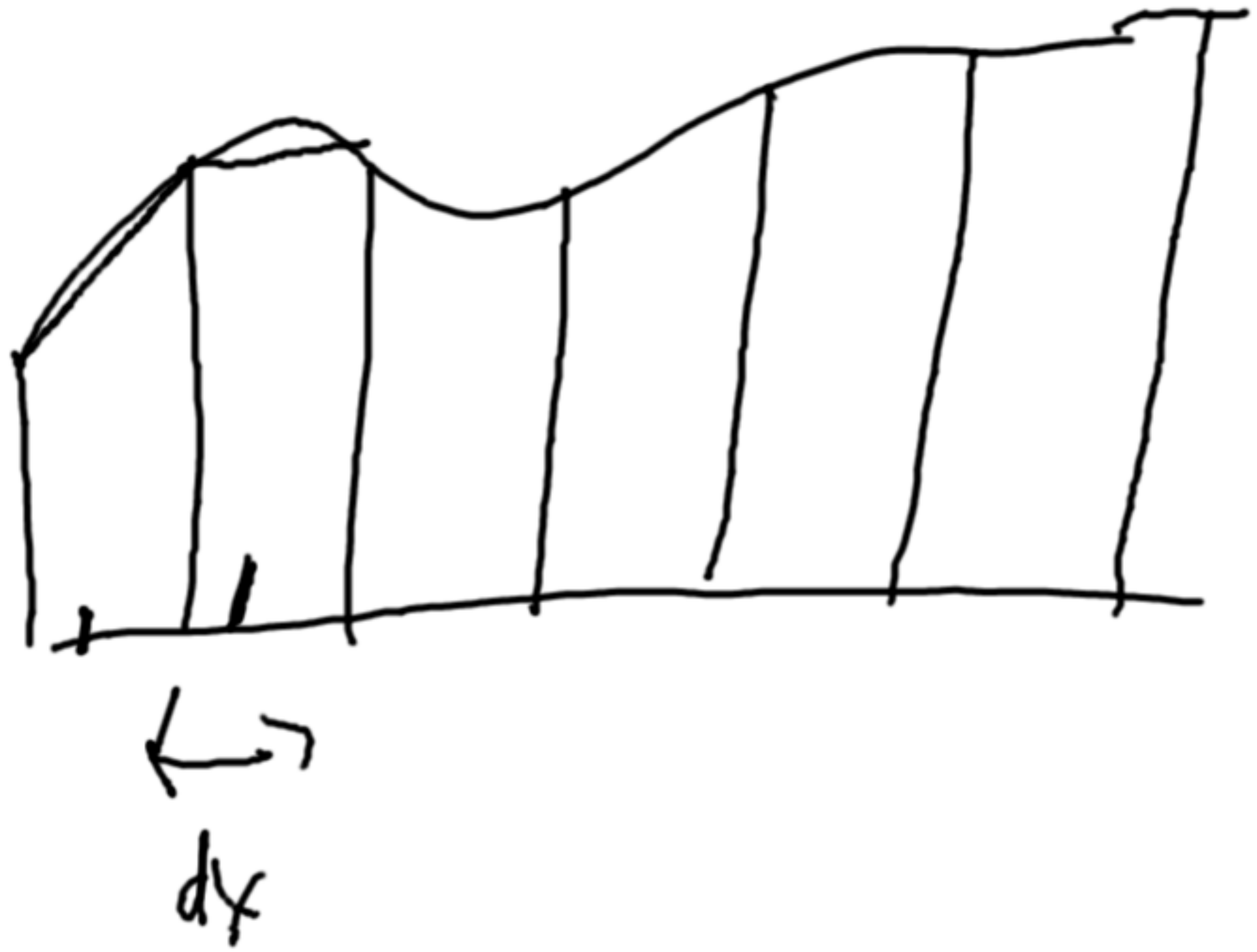


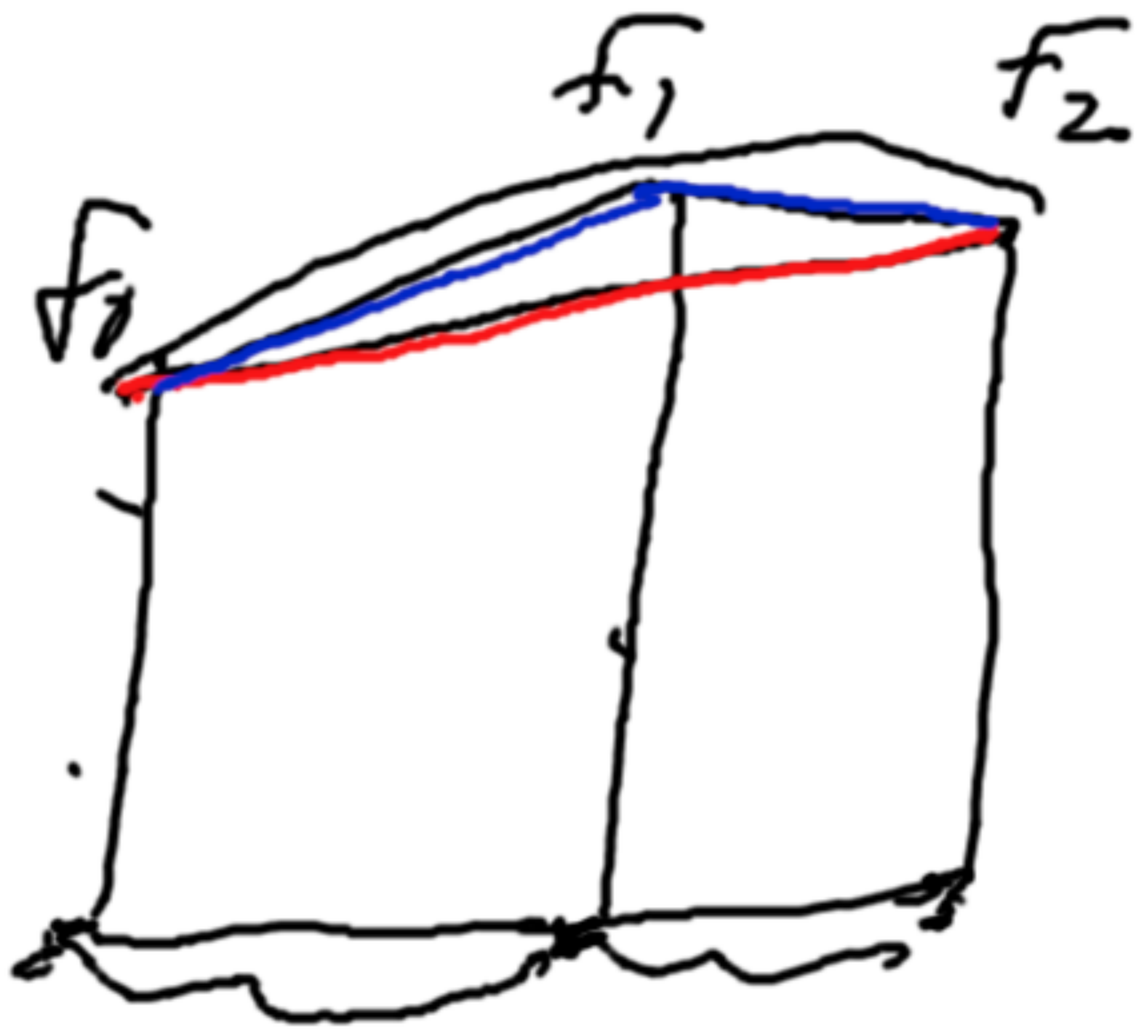
$f_l, f_r \Rightarrow area = ?$

$$height = \frac{f_l + f_r}{2}$$

$$area = \left(\frac{f_l + f_r}{2}\right) dx + \ldots$$

$$area_{next} = \frac{(f_{l\,next} + f_{right\,next})}{2}\, dx + \ldots$$

$$\Rightarrow \frac{f_0 + 2f_1 + 2f_2 + \ldots + 2f_{n-1} + f_n}{2}\, dx + \ldots$$



$$f(dx) = f_t + dx^2 \cdot a^2$$

$$F(2dx) = F_t + (2dx)^2 a^2$$

$$4f(dx) = 4F_t + 4dx\,a^2 + \cdots$$

$$F(2dx) = F_t + 4dx\,a^2 + \cdots$$

$$4f(x) - f(2dx) = 3F_t + \cdots$$

$$F(dx) = dx \cdot \frac{(f_0 + 2f_1 + f_2)}{2}$$

$$f(2dx) = 2dx \cdot \frac{(f_0 + f_2)}{2}$$

$$3f_t = 4f(dx) - f(2dx)$$

$$= dx(2f_0 + 4f_1 + 2f_2)$$

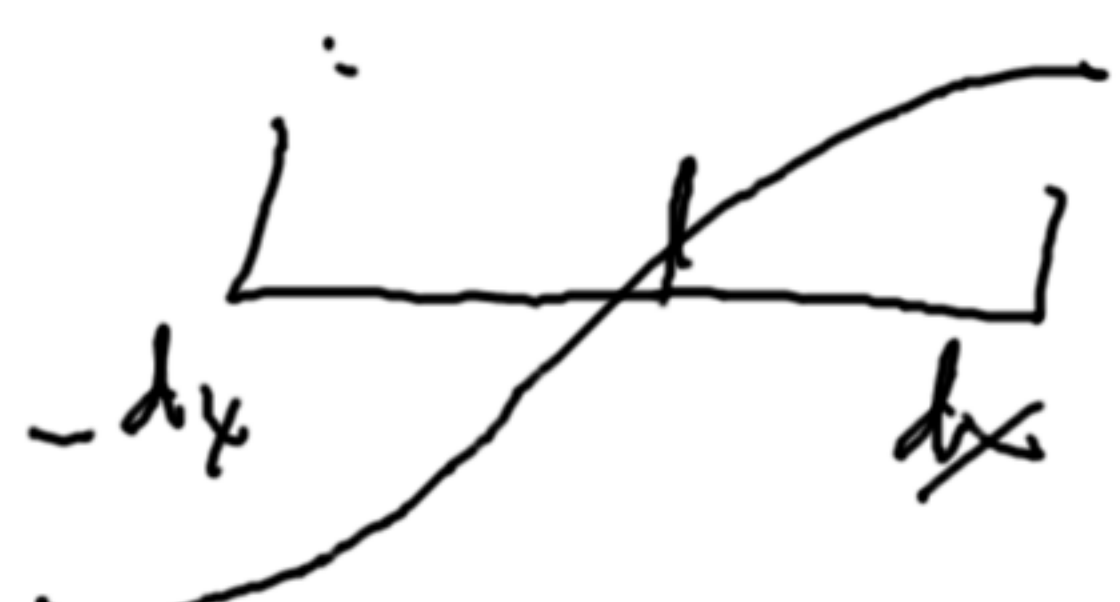
$$- dx(f_0 + f_2)$$

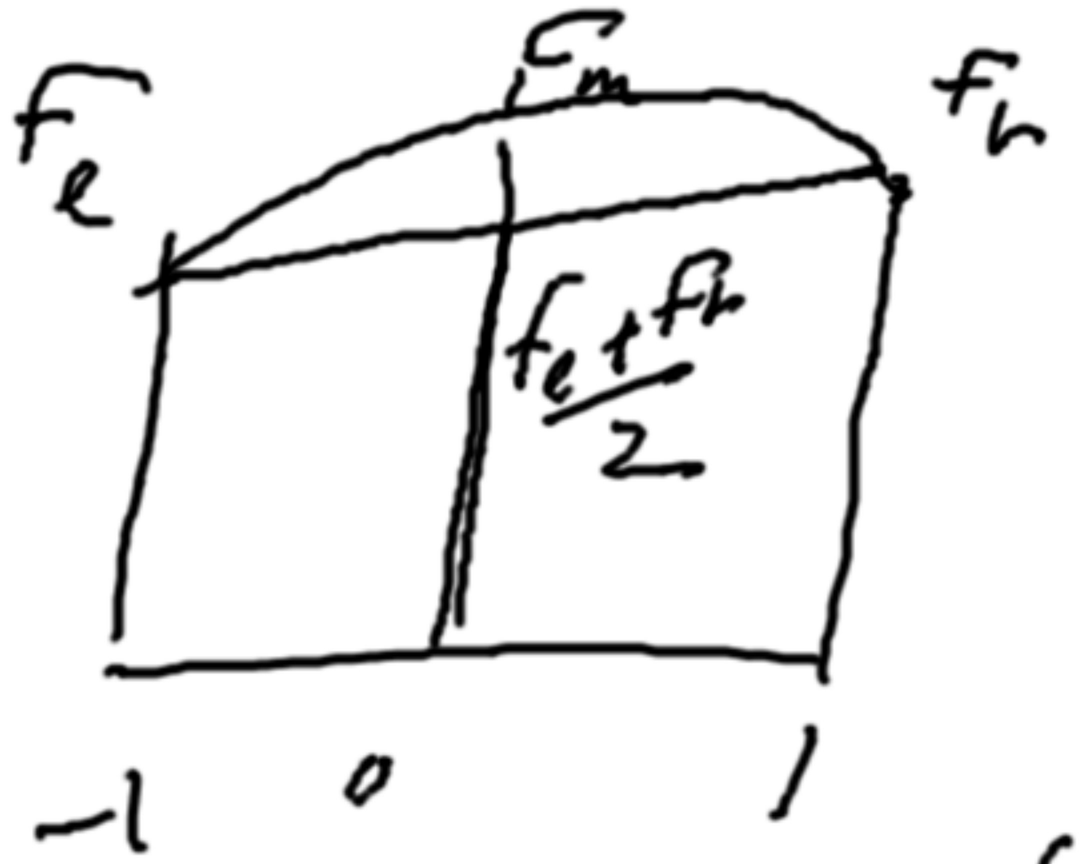
$$\Rightarrow \boxed{f_t = \frac{1}{3} dx (f_0 + 4f_1 + f_2)}$$

$$\frac{dx}{3}(f_0+4f_1+f_2)+\frac{dx}{3}(f_2+4f_3+f_4)+\dots$$

$$\Rightarrow \frac{dx}{3}(f_0+4f_1+2f_2+4f_3+2f_4+\dots+4f_{n-1}+f_n)$$

$$\int_{-1}^{1} x^3\,dx = 0$$





$$2\cdot\frac{(f_e+f_r)}{2} = f_e+f_r$$

$$A_{req} = \frac{2}{3}\left(f_m - \frac{(f_e+f_r)}{2}\right)\cdot 2$$

$$+\left(\frac{f_e+f_r}{2}\right)\cdot 2$$

$$f_m - \left(\frac{f_e+f_r}{2}\right)$$

$$= \frac{4}{3}f_m - \frac{2}{3}f_e - \frac{2}{3}f_r \text{ (trap)}$$

$$+ f_e + f_r \text{ (trap)}$$

$$(1-x^2)\left[f_m - \frac{(f_e+f_r)}{2}\right]$$

$$= \boxed{\frac{4}{3}f_m + \frac{1}{3}f_e + \frac{1}{3}f_r}$$

$$\int_{-1}^{1}(1-x^2) = x - \frac{x^3}{3}\Big|_{-1}^{1} = 2 - \frac{2}{3} = \frac{4}{3}$$

$$C = 4/3/2 = 2/3$$

$$\sum c_n x^n = c_0(x^0) + c_1(x^1) + \dots$$

$$= c_0 + c_1 x + c_2 x^2 + \dots$$

$$\int_a^b f(x)\,dx \Rightarrow u = \frac{1}{x} \quad du = -\frac{1}{x^2}dx$$

$$= -u^2 dx$$

$$dx = -\frac{du}{u^2}$$

$$\Rightarrow -\int_{1/a}^{1/b} \frac{f(\frac{1}{u})}{u^2}\,du$$

$$\Rightarrow \int_{1/b}^{1/a} \frac{f(\frac{1}{u})}{u^2}\,du \Rightarrow \int_a^\infty f(x)\,dx$$

$$\int_0^{\frac{1}{a}} \frac{f(\frac{1}{u})}{u^2}\,du$$