CMSC 330, Summer 2018 Quiz 4 Solution

Name ____

Instructions

- Do not start this quiz until you are told to do so.
- You have 20 minutes for this quiz.
- This is a closed book quiz. No notes or other aids are allowed.
- For partial credit, show all your work and clearly indicate your answers.

1. *(4 points)* Reduce the following λ term to normal form. If it does not have one, write "no normal form".

$$\begin{aligned}(\lambda x.\underline{\lambda y.x\ y\ y})\ (\lambda z.y)\ a &\rightarrow_\alpha \underline{(\lambda x.\lambda w.x\ w\ w)\ (\lambda z.y)}\ a\\ &\rightarrow_\beta \underline{(\lambda w.(\lambda z.y)\ w\ w)\ a}\\ &\rightarrow_\beta \underline{(\lambda z.y)\ a}\ a\\ &\rightarrow_\beta y\ a\end{aligned}$$

2. *(4 points)* Here are the call-by-value substitution semantics for the λ calculus.

$$(1)\ \frac{}{(\lambda x.t) \Downarrow_v (\lambda x.t)}$$

$$(2)\ \frac{t_1 \Downarrow_v (\lambda x.t_{12}) \qquad t_2 \Downarrow_v v_2 \qquad t_{12}[x \mapsto v_2] \Downarrow_v v}{(t_1\ t_2) \Downarrow_v v}$$

Write rules for a call-by-name λ calculus semantics. (Hint: You need only fill in the box for the hypotheses of rule (2).)

$$(1)\ \frac{}{(\lambda x.t) \Downarrow_n (\lambda x.t)}$$

$$(2)\ \frac{t_1 \Downarrow_v (\lambda x.t_{12}) \qquad t_{12}[x \mapsto t_2] \Downarrow_v v}{(t_1\ t_2) \Downarrow_n v}$$

3. *(4 points)* If possible, evaluate $(\lambda x.\lambda y.y)\ ((\lambda z.z\ z)\ (\lambda z.z\ z))$ with $\Downarrow_v$ (i.e. call-by-value evaluation) and prove your result with a derivation (a proof tree). If it does not evaluate explain why not.

Since $\Downarrow_v$ is call-by-value and requires the argument to be evaluated before the substitution, this entire thing cannot evaluate as $(\lambda z.z\ z)\ (\lambda z.z\ z)$ is the divergent combinator cannot be evaluated under $\Downarrow_v$.

4. *(4 points)* If possible, evaluate $(\lambda x.\lambda y.y)\ ((\lambda z.z\ z)\ (\lambda z.z\ z))$ with $\Downarrow_n$ (i.e. call-by-name evaluation) and prove your result with a derivation (a proof tree). If it does not evaluate explain why not.

$$\dfrac{\dfrac{}{(\lambda x.\lambda y.y) \Downarrow_n (\lambda x.\lambda y.y)} \qquad \dfrac{}{(\lambda y.y)[x \mapsto (\lambda z.z\ z)\ (\lambda z.z\ z)] = (\lambda y.y) \Downarrow_n (\lambda y.y)}}{(\lambda x.\lambda y.y)\ ((\lambda z.z\ z)\ (\lambda z.z\ z)) \Downarrow_n (\lambda y.y)}$$

5. *(4 points)* Describe the confluence property of the λ calculus. Are your results from (3) and (4) consistent with confluence?

Confluence states that there is a unique normal form, independent of reduction strategy. This is consistent since CBV did not yield a normal form and therefore confluence doesn't apply here.