The Remainder and Factor Theorems

Remainder Theorem

$$p(x) = (x - a) \cdot q(x) + r$$

When you divide a polynomial f(x) by a divisor d(x), you get a quotient q(x) and a remainder polynomial r(x).

This is written as $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$. The degree of the

remainder must be less than the degree of the divisor.

Example 1

Divide
$$2x^4 + 3x^3 + 5x - 1$$
 by $x^2 - 2x + 2$

$$\frac{2x^{2} + 7x + 10}{x^{2} - 2x + 2}$$

$$\frac{2x^{2} + 7x + 10}{2x^{4} + 3x^{3} + 0x^{2} + 5x - 1}$$

$$\frac{-(2x^{4} - 4x^{3} + 4x^{2})}{7x^{3} - 4x^{2} + 5x - 1}$$

$$\frac{-(7x^{3} - 14x^{2} + 14x)}{10x^{2} - 9x - 1}$$

$$\frac{10x^{2} - 20x + 20}{11x + 21}$$

$$\frac{2x^4 + 3x^3 + 5x - 1}{x^2 - 2x + 2} = 2x^2 + 7x + 10 + \frac{11x - 21}{x^2 - 2x + 2}$$

Remainder Theorem

If a polynomial f(x) is divided by x - k, then the remainder is r = f(k).

Synthetic Division: A process for division of a polynomial by a binomial.

Example 2: Divide

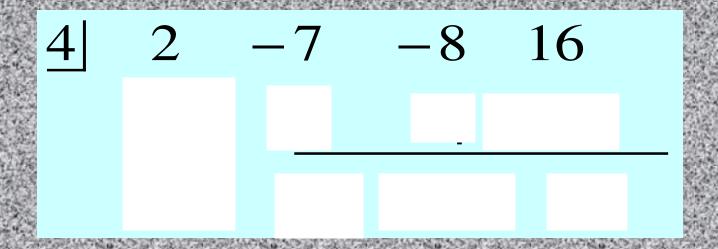
$$2x^3 - 7x^2 - 8x + 16$$
 by $x - 4$

Step 1: Write the terms of the polynomial in descending order.

$$2x^3 - 7x^2 - 8x + 16$$

Then write just the coefficients as shown. $2 - 7 - 8 \cdot 16$

Step 2: Write the constant k, of the divisor x – k to the left



Bring the first coefficient down.

Multiply the first coefficient by k and write the product under the second coefficient. Add the second column.

Write the sum as shown.

To write the result

$$\frac{2x^3 - 7x^2 - 8x + 16}{x - 4} = 2x^2 + x - 4$$

Example 3:

Divide:

$$x^3 + 2x^2 - 6x - 9$$
 by $x - 2$

$$\frac{x^3 + 2x^2 - 6x - 9}{x - 2} = x^2 + 4x + 2 - \frac{5}{x - 2}$$

Proof of Remainder Theorem

Let us assume that q(x) and 'r' are the quotient and the remainder respectively when a polynomial p(x) is divided by a linear polynomial (x - a). By division algorithm, Dividend = (Divisor × Quotient) + Remainder. Using this, $p(x) = (x - a) \cdot q(x) + r$. Substitute x = a $p(a) = (a - a) \cdot q(a) + r$ $p(a) = (0) \cdot q(a) + r$ p(a) = ri.e. the remainder = p(a).

Factor Theorem

$$p(x) = (x - a) \cdot q(x)$$

Factor Theorem

A polynomial f(x) has a factor x - k if and only if f(k) = 0

Example 4

One zero of
$$f(x) = x^3 - 2x^2 - 9x + 18$$
 is $x = 2$.

Find the other zeros of the function.

$$x = 2$$
 Subtract 2 from both sides
 $x - 2 = 0$ So, $x - 2$ is a factor

Because f(2) 1 -2 -9 18 -2 is a factor. Use syntheti 2 0-18 her factors.

Because f(2) = 0, you know that x - 2 is a factor. Use synthetic division to find the other factors.

The result gives the coefficients of the quotient.

$$f(x) = (x-2)(x^2-9)$$

$$=$$

b. $x^3 + 2x^2 - 6x - 9$ by x + 3

To find the value of k, rewrite the divisor in the form x-k. Because x+3=x-(-3), k=-3

$$\frac{x^3 + 2x^2 - 6x - 9}{x + 3} = x^2 - x - 3$$

Example 4

$$f(x) = 2x^3 + 11x^2 + 18x + 9$$
 given that $f(-3) = 0$

Because
$$f(-3) = 0$$
, you know that $x - (-3) = 0$
or $x + 3 = 0$ is a factor of $f(x)$.

dse synthetic division to find the other factors.

The result gives the coefficients guotemic

$$2x^{3} + 11x^{2} + 18x + 9 = (x+3)(2x^{2} + 5x + 3)$$
$$= (x+3)(2x+3)(x+1)$$

gcd(a,b) greatest common divisor of a and b

Arayabhata-Euclid's algorithm: How to find gcd(a,b), the greatest common divisor of a and b

Based on a single observation: if a = b q + r, then any divisor of a and b is also a divisor of r, and any divisor of b and r is also a divisor of a, so gcd(a,b) = gcd(b,r)

Euclid algorithm:

Example: gcd(55,35)

$$55 = 35*1 + 20$$
 so $gcd(55,35) = gcd(35,20)$

$$35 = 20*1 + 15$$
 so $gcd(35,20) = gcd(20,15)$

$$20 = 15*1 + 5$$
 so $gcd(20,15) = gcd(15,5)$

$$15 = 5*3 + 0$$
 so $gcd(15,5) = gcd(5,0)$