FIITJEE

Solutions to JEE(Main) -2024

Test Date: 1st February 2024 (Second Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*1. If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$ is $(-\infty,\alpha)\cup[\beta,\infty)$, then $\alpha^2 + \beta^3$ is equal to:

(1) 140 (2) 175
(3) 125 (4) 150

Ans. **(4)**
Sol. $x^2 - 25 \ge 0 \Rightarrow (x-5)(x+5) \ge 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$
$4 - x^2 \ne 0 \Rightarrow x \ne -2, 2$
$x^2 + 2x - 15 > 0 \Rightarrow (x+5)(x-3) > 0 \Rightarrow x \in (-\infty, -5) \cup (3, \infty)$
Taking intersection, we get $(-\infty, -5) \cup [5, \infty)$
$\therefore\ \alpha = -5, \beta = 5 \Rightarrow \alpha^2 + \beta^2 = 25 + 125 = 150$

*2. If z is a complex number such that $|z| \ge 1$, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is:

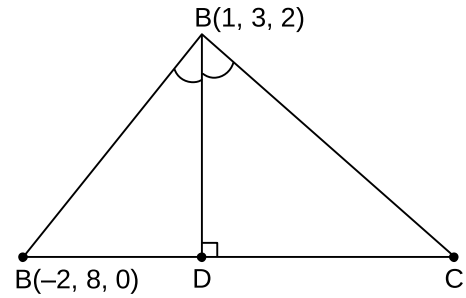
(1) 2 (2) $\frac{5}{2}$
(3) $\frac{3}{2}$ (4) 3

Ans. **(3)**
Sol. We know that $|z_1 + z_2| \ge ||z_1| - |z_2||$
$\Rightarrow \left|z+\left(\frac{3}{2}+2\hat{i}\right)\right| \ge \left||z|-\left|\frac{3}{2}+2\hat{i}\right|\right| \ge \left|1-\frac{5}{2}\right| \ge \frac{3}{2}$

3. Consider a ΔABC where A(1, 3, 2), B(– 2, 8, 0) and C(3, 6, 7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is:

(1) $\frac{37}{2\sqrt{38}}$ (2) $\sqrt{19}$
(3) $\frac{39}{2\sqrt{38}}$ (4) $\frac{\sqrt{38}}{2}$

Ans. **(1)**



Sol. $\left|\overrightarrow{AB}\right| = \left|-3\hat{i} + 5\hat{j} - 2\hat{k}\right| = \sqrt{38}$

$\left|\overrightarrow{AC}\right| = \left|2\hat{i} + 3\hat{j} + 5\hat{k}\right| = \sqrt{38}$

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{1} \Rightarrow BD = DC$

$\therefore\ D\left(\frac{1}{2}, 7, \frac{7}{2}\right) \Rightarrow \overrightarrow{AD} = -\frac{1}{2}\hat{i} + 4\hat{j} + \frac{3}{2}\hat{k}$

$\Rightarrow \overrightarrow{AD}\cdot\widehat{AC} = \left(-\frac{1}{2}\hat{i} + 4\hat{j} + \frac{3}{2}\hat{k}\right)\cdot\frac{\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)}{\sqrt{38}} = \frac{37}{2\sqrt{38}}$

4. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b)R_2(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then

(1) R_1 and R_2 both are equivalence relations
(2) only R_1 is an equivalence relation
(3) only R_2 is an equivalence relation
(4) neither R_1 nor R_2 is an equivalence relation

Ans. **(3)**

Sol. $a^2 + a^2 = 1 \Rightarrow a = \pm\frac{1}{\sqrt{2}}$ $\therefore$ $(a, a) \notin R_1\ \forall\ a \in R$ $\therefore$ R_1 is not reflexive

$\therefore$ R_1 is not equivalence relation

$a + b = b + a \Rightarrow (a, b), (a, b) \in R_2$ $\therefore$ R_2 is reflexive

If $(a, b)\ R_2\ (c, d) \Rightarrow a + d = b + c$

$\therefore$ If $a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d)\ R_2\ (a, b)$ $\therefore$ R_2 is symmetric

For transitive if we need to see it $(a, b)\ R_2\ (c, d)$ and $(c, d)\ R_2\ (e, f) \Rightarrow (a, b)\ R_2\ (e, f)$

If $(a, b)\ R_2\ (c, d) \Rightarrow a + d = b + c$ ..... (1)

If $(c, d)\ R_2\ (e, f) \Rightarrow c + f = d + e$ ..... (2)

Adding equation (1) + (2), we get $a + f = b + e \Rightarrow (a, b)\ R_2\ (e, f)$

$\therefore$ R_2 is transitive $\Rightarrow$ R_2 is equivalence relation

5. Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:

(1) 22 (2) 17
(3) 15 (4) 28

Ans. **(2)**

Sol. $D = 0 \Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$

$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$

$\therefore\ \lambda + 2\mu = 17$

6. If $\int_0^{\frac{\pi}{3}} \cos^4 x\,dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to:

(1) 2 (2) 1

(3) 3 (4) $\frac{3}{2}$

Ans. **(1)**

Sol. $\int_0^{\frac{\pi}{3}} \left(\frac{1+\cos 2x}{2}\right)^2 dx \Rightarrow \frac{1}{4}\int 1+\cos^2 2x + 2\cos 2x\,dx$

$= \frac{1}{4}\int \frac{3}{2}+\frac{\cos 4x}{2}+2\cos 2x\,dx = \frac{3x}{8}+\frac{\sin 4x}{32}+\frac{\sin 2x}{4}\Big|_0^{\frac{\pi}{3}}$

$= \frac{\pi}{8}+\frac{7}{64}\sqrt{3} \Rightarrow a = \frac{1}{8}, b = \frac{7}{64}$

$\Rightarrow 9a + 8b = \frac{9}{8}+\frac{7}{8} = 2$

*7. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is:

(1) 8 (2) 9

(3) $\frac{20}{3}$ (4) $\frac{80}{9}$

Ans. **(4)**

Sol. Let common ratio of G.P. be R

$\therefore \alpha, \beta$ are roots of $px^2 + pRx - pR^2 = 0$

$\therefore \alpha + \beta = -R$; $\alpha\beta = -R^2$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4} \Rightarrow \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4} \Rightarrow \frac{1}{R}=\frac{3}{4} \Rightarrow R=\frac{4}{3}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = R^2 + 4R^2 = 5P^2 = 5\times\frac{15}{9}=\frac{80}{9}$

8. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is:

(1) $\frac{9}{35}$ (2) $\frac{3}{35}$

(3) $\frac{24}{35}$ (4) $\frac{18}{35}$

Ans. **(4)**

Sol. Let A denote the event of AJAY appearing in JEE exam and B denote the event of VIJAY appearing in JEE exam

Given $P(\bar{A}) = \frac{2}{7} \Rightarrow P(A \cap B) = \frac{1}{5}$

We need to find $P(A \cap \bar{B})$

$P(A \cap \bar{B}) = P(A) = P(A \cap B) = \frac{5}{7} - \frac{1}{5} \Rightarrow \frac{18}{35}$

*9. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R and PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is:

(1) $\frac{13}{21}$
(2) $\frac{\sqrt{139}}{23}$
(3) $\frac{\sqrt{13}}{7}$
(4) $\frac{11}{19}$

Ans. (3)

Sol. $P(3\cos\theta, 2\sin\theta)$

Line passing through P and parallel to y-axis is $x = 3\cos\theta$

$\therefore Q(3\cos\theta, 3\sin\theta)$

Let R be (h, k)

$\Rightarrow h = \frac{12\cos\theta + 9\cos\theta}{7} \Rightarrow k = \frac{12\sin\theta + 6\sin\theta}{7}$

$\therefore$ Locus $\frac{7x^2}{21^2} + \frac{7y^2}{18^2} = 1$

$\therefore 18^2 = 21^2(1 - e^2) \Rightarrow e^2 = \frac{117}{21^2} \Rightarrow e = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$

*10. Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to:

(1) 2
(2) 1
(3) $\frac{5}{2}$
(4) $\frac{3}{2}$

Ans. (1)

Sol. $\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{2 + 10\alpha}{10} = 0.2 + \alpha$

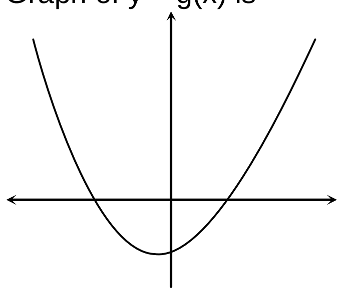
$\Rightarrow 0.2 + \alpha = 1.2 \Rightarrow \alpha = 1$

$\sigma^2 = \frac{\sum x_i^2}{10} - (\bar{x})^2 \Rightarrow \frac{84}{25} = \frac{40 - 10\beta^2 + 2\beta(2 + 10\alpha)}{10} - \frac{36}{25}$

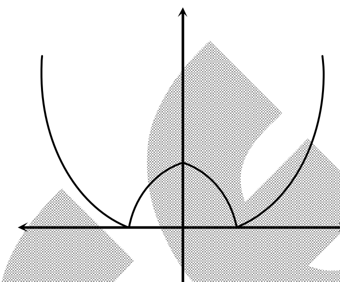
$\Rightarrow 5\beta^2 - 12\beta + 4 = 0 \Rightarrow \beta = 2$

11. let $f(x) = |2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to:
(1) 5 (2) 3
(3) 2 (4) 0

Ans. **(2)**
Sol. Let $g(x) = 2x^2 + 5x - 3 = (2x - 1)(x + 3)$
Graph of y = g(x) is



$\therefore$ Graph of y = f(x) = |g(|x|)| is



It is continuous everywhere
$\therefore$ m = 0
It is non differentiable at $x = -\frac{1}{2}$, $x = \frac{1}{2}$ and x = 0
$\therefore$ n = 3

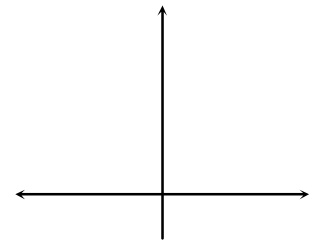
*12. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is:
(1) 0 (2) 3
(3) 1 (4) 2

Ans. **(1)**
Sol. Let $\cos x = t \Rightarrow 4t^3 + 4t^2 + 4t - 13 = 0$ where $t \in [-1, 1]$
Let $f(t) = 4t^3 + 4t^2 + 4t - 13$
$f'(t) = 12t^2 + 8t + 4 > 0 \ \forall \ t \in R$
$\therefore$ f(t) is always increasing
$f(-1) = -4 + 4 - 4 - 13 = -17$
$f(1) = 4 + 4 + 4 - 13 = -1$
$\therefore$ For $t \in [-1, 1]$ f(t) is never 0

*13. Let the locus of the midpoints of the chords of the circle $x^2 + (y - 1)^2 = 1$ drawn from the origin intersect the line x + y = 1 at P and Q. Then, the length of PQ is:
(1) $\frac{1}{2}$ (2) 1
(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Ans. **(3)**

Sol. Let the midpoint of chord be (h, k)



$\therefore$ Equation of chord is $T = S_1$
$\Rightarrow hx + ky - (y + k) = h^2 + k^2 - 2k$
$\because$ It passes through (0, 0) $\therefore -k = h^2 + k^2 - 2k$
$\therefore$ Locus is $x^2 + y^2 - y = 0$
For P and Q $(1 - y)^2 + y^2 - y = 0$
$\Rightarrow 2y^2 - 3y + 1 = 0 \Rightarrow y = 1, \frac{1}{2}$
$\therefore x = 0,\ x = \frac{1}{2}$
$\therefore P(0, 1)$ and $Q = \left(\frac{1}{2}, \frac{1}{2}\right)$ $\therefore PQ = \frac{1}{\sqrt{2}}$

14. Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim\limits_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to _______.
(1) 7
(2) 9
(3) 3
(4) 5

Ans. **(2)**
Sol. $\frac{dy}{dx} = \alpha y + 3 \Rightarrow \int \frac{dy}{\alpha y + 3} = \int dx \Rightarrow \frac{1}{\alpha} \ln|2\alpha + 3| = x + c$
$\Rightarrow \ln|\alpha y + 3| = \alpha(x + c)$,
at x = 0, y = 2
$\Rightarrow \ln|2\alpha + 3| = \alpha c \Rightarrow c = \frac{1}{\alpha}\ln|2\alpha + 3|$
$\Rightarrow \frac{1}{\alpha}\ln|\alpha y + 3| = x + \frac{1}{\alpha}\ln|2\alpha + 3|$
$\Rightarrow \ln|\alpha y + 3| = \alpha x + \ln|2\alpha + 3| \Rightarrow \frac{\alpha y + 3}{2\alpha + 3} = e^{\alpha x}$
$\Rightarrow \alpha y + 3 = (2\alpha + 3)e^{\alpha x}$ as $x \to -\infty$, $y = 1 \Rightarrow \alpha + 3 = 0 \Rightarrow \alpha = -3$
$\Rightarrow -3y + 3 = -3e^{-3x} \Rightarrow -y + 1 = -e^{-3x} \Rightarrow y = 1 + e^{-3x}$
At $x = -\log_{e^2}$; $y = 1 + e^{3\log_{e^2}} = 9$

15. Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is:
(1) 18
(2) 24
(3) 26
(4) 36

Ans. **(1)**
Sol. Any point on the line is $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$
It distance from R (1, 2, 3) is 6 units
$\sqrt{(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2} = 6$
$\Rightarrow 72\lambda^2 - 72\lambda + 36 = 36 \Rightarrow \lambda = 0, 1$

$\therefore$ P(–3, 4, –1) and Q(5, 6, 1)
Centroid (1, 4, 1) $\therefore$ $\alpha^2 + \beta^2 + \gamma^2 = 1 + 16 + 1 = 18$

16. The value of $\int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}} dx$ is equal to:

(1) – 1 (2) 2
(3) 0 (4) 1

Ans. **(3)**

Sol. Let $I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$

Using the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow I = \int_a^1 2(1-x)^3 - 3(1-x)^2 - ((1-x)+1)^{\frac{1}{3}} dx = \int_a^1 (-2x^3 + 3x^2 + x - 1)^{\frac{1}{3}} dx = -I$

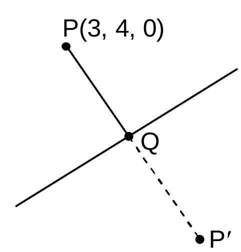
$\Rightarrow I = 0$

17. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ), then 14 ($\alpha + \beta + \gamma$) is:

(1) 102 (2) 138
(3) 132 (4) 108

Ans. **(4)**

Sol. Any point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is



Q(3λ + 1, 2λ – 1, λ + 2)
D.R of PQ (3λ – 2, 2λ – 5, λ – 7)
PQ is perpendicular to line 3(3λ – 2) + 2(2λ – 5) + λ – 7 = 0

$\Rightarrow 14\lambda - 23 = 0 \Rightarrow \lambda = \frac{23}{14}$

$\therefore$ $Q\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$ $\therefore$ Image $P'\left(\frac{62}{7}, \frac{4}{7}, -\frac{12}{7}\right)$ $\therefore$ 14($\alpha + \beta + \gamma$) = 108

*18. Let S_n denote the sum of the first n terms of an arithmetic progression. If S_{10} = 390 and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(1) 800 (2) 890
(3) 790 (4) 690

Ans. **(3)**

Sol. S[2a + 9d] = 390 $\Rightarrow$ 2a + 9d = 78

$\frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a - 3d = 0 \Rightarrow a = 3 \Rightarrow d = 8$

$\therefore$ $S_{15} - S_5 = \frac{15}{2}[6+112] - \frac{5}{2}[6+32] = 790$

*19. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is:

(1) $\frac{1}{9}$
(2) $\frac{1}{4}$
(3) $\frac{4}{9}$
(4) $\frac{9}{4}$

Ans. **(4)**

Sol. $n={}^{18}C_{12}\left(\frac{1}{3}\right)^{6}\left(\frac{1}{2}\right)^{12}$; $m={}^{18}C_{6}\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^{6}$

$\therefore\ \frac{n}{m}=\frac{3^6}{2^6}\Rightarrow\left(\frac{n}{m}\right)^{\frac{1}{3}}=\frac{9}{4}$

20. Let $f(x)=\begin{cases}x-1, & x \text{ is even,}\\ 2x, & x \text{ is odd,}\end{cases}$ $x\in N$. If for some $a\in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to:

(1) 169
(2) 121
(3) 225
(4) 144

Ans. **(4)**

Sol. $f(x)=\begin{cases}x-1 & ;\ x \text{ is even}\\ 2x & ;\ x \text{ is odd}\end{cases}$

If a is even $f(f(a)) = f(a-1) = 2(a-1)$
$\therefore f(f(f(a))) = 2(a-1)-1 \Rightarrow 2a-3$
$\Rightarrow 2a-3=21 \Rightarrow a=12$
If a is odd $f(f(a)) = f(2a) = 2a-1$
$f(f(f(a))) = 2(2a-1)$ thus not possible

$\therefore\ \lim\limits_{x\to 12^-}\frac{|x|^3}{a}-\left[\frac{x}{a}\right]=144$

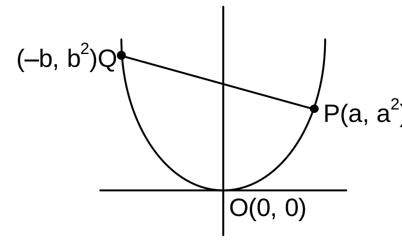
SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*21. Three points O(0, 0), $P(a, a^2)$, $Q(-b, b^2)$, $a>0$, $b>0$, are on the parabola $y=x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ________.

Ans. 7



Sol. Equation of PQ = $y - a^2 = \frac{a^2 - b^2}{a+b}(x - a)$

$\Rightarrow (a - b)x - y + ab = 0$

$\therefore S_1 = \int_{-b}^{a}\left((a-b)x + ab - x^2\right)dx$

$= (a+b)\left(\frac{(a-b)^2}{2} + ab - \frac{(a^2+b^2-ab)}{3}\right)$

$S_2 = \frac{1}{2}ab(a+b)$

$\therefore \frac{S_1}{S_2} = \frac{\frac{(a-b)^2}{2} + ab - \frac{(a^2+b^2-ab)}{3}}{ab/2} = \frac{(a+b)^2}{3ab} = \frac{1}{3}\left(\frac{a}{b} + \frac{b}{a} + 2\right)$

$\therefore$ minimum value of $\frac{S_1}{S_2}$ is $\frac{4}{3}$ $\left(\because \frac{a}{b} + \frac{b}{a} \ge 2\right)$ $\therefore m + n = 7$

22. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to _______.

Ans. **8**

Sol. $y^2 = \frac{k}{2}x$ and $\left(y - \frac{1}{k}\right)^2 = \frac{-2}{k}\left(x - \frac{1}{2k}\right)$

For point of intersection

$ky^2 = 2\left(y - \frac{2}{k}y^2\right) \Rightarrow y = 0 \text{ and } y = \frac{2k}{k^2+4}$

$\therefore$ Area bounded $\int_{0}^{\frac{2k}{k^2+4}}\left(\frac{2y - ky^2}{2} - \frac{2y^2}{k}\right)dy = \frac{2k^2}{3(k^2+4)^2} = \frac{2}{3\left(k^2 + \frac{16}{k^2} + 8\right)}$

For maximum area we have to minimize $k^2 + \frac{16}{k^2} + 8$

$\frac{k^2 + \frac{16}{k^2}}{2} \ge 4 \Rightarrow k^2 + \frac{16}{k^2} \ge 8$

It holds for $k = \pm 2$

23. If $y = \frac{\left(\sqrt{x}+1\right)\left(x^2 - \sqrt{x}\right)}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to ______.

Ans. **105**

Sol. $y = \frac{x^2\sqrt{x} - x + x^2 - \sqrt{x}}{x\sqrt{x} + x + \sqrt{x}} + \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$

$y = \frac{(x-1)\left(x\sqrt{x} + x + \sqrt{x}\right)}{x\sqrt{x} + x + \sqrt{x}} + \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$

$\frac{dy}{dx} = 1 - \cos^4 x \sin x + \cos^2 x \sin x$

$\therefore$ at x = $\frac{\pi}{6}, \frac{dy}{dx} = 1 - \frac{9}{32} + \frac{3}{8} = \frac{35}{32}$

$\therefore$ $96y'\left(\frac{\pi}{6}\right) = 105$

24. If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, x(1) = 1, then 5x(2) is equal to_________.

Ans. 5

Sol. $\frac{dx}{dy} - \frac{x}{y} = \frac{1}{y} - y$

Integrating factor = $e^{\int -\frac{1}{y}dy} = \frac{1}{y}$

$\therefore$ $\frac{x}{y} = \int\left(\frac{1}{y^2} - 1\right)dy \Rightarrow x = -1 - y^2 + cy$

at y = 1, x = 1 $\therefore$ c = 3 $\therefore$ x = –1 – y^2 + 3y

at y = 2, x = –1 – 4 + 6 = 1

$\therefore$ 5x(2) = 5

25. Let f: (0, ∞) → R and F(x) = $\int_0^x tf(t)dt$. If F(x^2) = x^4 + x^5, then $\sum_{r=1}^{12} f\left(r^2\right)$ is equal to ________.

Ans. 219

Sol. $F(x^2) = \int_0^{x^2} t f(t) dt$

Differentiating both sides w.r.t. x

$F'(x^2) \times 2x = x^2 f(x^2) \times 2x$

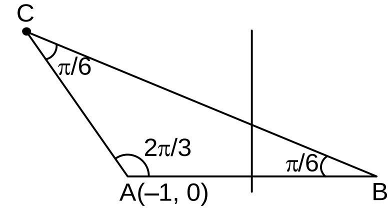
$\Rightarrow$ $f(x^2) = 2 + \frac{5}{2}x$

$\therefore$ $\sum_{r=1}^{12} f\left(r^2\right) = \sum_{r=1}^{12} 2 + \frac{5}{2}r = 24 + \frac{5}{2} \times \frac{12 \times 13}{2} = 219$

*26. Let ABC be an isosceles triangle in which A is at (– 1, 0), ∠A = $\frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If BC = $4\sqrt{3}$ and the line BC intersects the line y = x + 3 at (α, β), then $\frac{\beta^4}{\alpha^2}$ is _______ .

Ans. 36

Sol. Let AB = AC = x



Using sine rule

$$\frac{AB}{\sin 30^\circ} = \frac{BC}{\sin 120^\circ}$$

$$\Rightarrow\ 2x = \frac{4\sqrt{3}}{\sqrt{3}/2} \Rightarrow x = 4$$

$\therefore$ B(3, 0)

Equation of BC

$$y - 0 = -\frac{1}{\sqrt{3}}(x-3) \Rightarrow x+\sqrt{3}y-3=0$$

Put y = x + 3 $\Rightarrow\ x+\sqrt{x}+3\sqrt{3}-3=0$

$$\Rightarrow\ x = \frac{3\left(1-\sqrt{3}\right)}{\sqrt{3}+1},\ y = \frac{6}{\sqrt{3}+1}$$

$$\therefore\ \frac{\beta^4}{\alpha^2} = \frac{1296\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)^4 \times 9\left(1-\sqrt{3}\right)^2} = 36$$

27. Let A = $I_2 - 2\,MM^T$, where M is a real matrix of order 2 $\times$ 1 such that the relation $M^TM = I_1$ holds. if λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order 2 $\times$ 1, then the sum of squares of all possible values of λ is equal to __________.

Ans. **2**

Sol. Let A = $I_2 - 2MM^T$

M is real matrix 2 $\times$ 1 order

Let M = $\begin{bmatrix} a \\ b \end{bmatrix}$ a, b $\in$ R

$$\Rightarrow\ MM^T = \begin{bmatrix} a \\ b \end{bmatrix}[ab] = \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$$

$M^TM = [a^2 + b^2] = [1]\ \Rightarrow a^2 + b^2 = 1$

Now, $I_2 - 2MM^T = \begin{bmatrix} 1-2a^2 & -2ab \\ -2ab & 1-2a^2 \end{bmatrix} = A$

Now, $(A - \lambda I)\,X = 0$

$$\Rightarrow\ \begin{bmatrix} 1-\lambda-2a^2 & -2ab \\ -2ab & 1-\lambda-2b^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Since, X is non zero real matrix

$$\begin{vmatrix} 1-\lambda-2a^2 & -2ab \\ -2ab & 1-\lambda-2b^2 \end{vmatrix} = 0$$

$\Rightarrow\ (1-\lambda)^2 - 2(1-\lambda)(b^2 + a^2) = 0, \Rightarrow (1-\lambda)[1-\lambda-2] = 0$

$\lambda = \pm 1$

28. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = -\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c} = 4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b}\times\vec{a} = \vec{c}\times\vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i}+4\hat{j}+\hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is _______ .

Ans. **38**

Sol. $\vec{b}\times\vec{a}=\vec{c}\times\vec{a} \Rightarrow (\vec{c}-\vec{b})\times\vec{a}=0$

$\Rightarrow \vec{c}-\vec{b}$ is parallel to $\vec{a} \Rightarrow \vec{c}=\vec{b}+\lambda\vec{a}$

$\Rightarrow 4\hat{i}+c_2\hat{j}+c_3\hat{k}=(\lambda-1)\hat{i}+(\lambda-8)\hat{j}+(\lambda+2)\hat{k}$

$\Rightarrow \lambda = 5 \therefore c_2 = -3, c_3 = 7$

$\therefore \vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$

$\cos\theta=\sqrt{74}\sqrt{26} \Rightarrow \tan\theta=\frac{\sqrt{1875}}{7} \Rightarrow \tan^2\theta=\frac{1875}{49}=38.26$

*29. If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to _____

Ans. **1**

Sol. Let the length of the sides of a triangle are a, ar, ar^2

$\Rightarrow a + ar > ar^2 \Rightarrow r^2 - r - 1 < 0 \Rightarrow r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$\Rightarrow [r] = 1$ and $[-r] = -2 \Rightarrow 3[r] + [-r] = 3 + (-2) = 1$

*30. The lines $L_1, L_2, \ldots\ldots, L_{20}$ are distinct. For n = 1, 2, 3, …., 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots\ldots, L_{20}\}$ is equal to ______ .

Ans. **101**

Sol. Maximum number of point of intersection will occur if the point of concurrency of L_{2n} does not lie on the lines L_{2n-1} and none of these of the lines from L_{2n} is parallel to L_{2n-1}

Then number of point of intersection will be

(No. of ways of choosing 1 line from L_{2n}) $\times$ (No. of ways of choosing 1 line from L_{2n-1}) + 1 point of concurrency of L_{2n}

$= {}^{10}C_1\times{}^{10}C_1+1=101$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

31. From the statements given below:
(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of $\hbar$.
(B) Nuclear forces do not obey inverse square law.
(C) Nuclear forces are spin dependant.
(D) Nuclear forces are central and charge independent.
(E) Stability of nucleus is inversely proportional to the value of packing fraction.
Choose the correct answer from the options given below:
(1) (B), (C), (D), (E) only
(2) (A), (C), (D), (E) only
(3) (A), (B), (C), (E) only
(4) (A), (B), (C), (D) only

Ans. (3)

Sol. Theoretical

*32. A body of mass 4 kg experience two forces $\vec{F_1} = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F_2} = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is:
(1) $2\hat{i} + \hat{j} + \hat{k}$
(2) $4\hat{i} + 2\hat{j} + 2\hat{k}$
(3) $-2\hat{i} - \hat{j} - \hat{k}$
(4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Ans. (1)

Sol. $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 8\hat{i} + 4\hat{j} + 4\hat{k}$

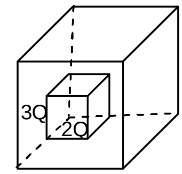
$\vec{a} = \frac{\vec{F}_{net}}{m} = 2\hat{i} + \hat{j} + \hat{k}$

33. Monochromatic light of frequency 6×10^{14}Hz is produced by a laser. The power emitted is 2×10^{-3}W . How many photons per second on an average are emitted by the source?
(1) 5×10^{15}
(2) 7×10^{16}
(3) 6×10^{15}
(4) 9×10^{18}

Ans. (1)

Sol. n(per second) = $\frac{P}{hr} = 5 \times 10^{15}$

34. C_1 and C_2 are two hollow concentric cubes enclosing charges 2Q and 3Q respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is:

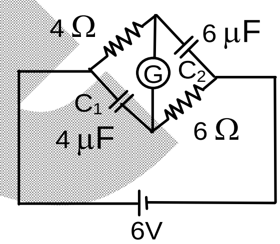


(1) 3 : 2 (2) 5 : 2
(3) 2 : 5 (4) 2 : 3

Ans. **(3)**

Sol. $\dfrac{Q_{c_1}}{Q_{c_2}} = \dfrac{2Q}{2Q+3Q} = \dfrac{2}{5}$

35. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is:



(1) 1 (2) $\dfrac{2}{3}$
(3) $\dfrac{3}{2}$ (4) $\dfrac{1}{2}$

Ans. **(4)**

Sol. $\dfrac{Q_1}{Q_2} = \dfrac{C_1V_1}{C_2V_2} = \dfrac{4(3)}{6(4)} = \dfrac{1}{2}$.

36. Match List-I with List-II.

List-I (Number)	**List-II (Significant figure)**
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below:
(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III) (2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV) (4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. **(1)**

Sol. Theoretical

*37. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:
(1) $\dfrac{1}{100}$th (2) $\dfrac{1}{10}$th
(3) 100 times (4) 10 times

Ans. **(2)**

Sol. $\frac{E_2}{E_1} = \frac{4\pi(10R)^2 \times T}{1000(4\pi)R^2 \times T} = \frac{1}{10}$

*38. A cricket player catches a ball of mass 120g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

(1) 30 (2) 24
(3) 12 (4) 25

Ans. (1)

Sol. F = $\frac{\Delta P}{\Delta t} = 30N$

39. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω, the balance length changes by:

(1) 62.5 cm (2) 22.5 cm
(3) 20 cm (4) 65 cm

Ans. (2)

Sol. $\frac{2}{40} = \frac{X}{60}$

X = 3 Ω

$\frac{2}{\ell'} = \frac{6/5}{100-\ell'}$, ℓ' = 62.5 cm, $\Delta\ell$ = 22.5 cm.

*40. A diatomic gas $(\gamma = 1.4)$ does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is:

(1) 800 J (2) 600 J
(3) 700 J (4) 850 J

Ans. (3)

Sol. $\frac{Q}{w} = \frac{nC_P\Delta T}{nR\Delta T} = \frac{C_P}{R} = \frac{7}{2}$

Q = 700 J

*41. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}):

(1) –50 and –30 (2) –50 and 30
(3) –30 and 50 (4) 50 and –30

Ans. (2)

Sol. $\vec{v}_{BA} = \vec{V}_B - \vec{V}_A$

V_{BA} = –30 – 20 (taking north direction as positive)

$= -50$ m/s.

$\vec{V}_{GB} = -\vec{V}_{BG}$ = 30 m/s

*42. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option:

(1) $T^2 \alpha R^{7/2}$
(2) $T^2 \alpha R^3$
(3) $T^2 \alpha R^{5/2}$
(4) $T^2 \alpha R^{3/2}$

Ans. (3)

Sol. $\frac{mv^2}{R} = \frac{GMm}{R^{3/2}}$

$V = \sqrt{\frac{GM}{R^{1/2}}}$

$T = \frac{2\pi R}{V} = \frac{2\pi R^{5/4}}{\sqrt{GM}}$

43. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be:

(1) 60^o
(2) 45^o
(3) 15^o
(4) 30^o

Ans. (1)

Sol. a sin $\theta = \lambda$

$\theta = 30^0$

Angular speed = $2\theta = 60^0$

*44. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is:

(1) 1.0
(2) 1.5
(3) 2.0
(4) 0.5

Ans. (4)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_0}}$

$V_{rms)}o_2 = \frac{(V_{rms})H_2}{4} = 0.5 \text{ km/s}$

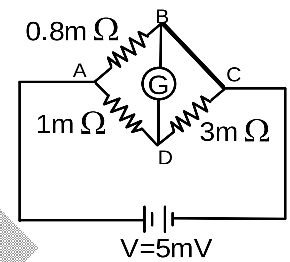
45. If frequency of electromagnetic wave is 60 MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

(1) 2.5
(2) 5
(3) 10
(4) 2

Ans. (2)

Sol. $C = \gamma \lambda$
$\lambda = 5m$

46. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement show in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25^oC and resistance of the semiconductor arm is $3m\Omega$. Arm BC is cooled at a constant rate of $2^oC/s$. If the galvanometer G shows no deflection after 10 s, then α is:



(1) $-1\times10^{-2}\ ^oC^{-1}$ (2) $-2\times10^{-2}\ ^oC^{-1}$
(3) $-2.5\times10^{-2}\ ^oC^{-1}$ (4) $-1.5\times10^{-2}\ ^oC^{-1}$

Ans. **(1)**

Sol. $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
$\Delta R = R\alpha\Delta T$
$\frac{0.8}{1} = \frac{x}{3}$
$x = 2.4\ m\Omega$
$\Delta x = -0.6\ m\Omega$
$\frac{\Delta x}{x} = \alpha\Delta T$
$\alpha = -1 \times 10^{-2}\ ^0C^{-1}$

47. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)eV$. The value of X is:
(Given, $h = 6.6\times10^{-34}$ Js, $e = 1.6\times10^{-19}$ C) .
(1) 11 (2) 13
(3) 15 (4) 21

Ans. **(3)**

Sol. $\frac{hc}{\lambda} = \frac{Xe}{8}$
X = 15

48. A transformer has an efficiency of 80% and work at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is:
(1) 1.33 A (2) 13.33 A
(3) 1.59 A (4) 15.1 A

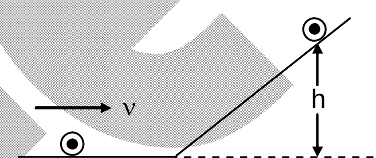
Ans. **(2)**
Sol. $P \times 0.8 = V_S I_S$.
$I_S = 13.33$ A

49. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be:
(1) 199 G
(2) 200 G
(3) $\frac{G}{200}$
(4) $\frac{G}{199}$

Ans. (none)

Sol. $\frac{G}{X} = \frac{95}{9}$
X = G/19
R = $\frac{GX}{G+X} = \frac{G}{20}$

*50. A disc of radius R and mass M is rolling horizontally without slipping with speed v, it then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is:



(1) $\frac{3}{4}\frac{v^2}{g}$
(2) $\frac{v^2}{g}$
(3) $\frac{2}{3}\frac{v^2}{g}$
(4) $\frac{1}{2}\frac{v^2}{g}$

Ans. (4)

Sol. $\frac{1}{2}mv^2 = mgh$ (Rotational K.E. remains constant)
H = $\frac{v^2}{2g}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

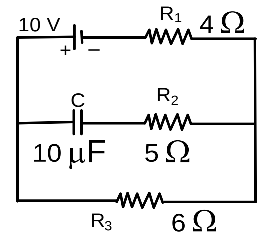
*51. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillation if a mass 9 m is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$ is_________.

Ans. 3

Sol. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$\frac{f_1}{f_2} = \sqrt{\frac{9m}{m}} = 3$$

52. In an electrical circuit drawn below the amount of charge stored in the capacitor is _______μC.



Ans. 60

Sol. At steady state

$I = \frac{10}{4+6} = 1A$

V = 6V

Q = 60 μC

*53. A particle initially at rest starts moving from reference point x = 0 along x – axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is __________ ms^{-2}

Ans. **8**

Sol. $a = \frac{Vdv}{dx}$

$= (4\sqrt{x})\left(4 \times \frac{1}{2\sqrt{x}}\right)$

$= 8\ m/s^2$

54. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}}\mu C$ where $x =$ ______.$\left[\text{use } g = 10\ m/s^2\right]$.

Ans. **3**

Sol. $\tan\theta = \frac{qE}{mg}$

$Q = \frac{1}{\sqrt{3}}\frac{mg}{E} = \frac{1}{\sqrt{3}}\frac{2\times10^{-3}\times10}{2\times10^4} = \frac{1}{\sqrt{3}}\mu C$

55. A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2. The magnetic field produced by the magnet is 0.01 T and the deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x\times10^{-5}$ N−m/rad. The value of x is ________

Ans. **4**

Sol. $G\theta = nIAB$
$N \times 10^{-5} \times 0.05 = 100 \times 10^{-2} \times 2 \times 10^{-4} \times 0.01$
$N = 4$

*56. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be _______
[Area of cross section of wire = 0.005 cm^2, $Y = 2 \times 10^{11}\ Nm^{-2}$ and $g = 10\,m/s^{-2}$].

Ans. **3**

Sol. $\varepsilon = \frac{T}{A\gamma}$
$\frac{\varepsilon_1}{\varepsilon_2} = \frac{T_1}{T_2} = 3$

57. In Young's double slit experiment, monochromatic light of wavelength $5000\mathring{A}$ is used. The slits are 1.0 mm apart and screen is placed at (1)0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is _________ $\times 10^{-6}$ m.

Ans. **125**

Sol. $I = I_{max} \cos^2 \frac{\pi dy}{\lambda D}$
$Y = 125 \times 10^{-6}$ m

58. A coil of 200 turns and area 0.20 m^2 is rotated at half a revolution per second and is placed in uniform magnetic field of 0.01 T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is __________.

Ans. **5**

Sol. $\varepsilon_{max} = nBA\omega$
$= 0.4\,\pi$
$\beta = 5$

*59. A uniform rod AB of mass 2 kg and length 30 cm at rest on a smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}$s, where x = ___________.

Ans. **4**

Sol. $\frac{J\ell}{2} = \frac{m\ell^2}{12}\omega$
$\frac{\pi}{2} = \omega\frac{\pi}{x}$
$x = 4$

60. A particular hydrogen-like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from n = 2 to n = 1. The frequency of radiation emitted in transition from n = 3 to n = 1 is $\frac{x}{9}\times10^{15}$ Hz, when x = ________.

Ans. 32

Sol. $h\nu = 13.6\, z^2 \left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$$\frac{\frac{x}{9}\times10^{15}}{3\times10^{15}} = \frac{1-\frac{1}{3^2}}{1-\frac{1}{2^2}}.$$

x = 32

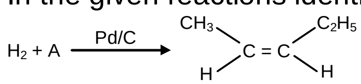
PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*61. In the given reactions identify A and B



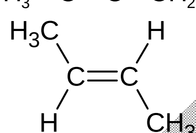
$CH_3-C\equiv C-CH_3+H_2 \xrightarrow{Na/Liquid\,NH_3} "B"$

(1) A : n – Pentane B : Cis – 2 – butene
(2) A : 2 – Pentyne B : Cis – 2 – butene
(3) A : n – Pentane B : trans – 2 – butene
(4) A : 2 – Pentyne B : trans – 2 – butene

Ans. (4)

Sol. $A=CH_3-C\equiv C-CH_2-CH_3$

B =



A will produce cis alkene and Na/Liquid NH_3 gives trans alkene.

*62. Solubility of calcium phosphate (molecular mass, M) in water is W_g per 100 mL at 25°C. Its solubility product at 25°C will be approximately.

(1) $10^7\left(\frac{W}{M}\right)^3$ (2) $10^3\left(\frac{W}{M}\right)^5$

(3) $10^7\left(\frac{W}{M}\right)^5$ (4) $10^5\left(\frac{W}{M}\right)^5$

Ans. (3)

Sol. $S=\frac{W}{M}\times 10\,/\,L$

$$Ca_3(PO_4)_2(s) \rightleftharpoons 3Ca^{++}+2PO_4^{---}$$
$$-\qquad 3S \qquad 2S$$

$$Ksp=\left[Ca^{++}\right]^3\left[PO_4^{---}\right]^2=(3S)^3(2S)^2=104\times(S)^5=104\left(\frac{W}{M}\times 10\right)^5\simeq 10^7\left(\frac{W}{M}\right)^5$$

*63. Given below are two statements:
Statement (I): SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.
Statement (II): Allotropic forms of carbon are due to preprety of catenation and $p\pi-d\pi$ bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Both Statement I and Statement II are true
(4) Statement I is true but Statement II is false

Ans. **(4)**

*64. The set of meta directing functional groups from the following sets is:
(1) $-CN, -NH_2, -NHR, -OCH_3$
(2) $-CN, -CHO, -NHCOCH_3, -COOR$
(3) $-NO_2, -NH_2, -COOH, -COOR$
(4) $-NO_2, -CHO, -SO_3H, -COR$

Ans. **(4)**

65. $[Co(NH_3)_6]^{3+}$ and $[CoF_3]^{3-}$ are respectively known as:
(1) Inner orbital complex, spin paired complex
(2) Spin paired complex, spin free complex
(3) Spin free complex, spin paired complex
(4) Outer orbital complex, inner orbital complex

Ans. **(2)**

Sol. NH_3 is strong ligand while F^- is weak ligand.

66. The transition metal having highest 3rd ionisation enthalpy is:
(1) Mn
(2) Fe
(3) Cr
(4) V

Ans. **(1)**

Sol. $Mn(25) = [Ar], 4s^2, 3d^5$
$Mn^{++} = [Ar]4s^0, 3d^5, d^5$ is stable due to half filled subshell.

67. Match List-I with List-II

List-I Compound		**List-II Use**	
(A)	Carbon tetrachloride	(I)	Paint remover
(B)	Methylene chloride	(II)	Refrigerators and air conditioners
(C)	DDT	(III)	Fire extinguisher
(D)	Freons	(IV)	Non Biodegradable insecticide

Choose the correct answer from the options given below:
(1) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
(2) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)
(3) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)
(4) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Ans. **(2)**

68. Match List-I with List-II

List-I Reactants		List-II Product	
(A)	Phenol, Zn/Δ	(I)	Salicylaldehyde
(B)	Phenol, $CHCl_3$, NaOH, HCl	(II)	Salicylic acid
(C)	Phenol, CO_2, NaOH, HCl	(III)	Benzene
(D)	Phenol, Conc. HNO_3	(IV)	Picric acid

Choose the correct answer from the options given below:
(1) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)
(2) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)
(3) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)
(4) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Ans. (2)

69. Given below are two statements:
Statement (I): Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.
Statement (II): Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Ans. (1)

70. $$C_2H_5Br \xrightarrow{\text{alc. KOH}} A \xrightarrow[CCl_4]{Br_2} B \xrightarrow[\text{Excess}]{KCN} C \xrightarrow[\text{Excess}]{H_3O^+} D$$

Acid D formed in above reaction is:
(1) Malonic acid
(2) Oxalic acid
(3) Succinic acid
(4) Gluconic acid

Ans. (3)

Sol. $A = CH_2 = CH_2$, $B = \underset{|}{\overset{Br}{}}CH_2 - \underset{|}{\overset{Br}{}}CH_2$, C= $\begin{array}{l} H_2C—CN \\ \ \ | \\ H_2C—CN \end{array}$, D = $\begin{array}{l} H_2C—COOH \\ \ \ | \\ H_2C—COOH \end{array}$

71. Lassaigne's test is used for detection of:
(1) Phosphorous and halogens only
(2) Nitrogen, sulphur and phosphorus only
(3) Nitrogen, sulphur, phosphorus and halogens
(4) Nitrogen and sulphur only

Ans. (3)

72. The strongest reducing agent among the following is:
(1) SbH_3
(2) NH_3
(3) BiH_3
(4) PH_3

Ans. (3)

73. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R)
Assertion (A): In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.
Reason (R): Extra stability to half filled electronic configuration is observed than incompletely filled electronic configfurations.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) (A) is true but (R) is false
(2) Both (A) and (R) are true and (R) is the correct explanation of (A)
(3) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(4) (A) is false but (R) is true

Ans. (2)

*74. The functional group that shows negative resonance effect is:
(1) –OH
(2) –OR
(3) –COOH
(4) $-NH_2$

Ans. (3)

*75. The number of radial node/s for 3p orbital is:
(1) 3
(2) 2
(3) 1
(4) 4

Ans. (3)

Sol. Radial node $=(n-\ell-1)=(3-1-1)=1$

*76. Which among the following has highest boiling point?
(1) $CH_3CH_2CH_2CH_2-OH$
(2) $CH_3CH_2CH_2CH_3$
(3) $CH_3CH_2CH_2CHO$
(4) $H_5C_2-O-C_2H_5$

Ans. (1)

Sol. Due to H-bonding

*77. Given below are two statements:
Statement (I): A π bonding MO has lower electron density above and below the inter-nuclear axis.
Statement (II): The π^* antibonding MO has a node between the nuclei.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

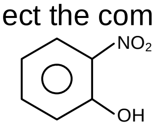
Ans. (4)

78. Given below are two statements:
Statement (I): Both metals and non-metals exist in p and d-block elements.
Statement (II): Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both statement I and statement II are false
(2) Both statement I and statement II are true
(3) Statement I is false but statement II is true
(4) Statement I is true but statement II is false

Ans. (3)

79. Which of the following compounds show colour due to d-d transition?
(1) $K_2Cr_2O_7$ (2) $CuSO_4.5H_2O$
(3) $KMnO_4$ (4) K_2CrO_4

Ans. (2)

*80. Select the compound from the following that will show intramolecular hydrogen bonding.
(1) 

(2) H_2O
(3) C_2H_5OH (4) NH_3

Ans. (1)

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

81. The number of tripeptides formed by three different amino acids using each amino acid once is ___.

Ans. 6

82. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at –24°C is _________ kg (Molar mass in g mol^{-1} for ethylene glycol 62, K_f of water = 1.86 K kg mol^{-1})

Ans. 15

Sol. $\Delta T = \frac{K_f \times w \times 1000}{MW}, 24 = \frac{1.86 \times w \times 1000}{62 \times 18.6 \times 1000}, w = 14.88\,kg$

*83. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is ___________ .

Ans. 6

Sol. $(I)\,CH_3-\underset{\underset{CH_3}{|}}{C}H-CH_2-CH_2-Cl\,(1)$

$(II)\,CH_3-\underset{\underset{CH_3}{|}}{C}H-\overset{\overset{Cl}{|}}{\underset{*}{C}}H-CH_3\,(1+1)$

$(III)\,CH_3-\overset{\overset{Cl}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_2-CH_3\,(1)$

$(IV)\,\overset{\overset{Cl}{|}}{C}H_2-\overset{*}{\underset{\underset{CH_3}{|}}{C}}H-CH_2-CH_3\,(1+1)$

84. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

$A(g) \rightarrow 2B(g) + C(g)$

S. No.	Time/s	Total pressure/(atm)
(1)	0	0.1
(2)	115	0.28

The rate constant of the reaction is __________ $\times 10^{-2}\ s^{-1}$ (nearest integer)

Ans. 2

Sol.

A	$\rightarrow$	$2B + C$		
0.1		0	0	$t = 0$
$(0.1 - P_x)$		$2P_x$	P_x	$t = 115$ sec

According to question

$0.28 = 0.1 - P_x + 2P_x + P_x$

$$P_x = \frac{(0.28 - 0.1)}{2} = 0.09$$

$$K = \frac{2.303}{115}\log\frac{0.1}{(0.1 - 0.09)} = 2 \times 10^{-2}$$

*85. For a certain reaction at 300K, K = 10, then ΔG° for the same reaction is ________ $\times 10^{-1}\,kJmol^{-1}$. (Given R = 8.314 $JK^{-1}\ mol^{-1}$)

Ans. 57

Sol. $$\Delta G^0 = -2.303RT\log K = -\frac{2.303 \times 8.314 \times 300 \times \log 10}{1000} = 5.74 = 57.4 \times 10^{-1} kJ/mol$$

86. The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is ____ $\times 10^5$ C.

Ans. 2

Sol. $H_2O \rightarrow \frac{1}{2}O_2 + 2H^+ + 2e^-$

$2 \times 96500 = 193000C = 1.93 \times 10^5$

$1.93 \approx 2$

*87. Following Kjeldahl's method, 1g of organic compound released ammonia, that neutralised 10 mL of 2M H_2SO_4. The percentage of nitrogen in the compound is _______ %.

Ans. **56**

Sol. $\%N = \frac{1.4 \times 10 \times 2 \times 2}{1} = 56$

88. Consider the following redox reaction:

$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$

The standard reduction potentials are given as below (E_{red}):

$E^o_{MnO_4^-/Mn^{2+}} = +1.51\text{ V}$

$E^o_{CO_2/H_2C_2O_4} = -0.49\text{ V}$

If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of x = _______ (nearest integer)

Ans. **338**

Sol. $E^0_{cell} = E^0_c - E^0_A = -0.49 - (1.51)$

$0 = E^0_{cell} - \frac{0.0591}{n}\log K$

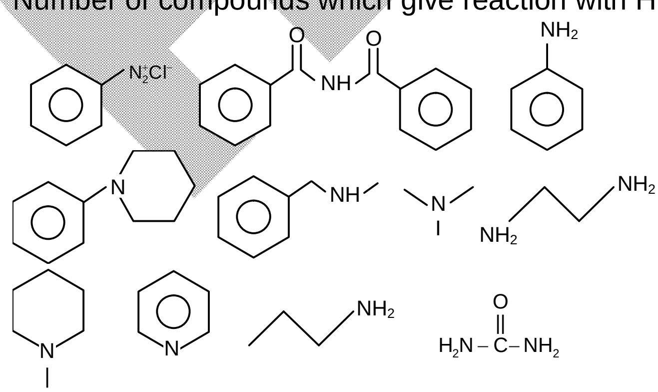
$E^0_{cell} = \frac{0.0591}{10}\log K_{eq}$

$-2 = \frac{0.0591}{10}\log K_{eq}$

$K_{eq} = 10^{338}$

x = 338

89. Number of compounds which give reaction with Hinsberg's reagent is __________ .



Ans. **5**

*90. 10 mL of gaseous hydrocarbon on combustion gives 40 mL of $CO_2(g)$ and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is __________.

Ans. **14**

Sol.

$$CxHy+\left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2+\frac{y}{2}H_2O$$

10 $\qquad\qquad\qquad\qquad$ 10x

$10x = 40$

$x = 4$

$\frac{10y}{2}=50$

$y=10$

C_4H_{10}

$x+y=4+10=14$