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Indian Institute of Technology Roorkee

CSN-353 Theory of Computation

End Semester Exam

Total Marks: 50

Time: 3 Hours

True/False Questions (10 Marks)

1. $L = \{\alpha\beta\alpha\gamma \mid \alpha, \beta, \gamma \in \Sigma^*, \alpha = \epsilon, |\beta| = |\gamma|\}$ is a Context-Free Language.
2. Let L be a context-free language (CFL), $x \in L$, and a proper prefix of x is also in L. L cannot be accepted by a deterministic pushdown automaton (DPDA) in empty stack mode.
3. If L is a context-free language (CFL) and $x \in L$ with $|x| \geq p$, where p is the pumping constant, then the number of strings in L is infinite.
4. If L_1 and L_2 are recognized by Turing machines (TMs) M_1 and M_2, then there exists a TM that recognizes L_1L_2.
5. Given a grammar G of length n, we can find an equivalent Chomsky-Normal-Form grammar for G in time $O(n)$ and the resulting grammar has length $O(n)$.
6. Neither the language TOTAL $= \{M \mid M$ halts on all inputs$\}$ nor its complement is recursively enumerable.
7. The class of recursively enumerable sets is closed under union and intersection.
8. A multi-tape Turing Machine can recognize a language that no single tape TM can recognize.
9. There exists a Language L for which there is an NDTM M to accept it, but there is no DTM to accept the same language L.
10. A context-free grammar is said to be linear if, in each production rule, at most, one non-terminal occurs on the right-hand side.

If you find any MCQ to be incorrect, explicitly mention it in your answer.

Multiple Choice Questions (20 Marks)

1. Consider the symmetric difference of two languages A and B (over the same alphabet), denoted by $A \triangle B$. Which of the following statements is/are **TRUE**?

   (a) If A and B are both context-free languages (CFLs), then $A \triangle B$ must be a CFL.

   (b) If A is a CFL and B is not a CFL, then $A \triangle B$ must be a CFL.

   (c) If A is a CFL and B is regular, then $A \triangle B$ must be a CFL.

(d) If A and B are regular languages, then $A \triangle B$ is always context-free.

2. Consider the languages:

$$L_1 = \{a^m b^m c^{m+n} \mid m, n > 1\},$$
$$L_2 = \{a^m b^n c^{m+n} \mid m, n > 1\}.$$

Which of the following statements is **TRUE**?

(a) Both L_1 and L_2 are context-free languages (CFLs).

(b) Neither L_1 nor L_2 is a context-free language.

(c) L_1 is not a CFL, but L_2 is a CFL.

(d) L_1 is a CFL, but L_2 is not a CFL.

3. Consider the two grammars G and G' with the start symbols S and S', and with the following productions:

- Productions of G:

$$S \to aS \mid B, \quad B \to bB \mid b.$$

- Productions of G':

$$S' \to aA' \mid bB', \quad A' \to aA' \mid B', \quad B' \to bB' \mid \epsilon.$$

Which of the following statements is **TRUE**?

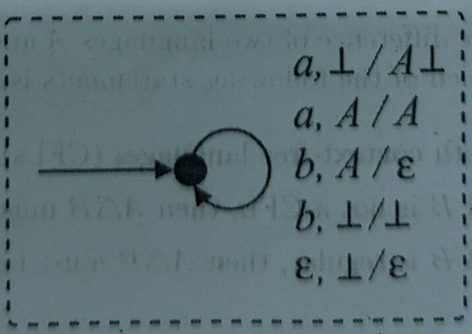
(a) $L(G) = L(G')$.

(b) $L(G)$ is strictly contained in $L(G')$.

(c) $L(G')$ is strictly contained in $L(G)$.

(d) Neither $L(G)$ is contained in $L(G')$ nor $L(G')$ is contained in $L(G)$.

4. What is the language over the alphabet $\{a, b\}$ that is accepted by the following PDA? The PDA accepts by empty stack. Here, $\perp$ is the initial bottom marker for the stack.



(a) $\{a^n b^n \mid n > 0\}$

(b) $\{a^m b^n \mid m, n \geq 0\}$

(c) $\{a^m b^n \mid m, n \geq 1\}$

(d) $L\{(a+b)^* b\}$

5. Let Σ_1 and Σ_2 be disjoint alphabets, $\Sigma = \Sigma_1 \cup \Sigma_2$, and $L \subseteq \Sigma^*$. Denote by L_1 the language over Σ_1 obtained by deleting all symbols of Σ_2 from the strings in L. Likewise, let L_2 denote the language over Σ_2 obtained by deleting all symbols of Σ_1 from the strings in L.

   For example, if $\Sigma_1 = \{a\}$, $\Sigma_2 = \{b\}$, and $L = \{abab^2ab^3...ab^n, \mid n \geq 1\}$, then we have:

$$L_1 = \{a^n \mid n \geq 1\}, \quad L_2 = \{b^{n(n+1)/2} \mid n \geq 1\}.$$

   Which of the following statements is/are **FALSE**?

   (a) If L is a DCFL, then both L_1 and L_2 must be DCFL.

   (b) If both L_1 and L_2 are DCFL, then L must be a DCFL.

   (c) If L_1 is a regular language and L_2 is a DCFL, then L must be a DCFL.

   (d) If L is a regular language, then both L_1 and L_2 must be regular languages.

6. Let M be a Turing machine over the alphabet Σ with $L(M) = L$. Let M' be the Turing machine obtained from M by swapping the roles played by the accept and reject states of M. Finally, let $L' = L(M')$, and $\sim L$ denote the complement of L (in Σ^*).

   Which of the following statements is/are always **TRUE**?

   (a) $L' = \sim L$

   (b) $L' \neq \sim L$

   (c) $L' \subseteq \sim L$

   (d) $\sim L \subseteq L'$

7. Which of the following statements about multi-tape Turing machines is **TRUE**?

   (a) Multi-tape Turing machines can recognize a strictly larger class of languages than single-tape Turing machines.

   (b) Every multi-tape Turing machine can be simulated by a single-tape Turing machine with only a quadratic increase in time complexity.

   (c) Multi-tape Turing machines require exponentially more states than single-tape Turing machines to recognize the same language.

   (d) The language classes recognized by single-tape and multi-tape Turing machines are fundamentally different.

8. Which of the following statements is/are **FALSE**?

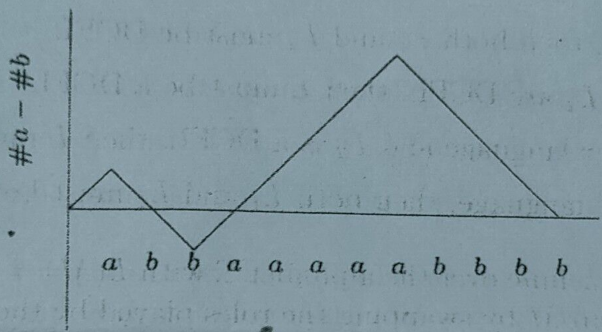
(a) For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.

(b) Turing recognizable languages are closed under union and complementation.

(c) Turing decidable languages are closed under intersection and complementation.

(d) Turing recognizable languages are closed under union and intersection.

9. The graph below shows the value $\#a - \#b$ plotted against prefixes of a word $x \in \{a, b\}^*$. Analyze the graph carefully and identify the language represented by it.



(a) $L = \{x \in \{a, b\}^* \mid \#a(x) > \#b(x)\}$

(b) $L = \{x \in \{a, b\}^* \mid \#a(x) < \#b(x)\}$

(c) $L = \{x \in \{a, b\}^* \mid \#a(x) = \#b(x)\}$

(d) $L = \{x \in \{a, b\}^* \mid \#a(x) + \#b(x) \text{ is even}\}$

10. What language is generated by the unrestricted grammar $G = (\{S, B, a, b, c\}, \{a, b, c\}, R, S)$, where R consists of the following productions?

$$S \to aBSccc \mid aBccc$$

$$Ba \to aB, \quad Bc \to bbc, \quad Bb \to bbb$$

(a) $\{a^n b^{3n} c^{3n} \mid n \geq 0\}$

(b) $\{a^n b^{2n} c^{3n} \mid n \geq 0\}$

(c) $\{a^n b^n c^n \mid n > 0\}$

(d) $\{a^n b^{2n} c^{3n} \mid n > 0\}$

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Descriptive Answer Type (20 Marks)

1. (a) Define a Turing Machine formally. [2]

   (b) Explain how a multitape Turing Machine can be simulated using a single-tape Turing Machine. [3]

2. Consider the language $L = \{a^n b^{n^2} \mid n \geq 0\}$. Use the Pumping Lemma for CFLs to determine whether L is a context-free language or not. Clearly explain your assumptions. [5]

3. Design a Turing Machine (TM) M that decides the language:

$$L = \{0^{2^n} \mid n \geq 0\}.$$

   Clearly explain the steps your Turing Machine takes to decide if the given string belongs to L. [5]

4. Consider the following language over $\Sigma = \{a, b, c\}$:

$$L_1 = \{a^i (bc)^j \mid i, j > 0 \text{ and } i > j\}.$$

   (a) Design a PDA $M = (Q, \Sigma, \Gamma, \delta, s, \perp, F)$ to accept L_1. M must contain at most two states and clearly mention whether it accepts by final state, empty stack, or both. [3]

   (b) Provide a detailed explanation of the transition function δ of your PDA, and describe how it ensures that $i > j$. [2]