Closure of a Set of Functional Dependencies

The set of all dependencies that include F (the set of functional dependencies that are specified on relation schema R) as well as all dependencies that can be inferred from F is called the closure of F; it is denoted by F+.

We shall use the notation F+ to denote the closure of the set F, that is, the set of all functional dependencies that can be inferred given the set F. F+ contains all of the functional dependencies in F.

$R(ABC)$ $\quad F = \{A \to B,\ B \to C\}$

$F^+ = ?$

ABC $\quad 2^3 = 8$

ϕ $\quad \cancel{\phi^+ = \phi,\ \phi \to \phi,\ 1}$

A $\quad A^+ = ABC$ $\quad 2^3 = 8$ $\quad A \to A,\ A \to B,\ A \to C,\ A \to BC,\ A \to AB,\ A \to AC,\ \cancel{A \to \phi},\ A \to ABC$ $\quad 8 - 1 = \boxed{7}$

B $\quad B^+ = BC$ $\quad B \to C$ $\quad 2^2 = 4 = \boxed{3}$

C $\quad C^+ = C$ $\quad 2^1 = 2$ $\quad C \to \phi,\ C \to C$ $\quad 2 - 1 = \boxed{1}$

AB $\quad (AB)^+ = ABC$ $\quad 8 - 1 = \boxed{7}$

BC $\quad (BC)^+ = BC$ $\quad 4 - 1 = \boxed{3}$

AC $\quad (AC)^+ = ABC$ $\quad 8 - 1 = \boxed{7}$

ABC $\quad (ABC)^+ = ABC$ $\quad 8 - 1 = \boxed{7}$

$F^+ = 43 - 5 = 35$

$F^+ = \{\ \cancel{43}\ 35$ FDs $\}$

Membership Test

To check whether the given FD X → Y is a member of F+ (closure of FD Set F) or not

We use the notation F |=X → Y to denote that the functional dependency X → Y is inferred from the set of functional dependencies F.

$F = \{ \ldots \}$

$X \to Y$

$F = \{A \to B, B \to C\} \models A \to C$ ✓

$A^+ = ABC$

$A \to C \in F^+$

$F = \{X \to Y, Y \to Z\} \models X \to Z$ ✓

$X \to Y, Y \to Z$

$X \to Z \in F^+$

$X^+ = XYZ$

$F: \{XY \to Z, Y \to W\} \models \{WX \to Z\}$ ✗

$(WX)^+ = WX$

$WX \to Z \notin F^+$

$\{XY \to Z, Z \to X\} \models Z \to Y$ ✗

$Z^+ = ZX$

$\{X \to Y, Y \to Z\} \models X \to YZ$ ✓

$X^+ = XYZ$

$\{XY \to Z, Z \to W\} \models \{X \to W\}$ ✗

$X^+ = X$

Cover

A set of functional dependencies F is said to cover another set of functional dependencies G if every FD in G is also in F+ ; that is, if every dependency in G can be inferred from F; alternatively, we can say that G is covered by F.

Equivalence

Two sets of functional dependencies F and G are equivalent if F+ = G+. Therefore, equivalence means that every FD in F can be inferred from G, and every FD in G can be inferred from F; that is, F is equivalent to G if both the conditions (F covers G) and (G covers F) hold.

① ②

$(F \equiv G) \iff (① \wedge ②) \iff (F^+ = G^+)$

$\boxed{(F \equiv G) \iff (F^+ = G^+)}$

① F Covers G : $F^+ \supseteq G$ All F.D. of G member of F Set

② G Covers F : $G^+ \supseteq F$

$F = \{A \to B,\ AB \to C,\ D \to AC,\ D \to E\}$

$G = \{A \to BC,\ D \to AE\}$

$F^+ \equiv F$ $G^+ \equiv G$

F Covers G :- $\boxed{F^+ \supseteq G}$ ✓

$F: \{A \to B,\ AB \to C,\ D \to AC,\ D \to E\} \models$ $A \to BC$ ✓ $D \to AE$ ✓

G Covers F :- $G: \{A \to BC,\ D \to AE\} \models$ $A \to B$ ✓ $AB \to C$ ✓ $D \to AC$ ✓ $D \to E$ ✓

$\boxed{G^+ \supseteq F}$ ✓

$(F \equiv G)$ $\boxed{F^+ = G^+}$ ✓

$$(F^+)^+ = F^+$$

$$(F^+ \equiv F) \qquad (F^+)^+ = F^+$$

Canonical cover or Minimal Cover

A canonical cover of F is a **Minimal** set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

$F:- \{A \to B,\ B \to C,\ A \to C\}$

$\equiv \{A \to B,\ B \to C\} : F_1$

$F \equiv F_1$

$\boxed{F^+ = F_1^+}$

$F_1^+ = F_2^+$

$F_1 \quad \{A \to B,\ A\boxed{D} \to B\}$

$\{A \to B\} \quad F_2$

$A \to B$

$AD \to BD$

$\boxed{AD \to B},\ AD \to D$

Extraneous Attributes:

Consider F, and a functional dependency, A B

"Extraneous": Are there any attributes in A or B that can be safely removed ?

- Without changing the constraints implied by F

An attribute of a functional dependency is said to be extraneous if we can remove it without changing the closure of the set of functional dependencies.

(I) $F = \{AB \to C,\ A \to B\}$

$A \to B$

$A\boxed{B} \to C$

$\equiv$

$A \to B$

$A \to C$

$\{ABCD \to E,\ AB \to CD\}$

$\{AB \to E,\ AB \to CD\} \equiv \boxed{\{AB \to ECD\}}$

$A^+ = ABC$

$(AB)^+ = ABC$

$A \to B$

$A \to C$

$AB \to C$

(II) $\{AB \to C,\ A \to C\} \equiv \{A \to C\}$

(III) $\{A \to BC,\ B \to C\} \equiv \{A \to B,\ B \to C\}$

$X \to BCD$

$BC \to D$

$\equiv$

$X \to BC$

$BC \to D$

(IV) $\{AB \to CD,\ BC \to D\} \equiv \{AB \to C,\ BC \to D\}$

$(AB)^+ = ABCD$

Q.1 $F: \{A \to BC,\ B \to C,\ AB \to C,\ A \to B\}$

$A \to BC,\ B \to C,\ AB \to C,\ \cancel{A \to B}$

$A \to BC,\ B \to C,\ AB \to C$

$A \to B,\ B \to C,\ AB \to C$

$A \to B$
$B \to C$
$A \to C$

$A \to B\boxed{C}$
$B \to C$

$A \to B$
$B \to C$

$\{A \to B,\ B \to C\}$

$F: \{ABCD \to E,\ E \to D,\ A \to B,\ AC \to D\}$

$ABCD \to E,\ E \to D,\ A \to B,\ AC \to D$

$AC \to E,\ E \to D,\ A \to B,\ AC \to D$

$AC \to DE,\ E \to D,\ A \to B$

$\{AC \to E,\ E \to D,\ A \to B\}$

$F: \{X \to YZ,\ Y \to XZ,\ Z \to X\}$

$\boxed{F^+ = F_1^+}$

$X \to YZ \quad Y \to Z \quad Z \to X$

$X \to Y,\ Y \to Z,\ Z \to X$ $\quad F_1$

$F: \{A \to BC, \quad B \to AC, \quad C \to AB\}$

$A \to B$
$A \to C$
$B \to AC$
$C \to AB$ ×

$A \to B$
$A \to C$
$B \to AC$ × ×
$C \to A$

$A \to B$
$A \to C$
$B \to C$
$C \to A$

$A \to B\cancel{C}$
$B \to C$
$C \to A$

$A \to B$
$B \to C$
$C \to A$

F_1

I

$A \to B$
$A \to C$
$B \to A$
$C \to A$

F_3 ✓

$A \to BC$
$B \to A$
$C \to A$

F_2

II