

End Term Exam-IBM 311 (Spring 2022)

Duration: 3 hours from 2:15 pm to 5:15 pm.

There are 8 sections in the exam.

Sections 1 to 6 are objective, has 41 questions of one mark each with one third negative marking for wrong answer. Unanswered questions will not be marked. Maximum marks in these 6 objective sections are 41.

Section 7 has 7 subjective questions of 5 marks each with word limit of 300 words. Max marks in this section are 35.

Section 8 has 6 long answer type questions of 7 marks each. Max marks in this section 42.

Attempt all questions in a section together.

Maximum Marks: 118.

The 41 objective questions are organized as follows:

Section 1 is General and has 15 questions.

Section 2 is Out of control detection has 7 questions.

Section 3 is Forecasting and has 4 questions.

Section 4 is Service level 1 and has 3 questions.

Section 5 is Inventory and has 5 questions.

Section 6 is Aggregate production planning in goods and has 7 questions.

Section 1: General

1. Which of the following relationships must always be correct?
 - (a) Tolerances > process variability > control limits
 - (b) Process variability > tolerances > control limits
 - ☒ (c) Tolerances > control limits > process variability
 - (d) Process variability > control limits > tolerances
 - (e) Process variability < tolerances < control limits
2. Studies on a bottle-filling machine indicate that it fills bottles to a mean of 16 ounces with a standard deviation of 0.10 ounces. What is the process specification, assuming the C_{pk} index of 1?
 - (a) lower spec = 0.1 ounces and upper spec = 1 ounce
 - (b) lower spec = 8.7 ounces and upper spec = 12.5 ounces
 - (c) lower spec = 12.0 ounces and upper spec = 21.4 ounces
 - ☒ (d) lower spec = 15.7 ounces and upper spec = 16.3 ounces
 - (e) lower spec = 14.3 ounces and upper spec = 17.7 ounces
3. Which of the following is not a characteristic of an effective product design process?
 - i) matches product characteristics with customer requirements
 - ii) maximizes the revisions necessary to make a design workable
 - iii) ensures that customer requirements are met in the least costly and simplest manner
 - iv) reduces the time required to design a new product or service

(a) only i ☒ (b) only ii (c) only ii and iii ☒ (d) only ii and iv

4. Consider an espresso stand with a single barista. Customers arrive at the stand at the rate of 28 per hour according to a Poisson distribution. Service times are exponentially distributed with a service rate of 35 customers per hour. The probability that the server is busy is 28/35
- a) 0.20 b) 0.40 c) 0.60 ☒ d) 0.80
5. Consider an espresso stand with a single barista. Customers arrive at the stand at the rate of 28 per hour according to a Poisson distribution. Service times are exponentially distributed with a service rate of 35 customers per hour. The probability that there are exactly 3 customers in the system is $(28/35)^3 (1 - (28/35))$
- a) 0 ☒ b) 0.1024 c) 0.4096 d) 0.5120
6. Consider an espresso stand with a single barista. Customers arrive at the stand at the rate of 28 per hour according to a Poisson distribution. Service times are exponentially distributed with a service rate of 35 customers per hour. The average time in minutes a customer spends waiting in line for service is $1 / (35 - 28)$
- a) 0.114 minutes ☒ b) 0.143 minutes c) 6.84 minutes d) 8.58 minutes
7. An electronic component in an airborne radar system has a useful life described by an exponential distribution with failure rate $10^{-4}/h$. The mean time to failure for this component isand the probability that this component will not fail before its expected life would be
- a) 10,000 hours; 0.5 b) 5,000 hours; 0.33 c) 10,000 hours; 0.63 ☒ d) 10,000 hours; 0.37
8. The goods-services continuum consists of which set of the following categories?
- ☒ a) No goods, some goods, even mix, some service, no service
☒ b) Pure goods, core goods, core services, pure services
c) No service, some service, good service, excellent service
d) Self-service, help desk service, face-to-face service, service-with-a-smile
e) None of these b
9. The correct order in the good-services continuum is
- a) pure services - core services - core goods - pure goods
b) pure goods - pure services - core services - core goods
c) pure goods - pure services - core goods - core services d
☒ d) pure goods - core goods - core services - pure services
e) core goods - core services - pure goods - pure services
10. The customer inter-arrival time at a service desk is exponentially distributed with a mean value of $1/\lambda$. Which of the following statements are true:
- ☒ a. Approximately 63% of the customers will have an arrival time less than ' $1/\lambda$ '.
b. Approximately 50% of the customers will have an arrival time of less than ' $1/\lambda$ '.
c. The percentage of customers having arrival time less than equal to mean value of inter-arrival time depends on the mean value of inter-arrival time.
☒ d. The percentage of customers having arrival time less than equal to the mean value of inter-arrival time does not depend on the mean value of the inter-arrival time.
e. Approximately 37% of the customers will have an arrival time of less than ' $1/\lambda$ '
- a) Only b) and c) are correct ☒ b) Only a) and c) are correct
c) Only e) and d) are correct ☒ d) Only a) and d) are correct
11. Which of the following statements about Deming's Red Bead experiment are true:
- ☒ a) Common causes can only be addressed by management action.

- b) Common causes can only be addressed by worker action.
 c) Rewards and firing decisions should be based on short term performance.
 d) Short term variation in performance could be because of the underlying randomness in the process.
 (a) a & c ☒ (b) a & d (c) a & b (d) c & d (e) c & b

12. XYZ manufacturing has received an order to produce a rod 5 inches in diameter $\pm .04$ inch. In sample runs, the machine tool that will be making the rod has been able to produce rods with a mean diameter of 4.99 inches and a standard deviation of 0.011 inch. Which of the following statements is true?

1. The process is capable of meeting design spec but is off-center.
 2. The process is capable of meeting design specs and is on-center.
 3. The process is incapable of meeting design specs.
☒ a) Only 1 b) Only 2 c) Only 3 d) Only 1 & 2
 e) Only 2 & 3 f) Only 1 & 3 g) All of them

13. Consider an espresso stand with a single barista. Customers arrive at the stand at the rate of 28 per hour according to a Poisson distribution. Service times are exponentially distributed with a service rate of 35 customers per hour. The probability that there are exactly 3 customers in the system is
 a) 0 ☒ b) 0.1024 c) 0.4096 d) 0.5120

14. Consider an espresso stand with a single barista. Customers arrive to the stand at the rate of 28 per hour according to a Poisson distribution. Service times are exponentially distributed with a service rate of 35 customers per minute. If the arrival rate remains at 28 customers per hour and the stand's manager wants to have the average time a customer spends in the system (i.e., wait time and service time) to be a maximum of 6 minutes on average, then the service rate must

- ☒ a) decrease by 2 to 33 customers per hour.
 b) decrease by 3 to 32 customers per hour.
☒ c) increase by 3 to 38 customers per hour.
 d) increase by 2 to 37 customers per hour.

$$1/(u-28) = 1/10$$

actually two server setup...

15. A service counter employs two servers. On average a server requires 8 minutes to process a customer and service times follow an exponential distribution. Customers arrive at the counter at the rate of 12 per hour according to a Poisson distribution. The average amount of time, in minutes, spent in the system (i.e., waiting and being served) is approximately

- a) 0.237 minutes ☒ b) 14.22 minutes c) 22.20 minutes d) 33.30 minutes

Section 2: Out of Control Detection

The Western Electric Handbook (1956) suggests a set of decision rules for detecting non-random patterns on control charts. Specifically, it suggests concluding that the process is out of control if either:

1. One-point plots outside the three-sigma control limits, or
2. Two out of three consecutive points plot beyond the two-sigma warning limits, or
3. Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line, or
4. Eight consecutive points plot on one side of the centerline.

Those rules apply to one side of the centerline at a time. Therefore, a point above the upper warning limit followed immediately by a point below the lower warning limit would not signal an out-of-control alarm.

Calculate the probability of each of these patterns occurring assuming the data points are independent but identically distributed with a normal distribution.

16. Calculate the probability of One-point plots outside the three-sigma control limits,
 (a) 0.00135 ☒ (b) 0.0027 (c) 0.027 (d) 0.0036
17. Calculate the probability of Two out of three consecutive points plot beyond the two-sigma warning limits
 (a) 0.00135 ☒ (b) 0.0027 (c) 0.0036 (d) 0.0055
18. Calculate the probability of Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line
 (a) 0.0036 (b) 0.0027 (c) 0.00135 (d) 0.0055
19. Calculate the probability of Eight consecutive points plot on one side of the centerline
 (a) 0.0039 (b) 0.0055 (c) 0.0027 (d) 0.0036
20. Modern Inc. manufactures low-cost tables at a processing cost of \$80 per table. The company produces 100 units per day and averages 90% good quality resulting in 10% defective items. Fifty percent of the defective units are reworked prior to shipment to Modern's distribution center. The rework cost is \$10 per unit. If the percent good quality increases from 90% to 95%, the quality productivity ratio (QPR) is
☒ (a) approximately 1.20. (b) approximately 1.40.
 (c) approximately 1.60. (d) approximately 180.
21. For the process to be capable of meeting design specification the process capability index must be
 (a) less than one (1.0)
☒ (b) equal to or greater than one (1.0) b
 (c) less than zero (0.0)
 (d) equal to or greater than zero (0.0)
22. Which of the following statement is correct?
 (1) With Six Sigma, a teacher and mentor are known as a Green Belt.
 (2) Six Sigma is a recognized quality program based strictly on statistical process control.
 (3) Companies that have adopted Six Sigma view it as a short-term strategy for incremental quality improvement.
☒ (4) The fundamental objective of Six Sigma is to focus on improvement by reducing process variation.
 (a) Option (1) and Option (2) only
 (b) Option (2) and Option (4) only
☒ (c) Option (4) only
 (d) Option (2) and Option (3) only

Section 3: Forecasting

A forecasting model has produced the following forecasts:

| Period | Demand | Forecast | Error |
|----------|--------|----------|-------|
| January | 120 | 110 | |
| February | 110 | 115 | |
| March | 115 | 120 | |
| April | 125 | 115 | |
| May | 130 | 125 | |

23. The mean absolute deviation (MAD) for the end of May is a
(a) 7.0. (b) 7.5. (c) 10.0. (d) 3.0

24. The mean absolute percent deviation (MAPD) for the end of May is b
(a) 0.0250. (b) 0.0583. (c) 0.5830. (d) 0.6670.

25. At the end of May, the average error would be c
(a) 7. (b) 5. (c) 3. (d) 1.

26. At the end of May, the tracking signal would be d
(a) 0.000. (b) 0.667. (c) 1.333. (d) 2.143.

Section 4: Service Level 1

The demand for an electronic component is normally distributed with an average daily demand of 500 units and a standard deviation of 50. The lead-time for the component is 9 days.

d 27. If a service level of 95% is desired then the company's reorder point for this component is approximately
(a) 3,785 units. (b) 4,500 units. (c) 4,627 units. (d) 4,747 units.

b 28. If a service level of 95% is desired, then the company's safety stock for this component is approximately
(a) 150 units. (b) 247 units. (c) 336 units. (d) 740 units.

b 29. If the company sets a reorder point of 4,650 for this component then its service level is approximately
(a) 50 percent. (b) 84 percent. (c) 92 percent. (d) 98 percent.

Section 5: Inventory

Annual demand for a product is 40,000 units. The product is used at a constant rate over the 365 days the company is open every year. The annual holding cost for the product is estimated to be \$2.50 per unit and the cost of placing each order is \$125.00.

a 30. If the company orders according to the economic order quantity (EOQ) formula then its optimal order size for this product is
(a) 2,000 units. (b) 4,000 units. (c) 20,000 units. (d) 40,000 units.

d 31. If the company orders according to the economic order quantity (EOQ) formula then _____ orders are placed annually.
(a) 5 (b) 10 (c) 15 (d) 20

a 32. If the company orders according to the economic order quantity (EOQ) formula, then the time between orders (order cycle time) is
(a) 18.25 days. (b) 24.33 days. (c) 36.5 days. (d) 73 days.

c 33. If the company orders according to the economic order quantity (EOQ) formula then its total annual inventory cost for this product is
(a) \$100,000. (b) \$50,000. (c) \$5,000. (d) \$2,500.

d 34. If the company orders according to the economic order quantity (EOQ) formula, then its average inventory level for this product is
(a) 20,000 units. (b) 10,000 units. (c) 2,500 units. (d) 1,000 units.

Section 6: Aggregate production planning in goods

The following information relates to a company's aggregate production planning activities:

| Quarter | Demand Forecast |
|---------|-----------------|
| 1 | 37,500 |
| 2 | 45,000 |
| 3 | 25,000 |
| 4 | 62,500 |

Handwritten notes:
 21,500 → 25,000 → 500
 20,000 → 40 → 50 → 55

Beginning Workforce = 125 workers

Production per Employee = 500 units per quarter

Hiring Cost = \$750 per worker

Firing Cost = \$1,500 per worker

Inventory Carrying Cost = \$10 per unit per quarter

- c 35. If a chase demand strategy is used then the number of workers hired at the start of quarter 4 is
 (a) 0 (b) 15 ✓ (c) 75 (d) 125
- b 36. If a chase demand strategy is used the number of workers fired at the start of quarter 3 is
 (a) 0 ✓ (b) 40 (c) 50 (d) 75
- c 37. If a chase demand strategy is used the total hiring and firing costs for the production plan is
 (a) \$67,500 (b) \$135,000 ✓ (c) \$202,500 (d) \$337,500
- a 38. If a level production strategy is used the number of units to produce each quarter is
 ✓ (a) 42,500 (b) 85,000 (c) 62,500 (d) 37,500
- c 39. If a level production strategy is used the number of workers required each quarter is
 (a) 50 (b) 75 ✓ (c) 85 (d) 125
- d 40. If a level production strategy is used the number of units in inventory at the end of quarter 3 is
 (a) 0 (b) 2,500 (c) 5,000 ✓ (d) 20,000
- c 41. If a level production strategy is used the total cost of the production plan (hiring cost, firing cost, and inventory cost) is
 (a) \$60,000 (b) \$275,000 ✓ (c) \$335,000 (d) \$610,000

Subjective:

Section 7: Short answer (word limit of 300 words)

- How would the ongoing pandemic effect the supply chains? Explain the underlying reasons for the effects. (5 Marks)
- What is a tracking signal? Explain the construction of a tracking signal. (5 Marks)
- Instantiate the extremes in production planning strategies across sectors. (5 Marks)
- State the production quantity model of inventory. List the conditions when (5 Marks)
 - the economic order quantity model results in periodic ordering
 - the economic order quantity model results in variable ordering frequency
- What is
 - Sampling distribution
 - Central limit theorem (5 Marks)
- The amount of Denim used daily by the Southwest Apparel Company in its manufacturing process to make jeans is normally distributed with an average of 4,000 yards of denim and a standard deviation of

600 yards. The lead time required to receive an order of denim from the textile mill is a constant 7 days. Determine the safety stock and reorder point if the company wants to limit the probability of a stockout and work stoppage to 5%. (5 Marks)

7. What is type 2 error, how can it be controlled? (5 Marks)

Section 11: Long answer

1. List the categories of variations one encounters in operating and managing the supply chains. How are these categories of variations dealt with? (7 Marks)

2. Briefly describe the Read Bead Experiment and the Funnel Experiment. What is the key take away from each of these experiments? (7 Marks)

3. Select an

a) existing or new product

b) existing or new service

that you would like to redesign/design. State your reasons for the selection and describe the redesign/design process for your selection. (7 Marks)

4. Describe the BWE. Demonstrate the various ways that conflict and complexity are driven into the supply chain and the challenges they pose for managers. (7 Marks)

5. Given in the following table are the tasks necessary for the assembly of Fine Cedar Chests, the length of time needed to perform each task, and the operations that must be completed prior to the subsequent operations. (7 Marks)

| Element | Precedence | Time (min) |
|---------|------------|------------|
| A | None | 2 |
| B | A | 4 |
| C | B | 5 |
| D | None | 5 |
| E | D | 3 |
| F | None | 1 |
| G | F | 2 |
| H | C, E, G | 4 |

a. Calculate the cycle time necessary to complete 300 cedar chests in a 35-hour week.

b. What is the minimum number of workstations that can be used on the assembly line and still reach the production quota? Balance the line and calculate the line's efficiency.

c. Rebalance the line with a cycle time of 9 minutes. How do the number of workstations, output, and line efficiency change? (7 Marks)

6. Fabulous Fit Fibres produces a line of clothes that exhibits a varying demand pattern. Given the following demand forecasts, production costs, and constraints answer the following questions: (7 Marks)

a. Design a production plan for Fabulous Fit using the transportation method of LP. Also, calculate the cost of production plan.

b. Formulate the production planning problem as LP.

| Period | Demand |
|--------|--------|
| Sep | 100 |
| Oct | 150 |
| Nov | 200 |
| Dec | 300 |

Maximum regular production 100 units/month

Maximum overtime production 50 units/month

Maximum subcontracting 50 units/month

Regular production costs \$10/unit

Overtime production costs \$25/unit

Subcontracting costs \$35/unit

Inventory holding costs \$5/unit/month

Beginning inventory 0

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -3.9 | .00005 | .00005 | .00004 | .00004 | .00004 | .00004 | .00004 | .00004 | .00003 | .00003 |
| -3.8 | .00007 | .00007 | .00007 | .00006 | .00006 | .00006 | .00006 | .00005 | .00005 | .00005 |
| -3.7 | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| -3.6 | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| -3.5 | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| -3.4 | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| -3.3 | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| -3.2 | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| -3.1 | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| -3.0 | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| -2.9 | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| -2.8 | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| -2.7 | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| -2.6 | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| -2.5 | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| -2.4 | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| -2.3 | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| -2.2 | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| -2.1 | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| -2.0 | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| -1.9 | .02872 | .02807 | .02743 | .02680 | .02619 | .02559 | .02500 | .02442 | .02385 | .02330 |
| -1.8 | .03593 | .03515 | .03438 | .03362 | .03288 | .03216 | .03144 | .03074 | .03005 | .02938 |
| -1.7 | .04457 | .04363 | .04272 | .04182 | .04093 | .04006 | .03920 | .03836 | .03754 | .03673 |
| -1.6 | .05480 | .05370 | .05262 | .05155 | .05050 | .04947 | .04846 | .04746 | .04648 | .04551 |
| -1.5 | .06681 | .06552 | .06426 | .06301 | .06178 | .06057 | .05938 | .05821 | .05705 | .05592 |
| -1.4 | .08076 | .07927 | .07780 | .07636 | .07493 | .07353 | .07215 | .07078 | .06944 | .06811 |
| -1.3 | .09680 | .09510 | .09342 | .09176 | .09012 | .08851 | .08691 | .08534 | .08379 | .08226 |
| -1.2 | .11507 | .11314 | .11123 | .10935 | .10749 | .10565 | .10383 | .10204 | .10027 | .09853 |
| -1.1 | .13567 | .13350 | .13136 | .12924 | .12714 | .12507 | .12302 | .12100 | .11900 | .11702 |
| -1.0 | .15866 | .15625 | .15386 | .15151 | .14917 | .14686 | .14457 | .14231 | .14007 | .13786 |
| -0.9 | .18406 | .18141 | .17879 | .17619 | .17361 | .17106 | .16853 | .16602 | .16354 | .16109 |
| -0.8 | .21186 | .20897 | .20611 | .20327 | .20045 | .19766 | .19489 | .19215 | .18943 | .18673 |
| -0.7 | .24196 | .23885 | .23576 | .23270 | .22965 | .22663 | .22363 | .22065 | .21770 | .21476 |
| -0.6 | .27425 | .27093 | .26763 | .26435 | .26109 | .25785 | .25463 | .25143 | .24825 | .24510 |
| -0.5 | .30854 | .30503 | .30153 | .29806 | .29460 | .29116 | .28774 | .28434 | .28096 | .27760 |
| -0.4 | .34458 | .34090 | .33724 | .33360 | .32997 | .32636 | .32276 | .31918 | .31561 | .31207 |
| -0.3 | .38209 | .37828 | .37448 | .37070 | .36693 | .36317 | .35942 | .35569 | .35197 | .34827 |
| -0.2 | .42074 | .41683 | .41294 | .40905 | .40517 | .40129 | .39743 | .39358 | .38974 | .38591 |
| -0.1 | .46017 | .45620 | .45224 | .44828 | .44433 | .44038 | .43644 | .43251 | .42858 | .42465 |
| -0.0 | .50000 | .49601 | .49202 | .48803 | .48405 | .48006 | .47608 | .47210 | .46812 | .46414 |

KEY FORMULAS

Single-Server Model

$$P_0 = 1 - \frac{\lambda}{\mu} \quad W = \frac{1}{\mu - \lambda}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \quad W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$L = \frac{\lambda}{\mu - \lambda} \quad \rho = \frac{\lambda}{\mu}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad I = 1 - \frac{\lambda}{\mu}$$

where λ = arrival rate
 μ = service rate

Single-Server Model with Finite Calling Population

$$P_0 = \frac{1}{\sum_{n=0}^N \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n} \quad W_q = \frac{L_q}{(N-L)\lambda}$$

$$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0) \quad W = W_q + \frac{1}{\mu}$$

$$L = L_q + (1 - P_0) \quad P_n = \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n P_0$$

where N = population size

Single-Server Model with Finite Queue

$$P_0 = \frac{1 - \lambda/\mu}{1 - (\lambda/\mu)^{M+1}} \quad W = \frac{L}{\lambda(1 - P_M)}$$

$$P_n = (P_0)(\lambda/\mu)^n, n \leq M \quad W_q = W - \frac{1}{\mu}$$

$$L = \frac{\lambda/\mu}{1 - \lambda/\mu} - \frac{(M+1)(\lambda/\mu)^{M+1}}{1 - (\lambda/\mu)^{M+1}}$$

$$L_q = L - \frac{\lambda(1 - P_M)}{\mu}$$

where M = maximum number allowed in the system

Single-Server Model with Constant Service Times

$$P_0 = 1 - \frac{\lambda}{\mu} \quad W_q = \frac{L_q}{\lambda}$$

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)} \quad W = W_q + \frac{1}{\mu}$$

$$L = L_q + \frac{\lambda}{\mu} \quad \rho = \frac{\lambda}{\mu}$$

Multiple-Server Model

$$P_0 = \frac{1}{\left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$P_n = \begin{cases} \frac{1}{s!s^{n-s}} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{for } n > s \\ \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{for } n \leq s \end{cases}$$

$$P_w = \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} P_0$$

$$L = \frac{\lambda\mu(\lambda/\mu)^s}{(s-1)!(s\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

$$W = \frac{L}{\lambda}$$

$$L_q = L - \frac{\lambda}{\mu}$$

$$W_q = W - \frac{1}{\mu}$$

where s = number of servers