n} W_i \, D_i$$

where

W_i = the weight for period i, between 0 and 100 percent

$\sum W_i = 1.00$

Weighted Moving Average Example

MONTH	*WEIGHT*	*DATA*
August	17%	130
September	33%	110
October	50%	90

November Forecast $$WMA_3 = \sum_{i=1}^{3} W_i \, D_i$$

Weighted Moving Average Example

MONTH	*WEIGHT*	*DATA*
August	17%	130
September	33%	110
October	50%	90

November Forecast $WMA_3 = \sum_{i=1}^{3} W_i D_i$

$= (0.50)(90) + (0.33)(110) + (0.17)(130)$

$= 103.4$ orders

Exponential Smoothing

- Averaging method
- Weights most recent data more strongly
- Reacts more to recent changes
- Widely used, accurate method
- Smoothing constant, α
  - applied to most recent data

Exponential Smoothing

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

where:

F_{t+1} = forecast for next period

D_t = actual demand for present period

F_t = previously determined forecast for present period

α = weighting factor, smoothing constant

Effect of Smoothing Constant

$$0.0 \leq \alpha \leq 1.0$$

If $\alpha = 0.20$, then $F_{t+1} = 0.20\, D_t + 0.80\, F_t$

If $\alpha = 0$, then $F_{t+1} = 0\, D_t + 1\, F_t = F_t$

Forecast does not reflect recent data

If $\alpha = 1$, then $F_{t+1} = 1\, D_t + 0\, F_t = D_t$

Forecast based only on most recent data

Exponential Smoothing (α=0.30)

PERIOD	MONTH	DEMAND
1	Jan	37
2	Feb	40
3	Mar	41
4	Apr	37
5	May	45
6	Jun	50
7	Jul	43
8	Aug	47
9	Sep	56
10	Oct	52
11	Nov	55
12	Dec	54

$$F_2 = \alpha D_1 + (1 - \alpha)F_1$$

$$F_3 = \alpha D_2 + (1 - \alpha)F_2$$

$$F_{13} = \alpha D_{12} + (1 - \alpha)F_{12}$$

Exponential Smoothing (α=0.30)

PERIOD	MONTH	DEMAND
1	Jan	37
2	Feb	40
3	Mar	41
4	Apr	37
5	May	45
6	Jun	50
7	Jul	43
8	Aug	47
9	Sep	56
10	Oct	52
11	Nov	55
12	Dec	54

$$F_2 = \alpha D_1 + (1 - \alpha)F_1$$
$$= (0.30)(37) + (0.70)(37)$$
$$= 37$$

$$F_3 = \alpha D_2 + (1 - \alpha)F_2$$
$$= (0.30)(40) + (0.70)(37)$$
$$= 37.9$$

$$F_{13} = \alpha D_{12} + (1 - \alpha)F_{12}$$
$$= (0.30)(54) + (0.70)(50.84)$$
$$= 51.79$$

Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST, F_{t+1} ($\alpha = 0.3$)	($\alpha = 0.5$)
1	Jan	37	–	–
2	Feb	40		
3	Mar	41		
4	Apr	37		
5	May	45		
6	Jun	50		
7	Jul	43		
8	Aug	47		
9	Sep	56		
10	Oct	52		
11	Nov	55		
12	Dec	54		
13	Jan	–		

Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST, F_{t+1} (α = 0.3)	(α = 0.5)
1	Jan	37	– N.A	–
2	Feb	40	37.00	37.00 F_1
3	Mar	41	37.90	38.50
4	Apr	37	38.83	39.75
5	May	45	38.28	38.37
6	Jun	50	40.29	41.68
7	Jul	43	43.20	45.84
8	Aug	47	43.14	44.42
9	Sep	56	44.30	45.71
10	Oct	52	47.81	50.85
11	Nov	55	49.06	51.42
12	Dec	54	50.84	53.21
13	Jan	–	51.79	53.61

Adjusted Exponential Smoothing

$\alpha D_t + (1-\alpha) F_t$

$$AF_{t+1} = F_{t+1} + T_{t+1}$$

where

T = an exponentially smoothed trend factor

$F_3 \quad F_2 \quad + (1-\beta) T_2$

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)\, T_t$$

$T_3 = \beta [F_3 - F_2]$

$T_2 = \beta (F_2 - F_1) + (1-\beta) T_1 = 0$

where

T_t = the last period trend factor

β = a smoothing constant for trend

$0 \leq \beta \leq 1$

Adjusted Exponential Smoothing (β=0.30)

PERIOD	MONTH	DEMAND
1	Jan	37
2	Feb	40
3	Mar	41
4	Apr	37
5	May	45
6	Jun	50
7	Jul	43
8	Aug	47
9	Sep	56
10	Oct	52
11	Nov	55
12	Dec	54

$$T_3 = \beta(F_3 - F_2) + (1 - \beta)\, T_2$$

$$AF_3 = F_3 + T_3$$

$$T_{13} = \beta(F_{13} - F_{12}) + (1 - \beta)\, T_{12}$$

$$AF_{13} = F_{13} + T_{13} =$$

Adjusted Exponential Smoothing (β=0.30)

PERIOD	MONTH	DEMAND
1	Jan	37
2	Feb	40
3	Mar	41
4	Apr	37
5	May	45
6	Jun	50
7	Jul	43
8	Aug	47
9	Sep	56
10	Oct	52
11	Nov	55
12	Dec	54

$$T_3 = \beta(F_3 - F_2) + (1 - \beta)\, T_2$$
$$= (0.30)(38.5 - 37.0) + (0.70)(0)$$
$$= 0.45$$

$$AF_3 = F_3 + T_3 = 38.5 + 0.45$$
$$= 38.95$$

$$T_{13} = \beta(F_{13} - F_{12}) + (1 - \beta)\, T_{12}$$
$$= (0.30)(53.61 - 53.21) + (0.70)(1.77)$$
$$= 1.36$$

$$AF_{13} = F_{13} + T_{13} = 53.61 + 1.36 = 54.97$$

Adjusted Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST F_{t+1}	TREND T_{t+1}	ADJUSTED FORECAST AF_{t+1}
1	Jan	37	37.00	–	–
2	Feb	40	37.00	0.00	37.00
3	Mar	41	38.50	0.45	38.95
4	Apr	37	39.75	0.69	40.44
5	May	45	38.37	0.07	38.44
6	Jun	50	38.37	0.07	38.44
7	Jul	43	45.84	1.97	47.82
8	Aug	47	44.42	0.95	45.37
9	Sep	56	45.71	1.05	46.76
10	Oct	52	50.85	2.28	58.13
11	Nov	55	51.42	1.76	53.19
12	Dec	54	53.21	1.77	54.98
13	Jan	–	53.61	1.36	54.96

Adjusted Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST F_{t+1}	TREND T_{t+1}	ADJUSTED FORECAST AF_{t+1}
1	Jan	37			
2	Feb	40			
3	Mar	41			
4	Apr	37			
5	May	45			
6	Jun	50			
7	Jul	43			
8	Aug	47			
9	Sep	56			
10	Oct	52			
11	Nov	55			
12	Dec	54			
13	Jan	–			

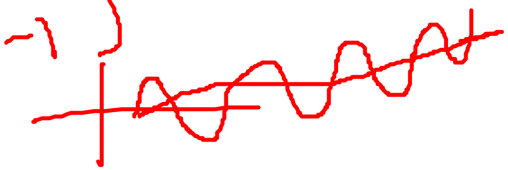
Techniques for Seasonality

- Seasonality – regularly repeating movements in series values that can be tied to recurring events
  - Expressed in terms of the amount that actual values deviate from the average value of a series
  - Models of seasonality
    - Additive
      - Seasonality is expressed as a quantity that gets added to or subtracted from the time-series average in order to incorporate seasonality
    - Multiplicative
      - Seasonality is expressed as a percentage of the average (or trend) amount which is then used to multiply the value of a series in order to incorporate seasonality

Seasonal Relatives

- **Seasonal relatives**
  - The seasonal percentage used in the multiplicative seasonally adjusted forecasting model

$(t_i - t_{i-1})$



- Using seasonal relatives
  - To *deseasonalize* data
    - Done in order to get a clearer picture of the nonseasonal (e.g., trend) components of the data series
    - Divide each data point by its seasonal relative
  - To *incorporate seasonality* in a forecast

    1. Obtain trend estimates for desired periods using a trend equation
    2. Add seasonality by multiplying these trend estimates by the corresponding seasonal relative

Seasonal Adjustments

- Repetitive increase/ decrease in demand
- Use seasonal factor to adjust forecast

$$\text{Seasonal factor} = S_i = \frac{D_i}{\sum D}$$

Seasonal Adjustment

DEMAND (1000'S PER QUARTER)

YEAR	*1*	*2*	*3*	*4*	*Total*
2002	12.6	8.6	6.3	17.5	
2003	14.1	10.3	7.5	18.2	
2004	15.3	10.6	8.1	19.6	
	____	____	____	____	____

$$S_1 = \frac{D_1}{\sum D} =$$

$$S_3 = \frac{D_3}{\sum D} =$$

$$S_2 = \frac{D_2}{\sum D} =$$

$$S_4 = \frac{D_4}{\sum D} =$$

Seasonal Adjustment

For 2005

$y =$

$SF_1 = (S_1)\ (F_5) =$
$SF_2 = (S_2)\ (F_5) =$
$SF_3 = (S_3)\ (F_5) =$
$SF_4 = (S_4)\ (F_5) =$

Seasonal Adjustment

DEMAND (1000'S PER QUARTER)

YEAR	*1*	*2*	*3*	*4*	*Total*
2002	12.6	8.6	6.3	17.5	45.0
2003	14.1	10.3	7.5	18.2	50.1
2004	15.3	10.6	8.1	19.6	53.6
Total	42.0	29.5	21.9	55.3	148.7

$$S_1 = \frac{D_1}{\sum D} = \frac{42.0}{148.7} = 0.28$$

$$S_3 = \frac{D_3}{\sum D} = \frac{21.9}{148.7} = 0.15$$

$$S_2 = \frac{D_2}{\sum D} = \frac{29.5}{148.7} = 0.20$$

$$S_4 = \frac{D_4}{\sum D} = \frac{55.3}{148.7} = 0.37$$

Seasonal Adjustment

For 2005

$$y = 40.97 + 4.30x = 40.97 + 4.30(4) = 58.17$$

$SF_1 = (S_1)(F_5) = (0.28)(58.17) = 16.28$

$SF_2 = (S_2)(F_5) = (0.20)(58.17) = 11.63$

$SF_3 = (S_3)(F_5) = (0.15)(58.17) = 8.73$

$SF_4 = (S_4)(F_5) = (0.37)(58.17) = 21.53$

Associative Forecasting Techniques

- Associative techniques are based on the development of an equation that summarizes the effects of predictor variables
  - **Predictor variables** - variables that can be used to predict values of the variable of interest
    - Home values may be related to such factors as home and property size, location, number of bedrooms, and number of bathrooms

I.V = R.H.S

Simple Linear Regression

- Regression - a technique for fitting a line to a set of data points
  - Simple linear regression - the simplest form of regression that involves a linear relationship between two variables
    - The object of simple linear regression is to obtain an equation of a straight line that minimizes the sum of squared vertical deviations from the line (i.e., the *least squares criterion*)

Least Squares Line

$$y_c = a + bx$$

where

y_c = Predicted (dependent) variable

x = Predictor (independent) variable

b = Slope of the line

a = Value of y_c when $x = 0$ (i.e., the height of the line at the y intercept)

and

$$b = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

$\hat{y}_i = m x_i + c$

$$a = \frac{\sum y - b\sum x}{n} \text{ or } \bar{y} - b\bar{x}$$

$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

where

n = Number of paired observations

Correlation Coefficient

- Correlation, *r*
  - A measure of the strength and direction of relationship between two variables
  - Ranges between -1.00 and +1.00

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

- r^2, square of the correlation coefficient
  - A measure of the percentage of variability in the values of *y* that is "explained" by the independent variable
  - Ranges between 0 and 1.00

$\bar{x} = 1 \cdot \bar{X}$

$= [X - \bar{X}]$

$[Y - \bar{Y}]$

$[Y - \bar{Y}] \cdot [X - \bar{X}] = \|Y - \bar{Y}\| \, \|X - \bar{X}\| \cos\theta$

$$\cos\theta = \frac{[Y - \bar{Y}][X - \bar{X}]}{\|Y - \bar{Y}\| \, \|X - \bar{X}\|}$$

Linear Regression Example

x (WINS)	y (ATTENDANCE)	xy	x^2
4	36.3	145.2	16
6	40.1	240.6	36
6	41.2	247.2	36
8	53.0	424.0	64
6	44.0	264.0	36
7	45.6	319.2	49
5	39.0	195.0	25
7	47.5	332.5	49
49	346.7	2167.7	311

Linear Regression Example

$$\bar{x} = \frac{49}{8} = 6.125$$

$$\bar{y} = \frac{346.9}{8} = 43.36$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}^2}{\sum x^2 - n\bar{x}^2}$$

$$= \frac{(2{,}167.7) - (8)(6.125)(43.36)}{(311) - (8)(6.125)^2}$$

$$= 4.06$$

$$a = \bar{y} - b\bar{x}$$

$$= 43.36 - (4.06)(6.125)$$

$$= 18.46$$

Computing Correlation

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{[n\sum x^2 - (\sum x)^2]\,[n\sum y^2 - (\sum y)^2]}}$$

Computing Correlation

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2]\,[n\sum y^2 - (\sum y)^2]}}$$

$$r = \frac{(8)(2{,}167.7) - (49)(346.9)}{\sqrt{[(8)(311) - (49)^2]\,[(8)(15{,}224.7) - (346.9)^2]}}$$

$$r = 0.947$$

Coefficient of determination

$$r^2 = (0.947)^2 = 0.897$$

Simple Linear Regression Assumptions

1. Variations around the line are random
2. Deviations around the average value (the line) should be normally distributed
3. Predictions are made only within the range of observed values

Multiple Linear Regression

Causation vs Correlation

Correlation

A implies B

A	B	C
1	1	1
1	0	0
0	0	1
0	1	1

B implies A

A	B	C
1	1	1
1	0	1
0	0	1
0	1	0

Causation

A causes B

A	B	C
1	1	1
1	0	0
0	0	1
0	1	0

Necessity and Sufficiency

Issues to consider:

- Always plot the line to verify that a linear relationship is appropriate
- The data may be time-dependent.
  - If they are
    - use analysis of time series
    - use time as an independent variable in a multiple regression analysis
- A small correlation may indicate that other variables are important

Monitoring the Forecast

- Tracking forecast errors and analyzing them can provide useful insight into whether forecasts are performing satisfactorily
- Sources of forecast errors:
  - The model may be inadequate due to
    a. omission of an important variable
    b. a change or shift in the variable the model cannot handle
    c. the appearance of a new variable
  - Irregular variations may have occurred C. L
    A C
  - Random variation
- Control charts are useful for identifying the presence of non-random error in forecasts
- Tracking signals can be used to detect forecast bias

Forecast Control

- Tracking signal
  - monitors the forecast to see if it is biased high or low

- 1 MAD ≈ 0.8σ Control limits of 2 to 5 MADs are used most frequently

$$\text{Tracking signal} = \frac{\sum (D_t - F_t)}{MAD} = \frac{E}{MAD}$$

$3 \times \frac{MAD}{8}$

$= 3.75\ MAD$

$\sum_t \frac{|D_t - f_t|}{t}$

3.75

Tracking Signal Values

PERIOD	DEMAND D_t	FORECAST, F_t	ERROR $D_t - F_t$	$\sum E = \sum(D_t - F_t)$	MAD
1	37	37.00	–	–	–
2	40	37.00	3.00	3.00	3.00
3	41	37.90	3.10	6.10	3.05
4	37	38.83	-1.83	4.27	2.64
5	45	38.28	6.72	10.99	3.66
6	50	40.29	9.69	20.68	4.87
7	43	43.20	-0.20	20.48	4.09
8	47	43.14	3.86	24.34	4.06
9	56	44.30	11.70	36.04	5.01
10	52	47.81	4.19	40.23	4.92
11	55	49.06	5.94	46.17	5.02
12	54	50.84	3.15	49.32	4.85

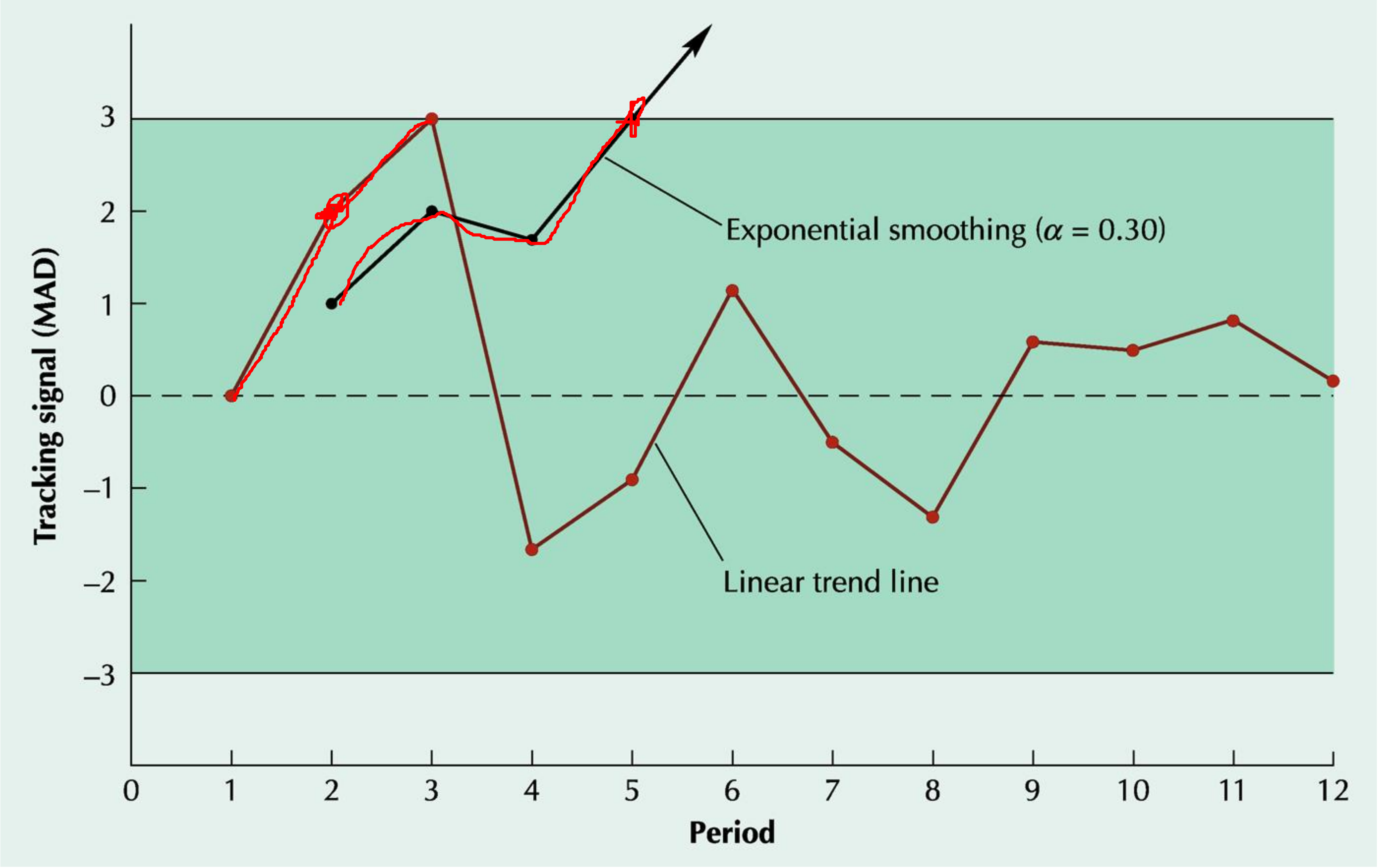
Tracking Signal Values

PERIOD	DEMAND D_t	FORECAST, F_t	ERROR $D_t - F_t$	$\sum E = \sum(D_t - F_t)$	MAD	**TRACKING SIGNAL**
1	37	37.00	–	–	–	**–**
2	40	37.00	3.00	3.00	3.00	**1.00**
3	41	37.90	3.10	6.10	3.05	**2.00**
4	37	38.83	-1.83	4.27	2.64	**1.62**
5	45	38.28	6.72	10.99	3.66	**3.00**
6	50	40.29	9.69	20.68	4.87	**4.25**
7	43	43.20	-0.20	20.48	4.09	**5.01**
8	47	43.14	3.86	24.34	4.06	**6.00**
9	56	44.30	11.70	36.04	5.01	**7.19**
10	52	47.81	4.19	40.23	4.92	**8.18**
11	55	49.06	5.94	46.17	5.02	**9.20**
12	54	50.84	3.15	49.32	4.85	**10.17**

$$TS_3 = \frac{6.10}{3.05} = 2.00$$

6.

Tracking Signal Plot



Statistical Control Charts

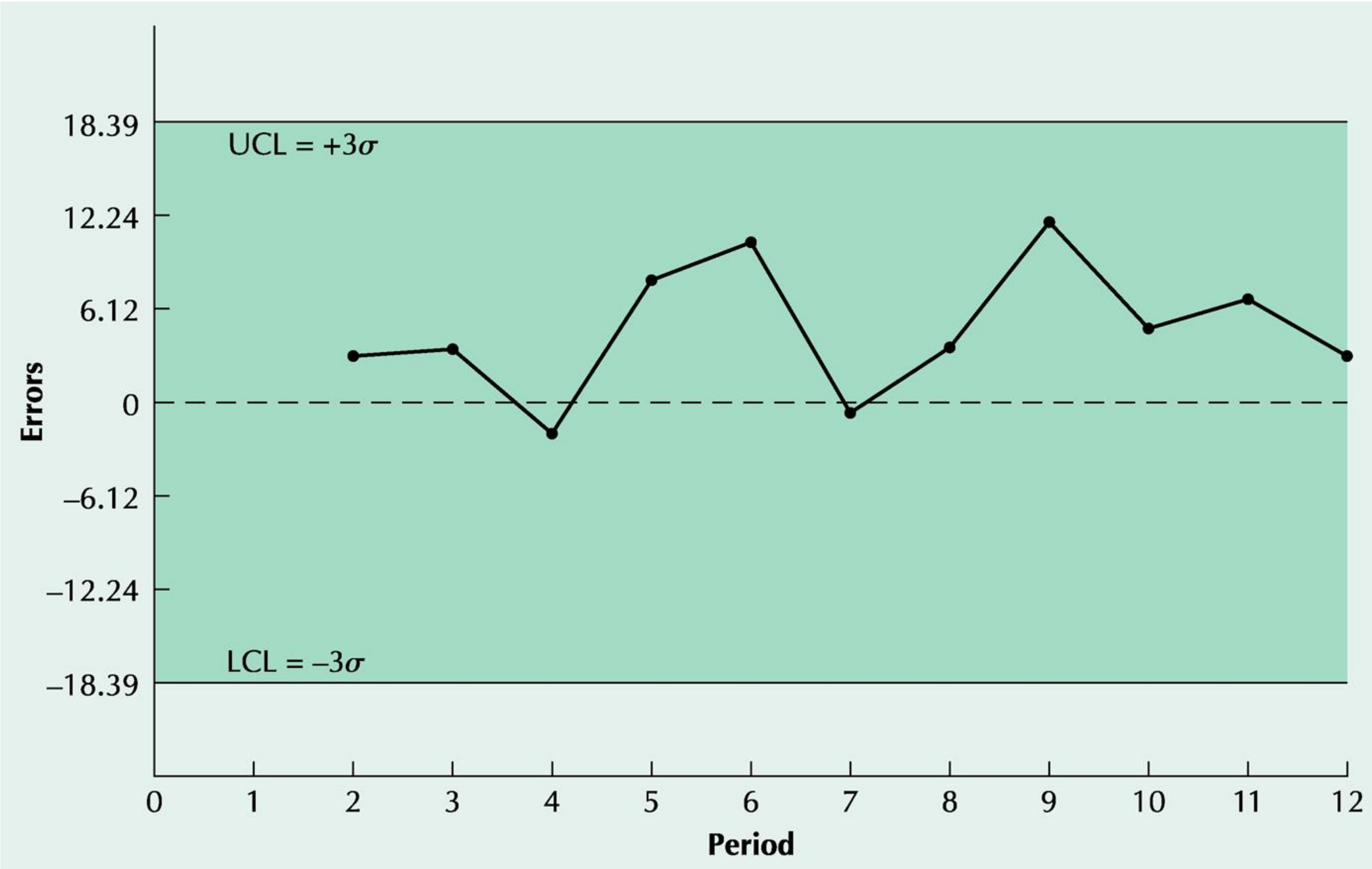
- Using σ we can calculate statistical control limits for the forecast error
- Control limits are typically set at ± 3σ

$$\sigma = \sqrt{\frac{\sum (D_t - F_t)^2}{n - 1}}$$

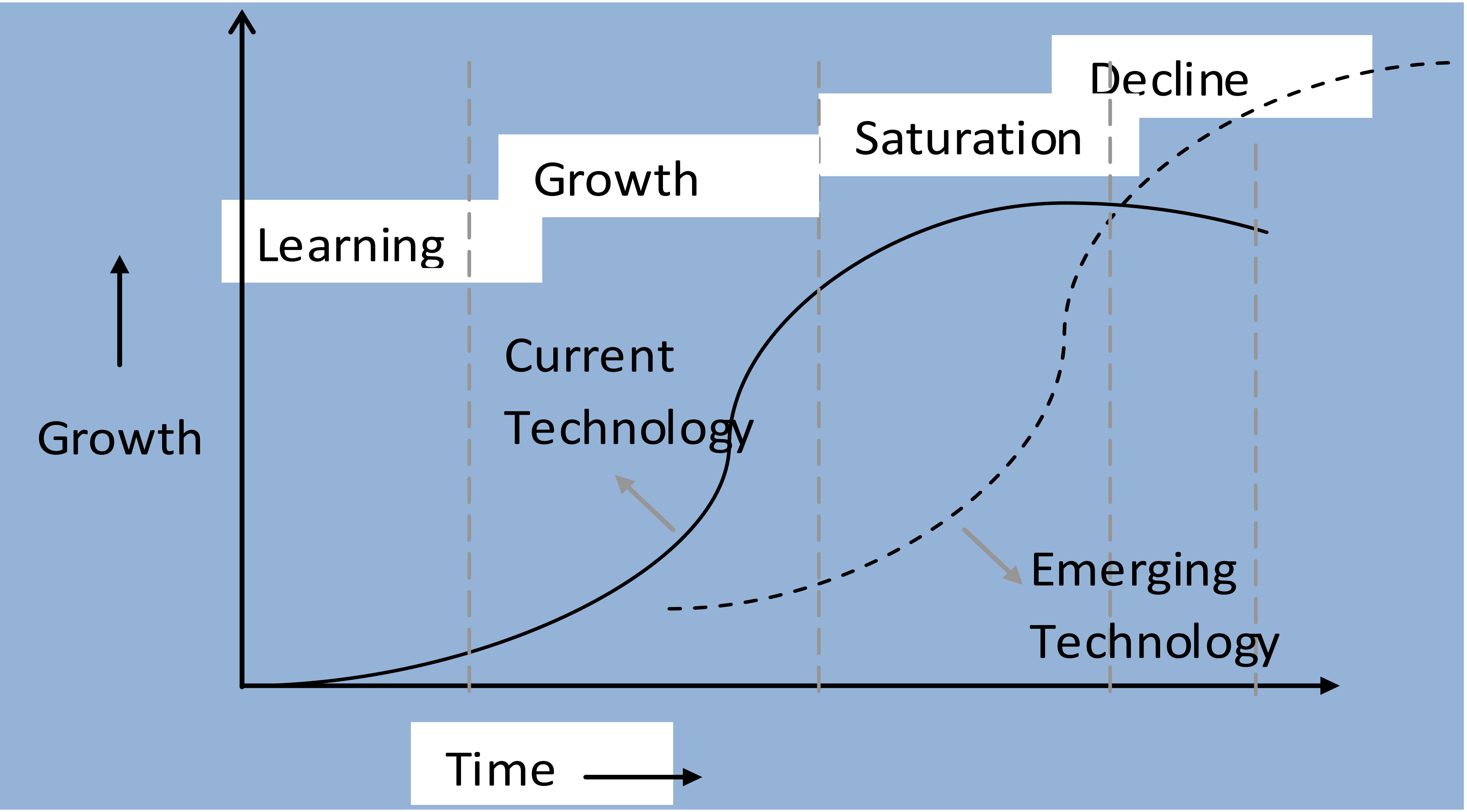
$$\sum_{i=1} (e_i - \bar{e})^2$$

- Mean squared error (MSE)
  - Average of squared forecast errors

Statistical Control Charts



S-curve of Technology Diffusion

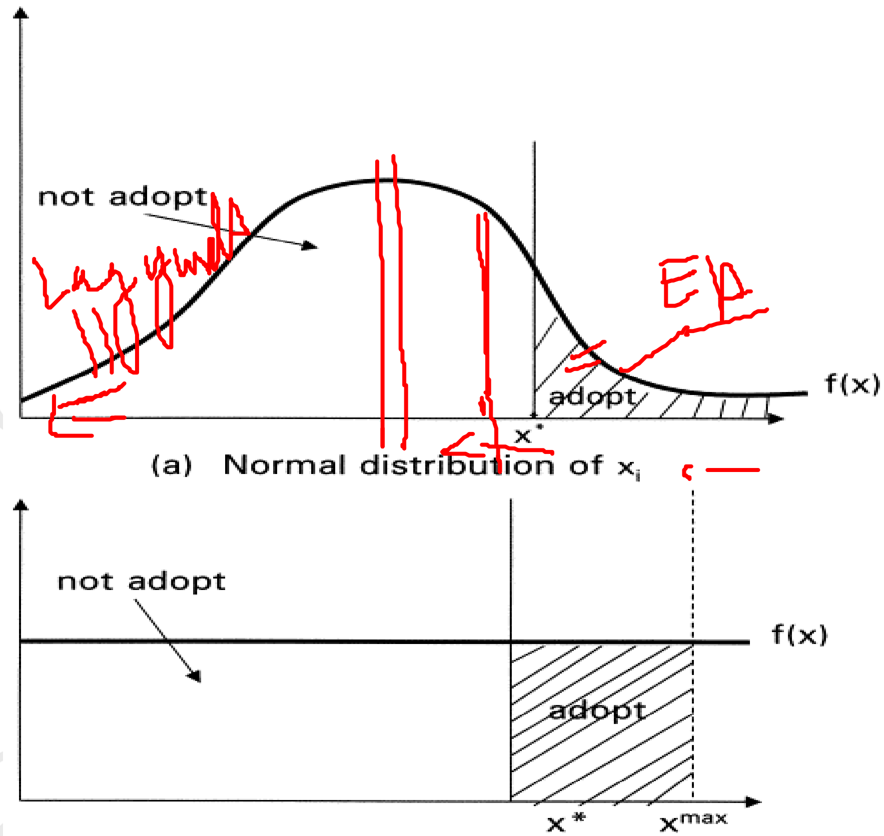


Technology Diffusion

- Technology diffusion is typically modelled as an S-shaped curve over time
- Assumption is that there is an upper limit to the growth of a technology
- Growth pattern follows a logistic path
- Each technology undergoes four different phases: learning, growth, saturation, and decline

Two distributions of f(x) with thresholds separating adopters from non-adopters

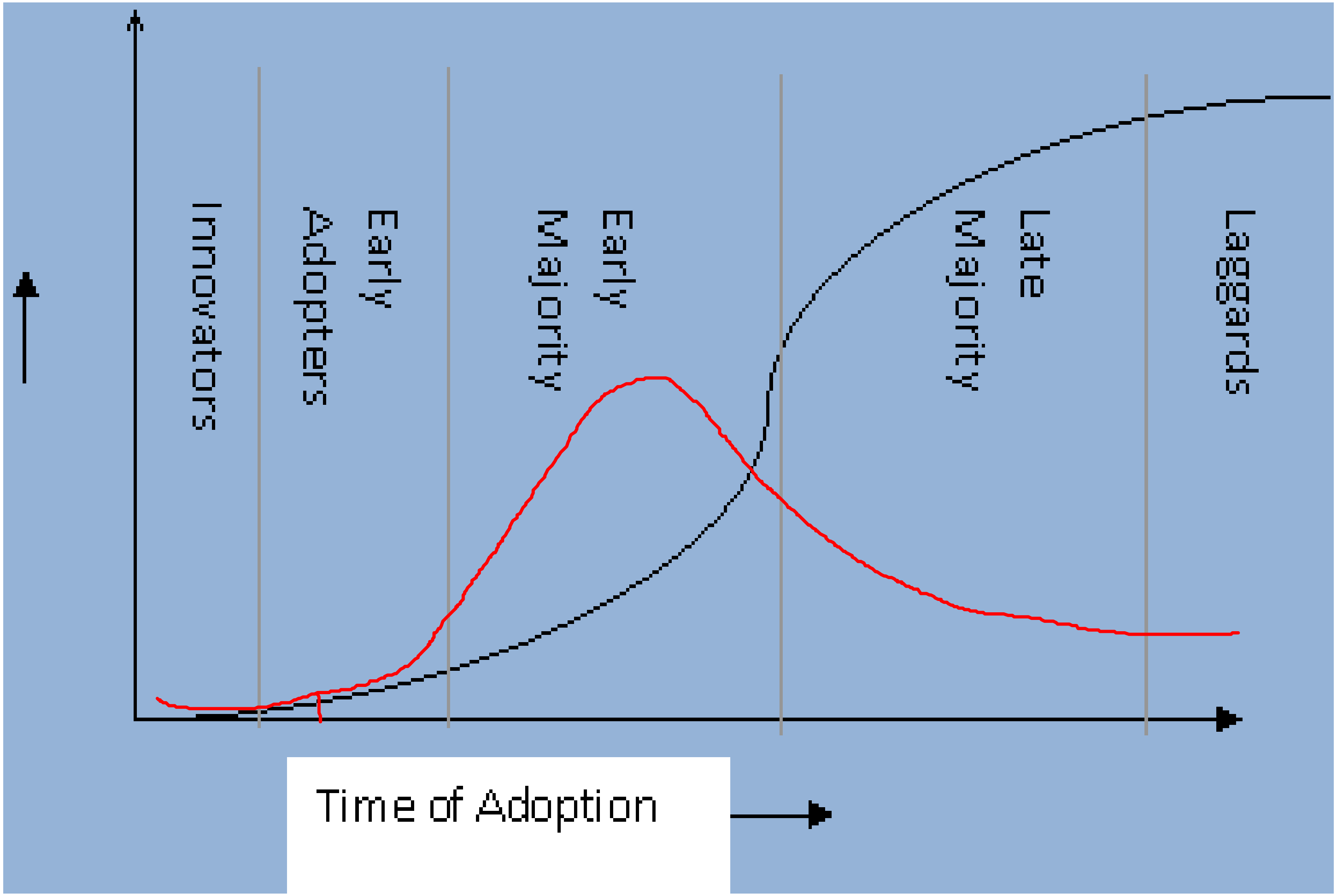
x = Characteristic determining the profitability of adopting a technology.



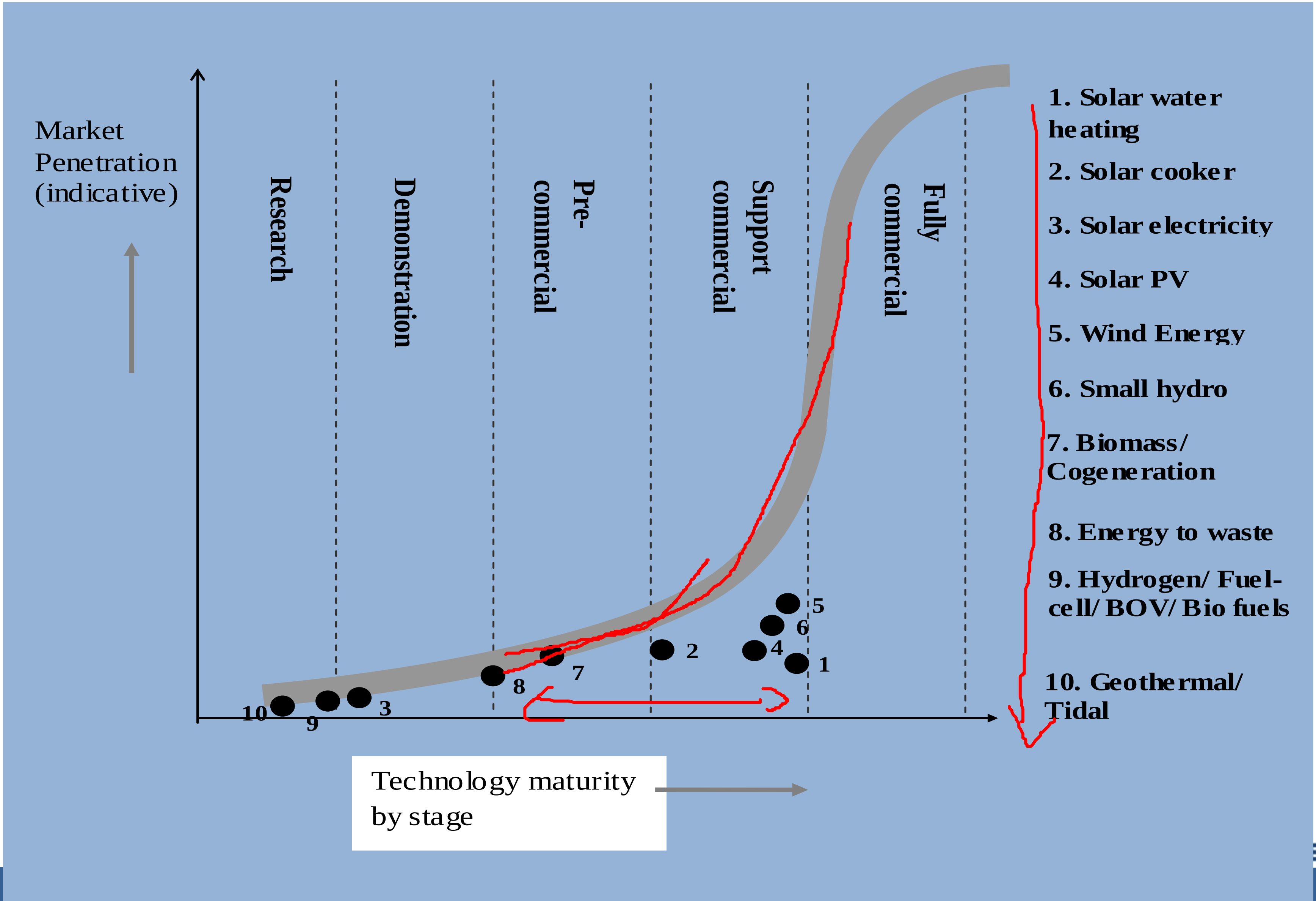
(b) rectangular distribution of x_i

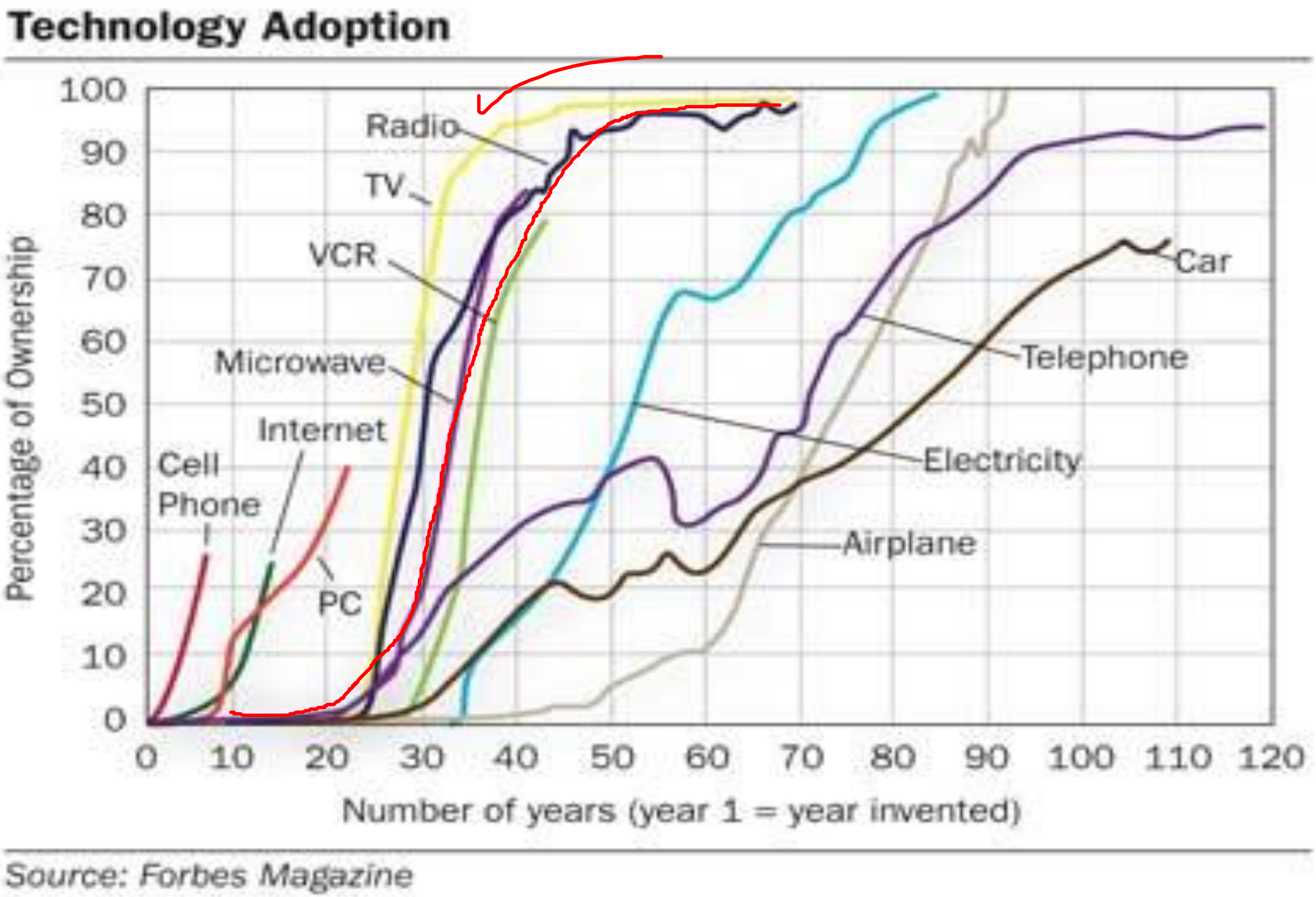
[Source; Geroky, 2000](https://doi.org/10.1016/S0048-7333(99)00092-X)
https://doi.org/10.1016/S0048-7333(99)00092-X

Rate of adoption of the innovation



Position of individual SET relative to market penetration




Technology Adoption
Percentage of Ownership
100
90
80
70
60
50
40
30
20
10
0
Radio
TV
VCR
Microwave
Internet
Cell Phone
PC
Telephone
Electricity
Airplane
Car
0
10
20
30
40
50
60
70
80
90
100
110
120
Number of years (year 1 = year invented)
Source: Forbes Magazine

Choosing a Forecasting Technique

- Factors to consider
  - Cost
  - Accuracy
  - Availability of historical data
  - Availability of forecasting software
  - Time needed to gather and analyze data and prepare a forecast
  - Forecast horizon

Operations Strategy

- The better forecasts are, the more able organizations will be to take advantage of future opportunities and reduce potential risks
  - A worthwhile strategy is to work to improve short-term forecasts
    - Accurate up-to-date information can have a significant effect on forecast accuracy:
      - Prices
      - Demand
      - Other important variables
  - Reduce the time horizon forecasts have to cover
  - Sharing forecasts or demand data through the supply chain can improve forecast quality