Statistical Process Control

Lecture Outline

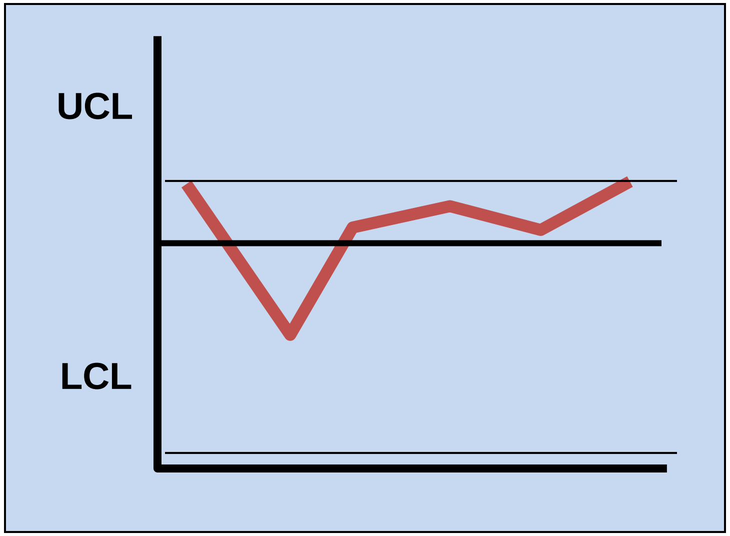
- Basics of Statistical Process Control
- Control Charts
- Control Charts for Attributes
- Control Charts for Variables
- Control Chart Patterns
- SPC with Excel and R
- Process Capability

Learning Objectives

- Explain when and how to use statistical process control to ensure the quality of products and services
- Discuss the rationale and procedure for constructing attribute and variable control charts
- Utilize appropriate control charts to determine if a process is in-control
- Identify control chart patterns and describe appropriate data collection
- Assess the process capability of a process

Statistical Process Control (SPC)

- Statistical Process Control
  - monitoring production process to detect and prevent poor quality
- Sample
  - subset of items produced to use for inspection
- Control Charts
  - process is within statistical control limits



Process Variability

- <u>Random</u>
  - inherent in a process
  - depends on equipment and machinery, engineering, operator, and system of measurement
  - natural occurrences
- <u>Non-Random</u>
  - special causes
  - identifiable and correctable
  - include equipment out of adjustment, defective materials, changes in parts or materials, broken machinery or equipment, operator fatigue or poor work methods, or errors due to lack of training

Chance and assignable causes of variation

- A process is operating with only **chance causes of variation** present is said to be **in statistical control**.
- A process that is operating in the presence of **assignable causes** is said to be **out of control**.

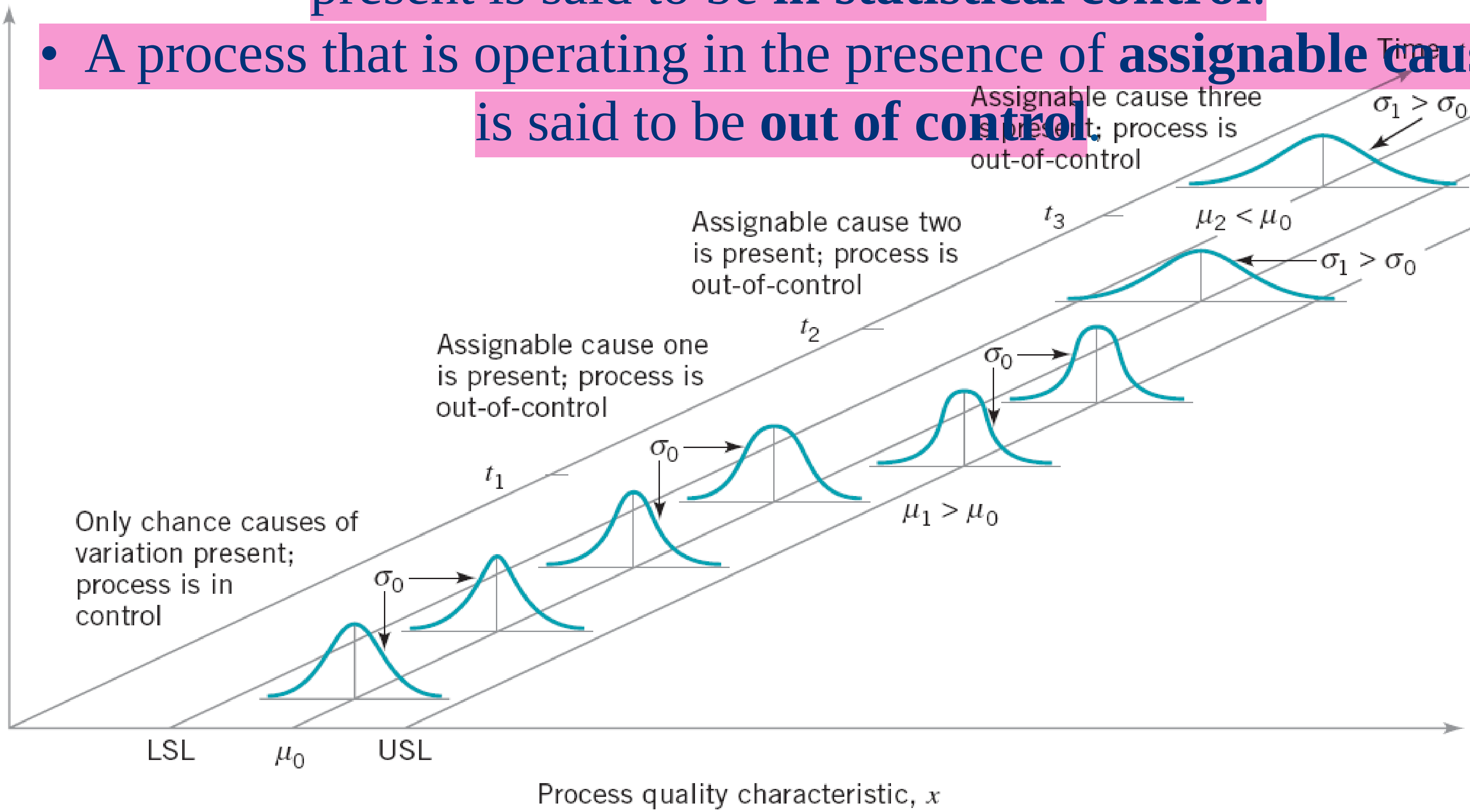


FIGURE 5.1 Chance and assignable causes of variation.

SPC in Quality Management

- SPC uses
  - Is the process in control?
  - Identify problems in order to make improvements
  - Contribute to the TQM goal of continuous improvement

Quality Measures: Attributes and Variables

- Attribute
  - A characteristic which is evaluated with a discrete response
  - good/bad; yes/no; correct/incorrect
- Variable measure
  - A characteristic that is continuous and can be measured
  - Weight, length, voltage, volume

SPC Applied to Services

- Nature of defects is different in services
- Service defect is a failure to meet customer requirements
- Monitor time and customer satisfaction

SPC Applied to Services

- Hospitals
  - timeliness & quickness of care, staff responses to requests, accuracy of lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance & checkouts
- Grocery stores
  - waiting time to check out, frequency of out-of-stock items, quality of food items, cleanliness, customer complaints, checkout register errors
- Airlines
  - flight delays, lost luggage & luggage handling, waiting time at ticket counters & check-in, agent & flight attendant courtesy, accurate flight information, cabin cleanliness & maintenance

SPC Applied to Services

- Fast-food restaurants
  - waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy
- Catalogue-order companies
  - order accuracy, operator knowledge & courtesy, packaging, delivery time, phone order waiting time
- Insurance companies
  - billing accuracy, timeliness of claims processing, agent availability & response time

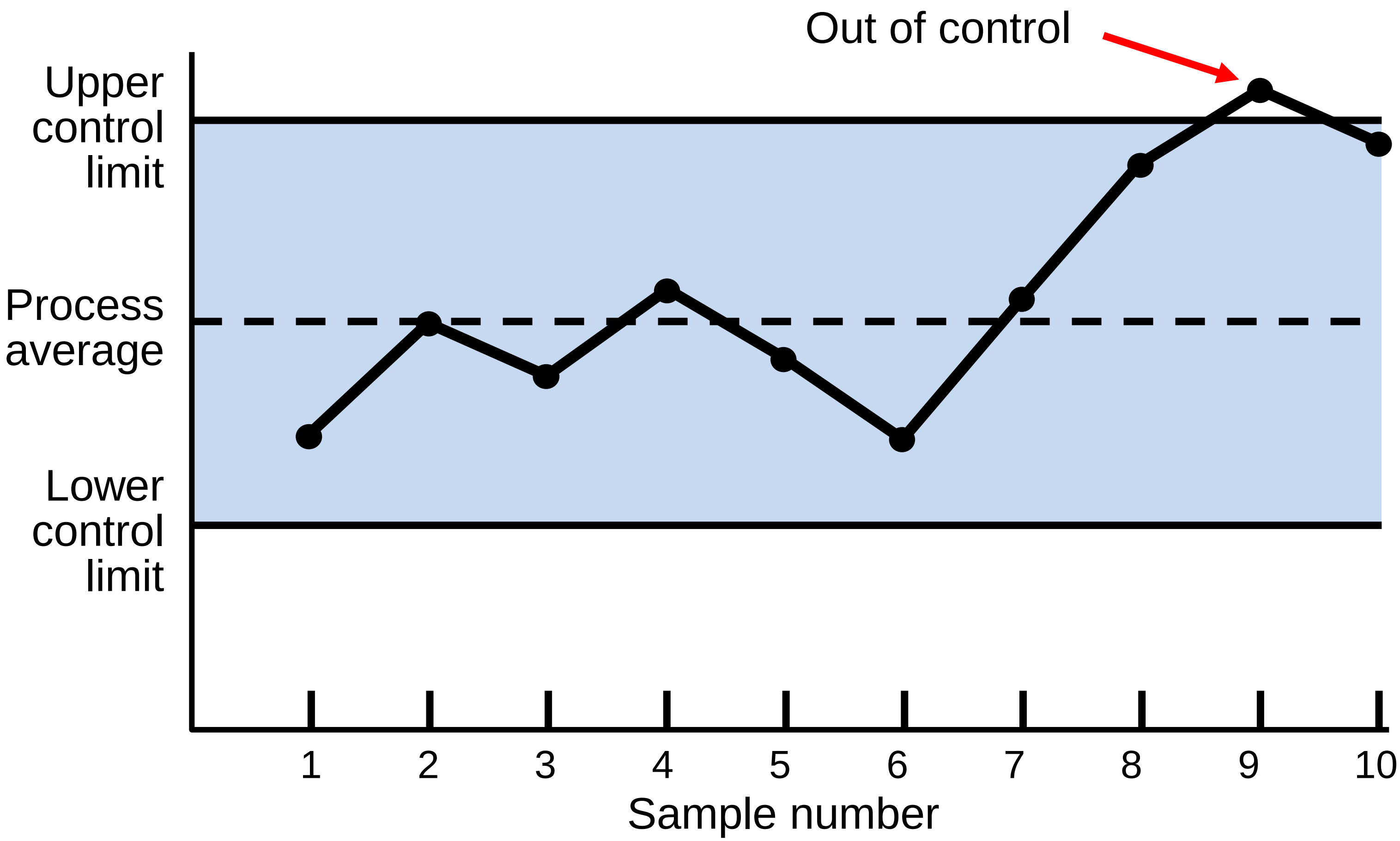
Where to Use Control Charts

- Process
  - Has a tendency to go out of control
  - Is particularly harmful and costly if it goes out of control
- Examples
  - At beginning of process because of waste to begin production process with bad supplies
  - Before a costly or irreversible point, after which product is difficult to rework or correct
  - Before and after assembly or painting operations that might cover defects
  - Before the outgoing final product or service is delivered

Control Charts

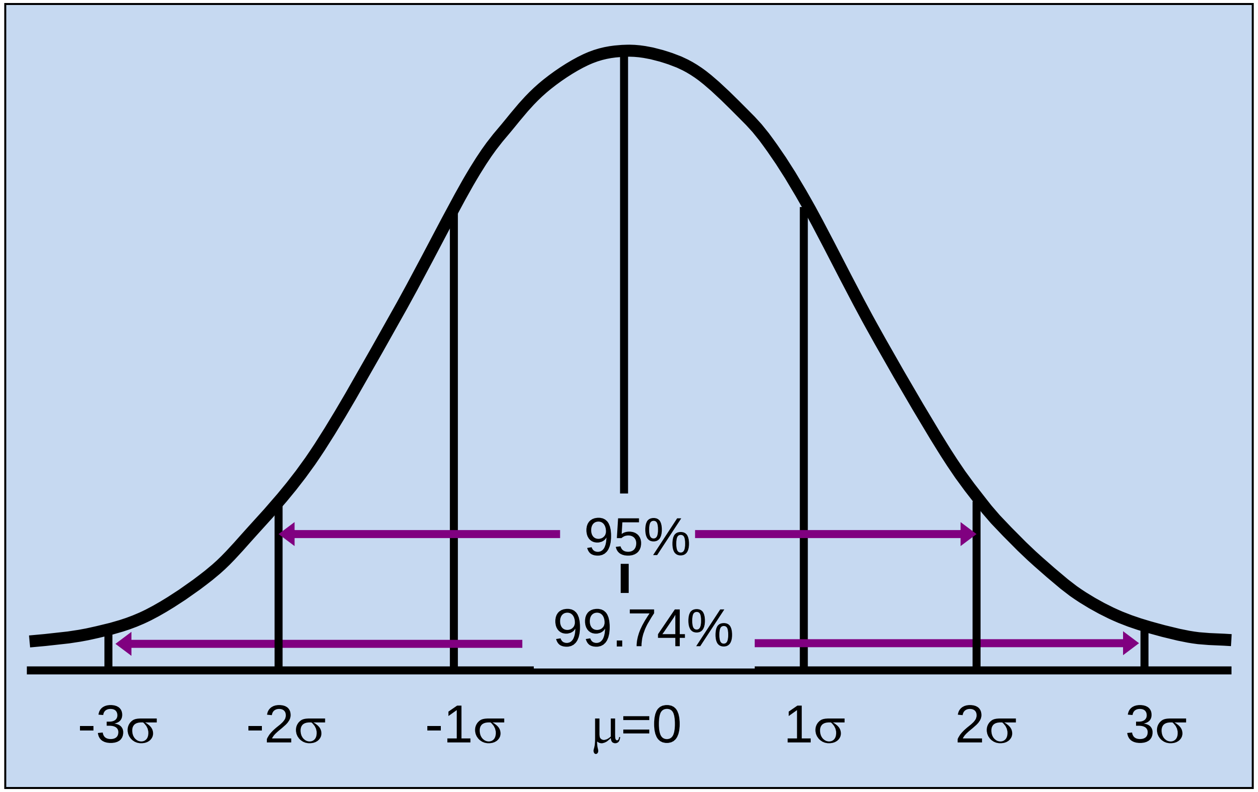
- A graph that monitors process quality
- Control limits
  - upper and lower bands of a control chart
- Attributes chart
  - p-chart
  - c-chart
- Variables chart
  - mean (x bar – chart)
  - range (R-chart)

Process Control Chart



Normal Distribution

- Probabilities for Z= 2.00 and Z = 3.00



A Process Is in Control If …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed

Control Charts for Attributes

- p-chart
  - uses portion defective in a sample

- c-chart
  - uses number of defects (non-conformities) in a sample

p-Chart

$$\mathrm{UCL} = \bar{p} + z\sigma_p$$

$$\mathrm{LCL} = \bar{p} - z\sigma_p$$

z = number of standard deviations from process average

$\bar{p}$ = sample proportion defective; estimates process mean

σ_p = standard deviation of sample proportion

$$\sigma_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Construction of p-Chart

SAMPLE #	NUMBER OF DEFECTIVES	PROPORTION DEFECTIVE
1	6	.06
2	0	.00
3	4	.04
:	:	:
:	:	:
20	18	.18
	200	

20 samples of 100 pairs of jeans

Construction of p-Chart

$$\bar{p} = \frac{\text{total defectives}}{\text{total sample observations}} =$$

$$\text{UCL} = \bar{p} + z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} =$$

UCL =

$$\text{LCL} = \bar{p} - z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} =$$

LCL =

Construction of p-Chart

$$\bar{p} = \frac{\text{total defectives}}{\text{total sample observations}} = 200 / 20(100) = 0.10$$

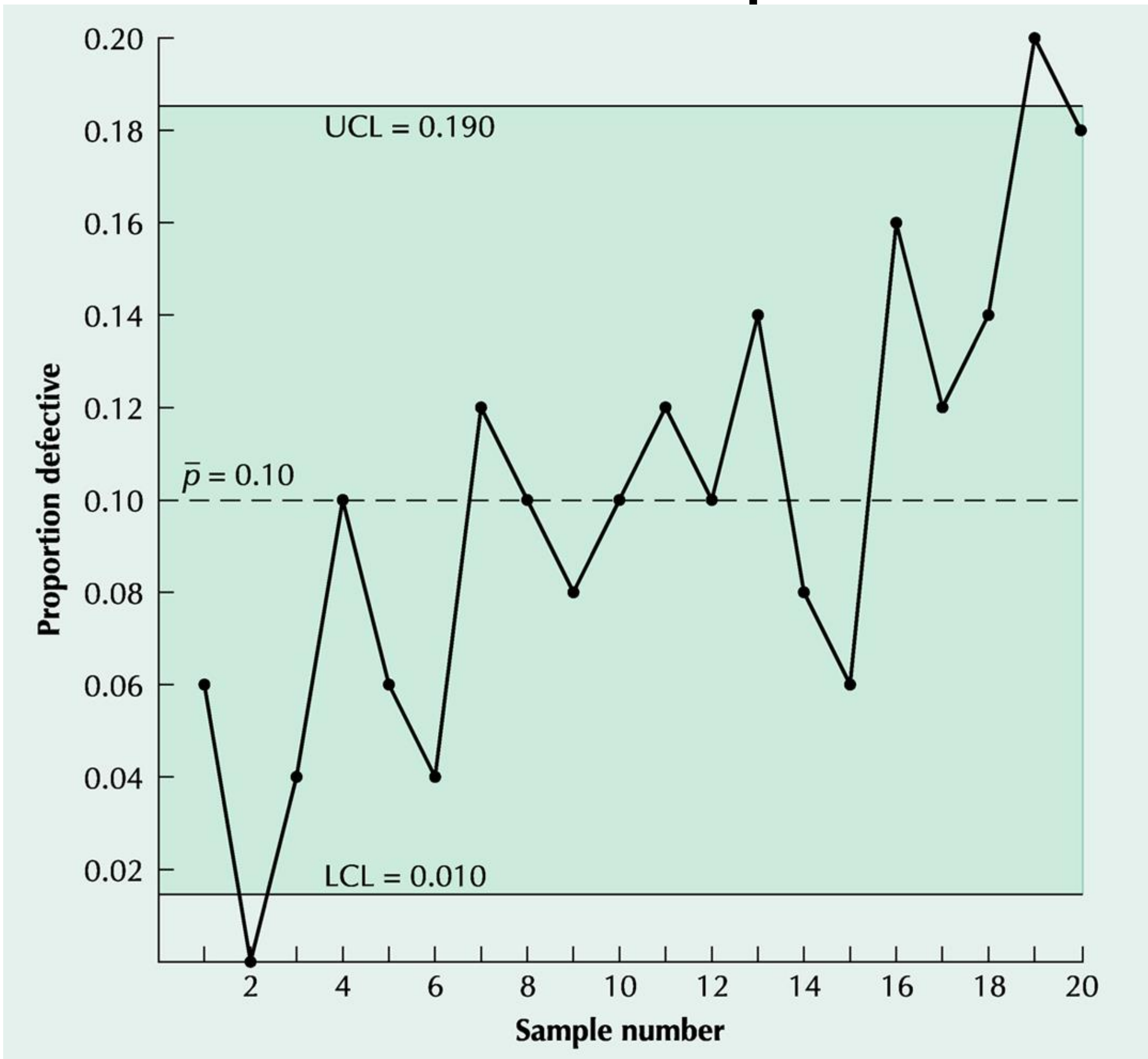
$$\text{UCL} = \bar{p} + z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 + 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

UCL = 0.190

$$\text{LCL} = \bar{p} - z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 - 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

LCL = 0.010

Construction of p-Chart



p-Chart in Excel

Click on "Insert" then "Charts" to construct control chart

Example 3.1. Construction of a *p*-Chart

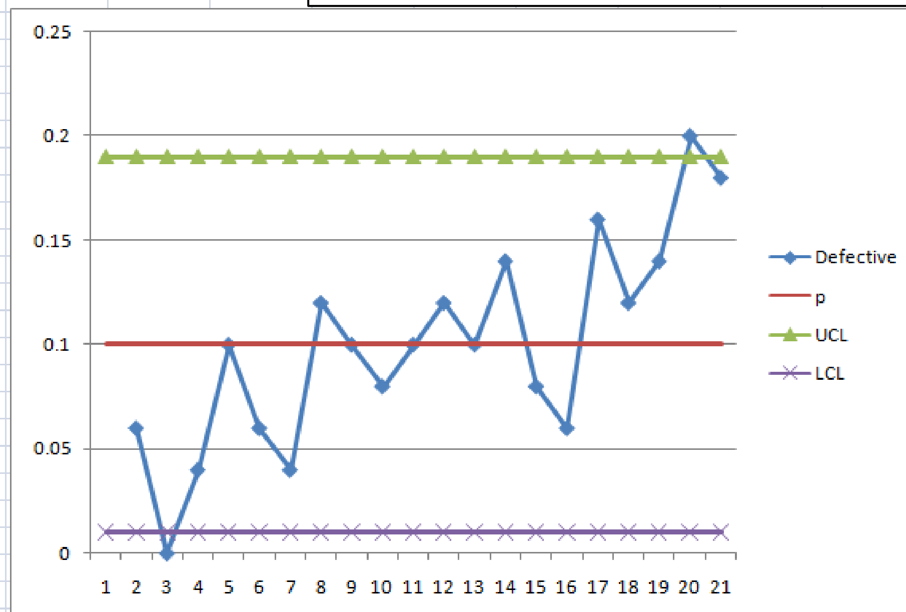
Sample	Proportion Defective	$\bar{p}$	UCL	LCL	Number of Defectives
0		0.10	0.19	0.01	
1	0.06	0.10	0.19	0.01	6
2	0.00	0.10	0.19	0.01	0
3	0.04	0.10	0.19	0.01	4
4	0.10	0.10	0.19	0.01	10
5	0.06	0.10	0.19	0.01	6
6	0.04	0.10	0.19	0.01	4
7	0.12	0.10	0.19	0.01	12
8	0.10	0.10	0.19	0.01	10
9	0.08	0.10	0.19	0.01	8
10	0.10	0.10	0.19	0.01	10
11	0.12	0.10	0.19	0.01	12
12	0.10	0.10	0.19	0.01	10
13	0.14	0.10	0.19	0.01	14
14	0.08	0.10	0.19	0.01	8
15	0.06	0.10	0.19	0.01	6
16	0.16	0.10	0.19	0.01	16
17	0.12	0.10	0.19	0.01	12
18	0.14	0.10	0.19	0.01	14
19	0.20	0.10	0.19	0.01	20
20	0.18	0.10	0.19	0.01	18
					200

Column values copied from I5 and I6

$\bar{p}$ =	0.10
UCL =	0.19
LCL =	0.01

I4 + 3*SQRT(I4*(1-I4)/100)

I4 - 3*SQRT(I4*(1-I4)/100)



c-Chart

$$\text{UCL} = \bar{c} + z\sigma_c$$

$$\text{LCL} = \bar{c} - z\sigma_c$$

$$\sigma_c = \sqrt{\bar{c}}$$

where

c = number of defects per sample

c-Chart

Number of defects in 15 sample rooms

SAMPLE	NUMBER OF DEFECTS
1	12
2	8
3	16
:	:
:	:
15	15
	190

$\bar{c} =$

$$UCL = c + z\sigma_c$$

$$LCL = c - z\sigma_c$$

c-Chart

Number of defects in 15 sample rooms

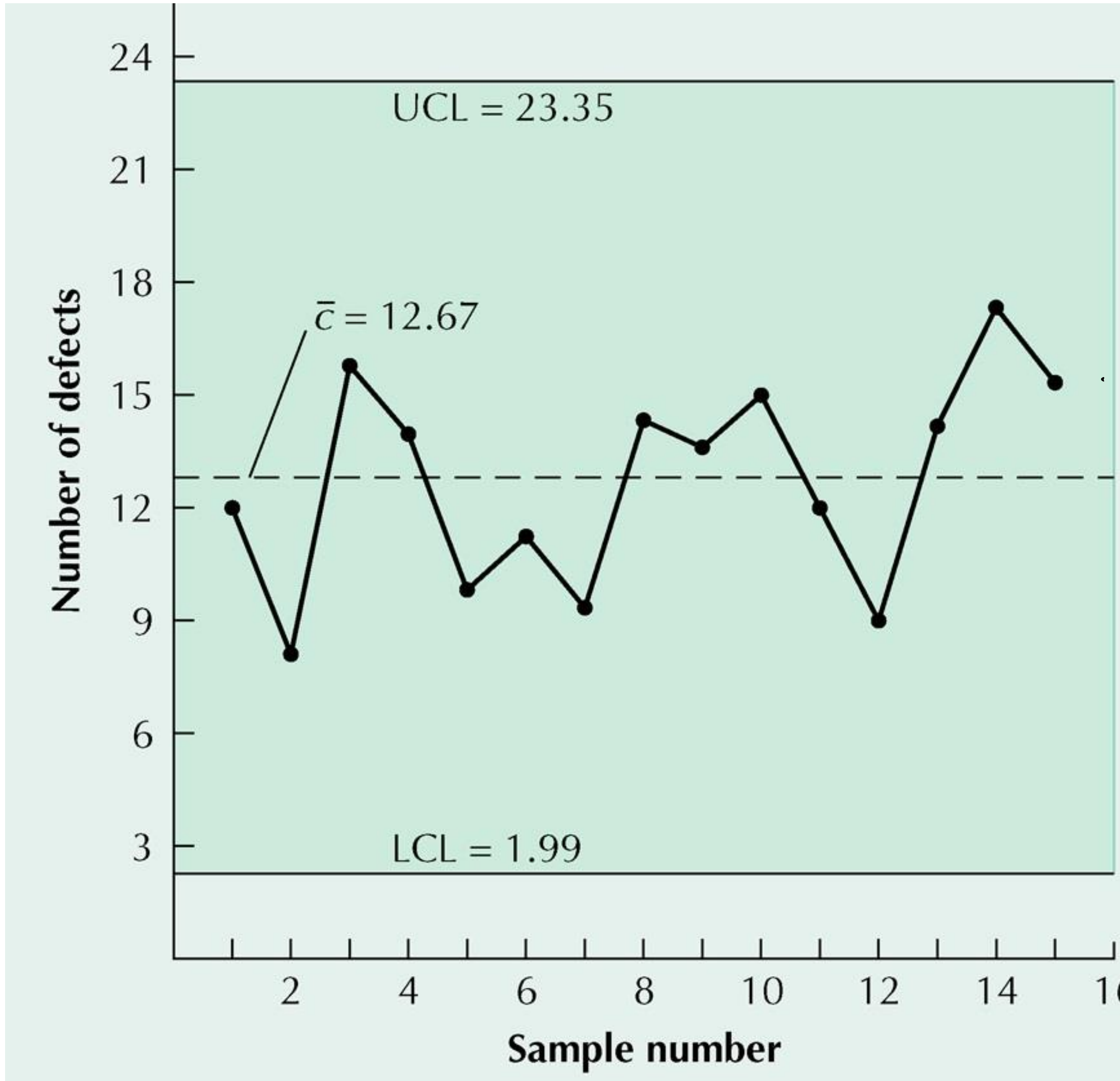
SAMPLE	NUMBER OF DEFECTS
1	12
2	8
3	16
:	:
:	:
15	15
	190

$$\bar{c} = \frac{190}{15} = 12.67$$

$$\begin{aligned} \text{UCL} &= \bar{c} + z\sigma_c \\ &= 12.67 + 3\sqrt{12.67} \\ &= 23.35 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{c} - z\sigma_c \\ &= 12.67 - 3\sqrt{12.67} \\ &= 1.99 \end{aligned}$$

c-Chart



Phase

Control Charts for Variables

- Range chart (R-Chart)
  - Plot sample range (variability)
- Mean chart ($\bar{x}$ -Chart)
  - Plot sample averages

x-bar Chart: σ Known

$$UCL = \bar{\bar{x}} + z\,\sigma_{\bar{x}}$$
$$LCL = \bar{\bar{x}} - z\,\sigma_{\bar{x}}$$

Where

$$\bar{\bar{X}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_k}{k}$$

σ = process standard deviation

σ_x = standard deviation of sample means $= \sigma/\sqrt{n}$

k = number of samples (subgroups)

n = sample size (number of observations)

x-bar Chart Example: σ Known

	Observations(Slip-Ring Diameter, cm) n					
Sample k	1	2	3	4	5	$\bar{x}$
1	5.02	5.01	4.94	4.99	4.96	4.98
2	5.01	5.03	5.07	4.95	4.96	5.00
3	4.99	5.00	4.93	4.92	4.99	4.97
4	5.03	4.91	5.01	4.98	4.89	4.96
5	4.95	4.92	5.03	5.05	5.01	4.99
6	4.97	5.06	5.06	4.96	5.03	5.01
7	5.05	5.01	5.10	4.96	4.99	5.02
8	5.09	5.10	5.00	4.99	5.08	5.05
9	5.14	5.10	4.99	5.08	5.09	5.08
10	5.01	4.98	5.08	5.07	4.99	5.03
						50.09

We know σ = .08

x-bar Chart Example: σ Known

$$\bar{\bar{X}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_k}{k}$$

$$\text{UCL} = \bar{\bar{x}} + z\,\sigma_{\bar{x}}$$

$$\text{LCL} = \bar{\bar{x}} - z\,\sigma_{\bar{x}}$$

x-bar Chart Example: σ Known

$$\bar{\bar{x}} = \frac{50.09}{10} = 5.01$$

$$\text{UCL} = \bar{\bar{x}} + z\,\sigma_{\bar{x}}$$
$$= 5.01 + 3(.08/\sqrt{5})$$
$$= 5.12$$

$$\text{LCL} = \bar{\bar{x}} - z\,\sigma_{\bar{x}}$$
$$= 5.01 - 3(.08/\sqrt{5})$$
$$= 4.90$$

x-bar Chart Example: σ Unknown

$$UCL = \bar{\bar{x}} + A_2\bar{R} \qquad LCL = \bar{\bar{x}} - A_2\bar{R}$$

where

$\bar{\bar{x}}$ = average of the sample means

$\bar{R}$ = average range value

Control Chart Factors

Sample Size	Factor for X-chart	Factors for R-chart	
n	A2	D3	D4
2	1.880	0.000	3.267
3	1.023	0.000	2.575
4	0.729	0.000	2.282
5	0.577	0.000	2.114
6	0.483	0.000	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.283	1.717
13	0.249	0.307	1.693
14	0.235	0.328	1.672
15	0.223	0.347	1.653
16	0.212	0.363	1.637
17	0.203	0.378	1.622
18	0.194	0.391	1.609
19	0.187	0.404	1.596
20	0.180	0.415	1.585
21	0.173	0.425	1.575
22	0.167	0.435	1.565
23	0.162	0.443	1.557
24	0.157	0.452	1.548
25	0.153	0.459	1.541

x-bar Chart Example: σ Unknown

SAMPLE *k*	OBSERVATIONS (SLIP- RING DIAMETER, CM) 1	2	3	4	5	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
					Totals	50.09	1.15

Control Limits for the $\bar{x}$ Chart

$$\mathrm{UCL} = \bar{\bar{x}} + A_2\bar{R}$$

$$\text{Center line} = \bar{\bar{x}} \tag{6.4}$$

$$\mathrm{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

The constant A_2 is tabulated for various sample sizes in Appendix Table VI.

Control Limits for the R Chart

$$\mathrm{UCL} = D_4\bar{R}$$

$$\text{Center line} = \bar{R} \tag{6.5}$$

$$\mathrm{LCL} = D_3\bar{R}$$

The constants D_3 and D_4 are tabulated for various values of n in Appendix Table VI.

$$\hat{\sigma} = \frac{\overline{R}}{d_2} \tag{6.6}$$

If we use $\overline{\overline{x}}$ as an estimator of μ and $\overline{R}/d_2$ as an estimator of σ, then the parameters of the $\overline{x}$ chart are

$$\begin{aligned} \text{UCL} &= \overline{\overline{x}} + \frac{3}{d_2\sqrt{n}}\overline{R} \\ \text{Center line} &= \overline{\overline{x}} \\ \text{LCL} &= \overline{\overline{x}} - \frac{3}{d_2\sqrt{n}}\overline{R} \end{aligned} \tag{6.7}$$

If we define

$$A_2 = \frac{3}{d_2\sqrt{n}} \tag{6.8}$$

then equation (6.7) reduces to equation (6.4).

x-bar Chart Example: σ Unknown

$$\bar{R} = \frac{\sum R}{k}$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{k}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

x-bar Chart Example: σ Unknown

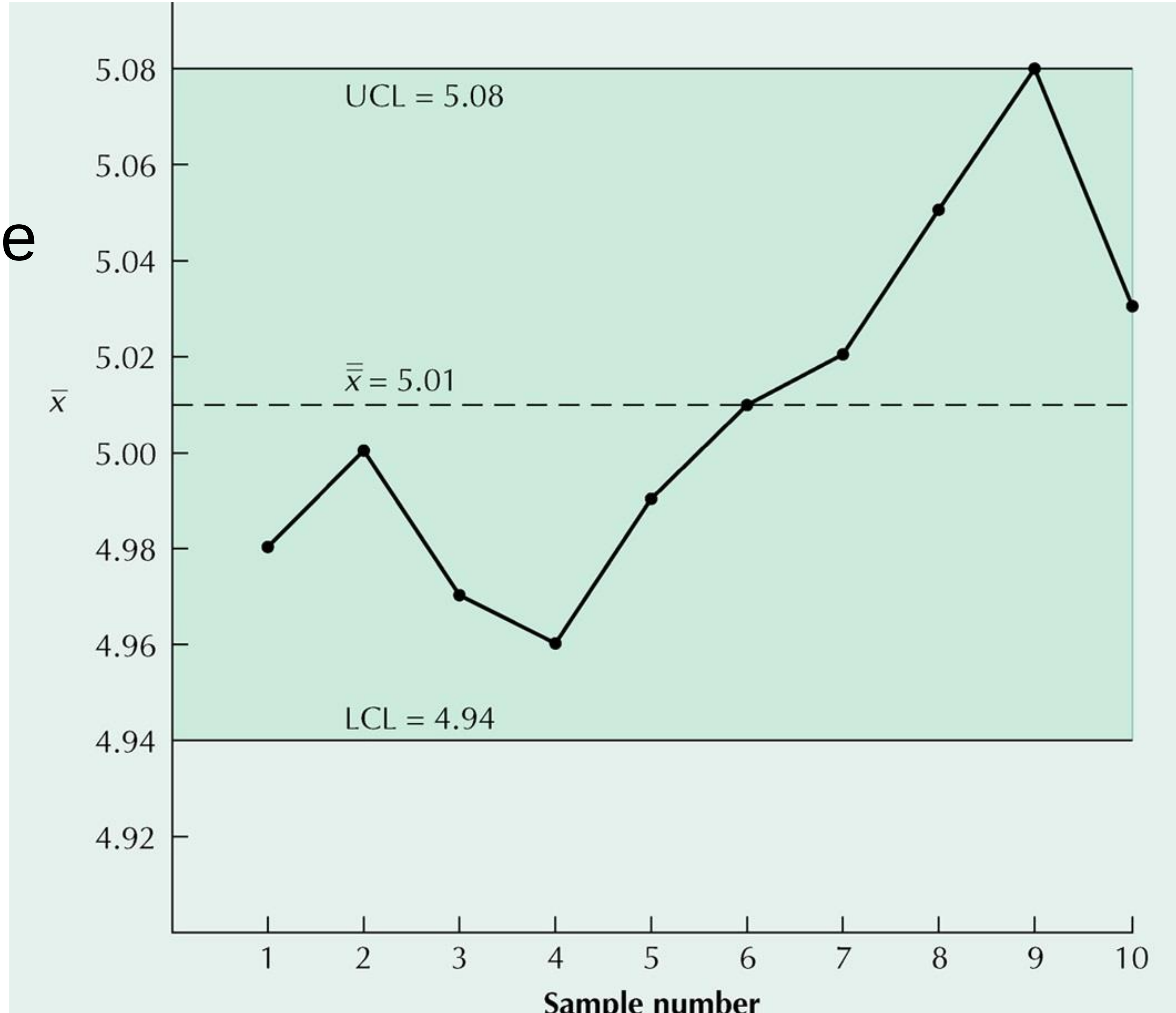
$$\bar{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 5.01 + (0.58)(0.115) = 5.08$$

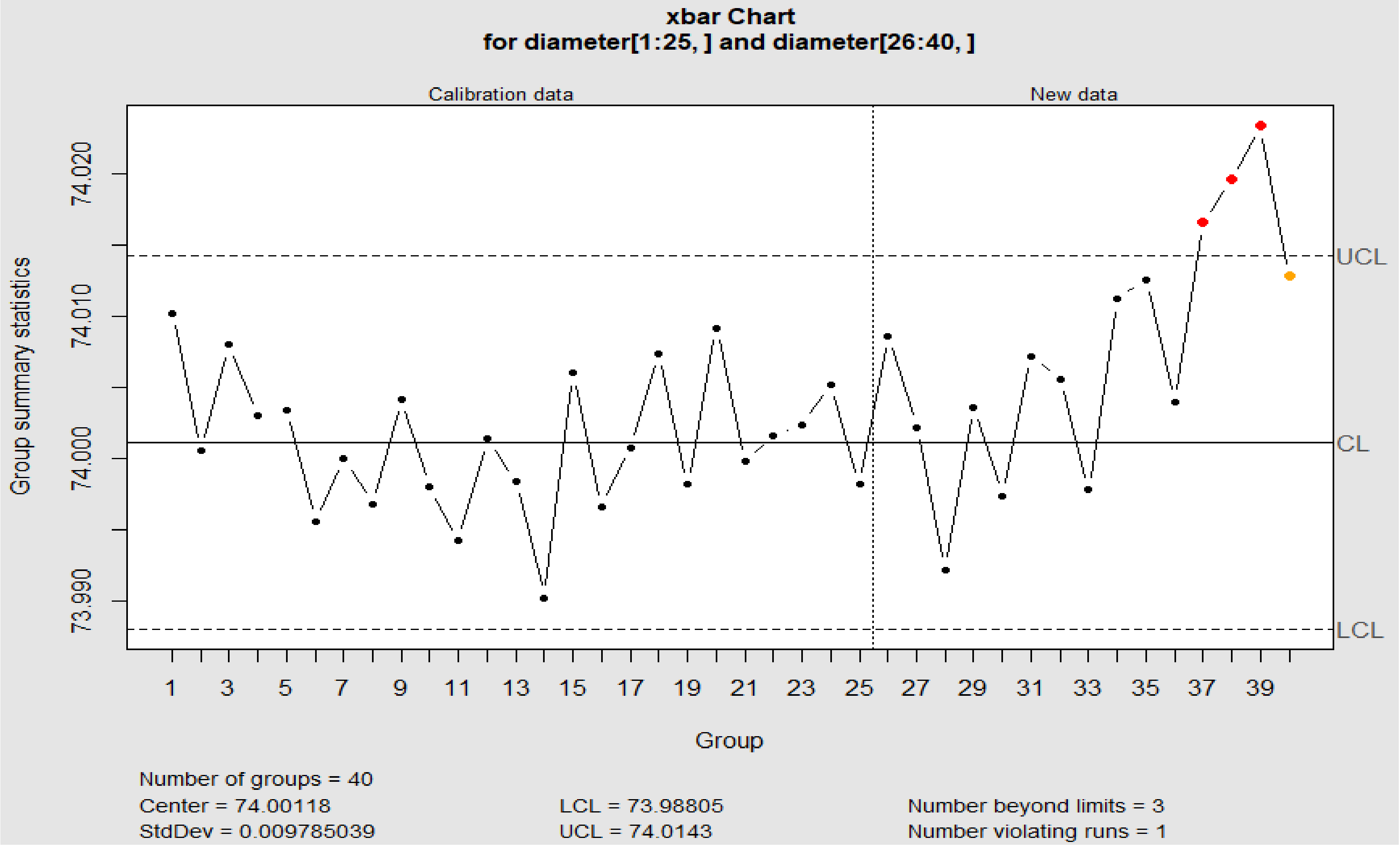
$$LCL = \bar{\bar{x}} - A_2\bar{R} = 5.01 - (0.58)(0.115) = 4.94$$

x- bar Chart Example



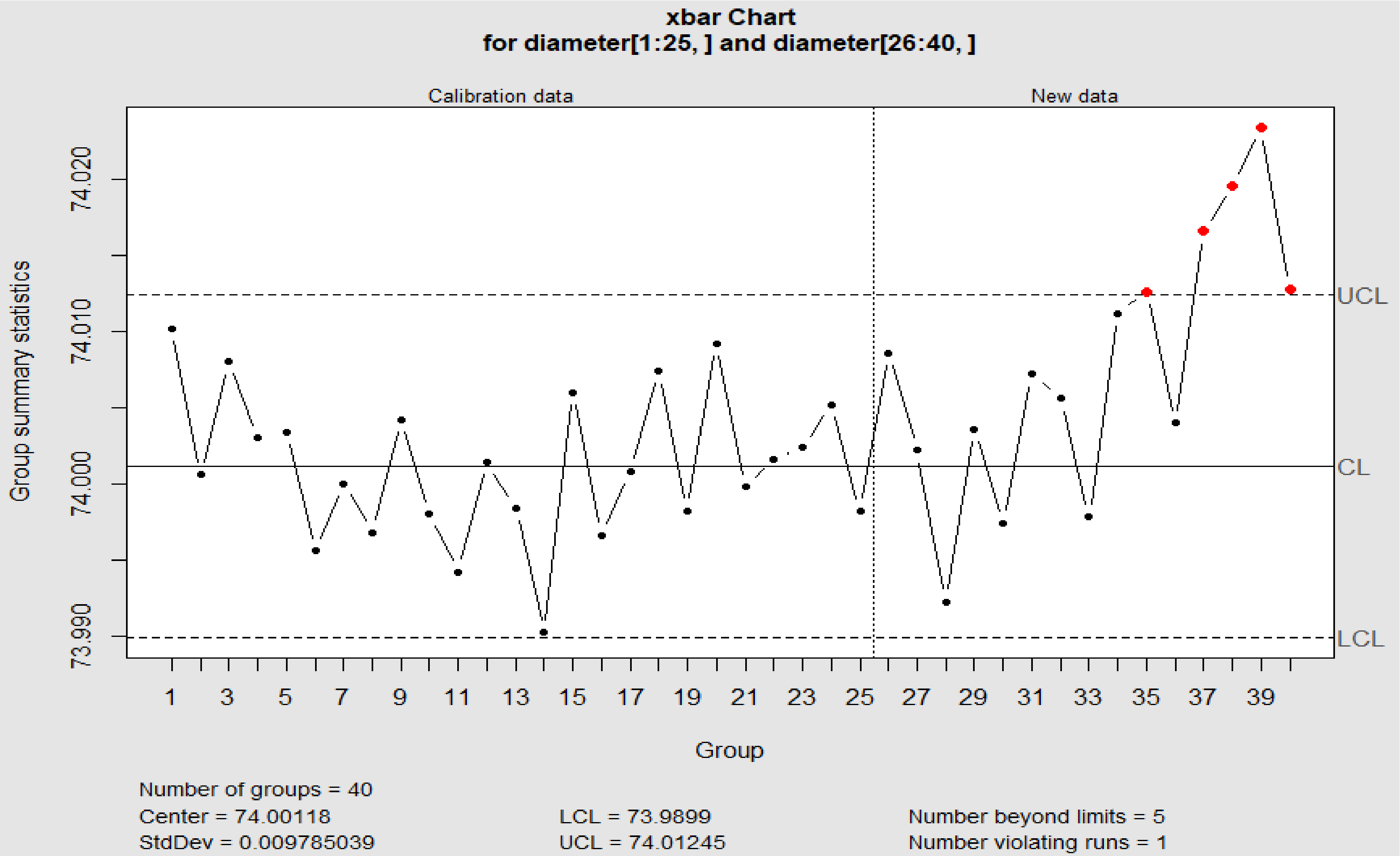
Piston rings

Control Limits = 3 sigma



Piston rings

Control limit .99 Confidence level



R- Chart

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R}$$

$$\bar{R} = \frac{\sum R}{k}$$

Where

R = range of each sample

k = number of samples (sub groups)

Control Limits for the $\bar{x}$ Chart

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$\text{Center line} = \bar{\bar{x}} \qquad (6.4)$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

The constant A_2 is tabulated for various sample sizes in Appendix Table VI.

Control Limits for the *R* Chart

$$\text{UCL} = D_4\bar{R}$$
$$\text{Center line} = \bar{R} \qquad (6.5)$$
$$\text{LCL} = D_3\bar{R}$$

The constants D_3 and D_4 are tabulated for various values of n in Appendix Table VI.

Now consider the R chart. The center line will be $\overline{R}$. To determine the control limits, we need an estimate of σ_R. Assuming that the quality characteristic is normally distributed, $\hat{\sigma}_R$ can be found from the distribution of the relative range $W = R/\sigma$. The standard deviation of W, say d_3, is a known function of n. Thus, since

$$R = W\sigma$$

the standard deviation of R is

$$\sigma_R = d_3\sigma$$

Since σ is unknown, we may estimate σ_R by

$$\hat{\sigma}_R = d_3 \frac{\overline{R}}{d_2} \tag{6.9}$$

Consequently, the parameters of the R chart with the usual three-sigma control limits are

$$\begin{aligned} \text{UCL} &= \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3d_3 \frac{\overline{R}}{d_2} \\ \text{Center line} &= \overline{R} \\ \text{LCL} &= \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3d_3 \frac{\overline{R}}{d_2} \end{aligned} \tag{6.10}$$

If we let

$$D_3 = 1 - 3\frac{d_3}{d_2} \quad \text{and} \quad D_4 = 1 + 3\frac{d_3}{d_2}$$

equation (6.10) reduces to equation (6.5).

R-Chart Example

	OBSERVATIONS (SLIP- RING DIAMETER, CM)						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
					Totals	50.09	1.15

R-Chart Example

$UCL = D_4\bar{R} =$

$LCL = D_3\bar{R} =$

Retrieve chart factors D_3 and D_4

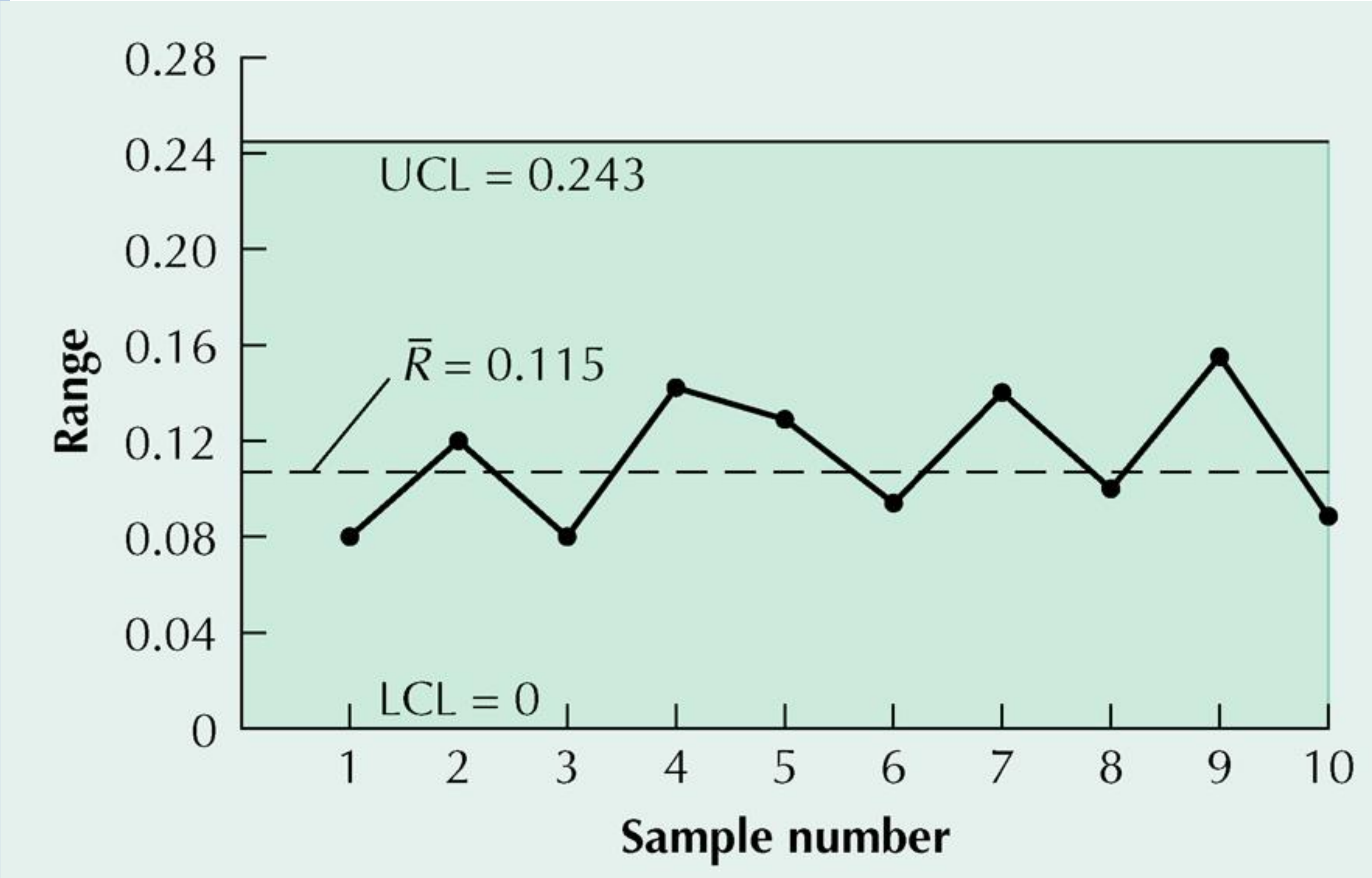
R-Chart Example

$$UCL = D_4\bar{R} = 2.11(0.115) = 0.243$$

$$LCL = D_3\bar{R} = 0(0.115) = 0$$

Retrieve chart factors D_3 and D_4

R-Chart Example



Using x- bar and R-Charts Together

- Process average and process variability must be in control
- Samples can have very narrow ranges, but sample averages might be beyond control limits
- Or, sample averages may be in control, but ranges might be out of control
- An R-chart might show a distinct downward trend, suggesting some nonrandom cause is reducing variation

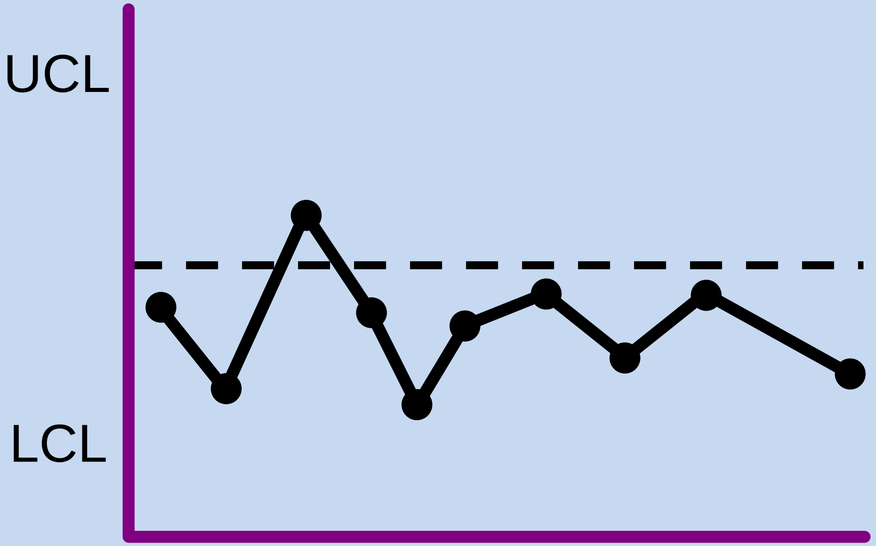
Control Chart Patterns

- Run
  - sequence of sample values that display same characteristic
- Pattern test
  - determines if observations within limits of a control chart display a nonrandom pattern

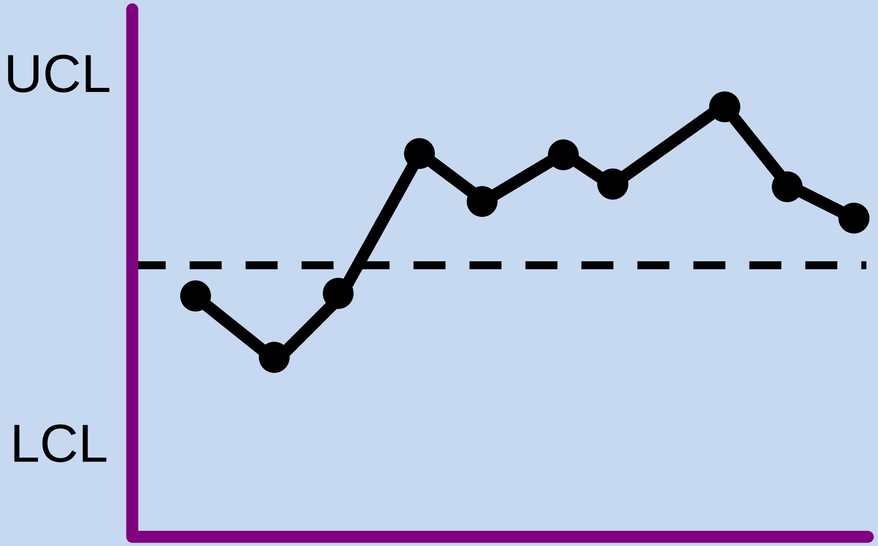
Control Chart Patterns

- To identify a pattern look for:
  - 8 consecutive points on one side of the center line
  - 8 consecutive points up or down
  - 14 points alternating up or down
  - 2 out of 3 consecutive points in zone A (on one side of center line)
  - 4 out of 5 consecutive points in zone A or B (on one side of center line)

Control Chart Patterns

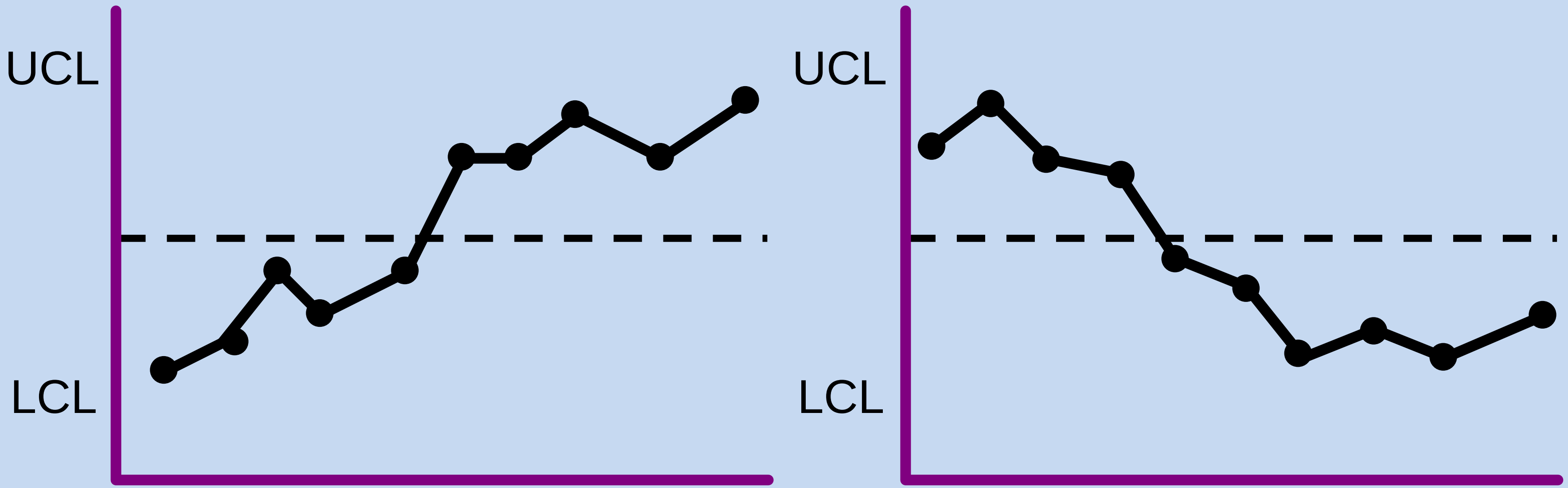


Sample observations consistently below the center line



Sample observations consistently above the center line

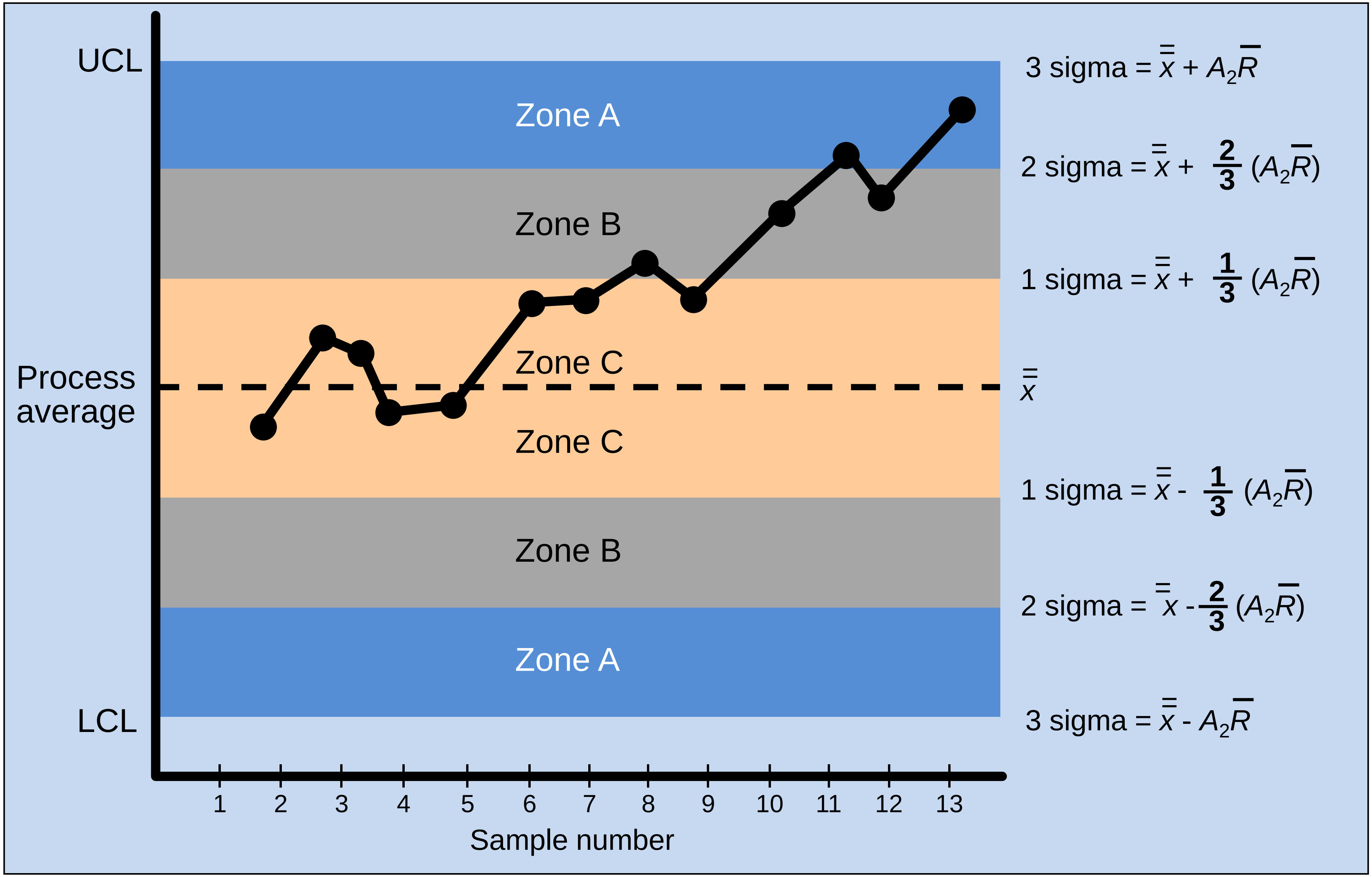
Control Chart Patterns



Sample observations consistently increasing

Sample observations consistently decreasing

Zones for Pattern Tests



Performing a Pattern Test

SAMPLE	$\bar{x}$	ABOVE/BELOW	UP/DOWN	ZONE
1	4.98	B	—	B
2	5.00	B	U	C
3	4.95	B	D	A
4	4.96	B	D	A
5	4.99	B	U	C
6	5.01	—	U	C
7	5.02	A	U	C
8	5.05	A	U	B
9	5.08	A	U	A
10	5.03	A	D	B

Sample Size Determination

- Attribute charts require larger sample sizes
  - 50 to 100 parts in a sample

- Variable charts require smaller samples
  - 2 to 10 parts in a sample

SPC with Excel

Click on “Insert” then “Line” to construct control chart

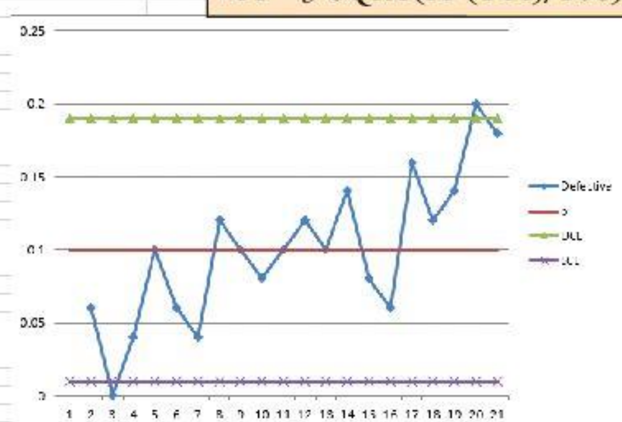
Example 3.1. Construction of a *p*-Chart

Sample	Proportion Defective	$\overline{p}$	UCL	LCL	Number of Defectives
0		0.10	0.19	0.01	
1	0.06	0.10	0.19	0.01	6
2	0.00	0.10	0.19	0.01	0
3	0.04	0.10	0.19	0.01	4
4	0.10	0.10	0.19	0.01	10
5	0.06	0.10	0.19	0.01	6
6	0.04	0.10	0.19	0.01	4
7	0.12	0.10	0.19	0.01	12
8	0.10	0.10	0.19	0.01	10
9	0.08	0.10	0.19	0.01	8
10	0.10	0.10	0.19	0.01	10
11	0.12	0.10	0.19	0.01	12
12	0.10	0.10	0.19	0.01	10
13	0.14	0.10	0.19	0.01	14
14	0.08	0.10	0.19	0.01	8
15	0.06	0.10	0.19	0.01	6
16	0.16	0.10	0.19	0.01	16
17	0.12	0.10	0.19	0.01	12
18	0.14	0.10	0.19	0.01	14
19	0.20	0.10	0.19	0.01	20
20	0.18	0.10	0.19	0.01	18
					200

$\overline{p}$ =	0.10
UCL =	0.19
LCL	0.01

=I4 + 3*SQRT(I4*(1-I4)/100)

=I4 – 3*SQRT(I4*(1-I4)/100)



Process Capability Analysis

Prof. Tarun Sharma

Process Capability

Process variability

Design specifications

- Are they aligned with each other?

Definitions

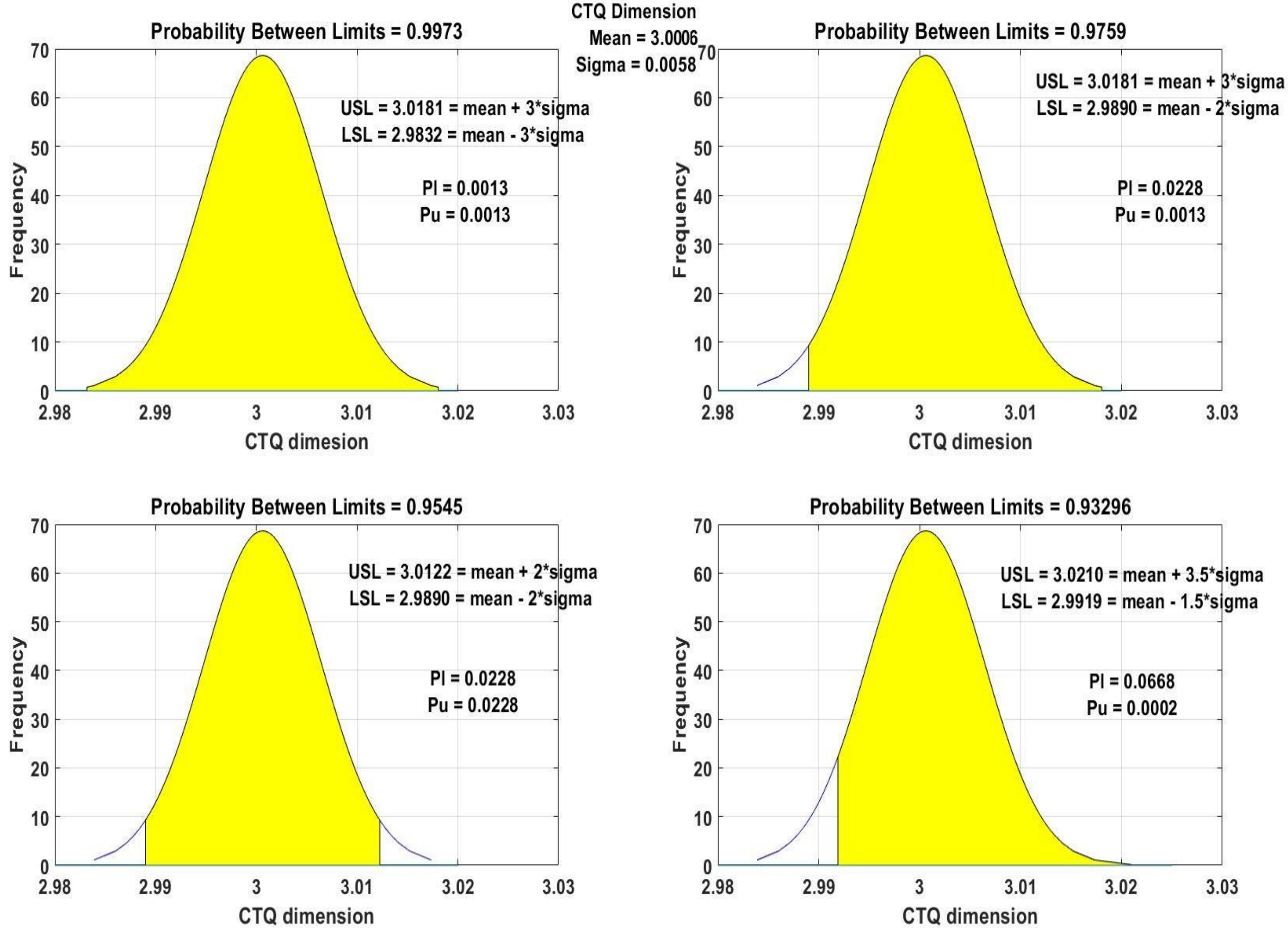
- Process variability
  - Process Range = 6*sigma

  Where sigma is the standard deviation of the CTQ dimension

- Design Specifications
  - Upper Specification Limit (USL)
  - Lower Specification Limit (LSL)
  - Tolerance Range = USL - LSL

Definitions

- Process fallout
  - Fraction of the process output which does not meet the specification, i.e., the CTQ dimension is outside of the design specifications

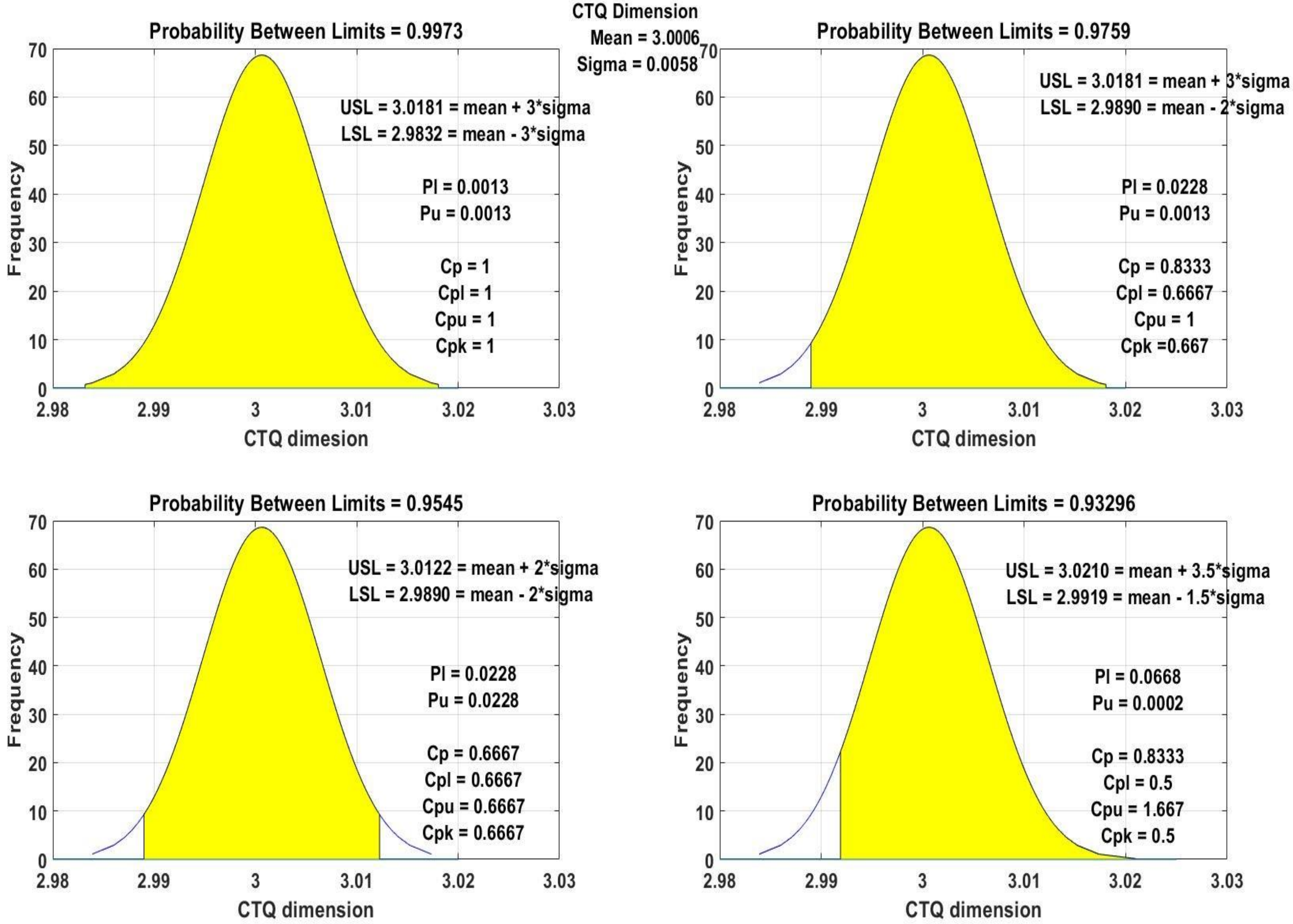


CTQ Dimension
Mean = 3.0006
Sigma = 0.0058
Probability Between Limits = 0.9973
USL = 3.0181 = mean + 3*sigma
LSL = 2.9832 = mean - 3*sigma
Pl = 0.0013
Pu = 0.0013
Frequency
CTQ dimesion
Probability Between Limits = 0.9759
USL = 3.0181 = mean + 3*sigma
LSL = 2.9890 = mean - 2*sigma
Pl = 0.0228
Pu = 0.0013
Frequency
CTQ dimesion
Probability Between Limits = 0.9545
USL = 3.0122 = mean + 2*sigma
LSL = 2.9890 = mean - 2*sigma
Pl = 0.0228
Pu = 0.0228
Frequency
CTQ dimension
Probability Between Limits = 0.93296
USL = 3.0210 = mean + 3.5*sigma
LSL = 2.9919 = mean - 1.5*sigma
Pl = 0.0668
Pu = 0.0002
Frequency
CTQ dimension

Numeric measure of Process Capability

- Process Capability Ratio (Cp) = Tolerance Range/ Process variation

- Process Capability Index (Cpk) = Min{(Mean – LSL)/(3*sigma), (USL-Mean)/(3*sigma)}

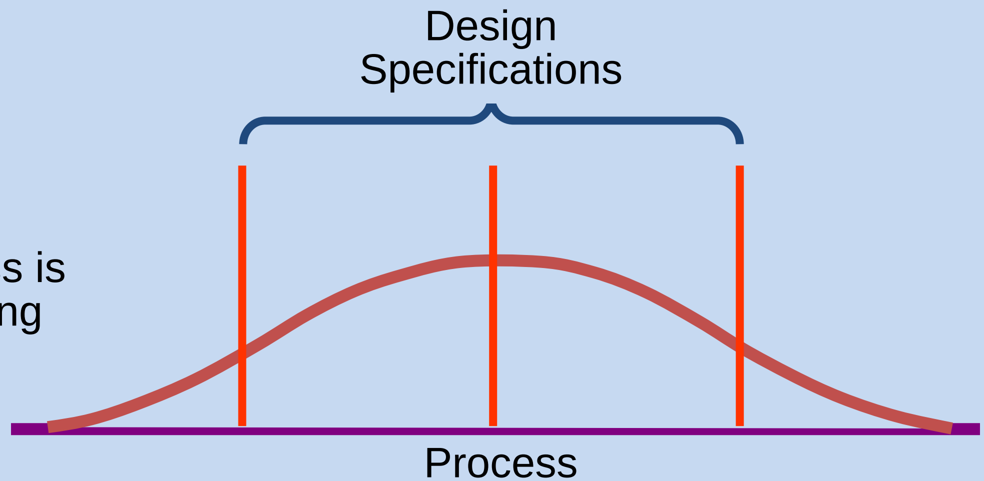

CTQ Dimension
Mean = 3.0006
Sigma = 0.0058
Probability Between Limits = 0.9973
USL = 3.0181 = mean + 3*sigma
LSL = 2.9832 = mean - 3*sigma
Pl = 0.0013
Pu = 0.0013
Cp = 1
Cpl = 1
Cpu = 1
Cpk = 1
Frequency
CTQ dimesion
Probability Between Limits = 0.9759
USL = 3.0181 = mean + 3*sigma
LSL = 2.9890 = mean - 2*sigma
Pl = 0.0228
Pu = 0.0013
Cp = 0.8333
Cpl = 0.6667
Cpu = 1
Cpk =0.667
Frequency
CTQ dimesion
Probability Between Limits = 0.9545
USL = 3.0122 = mean + 2*sigma
LSL = 2.9890 = mean - 2*sigma
Pl = 0.0228
Pu = 0.0228
Cp = 0.6667
Cpl = 0.6667
Cpu = 0.6667
Cpk = 0.6667
Frequency
CTQ dimension
Probability Between Limits = 0.93296
USL = 3.0210 = mean + 3.5*sigma
LSL = 2.9919 = mean - 1.5*sigma
Pl = 0.0668
Pu = 0.0002
Cp = 0.8333
Cpl = 0.5
Cpu = 1.667
Cpk = 0.5
Frequency
CTQ dimension

Process Capability

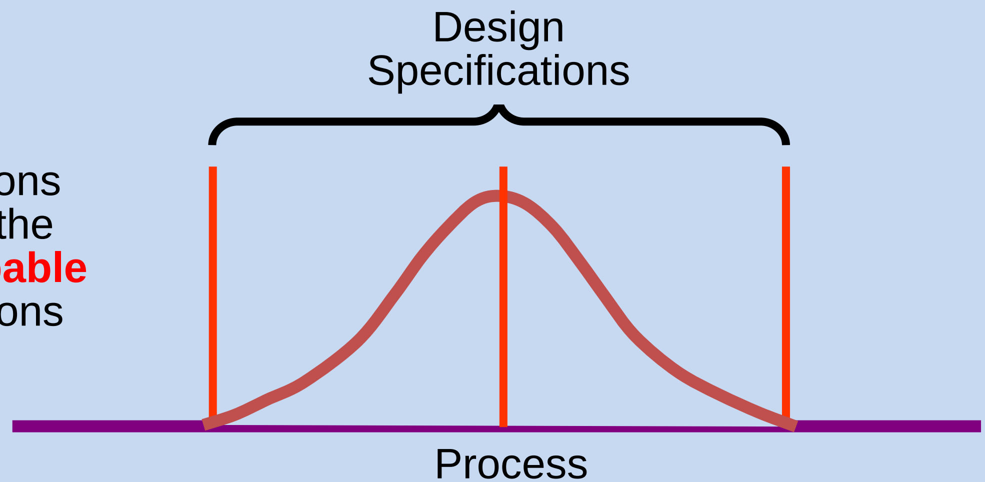
- Compare natural variability to design variability
- Natural variability
  - What we measure with control charts
  - Process mean = 8.80 oz, Std dev. = 0.12 oz
- Tolerances
  - Design specifications reflecting product requirements
  - Net weight = 9.0 oz $\pm$ 0.5 oz
  - Tolerances are $\pm$ 0.5 oz

Process Capability

(a) Natural variation exceeds design specifications; process is **not capable** of meeting specifications all the time.

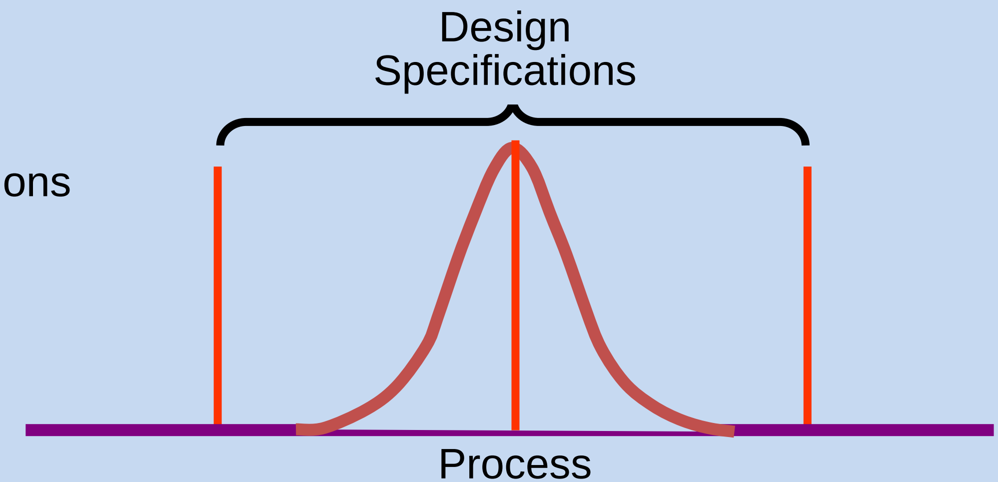


(b) Design specifications and natural variation the same; process is **capable** of meeting specifications most of the time.

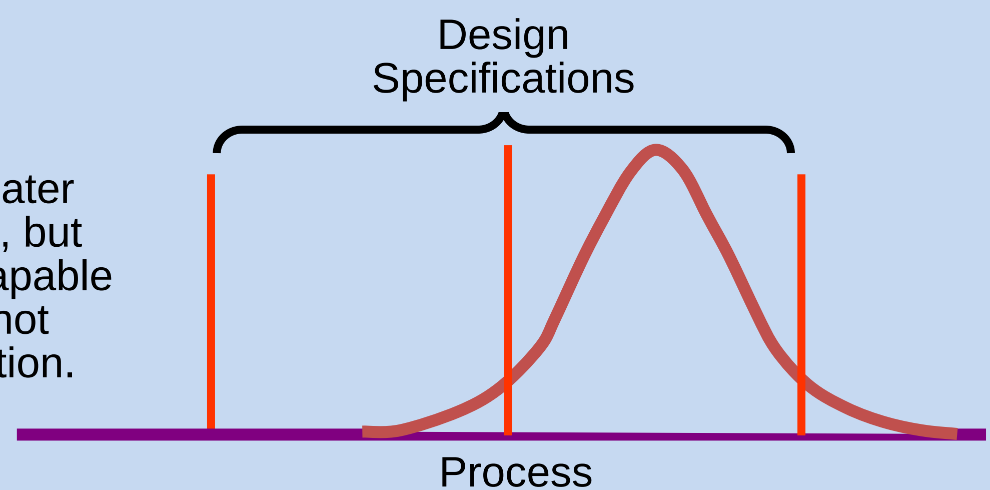


Process Capability

(c) Design specifications greater than natural variation; process is capable of always conforming to specifications.



(d) Specifications greater than natural variation, but process off center; capable but some output will not meet upper specification.



Process Capability Ratio

$$C_p = \frac{\text{tolerance range}}{\text{process range}}$$

$$= \frac{\text{upper spec limit - lower spec limit}}{6\sigma}$$

Computing C_p

Net weight specification = 9.0 oz $\pm$ 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_p = \frac{\text{upper specification limit - lower specification limit}}{6\sigma}$$

Computing C_p

Net weight specification = 9.0 oz $\pm$ 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_p = \frac{\text{upper specification limit - lower specification limit}}{6\sigma}$$

$$= \frac{9.5 - 8.5}{6(0.12)} = 1.39$$

Process Capability Index

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma},\ \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

Computing C_{pk}

Net weight specification = 9.0 oz $\pm$ 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

Computing C_{pk}

Net weight specification = 9.0 oz $\pm$ 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

$$= \text{minimum}\left[\frac{8.80 - 8.50}{3(0.12)}, \frac{9.50 - 8.80}{3(0.12)}\right] = 0.83$$

Impact of Process Capability Studies on management decision problems

- Make or buy decision
- Plant and process improvements to reduce process variability
- Contractual agreements with customers or vendors regarding product quality

Process Capability With Excel