Anway Prasad Shirgaonkar

RSN LAB 4 - IMU Dead Reckoning

Magnetometer Calibration

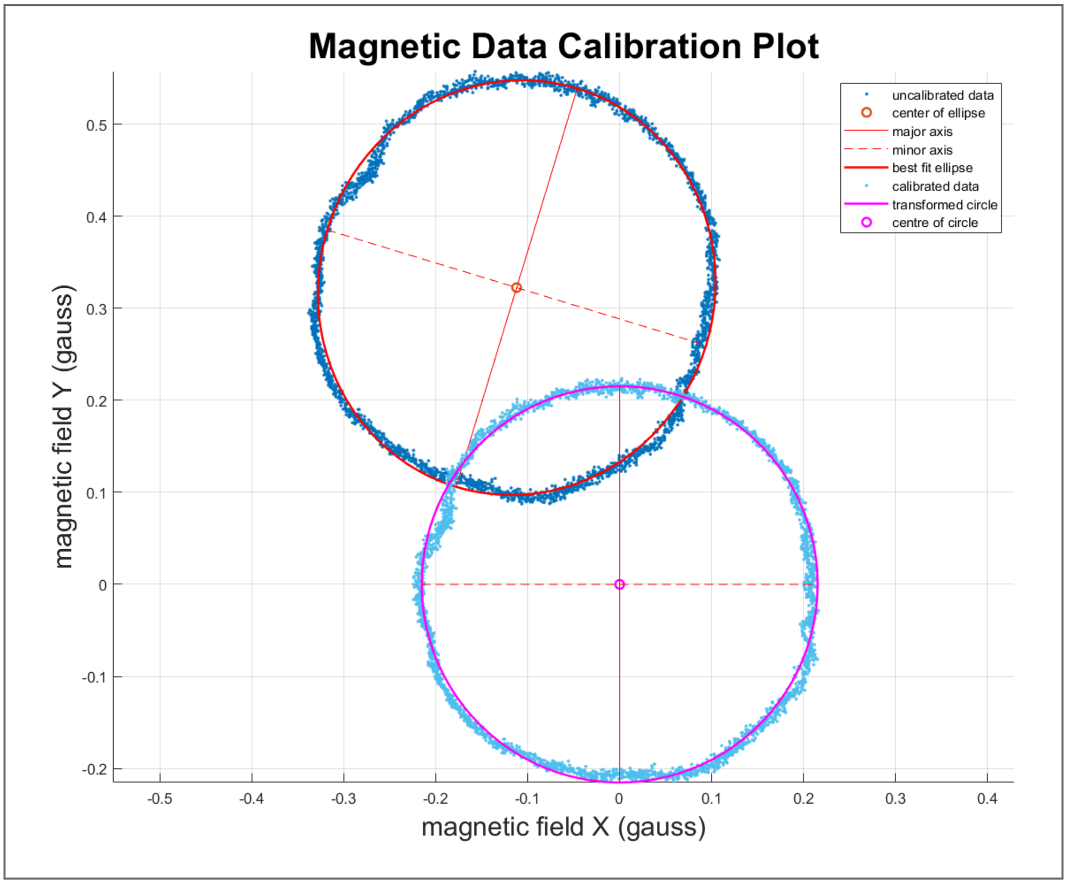

Fig. 1: Magnetometer data before and after calibration

Magnetometer calibration was done to rectify the hard and soft iron distortions. The IMU was mounted in the car and was driven 4-5 times in circles. Plotting x-y data results in an elliptical shape centered away from the origin. Ideally, the x-y plot of a rotating magnetometer should be a circle. The hard iron errors result in the shift of the center of this circle, and the soft iron errors result in the stretching of this circle thus forming an ellipse. **The uncalibrated data was modeled by a best-fit ellipse,** on which the linear transformations can be applied to transform it into a circle.

The transformations performed on the ellipse (in order) are

1. Shifting the center of the ellipse to the origin (subtract x_{shift}, y_{shift})

$$x_{shift}, y_{shift} = (-0.1120,\ 0.3225)$$

2. Rotating the ellipse so that the minor axis of the ellipse is parallel to the x-axis

$$R1 = [[0.9567\ \ -0.2911]\ ;\ [0.2911\ 0.9567]]$$

3. Scaling the major axis by a factor to equate the length of the major and minor axes

$$factor_{scale} * axis_{major} = axis_{minor}$$

$$R2 = [[1\quad 0]\ ;\ [0\quad 0.9567]]$$

Yaw Correction and Implementing Complementary Filters

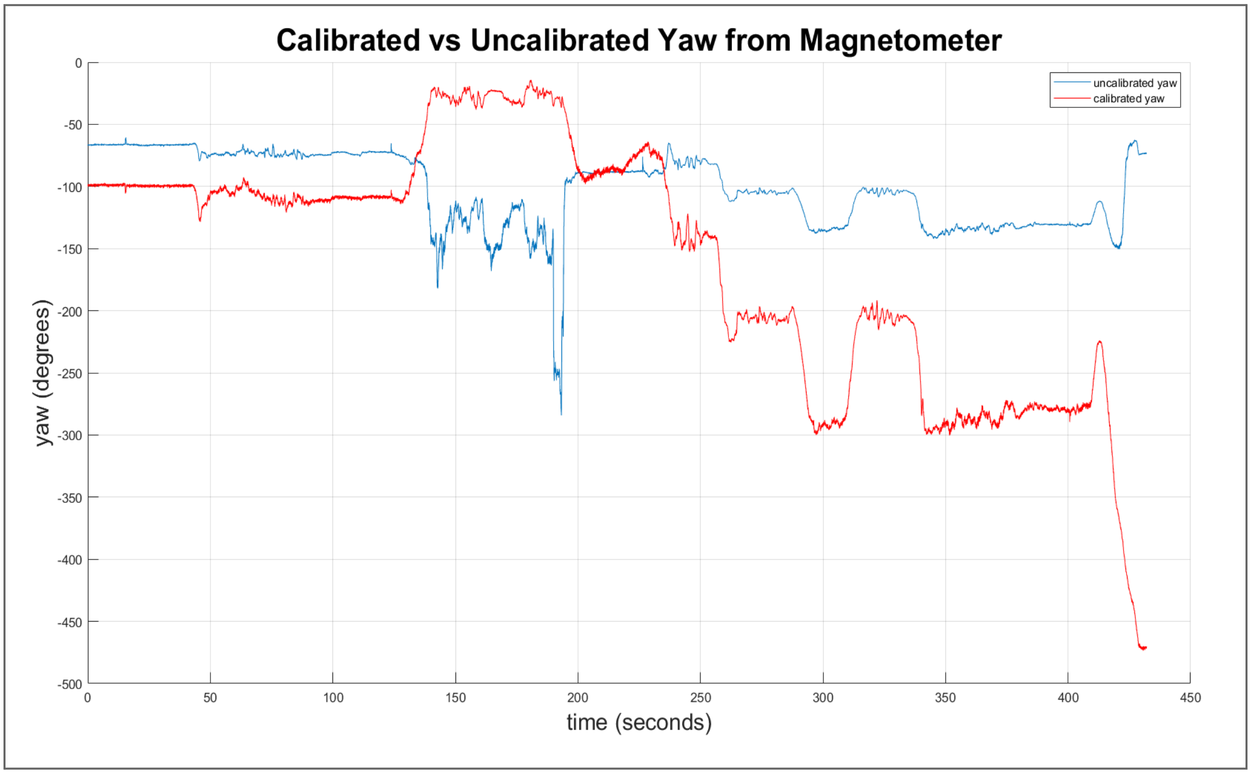

Fig. 2: Calibrated vs Uncalibrated Yaw from Magnetometer

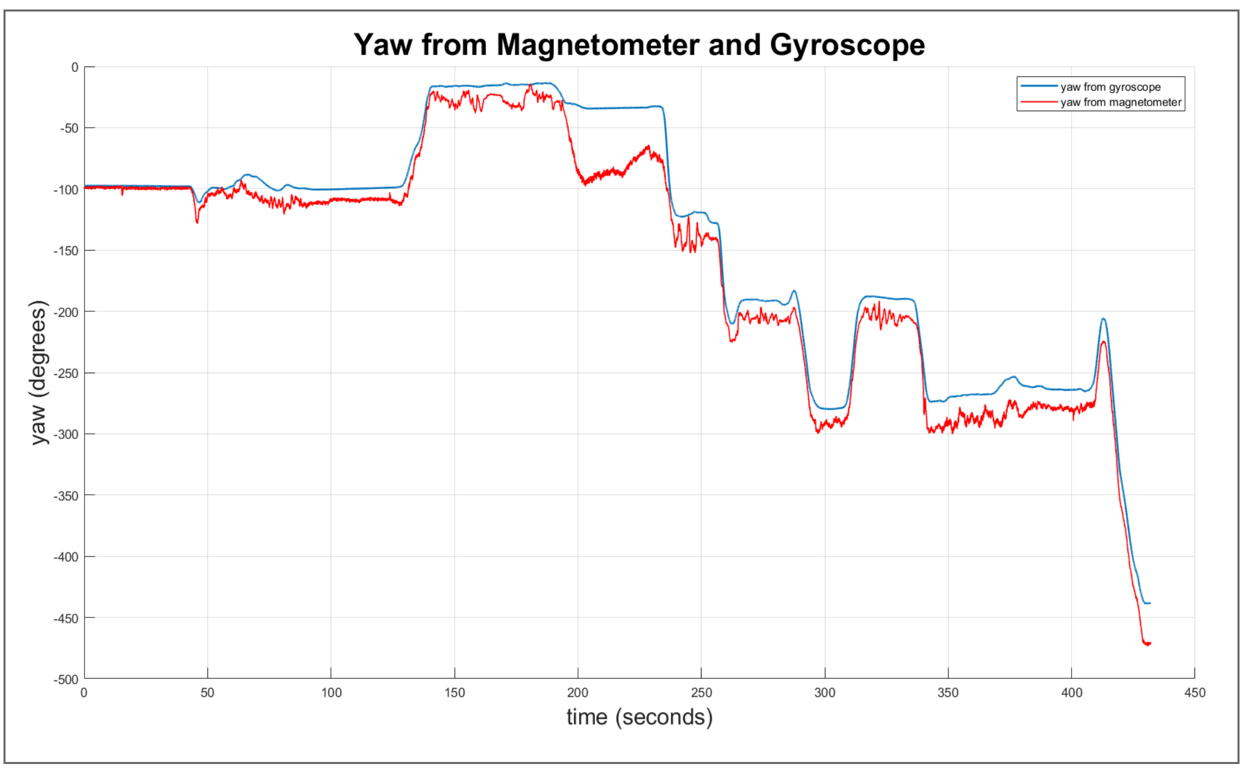

Fig. 3: Yaw from magnetometer vs Yaw from gyroscope

It is observed in Fig.3 that the yaw from the magnetometer and the yaw from the gyroscope is approximately in agreement with each other except from **time ≈ 190 to 230 secs when distortion is seen in the magnetometer yaw**. As there may be numerous unknown magnetic and ferromagnetic sources while driving in a city that can cause this error, exactly pinpointing the source of this error is very difficult if not impossible.

Another observation that can be done is that the **magnetometer yaw has a lot of high-frequency components** over a short duration of time. This property makes the **magnetometer unreliable to measure yaw over a short period** but is useful when estimating yaw over a long duration of time as it is not affected by drift.

The **gyroscope gives an excellent estimate of yaw over a short duration of time**. That means that the gyroscope can very quickly respond to abrupt changes in the yaw and give an accurate estimate of the yaw. The downside is that **the gyroscope tends to drift over time (bias instability)** which makes the gyroscope unreliable to measure yaw over a long duration.

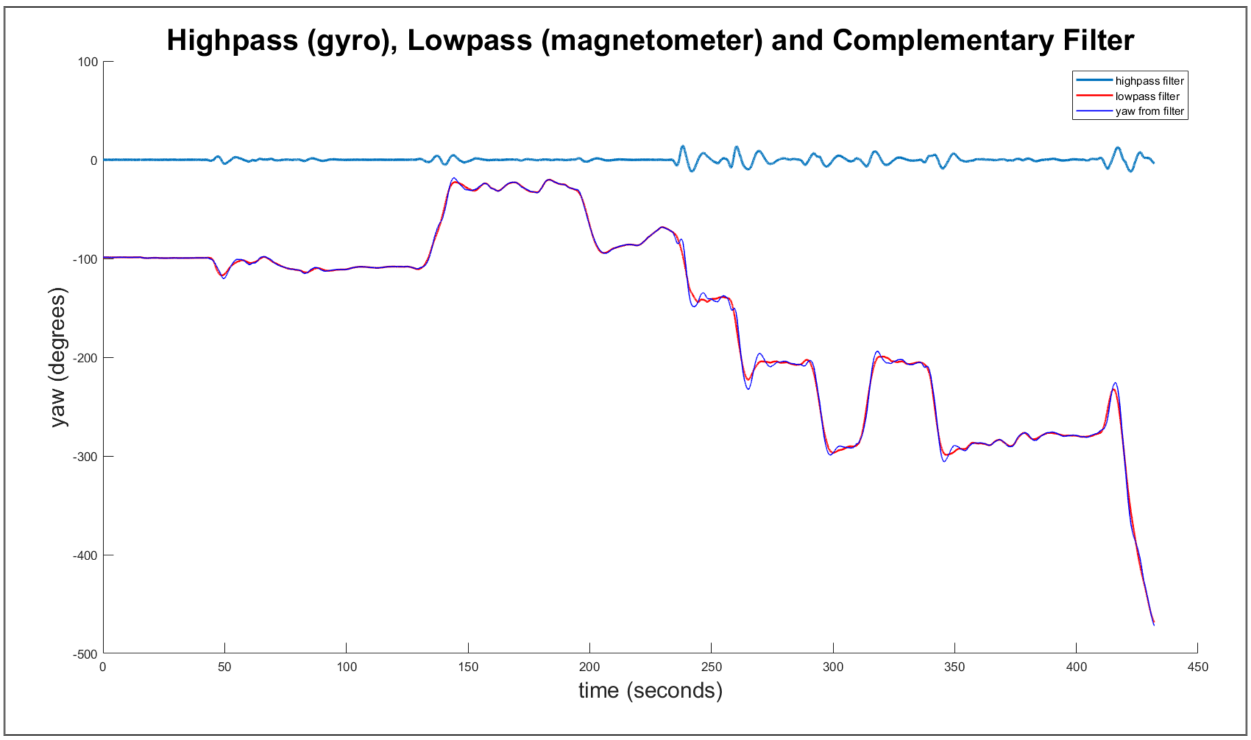

Fig. 4: Highpass (gyro) Lowpass (magnetometer) and Complementary filter output

In order to take the advantage of the magnetometer and the gyroscope, a complementary filter can be implemented. As discussed above, the magnetometer is reliable over a long duration of time and contains high-frequency noise whereas the gyroscope may drift over time. Hence, **The magnetometer data is passed through a lowpass filter to smoothen out the high-frequency noise and the gyroscope data is passed through a highpass filter to add the high-frequency components from the gyroscope.** The output of the lowpass and the highpass filter is added to obtain the complementary filter output. The characteristics of lowpass and highpass filters are as follows:

Characteristics	Lowpass Filter	Highpass Filter
Type	IIR Butterworth Filter	IIR Butterworth Filter
Cut off Frequency	0.1 Hz	0.1 Hz
Filter Order	3	5
Sampling Rate	40 Hz	40 Hz

The cut-off frequency was determined empirically with the help of the Fast Fourier Transform (FFT) of the signals.

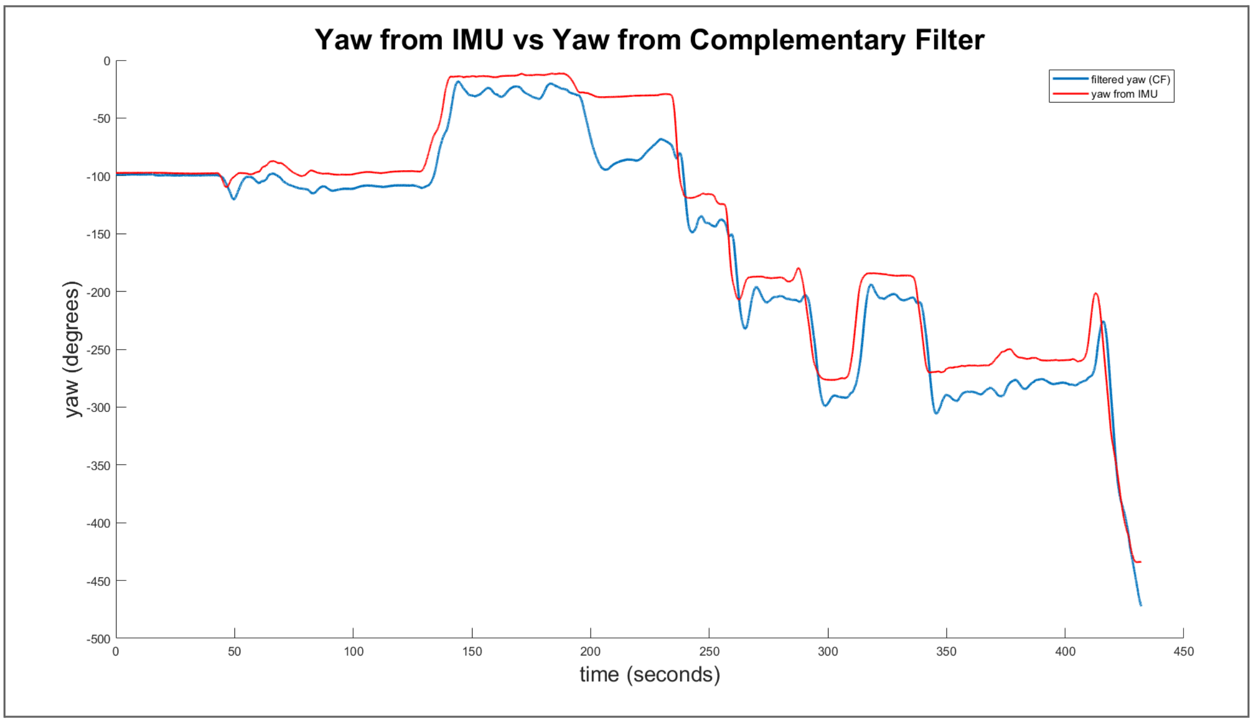

Fig. 5: Comparison of yaw from IMU and yaw from the complementary filter

Fig. 5 shows the comparison of the yaw obtained from IMU and yaw from the complementary filter. We can see that by combining the output of the lowpass and the highpass filters, the output of the complementary filter closely follows the IMU yaw (except for the part when there is a disturbance in magnetometer data as discussed earlier).

Evidently, the output of **the complementary filter does take advantage of the long-term stability of the magnetometer and the short-term accuracy of the gyroscope**. Even then, a slight offset is present in the IMU yaw and complementary filter yaw. One possible reason for this is that IMU yaw is calculated based on Extended Kalman Filter which is a more sophisticated algorithm to estimate position than a simple complementary filter.

The total duration of this collected data is not long enough to see a drift in the gyroscope, and hence the IMU yaw very closely follows the yaw from the gyroscope. Nevertheless, when navigating for a long duration, a yaw estimated from a complementary filter can be trusted more than either the yaw from the magnetometer or the yaw from the gyroscope.

Estimation of Velocity by Integrating Acceleration

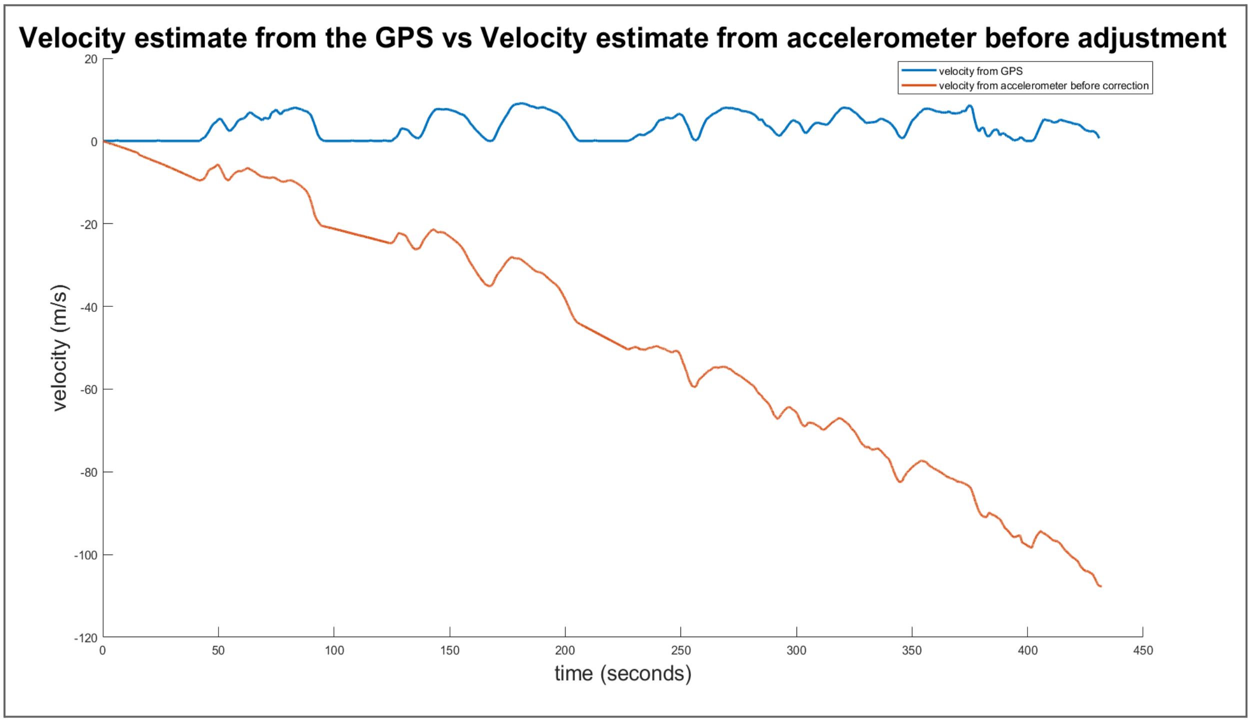

Fig. 6: Comparison of velocity estimates from the GPS and the accelerometer before adjustment

Fig. 6 shows the comparison of velocity obtained from integrating the acceleration x (only along the x-axis as acceleration in the y-axis will be negligible). It is clear from the graph that both the velocities are far apart in magnitude from each other. **The velocity obtained from integrating raw acceleration x drifts apart very quickly due to the biases in the accelerometer**. Although small in magnitude, the biases quickly add up and produce a large offset in the velocity after integration.

The first and natural approach to eliminate the bias was to assume that the bias is constant. This assumption leads to finding a best-fit line for the velocity data, whose slope would be the constant bias in the acceleration. Subtracting this constant bias from the acceleration and then integrating it again yielded better results, but there was still a considerable offset in the velocities, proving that **the bias in the accelerometer is NOT constant**.

To calculate and eliminate bias from the accelerometer, **the sections where the car is stationary are identified based on the value of the norm of acceleration** (in the x, y, and z axes). In the section where the car is stationary, the norm of acceleration will ideally be equal to the acceleration due to gravity ($9.81\ m/s^2$). The array of the norm of acceleration is calculated and a moving average with a window size of 5 is applied to get a better estimation of when the vehicle is stationary. Further, the windows of time are calculated when the norm of acceleration is in the range of $9.65\ -\ 9.85\ m/s^2$ for a minimum of 3 seconds.

Further, the start time and the end time of these sections are obtained. The procedure followed after this is

1. Calculate the mean of the values of acceleration x in the current section (values lying in the corresponding start time and end time).
2. Make the values of acceleration x in the current section 0
3. Subtract the value of the mean from the acceleration x-axis beginning from the end of the current section to the start of the next section
4. Repeat steps 1 to 3 until the end of the array (acceleration x) is reached.

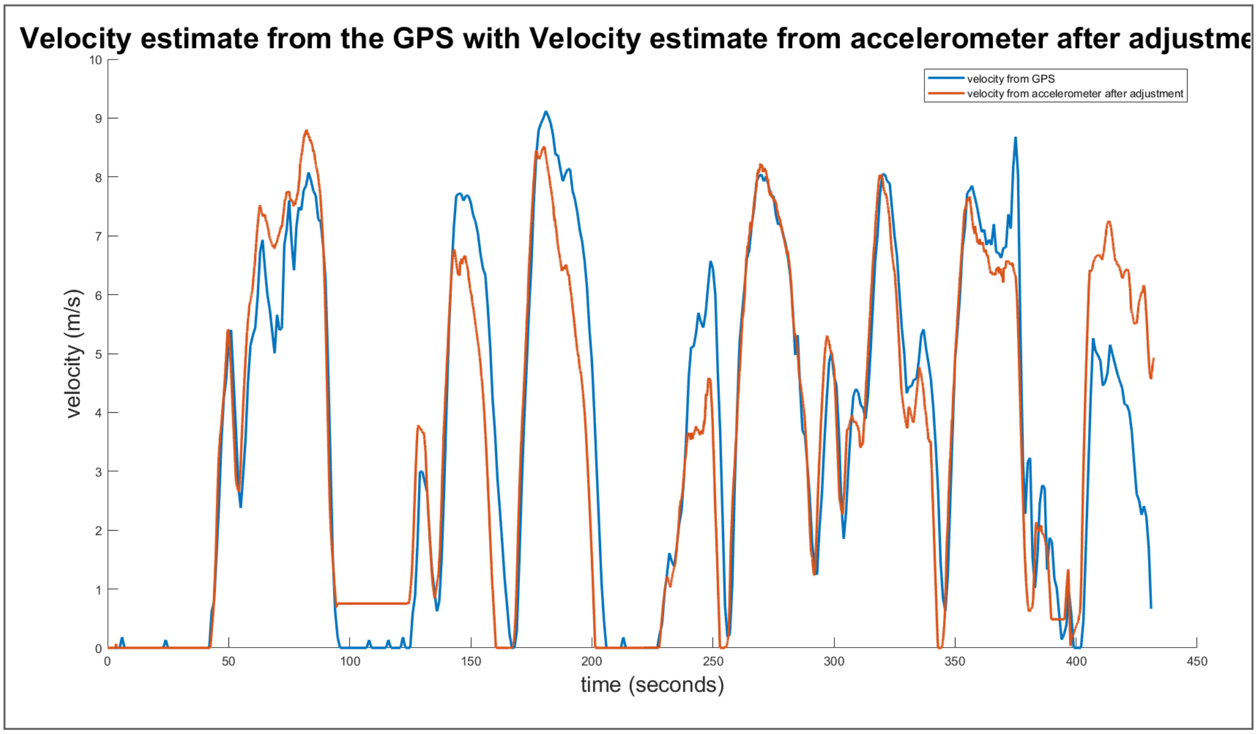

Fig. 7: Comparison of velocity estimates from the GPS and the accelerometer after adjustment

After this, the adjusted acceleration x array is obtained. It is integrated using a custom integration function. The feature of this custom function is that when it encounters a value of acceleration that takes the value of velocity below 0 (which should not be possible as the vehicle was not driven in reverse) then the function sets the velocity as 0.

Fig. 7 is the comparison of the velocities from the GPS and the accelerometer after subtracting the bias from the acceleration. It is clear from the graph that the velocities are much more in agreement with each other. However, there are still sections of data where the velocities do not match. This is due to the fact that **the perfect values of biases of the accelerometer cannot be calculated. Hence, even the tiniest of leaking biases will lead to a significant offset in the velocities after integration.**

Dead Reckoning with IMU

Let the center of the vehicle be $(X, Y, 0)$, its rotation rate about the center of mass be $(0, 0, \omega)$, and the position of the IMU in the vehicle frame be $(x_c, 0, 0)$. Then the acceleration in y-axis measured by the IMU is

$$y'' = Y'' + \omega X' + \omega' x_c$$

Assuming that $x_c = 0$, i.e the IMU is mounted on the center of the vehicle, and $Y' = 0$ (vehicle is not slipping sideways), we can equate $y'' = \omega X'$. The graph of y'' and $\omega X'$ plotted vs time is shown in Fig. 8

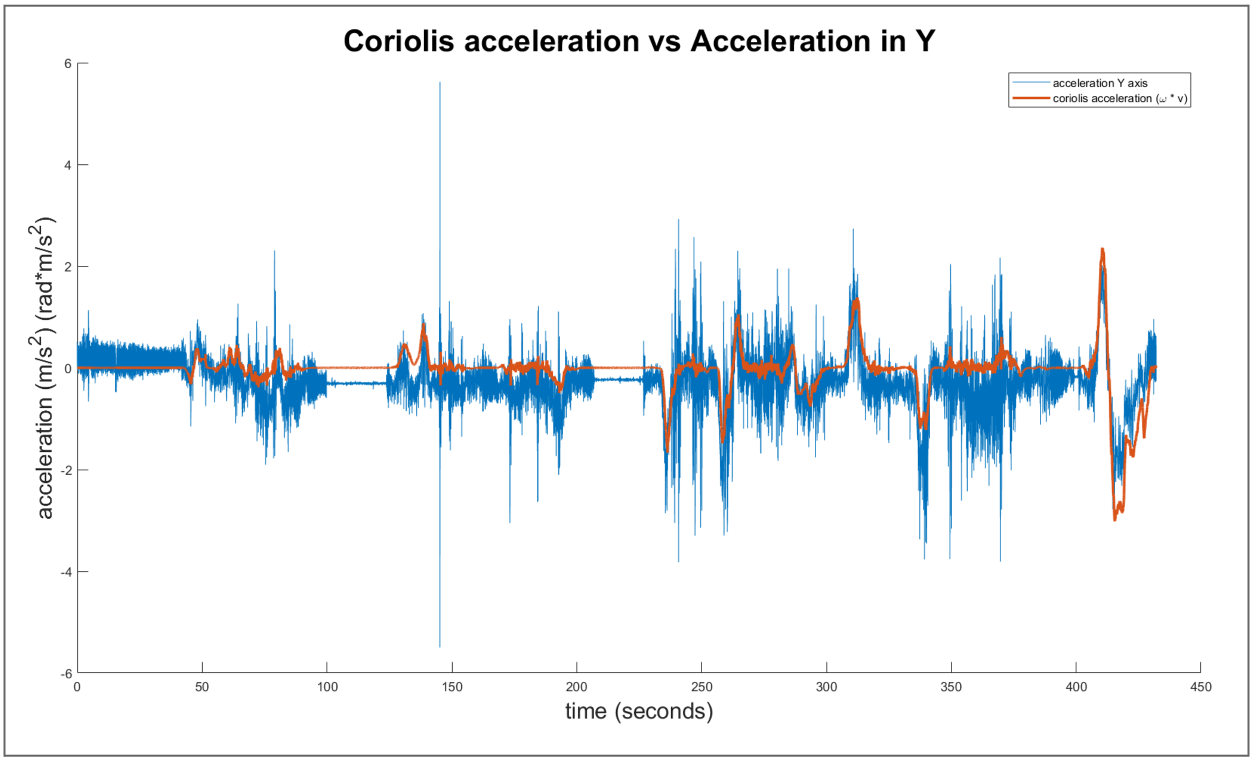

Fig. 8: Comparison of acceleration in y-axis with the coriolis acceleration

It is observed in Fig. 8 that the shape of $\omega X'$ and y'' roughly matches each other, but there are lot of **high-frequency components present additionally in the acceleration in the y-axis which affect the comparison of values**. To compensate for that, I **applied a moving mean** on the data with a window size of 15 (which is essentially a low pass filter) and then compared the results as seen in Fig. 9

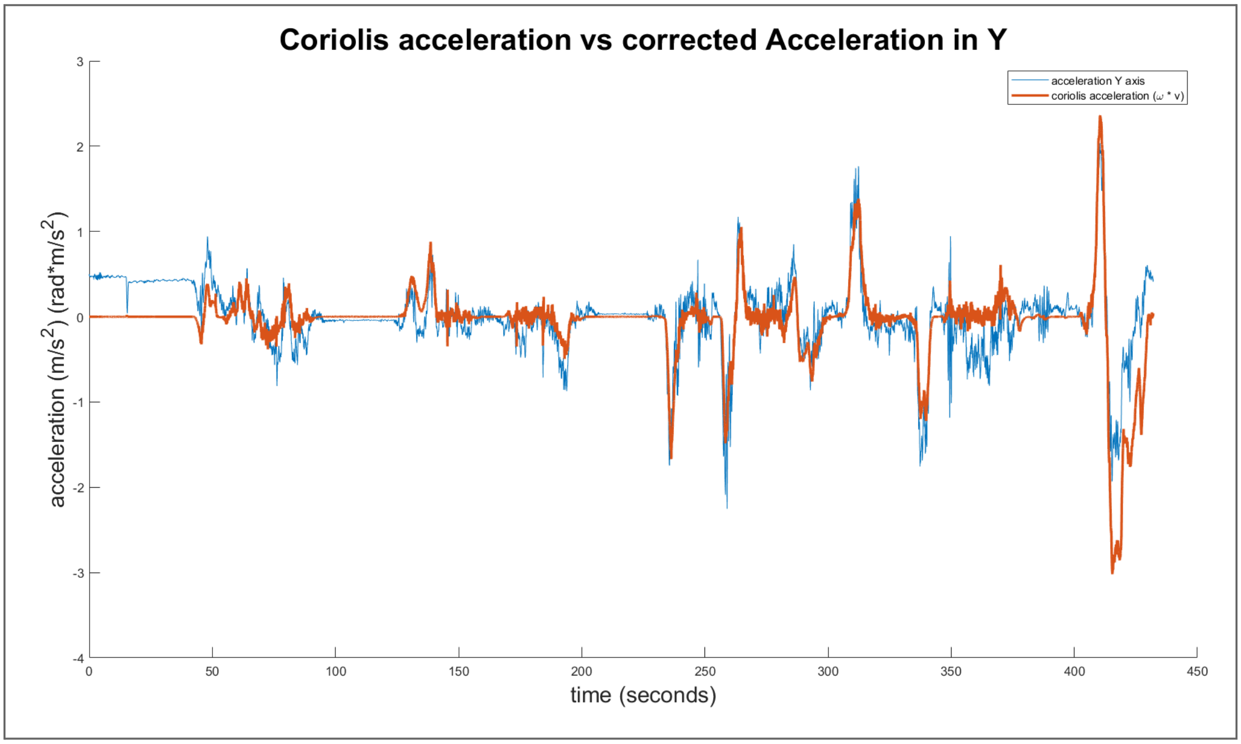

Fig. 9: Comparison of acceleration in y-axis with the coriolis acceleration after moving mean

In Fig.9, it is pretty evident that both the quantities are close in magnitude. **The offset present in the values can be attributed to the value of $\omega' x_c$ which is assumed to be 0, but may not be 0.**

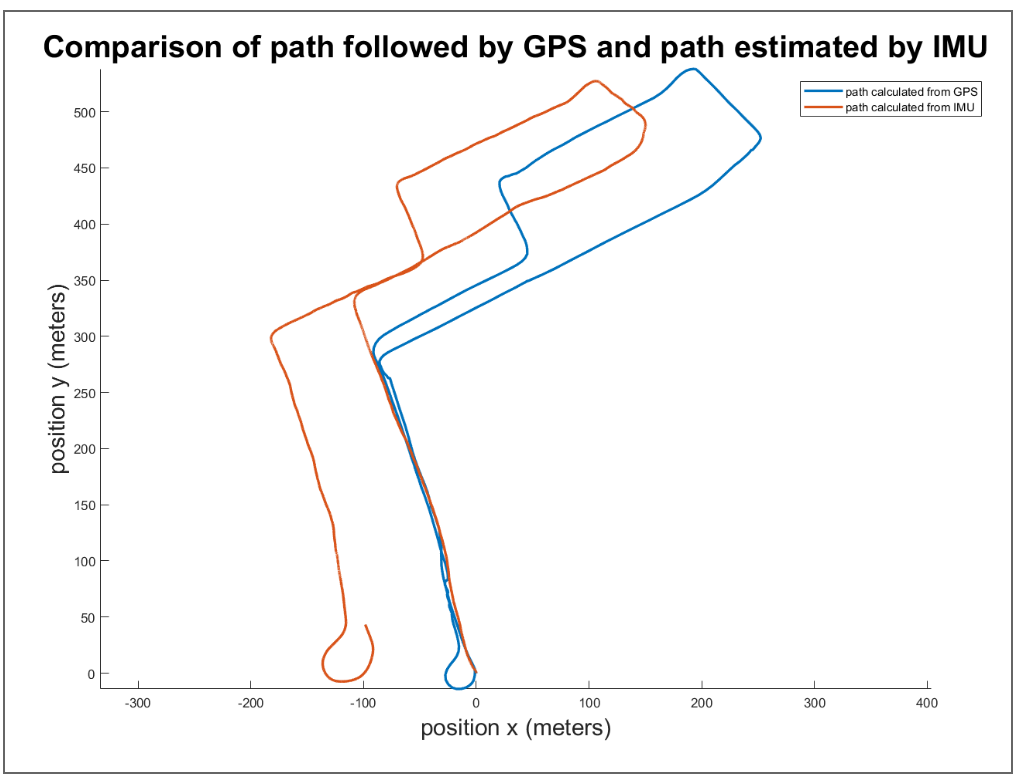

Fig. 10: Comparison of path calculated by GPS and path estimated by IMU

Fig. 10 shows the plot of the dead reckoning performed with the IMU. **The plot is NOT scaled by any factor, and the starting point and the heading are matched**. Clearly, the starting point and the ending point of the path estimated by IMU do not match.

The estimation of the position by GPS and by IMU match closely within 2 meters only for 52.275 seconds.

The accumulated error equals 31.07 meters within 10 seconds which is significantly greater than the 0.7 meters, which is mentioned in the datasheet of Vectornav VN-100

The reasons for the errors observed in dead reckoning are:

1. Improper estimation of bias in the acceleration, which leads to offsets in the velocities of GPS and IMU
2. Integration of errors in velocity which give rise to large offsets in the position estimation

Estimation of Xc

We have the equation

$$y'' = Y'' + \omega X' + \omega' x_c$$

In this equation, X' can be assumed to be the velocity calculated by integrating adjusted acceleration. Y'' is assumed to be 0. Thus we have only one unknown x_c, which converts the above equation to the linear algebraic form with one unknown, and 17282 equations.

$$Ax_c = B$$

x_c can be estimated with least squared eros formula $x_c = (A^T A)^{-1} A^T B$

Plugging this equation in MATLAB, x_c is **estimated to be about 0.529 meters or 5.29 cm.**