Solenoidal and Irrotational Vector

29/11/2022

(a) Solenoidal vector fields:-

Solenoidal vector field $\vec{A_s}$:-

$$\boxed{\vec{\nabla}\cdot\vec{A_s} = 0}$$

(b) Irrotational vector fields:-

Irrotational vector field $\vec{A_I}$:-

$$\boxed{\vec{\nabla}\times\vec{A_I} = 0}$$

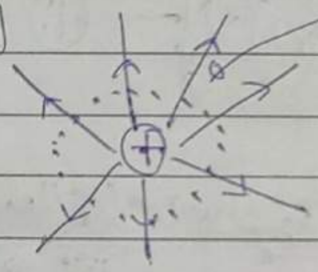
(c) Helmholtz theorem :-

⇒ If $\vec{\nabla}\cdot\vec{A} = \rho_v \neq 0$ and $\vec{\nabla}\times\vec{A} = \vec{\rho_s} \neq 0$

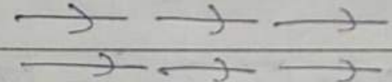
$\rho_v,\ |\vec{\rho_s}| \xrightarrow{|\vec{r}|\to\infty} 0$, then

$$\boxed{\vec{A} = \vec{A_I} + \vec{A_s}}$$

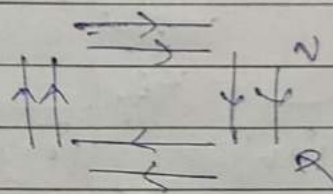
where $\vec{r}$ = position vector.

(i) 

through this surface, nonzero field lines are crossing so this is an example of non-solenoidal surface or non-solenoidal vector fields.

(ii) 

} net field lines crossing is zero; ⇒ solenoidal vector field.

(iii) 

⇒ rotational vector field.

$\vec{\nabla}\times\vec{v} \neq 0$

Electrostatics

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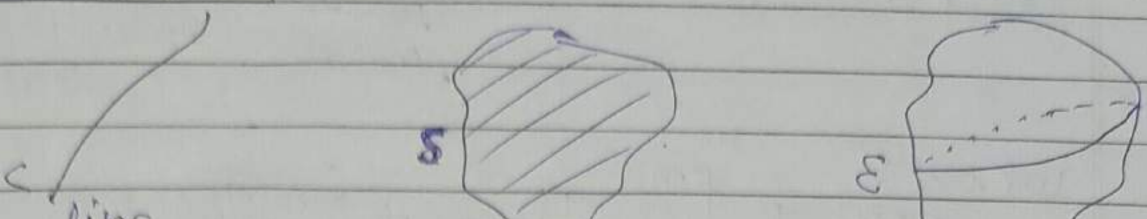
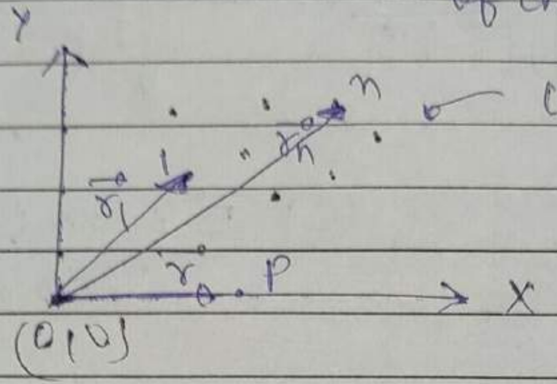
· Notation:-



line distribution of charges

surface distribution of charges.

volume distribution of charges.



collection of discrete charges.

· For line distribution, for an element we took position vector as $\lambda(\vec{r}')$; for surface; $\sigma(\vec{r}'')$; and for volume, $\rho(\vec{r}''')$.

Coulomb's law:-

(a) $$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\sum_{i=1}^{n} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3} + \int_C \frac{\lambda(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dl' + \iint_S \frac{\sigma(\vec{r}'')(\vec{r}-\vec{r}'')}{|\vec{r}-\vec{r}''|^3}\,d^2x'' + \iiint_V \frac{\rho(\vec{r}''')(\vec{r}-\vec{r}''')}{|\vec{r}-\vec{r}'''|^3}\,d^3x'''\right]$$

where: $d^2x'' = dx''\cdot dy''$
$d^3x''' = dx'''\,dy'''\,dz'''$.

(b) $\vec{\nabla}_r \times \vec{E}(\vec{r})$:- curl of electric field.

$\boxed{\text{Note}}$:- There always exists $\phi(\vec{r})$ = Electrostatic potential

$\Rightarrow \boxed{\vec{\nabla}_r f(r) = f'(r) \cdot \hat{a}_r}$

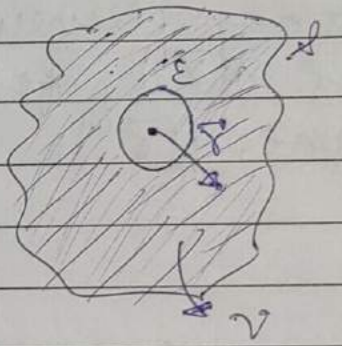
such that $\boxed{\vec{E}(\vec{r}) = -\vec{\nabla}(\underbrace{\phi(\vec{r})}_{\text{scalar}})}$ = negative of a gradient of a scalar.

where:

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\sum_{i=1}^{n} \frac{q_i}{|\vec{r}-\vec{r}_i|} + \int_C \frac{\lambda(\vec{r}')}{|\vec{r}-\vec{r}'|} dl' + \iint_S \frac{\sigma(\vec{r}'')}{|\vec{r}-\vec{r}''|} d^2x'' + \iiint_V \frac{\rho(\vec{r}''')}{|\vec{r}-\vec{r}'''|} d^3x'''\right]$$

(b) $\boxed{\vec{\nabla}^2\left(\frac{1}{r}\right) = -4\pi\delta^{(3)}(\vec{r})}$; (C) $\vec{E}$ is an irrotational vector.

Example:-



We have to calculate the flux of electric field through the shaded surface. means $\oiint \vec{E}\cdot\vec{ds}$.

$$\Rightarrow \oiint_{\epsilon+s} \vec{E}\cdot\vec{ds} = \oiint_{\epsilon+s}\left(\frac{q}{4\pi\epsilon_0}\frac{\vec{r}}{r^3}\right)\vec{ds} = \frac{q}{4\pi\epsilon_0}\iiint_V \underbrace{\left(\vec{\nabla}\cdot\frac{\vec{r}}{r^3}\right)}_{\text{when } r\neq 0 \Rightarrow \text{ it will be zero.}} d^3x$$

$$\Rightarrow \text{So: } \oiint_{\epsilon+s} \ldots = 0 \Rightarrow \oiint_{s} \vec{E}\cdot\vec{ds} = -\oiint_{\epsilon} \vec{E}\cdot\vec{ds}$$

When: $\lim_{\epsilon\to 0} \left(\oiint_{s} \vec{E}\cdot\vec{ds} = -\oiint_{\epsilon} \vec{E}\cdot\vec{ds}\right)$

$$\Rightarrow \boxed{\oiint_{s} \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}} \Rightarrow \text{(Gauss law)}$$

Dirac-delta function:- (Example of distribution function).

$$\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$$

$\Rightarrow \vec{\nabla}\cdot(\phi \vec{A}) = \vec{\nabla}\phi \cdot \vec{A} + \phi \vec{\nabla}\cdot\vec{A}$

let $\phi = \frac{1}{r^3}$ and $\vec{A} = \vec{r} \Rightarrow \vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = \vec{\nabla}\left(\frac{1}{r^3}\right)\cdot\vec{r} + \frac{\vec{\nabla}\cdot\vec{r}}{r^3}$

$\Rightarrow$ which will come out to be zero.

Note

$\Rightarrow \boxed{\nabla^2\left(\frac{1}{r}\right) = -4\pi\,\delta^{(3)}(\vec{r}) = -4\pi\,\delta(x)\,\delta(y)\,\delta(z).}$

where δ = dirac delta function