2 Introduction

The EPR paradox was proposed by Einstein, Podolsky, and Rosen in 1935 as a thought experiment to show that quantum mechanics was incomplete [1].

Bell's theorem, formulated by physicist John Bell in 1964, is a mathematical theorem that provides a way to test the compatibility of quantum mechanics with local realism. It provides a criterion to distinguish between the predictions of quantum mechanics and the predictions of local realistic(classical) theories.

The CHSH inequality was derived by John Clauser, Michael Horne, Abner Shimony, and Richard Holt in 1969, based on an earlier work by John Stewart Bell. We will also show this result using IBM Quantum Lab.

We will also discuss some of the main interpretations of quantum mechanics that attempt to resolve the paradoxes.

3 EPR Paradox

The Einstein-Podolsky-Rosen (EPR) paradox is a thought experiment proposed by Albert Einstein, Boris Podolsky, and Nathan Rosen in 1935. It was designed to challenge the completeness and implications of quantum mechanics, particularly the concept of entanglement and the uncertainty principle.

The EPR paper [1] first assumed Local Realism and, using mathematical deduction , argued that the quantum-mechanical description of physical reality given by wave function is incorrect and not complete.

But we will just try to understand it's anology :

The paradox involves a pair of particles that are entangled, meaning when we measure a property of one of the particle, we simultaneously know the measurment of other's same property. For having an intitutive understanding of entaglement let's considers the decay of a particle called the pi meson. When this particle decays, it produces an electron and a positron that have opposite spin and are moving away from each other. Therefore, if the electron spin is measured to be up, then the measured spin of the positron could only be down, and vice versa. This is true even if the particles are billions of miles apart. This is what it means about entanglement of particles.

The thought experiment involves a pair of particles that are prepared in a special state called entanglement. According to Einstein, Podolsky, and Rosen, if the position of one particle is measured, it allows for the prediction of the position of the other particle. Similarly, if the momentum of one particle is measured, it enables the prediction of the momentum of the other particle. The crucial point they raised was that no immediate influence or communication between the particles could occur, as this would violate the theory of relativity by transmitting information faster than the speed of light.

This result was later known as 'EPR criterion of reality' , which said "If, without in any way disturbing a system, we can predict with certainty (means with probability of unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity".

From this they infered that the second particle has both position and momentum well defined even before measuring, but in quantum mechanics these tow properties's operatores are non-commute which means both properties can't have simultanious reality.

Using this argument EPR paper proposed that the wave function of particles in quantum mechanics doesn't provide complete discription of physical reality.

Although there was further debate between Bohr and Einstein on this topic which came to be known as Bohr-Einstein debates.

4 Bell's Theorem

Bell's theorem is a family of results showing that any theory that assumes local causality and hidden variables cannot account for the correlations between entangled particles predicted by quantum mechanics. Local causality means that distant events can only influence each other through signals that travel at or below the speed of light.

Hidden variables are hypothetical properties of quantum particles that are not captured by quantum theory but determine the outcomes of measurements. Bell's theorem proves that any theory that satisfies these assumptions must obey certain inequalities, called Bell inequalities, that limit the correlations between measurements performed on entangled particles. However, quantum mechanics predicts, and experiments confirm that these inequalities can be violated, implying that either local causality or hidden variables must be given up. Bell's theorem reveals a fundamental tension between quantum mechanics and our classical intuition about physical reality.

In history, there were many derived Inequalities from Bell's Original; some of the notable ones are Clauser-Horne-Shimony-Holt (CHSH) inequality, CH (Clauser-Horne) inequality, Bell-CHSH-Mermin inequality and GHZ (Greenberger-Horne-Zeilinger) inequalities. But we will look at the CHSH Inequality.

4.1 The CHSH Inequality

The CHSH inequality is a mathematical expression that can test the validity of local hidden-variable theories, which are alternative explanations of quantum phenomena that do not rely on quantum entanglement. The inequality was derived by John Clauser, Michael Horne, Abner Shimony, and Richard Holt in 1969, based on an earlier work by John Stewart Bell. The inequality limits the correlation between two distant measurements of entangled particles if the outcomes are determined by some local hidden variables independent of the measurement settings. Quantum mechanics predicts that certain entangled states can violate the CHSH inequality, which implies that either local hidden variables do not exist or that non-local effects influence them. Many experiments have confirmed the violation of the CHSH inequality, thus supporting the quantum mechanical view of entanglement and challenging the notion of local realism.

Now for this experiment, we will create an pair of entangled particles and measure them in different basis and then calculate the CHSH Inequality. We will take two qubit and denote them as A and B. We will label bases for qubit A as X and x, for qublit B as Y and y. We will denote the measurement outcomes as +1 and -1. We will denote the expectation value of the product of the measurement outcomes as $\langle AB \rangle$.

Now we will write the CHSH Inequality as : $S_1 = X(Y - y) + x(Y + y)$.

Each of observation is either +1 or -1, thus any one of terms (Y+y) or (Y-y) is always 0 and other is ± 2. Thus the average value of S is always less than or equal to 2. Thus we get the first inequality as :

$$|\langle S_1 \rangle| \leq 2. \tag{1}$$

Now Expanding the above equation in terms of A, B, a and b we get :

$$|\langle S_1 \rangle| = |\langle XY \rangle - \langle Xy \rangle + \langle xY \rangle + \langle xy \rangle| \leq 2 \tag{2}$$

This will give us another Inequality as : $S_2 = X(Y + y) + x(Y - y)$.

Applying similar steps as above we get :

$$|\langle S_2 \rangle| = |\langle XY \rangle + \langle Xy \rangle - \langle xY \rangle + \langle xy \rangle| \leq 2 \tag{3}$$

If quantum mechanics can be described by local hidden variable theories, the previous inequalities must hold true. However we are going to demonstrate using IBM Quantum Lab, these inequalities can be violated in a quantum computer. Therefore, quantum mechanics is not compatible with local hidden variable theories.

4.2 The IBM Quantum Lab for CHSH Inequality

You can find the pdf format of the Code attached with this report.

Link to the Original Jupiter N/B : Quantum Lab on CHSH Inequality

The final result of the experiment is shown below :

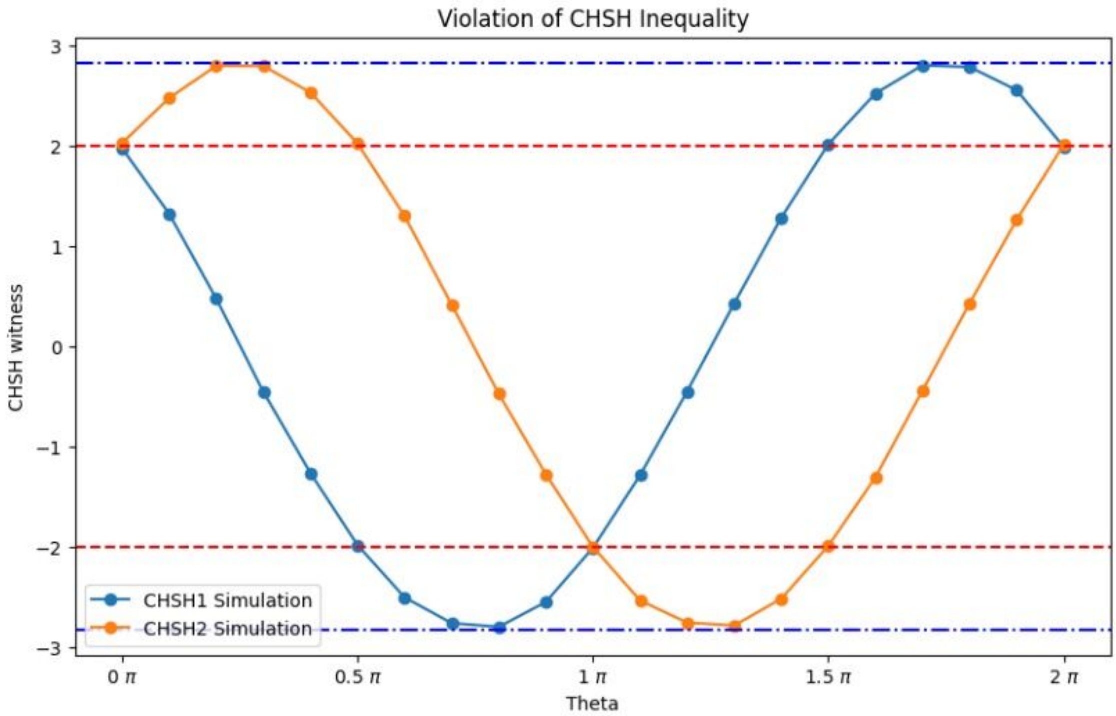

Figure 1: CHSH Inequality

As we can see that result of quantum computer has a max of $2\sqrt[2]{2}$ which was also showed by Bell's Original Paper.[2]

5 Interpretations of Quantum Mechanics

5.1 The Copenhagen Interpretation

The Copenhagen interpretation of quantum mechanics, formulated by a group of physicists led by Niels Bohr and Werner Heisenberg in the 1920s and 1930s, is one of the most influential interpretations of quantum theory. At its core, the Copenhagen interpretation recognizes the wave-particle duality of particles, acknowledging that they can exhibit both wave-like and particle-like behavior depending on the experimental context. It embraces the idea of quantum superposition, where a quantum system can exist in multiple states simultaneously. However, when a measurement is made, the system collapses into a single state, and the outcome of the measurement is probabilistic.

A key concept in the Copenhagen interpretation is the role of the observer. According to this interpretation, the act of observation or measurement is fundamental in determining the properties of a quantum system. Before measurement, a system is described by a wave function that represents the probabilities of different outcomes. But it is only through the process of measurement that a particular outcome is realized, and the wave function collapses into a definite state. This observer-dependent nature of quantum reality distinguishes the Copenhagen interpretation from classical physics, where properties of objects are assumed to exist independently of measurement.

Complementarity is another central idea in the Copenhagen interpretation. It recognizes that certain physical properties, such as position and momentum, or wave and particle nature, are complementary and cannot be simultaneously observed or measured with arbitrary precision. This principle suggests that our ability to observe or measure one property necessarily introduces uncertainty or disturbance in the measurement of the other. Thus, there are inherent limitations in our knowledge of the complete set of properties of a quantum system.

While the Copenhagen interpretation provides a mathematical framework that successfully predicts the statistical behavior of quantum systems, it is often criticized for its lack of a clear physical interpretation. It does not provide a complete description of the underlying physical reality of quantum phenomena and leaves unanswered questions about the nature of wave function collapse and the origin of quantum probabilities.

Nonetheless, the Copenhagen interpretation remains a foundational framework for understanding and applying quantum mechanics. Its emphasis on the probabilistic nature of quantum phenomena, the role of measurement, and the notion of complementarity has shaped the development of quantum theory and influenced subsequent interpretations and research in the field.

5.2 The Many-Worlds Interpretation

The Many-worlds interpretation (MWI) is a fascinating and thought-provoking interpretation of quantum mechanics proposed by physicist Hugh Everett in the 1950s. In the Many-worlds interpretation, it is posited that every quantum event, such as a measurement or observation, causes the universe to branch into multiple parallel universes, each representing a different outcome. These parallel universes, also known as "worlds" or "branches," coexist alongside our own but remain unobservable and inaccessible due to their lack of interaction.

According to the Many-worlds interpretation, when a quantum measurement is made, the observer and the observed system become entangled, resulting in the observer perceiving a specific outcome. However, all other possible measurement outcomes exist in other parallel universes, each corresponding to a different branch of the multiverse. This interpretation suggests that all possible outcomes of quantum events occur in different parallel universes, with each universe following its unique trajectory.

One of the critical implications of the Many-worlds interpretation is the resolution of the measurement problem in quantum mechanics. In traditional arrangements, the collapse of the wave function upon measurement raises questions about when and why it occurs. In the Many-worlds interpretation, however, there is no wave function collapse. Instead, the wave function evolves deterministically, with every possible measurement outcome realised in a separate universe. This interpretation provides a consistent and coherent explanation of quantum behaviour without needing an ad hoc collapse postulate.

While the Many-worlds interpretation offers an elegant solution to the measurement problem and provides a comprehensive account of quantum phenomena, it poses challenges and philosophical questions. Critics argue that an infinite number of parallel universes raises issues of empirical testability and Occam's razor. Furthermore, the interpretation leaves open the question of how the observer's consciousness is associated with a particular branch of the multiverse.

In conclusion, the Many-worlds interpretation presents a compelling and radical perspective on the nature of reality, suggesting that our universe is one among infinite parallel universes. This interpretation offers an alternative framework for understanding the behaviour of quantum systems and opens up new avenues for exploring the fundamental mysteries of the quantum world.

References

[1] A. Einstein, B. Podolsky and N. Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* (Physical Review Vol. 47, 777-780 (15th May 1935)).

[2] J. S. Bell, *ON THE EINSTEIN PODOLSKY ROSEN PARADO* (Physics Vol. 1, No. 3, pp. 195—200, 1964 Physics Publishing Co. Printed in the United State).

[3] Reference Taken form Qiskit Learning Notebooks.

[4] Resources form Brilliant.org : Bell's Theorem and Quantum Entanglement.

[5] Wikipedia Resources : EPR Paradox, Bohr–Einstein debates, CHSH Inequlity, Copenhagen Interpretation and Many-worlds Interpretation.

[6] Youtube Resources : Spooky Action at a Distance and A lecture of MIT OCW