Indian Institute of Technology Roorkee
Optimization Techniques (MAN-010)

Exercise – 3

1. Consider the graphical representations of the following linear program:
   Maximize (or minimize) $z = 5x_1 + 3x_2$
   Subject to $x_1 + x_2 \le 6$, $x_1 \ge 3$, $x_2 \ge 3$, $2x_1 + 3x_2 \ge 3$, $x_1, x_2 \ge 0$.

   (a) In each of the following cases indicate if the feasible region has one point, infinite number of points, or no point.
      (i) The constraints are as given above. (one)
      (ii) The constraint $x_1 + x_2 \le 6$ is changed to $x_1 + x_2 \le 5$.. (none)
      (iii) The constraint $x_1 + x_2 \le 6$ is changed to $x_1 + x_2 \le 7$. (infinite)

   (b) For each case in (a), determine the number of feasible extreme points, if any.
      (one, none, three)

   (c) For the cases in (a) in which a feasible solution exists, determine the maximum and minimum values of z and their associated extreme points
      (Max z = 24 = min z , min z = 24, max z = 29).

2. Solve graphically
   (i) Maximize (and minimize) $z = 10x_1 + 8x_2$
      Subject to $x_1 + x_2 \ge 2$, $4x_1 + 5x_2 \le 20$, $5x_1 + 4x_2 \le 20$, $x_1, x_2 \ge 0$.
   (ii) Maximize $z = 3x_1 + 4x_2$
      Subject to $x_1 + 2x_2 \le 6$, $x_1 - 2x_2 \le 3$, $2x_1 - x_2 \ge -2$, $x_1 \le 4$, $x_1 \ge 0$.

3. Consider the following problem:
   Maximize $z = -4x_1 + 6x_2$, s/t $2x_1 - 3x_2 \ge -6$, $-x_1 + x_2 \le 1$, $x_1, x_2 \ge 0$.
   Show graphically that the variables x_1 and x_2 can be increased indefinitely while the optimal value of the objective function remains constant.

4. Show graphically that the following problem has unbounded solution
   Maximize $z = 3x_1 + 4x_2$, s/t $2x_1 - 3x_2 \le 6$, $x_1 \le 5$, $x_1, x_2 \ge 0$.

5. Show the correspondence between extreme point and basic feasible solutions of the following problems:
   (i) Maximize $z = 3x_1 + 4x_2$
      Subject to $x_1 + 2x_2 \le 8$, $3x_1 + 2x_2 \le 12$, $x_1, x_2 \ge 0$.
   (ii) Maximize $z = 3x_1 + 4x_2$
      Subject to $x_1 + 2x_2 \le 4$, $3x_1 + 2x_2 \le 12$, $x_1, x_2 \ge 0$.

6. Find all basic feasible solutions and hence optimal solutions for the problems:
   (i) Maximize $z = 3x_1 + 2x_2 - x_4$
      Subject to $x_1 + 2x_2 + 2x_3 = 4$, $3x_1 - x_2 + 6x_3 + x_4 = 5$, $x_1, x_2, x_3, x_4 \ge 0$.
   (ii) Maximize $z = x_1 - 2x_2 + 3x_3$
      Subject to
   $2x_1 + 2x_2 + 2x_3 + x_4 = 6$, $4x_1 + 5x_2 + 2x_3 + 2x_3 = 12$, $x_1, x_2, x_3, x_4 \ge 0$.