Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)

Sheet-2

1. Write the standard form of the LPP

(i) Max $Z = 2x_1 + x_2 + x_3$

s. t. $x_1 - x_2 + 2x_3 \geq 2$, $|2x_1 + x_2 - x_3| \leq 4$, $3x_1 - 2x_2 - 7x_3 \leq 3$

$x_1, x_3 \geq 0$, $x_2 \leq 0$

(ii) Max $Z = x_1 + 2x_2 - x_3$

s. t. $x_1 + x_2 - x_3 \leq 5$, $-x_1 + 2x_2 + 3x_3 \geq -4$, $2x_1 + 3x_2 - 4x_3 \geq 3$,

$x_1 + x_2 + x_3 = 2$, $x_1 \geq 0$, $x_2 \geq p, x_3$ is unrestricted in sign.

(iii) Min $Z = 2x_1 - x_2 + 2x_3$

s. t. $-x_1 + x_2 + x_3 = 4$, $-x_1 + x_2 - x_3 \leq 6$, $x_1 \leq 0$, $x_2 \geq 0, x_3$ is unrestricted in sign.

2. Prove that intersection of any collection (finite or infinite) of convex sets in R^n is a convex set.

3. Prove that the feasible region of a linear programming problem is a convex set.

4. Prove that the set of all optimal solutions of a linear programming problem is a convex set.

5. Examine whether the following sets are convex or not:

(a) $S = \{(x_1, x_2) \mid 2x_1 + 5x_2 \leq 20, x_1 + 2x_2 \geq 6\} \subset R^2$,

(b) $S = \{(x_1, x_2, x_2) \mid x_1 - 2x_2 + 3x_3 \leq 12\} \subset R^3$,

(c) $S = \{(x_1, x_2) \mid x_2^2 \leq 2x_1\} \subset R^2$,

(d) $S = \{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3^2 \leq 9\} \subset R^3$,

(e) $S = \{(x_1, x_2) \mid x_1 x_2 \geq 4, x_1, x_2 \geq 0\} \subset R^2$,

(f) $S = \{(x_1, x_2) \mid x_1 x_2 \leq 4, x_1, x_2 \geq 0\} \subset R^2$.

(h) $S = \{(x_1, x_2) : 0 < x_1^2 + x_2^2 \leq 4\} \subset R^2$.

6. Find all the extreme points of the set $S = \{(x_1, x_2) \mid x_1 + 2x_2 \geq -2, -x_1 + x_2 \leq 4, x_1 \leq 4\}$ and represent the point (2, 3) as the convex combination of the extreme points of S.

7. Show that a linear program with bounded feasible region is bounded and give a counter example to show that the converse need not be true.

8. Prove that the minimum of a LPP occurs on some extreme point (vertex).

9. Prove that half space $\{\mathbf{X} \in \mathbf{R}^n : \mathbf{a}^{\mathrm{T}}\mathbf{X} \geq \alpha\}$ is a convex set.