Inventory Management

IBM 311

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Contents

- Elements of Inventory Management
- Inventory Control Systems
- Economic Order Quantity Models
- Quantity Discounts
- Reorder Point
- Periodic Inventory System

Inventory

- Stock of items to meet demand
  - Internal (Dependent)
  - External (Independent)

- Inventory management:
  - How much to order?
  - When to order?

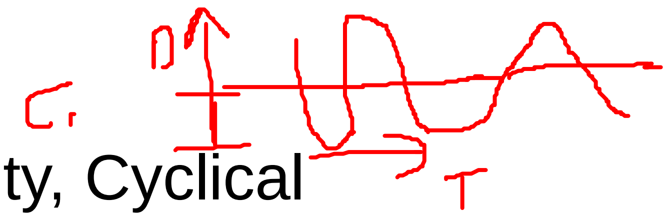
e.g. Raw materials, purchased parts, work in progress, items being transported, tool and equipment etc.

Dependent demand: items are used internally to produce a final product.
e.g. demand of wheel is depend on demand of cars.

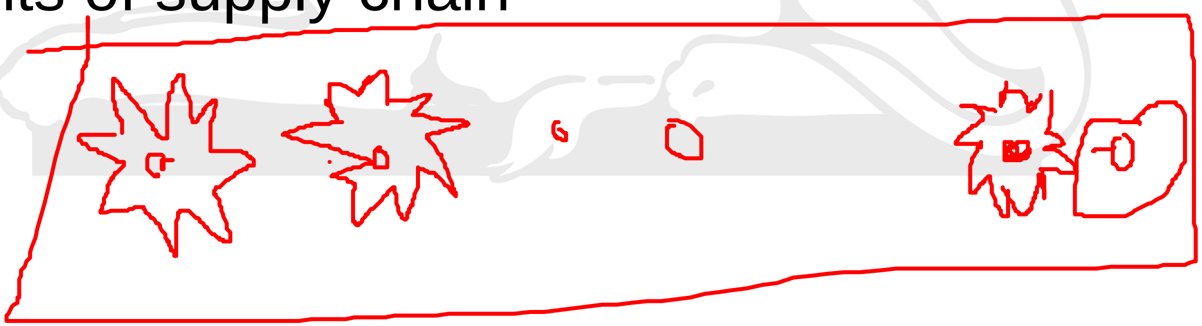
Independant demand: items are final products demanded by external customers.
e.g. Demand of Cars

Supply Chain Management

- BWE Bullwhip effect
  - Distortions (amplitude, phase lag) in demand information from customer to producer to supplier
  - How to deal with it?



- Variations in demand: Seasonality, Cyclical
- Independence from vendors
- Price discounts on bulk purchases
- Avoids disruptions by providing independence across the components of supply chain



Quality Management in the Supply Chain

- Lean vs Inventory
- Relationship between Quality and Inventory

Inventory of:

- Raw Materials
- Purchased parts and supplies
- Work-in-process products
- Items being transported
- Tools and equipment

Inventory

- Stock of items to meet demand
  - Internal (Dependent)
  - External (Independent)

- Inventory management:
  - How much to order?
  - When to order?

Inventory costs

- Carrying/holding costs the costs of holding items in inventory
  - Level of inventory
  - Duration of inventory
  - e.g. Storage facility (Rent, power, security, taxes, insurance)

    Material handling, Labor, Record keeping

    Borrowing to purchase inventory (interest on loans, taxes, insurance)

    Product deterioration, spoilage, breakage, obsolescence, pilferage
- Ordering costs the costs associated with replenishing the stock of inventory being held
  - Usually independent of the order size
  - E.g. requisition and purchase orders, transportation, receiving, inspection
  - Ordering cost ∝ 1/ Carrying costs

    as the order size increases, ordering costs decrease and carrying costs increase.
- Shortage/Stockout costs occur when customer demand cannot be met because of insufficient inventory.
  - Shortage costs ∝ 1/ Carrying costs

Inventory Control Systems

An inventory system controls the level of inventory by determining how much to order (the level of replenishment) and when to order.

- Continuous (fixed-order-quantity)
  - Continuous monitoring
  - Reorder point
  - Economic order quantity

Whenever the inventory on hand decreases to a predetermined level, referred to as the reorder point, a new order is placed to replenish the stock of inventory.

- Periodic (fixed-time-period)
  - Stock taking at specific time intervals

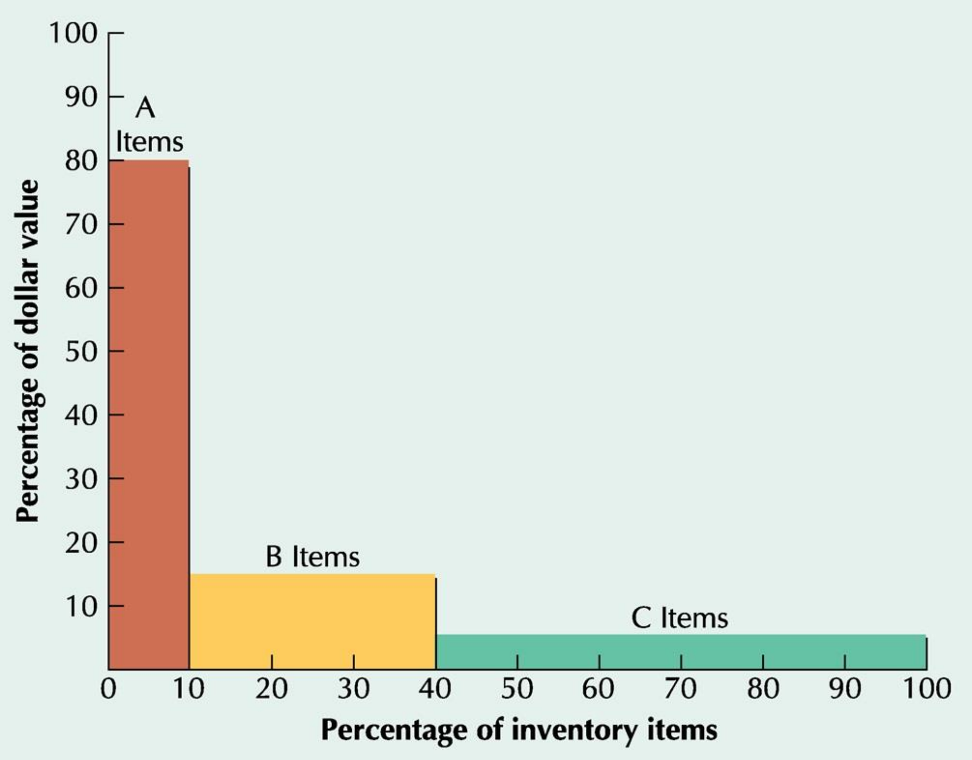
the inventory on hand is counted at specific time intervals—for example, every week or at the end of each month. After the inventory in stock is determined, an order is placed for an amount that will bring inventory back up to a desired level.

- ABC classification system
  - Cost of monitoring

The ABC system is a method for classifying inventory according to several criteria, including its dollar value to the firm

ABC Classification

- Class A
  - 5 – 15 % of units
  - 70 – 80 % of value
- Class B
  - 30 % of units
  - 15 % of value
- Class C
  - 50 – 60 % of units
  - 5 – 10 % of value



ABC Classification

PART	UNIT COST	ANNUAL USAGE
1	$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170
8	320	50
9	510	60
10	20	120

ABC Classification

Total value=unit cost*annual usage

Total quantity=Annual usage

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY		% CUMMULATIVE
9	$30,600	35.9	6.0	A	6.0
8	16,000	18.7	5.0		11.0
2	14,000	16.4	4.0		15.0
1	5,400	6.3	9.0	B	24.0
4	4,800	5.6	6.0		30.0
3	3,900	4.6	13.0		43.0
6	3,600	4.2	18.0	C	61.0
5	3,000	3.5	13.0		71.0
10	2,400	2.8	12.0		83.0
7	1,700	2.0	17.0		100.0
	$85,400				

ABC Classification

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	28.0
C	6, 5, 10, 7	12.5	60.0

Example 10.1

Economic Order Quantity (EOQ) Models

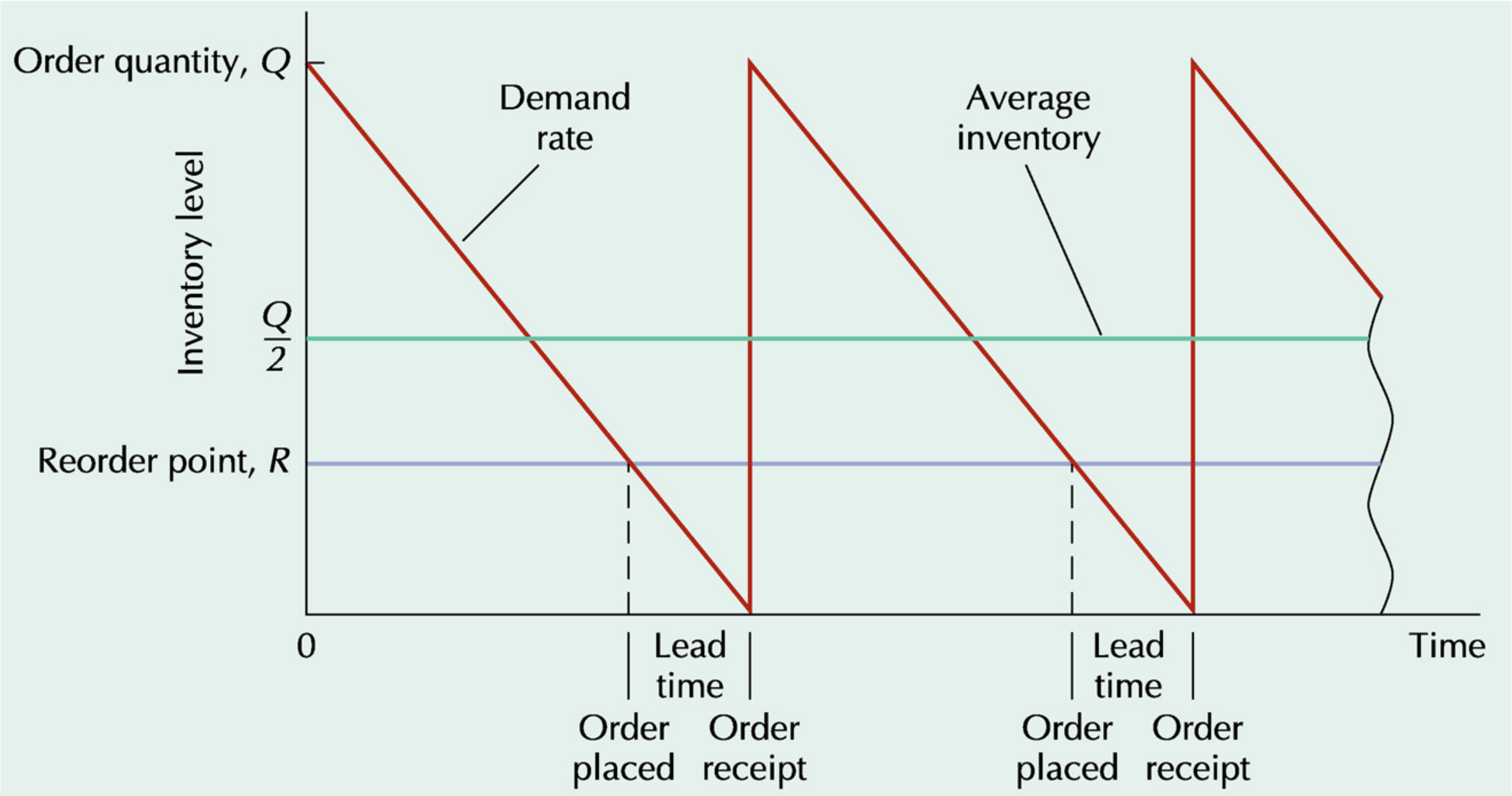
- EOQ the optimal order quantity that will minimize total inventory costs.
  - continuous inventory system
  - optimal order quantity that will minimize total inventory costs
- Basic EOQ model
- Production quantity model
- Order cycle
  - the time between receipt of orders in an inventory system

Assumptions of Basic EOQ Model

- Demand is known with certainty and is constant over time
- No shortages are allowed
- Lead time for the receipt of orders is constant
- Order quantity is received all at once

Lead time: The difference between the point when need for the materials has emerged and actual receiving of materials.

Inventory Order Cycle



The order is received all at once just at the moment when demand depletes the entire stock of inventory—the inventory level reaches 0—so there will be no shortages. This cycle is repeated continuously for the same order quantity, reorder point, and lead time.

EOQ Cost Model

C_o - cost of placing order

C_c - annual per-unit carrying cost

D - annual demand

Q - order quantity

$$\text{Annual ordering cost} = \frac{C_o D}{Q}$$

D/Q=no. of orders per year

$$\text{Annual carrying cost} = \frac{C_c Q}{2}$$

Q/2= Avg. inventory level

$$\text{Total cost} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

EOQ Cost Model

Deriving Q_{opt}

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$\frac{\partial TC}{\partial Q} = -\frac{C_o D}{Q^2} + \frac{C_c}{2}$$

$$0 = -\frac{C_0 D}{Q^2} + \frac{C_c}{2}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

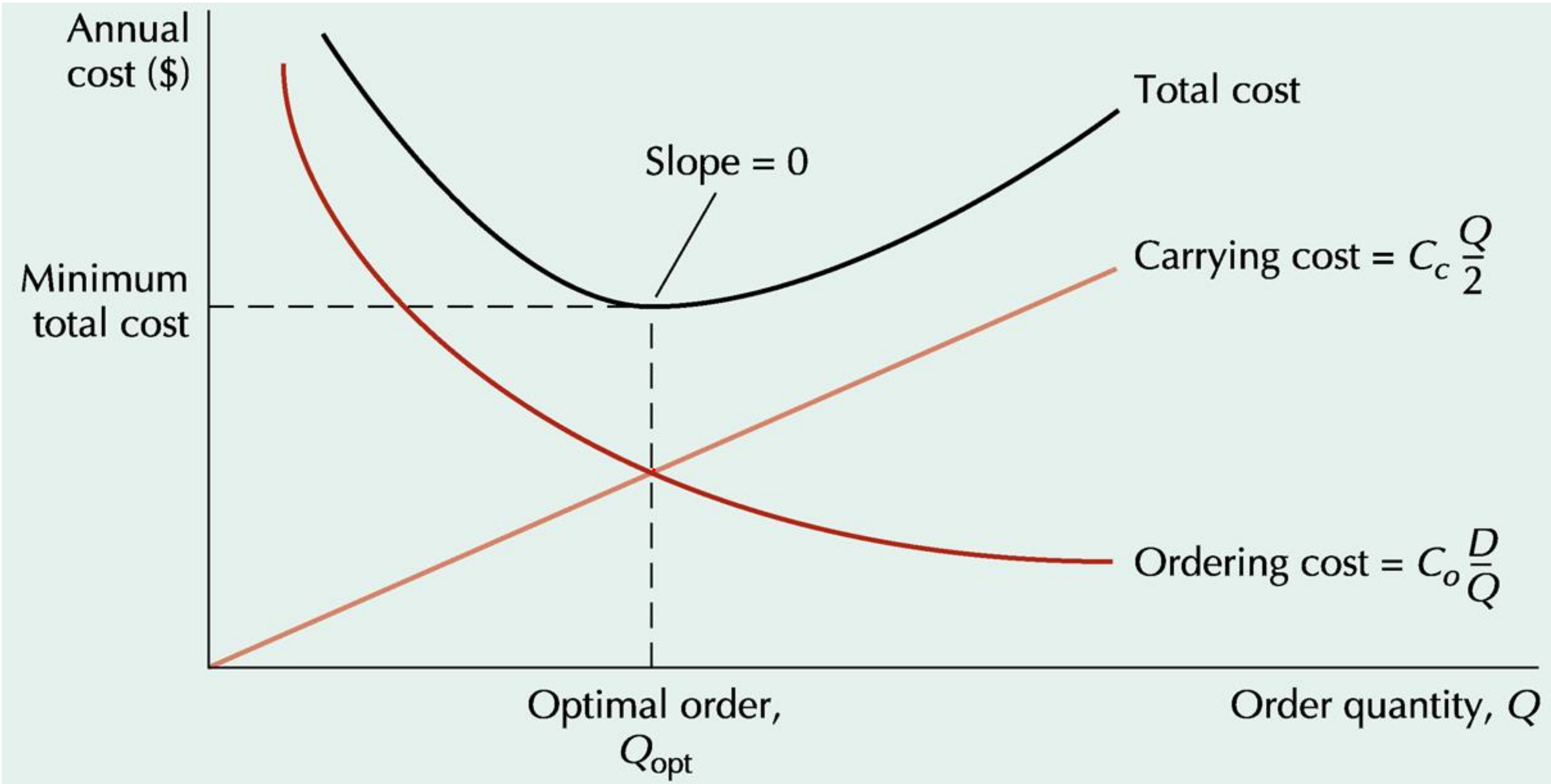
Proving equality of costs at optimal point

$$\frac{C_o D}{Q} = \frac{C_c Q}{2}$$

$$Q^2 = \frac{2C_o D}{C_c}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

EOQ Cost Model



EOQ Example

C_c = \$0.75 per gallon C_o = \$150 D = 10,000 gallons

$$Q_{\text{opt}} = \sqrt{\frac{2C_oD}{C_c}}$$

$$TC_{\min} = \frac{C_oD}{Q} + \frac{C_cQ}{2}$$

Q_{opt} =

$TC_{\min}$ =

Q_{opt} =

$TC_{\min}$ =

Orders per year = D/Q_{opt}

Order cycle time =

EOQ Example

C_c = \$0.75 per gallon C_o = \$150 D = 10,000 gallons

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} \qquad TC_{min} = \frac{C_oD}{Q} + \frac{C_cQ}{2}$$

$$Q_{opt} = \sqrt{\frac{2(150)(10,000)}{(0.75)}} \qquad TC_{min} = \frac{(150)(10,000)}{2,000} + \frac{(0.75)(2,000)}{2}$$

$$Q_{opt} = 2,000 \text{ gallons} \qquad TC_{min} = \$750 + \$750 = \$1,500$$

Orders per year = D/Q_{opt}
= 10,000/2,000
= 5 orders/year

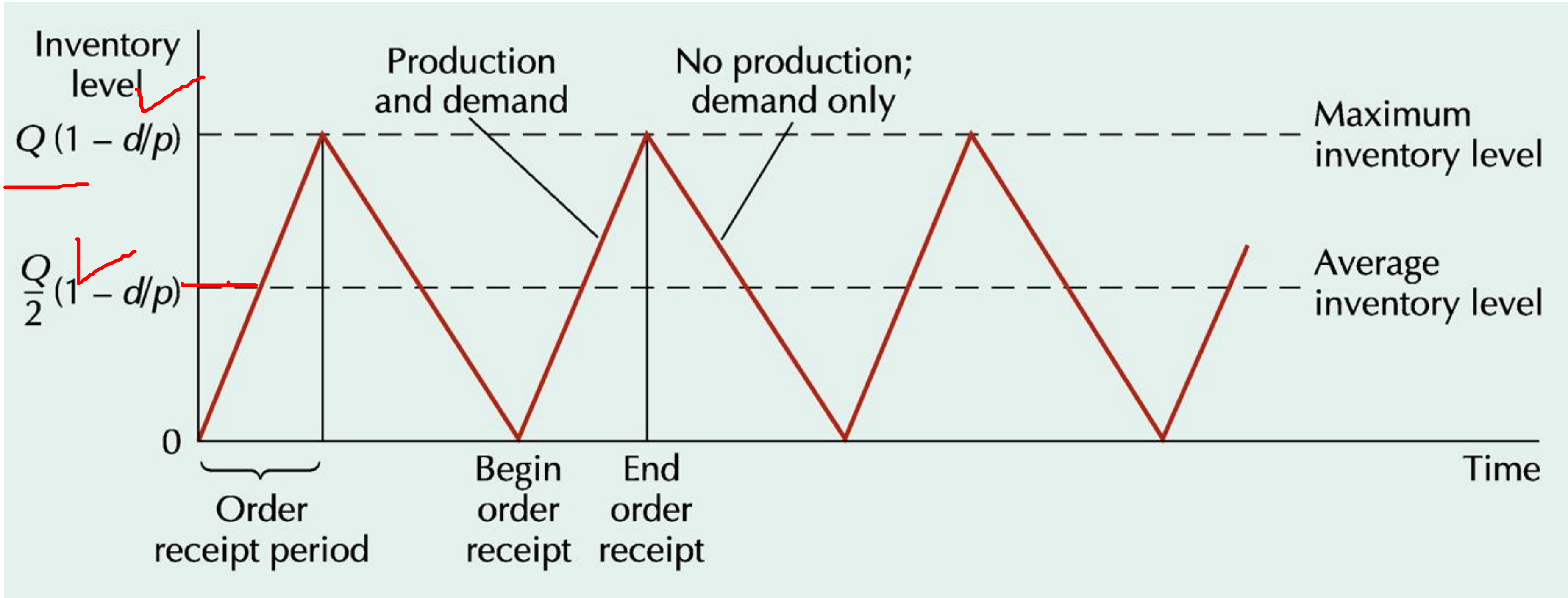
Order cycle time = 311 days/(D/Q_{opt})
= 311/5
= 62.2 store days

Production Quantity Model

- Order is received gradually, as inventory is simultaneously being depleted
  - non-instantaneous receipt model
  - assumption that *Q* is received all at once is relaxed
- *p* - daily rate at which an order is received over time, the *production rate*
- *d* - daily rate at which inventory is demanded

In this model variation, the maximum inventory level is not simply Q; it is an amount somewhat lower than Q, adjusted for the fact the order quantity is depleted during the order receipt period.

Production Quantity Model



Q/p = time required to complete the order

$(Q - Q/p * d) = Q(1 - d/p)$

Production Quantity Model

p = production rate

d = demand rate

$$\text{Maximum inventory level} = Q - \frac{Q}{p}d = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Average inventory level} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right)$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c\left(1 - \frac{d}{p}\right)}}$$

Production Quantity Model

C_c = \$0.75 per gallon $\quad C_o$ = \$150 $\quad D$ = 10,000 gallons

d = 10,000/311 = 32.2 gallons per day $\quad p$ = 150 gallons per day

$$Q_{\text{opt}} = \sqrt{\frac{2C_oD}{C_c\left(1 - \dfrac{d}{p}\right)}} =$$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2}\left(1 - \frac{d}{p}\right) =$$

$$\text{Production run} = \frac{Q}{p} =$$

Production Quantity Model

$$\text{Number of production runs} = \frac{D}{Q} =$$

$$\text{Maximum inventory level} = Q\left(1 - \frac{d}{p}\right) =$$

Production Quantity Model

C_c = \$0.75 per gallon $\quad$ C_o = \$150 $\quad$ D = 10,000 gallons

d = 10,000/311 = 32.2 gallons per day $\quad$ p = 150 gallons per day

$$Q_{\text{opt}} = \sqrt{\frac{2C_oD}{C_c\left(1 - \dfrac{d}{p}\right)}} = \sqrt{\frac{2(150)(10{,}000)}{0.75\left(1 - \dfrac{32.2}{150}\right)}} = 2{,}256.8 \text{ gallons}$$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2}\left(1 - \frac{d}{p}\right) = \$1{,}329$$

$$\text{Production run} = \frac{Q}{p} = \frac{2{,}256.8}{150} = 15.05 \text{ days per order}$$

Production Quantity Model

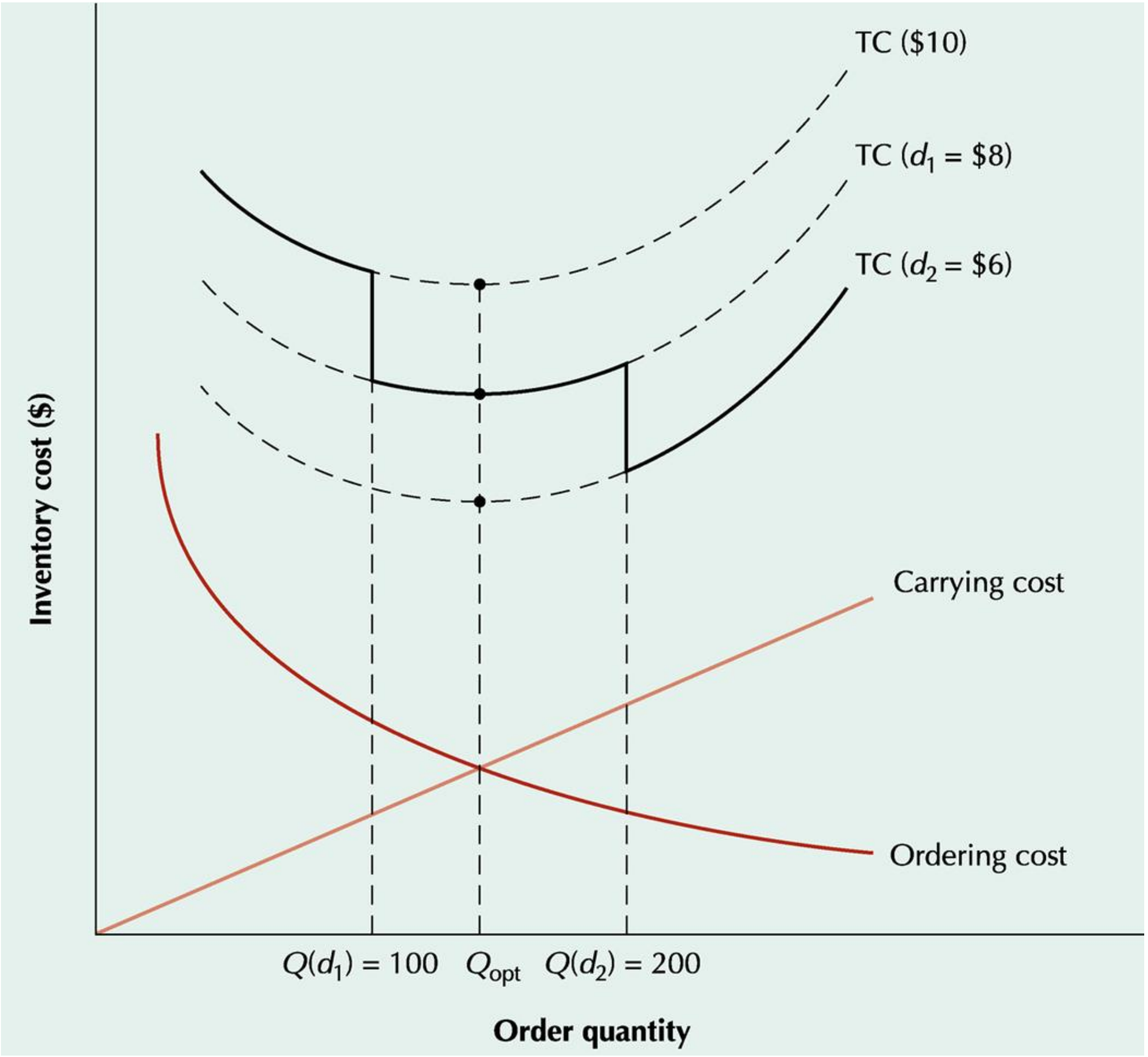
$$\text{Number of production runs} = \frac{D}{Q} = \frac{10{,}000}{2{,}256.8} = 4.43 \text{ runs/year}$$

$$\text{Maximum inventory level} = Q\left(1 - \frac{d}{p}\right) = 2{,}256.8\left(1 - \frac{32.2}{150}\right)$$

$$= 1{,}772 \text{ gallons}$$

A quantity discount is a price discount on an item if predetermined numbers of units are ordered

Quantity Discount Model



Quantity Discount

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

C_o = $2,500
C_c = $190 per TV
D = 200 TVs per year

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} =$$

For Q =

$$TC = \frac{C_oD}{Q_{opt}} + \frac{C_cQ_{opt}}{2} + PD =$$

For Q = 90

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2} + PD =$$

Quantity Discount

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

C_o = $2,500
C_c = $190 per TV
D = 200 TVs per year

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ TVs}$$

For $Q = 72.5$

$$TC = \frac{C_oD}{Q_{opt}} + \frac{C_cQ_{opt}}{2} + PD = \$233{,}784$$

For $Q = 90$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2} + PD = \$194{,}105$$

Reorder Point

- Inventory level at which a new order is placed

$$R = dL$$

where

d = demand rate per period
L = lead time

Reorder Point

Demand = 10,000 gallons/year

Store open 311 days/year

Daily demand =

Lead time = L = 10 days

R = dL =

Reorder Point

Demand = 10,000 gallons/year

Store open 311 days/year

Daily demand = 10,000 / 311 = 32.154 gallons/day

Lead time = L = 10 days

R = dL = (32.154)(10) = 321.54 gallons

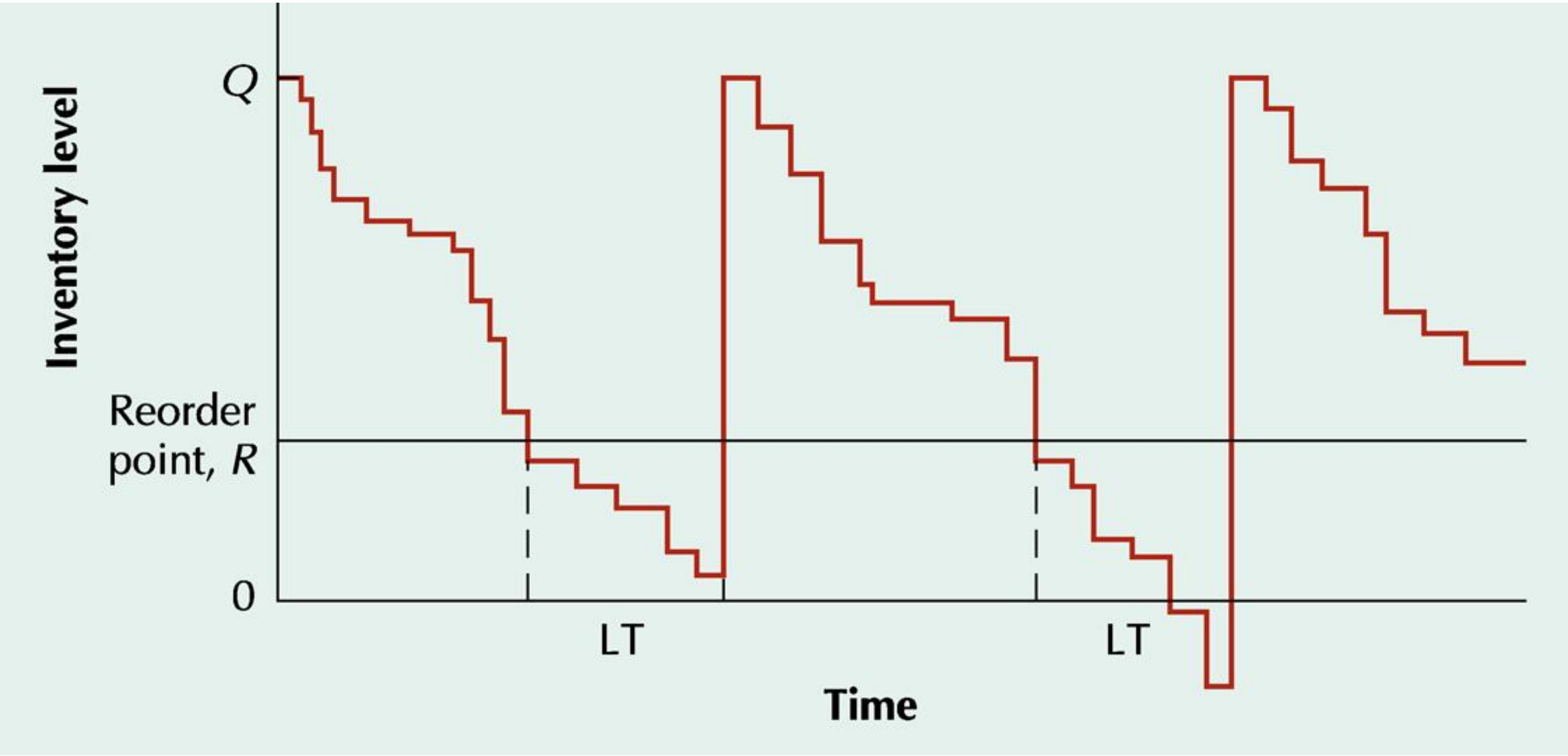
Safety Stock

a buffer added to the inventory on hand during lead time.

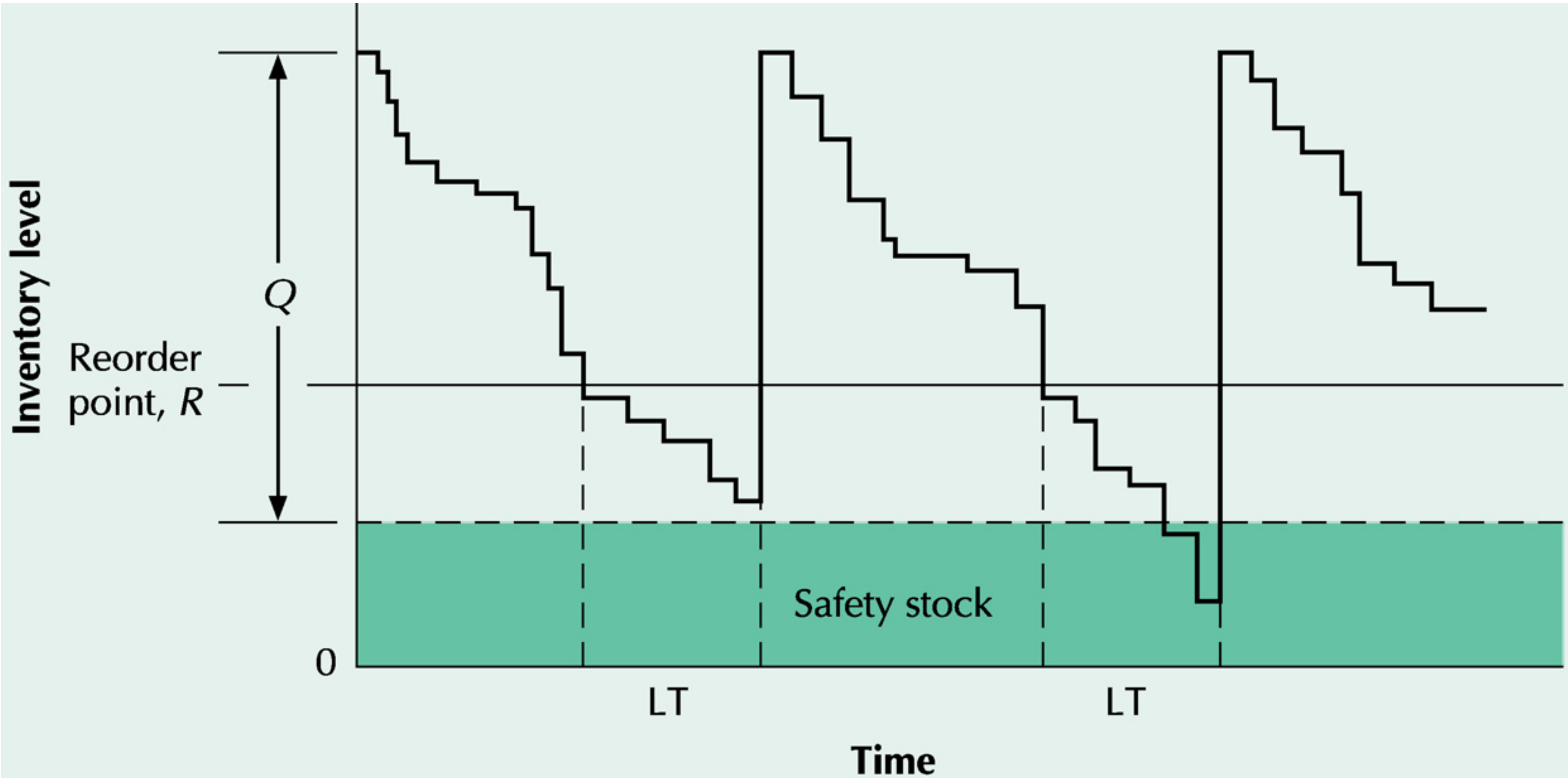
- Safety stock an order is made when the inventory level reaches the reorder point
  - buffer added to on hand inventory during lead time
- Stockout When demand exceeds the available inventory in stock.
  - an inventory shortage
- Service level the probability that a stockout will not occur.
  - probability that the inventory available during lead time will meet demand
  - P(Demand during lead time <= Reorder Point)

A service level of 90% means that there is a 0.90 probability that demand will be met during the lead time, and the probability that a stockout will occur is 10%

Variable Demand With Reorder Point



Reorder Point With Safety Stock



Reorder Point With Variable Demand

$$R = \bar{d}L + z\sigma_d \sqrt{L}$$

where

$\bar{d}$ = average daily demand

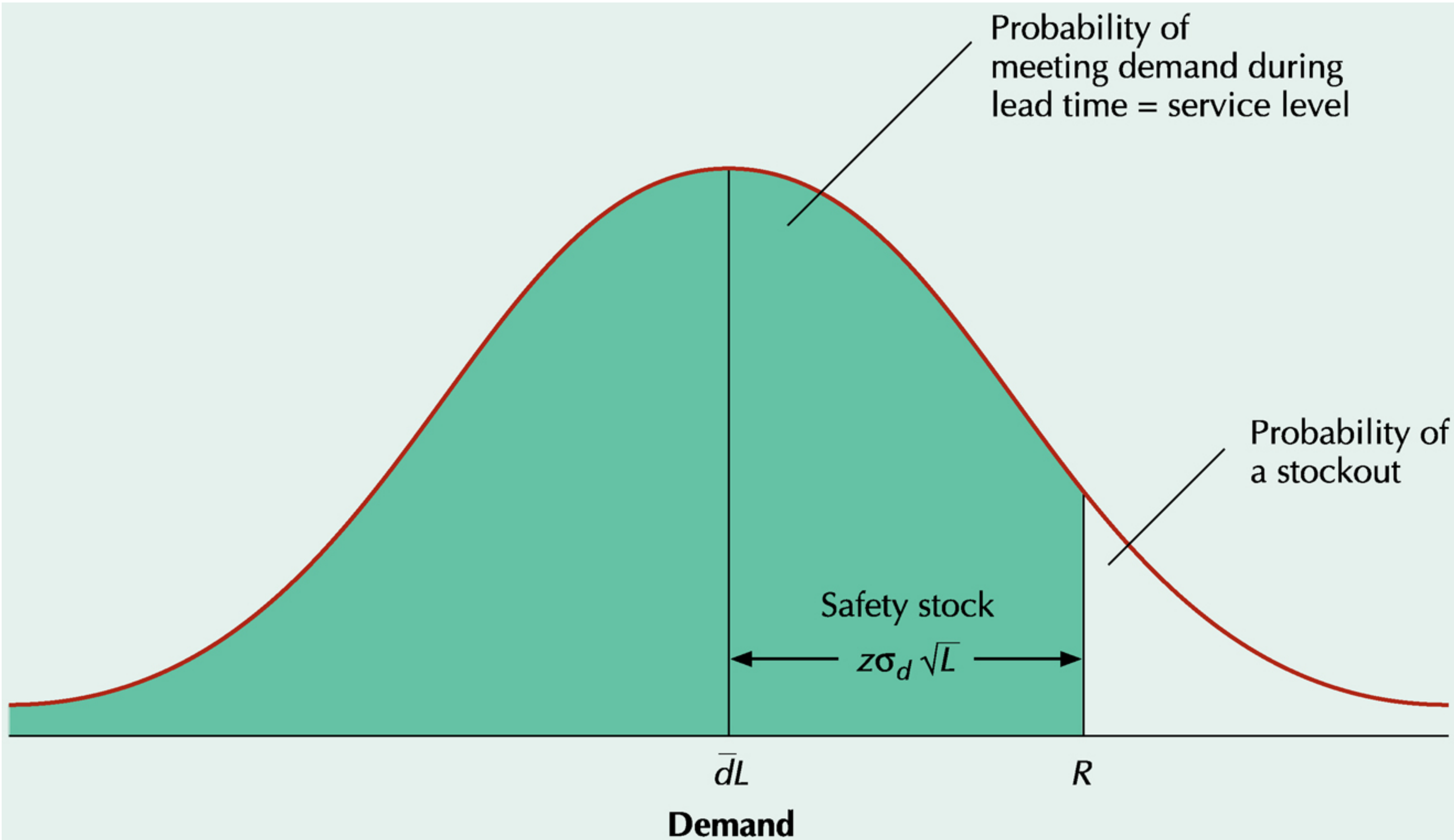
L = lead time

σ_d = the standard deviation of daily demand

z = number of standard deviations corresponding to the service level probability

$z\sigma_d\sqrt{L}$ = safety stock

Reorder Point For a Service Level



Reorder Point For Variable Demand

The paint store wants a reorder point with a 95% service level and a 5% stockout probability

$\bar{d}$ = 30 gallons per day
L = 10 days
σ_d = 5 gallons per day

For a 95% service level, $z = 1.65$

$R = \bar{d}L + z\,\sigma_d\sqrt{L}$

Safety stock $= z\,\sigma_d\sqrt{L}$

Reorder Point For Variable Demand

The paint store wants a reorder point with a 95% service level and a 5% stockout probability

$\bar{d}$ = 30 gallons per day
L = 10 days
σ_d = 5 gallons per day

For a 95% service level, z = 1.65

$$R = \bar{d}L + z\,\sigma_d\sqrt{L}$$
$$= 30(10) + (1.65)(5)(\sqrt{10})$$
$$= 326.1 \text{ gallons}$$

$$\text{Safety stock} = z\,\sigma_d\sqrt{L}$$
$$= (1.65)(5)(\sqrt{10})$$
$$= 26.1 \text{ gallons}$$

Order Quantity for a Periodic Inventory System

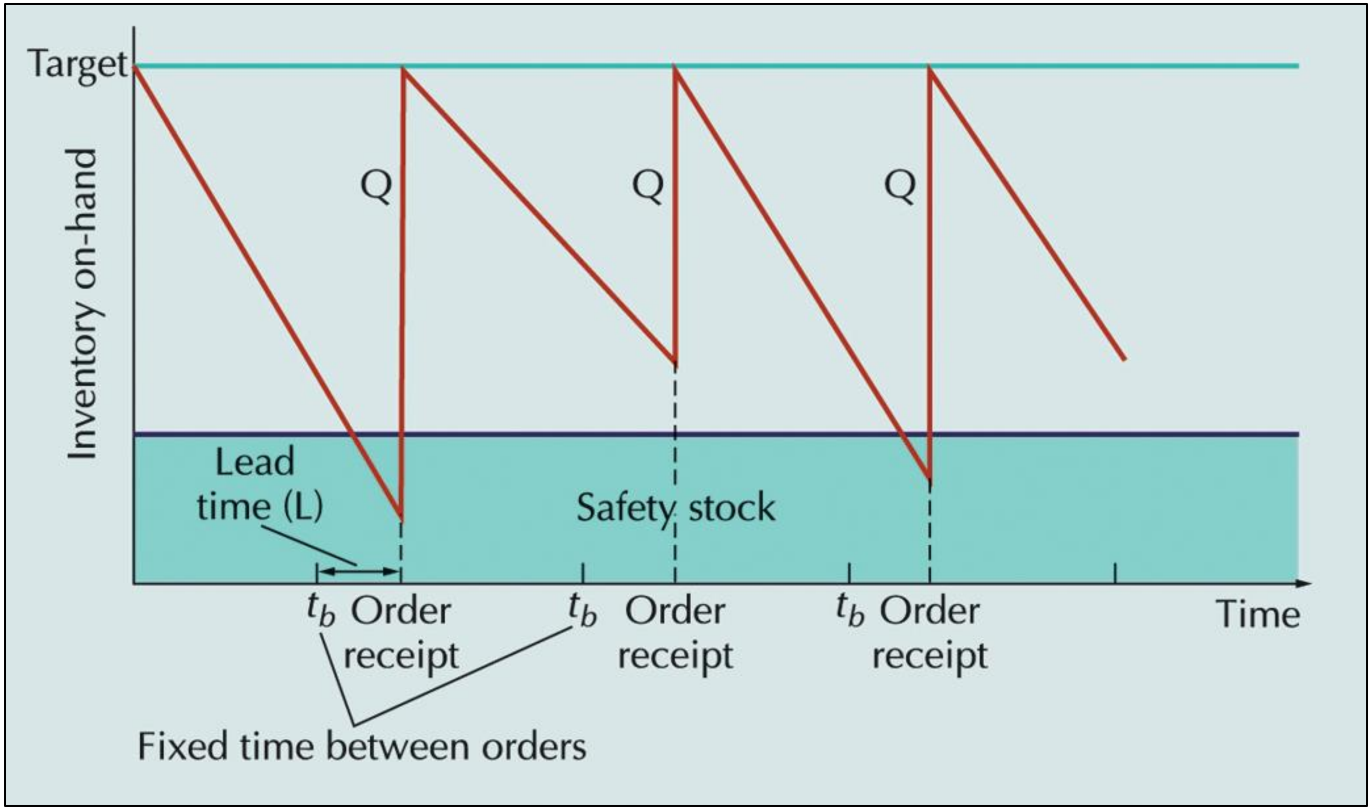
$$Q = \bar{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

where

d = average demand rate
t_b = the fixed time between orders
L = lead time
σ_d = standard deviation of demand
$z\sigma_d \sqrt{t_b + L}$ = safety stock
I = inventory level

Fixed-time-period inventory system tem is one in which the time between orders is constant and the order size varies. Small retailers often use this sytem. Drugstores are one example.

Periodic Inventory System



Fixed-Period Model With Variable Demand

d = 6 packages per day
σ_d = 1.2 packages
t_b = 60 days
L = 5 days
I = 8 packages
z = 1.65 (for a 95% service level)

$$Q = \bar{d}(t_b + L) + z\sigma_d\sqrt{t_b + L} - I$$

Fixed-Period Model With Variable Demand

$\bar{d}$ = 6 packages per day
σ_d = 1.2 packages
t_b = 60 days
L = 5 days
I = 8 packages
z = 1.65 (for a 95% service level)

$$Q = \bar{d}(t_b + L) + z\sigma_d\sqrt{t_b + L} - I$$

$$= (6)(60 + 5) + (1.65)(1.2)\sqrt{60 + 5} - 8$$

$$= 397.96 \text{ packages}$$