FIITJEE

Solutions to JEE(Main) -2024

Test Date: 6th April 2024 (Second Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q1. Let $0 \le r \le n$ If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then 2n + 5r is equal to

(A) 50 (B) 55
(C) 62 (D) 60

Ans. A

Sol. $\frac{{}^{n+1}C_{r+1}}{{}^{n}C_{r}} = \frac{55}{35} \Rightarrow \frac{n+1}{r+1} = \frac{11}{7} \Rightarrow 7n - 11r = 4$ …….. (i)

$\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{35}{21} \Rightarrow \frac{n}{r} = \frac{5}{3} \Rightarrow 3n = 5r$ …………(ii)

By solving (i) and (ii) we get n =10 and r = 6

$\therefore$ 2n + 5r = 20 + 30 = 50.

Q2. A software company sets up m number of computer system to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment The value of m is equal to

(A) 125 (B) 180
(C) 160 (D) 150

Ans. D

Sol. since the total workers finished the work = 17m days

Now according the questions,

m + (m – 4) + (m – 8) + (m – 12) + (m – 16) + …….+ (m - 96) = 17m

$\Rightarrow$ 25m – (4 + 8 + 12 + 16 + ……..+ 96) = 17m

$\Rightarrow$ 25m – 4(1 + 2 + 3 + 4 +………..+ 24) = 17m

$\Rightarrow$ m = 150

Q3. Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$ represents a circle passing through origin. Then radius of the circle is

(A) $\sqrt{17}$ (B) 2
(C) $\frac{\sqrt{17}}{2}$ (D) $\frac{1}{2}$

Ans. C

Sol. $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)} \Rightarrow \left(\beta x - 2\alpha y - (\beta\gamma - 4\alpha)\right)dy = \left((2+\alpha)x - \beta y + 2\right)dx$

$\Rightarrow \beta(xdy + ydx) - (2\alpha + \beta)ydy + 4\alpha dy = (2 + \alpha)xdx + 2dx$

$\Rightarrow \beta d(xy) - (2\alpha + \beta)ydy + 4\alpha dy = (2 + \alpha)xdx + 2dx$

$\Rightarrow \beta xy - (2\alpha + \beta)\frac{y^2}{2} + 4\alpha y = (2 + \alpha)\frac{x^2}{2} + 2x + c$

$\Rightarrow (2 + \alpha)\frac{x^2}{2} - \beta xy + (2\alpha + \beta)\frac{y^2}{2} - 4\alpha y + 2x + c = 0$

$\Rightarrow (2+\alpha)x^2 - 2\beta xy + (2\alpha+\beta)y^2 - 8\alpha y + 4x + 2c = 0$ .Hence it represents a circle therefore $(2+\alpha) = (2\alpha+\beta)$ and $\beta = 0 \Rightarrow \alpha = 2$, $\beta = 0$
Also $(2+\alpha)x^2 - 2\beta xy + (2\alpha+\beta)y^2 - 8\alpha y + 4x + 2c = 0$ passes through origin , therefore $c = 0$.

$x^2 + y^2 + x - 4y = 0 \Rightarrow r = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$.

Q4. If P (6, 1) be the orthocenter of the triangle whose vertices are A (5,-2), B (8, 3) and C (h ,k) then the point C lies on the

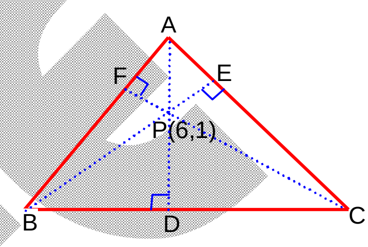
(A) $x^2 + y^2 - 52 = 0$ (B) $x^2 + y^2 - 74 = 0$
(C) $x^2 + y^2 - 61 = 0$ (D) $x^2 + y^2 - 65 = 0$

Ans. D

Sol. Slope AD = slope of AP = 3

Slope BC = $-\frac{1}{3}$

Equation of BC is $x + 3y - 17 = 0$
slope of BE = 1
Slope of AC = –1
Equation of AC is $x + y - 3 = 0$ point C is (–4, 7). Hence it is satisfying the option D



Q5. Let $A = \{1,2,3,4,5\}$ .Let R be a relation on A defined by xRy if and only $4x \le 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(A) 24 (B) 25
(C) 26 (D) 23

Ans. B

Sol. Given that $4x \le 5y$ then
R={(1,1),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4)(2,5),(3,3),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5)}
i.e. 16 elements. i.e. m = 16
Now to make R a symmetric relation {(2,1),(3,2),(4,3),(3,1),(4,2),(5,3),(4,1),(5,2),(5,1)}
i.e. n = 9 , So m + n = 25.

Q6. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$;$x > 0$ attains the maximum value at $x = \frac{1}{e}$ then

(A) $e^{2\pi} < (2\pi)^e$ (B) $(2e)^\pi > \pi^{(2e)}$
(C) $e^\pi > \pi^e$ (D) $e^\pi < \pi^e$

Ans. C

Sol. Let $f(x) = \left(\frac{1}{x}\right)^{2x}$;$x > 0$. Taking log on both the sides

$\ln f(x) = -2x\ln x$, differentiating with respect to x , we get, $\frac{1}{f(x)} f'(x) = -2 - 2\ln x$

for $x > \frac{1}{e}$, function is increasing . so, $e < \pi$, $\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi} \Rightarrow e^\pi > \pi^e$

Q7. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$ .Then the square of the projection of $\vec{a}$ on $\vec{b}$ is

(A) 2 (B) $\frac{2}{3}$

(C) $\frac{1}{5}$ (D) $\frac{1}{3}$

Ans. A

Sol. $\vec{a} = 2\hat{i} + \hat{j} - \hat{k},\ \ \vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$

$((\vec{a} \times (\hat{i} + \hat{j})) = (2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} + \hat{j}) = \hat{i} - \hat{j} + \hat{k}$

$((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i})\ = (\hat{i} - \hat{j} + \hat{k}) \times \hat{i} = \hat{j} + \hat{k}$

Now, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}\ = \hat{j} - \hat{k}$

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\hat{a}.\hat{b}}{|\hat{b}|} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

Q8. If A is a square matrix of order 3 such that det (A) = 3 and $\det(\text{adj}(-4\text{adj}.(-3\text{adj}(3\text{dj}((2A)^{-1})))) = 2^m 3^n$. Then $m + 2n$ is equal to

(A) 3
(B) 2
(C) 4
(D) 6

Ans. C

Sol. $(\text{adj}(-4\text{adj}(-3\text{adj}(3\text{dj}((2A)^{-1})))) = \left|-4\text{adj}(-3\text{adj}(3\text{dj}((2A)^{-1}))\right|^2 = 4^6\left|\text{adj}(-3\text{adj}(3\text{dj}((2A)^{-1}))\right|^2$

$= 2^{12}.3^{12}\left|3\text{dj}((2A)^{-1})\right|^8 = 2^{12}.3^{12}.3^{24}\left|\text{adj}((2A)^{-1})\right|^8 = 2^{12}.3^{36}\left|(2A)^{-1})\right|^{16} = 2^{12}.3^{36}\dfrac{1}{\left|2A\right|^{16}}$

$= 2^{12}.3^{36}\dfrac{1}{2^{48}\left|A\right|^{16}} = 2^{12}.3^{36}\dfrac{1}{2^{48}.3^{16}} = 2^{-36}.3^{20}$

Hence m = – 36 and n = 20
m + 2n = 4

Q9. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a is vector such that $|\vec{c}| \geq 6$, $\vec{a}.\vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 60° then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

(A) $\dfrac{3}{2}\sqrt{3}$
(B) $\dfrac{9}{2}(6 - \sqrt{6})$
(C) $\dfrac{9}{2}(6 + \sqrt{6})$
(D) $\dfrac{3}{2}\sqrt{6}$

Ans. C

Sol. $\vec{a} \times \vec{b} = \hat{i} - \hat{j} + 5\hat{k} \Rightarrow |\vec{a} \times \vec{b}| = \sqrt{27}$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = |(\vec{a} \times \vec{b})| \times |\vec{c}| = |\vec{a} \times \vec{b}||\vec{c}|\cos 60^0 = \sqrt{27}\,|\vec{c}|\,\dfrac{\sqrt{3}}{2} = \dfrac{9|\vec{c}|}{2}$ ……….(i)

$|\vec{c} - \vec{a}|\ = 2\sqrt{2} \Rightarrow |\vec{c} - \vec{a}|^2\ = (2\sqrt{2})^2 \Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 38 - 12|\vec{c}| = 8 \Rightarrow |\vec{c}|^2 - 12|\vec{c}| + 30 = 0 \Rightarrow |\vec{c}| = 6 \pm \sqrt{6}$

From (i), $|(\vec{a} \times \vec{b}) \times \vec{c}| = \dfrac{9|\vec{c}|}{2} = \dfrac{9(6 + \sqrt{6})}{2}$

Q10. $\lim\limits_{n\to\infty}\dfrac{(1^2 - 1)(n-1) + (2^2 - 2)(n-2) + \ldots\ldots\ldots + ((n-1)^2 - (n-1)).1}{(1^3 + 2^3 + \ldots + n^2) - (1^2 + 2^2 + \ldots + n^2)}$ is equal to

(A) $\dfrac{3}{4}$
(B) $\dfrac{2}{3}$
(C) $\dfrac{1}{3}$
(D) $\dfrac{1}{2}$

Ans. C

Sol. $\lim\limits_{n\to\infty}\dfrac{(1^2-1)(n-1)+(2^2-2)(n-2)+........+((n-1)^2-(n-1)).1}{(1^3+2^3+...+n^2)-(1^2+2^2+...+n^2)}\ \lim\limits_{n\to\infty}\dfrac{\sum\limits_{r=1}^{n}(r^2-r)(n-r)}{\sum\limits_{r=1}^{n}r^3-\sum\limits_{r=1}^{n}r^2}$

$$=\lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n}(n+1)r^2-r^3-nr)}{\sum\limits_{r=1}^{n}r^3-\sum\limits_{r=1}^{n}r^2}=\lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n}(n+1)r^2-r^3-nr)}{\sum\limits_{r=1}^{n}r^3-\sum\limits_{r=1}^{n}r^2}$$

$$=\lim_{n\to\infty}\frac{(n+1)\left(\frac{n(n+1)(2n+1)}{6}\right)-\left\{\frac{n(n+1)}{2}\right\}^2-n\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)(2n+1)}{6}}$$

$$=\lim_{n\to\infty}\frac{(n+1)\left(\frac{n(n+1)(2n+1)}{6}\right)-\left\{\frac{n(n+1)}{2}\right\}^2-n\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)(2n+1)}{6}}$$

$$=\lim_{n\to\infty}\frac{\frac{n(n+1)}{2}\left(\frac{2n^2+3n+1}{3}-\frac{n^2+n}{2}-n\right)}{\frac{n(n+1)}{2}\left(\frac{n^2+n}{2}-\frac{2n+1}{3}\right)}=\lim_{n\to\infty}\frac{\left(\frac{4n^2+6n+2-3n^2-3n-6n}{6}\right)}{\left(\frac{3n^2+3n-4n-2}{6}\right)}$$

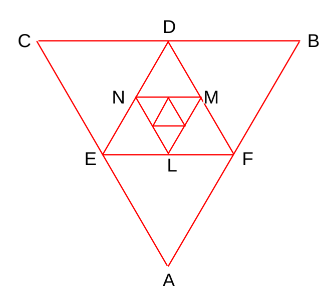
$$=\lim_{n\to\infty}\frac{\left(\frac{n^2+2-3n}{6}\right)}{\left(\frac{3n^2-n-2}{6}\right)}=\lim_{n\to\infty}\frac{n^2+2-3n}{3n^2-n-2}=\frac{1}{3}$$

Q11. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of area of all the triangles formed in this process, then

(A) $P^2=6\sqrt{3}Q$
(B) $P^2=36\sqrt{3}\,Q$
(C) $P=36\sqrt{3}\,Q^2$
(D) $P=72\sqrt{3}\,Q$

Ans. **B**

Sol. Let 'a' be the side of the equilateral ΔABC and D , E , F are the mid points of the ΔABC and so on



Area of $\Delta ABC=\dfrac{\sqrt{3}a^2}{4}$

Area of $\Delta DEF=\dfrac{\sqrt{3}}{4}\times\dfrac{a^2}{4}=\dfrac{\sqrt{3}}{16}a^2$

Area of $\Delta LMN=\dfrac{\sqrt{3}}{4}\times\dfrac{a^2}{16}=\dfrac{\sqrt{3}}{64}a^2$ ……… and so on

Sum of areas = Q = $\dfrac{\sqrt{3}a^2}{4}+\dfrac{\sqrt{3}}{16}a^2+\dfrac{\sqrt{3}}{64}a^2+..........$

$$=\frac{\sqrt{3}a^2}{4}\left(1+\frac{1}{4}+\frac{1}{16}+..........\right)=\frac{\sqrt{3}a^2}{4}\times\frac{1}{1-\frac{1}{4}}=\frac{a^2}{\sqrt{3}}$$

$\therefore Q = \dfrac{a^2}{\sqrt{3}}$ ....(i)

$P = 3a + \dfrac{3a}{2} + \dfrac{3a}{4} + \dfrac{3a}{8} + ... = 3a\left(1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + ...\right)$

$\therefore P = 6a \Rightarrow a = \dfrac{P}{6} \Rightarrow a^2 = \dfrac{P^2}{36}$ ....(ii)

From (i) and (ii) we get,

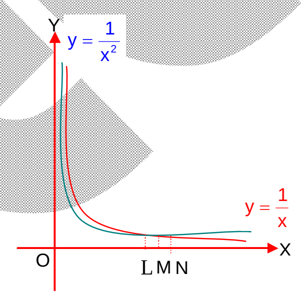
$\therefore Q = \dfrac{P^2}{36\sqrt{3}} \Rightarrow P^2 = 36\sqrt{3}Q$

Q12. If the area of the region $\{(x, y) : \dfrac{a}{x^2} \le y \le \dfrac{1}{x}, 1 \le x \le 2, 0 < a < 1\}$ is $(\log_e 2) - \dfrac{1}{7}$ then the value of $7a - 3$

(A) 0 (B) – 1
(C) 2 (D) 1

Ans. **B**

Sol. $y \ge \dfrac{a}{x^2}, y \le \dfrac{1}{x}, 1 \le x \le 2, 0 < a < 1\}$



Area = $\displaystyle\int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right)dx = \ln x + \frac{a}{x}\Big|_1^2$ $= \ln 2 + \dfrac{a}{2} - a$

$= \ln 2 - \dfrac{a}{2}$

$\therefore a = \dfrac{2}{7} \Rightarrow 7a = 2 \Rightarrow 7a - 3 = -1$

Q13. If all words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is

(A) NRAGPU (B) NRAPGU
(C) NRAGUP (D) NRAPUG

Ans. **C**

Sol. N A G P U R
Starting with
A = 5! = 120
G = 5! = 120
NA = 4! = 24
NG = 4! = 24
NP = 4! = 24
NRAGPU = 1
Total = 120 + 120 + 24 + 24 + 24 + 1 = 313
NRAGPU = 314
NRAPGUP = 315. Hence 315 th word is NRAPGU.

Q14. Suppose for a function h, h (0) = 0, h (1) =1 and $h'(0) = h'(1) = 2$ .If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to

(A) 5 (B) 3
(C) 8 (D) 4

Ans. **D**

Sol. $g(x) = h(e^x)e^{h(x)} \Rightarrow g'(x) = h'(e^x)e^x e^{h(x)} + h(e^x)e^{h(x)}h'(x)$

$\Rightarrow g'(0) = h'(e^0)e^0 e^{h(0)} + h(e^0)e^{h(0)}h'(0)$

$\Rightarrow g'(0) = h'(1)e^{h(0)} + h(1)e^{h(0)}h'(0) \Rightarrow g'(0) = 2 + 2 = 4$.

Q15. Let $P(\alpha,\beta,\gamma)$ be the image of the point Q (3, – 3, 1) in the line $\frac{x-0}{1}=\frac{y-3}{1}=\frac{z-1}{-1}$ is and R be the point (2, 5, –1). If the area of the triangle $\lambda^2 = 74K$ then K is equal to

(A) 36 (B) 18

(C) 81 (D) 72

Ans. C

Sol. $\overrightarrow{RQ} = \hat{i} - 8\hat{j} + 2\hat{k}$, $\overrightarrow{RS} = \hat{i} + \hat{j} - \hat{k}$ and $RQ = \sqrt{1+64+4} = \sqrt{69}$

$$\cos\theta = \frac{\overrightarrow{RQ}.\overrightarrow{RS}}{\left|\overrightarrow{RQ}\right|\left|\overrightarrow{RS}\right|} = \frac{\left(\hat{i}-8\hat{j}+2\hat{k}\right).\left(\hat{i}+\hat{j}-\hat{k}\right)}{\sqrt{69}\sqrt{3}} = \left|\frac{1-8-2}{\sqrt{207}}\right| = \frac{9}{3\sqrt{23}} = \frac{3}{\sqrt{23}}$$

$$\cos\theta = \frac{3}{\sqrt{23}} \Rightarrow \sin\theta = \frac{QS}{3\sqrt{23}} = \frac{\sqrt{14}}{\sqrt{23}} \Rightarrow QS = \sqrt{42}$$

Area of Triangle PQR = $\frac{1}{2}.2QS.RS = \sqrt{42}.3\sqrt{3} = 9\sqrt{14}$

$\therefore \lambda^2 = 81.14 = 14K \qquad \therefore K = 81$

Q16. If z_1 , z_2 are two distinct complex number such $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right| = 2$, then

(A) Both z_1 and z_2 lie on the same circle

(B) Either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1

(C) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1

(D) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

Ans. D

Sol. $$\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right| = 2 \Rightarrow \left|z_1-2z_2\right|^2 = 4\left|\frac{1}{2}-z_1\bar{z}_2\right|^2 \Rightarrow (z_1-2z_2)(\overline{z_1}-2\overline{z_2}) = 4(\frac{1}{2}-z_1\bar{z}_2)(\frac{1}{2}-z_2\bar{z}_1)$$

$$\Rightarrow \left|z_1\right|^2 - 2z_1\overline{z_2} - 2z_2\overline{z_1} + 4\left|z_2\right|^2 = 4\left(\frac{1}{4} - \frac{1}{2}z_2\bar{z}_1 - \frac{1}{2}z_1\bar{z}_2 + \left|z_2\right|^2\left|z_1\right|^2\right)$$

$$\Rightarrow \left|z_1\right|^2 - 2z_1\overline{z_2} - 2z_2\overline{z_1} + 4\left|z_2\right|^2 = 1 - 2z_2\bar{z}_1 - 2z_1\bar{z}_2 + 4\left|z_2\right|^2\left|z_1\right|^2$$

$$\Rightarrow (\left|z_1\right|^2 - 1)(1 - 4\left|z_2\right|^2) = 0 \Rightarrow (\left|z_1\right|^2 - 1) - 4\left|z_2\right|^2(\left|z_1\right|^2 - 1) = 0 \Rightarrow \left|z_1\right| = 1, \left|z_2\right| = \frac{1}{2}$$

Q17. If $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{1}{12}\tan^{-1}(3\tan x) + \text{constant}$, then the maximum value of a sinx + b cosx , is

(A) 41 (B) $\sqrt{39}$

(C) $\sqrt{40}$ (D) $\sqrt{42}$

Ans. C

Sol. $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{1}{a^2}\int \frac{\sec^2 x dx}{\frac{b^2}{a^2} + \tan^2 x}$ put tanx = t $\Rightarrow$ $\sec^2 x dx$ = dt

$\frac{1}{a^2}\int \frac{dt}{\frac{b^2}{a^2} + t^2} = \frac{1}{ab}\tan^{-1}(\frac{a}{b}\tan x) + c \Rightarrow \frac{1}{ab} = \frac{1}{12} \Rightarrow ab = 12$ ……….(i)

and $\frac{a}{b} = 3 \Rightarrow a = 3b \Rightarrow 3b^2 = 12 \Rightarrow b = 2$, a=6

The maximum value of a sinx + b cosx = $\sqrt{a^2 + b^2} = \sqrt{40}$

Q18. Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(A $\left[\frac{1}{7}, \frac{1}{5}\right]$ (B) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(C) $\left[\frac{1}{7}, \frac{1}{6}\right]$ (D) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Ans. **D**

Sol. $f(x) = \frac{1}{7 - \sin 5x} \Rightarrow f_{max} = \frac{1}{6}$ and $f_{min} = \frac{1}{8}$

Q19. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is

(A) $\frac{18}{25}$ (B) $\frac{4}{25}$

(C) $\frac{12}{25}$ (D) $\frac{6}{25}$

Ans. **C**

Sol. Total number of ways = 5^3

Favorable ways $= {}^5C_3\ (2^3 - 2) = 60$

$\therefore$ Required probability = $\frac{60}{125} = \frac{12}{25}$

Q20. If the locus of the point, whose distance from the point (2,1) and (1, 3) are in the ratio 5:4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is

(A) 437 (B) – 27

(C) 37 (D) 5

Ans. **C**

Sol. as per problems , $\frac{\sqrt{(x-2)^2 + (y-1)^2}}{\sqrt{(x-1)^2 + (y-3)^2}} = \frac{5}{4} \Rightarrow \frac{(x-2)^2 + (y-1)^2}{(x-1)^2 + (y-3)^2} = \frac{25}{16}$

$\Rightarrow \frac{x^2 + y^2 - 4x - 2y + 5}{x^2 + y^2 - 2x - 6y + 10} = \frac{25}{16} \Rightarrow 9x^2 + 9y^2 + 14x - 118y + 170 = 0$

a = 9 b = 9 , c = 0 , d = 14 , e = - 118

$a^2 + 2b + 3c + 4d + e$ = 81 + 18 + 0 + 56 – 118 = 37

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q21. In a triangle ABC, BC = 7, AC = 8, AB = $\alpha \in N$ and $\cos A = \frac{2}{3}$.If 49cos(3C) + 42 = $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to__________.

Ans. **39**

Sol. $\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{2}{3} = \frac{8^2 + c^2 - 7^2}{2 \times b \times c} \Rightarrow c = 9$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$

49 cos3C + 42 $\Rightarrow$ 49(4 cos^{3}C – 3cosC) + 42$\Rightarrow 49\left(4\left(\frac{2}{7}\right)^3 - 3\left(\frac{2}{7}\right)\right) + 42 = \frac{32}{7} = \frac{m}{n}$

m + n = 32 + 7 = 39

Q22. If the system of equations

$2x + 7y + \lambda z = 3$

$3x + 2y + 5z = 4$. Has infinitely many solutions, and then $(\lambda - \mu)$ is equal to_______.

$x + \mu y + 32z = -1$

Ans. **38**

Sol. $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Rightarrow \Delta_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0 \Rightarrow \mu = -39$ And $\Delta = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0 \Rightarrow \lambda = -1$

$\therefore \lambda - \mu = 38$

Q23. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items n the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$ where gcd (m,n) = 1 then n – m is equal to_______.

Ans. **71**

Sol. Given a lot of 12 items, 3 are defective.

Good items, 12 − 3 = 9

Let X be the number of defective items.

So, value of X = 0,1,2,3

A sample of S items is drawn.

No defective

P(X = 0) = $\frac{{}^3C_0 \times {}^9C_5}{{}^{12}C_5} = \frac{7}{44}$

One defective

$P(X = 1) = \frac{{}^3C_1 \times {}^9C_4}{{}^{12}C_5} = \frac{21}{44}$

Two defective

$P(X=2) = \frac{{}^3C_2 \times {}^9C_3}{{}^{12}C_5} = \frac{7}{22}$

$P(X=3) = \frac{{}^3C_3 \times {}^9C_2}{{}^{12}C_5} = \frac{1}{22}$

Mean $\overline{X} = \sum P_i X_i$ = P(X=0)×0 + P(X=1)×1+ P(X=2)×2+ P(X=3)×3= $\frac{5}{4}$

Variance = $\sum P_i X_i^2 - \left(\sum P_i X_i\right)^2 = \frac{21}{44}\times 1 + \frac{7}{22}\times 4 + \frac{1}{22}\times 9 - \frac{25}{16} = \frac{105}{176} = \frac{m}{n}$

m – n = 176 – 105 = 71

Q24. $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + 3(1+x)^4 + ... + 60(1+x)^{60}$ $x \neq 0$
and $(60)^2 S(60) = a(b)^b + b$, $a, b \in N$ then (a + b) equal to______.

Ans. **3660**

Sol. $S(x) \quad = (1+x) + 2(1+x)^2 + 3(1+x)^3 + 3(1+x)^4 + ... + 60(1+x)^{60}$ …………(i)

$(1+x)S(x) = \quad (1+x)^2 + 2(1+x)^3 + 3(1+x)^4 + ...59(1+x)^{60} + 60(1+x)^{61}$ ……..(ii)

- $xS(x) \quad = \quad (1+x) + (1+x)^2 + (1+x)^3 + ...(1+x)^{60} - 60(1+x)^{61}$

$S(x) \quad = (1+x)\frac{1-(1+x)^{60}}{x^2} + \frac{60(1+x)^{61}}{x}$

$S(60) = (1+60)\frac{1-(1+60)^{60}}{60^2} + \frac{60(1+60)^{61}}{60} = \frac{61(1-61^{60}) + 60^2 61^{61}}{60^2}$

$\Rightarrow 60^2 S(60) = 61(1-61^{60}) + 60^2 61^{61} = 61 - 61^{61} + 60^2 . 61^{61}$

$\Rightarrow (60)^2 S(60) = 3599.61^{61} + 61 = a(b)^b + b$

a = 3599 and b = 61

a + b = 3599 + 61 = 3660

Q25. Let [t] denote the largest integer less or equal to t.
If $\int_0^3 \left(\left[x^2\right] + \left[\frac{x^2}{2}\right]\right)dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7}$ where $a, b, c \in Z$, then a + b + c is equal to________.

Ans. **23**

Sol. $\int_0^3 \left([x^2] + \left[\frac{x^2}{2}\right]\right)dx$

$\int_0^3 \left([x^2]\right)dx + \int_0^3 \left(\left[\frac{x^2}{2}\right]\right)dx$

$\int_0^3 \left([x^2]\right)dx = \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx + \int_{2}^{\sqrt{5}} 4.dx + \int_{\sqrt{5}}^{\sqrt{6}} 5.dx + \int_{\sqrt{6}}^{\sqrt{7}} 6.dx + \int_{\sqrt{7}}^{\sqrt{8}} 7.dx + \int_{\sqrt{8}}^{3} 8.dx$

$= \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx + \int_{2}^{\sqrt{5}} 4.dx + \int_{\sqrt{5}}^{\sqrt{6}} 5.dx + \int_{\sqrt{6}}^{\sqrt{7}} 6.dx + \int_{\sqrt{7}}^{\sqrt{8}} 7.dx + \int_{\sqrt{8}}^{3} 8.dx$

$= 21 - 3\sqrt{2} - \sqrt{3} - \sqrt{5} - \sqrt{6} - \sqrt{7}$

And other one

$\int_0^3 \left(\left[\frac{x^2}{2}\right]\right)dx = \int_0^{\sqrt{2}} 0.dx + \int_{\sqrt{2}}^{2} 1.dx + \int_{2}^{\sqrt{6}} 2.dx + \int_{\sqrt{6}}^{\sqrt{8}} 3.dx + \int_{\sqrt{8}}^{3} 4.dx$

$$= \int_0^{\sqrt{2}} 0.dx + \int_{\sqrt{2}}^{2} 1.dx + \int_2^{\sqrt{6}} 2dx + \int_{\sqrt{6}}^{\sqrt{8}} 3.dx + \int_{\sqrt{8}}^{3} 4.dx$$

$$= 0 + \left(2-\sqrt{2}\right) + 2\left(\sqrt{6}-2\right) + 3\left(\sqrt{8}-\sqrt{6}\right) + 4\left(3-\sqrt{8}\right)$$

$$= 2-\sqrt{2}+2\sqrt{6}-4+6\sqrt{2}-3\sqrt{6}+12-8\sqrt{2} = 10-3\sqrt{2}-\sqrt{6}$$

$$\int_0^3 \left(\left[x^2\right]\right)dx + \int_0^3 \left(\left[\frac{x^2}{2}\right]\right)dx$$

$$= 21-3\sqrt{2}-\sqrt{3}-\sqrt{5}-\sqrt{6}-\sqrt{7}+10-3\sqrt{2}-\sqrt{6} = 31-6\sqrt{2}-\sqrt{3}-\sqrt{5}-2\sqrt{6}-\sqrt{7}$$

$$= 31-4\sqrt{2}-\sqrt{3}-\sqrt{5}-2\sqrt{6}-\sqrt{7} = a+b\sqrt{2}-\sqrt{3}-\sqrt{5}+c\sqrt{6}-\sqrt{7}$$

Hence a = 31 , b = – 6 , c = – 2

a + b + c = 31 – 6 – 2 = 23

Q26. Let [t] denote the greatest integer less than or to t. Let $f:[0,\infty)\to R$ be a function defined by $f(x) = \left[\frac{x}{2}+3\right] - [\sqrt{x}]$ .Let S be the set of all points in the interval [0, 8] at which f is not continuous. Then $\sum_{a\in S} a$ is equal to_______.

Ans. **17**

Sol. $f(x) = \left[\frac{x}{2}+3\right] - \left[\sqrt{x}\right] = \left[\frac{x}{2}\right] - \left[\sqrt{x}\right] + 3 \because\ x\in[0,8],\ \frac{x}{2}\in[0,4],\quad \sqrt{x}\in\left[0,2\sqrt{2}\right]$

Point of discontinuity,

When $\frac{x}{2} = 0,1,2,3,4$ and $\sqrt{x}$ = 0 , 2 , 4 , 6 , 8

$x = \underset{\text{right cont}}{0}, \underset{\text{discontinuity}}{1,2,3,4}$ and $x = \underset{\text{cont}}{0}, \underset{\text{discontinuity}}{1,4,9}$

Now we want to check discontinuity at x= 4

$$f(4^+) = \left[\frac{4^+}{2}\right] - \left[\sqrt{4^+}\right] + 3 = 2-2+3 = 3$$

$$f(4^-) = \left[\frac{4^-}{2}\right] - \left[\sqrt{4^-}\right] + 3 = 1-1+3 = 3$$

$$f(4) = \left[\frac{4}{2}\right] - \left[\sqrt{4}\right] + 3 = 2-2+3 = 3$$

f(x) is continuous at x = 4

Point of discontinuity = 1 , 2 , 6 , 8 { Continuous + Discontinuous = Discontinuous}

Sum = 1 + 2 + 6 + 8 = 17

Q27. If the solution y(x) of the given differential equation $(e^y+1)\cos x dx + e^y \sin x dy = 0$ passes through the point equal to $\left(\frac{\pi}{2}, 0\right)$ then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to __________.

Ans. **3**

Sol. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0\ ) \Rightarrow d((e^y + 1)\sin x) = 0 \Rightarrow (e^y + 1)\sin x = c$

It passes through the point $\left(\frac{\pi}{2}, 0\right) \Rightarrow c = 2 \quad \therefore\ (e^y + 1)\sin x = 2$

$$\Rightarrow e^{y\left(\frac{\pi}{6}\right)} = 2\operatorname{cosec} x - 1 \Rightarrow e^y = 4-1 = 3$$

Q28. Let α,β be roots of $x^2+\sqrt{2}x-8=0$. If $U_n=\alpha^n+\beta^n$ then $\frac{U_{10}+\sqrt{2}U_9}{2U_8}$ is equal to______.

Ans. **4**

Sol. $x^2+\sqrt{2}x-8=0 \quad \Rightarrow \alpha^2+\sqrt{2}\alpha-8=0$ ………(i)

and $\beta^2+\sqrt{2}\beta-8=0$ ………..(ii)

$\alpha^{10}+\sqrt{2}\alpha^9 = \ 8\alpha$ ....(iii)

$\beta^{10}+\sqrt{2}\beta^9 = \ 8\beta$ ....(iv)

Adding (iii) and (iv) we get

$\alpha^{10}+\beta^{10}+\sqrt{2}(\alpha^9+\beta^9)=8(\alpha^8+\beta^8)$

$\Rightarrow \frac{\alpha^{10}+\beta^{10}+\sqrt{2}(\alpha^9+\beta^9)}{(\alpha^8+\beta^8)}=8 \Rightarrow \frac{U_{10}+\sqrt{2}U_9}{2U_8}=4$

Q29. If the shortest distance between the lines $\frac{x+2}{-3}=\frac{y+5}{2}=\frac{z-4}{4}$ to $\frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$ and is $\frac{44}{\sqrt{30}}$ then the largest possible value of $|\lambda|$ is equal______.

Ans. **43**

Sol. Let $\vec{r_1}=\vec{a_1}+t_1\vec{b_1}=-2\hat{i}-5\hat{j}+4\hat{k}+t_1(-3\hat{i}+2\hat{j}+4\hat{k})$

$\vec{r_2}=\vec{a_2}+t_2\vec{b_2}=\lambda\hat{i}+2\hat{j}+\hat{k}+t_2(3\hat{i}-\hat{j}+\hat{k})$

$$SD=\frac{\left|(\vec{a_1}-\vec{a_2}).(\vec{b_1}\times\vec{b_2})\right|}{\left|\vec{b_1}\times\vec{b_2}\right|}=\frac{\left|\left((-2\hat{i}-5\hat{j}+4\hat{k})-(\lambda\hat{i}+2\hat{j}+\hat{k})\right).(-3\hat{i}+2\hat{j}+4\hat{k})\times(3\hat{i}-\hat{j}+\hat{k})\right|}{\left|(-3\hat{i}+2\hat{j}+4\hat{k})\times(3\hat{i}-\hat{j}+\hat{k})\right|}$$

$$=\frac{\left|\left((2+\lambda)\hat{i}+7\hat{j}-3\hat{k}\right).(6\hat{i}+15\hat{j}+3\hat{k})\right|}{\left|6\hat{i}+15\hat{j}+3\hat{k}\right|}=\frac{\left|-6\lambda-12-105-9\right|}{\sqrt{36+225+9}}$$

$$\Rightarrow \frac{\left|6\lambda+126\right|}{3\sqrt{30}}=\frac{44}{\sqrt{30}} \Rightarrow \lambda=1,\ -43$$

$|\lambda|=43$

Q30. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x=\pm\frac{4}{\sqrt{3}}$ respectively. Let the line $y-\sqrt{3}x+\sqrt{3}=0$ touch this hyperbola at (x_0,y_0). If m is the product of the focal distance of the point (x_0,y_0) then $4e^2$ +m is equal to____.

Ans. **bonus**

Sol. Given $\frac{2b^2}{a}=9$ and $\frac{a}{e}=\pm\frac{4}{\sqrt{3}}$ also equation of tangent $y-\sqrt{3}x+\sqrt{3}=0$

Let slope m = $\sqrt{3}$ and C = – $\sqrt{3}$

By condition of tangency,

$\Rightarrow 6=6a^2-9a \Rightarrow a=2,\ b^2=9$

Equation of Hyperbola is $\frac{x^2}{4}-\frac{y^2}{9}=1$

and for tangent Point of contact is $(4,\ 3\sqrt{3}) = (x_0,\ y_0)$

therefore eccentricity $e = \sqrt{1+\frac{9}{4}} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$

Again product of focal distances

m = $(x_0e + a)(x_0e - a) \Rightarrow m + 4e^2 = 20e^2 - a^2 = 20 \times \frac{13}{4} - 4 = 61$

(There is a printing mistake in the equation of directrix $x = \pm\frac{4}{\sqrt{3}}$

The corrected equation is $x = \pm\frac{4}{\sqrt{13}}$ for directrix, as eccentricity must be greater than one, so question must be bonus).

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q31. The number of electrons per second in the filament of a 110W bulb operating at 220V is : (Given $e = 1.6\times10^{-19}C$)

(A) 6.25×10^{18} (B) 1.25×10^{19}

(C) 6.25×10^{17} (D) 31.25×10^{17}

Ans. **D**

Sol. $i = \frac{P}{V}$

$i = \frac{110}{220}$

$i = 0.5A$

$\frac{ne}{t} = 0.5$

$\frac{n}{t} = \frac{0.5}{1.6\times10^{-19}} = 31.25\times10^{17}$

Q32. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be : Given hc = 1242 eV nm.

(A) 207 nm (B) 103.5 nm

(C) 414 nm (D) 407 nm

Ans. **A**

Sol. $E = \frac{1240}{\lambda(nm)}ev$

$\lambda = \frac{1240}{6} = 207nm$

Q33. For the thin convex lens, the radii of curvature are at 15cm and 30cm respectively. The focal length the lens is 20cm. The refractive index of the material is :

(A) 1.8 (B) 1.4

(C) 1.5 (D) 1.2

Ans. **C**

Sol. $\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{20} = \left(\frac{\mu}{1} - 1\right)\left(\frac{1}{15} - \frac{1}{30}\right)$

$\Rightarrow \mu - 1 = \frac{1}{2}$

$\Rightarrow \mu = 1.5$

Q34. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is :

(A) 1 N
(B) 12 N
(C) 4 N
(D) 6 N

Ans. D

Sol.

$(P)^Q \quad (S)^Q$

$$F = \frac{KQ^2}{r^2} = 16$$

When third identical sphere contact with P and S , then charge on P & S are

$(P)^{\frac{Q}{2}} \quad (S)^{\frac{3Q}{4}}$

$$F' = \frac{K\frac{Q}{2}\times\frac{3Q}{4}}{r^2} = \frac{16\times3}{8} = 6N$$

Q35. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is :

(A) $\sqrt{t_1t_2}$
(B) $\sqrt{\frac{t_1}{t_2}}$
(C) $\sqrt{t_1-t_2}$
(D) $\sqrt{t_1+t_2}$

Ans. A

Sol.

$$h = ut_1 - \frac{1}{2}gt_1^2 \text{ ....(1)}$$

$$h = -ut_2 - \frac{1}{2}gt_2^2 \text{ ....(2)}$$

$$h = -\frac{1}{2}gt^2 \text{ ....(3)}$$

From equation (1), (2) & (3)

$$t = \sqrt{t_1t_2}$$

Q36. A car of 800 kg is taking turn on a banked road of radius 300m and angle of banking 30°. If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely: ($g = 10m/s^2, \sqrt{3} = 1.73$)

(A) 264 m/s
(B) 51.4 m/s
(C) 70.4 m/s
(D) 102.8 m/s

Ans. B

Sol. Maximum speed on banking road for safe turning

$$V_{max} = \sqrt{Rg\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)}$$

$$V_{max} = \sqrt{300\times10\left(\frac{\tan30+0.2}{1-0.2\times\tan30}\right)}$$

$$V_{max} = 51.4m/s$$

Q37. In the given electromagnetic wave $E_y = 600\sin(\omega t - kx)\,Vm^{-1}$, intensity of the associate light beam is (in W/m^2 : (Given $\epsilon_0 = 9\times10^{-12}\,C^2N^{-1}m^{-2}$)

(A) 972 (B) 729
(C) 243 (D) 486

Ans. A

Sol. $I = \frac{1}{2}\varepsilon_0 E^2 C$

$I = \frac{1}{2}\times 9\times10^{-12}\times(600)^2\times 3\times10^8$

$I = 486\,W/m^2$

Q38. Energy of 10 non rigid diatomic molecules at temperature T is :

(A) 70 K_BT (B) 35 RT
(C) $\frac{7}{2}RT$ (D) 35 K_BT

Ans. D

Sol. D.O.f of diatomic molecule =7

$E = \frac{f}{2}NK_BT = \frac{7}{2}\times10\times K_BT = 35K_BT$

Q39. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be :

(A) 6 % (B) 500 %
(C) 600 % (D) 60 %

Ans. B

Sol. $KE = \frac{P^2}{2m}$

$P \propto \sqrt{KE}$

$\frac{P_2}{P_1} = \sqrt{\frac{KE_2}{KE_1}}$

$\frac{P_2}{P_1} = \sqrt{36}$

$\frac{P_2}{P_1} = 6$

$\frac{P_2}{P_1} - 1 = 6 - 1$

$\frac{P_2 - P_1}{P_1}\% = 5\times100 = 500\%$

Q40. The longest wavelength associated with Paschen series is : (Given $R_H = 1.097\times10^7$ SI unit)

(A) $1.094\times10^{-6}m$ (B) $3.646\times10^{-6}m$
(C) $2.973\times10^{-6}m$ (D) $1.876\times10^{-6}m$

Ans. A

Sol. $\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Longest wavelength for paschan series

$n_1 = 3$

$n_2 = 4$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2} \right)$$

$\lambda = 1.876 \times 10^{-6} m$

Q41. Given below are two statements :

Statement (I) : Dimensions of specific heat is $\left[L^2 T^{-2} K^{-1}\right]$.

Statement (II) : Dimensions of gas constant is $\left[ML^2 T^{-1} K^{-1}\right]$.

In the light of the above statements, choose the most appropriate answer from the options given below.
(A) Statement (I) is correct but statement (II) is incorrect
(B) Both statement (I) and statement (II) are incorrect
(C) Both statement (I) and statement (II) are correct
(D) Statement (I) is incorrect but statement (II) is correct

Ans. A

Sol. $[S] = \left[\frac{Q}{m\Delta T}\right] = \left[L^2 T^{-2} K^{-1}\right]$

$[R] = \left[\frac{PV}{nT}\right] = \left[ML^2 T^{-2} mole^{-1} k^{-1}\right]$

(i) correct
(ii) incorrect

Q42. Pressure inside a soap bubble is greater than the pressure outside by an amount :
(given : R = Radius of bubble, S = Surface tension of bubble)

(A) $\frac{4S}{R}$ (B) $\frac{2S}{R}$

(C) $\frac{S}{R}$ (D) $\frac{4R}{S}$

Ans. A

Sol. Both side air

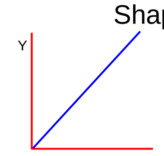
$\Delta P = \frac{4S}{R}$

Q43. Match List-I with List-II :

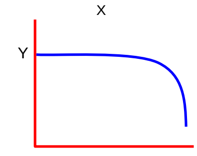
List – I
Y vs X

List – II
Shape of Graph

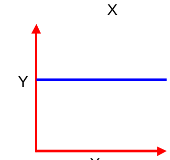
(a) Y = magnetic susceptibility
X = magnetising field

(I) 

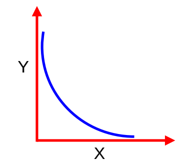
(b) Y = magnetic field
X = distance from centre of a current carrying wire for x < a
(where a = radius of wire)

(II) 

(c) Y = magnetic field
X = distance from centre of a current carrying wire for x > a
(where a = radius of wire)

(III) 

(d) Y = magnetic field inside solenoid (IV)
X = distance from centre



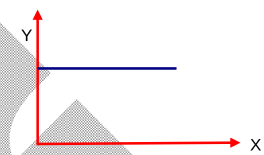
Choose the correct answer from the options given below :

(A) (a – I), (b – III), (c – II), (d – IV)
(B) (a – III), (b – IV), (c – I), (d –II)
(C) (a – IV), (b – I), (c – III), (d –II)
(D) (a – III), (b – I), (c – IV), (d – II)

Ans. D

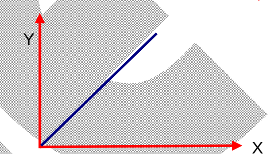
Sol.

(a) Magnetic susceptibility vs magnetic field



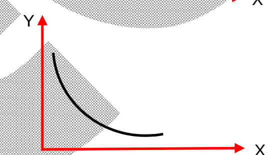
(b) $x < a$

$B = \dfrac{\mu_0 ix}{2\pi a}$

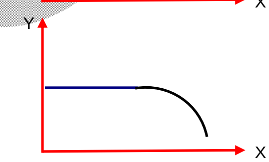


(c) $x > a$

$B = \dfrac{\mu_0 i}{2\pi x}$



(d) Magnetic field inside solenoid form centre of solenoid



Q44. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4th division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1cm ?

(A) 10
(B) 5
(C) 40
(D) 20

Ans. D

Sol. L.C = 1M.S.D – 1 V.S. D = $\dfrac{1}{50}$MS.D

Zero error = 0.04 mm

$4 \times L.C = 0.04$

$LC = 0.01mm$

$\dfrac{1}{50} M.S.D = 0.01mm$

$1 M.S.D = 0.5\,mm$

0.5 mm = 1 M.S.D

So 1cm = 10mm = 20 M.S.D

Q45. When UV light of wavelength 300nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is :

(Given hc = 1240 eV nm)

(A) 4 V
(B) 1.5 V
(C) 2 V
(D) 4.1 V

Ans. C

Sol. $h_f = \phi + eV$

$$\frac{1240}{300} = 2.13 + eV$$

$eV = 4.13 - 2.13 = 2eV$

$V = 2V$

Q46. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division = 49 MSD; 20 divisions on main scale in each cm

For mark on paper

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab

MSR = 7.12 cm, VC = 41

For powder particle on the top surface of the glass slab

MSR = 4.05cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is :

(A) 1.35 (B) 1.42

(C) 1.24 (D) 1.52

Ans. B

Sol. L = M.S.R + L.C $\times$ V.S.R

L.C = 1 M.S.D –1V.S.D

L.C = 0.001 CM

$L_1 = 8.45 + 26 \times 0.001 = 84.76\text{mm}$

$L_2 = 7.12 + 41 \times 0.001 = 71.61\text{mm}$

$L_3 = 4.05 + 1 \times 0.001 = 40.51\text{mm}$

$$\mu = \frac{L_1 - L_3}{L_2 - L_3} = \frac{84.76 - 40.51}{71.61 - 40.51} = 1.42$$

Q47. Assuming the earth to be a sphere of uniform mass density, a body weighed 300N on the surface of earth. How much it would weigh at R/4 depth under surface of earth ?

(A) 375 N (B) 300 N

(C) 75 N (D) 225 N

Ans. D

Sol. Gravitational acceleration at depth

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R/4}{R}\right) = \frac{3g}{4}$$

$$W = mg = \frac{30 \times 3g}{4} = 225\text{N}$$

$$\left(m = \frac{300}{g} = \frac{300}{10} = 30\text{kg}\right)$$

Q48. A total of 48J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is :

Given, R = 8.3JK^{-1} mol^{-1}.

(A) 72.9 J (B) 24.9 J

(C) 48 J (D) 23.1 J

Ans. D

Sol. $\Delta Q = \Delta U + W$

$$\Delta Q = n\left(\frac{fR}{2}\right)\Delta T + w$$

$$48 = \frac{1 \times 3 \times 8.3}{2} \times 2 + w$$
$$w = 48 - 24.9 = 23.1J$$

Q49. In a coil, the current changes from -2 A to +2A in 0.2s and induces an emf of 0.1 V. The self inductance of the coil is :
(A) 1 mH (B) 2.5 mH
(C) 4 mH (D) 5 mH

Ans. **D**

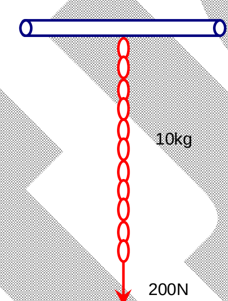
Sol. $\in = -L\frac{di}{dt}$

$$0.1 = L \times \frac{4}{0.2}$$
$$L = 5mH$$

Q50. A body of weight 200N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if g = 10 m/s^2) :
(A) 100 N (B) 200 N
(C) 300 N (D) 150 N

Ans. **C**

Sol. F=200+mg
F=200+10×10=300N



SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q51. For a given series LCR circuit it is found that maximum current is drawn when value of variable capacitance is 2.5nF. If resistance of 200Ω and 100mH inductor is being used in the given circuit. The frequency of ac source is _______ $\times 10^3$Hz. (given $\pi^2 = 10$)

Ans. **10**

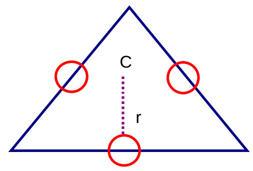
Sol. Current is maximum at resonance

$$f = \frac{1}{2\pi\sqrt{LC}}$$
$$f = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 25 \times 10^{-9}}}$$
$$f = 10 \times 10^3 \text{ Hz}$$

Q52. Three balls of masses 2kg, 4kg and 6kg respectively are arranged at centre of the edges of an equilateral triangle of side 2m. The moment of intertia of the system about an axis through the centroid and perpendicular to the plane, of triangle, will be _______kg m^2.

Ans. **4**

Sol. $r = \frac{a}{2\sqrt{3}} = \frac{1}{\sqrt{3}}m$



$I = m_1r^2 + m_2r^2 + m_3r^2$

$I = r^2(m_1 + m_2 + m_3)$

$I = \left(\frac{1}{\sqrt{3}}\right)^2(2+4+6)$

$I = 4kgm^2$

Q53. Two open organ pipes of lengths 60cm and 90cm resonate at 6^{th} and 5^{th} harmonics respectively. The difference of frequency for the given modes is _____Hz.
(Velocity of sound in air = 333 m/s)

Ans. **740**

Sol. $f = \frac{nv}{2L}$

$\Delta f = \frac{v}{2}\left(\frac{n_1}{L_1} - \frac{n_2}{L_2}\right)$

$\Delta_f = \frac{333}{2}\left(\frac{6}{0.6} - \frac{5}{0.9}\right)$

$\Delta f = 740Hz$

Q54. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n} where n = ________.

Ans. **3**

Sol. $x^2 = 1 + t^2 \Rightarrow x^2 - t^2 = 1$

$2x\frac{dx}{dt} = 2t$

$xv = t$

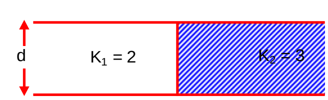
$\frac{dx}{dt}.v + x.\frac{dv}{dt} = 1$

$\Rightarrow v^2 + xa = 1$

$\Rightarrow a = \frac{1-v^2}{x} = \frac{1-\frac{t^2}{x^2}}{x}$

$\Rightarrow a = \frac{x^2 - t^2}{x^3} = \frac{1}{x^3} = x^{-3}$

Q55. A capacitor of $10\mu F$ capacitance whose plates are separated by 10 mm through air and each plate has area $4cm^2$ is now filled equally with two dielectric media of $K_1 = 2, K_2 = 3$ respectively as shown in figure. If new force between the plates is 8N. The supply voltage is V.

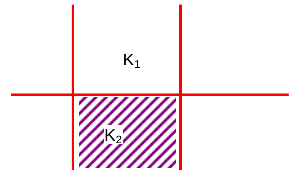


Ans. 0.02

Sol. $C_1 = \frac{A \in_0 \times 2}{d} = 10\mu F$

$C_2 = \frac{A \in_0 \times 3}{d} = 5 \times 3 = 15\mu F$

$C_{eqv} = C_1 + C_2 = 25\mu F$



$Q = Q_1 + Q_2$
$Q_1 = 10V\mu C$
$Q_2 = 15V\mu C$

$$\frac{Q_1^2}{1A \in_0 k_1} + \frac{Q_2^2}{2A \in_0 k} = 8$$

$$\frac{100 \times 10^{-12} v^2}{2} + \frac{225 \times 10^{-12} v^2}{3} = 8 \times 2A \in_0$$

$$125 \times 10^{-12} v^2 = 8 \times 2 \times 4 \times 10^{-4} \times 8.85 \times 10^{-12}$$

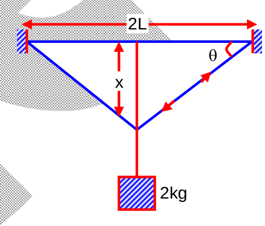
$$v = \sqrt{\frac{8 \times 2 \times 4 \times 8.85 \times 10^{-4}}{125}}$$

$$v = \sqrt{4.5} \times 10^{-2}$$

$$v = 2.12 \times 10^{-2} = 0.02V$$

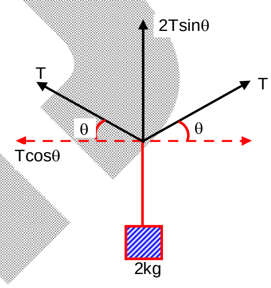
Answer not match with NTA

Q56. A wire of cross sectional area A, modulus of elasticity $2 \times 10^{11} Nm^{-2}$ and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle it sags lower from its original position making angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is _______ $\times 10^{-4} m^2$ (consider x << L).
(given g = 10 m/s^2)



Ans. 1

Sol. $2T\sin\theta = mg$



$$2 \times T \times \frac{1}{100} = 2 \times 10$$

$$T = 1000N$$

$$\Delta L = 2\sqrt{x^2 + L^2} - 2L$$

$$\text{Strain} = \frac{\Delta L}{2L} = \frac{2\sqrt{x^2+L^2} - 2L}{2L}$$

$$\text{Strain} = \frac{1}{L} \times L\sqrt{1 + \frac{x^2}{L^2}} - 1$$

$$= 1 + \frac{1}{2}\left(\frac{x^2}{L^2}\right) - 1$$

$$= \frac{1}{2}\frac{x^2}{L^2}$$

$$\text{Modules of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

$$2 \times 10^{11} = \frac{1000/A}{\frac{1}{2} \times \left(\frac{1}{100}\right)^2}$$

$$\Rightarrow A = \frac{1000 \times 10000 \times 2}{2 \times 10^{11}}$$

$$\Rightarrow A = 1 \times 10^{-4} m^2$$

Q57. Two coherent monochromatic light beams of intensities I and 4I are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is x I. The value of x is______.

Ans. **8**

Sol. $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{I}\sqrt{4I}\right)^2 = 9I$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{I} - \sqrt{4I}\right)^2 = I$

$I_{max} - I_{min} = 9I - I = 8I$

Q58. A coil having 100 turns, area of $5\times10^{-3}m^2$, carrying current of 1 mA is placed in uniform magnetic field of 0.20 T such a way that plane of coil plane of coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is _______ μJ.

Ans. **100**

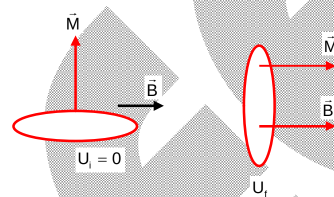
Sol. $\Delta U = U_f - U_i = U_f - 0 = U_f$

$\Delta w = \Delta U = U_f = \vec{M}.\vec{B}$

$\Delta w = N(iA)B$

$\Delta w = 100\times1\times10^{-3}\times5\times10^{-3}\times0.2$

$\Delta w = 100\mu J$



Q59. In Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at 10.2 V. The wavelength of light emitted by hydrogen atom when excited to the first excitation level is ______nm. (Given hc = 1245 eV nm, e = $1.6\times10^{-19}C$).

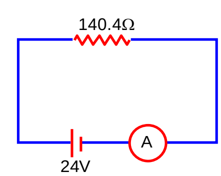
Ans. **122**

Sol. $E = \frac{hc}{\lambda}$

$10.2ev = \frac{1245}{\lambda(nm)}ev$

$\lambda_{nm} = \frac{1245}{10.2} = 122nm$

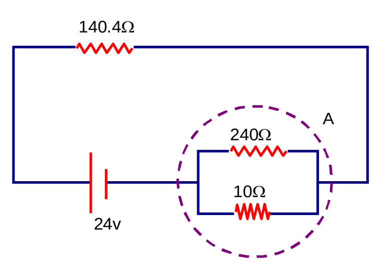
Q60. In the given figure an ammeter A consists of a 240Ω coil connected in parallel to a 10Ω shunt. The reading of the ammeter is______mA.



Ans. **160**

Sol. $i = \frac{V}{R_{eqv}} = \frac{24}{140.4 + \left(\frac{240\times10}{240+10}\right)}$

$i = \frac{24}{150} = 160mA$



PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q61. Arrange the following in the increasing order of number of unpaired electron in it.
(a) Sc
(b) Cr
(c) V
(d) Ti
(e) Mn
Choose the correct answer from the options given below:
(A) (a) < (d) < (c) < (d) <(e)
(B) (a) < (d) < (c) < (e) < (b)
(C) (c) < (e) < (b) < (a) < (d)
(D) (b) < (c) < (d) < (e) < (a)

Ans. **B**

Sol.

	No. of unpaired electrons
$Sc-[Ar]4s^2 3d^1$	1
$Cr-[Ar]4s^1 3d^5$	6
$V-[Ar]4s^2 3d^3$	3
$T_i-[Ar]4s^2 3d^2$	2
$Mn-[Ar]4s^2 3d^5$	5

Q62. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dil. HNO_3 and then $AgNO_3$ solution was added. The indicates presence of:
(A) Br^-
(B) Cl^-
(C) I^-
(D) CO_3^{2-}

Ans. **A**

Sol. AgBr is pale yellow precipitate and is partially soluble in NH_4OH solution.

Q63. The ratio $\frac{K_P}{K_C}$ for the reaction:

$$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons CO_{2(g)}$$

(A) $(RT)^{1/2}$
(B) $\frac{1}{\sqrt{RT}}$
(A) 1
(D) RT

Ans. **B**

Sol. $CO(g) + \frac{1}{2}O_2(9) \rightleftharpoons CO_2(g)$

$$\Delta n_g = 1-\left(1+\frac{1}{2}\right) = -\frac{1}{2}$$

$$\frac{k_P}{k_C} = (RT)^{\Delta ng} = (RT)^{-1/2} = \frac{1}{\sqrt{RT}}$$

Q64. Molality (m) of 3M aqueous solution of NaCl is:
(Given: Density of solution =1.25 g mL^{-1}. Molar mass in g mol^{-1}: Na-23, Cl-35.5)
(A) 3.85
(B) 2.90
(C) 2.79
(D) 1.90 m

Ans. **C**

Sol. $d = \frac{M}{m} + \left(\frac{M \times (MM)_{Solute}}{1000}\right)$

d = 1.25gm/ml , Molarity = 3M, $(MM)_{Solute}$ = 58.5 gm mol^{-1}

Molality (m) $= \frac{3 \times 1000}{(1.25 \times 1000) - (3 \times 58.5)} = 2.79\,m$

Q65. Match **List – I** with **List – II**

List –I	**List – II**
Alkali Metal	**Emission Wavelength in nm**
(a) Li	(I) 589.2
(b) Na	(II) 455.2
(c) Rb	(III) 670.8
(d) Cs	(IV) 780.0

Choose the **correct** answer from the options given below:
(A) (a)-(III), (b)-(I), (c)-(IV), (d)-(II)
(B) (a)-(IV), (b)-(II), (c)-(I), (d)-(III)
(C) (a)-(I), (b)-(IV), (c)-(III), (d)-(II)
(D) (a)-(II), (b)-(IV), (c)-(III), (d)-(I)

Ans. **A**

65. Fact based

Q66. Consider the given reaction, identify the major product P.

$$CH_3-COOH \xrightarrow[\text{(iv) } H_2O\ /\ \bar{O}H,\ \Delta]{\text{(i) } LiAlH_4 \text{ (ii) PCC (iii) } HCN\ /\ \bar{O}H}$$

(A) $CH_3-CH_2-CH_2-OH$
(B) $CH_3-CH_2-\overset{\overset{\large O}{\|}}{C}-OH$
(C) $CH_3-\overset{\overset{\large O}{\|}}{C}-CH_2CH_3$
(D) $CH_3-\overset{\overset{\large OH}{|}}{CH}\text{-}COOH$

Ans. **D**

Sol.

$$CH_3COOH \xrightarrow{LiAlH_4} CH_3CH_2OH \xrightarrow{PCC} CH_3-\overset{\overset{\large O}{\|}}{C}-H \xrightarrow[{}^-OH]{HCN} CH_3-\overset{\overset{\large OH}{|}}{CH}-CN$$

$$\xrightarrow[\Delta]{{}^-OH/H_2O} CH_3-\overset{\overset{\large OH}{|}}{CH}-COOH$$

Q67. How can an electrochemical cell be converted into an electrolytic cell?
(A) Applying an external opposite potential lower than E^0_{cell}
(B) Reversing the flow of ions in salt bridge.
(C) Applying an external opposite potential greater than E^0_{cell}
(D) Exchanging the electrodes at anode and cathode.

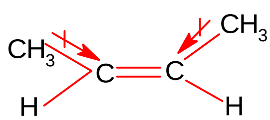
Ans. **C**

Sol. Applying an external opposite potential greater then electromotive force of the cell reverses polarity of electrodes.

Q68. The incorrect statement regarding the geometrical isomers of 2- butene is:
(A) Cis-2- butene has less dipole moment than trans-2- butene.
(B) trans-2-butene is more stable than cis-2-butene.
(C) cis-2-butene and trans-2-butene are stereoisomers.
(D) cis-2-butene and trans-2-butene are not interconvertible at room temperature.

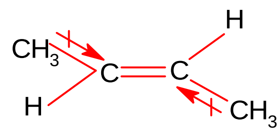
Ans. **A**

Sol.



(Cis)

$\mu_{net} \neq 0$



(trans)

$\mu_{net} = 0$

Cis – 2 – Butene has more dipole moment then trans – 2 – Butene

Q69. The number of ions from the following that are expected to behave as oxidizing agent is:
$Sn^{4+}, Sn^{2+}, Pb^{2+}, Ti^{3+}, Pb^{4+}, Ti^{+}$
(A) 1 (B) 4
(C) 2 (D) 3

Ans. **C**

Sol.
$\left.\begin{array}{l} Tl^{3+} \longrightarrow Tl^{+} \\ Pb^{4+} \longrightarrow Pb^{2+} \end{array}\right\}$ More stable due to inert pair effect

Q70. Given below are two statements:
Statement I : PF_5 and BrF_5 both exhibit sp^3d hybridisation.
Statement II: Both SF_6 and $[Co(NH_3)_6]^{3+}$ exhibit sp^3d^2 hybridisation.
In the light of the above statements, choose the **correct** answer from the options given below:
(A) Both **Statement I** and **Statement II** are true.
(B) **Statement I** is false but **Statement II** is true.
(C) **Statement I** is true but **Statement II** is false.
(D) Both **Statement I** and **Statement II** are false

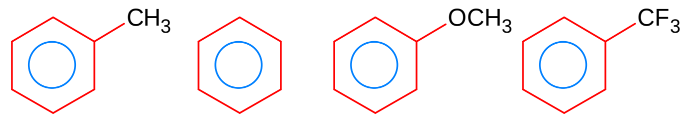
Ans. **D**

Sol.
$PF_5 \longrightarrow sp^3d$ Hybridisation
$BrF_5 \longrightarrow sp^3d^2$ Hybridisation
$SF_6 \longrightarrow sp^3d^2$ Hybridization
$[Co(NH_3)_6]^{3+}$ - d^2sp^3 Hybridization
Both statement (I) & statement (II) are false.

Q71. The incorrect statements regarding enzymes are:
(a) Enzymes are biocatalysts.
(b) Enzymes are non- specific and can catalyse different kinds of reactions.
(c) Most Enzymes are globular proteins.
(d) Enzyme-oxidase catalyes the hydrolysis of maltose into glucose
Choose the correct answer from the option given below:
(A) (b) and (d) (B) (b) and (c)
(C) (a), (b), and (c) (D) (b), (c) and (d)

Ans. **A**

Sol. Enzymes are specific.
Maltase catalyses hydrolysis of maltose into D-glucose.

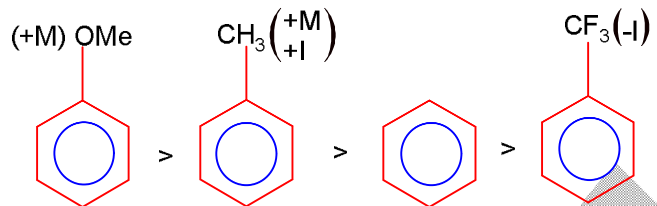
Q72. 

(I) (II) (III) (IV)

The correct arrangement for decreasing order of electrophilic substitution for above compounds is:

(A) (IV) > (I) > (II) > (III)
(B) (II) > (IV) > (III) > (I)
(C) (III) > (IV) > (II) > (I)
(D) (III) > (I) > (II) > (IV)

Ans. **D**

Sol. 

Q73. Match **List – I** with **List – II**

List –I **Tetrahedral Complex**	**List – II** **Electronic configuration**
(a) $TiCl_4$	(I) e^2, t_2^0
(b) $[FeO_4]^{2-}$	(II) e^4, t_2^3
(c) $[FeCl_4]^-$	(III) e^0, t_2^0
(d) $[CoCl_4]^{2-}$	(IV) e^2, t_2^3

Choose the **correct** answer from the options given below:

(A) (a) – (I), (b) – (III), (c) – (IV), (d) – (II)
(B) (a) – (III), (b) – (IV), (c) – (II), (d) – (I)
(C) (a) – (III), (b) – (I), (c) – (IV), (d) – (II)
(D) (a) – (IV), (b) – (III), (c) – (I), (d) – (II)

Ans. **C**

Sol. $TiCl_4 - Ti^{4+} - eg^0 t_{2g}{}^0$

$[FeO_4]^{2-} - Fe^{6+} - eg^2\, t_{2g}{}^0$

$[FeCl_4]^- - Fe^{3+} - eg^2 t_{2g}{}^3$

$[CoCl_4]^{2-} - Co^{2+} - eg^4 t_{2g}{}^3$

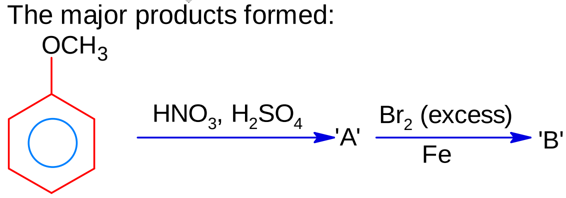
Q74. The IUPAC name of $[PtBr_2(PMe_3)_2]$ is:

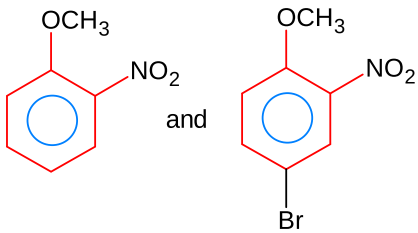
(A) bis [bromo (trimethylphosphine)] plantium (II)
(B) dibromodi (trimethyl phosphine) platinum(II)
(C) bis (trimethyl phosphine) dibromoplatinum(II)
(D) dibromobis (trimethyl phosphine) platinum (II)

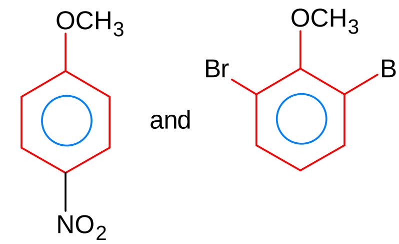
Sol. **D**

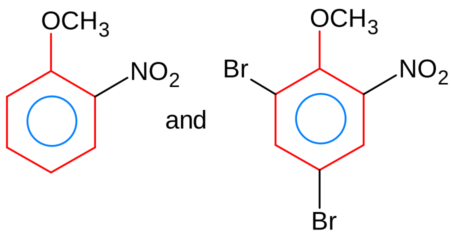
Sol. Dibromo-bis (trimethyl phosphine) platinum (II)

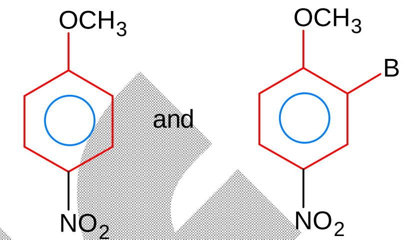
Q75. The major products formed:



(A) 

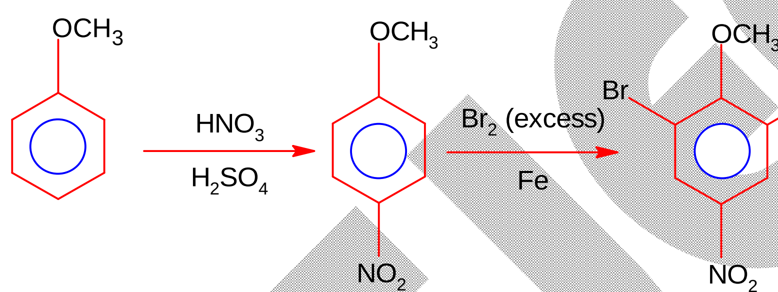
(B) 

(C) 

(D) 

Ans. B

Sol.



Q76. Match **List – I** with **List – II**

List –I **Reaction**	**List – II** **Type of redox reaction**
(a) $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$	(I) Decomposition
(b) $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$	(II) Displacement
(c) $2Na_{(s)} + 2H_2O_{(l)} \rightarrow 2NaOH_{(aq)} + H_{2(g)}$	(III) Disproportionation
(d) $2NO_{2(g)} + 2^{-}OH_{(aq)} \rightarrow NO^{-}_{2(aq)} + H_2O_{(l)} + NO^{-}_{3(aq)}$	(IV) Combination

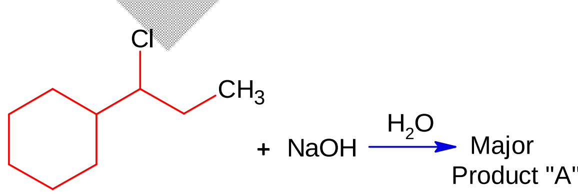
Choose the **correct** answer from the options given below:

(A) (a) – (II), (b) – (III), (c) – (IV), (d) – (I)
(B) (a) – (III), (b) – (II), (c) – (I), (d) – (IV)
(C) (a) – (IV), (b) – (I), (c) – (II), (d) – (III)
(D) (a) – (I), (b) – (II), (c) – (III), (d) – (IV)

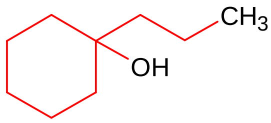
Ans. C

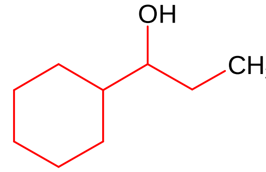
Sol. (a) – Combination
(b) – Decomposition
(c) – Displacement
(d) – Disproportionation

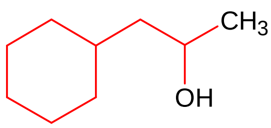
Q77.

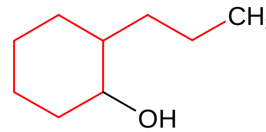


Consider the above chemical reaction, product “A” is

(A) 

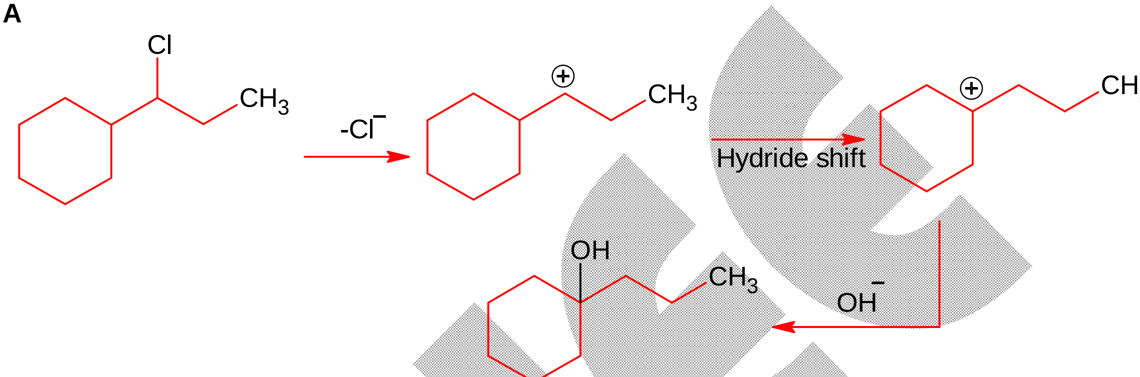
(B) 

(C) 

(D) 

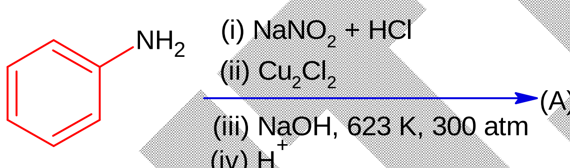
Ans. A

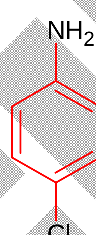
Sol.

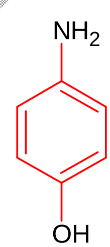


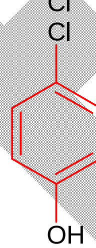
Due to presence of bulkier group, SN^1 mechanism prevails.

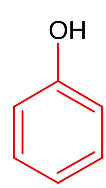
Q78. Identify the product (A) in the following reaction.



(A) 

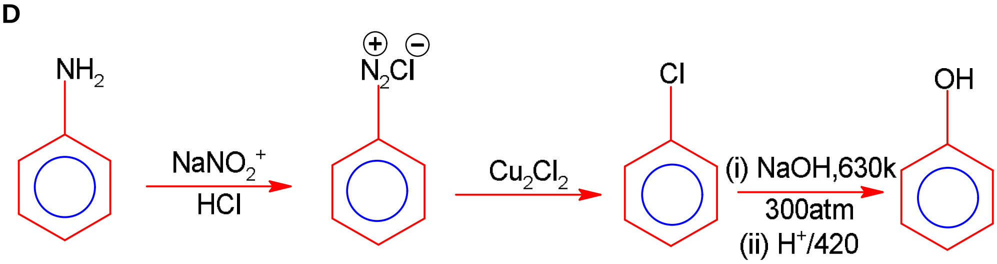
(B) 

(C) 

(D) 

Ans. D

Sol.



Q79. The correct statement among the following, for a “chromatography” purification method is:
(A) R_f is an integral value
(B) Non-polar compounds are retained at top and polar compounds come down in column chromatography.
(C) R_f of a polar compound is smaller than that of a non- polar compound.
(D) Organic compounds run faster than solvent in the thin layer chromatographic plate.

Ans. **C**

Sol. R_f of polar compound is less compared to R_f of non-polar compounds.

Q80. Evaluate the following statements related to group 14 elements for their correctness.
(a) Covalent radius decreases down the group from C to Pb in a regular manner.
(b) Electro negativity decreases from C to Pb down the group gradually.
(c) Maximum covalence of C is 4 whereas other elements can expand their covalance due to presence of d orbitals.
(d) Heavier elements do not form $P\pi$-$P\pi$ bonds.
(e) Carbon can exhibit negative oxidation states.
Choose the correct answer from the options given below:
(A) (c), (d) and (e) only
(B) (a), (b) and (c) only
(C) (a) and (b) only
(D) (c) and (d) only

Ans. **A**

Sol. (A) Down the group. Covalent radius increases
(B) Electronegativty does not decreases gradually from C to Pb.

SECTION - B

(Numerical Answer Type)

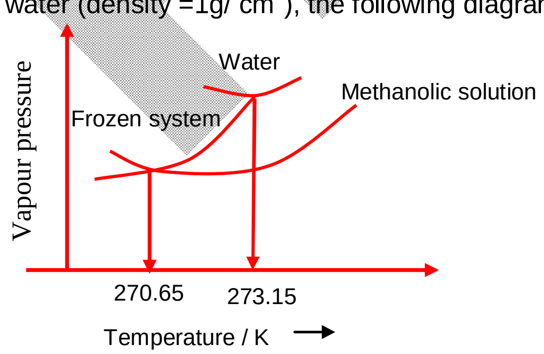
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q81. Among VO_2^+, MnO_4^- and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidizing ability is________ BM (Nearest integer).
(Given atomic V=23, Mn = 25, Cr=24)

Ans. **0**

Sol. Oxidizing ability : $V^{5+} < Cr^{6+} < Mn^{7+}$
$V^{5+} - [Ar]4s^0\,3d^0$
Number of unpaired electrons = 0

Q82. When $'x'\times10^{-2}$ mL methanol (molar mass = 32g; density = 0.792 g/ cm^3) is added to 100 mL water (density =1g/ cm^3), the following diagram is obtained.



[Given: Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol^{-1}]

Ans. **543**

Sol. $\Delta T_f = 273.15 - 270.65 = 2.5k$

$\Delta T_f = k_f \times m$

$\Rightarrow 2.5 = 1.86 \times \dfrac{n}{0.1}$

$\Rightarrow n = 0.1344 \text{moles}$

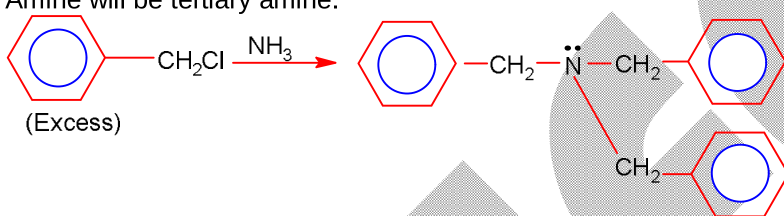
$\text{Mass} = 0.1344 \times 32 = 4.3 \text{gm}$

$\text{Volume} = \dfrac{\text{Mass}}{\text{density}} = \dfrac{4.3\text{gm}}{0.792\text{gm}/\text{m}\ell} = 5.43\text{m}\ell = 543 \times 10^{-2}\text{m}\ell$

Q83. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it the solution remains clear. Molar mass of the amine (X) formed is_______.
(Given: molar mass in gmol^{-1} C:12, H:1, O:16,N:14)

Ans. **287**

Sol. Amine will be tertiary amine.



Molar Mass of (X) = 287

Q84. For hydrogen atom, energy of an electron in first excited state is -3.4eV, K.E of the same electron of hydrogen atom is x eV. Value of x is __________ $\times 10^{-1}$eV (Nearest integer).

Ans. **34**

Sol. Kinetic energy of electron = -1× Energy of orbit

$= 3.4\text{eV}$

$= 34 \times 10^{-1}\text{eV}$

Q85. Total number of species from the following with central atom utilizing sp^2 hybrid orbitals for bonding is_________________.
$NH_3, SO_2, SiO_2, BeCl_2, C_2H_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3, C_2H_4Cl_2$

Ans. **6**

Sol. Central atom having sp^2 hybridisation,

$So_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3$

$SiO_2, NH_3, C_2H_4Cl_2 \longrightarrow sp^3$ hybridization

Q86. Consider the two different first order reactions given below
A + B →C (Reaction 1)
P→Q (Reaction 2)
The ratio of the half life of Reaction 1: Reaction 2 is 5:2. If t_1 and t_2 represent the time take to complete 2/3rd and 4/5th of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1:t_2$ is ______________ $\times 10^{-1}$ (nearest integer)
[Given: $\log_{10}(3)=0.477$ and $\log_{10}(5)=0.699$]

Ans. **17**

Sol.

$t_{1/2} = \dfrac{\ell n 2}{k}$ (for first order reaches)

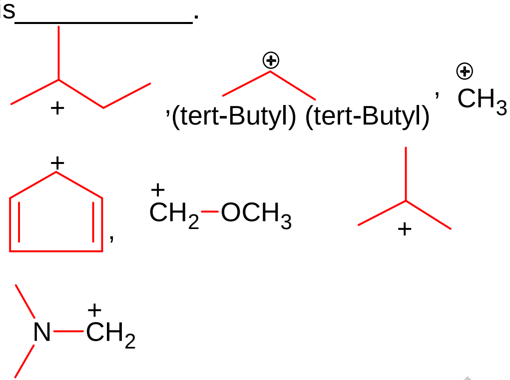
$\dfrac{(t_{1/2})I}{(t_{1/2})II} = \dfrac{k_2}{k_1} = \dfrac{5}{2}$

$t_1 = \dfrac{1}{k_1} \ell n \dfrac{1}{(1-2/3)} = \dfrac{\ell n 3}{k_1}$

$$t_2 = \frac{1}{k_2}\ell n\frac{1}{(1-4/5)} = \frac{\ell n5}{k_2}$$

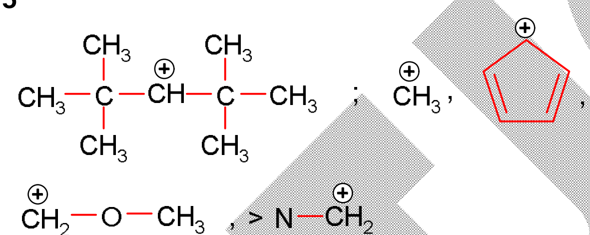
$$\frac{t_1}{t_2} = \frac{\ell n3}{\ell n5}\times\frac{k_2}{k_1} = 1.7 = 17\times10^{-1}$$

Q87. Number of carbocations from the following that are not stabilized by hyperconjugation is____________.



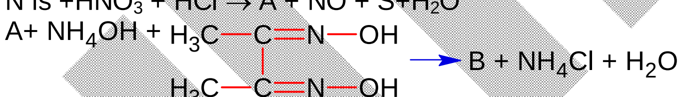
Ans. 5

Sol.



Q88. Consider the following reactions

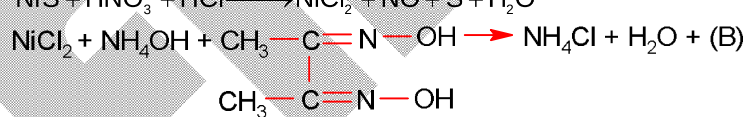
N is $+HNO_3 + HCl \rightarrow A + NO + S + H_2O$



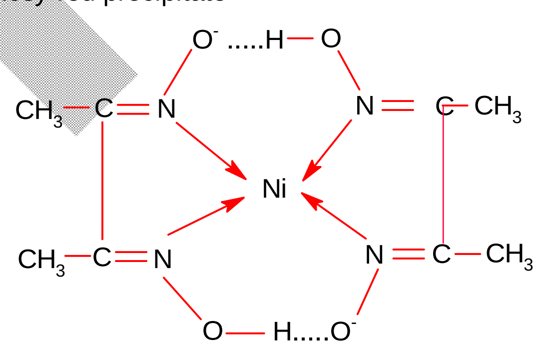
The number of protons that do not involve in hydrogen bonding in the product B is ___________

Ans. 12

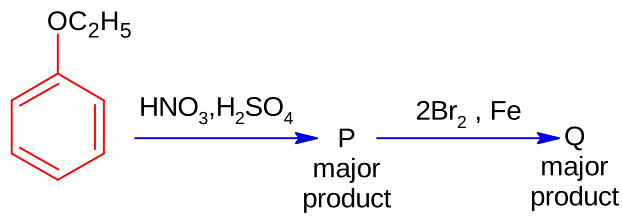
Sol. $NiS + HNO_3 + HCl \longrightarrow NiCl_2 + NO + S + H_2O$



(B) $\rightarrow$ Rosy-red precipitate



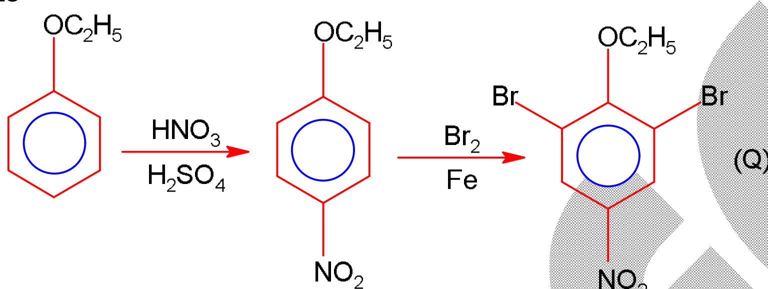
Q89.



The ratio of number of oxygen atoms to bromine atoms in the product Q is ________ $\times 10^{-1}$

Ans. **15**

Sol.



$$\frac{\text{no. of O}-\text{atoms}}{\text{no. of Br}-\text{atoms}}=\frac{3}{2}=1.5=15\times10^{-1}$$

Q90. For the reaction at 298K, 2A+B→C, ΔH=400kJ mol^{-1} and ΔS = 0.2 kJ mol^{-1} K^{-1}. The reaction will become spontaneous above_______ K.

Ans. **2000**

Sol. For feasibility

$\Delta G < O$

$\Delta H - T\Delta S < O$

$\Rightarrow T\Delta S > \Delta H$

$\Rightarrow T > \dfrac{\Delta H}{\Delta S}$

$\Rightarrow T > 2000k$