Lecture 11 **17.2.2025**

Today's agenda:

Boolean operations:

true = λx. λy. x

false = λx. λy. y

OR = λx. λy. x true y [version 1]

OR = λx. λy. x x y [version 2] does not involve any other function; thus better

AND = λx. λy. x y false [version 1]

AND = λx. λy. x y x [version 2] does not involve any other function; thus better

NOT = λx. x false true

XOR = λx. λy. x (NOT y) y

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Idea: use the functions true and false appropriately.

consider OR: x OR y [version 1]

λx. λy. x ___ y

When we pass the actual parameter say x=true, we get true ____ y

By the property, "_____" would be returned. Since we know the answer as true so "___" is true.

What if x=false? Then the answer is whatever is the value of y. so the second argument should be y and our choice is correct.

So we get OR: λx. λy. x true y

Take a closer look. What if x is true? It is true which is x. So we have

OR: λx. λy. x x y now the body of OR does not contain any constant (true/false) [version 2]

Now we can design AND easily

What if x=false? Then the answer is false irrespective of the value of y. Otherwise, it is the value of y. [version 1]

Take a closer look. What if x is false? It is false which is x. So we have

AND: λx. λy. x y x [version 2]

Pair: (a, b)

The function for Pair should specify the two components and the projection function.

pair = $\lambda x.\ \lambda y.\ \lambda z.\ (z\ x\ y)$ x,y are variables for the actual parameters e.g., a, b

fst = $\lambda p.\ p\ (\lambda x.\ \lambda y.\ x)$ first component of the pair [NB: *fst* is not the same as *first*]

snd = $\lambda p.\ p\ (\lambda x.\ \lambda y.\ y)$ second component of the pair [*snd* is not the same as *second*]

let us compute fst (pair a b) and snd (pair a b):

pair a b = $(\lambda x.\ \lambda y.\ \lambda z.\ (z\ x\ y))\ a\ b$

$= (\lambda y.\ \lambda z.\ (z\ a\ y))\ b$ "=" means beta-reduction

$= \lambda z.\ (z\ a\ b)$

= N (say)

fst N = $(\lambda p.\ p\ (\lambda x.\ \lambda y.\ x))\ N$

$= (\lambda p.\ p\ (\lambda x.\ \lambda y.\ x))\ (\lambda z.\ (z\ a\ b)\ )$ see the role of *p* and *z*.

$= (\lambda z.\ (z\ a\ b)\ )\ (\lambda x.\ \lambda y.\ x)$ *z* may be replaced with either *first* or *second*

$= (\lambda x.\ \lambda y.\ x)\ a\ b$ *first a b*

$= (\lambda y.\ a\ )\ b$

= a

snd N = $\lambda p.\ p\ (\lambda x.\ \lambda y.\ y)\ N$

$= \lambda p.\ p\ (\lambda x.\ \lambda y.\ y)\ (\lambda z.\ (z\ a\ b)\ )$

$= (\lambda z.\ (z\ a\ b)\ )\ (\lambda x.\ \lambda y.\ y)$

$= (\lambda x.\ \lambda y.\ y)\ a\ b$ *second a b*

$= (\lambda y.\ y)\ b$

= b

End of lecture