Department of Computer Science and Engineering
IIT Roorkee

ETE Spring Semester 2024-25 **10.5.2025**
CSN312 Principles of Programming Languages **FM: 40** **Duration: 180 min**

Do not write anything on the question paper.

Answer all the questions. Answer for each question should begin on a new page.
Zero mark would be given for correct answers with no steps/unjustified steps/ incorrect justifications.
The answers should appear in order of the questions.

1. **(i)** State the axioms and deduction rules of the system TA_λ.
(ii) Give a deduction of $\mapsto C : \tau$ where $C \equiv \lambda x.\lambda y.\lambda z.I(y(Ixz))$ $I \equiv \lambda x.x$ and $\tau = (a \to b) \to (b \to c) \to a \to c$, using the rules given in **1. (i)**. Mention clearly the starting step and the last step of the deduction. Justify how each new step is obtained from the previous steps. [3+5]

2. Apply the Principal Type (PT) algorithm to obtain the principal deduction for $M \equiv P\ I$ where $P \equiv \lambda x.\lambda y.\lambda z.K\,(xy)(xz)$ $I \equiv \lambda x.x$ $K = \lambda x.\lambda y.x$. The principal deduction should be shown clearly at the end. All the steps leading to the principal deduction should be clearly shown and justified. [8]

3. Give a proof of $\tau \equiv (a \to b \to b \to c) \to a \to b \to c$ in Intuitionist Implicational Logic. From the proof of τ, use the logic to lambda mapping to obtain a term M such that $M : \tau$. Justify the mapping of each step. [8]

4. **(i)** Find the most general unifier (m.g.u) **U** of $\langle \rho, \tau \rangle$ where
$\rho \equiv a \to (b \to b)$, $\tau \equiv (c \to c) \to a$, and give the corresponding most general unification. Prove that **U** is the m.g.u.
(ii) Establish the construction tree of the type
$\tau \equiv \big(a \to (b \to c)\big) \to ((a \to b) \to (a \to c))$. All the steps leading to the final tree should be clearly shown and justified. [4+4]

5. Consider the *List* data structure as discussed in the class. Give a pure lambda calculus-based encoding for *List*. All the steps should be clearly justified. [8]

END