



Lecture 13

Semantics Analysis

Awanish Pandey

Department of Computer Science and Engineering
Indian Institute of Technology
Roorkee

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Take aways from the last class

- LR(1) parse table

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- LALR Parse Table

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- Parser Generator

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 - ▶ An identifier may be usable in one part of the program but not another

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- Methods in a class are not multiply defined

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- It may store information in symbol table

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- The synthesized attribute of Node N can be defined using inherited attributes of Node N
- Inherited attribute of Node N can not be defined using attribute of child of Node N
- Terminal can have only synthesized attributes (calculated from lexical phase). No SDD rules for computing attributes of terminal

Example

- Consider a grammar for signed binary numbers

$Number \rightarrow sign \ list$

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Symbol	Attribute
number	value
sign	negative
list	position, value
bit	position, value

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$\text{list} \rightarrow \text{bit}$	$\text{bit.position} \leftarrow \text{list.position}$ $\text{list.value} \leftarrow \text{bit.value}$
$\text{list}_0 \rightarrow \text{list}_1 \text{ bit}$	$\text{list}_1.\text{position} \leftarrow \text{list}_0.\text{position} + 1$ $\text{bit.position} \leftarrow \text{list}_0.\text{position}$ $\text{list}_0.\text{value} \leftarrow \text{list}_1.\text{value} + \text{bit.value}$

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$list_0 \rightarrow list_1 bit$	$list_1.position \leftarrow list_0.position + 1$ $bit.position \leftarrow list_0.position$ $list_0.value \leftarrow list_1.value + bit.value$
bit $\rightarrow 0$	bit.value $\leftarrow 0$

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$\text{bit} \rightarrow 0$	$\text{bit.value} \leftarrow 0$
$\text{bit} \rightarrow 1$	$\text{bit.value} \leftarrow 2^{\text{bit.position}}$

Parse tree and the dependence graph

