Bose-Einstein Condensation

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1)What was the reason for choosing Bose-Einstein Condensation(BEC) as the topic for my PHN006 project?

Bose-Einstein condensate was discovered for the first time recently in 1995 and thus continues to be an active area of research. Therefore, choosing Bose-Einstein condensation as the topic would allow me to explore the latest developments in the field.

This topic has both experimental and theoretical significance. Therefore, it allows me to explore both the experimental techniques used for creating and studying Bose-Einstein Condensate and simultaneously learn the mathematical and theoretical framework which explains BECs.

The experimental realization of BECs in 2001 earned it a Nobel Prize in Physics!

Therefore, choosing BEC as my topic would allow me to understand the prestigious work that led to it winning the Nobel Prize.

2)How relevant is this topic to the PHN006 course in Quantum and Statistical Mechanics?

Bose-Einstein Condensation is a **fundamental Quatum phenomenon** that occurs at very low temperatures. At the same time, we have Bose-Einstein Distribution as a part of this course. Hence, the chosen topic would allow me to dive deeper into the study of Quantum mechanics w.r.t Bose-Einstein distribution and Bose-Einstein Condensates.

1. <u>Introduction</u>:

1.1 What are Bosons?

A particle whose spin quantum number has non-negative integer values, i.e., 0,1,2,3…. is called a Boson.

Bosons are identical particles that are indistinguishable because their wave functions nearly overlap.

Bosons are classified into two types:

(i)<u>Elementary Bosons</u>: The Bosons which act as force carriers or give rise to the phenomenon of mass.

Example: 1)Photons because they act as force carriers of the electromagnetic field.
2)Higgs Bosons: A Boson, which is also a force carrier of Higgs Field and is responsible for granting mass to other particles! Hence it is also called the *'God Particle.'*

(ii)<u>Composite Bosons: </u>A composite particle comprising an **even** number of Fermions.

Fermions have a spin quantum number of odd multiples of half (1/2, 3/2, 5/2…..).
Therefore even number of Fermions is necessary.
Example: Stable nuclei of even <u>mass number</u> such as Deuterium(D), Helium-4 (He-4).

1.2 What is Bose-Einstein Condensate (BEC)?

A Bose-Einstein Condensate is a state of matter created when Bosons are cooled to near absolute zero (-273.15K).

At such low temperatures, the energies of Bosons become so low that their <u>*quantum properties do not interfere*</u>, resulting in the formation of a single entity consisting of nearly all Bosons occupying the **same quantum state at the lowest available energy.**

2. Theoretical Framework:

2.2 Bose-Einstein Distribution and its Derivation-

The probability that a Boson at temperature T (in K) occupies an energy state with energy ε is given by.

$$f(\varepsilon) = \frac{1}{Ae^{\varepsilon/kT} - 1} \qquad ......(1)$$

where k=Boltzmann Constant(in JK^{-1})

T=Absolute Temperature(in K)

A=constant whose value depends on temperature and the chemical

the potential of Bosons. ($A = e^{-\frac{\mu}{kT}}$)(Unit-less)

(It is to be noted that the chemical potential(μ)(SI Unit- J) of species

in a mixture is defined as the rate of change of free energy of the

system concerning changes in the number of atoms or molecules

of the species added to the system and thus the Bosons that we are

talking about are the **Composite Bosons.** However the same

relation holds for elementary Bosons, too, except that the

constant depends on Temperature and other factors instead of

chemical potential)

Derivation:

The number of ways n identical particles can be distributed into g distinct states is given by.

$$W = \frac{(n+g-1)!}{n!(g-1)!}$$

Let us consider that-

n_1 particles occupy a state g_1 which corresponds to energy ε_1 (W1 ways)

n_2 particles occupy a state g_2 which corresponds to energy ε_2 (W2 ways)

and so on….

Hence the Total Number of ways of this distribution is given by-

W=W1 x W2 x W3……

Approximation- g_k-1 is nearly equal to g_k because the number of microstates are large

$$W = \prod_k \frac{(n_k + g_k)!}{n_k!g_k!}$$

From the basics of Statistical mechanics, the greater the number of ways of distribution W of particles into a given state more probable is the distribution.

We have to maximize the above expression subject to.

$$\sum_k n_k = N$$

$$\sum_k \varepsilon_k n_k = U$$

where N is the total number of particles, and U is the total Energy of particles.
This maximization can be done by using the **Sterling formula** for the Factorials.

We will maximize W through the maximization of ln W because ln
The function is a strictly increasing function.

$$\ln W = \sum_k ((n_k + g_k)\ln(n_k + g_k) - n_k \ln n_k - g_k \ln g_k)$$

$$d(\ln W) = \sum_k (\ln(n_k + g_k) - \ln n_k) dn_k$$

Also,

$$\sum_k dn_k = 0$$

$$\sum_k \varepsilon_k dn_k = 0$$

Hence,

$$d(\ln W) = \sum_k (\ln \frac{(n_k + g_k)}{n_k}) dn_k + \sum_k \alpha dn_k + \sum_k \beta \varepsilon_k dn_k$$

$$d(\ln W) = \sum_k ((\ln \frac{(n_k + g_k)}{n_k}) + \alpha + \beta \varepsilon_k) dn_k$$

It is found that for appropriate values of α and β (called the Lagrange Multipliers), each of the above terms vanishes.

$$\ln\frac{(n_k+g_k)}{n_k}+\alpha+\beta\varepsilon_k=0$$

$$n_k=\frac{g_k}{e^{-\alpha}e^{-\beta\varepsilon_k}-1}$$

Hence,

$$f(\varepsilon_k)=\frac{1}{e^{-\alpha}e^{-\beta\varepsilon_k}-1}$$

Which is the same as the Boson-Einstein Distribution function.

(beta is found to be equal to -1/kT)

2.2- Density of States:

Consider a particle in an **Anisotropic (i.e., the force constants in 3-Dimensions are not the same)** 3-Dimension harmonic oscillator potential with potential energy given by-

$$U(\vec{r})=\frac{1}{2}(K_x x^2+K_y y^2+K_z z^2)$$

The energy levels $\varepsilon(n_x, n_y, n_z)$ are given by-

$$\varepsilon(n_x,n_y,n_z)=(n_x+\frac{1}{2})hv_x+(n_y+\frac{1}{2})hv_y+(n_z+\frac{1}{2})hv_z$$

where nx,ny,nz ={0,1,2,3.....}

We now determine the number of states G(ε) with energy less than a given value ε. **Assumption**: For energies considerable compared to hv, we treat n as continuous rather than discrete integers.

We now consider a coordinate system whose axes are given by-

$\varepsilon_i=n_ihv_i$ where i={x,y,z}.

Therefore G(ε) is the volume enclosed between the three axes and the plane $\varepsilon=\varepsilon_x+\varepsilon_y+\varepsilon_z$.

$$G(\varepsilon) = \frac{1}{h^3 v_1 v_2 v_3} \int_0^{\varepsilon} d\varepsilon_x \int_0^{\varepsilon-\varepsilon_x} d\varepsilon_y \int_0^{\varepsilon-\varepsilon_x-\varepsilon_y} d\varepsilon_z = \frac{\varepsilon^3}{6h^3 v_1 v_2 v_3}$$

Let g(ε) be the density of states, then g(ε)dε gives the number of states between

Energy levels ε and ε+dε.

$$g(\varepsilon)\, d\varepsilon = G(\varepsilon)$$

$$Hence,$$

$$g(\varepsilon) = \frac{\mathrm{d}G(\varepsilon)}{\mathrm{d}\varepsilon}$$

$$g(\varepsilon) = \frac{\varepsilon^2}{2h^3 v_1 v_2 v_3}$$

Hence for an α-dimensional system, we can write-

$$g(\varepsilon) = C_\alpha e^{\alpha-1} \quad \text{.........(2)}$$

Where C_α is a constant.

2.3 Transition Temperature:

(i) Transition Temperature is the maximum temperature at which macroscopic occupation by the Bosons of the **lowest-energy state** appears.

(ii)At temperatures below this temperature, **Bose-Einstein condensate** is said to be formed.

(iii) Assumption: The lowest-energy state corresponds to the energy level with nearly zero energies.

(iv) Therefore, all the states with higher energy levels are considered to be **excited states.**

Therefore the number of Bosons in excited states is given by.

$$N_e = \int_{0+}^{\infty} g(\varepsilon) f(\varepsilon)\, d\varepsilon$$

Using the expressions of f(ε), g(ε) from (1) and (2) and evaluating the integral.

The constant A in the function f(ε) becomes 1 for the case of getting maximum Ne

because chemical potential is a non-positive quantity, and hence f(ε) becomes maximum when chemical potential=0

$$N_e = C_\alpha \Gamma(\alpha)\, \zeta(\alpha)\, (kT)^{\alpha}$$

where α denotes the number of dimensions

Γ(α) denotes the Gamma Function

ζ(α) denotes the Riemann-Zeta Function.

Just above the critical temperature Tc, the number of excited Bosons must

be equal to the Total number of Bosons since the number of bosons occupying 0

energy state is nearly 0 above Tc.

For a uniform Bose-gas in a 3-Dimensional box of volume V, the index of α=1.5.

$$N = N_e$$

$$T = T_c$$

$$C_{3/2} = \frac{Vm^{3/2}}{\sqrt{2}\pi^2 \left(\frac{h}{2\pi}\right)^{3/2}}$$

$$\Gamma\left(3/2\right) = \frac{\sqrt{\pi}}{2}$$

Hence we get-

$$T_c = \left(\frac{n}{\zeta(3/2)}\right)^{2/3} \frac{2\pi\hbar^2}{mk_B} \approx 3.3125\ \frac{\hbar^2 n^{2/3}}{mk_B}$$

$$\boxed{T_c = 3.31 \frac{n^{2/3} h^2}{4\pi^2 m k_B}} \quad \text{............(3)}$$

where,

Tc is the Critical Temperature

n is the number density of particles

m is mass per boson

k_B is the Boltzmann Constant

ζ is Riemann Zeta Function and ζ(3/2) ≈ 2.6124

2.4 Condensate Fraction:

<u>Objective of the section</u>: To determine the fraction of bosons in the condensate.

(i) For condensate to be formed-

$$T < T_c$$

(ii) Consider a system of Boson particles at temperatures T then the number of excited Bosons i.e., whose energies are greater than the lowest energy level, is given by following the equation derived earlier-

$$N_e = C_\alpha \Gamma(\alpha)\, \zeta(\alpha)\, (kT)^{\alpha}$$

(iii) Hence the fraction of Bosons present in the condensate is given by-

$$\frac{Number\ of\ \ Bosons\ present\ in\ the\ condensate}{Total\ number\ of\ \ Bosons}$$

$$= \frac{N - N_e}{N}$$

$$= \frac{C_\alpha \Gamma(\alpha)\,\zeta(\alpha)\left(kT_c\right)^{\alpha} - C_\alpha \Gamma(\alpha)\,\zeta(\alpha)\,(kT)^{\alpha}}{C_\alpha \Gamma(\alpha)\,\zeta(\alpha)\left(kT_c\right)^{\alpha}}$$

$$= 1 - \left(\frac{T}{T_c}\right)^{\alpha}$$

(iv) For particles in a box in 3 Dimensions, α is 3/2. Hence the above fraction becomes-

$$Fraction\ of\ \ Bosons\ in\ Condensate\ = 1 - \left(\frac{T}{T_c}\right)^{3/2} \qquad ....(4)$$

3. Experimental Realization of BEC:

We have looked at Bose-Einstein distribution, its derivation, and various parameters related to Bose-Einstein Condensate. We will now be looking at the experimental formation of Bose-Einstein Condensate.

3.1 Laser Cooling (Doppler Cooling):

(i) Doppler Cooling is by far the most common type of Laser cooling used for cooling atoms to temperatures low enough to form the Bose-Einstein condensate (Note that we are talking about **Composite Bosons**)

(ii) The technique involves bombarding atoms with Laser light with frequency **ν_0 slightly less than ν** (where ν is the frequency corresponding to the photon with energy equal to the energy required for excitation of the atom) such that the direction of motion of atoms and laser light are opposite.

(iii) The frequency observed by the atom absorbing the light will be **greater than**

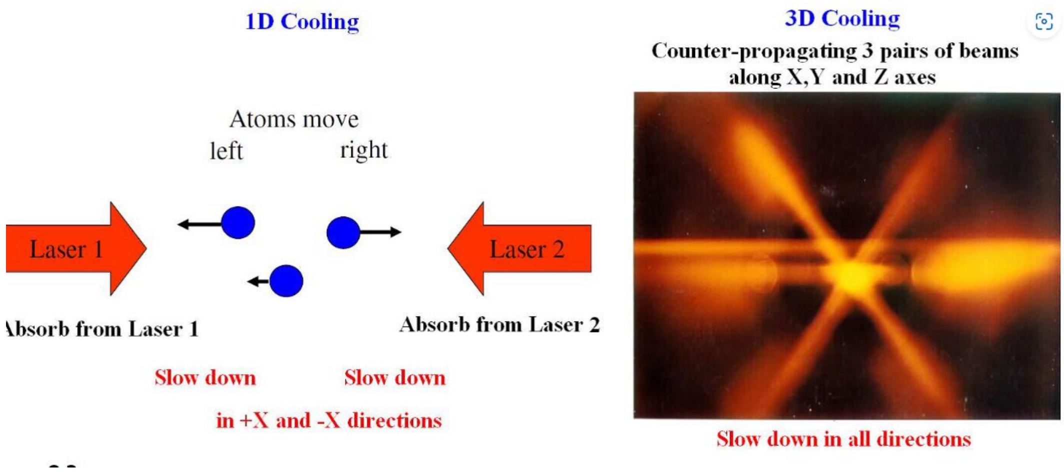
ν_0 (and thus can be greater than ν) because of the Doppler effect, and the expression for the same is-

$$\nu' = \nu_0 \left(\frac{c + v_{atom}}{c} \right)$$

(Under the assumption that the source of the laser is stationary)

(iv) The atom absorbs laser light in a direction opposite to its motion and re-emits it randomly. The net result is, therefore, **loss of energy**.

(v) As the Average energy decreases, so does the system's temperature, so cooling occurs.



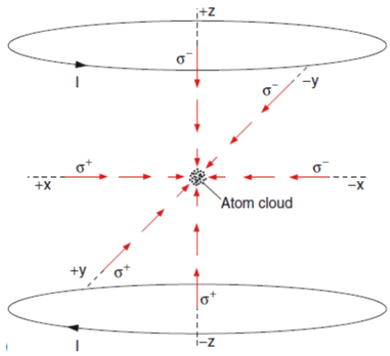
3.2 Magneto-Optical Trapping(MOT) of Atoms:

(i) Doppler Cooling certainly cools the atoms but not so much that it forms the condensate.

(ii) Cooling through Magneto-Optical Trapping of Atoms is a common technique accompanying Doppler Cooling.

(iii) Apparatus is set as shown below.

Positive Z-direction



(iv) Apparatus consists of Anti-Helmholtz coils (i.e., they carry current in opposite directions) and 6 – Laser Beams directed towards the center of the coils.

Three beams are Left-Circularly Polarized (shown by σ_+).

The rest three are Right-Circularly Polarized (shown by σ_-).

Each carries an angular momentum whose value is $h/2\pi$

(v) The magnetic field at the center of the coil is 0, and it varies linearly with z- for a small range near the center.

(vi) It is clear from the diagram that-

Atoms present in z>0 region will absorb σ_- light and undergo the transition $\Delta m=-1$

Atoms present in z<0 region will absorb σ_+ light and undergo the transition $\Delta m=+1$

Atoms present at the z=0 region will absorb both Left and Right Circularly Polarized light with equal probability.

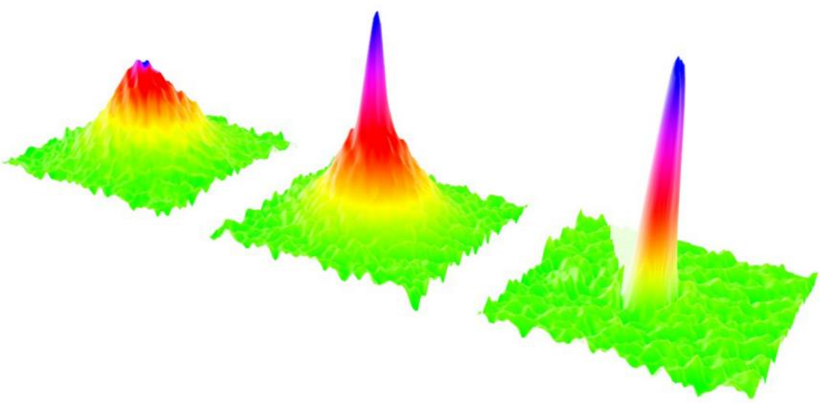
(vi) The atoms in z>0 region undergo the transition $\Delta m=-1$ which can cause

their magnetic moments to make an obtuse angle with a magnetic field and thus have higher potential energies by

$$U = -\vec{m}.\vec{B}$$

Similar reasoning can be applied to the atoms in the z<0 region too.

(vii) The result is that all atoms tend to stay near the origin and, at the same time, undergo Doppler Cooling. This favors the formation of BEC.



A diagram denoting the density of atoms in a region. Left-most diagram is

partly BEC, while the right-most chart is **nearly 100% BEC**.

4. Physical Property of BEC- Superfluidity:

(i) No direct correlation exists between a Bose-Einstein Condensate and Superfluid. However, we shall discuss this property concerning a composite

Bose-Einstein Condensate Helium-4 (He-4), which is a super-fluid.

(ii) Super-Fluidity is a characteristic property of a fluid with **zero viscosity** which allows the liquid to flow without loss of kinetic energy.

Reason:

The viscous force acting on any section of fluid of area A is given by-

$$\vec{F}_{viscous} = -\eta A \frac{\mathrm{d}\vec{v}}{\mathrm{d}x}$$

As η=0 for super-fluids, therefore, $F_{viscous}$ vanishes.

(iii) The transition temperature of He-4 for BEC formation is also known as the **Lamda point**. This value equals 2.17K for He-4

(iv) The remarkable properties of super-fluid resulted in introducing two-fluid descriptions of Hydrodynamics.

(v) The two fluids are- the normal and the super components, which are inter-penetrating.

(vi) At **very low temperatures,** the contribution of the normal component vanishes, while the density of the super-fluid component approaches the density of the fluid.

(vii) At **transition temperature and beyond,** the contribution of the super-fluid component vanishes, and the density of the normal component of fluid approaches the total density of the fluid.

(viii) Super-Fluid He-4 finds applications in MRI(Magnetic Resonance Imaging),

Cryogenics, Aerostatics, etc.

5. Application of BEC- Atom Laser:

(i) An Atomic Laser is a beam comprising a **coherent state of propagating atoms**

and thus, such a beam can be used in the study of quantum optics topics analogous to

photon.

(ii) It can be obtained from Bose-Einstein Condensate (BEC) because BEC consists of Bosons, all of which occupy the **same lowest-energy state** and are coherent

i.e., the phase difference between their wave functions remains constant with time.

(iii) We know that a MOT (Magneto-Optical Trap) can be used for BEC formation at the center of the coils. If the **trap is turned off,** the atoms present in BEC will fall as a monochromatic beam of atoms

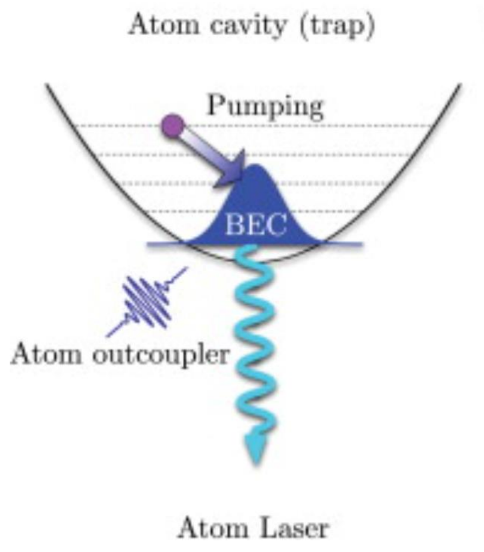
(iv) The challenge here is to eliminate the atoms which are **not present in BEC**

because they are not coherent with the atoms already present in BEC.

(v) A process known as **Output Coupling (or simply Outcoupling)** is employed in which a **RadioFrequency wave (RF wave)** of strong intensity is directed towards the region between the coils. This causes the flipping of spins of atoms not present in BEC and eliminates them from the trap. (Note that atoms in BEC have their spins opposite the direction of the magnetic field since they are confined to a small region near the center of the coil; therefore, flipping their spin will cause atoms in BEC present in $z>0$ region to move to the $z<0$ region and vice versa, however, the BEC remains intact)

(vi) Finally, the trap is turned off, and the atoms present in BEC, which are all coherent, are accelerated by a special process, resulting in Atom laser formation.

(vii) The limitation of an Atom-Laser compared to an ordinary Laser is the **flux. A typical** Laser pointer has a flux nearly 10^8 times the atomic laser produced by the out-coupling process.



A diagram indicating the out-coupling process and acceleration of BEC atoms resulting in Atom Laser Formation.

6. Summary:

What did I finally learn about Bose-Einstein Condensates in short?

(i) We started by discussing what bosons and BECs are.

(ii) We then looked at the theoretical framework and derived the results related to Bose-Einstein distribution and Density of States and various properties of BECs.

BECs begin to form at a particular temperature called the transition temperature(T_c), and as we reduce the temperature($T<T_c$) further, the fraction of atoms in BEC increases.

$$T_c = 3.31 \frac{n^{2/3} h^2}{4\pi^2 m k_B}$$

$$Fraction\ of\ Bosons\ in\ Condensate = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

(iii) We then looked at how bose-einstein condensates are formed experimentally where we discussed two techniques- (I) Doppler Cooling, which involves reducing the average energy of atoms present in the system through Laser light with a frequency of slightly less than that required for excitation. (II) Magneto-Optical Trap(MOT) which uses Doppler Cooling in addition to Anti-Helmholtz coils which carry current in opposite directions, and 6-Laser beams are directed towards the center of the coil (three of which are Left-Circularly Polarized and the rest three are Right-Circularly Polarized). This results in BEC formation at the center of the coil.

(iv) We then discussed a remarkable physical property related to the BEC of He-4: Superfluidity which makes He-4 have a zero-coefficient of viscosity below the Lambda point (Temperature of 2.17K) because of which He-4 as a superfluid finds many applications.

(v) Finally, we discussed Atom-Laser, an application of Bose-Einstein Condensate

which acts similar to a Laser except that it consists of atoms instead of photons and that its flux is much lesser than photon flux; further research in this regard and study of quantum optics through atomic laser is being carried out.

THE END

Thank you, sir, for your patience!

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