Boolean Algebra

LOGIC GATES

Formal logic: In formal logic, a statement (proposition) is a declarative sentence that is either true(1) or false (0).

It is easier to communicate with computers using formal logic.

- **Boolean variable:** Takes only two values – either true (1) or false (0).

They are used as basic units of formal logic.

Boolean function and logic diagram

- **Boolean function:** Mapping from Boolean variables to a Boolean value.

- **Truth table**:
  - Represents relationship between a Boolean function and its binary variables.
  - It enumerates all possible combinations of arguments and the corresponding function values.

Boolean function and logic diagram

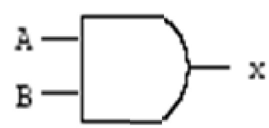
- **Boolean algebra**: Deals with binary variables and logic operations operating on those variables.

- **Logic diagram**: Composed of graphic symbols for logic gates. A simple circuit sketch that represents inputs and outputs of Boolean functions.

Gates

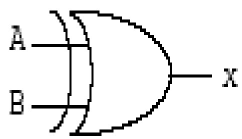
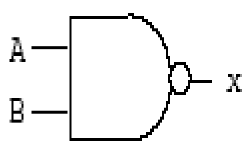
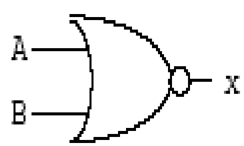
- Refer to the hardware to implement Boolean operators.

- The most basic gates are

Name	Graphic symbol	Algebraic function	Truth table	
Inverter		x = A'	A | x 0 | 1 1 | 0	
AND		x = AB	A B | x 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1	True if both are true.
OR		x = A + B	A B | x 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1	True if either one is true.

Boolean function and truth table

- Other common gates include:

Name	Graphic symbol	Algebraic function	Truth table	
Exclusive-OR (XOR)		$x = A \oplus B$ $= A'B + AB'$	A B | x 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0	Parity check: True if only one is true.
NAND		$x = (AB)'$	A B | x 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0	Inversion of AND.
NOR		$x = A + B$	A B | x 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0	Inversion of OR.

BASIC IDENTITIES OF BOOLEAN ALGEBRA

- ***Postulate 1 (Definition)***: A Boolean algebra is a closed algebraic system containing a set *K* of two or more elements and the two operators · and + which refer to logical AND and logical OR

Basic Identities of Boolean Algebra (Existence of 1 and 0 element)

(1) $x + 0 = x$

(2) $x \cdot 0 = 0$

(3) $x + 1 = 1$

(4) $x \cdot 1 = 1$

(Table 1-1)

Basic Identities of Boolean Algebra (Existence of complement)

(5) $x + x = x$

(6) $x \cdot x = x$

(7) $x + x' = x$

(8) $x \cdot x' = 0$

Basic Identities of Boolean Algebra (Commutativity):

(9) $x + y = y + x$

(10) $xy = yx$

Basic Identities of Boolean Algebra (Associativity):

(11) $x + (y + z) = (x + y) + z$

(12) $x (yz) = (xy) z$

Basic Identities of Boolean Algebra (Distributivity):

(13) $x(y + z) = xy + xz$

(14) $x + yz = (x + y)(x + z)$

Basic Identities of Boolean Algebra (DeMorgan's Theorem)

(15) $(x + y)' = x' y'$

(16) $(xy)' = x' + y'$

Basic Identities of Boolean Algebra (Involution)

(17) $(x')' = x$

Function Minimization using Boolean Algebra

- ***Examples***:

(a) $a + ab = a(1+b)=a$

(b) $a(a + b) = a.a +ab=a+ab=a(1+b)=a.$

(c) $a + a'b = (a + a')(a + b)=1(a + b) =a+b$

(d) $a(a' + b) = a. a' +ab=0+ab=ab$

Try

- $F = abc + abc' + a'c$

The other type of question

Show that;

1- $ab + ab' = a$

2- $(a + b)(a + b') = a$

1- $ab + ab' = a(b+b') = a.1=a$

2- $(a + b)(a + b') = a.a + a.b' + a.b + b.b'$

$= a + a.b' + a.b + 0$

$= a + a.(b' + b) + 0$

$= a + a.1 + 0$

$= a + a = a$

More Examples

- Show that;
  (a) $ab + ab'c = ab + ac$
  (b) $(a + b)(a + b' + c) = a + bc$

  (a) $ab + ab'c = a(b + b'c)$
  $= a((b+b').(b+c))=a(b+c)=ab+ac$

  (b) $(a + b)(a + b' + c)$
  $= (a.a + a.b' + a.c + ab +b.b' +bc)$
  $= \ldots$

DeMorgan's Theorem

(a) $(a + b)' = a'b'$

(b) $(ab)' = a' + b'$

Generalized DeMorgan's Theorem

(a) $(a + b + \ldots z)' = a'b' \ldots z'$

(b) $(a.b \ldots z)' = a' + b' + \ldots z'$

DeMorgan's Theorem

- $F = ab + c'd'$
- $F' = ??$

- $F = ab + c'd' + b'd$
- $F' = ??$

DeMorgan's Theorem

Show that: $(a + b.c)' = a'.b' + a'.c'$

More ***DeMorgan's*** example

Show that: $(a(b + z(x + a')))' = a' + b'(z' + x')$

$$
\begin{aligned}
(a(b + z(x + a')))' &= a' + (b + z(x + a'))' \\
&= a' + b'(z(x + a'))' \\
&= a' + b'(z' + (x + a')') \\
&= a' + b'(z' + x'(a')') \\
&= a' + b'(z' + x'a) \\
&= a' + b'z' + b'x'a \\
&= (a' + b'x'a) + b'z' \\
&= (a' + b'x')(a + a') + b'z' \\
&= a' + b'x' + b'z' \\
&= a' + b'(z' + x')
\end{aligned}
$$

More Examples

$$(a(b + c) + a'b)' = b'(a' + c')$$

$$ab + a'c + bc = ab + a'c$$

$$(a + b)(a' + c)(b + c) = (a + b)(a' + c)$$