CSN-212 Design and Analysis of Algorithms
[B] End Term Exam (Max Marks 200)

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Instructions: Please read the following instructions **very carefully** before attempting the questions.

1. Among the choices in a question, only the first UNCROSSED answer will be evaluated, the rest ignored.
2. Add page numbers to all used pages, and clearly mark the END and Rough Work. Leave the first left page to describe the attempted CHOICE for each question, and the page numbers it appears on.
3. For each answer not written completely in a single place, clearly mention *"continued on page X"*.
4. PENCIL should NOT be used in the answer sheet, else it will be considered as a case of CHEATING.
5. You can use concepts from the class as a BLACK BOX, unless the same is asked in the question.

Answer the following questions, carefully notice the choices amongst the subparts of a question.

1. Answer exactly **ONE** of the following **TWO** questions: **Maximum Marks: 20**

   (a) *b-ary counter.* Consider a b-ary counter where each literal ranges from $(0, 1, ..., b-1)$. *Starting from* $(0)_b$, show that the amortized number of literal changes for each *increment is constant.*

   (b) *Fractional Knapsack.* Given n items having weight w_i and value v_i, describe and analyze an algorithm to fill a knapsack with weight capacity W with items having maximum total value, where each item can be selected fractionally $(0-1)$. Also, prove the correctness of the algorithm.

2. Answer exactly **ONE** of the following **TWO** questions: **Maximum Marks: 25**

   (a) *Minimum Weight Shortest Path Tree.* Given a graph G and a source s, describe and analyze an algorithm to compute its shortest path tree (SPT) from s which is also its minimum spanning tree (if exists). Also, prove the correctness of the algorithm (assuming correctness of SPT).

   (b) *Disease Spread.* Given an undirected graph having some infected nodes, design and analyze an algorithm to find the vertex whose deletion results in most vertices being disconnected from all the infected vertices, and hence saved. Also, prove the correctness of the algorithm.

3. Answer exactly **ONE** of the following **TWO** questions: **Maximum Marks: 30**

   (a) *Edmond Karp's Algorithm.* Describe and analyze the shortest path variant of Ford Fulkerson algorithm for computing the maximum flow of a weighted network. Describe the proofs related to the analysis in detail assuming the correctness of Ford Fulkerson's algorithm.

   (b) *Subset Sum is NP Complete.* Given a set A of n numbers and a target t, the subset sum problem find if there exists a subset $S \subseteq A$ whose sum of elements is t. Show that Subset Sum is NP Complete by reducing from 3D Matching, which reports whether a subset of a given set of triplets from $X \times Y \times Z$ exists which covers all of $X \cup Y \cup Z$ without overlapping with each other.

4. **Surviving Cyclones [25 Marks].** The Indian Navy is fighting Somali pirates in the Arabian sea. Given a weighted (possibly negative) directed graph, where vertices represent locations in the sea, and edges represent the winds between the locations. The sudden drop in temperature turned every negative weighted cycle into a cyclone. Given the navigation system forces each ship to follow the shortest path starting from a node s, find which locations are safe to travel avoiding cyclones. Also, prove the correctness of the algorithm (use only classical algorithm as black box, not its applications).

5. **Degree assignment [30 Marks].** Given a list of natural numbers $d_0, d_1, ..., d_n$, design and analyze a greedy algorithm to compute (if exists) a simple undirected graph (no self loops or parallel edges) having n vertices with the above degree sequence. Also, prove the correctness of the algorithm.

6. **Handling debts [35 Marks].** A group of n friends regularly pay for each other such that they have accumulated debts $D[n][n]$ such that for any i, j at least one of $D[i][j]$ or $D[j][i]$ is zero (zero debt). Design and analyze an algorithm to compute $< n$ transfers which would clear all mutual debts, where a transfer from i to j is possible only when $D[i][j] > 0$. Also, prove the correctness of the algorithm.

7. **Hitting Set [35 Marks].** Given a universe of n elements $U = \{a_1, \cdots, a_n\}$ and m sets $S_i \subseteq U$, find the smallest hitting set of U, i.e., a collection S_i whose union is U. Prove that computing the smallest hitting set is NP Hard by reducing from the vertex cover problem, which reports the smallest set of vertices having at least one end point from every edge in the graph.