

## Exclusive-OR Operator ( ^ ).

Exclusive-OR returns a bit value of 1 if both bits are of opposite ( different ), nature , otherwise Exclusive-OR returns 0

• Example :

$\begin{array}{r} 0000 \\ \wedge 0000 \\ \hline 0000 \end{array}$	$\begin{array}{r} 1111 \\ \wedge 0000 \\ \hline 1111 \end{array}$	$\begin{array}{r} 1111 \\ \wedge 1111 \\ \hline 0000 \end{array}$	$\begin{array}{r} 1100 \\ \wedge 1010 \\ \hline 0110 \end{array}$
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XOR is really a surprising operator. You can't imagine the things that make it possible for us to do.

Remember the properties of XOR operator:

- If you take xor of a number with 0 (zero), it would return the same number again.

Means,  $n \wedge 0 = n$

Example:

$$6 \wedge 0 = 6 \text{ ( because } 110 \wedge 000 = 110 \text{ )}$$

$$27 \wedge 0 = 27 \text{ (because } 11011 \wedge 00000 = 11011 \text{ )}$$

- If you take xor of a number with itself, it would return 0 ( zero ).

Means,  $n \wedge n = 0$

Example:

$$6 \wedge 6 = 0 \text{ ( because } 110 \wedge 110 = 000 \text{ )}$$

$$27 \wedge 27 = 0 \text{ (because } 11011 \wedge 11011 = 00000 \text{ )}$$

1. We can swap the values of two variables without using any third ( temp ) variable.

Example:  $a = 5, b = 10$

$$\text{temp} = a \mid a = a \wedge b \text{ ( } a = 5 \wedge 10 = 0101 \wedge 11010 = 1111 = 15 \text{ )}$$

$$a = b \mid b = a \wedge b \text{ ( } b = 15 \wedge 10 = 1111 \wedge 1010 = 0101 = 5 \text{ )}$$

$$b = \text{temp} \mid a = a \wedge b \text{ ( } a = 15 \wedge 5 = 1111 \wedge 0101 = 1010 = 10 \text{ )}$$

$$a = 10, b = 5$$

**2. Toggling (Changing/Flipping ) the k-th Bit ( from right ) of a binary number :**

**Question :** For a given number “n”, toggle the k-th bit (from right)

**Solution :** Let, n = 27 (Binary Representation of n = 11011 ) K = 3

We'll use this expression for toggling:  $n \wedge (1 \ll (k-1))$ .

Means,  $11011 \wedge (00001 \ll 2)$

$11011 \wedge (00100)$

$11011 \wedge 00100$

11111 (Decimal of 11111 = 31 )

**3. Find the Missing number from the list of numbers :**

**Question :** You are given a list of n-1 integers , And these integers are in the range of 1 to n. There are no duplicates in the list. One of the integers is missing in the list. Now, we need to find that missing number.

• **Method 1:** By finding the sum of first “n” natural numbers.

**Step 1:** First , find the sum of all numbers from 1 to n ( means find the sum of first “n” natural numbers )

**Step 2:** Subtract each element ( all elements ) of the given list from the sum. And we'll get the missing number.

\* There might be an integer overflow while adding large numbers.

• **Method 2 :** Using XOR operator.

**Step 1 :** We'll take the xor of all numbers from 1 to n ( Means , we'll take xor of the first “n” natural numbers ).

**Step 2:** We'll take the xor of all elements of the given array ( list ).

**Step 3:** xor of Step 1 and Step 2 will give the required missing number.

**Example:** Given list arr = [ 4, 2, 1, 6, 8, 5, 3, 9] n = 9 ( given )

**Step 1:**  $\text{Step1\_result} = 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9 = 1$

**Step 2:**  $\text{Step2\_result} = 4 \wedge 2 \wedge 1 \wedge 6 \wedge 8 \wedge 5 \wedge 3 \wedge 9 = 6$

**Step 3:**  $\text{Final Result} = \text{Step1\_result} \wedge \text{Step2\_result} = 1 \wedge 6 = 7$

But , How Final\_Result calculated the missing number?

$$\text{Final\_Result} = (1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9) \wedge (4 \wedge 2 \wedge 1 \wedge 6 \wedge 8 \wedge 5 \wedge 3 \wedge 9).$$

Remember these properties :  $n \wedge n = 0$  and  $n \wedge 0 = n$

So, here,

$$\begin{aligned}\text{Final\_Result} &= (1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9) \wedge (4 \wedge 2 \wedge 1 \wedge 6 \wedge 8 \wedge 5 \wedge 3 \wedge 9). \\ &= 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9 \wedge 4 \wedge 2 \wedge 1 \wedge 6 \wedge 8 \wedge 5 \wedge 3 \wedge 9. \\ &= (1 \wedge 1) \wedge (2 \wedge 2) \wedge (3 \wedge 3) \wedge (4 \wedge 4) \wedge (5 \wedge 5) \wedge (6 \wedge 6) \wedge (7) \wedge (8 \wedge 8) \wedge (9 \wedge 9) \\ &= 0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 \wedge 0 \wedge 7 \wedge 0 \wedge 0 \text{ (property applied)} \\ &= 0 \wedge 7 \text{ (because we know } 0 \wedge 0 = 0\text{).} \\ &= 000 \wedge 111 \\ &= 111 \\ &= 7 \text{ (Required Result )}\end{aligned}$$

4. Find the duplicate element in the given array of all positive integers.

Input\_arr = [23, 21, 24, 27, 22, 27, 26, 25]    Output = 27

Method 1: By finding “min”, “max” and “sum”.                      — o(n)

Step 1: First , find the sum of all numbers from “min” to “max”.

Step 2 : Subtract each element ( all elements ) of the given array from the sum.

And At last, we would be having a negative of the duplicate element in sum. Then , Just return the absolute value of that negative value in sum.

Method 2: Using XOR Operator

Step 1: Find “min” and “max” values in the given array.                      — O(n)

Step 2: Find XOR of all integers from range “min” to “max” (inclusive).

Step 3: Find XOR of all elements of the given array.

Step 4: XOR of Step 2 and Step 3 will give the required duplicate number.

5. Construct an array from XOR of all elements of the array except elements at the same index.

Given an array A[] having “n” positive elements. The task is to create another array B[] such as B[i] is XOR of all elements of the array A[] except A[i].

Example : A[] = { 2 , 1 , 5 , 9 }      O/P = B[] = { 13 , 14 , 10 , 63 }

$1^5^9$        $2^5^9$        $2^1^9$        $2^1^5$   
 $2^5^9$        $2^1^9$

**Efficient Solution : Using XOR Operator**

**Step 1:** Find XOR of all elements of the given array.

**Step 2:** Now, for each element of A[], Calculate  $A[i] = \text{Step1\_result} \wedge A[i]$

**Time Complexity : O(n)** [because for loop will run for each element of the array]

```
int Step1_result = A[0];
int n = sizeof(A) / sizeof(A[0]);    // Calculating size of the array A
for(int i=1;i<n;i++)                  // Step 1
    Step1_result = Step1_result ^ A[i];

for(int i=0;i<n;i++)                  // Step 2
    A[i] = Step1_result ^ A[i];

return A;
```

Remember

- $n \wedge n = 0$
- $n \wedge 0 = n$

• Example : A[] = { 2 , 1 , 5 , 9 }      n = 4      O/P = B[] = { 13 , 14 , 10 , 63 }

Step1 :      Step1\_result =  $2 \wedge 1 \wedge 5 \wedge 9 = 15$

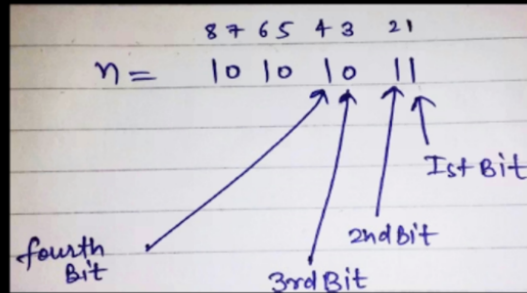
Step2 :      for i = 0 , A[0] =  $15 \wedge A[0] = 15 \wedge 2 = 13$

$$(2 \wedge 1 \wedge 5 \wedge 9) \wedge 2 = (2 \wedge 2) \wedge 1 \wedge 5 \wedge 9 = 0 \wedge 1 \wedge 5 \wedge 9 = 1 \wedge 5 \wedge 9 = 13$$

Similarly , for each element of A[]

Question : Given a Binary number  $n = 10101011$  , Write a program to check whether  $k$ -th bit ( from right ) is set ( Means , 1 at  $k$ -th position ) or not ?

Solution :  $n = 10101011$  , Let  $k = 4$  ( Given in Question )



So ,  $n = 1010\mathbf{1}011$  ( the 1 which is bigger is 4<sup>th</sup>-bit from right )

- The idea is very simple and very efficient :

We need to check only whether 4<sup>th</sup> bit ( from right ) is set or not. We do not need to think about all rest bits ( except 4<sup>th</sup> one ).

So , the approach is to make all the rest bits of "n" as 0 ( except the 4<sup>th</sup> bit ).

Handwritten calculation showing the binary number  $n = 10101011$  with a checkmark above it. Below it is  $8\ 00001000$ , followed by an arrow pointing to "let say B". The result is  $00001000$ .

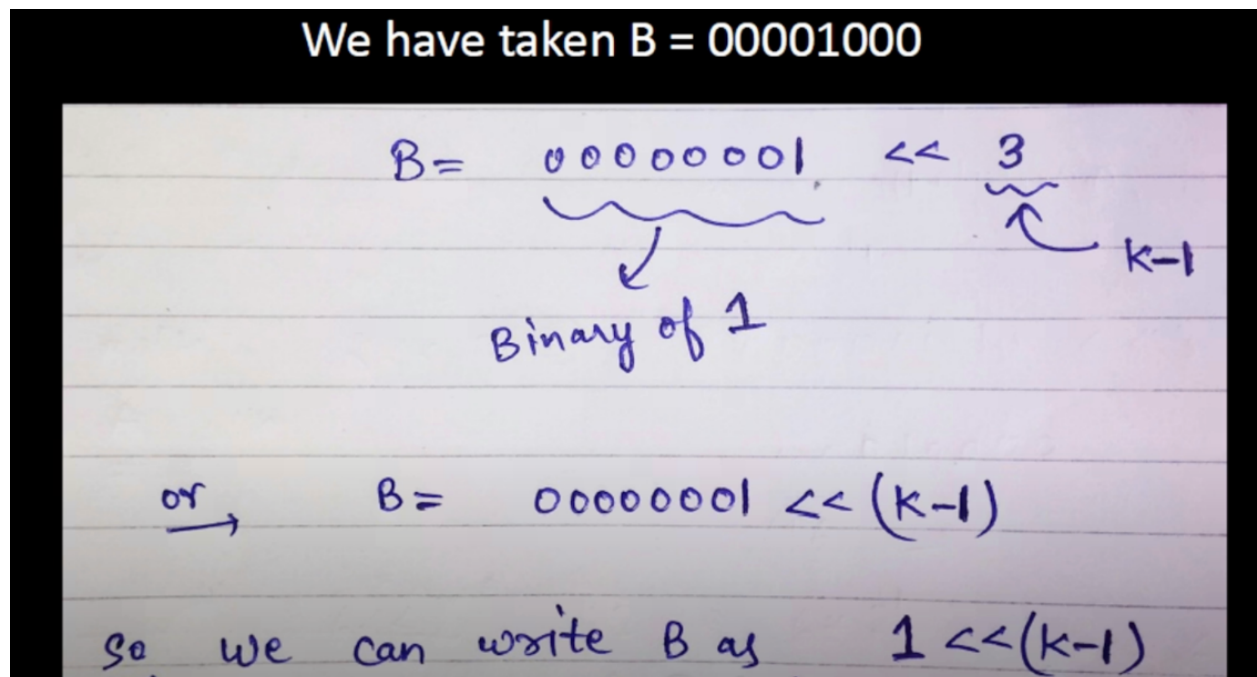
So, If we notice the result :

result = 00001000

And as we know that, after performing "& operation" on two numbers, the result will be 0 (zero) only if all the bits of the result are zero. Otherwise, the result will be non-zero.

Means , Here , result will return "non-zero value" because 4th bit ( from right is not zero.

So, It means, 4th bit is set 4th bit is 1) in given n = 10101011



```
1 result = n & (1<<(k-1));  
2 if(result != 0 ){  
3   System.out.println("k-th bit is SET");  
4 }else{  
5   System.out.println("k-th bit is NOT SET");  
6 }
```

### Bitwise Left Shift Operator ( << )

- Bitwise Left Shift Operator takes two operands , like :  $X \ll k$   
Here , X is the first operand , k is the second operand
- Then , bitwise left shift operator shifts bits of first operand to the left by kth positions and fills empty / vacated bit positions with 0

Bitwise Left Shift Operator moves / shifts all the bits of a number to the left by the specified position and fills empty / vacated bit positions with 0

**Example :  $X \ll k$**  (X is a Decimal number, k is the specified position)

**X= 23 ( Given Decimal Number), k= 2 ( specified position )**

**Binary Equivalent of 23 is : 10111**

**And if number X is stored as a 32-bit "int" , then**

Binary Equivalent would be : **00**000000 00000000 00000000 00010111

Now, shift 10111 to the left by 2 ( specified position )

So,  $10111 \ll 2$  : 00000000 00000000 00000000 01011100

Now , Decimal Equivalent of  $10111 \ll 2$  would be : 92

### **Remember**

- Integers are stored in the memory as a series of bits. So, for example , if integer number 6 is stored as a 32-bit “int”, then

Binary Equivalent of 6 would be : 00000000 00000000 00000000 00000110

- **Bitwise Left Shift Operator should not be used for negative numbers ( Means if Any of the operand of Bitwise left shift operand is negative), otherwise it might result undefined value.**

- **This Bitwise Left Shift Operator can be applied to int, long , and possibly short, byte , or char**

- If Any number is shifted more than the size of integers, then the behavior /result might also be undefined.

Example: If the integers are stored using 32-bits ,

then  $X \ll 33$ ,  $X \ll 34$ ,  $X \ll 35$ ,  $X \ll 36$  , ... and so on might result in undefined value.

## Inference / Conclusion

- |                    |  |   |
|--------------------|--|---|
| 1. $5 \ll 3 = 40$  |  | $40 = 5 * 8 = 5 * (2^3)$ (^ mean power)   |
| 2. $2 \ll 3 = 16$  |  | $16 = 2 * 8 = 2 * (2^3)$ (^ mean power)   |
| 3. $23 \ll 2 = 92$ |  | $92 = 23 * 4 = 23 * (2^2)$ (^ mean power) |
| 4. $6 \ll 1 = 12$  |  | $12 = 6 * 2 = 6 * (2^1)$ (^ mean power)   |
| 5. $6 \ll 3 = 48$  |  | $48 = 6 * 8 = 6 * (2^3)$ (^ mean power)   |

• In  $X \ll k$ , Bitwise Left Shift Operator multiplies the number X with  $(2^k)$