FIITJEE

Solutions to JEE(Main) -2024

Test Date: 30th January 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*1. A line passing through the point A(9, 0) makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is

(1) $\frac{y}{\sqrt{3}-2}+x=9$ (2) $\frac{x}{\sqrt{3}-2}+y=9$

(3) $\frac{x}{\sqrt{3}+2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

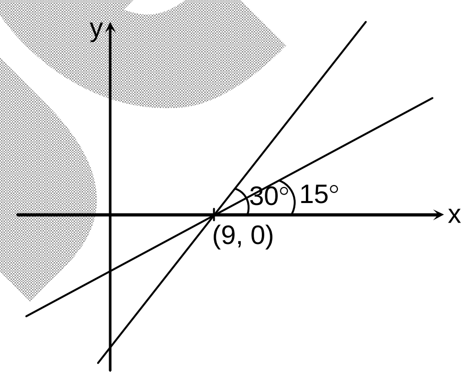
Ans. (1)

Sol. Equation of new line will be

y – 0 = tan 15° (x – 9)

$y=\left(2-\sqrt{3}\right)\left(x-9\right)$

$\Rightarrow\ x+\frac{y}{\sqrt{3}-2}=9$



*2. Let S_n denote the sum of first n terms of an arithmetic progression. If S_{20} = 790 and S_{10} = 145, then $S_{15}-S_5$ is

(1) 395 (2) 390

(3) 405 (4) 410

Ans. (1)

Sol. $S_{20}=\frac{20}{2}\left(2a+\left(20-1\right)d\right)=790$

$\Rightarrow$ 2a + 19d = 79 ….. (1)

$S_{10}=\frac{10}{2}\left(2a+\left(10-1\right)d\right)=145$

$\Rightarrow$ 2a + 9d = 29 ….. (2)

d = 5

a = –8

$S_{15}-S_5=\frac{15}{2}\left(-16+14\times 5\right)-\frac{5}{2}\left(-16+4\times 5\right)$ = 15 × 27 – 5 × 2 = 405 – 10 = 395

*3. If z = x + iy, xy $\neq$ 0, satisfies the equation $z^2 + i\bar{z} = 0$, then $\left|z^2\right|$ is equal to

(1) 9
(2) 1
(3) 4
(4) $\frac{1}{4}$

Ans. (2)

Sol. $z^2 = -i\bar{z}$

$\left|z^2\right| = \left|-i\bar{z}\right|$

$\left|z^2\right| = |z|$

|z| = 0 or 1
But |z| = 0 (Given)
So, |z| = 1

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$, $\vec{a}\cdot\vec{b} = 2$ and $\left|\vec{b}\right| = 4$. If $\vec{c} = 2\left(\vec{a}\times\vec{b}\right) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Ans. (3)

Sol. $\vec{a}\cdot\vec{b} = |\vec{a}|\left|\vec{b}\right|\cos\theta = 2$

$\cos\theta = \frac{1}{2}; \theta = \frac{\pi}{3}$

$\vec{c} = 2\left(\vec{a}\times\vec{b}\right) - 3\vec{b}$

$|\vec{c}|^2 = \left|2\left(\vec{a}\times\vec{b}\right) - 3\vec{b}\right|^2 = 4\left|\vec{a}\times\vec{b}\right|^2 + 9\left|\vec{b}\right|^2 - 12\left(\vec{a}\times\vec{b}\right)\cdot\vec{b}$

$|\vec{c}|^2 = 4\left(1\cdot 4\cdot\frac{\sqrt{3}}{2}\right)^2 + 9\times 4^2 - 0$

$|\vec{c}|^2 = 48 + 144 = 192$

Now, $\vec{c}\cdot\vec{b} = 2\left(\vec{a}\times\vec{b}\right)\cdot\vec{b} - 3\vec{b}\cdot\vec{b}$

$|\vec{c}|\left|\vec{b}\right|\cos\alpha = -3\times 4^2$

$\cos\alpha = \frac{-3\times 4^2}{4\times\sqrt{192}} = \frac{-3\times 4^2}{4\times 8\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\alpha = 150° = \frac{5\pi}{6}$

5. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (–x, y), where y > 0, is

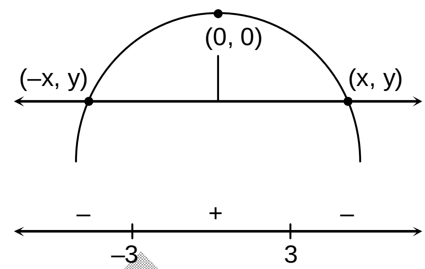
(1) 88
(2) 122
(3) 92
(4) 108

Ans. **(4)**

Sol. $y = -2x^2 + 54$

$2x^2 = 54 - y$

$x^2 = \frac{54-y}{2}$



Area of $\Delta = \frac{1}{2}|y|(2x)$

$f(x) = \Delta = \frac{1}{2}\left(54 - 2x^2\right) \times 2x$

$f'(x) = 54 - 6x^2 = 0$

$\Rightarrow 6(9 - x^2)$; $x = \pm 3$

f(x) will be minimum at 3

$f(3) = (54 - 2 \times 3^2) \times 3 = 108$

6. The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{\left(n^2+k^2\right)\left(n^2+3k^2\right)}$ is

(1) $\frac{\left(2\sqrt{3}+3\right)\pi}{24}$ (2) $\frac{13\pi}{8\left(4\sqrt{3}+3\right)}$

(3) $\frac{13\left(2\sqrt{3}-3\right)\pi}{8}$ (4) $\frac{\pi}{8\left(2\sqrt{3}+3\right)}$

Ans. **(2)**

Sol. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{n^4\left(1+\left(\frac{k}{n}\right)^2\right)\left(1+3\left(\frac{k}{n}\right)^2\right)}$

$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\frac{1}{\left(1+\left(\frac{k}{n}\right)^2\right)\left(1+3\left(\frac{k}{n}\right)^2\right)} = \int_0^1\frac{1}{\left(1+x^2\right)\left(1+3x^2\right)}dx$

$= \frac{1}{2}\int_0^1\frac{3\left(1+x^2\right)}{\left(1+x^2\right)\left(1+3x^2\right)} - \frac{\left(1+3x^2\right)}{\left(1+x^2\right)\left(1+3x^2\right)}dx = \frac{1}{2}\int_0^1\frac{3}{1+3x^2} - \frac{1}{1+x^2}dx$

$= \frac{1}{2}\left[\frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{1}\right) - \tan^{-1}x\right]_0^1 = \frac{1}{2}\left[-\frac{3}{\sqrt{3}}\times\frac{\pi}{3} - \frac{\pi}{4}\right] = \frac{\pi}{24}\left(4\sqrt{3}-3\right) = \frac{\pi}{24}\frac{(39)}{\left(4\sqrt{3}+3\right)} = \frac{13\pi}{8\left(4\sqrt{3}+3\right)}$

7. Let g : R → R be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x) = \frac{1}{2}\left[g(x) + g(2-x)\right]$, then

(1) $f''(x) = 0$ for atleast two x in (0, 2) (2) $f''(x) = 0$ for exactly one x in (0, 1)

(3) $f''(x) = 0$ for no x in (0, 1) (4) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

Ans. (1)

Sol. $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$

$f(x) = \frac{1}{2}\left(g(x) + g(2-x)\right)$

$f'(x) = \frac{1}{2}\left(g'(x) - g'(2-x)\right)$

$f'\left(\frac{1}{2}\right) = \frac{1}{2}\left(g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)\right) = 0$ and $f'\left(\frac{3}{2}\right) = 0$

Using Rolle's Theorem at least one root $\in \left(\frac{1}{2}, 1\right)$ and at least one root (1, 2)

$f''(x)$ have at least two roots in (0, 2)

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x - 2)$ and the line
$y = 2x - 8$, is

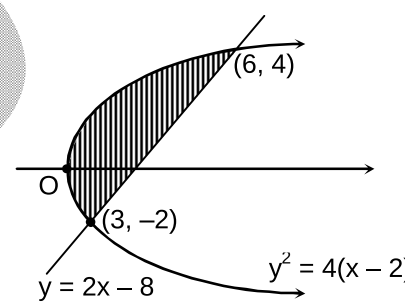
(1) 8 (2) 9
(3) 6 (4) 7

Ans. (2)

Sol. $y^2 = 4(x - 2)$
$y = 2x - 8$
$y^2 = 2y + 8$
$(y - 1)^2 = 9$
$Y = 1 \pm 3$

Required area = $\int_{-2}^{4}\left(\frac{y+8}{2} - \left(\frac{y^2}{4} + 2\right)\right)dy$

= 9



9. Let y = y(x) be the solution of the differential equation
sec x dy + {2(1 – x)tan x + x(2 – x)} dx = 0 such that y(0) = 2. Then y(2) is equal to

(1) 2 (2) 2{1 – sin(2)}
(3) 2{sin(2) + 1} (4) 1

Ans. (1)

Sol. $dy = \left(\frac{2(x-1)\tan x}{\sec x} + \frac{(x-2)x}{\sec x}\right)dx$

$y = \int\left(2(x-1)\sin x + (x^2 - 2x)\cos x\right)dx$

$= \int 2(x-1)\sin x\, dx + \left[(x^2 - 2x)\sin x\right] - \int 2(x-1)\sin x\, dx$

$y = (x^2 - 2x)\sin x + c$

f(0) = c = 2
f(2) = 0 + c = 2

10. Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Then 19(α + β + γ) is equal to

(1) 102 (2) 101
(3) 99 (4) 100

Ans. (2)

Sol. dr' of PM
$5\mu - 4, 2\mu - 1, 3\mu - 7$
$5(5\mu - 4) + 2(2\mu - 1) + 3(3\mu - 7) = 0$
$\Rightarrow 38\mu - 43 = 0 \Rightarrow \mu=\frac{43}{38}$
$\alpha + \beta + \gamma = 10\mu - 6 = 10\times\frac{43}{38}-6=\frac{215-114}{16}=\frac{101}{19}=101$

11. Two integers x and y chosen with replacement from the set {0, 1, 2, 3, …, 10}. Then the probability that |x – y| > 5, is

(1) $\frac{30}{121}$ (2) $\frac{62}{121}$
(3) $\frac{60}{121}$ (4) $\frac{31}{121}$

Ans. (1)

Sol. $|x - y| > 5$
Number of ways to select (x, y) such that
$|x - y| = 6$ is 10
$|x - y| = 7$ is 8
$|x - y| = 8$ is 6
$|x - y| = 9$ is 4
$|x - y| = 10$ is 2

Total: 30

Required probability = $\frac{30}{121}$

12. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then α + β + γ is equal to

(1) 12 (2) 9
(3) 11 (4) 8

Ans. (3)

Sol. $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\{\log_e(3-x)\}^{-1}$

Condition-I

$-1 \le \frac{2-|x|}{4} \le 1$

$-4 \le 2 - |x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$x \in [-6, 6]$

$x \in [-6, 3) - \{2\}$

$\alpha + \beta + \gamma = 6 + 3 + 1 = 11$

Condition-II

$3 - x > 0$

$x < 3$

$3 - x \ne 1$

$x \ne 2$

13. Consider the system of linear equations $x + y + z = 4\mu$, $x+ 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda, \mu \in R$. Which one of the following statements in NOT correct?

(1) The system has unique solution if $\lambda \ne \frac{1}{2}$ and $\mu \ne 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \ne 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \ne \frac{1}{2}$

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2\lambda - 1 \\ 0 & 1 & 4\lambda^2 - 2\lambda \end{vmatrix}$

$\Delta = 2\lambda(2\lambda - 1) - (2\lambda - 1) = (2\lambda - 1)^2$

$\Delta_z = \begin{vmatrix} 1 & 1 & 4\mu \\ 1 & 2 & 10\mu \\ 1 & 3 & \mu^2 + 15 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 4\mu \\ 0 & 1 & 6\mu \\ 0 & 1 & \mu^2 - 10\mu + 15 \end{vmatrix} = \Delta_x = \Delta_y$

$\Delta_x = \mu^2 - 16\mu + 15 = 0$

$\mu = 15, 1$

*14. If the circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Ans. (3)

Sol. $r_1 = r$, $C_1(-1, -2)$

$r_2 = 2$, $C_2(2, 2)$

Circle intersect exactly at two distinct points if $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

Now, $C_1C_2 = \sqrt{3^2 + 4^2} = 5$

$|r - 2| < 5$ and $5 < r + 2$

$-5 < r - 2 < 5$ $\quad$ $r > 3$

$-3 < r < 7$

Hence, $3 < r < 7$

*15. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is

(1) $\frac{\sqrt{5}}{3}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{5}}$

Ans. (4)

Sol. $2b = \frac{1}{2}(2ae) \Rightarrow e = \frac{2}{\sqrt{5}}$

*16. Let M denote the median of the following frequency distribution

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	9	10	8	6

Then 20M is equal to

(1) 416 (2) 104

(3) 52 (4) 208

Ans. (4)

Sol. l = 8, N = 36

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	9	10	8	6

$f_m = 10$

$h = 4$

$cf_{m-1} = 12$

$$M = \ell + \left(\frac{\frac{N}{2} - cf_{m-1}}{f_m}\right)h = 8 + \left(\frac{18-12}{10}\right) \times 4$$

$$M = 8 + \frac{24}{10} = 10.4$$

20M = 208

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 2+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$, then $\frac{1}{5}f'(0) =$ is equal to

(1) 0 (2) 1

(3) 2 (4) 6

Ans. (1)

Sol. $$f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 2+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix} = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 2+\sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix} = 45$$

$f'(x) = 0$

18. Let A(2, 3, 5) and C(–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to

(1) $\frac{1}{2}\sqrt{410}$
(2) $\frac{1}{2}\sqrt{474}$
(3) $\frac{1}{2}\sqrt{586}$
(4) $\frac{1}{2}\sqrt{306}$

Ans. (2)

Sol. Required area = $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = \frac{\sqrt{474}}{2}$

*19. If 2 $\sin^3 x + \sin 2x \cos x + 4 \sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then the roots of the equation $x^2 + nx + (n - 3) = 0$ belong to

(1) $(0, \infty)$
(2) $(-\infty, 0)$
(3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$
(4) Z

Ans. (2)

Sol. $2 \sin^3 x + \sin 2x \cos x + 4 \sin x - 4 = 0$
$\Rightarrow 2 \sin^3 x + 2 \sin x (1 - \sin^2 x) + 4 \sin x - 4 = 0$
$\Rightarrow 6 \sin x = 4$
$\sin x = \frac{2}{3}$ has exactly, 3 solution in $\left[0, \frac{n\pi}{2}\right]$
So, n = 5
Now, $x^2 + 5x + 2 = 0$
$x = \frac{-5 \pm \sqrt{25-8}}{2}$ hence, roots are negative (–)

20. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the $\lim_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$, then $8\alpha^2$ is equal to

(1) 16
(2) 2
(3) 1
(4) 4

Ans. (2)

Sol. $\alpha = \lim_{x \to 0} \frac{\int_0^x f(t)dt \cdot 1 + x \cdot f(x) \cdot 1 - 0}{e^{x^2}(2x)}$

$\alpha = \frac{1}{2}$

SECTION - B

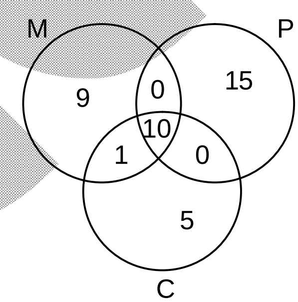
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*21. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _____

Ans. 10

Sol. $n(S) = 40$



$n(P \cup C \cup M) = 40$
$n(M) = 20$
$n(P) = 25$
$n(C) = 16$
$n(P \cap C) \le 15$
$n(M \cap C) \le 15$
$n(P \cap M) \le 11$
$n(P \cap M \cap C) = ?$
$\Rightarrow$ Maximum $n(P \cap M \cap C) \le$ minimum $\{n(P \cap C), n(P \cap M), n(C \cap M)\} = 11$
But 11 not satisfying given condition
Hence, required solution is 10

22. If d_1 is the shortest distance between the lines $x + 1 = 2y = -12z$, $x = y + 2 = 6z - 6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is _____

Ans. 16

Sol. $$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} = \frac{2\hat{i}}{12} - \frac{3\hat{j}}{12} + \frac{k}{2}$$

$$d_1 = \left| \frac{\left(\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(\frac{2\hat{i}}{12} - \frac{3\hat{j}}{12} + \frac{\hat{k}}{2}\right)}{\left|\frac{\hat{i}}{12} - \frac{3\hat{i}}{12} + \frac{\hat{k}}{2}\right|} \right| = \frac{\frac{14}{12}}{\frac{\sqrt{49}}{12}} = 2$$

Again, $\vec{b_1'} \times \vec{b_2'} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = 16\hat{i} + 16\hat{j} + 16\hat{k}$

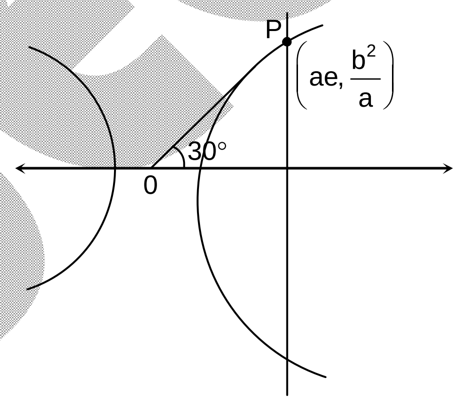
$d_2 = \left| \frac{(10\hat{j} + 2\hat{k}) \cdot (16\hat{i} + 16\hat{j} + 16\hat{k})}{16\sqrt{3}} \right| = \frac{16 \times 12}{16\sqrt{3}} = 4\sqrt{3}$

Now, $\frac{32\sqrt{3}d_1}{d_2} = \frac{32 \times \sqrt{3} \times 2}{4\sqrt{3}} = 16$

*23. Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{\ell}{m}(1 + \sqrt{n})$, where ℓ and m are co-prime numbers, than $\ell^2 + m^2 + n^2$ is equal to _____

Ans. **182**

Sol. Slope of $OP = \frac{b^2}{a^2 e} = \tan 30°$



$\frac{b^2}{9e} = \frac{1}{\sqrt{3}}$

$3b^4 = 81e^2 = 81\left(1 + \frac{b^2}{9}\right)$

$3b^4 = 81 + 9b^2$

$B^4 - 3b^2 - 27 = 0$

$b = \frac{3 + \sqrt{9 + 108}}{2} = \frac{3}{2}(1 + \sqrt{13})$

So, $\ell = 3$, m = 2, n = 13

$\ell^2 + m^2 + n^2 = 182$

24. Let A = {1, 2, 3, ….., 7} and let P(A) denote the power set of A. If the number of functions f : A $\to$ P(A) such that a $\in$ f(a), $\forall$ a $\in$ A is m^n, m and n $\in$ N and m is least, then m + n is equal to _____

Ans. **44**

Sol. Number of elements in power set of A = 2^7

Number of ways to choose an element of P(A) such that a $\in$ f(a) = $2^7 - 2^6 = 2^6$

Number of function f : A $\to$ P(A) such that a $\in$ f(a) = $(2^6)^7 = 2^{42}$

$\Rightarrow m^n = 2^{42}$ $\therefore$ m + n = 44

25. The value of $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$, where [t] denotes the greatest integer less than or equal to t, is _____

Ans. **155**

Sol. $\frac{10x}{x+1} = 1 \Rightarrow x = \frac{1}{9}$

$\frac{10x}{x+1} = 4 \Rightarrow x = \frac{2}{3}$

$\frac{10x}{x+1} = 9 \Rightarrow x = 9$

$I = \int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} 1dx + \int_{2/3}^{9} 2\,dx = \frac{155}{9}$

*26. Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to _____

Ans. **138**

Sol. LCM of (2, 6) = 6

Required number of terms = $\left[\frac{824}{6}\right] + 1 = 137 + 1 = 138$

(Where [.] represents greatest integer function)

27. Let y = y(x) be the solution of the differential equation $\left(1-x^2\right)dy = \left[xy + \left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$, –1 < x < 1, y(0) = 0. Id $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to _____

Ans. 9**7**

Sol. $\left(1-x^2\right)dy = \left[xy + \left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$

$\frac{dy}{dx} + \frac{x}{x^2-1}y = \frac{\left(x^3+2\right)\sqrt{3}}{\sqrt{1-x^2}}$

I.f = $e^{-\int \frac{x}{-x^2+1}dx} = e^{\ln\sqrt{1-x^2}} = \sqrt{1-x^2}$

$\Rightarrow d\left(y\sqrt{1-x^2}\right) = \int \sqrt{3}\left(x^3+2\right)dx$

$y\left(\sqrt{1-x^2}\right) = \sqrt{3}\left(\frac{x^3}{3} + 2x\right) + c$

y(0) = 0 ⇒ y = c = 0

$f\left(\frac{1}{2}\right) = \frac{65}{32} = 97$

*28. Let α, β ∈ N be roots of the equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. If λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to _____

Ans. **60**

Sol. $\alpha + \beta = 70$
$\alpha\beta = \lambda$
$f(\alpha) = \lambda = \alpha(70 - \alpha)$
$\alpha = 1, \beta = 69 \Rightarrow \lambda = 69$
$\alpha = 5 \Rightarrow \beta = 65 \Rightarrow \lambda = 325$
$\therefore$ Required result = $\frac{(2+8)(325+35)}{60} = 60$

29. If the function $f(x) = \begin{cases} \frac{1}{|x|} & ; \quad |x \geq 2| \\ ax^2 + 2b & ; \quad |x| < 2 \end{cases}$ is differentiable on R, then 48(a + b) is equal to _____

Ans. **15**

Sol. $f(x) = \begin{cases} -\frac{1}{x} & ; \quad x \leq -2 \\ ax^2 + 2b & ; \quad -2 < x < 2 \\ \frac{1}{x} & ; \quad x \geq 2 \end{cases}$

At, x = –2

$4a + 2b = \frac{1}{2}$ ..... (1)

$f'(x) = \begin{cases} \frac{1}{x^2} & ; \quad x \leq -2 \\ 2ax & ; \quad -2 < x < 2 \\ -\frac{1}{x^2} & ; \quad x \geq 2 \end{cases}$

At, x = 2

$4a = -\frac{1}{4} \Rightarrow a = -\frac{1}{16} \Rightarrow b = \frac{5}{16}$

48(a + b) = 15

*30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$.
If $4\alpha - \beta = 55k + 40$, then k is equal to _____

Ans. **353**

Sol. $\alpha = \sum_{r=1}^{10} \frac{\left(r^2 + 3r - 2\right)^2}{4}$

$4\alpha = \sum_{r=1}^{10} \left(r^4 + 9r^2 + 4 + 6r^3 - 4r^2 - 12r\right)$

$4\alpha = \sum_{r=1}^{10} r^4 + 6\sum_{r=1}^{10} r^3 + 5\sum_{r=1}^{10} r^2 - 12\sum_{r=1}^{10} r + \sum_{r=1}^{10} 4$

$$4\alpha = \beta + 6\left(\frac{10\times11}{2}\right)^2 + 5\left(\frac{10\times11\times21}{6}\right) - 12\left(\frac{10\times11}{2}\right) + 40$$

$4\alpha - \beta = 6 \times 55^2 + 35 \times 55 - 12 \times 55 + 40$

$4\alpha - \beta = 55(330 + 35 - 12) + 40$

$4\alpha - \beta = 55(353) + 40 = 55k + 40$

$k = 353$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

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31. Match List - I with List - II.

List – I	List - II
(A) Coefficient of viscosity	(I) $[M L^2 T^{-2}]$
(B) Surface tension	(II) $[M L^2 T^{-1}]$
(C) Angular momentum	(III) $[M L^{-1} T^{-1}]$
(D) Rotational kinetic energy	(IV) $[M L^0 T^{-2}]$

Choose the correct answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Ans. **(3)**

Sol. (A) $F = \eta A \frac{dv}{dy}$

$[\eta] = [ML^{-1}T^{-1}]$

(B) Surface tension = force / length

$\therefore [ML^0T^{-2}]$

(C) Angular momentum

$L = mvr$

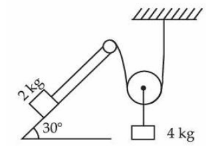
$\therefore [ML^2T^{-1}]$

(D) Rotational kinetic energy

K.E. = $\frac{1}{2}I\omega^2$

$\therefore [ML^2T^{-2}]$

*32. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2kg is



(1) g
(2) $\frac{g}{3}$
(3) $\frac{g}{2}$
(4) $\frac{g}{4}$

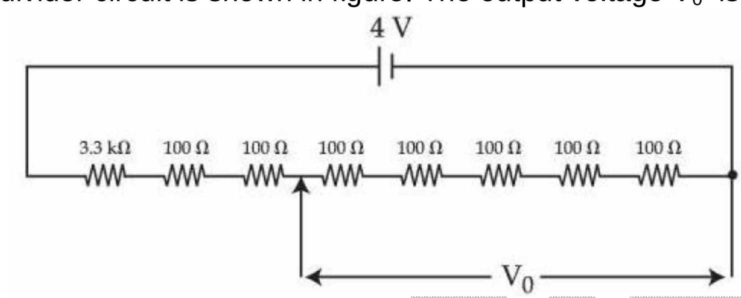
Ans. **(2)**

Sol. $2g \sin 30° - T = 2a$

$2T - 4g = 4\left(\frac{a}{2}\right)$

$\therefore\ a = g/3$

33. A potential divider circuit is shown in figure. The output voltage V_0 is:



(1) 4V
(2) 2 mV
(3) 0.5 V
(4) 12 mV

Ans. **(3)**

Sol. $R_{eq} = 3300 + 700 = 4000\ \Omega$

$V_0 = \frac{4}{4000}(500) = 0.5\ V$

*34. Young's modulus of material of a wire of length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modulus will be:

(1) $\frac{Y}{4}$
(2) 4Y
(3) Y
(4) 2Y

Ans. **(3)**

Sol. Young's modulus does not depend on dimension of the object. It depends on the material.

35. The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately;

(1) 215 nm
(2) 414 nm
(3) 400 nm
(4) 200 nm

Ans. **(2)**

Sol. $\lambda_{max} = \frac{hc}{\text{work function}}$

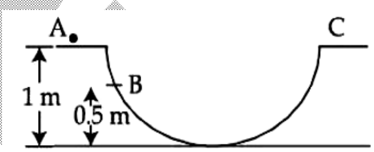
36. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is :

(1) 4
(2) $\frac{1}{4}$
(3) $\frac{1}{2}$
(4) 1

Ans. (3)

Sol. $\frac{\text{K.E.}}{\text{P.E.}} = \frac{1}{2}$

*37. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed towards right. The speed of the particle when it reaches the point B is: (Take $g=10\ m/s^2$),



(1) 20 m/s
(2) $\sqrt{10}$ m/s
(3) $2\sqrt{10}$ m/s
(4) 10 m/s

Ans. (2)

Sol. $\frac{1}{2}mv^2 = mg\left(\frac{1}{2}\right)$

$v^2 = g$

$v = \sqrt{10}$ m/s

38. The electric field of an electromagnetic wave in free space is represented as $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$. The corresponding magnetic induction vector will be:

(1) $\vec{B} = E_0 C \cos(\omega t - kz)\hat{j}$
(2) $\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\hat{j}$
(3) $\vec{B} = E_0 C \cos(\omega t + kz)\hat{j}$
(4) $\vec{B} = \frac{E_0}{C}\cos(\omega t + kz)\hat{j}$

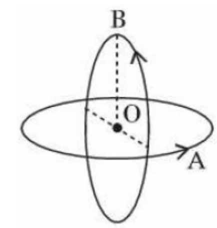
Ans. (2)

Sol. $B_0 = \frac{E_0}{C}$

$\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\hat{j}$

Also, $\hat{C} = \hat{E} \times \hat{B}$

39. Two insulated circular loop A and B of radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in the figure. The magnitude of the magnetic induction at the centre will be:



(1) $\frac{\sqrt{2}}{a}\mu_0 I$ (2) $\frac{\mu_0 I}{2a}$

(3) $\frac{\mu_0 I}{\sqrt{2}a}$ (4) $\frac{2\mu_0 I}{a}$

Ans. (3)

Sol. $B_{net} = \frac{\mu_0 i}{2a}\sqrt{2}$

As magnetic field due to both the rings will be perpendicular to each other at the centre.

40. The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be :

(1) 2 mm (2) 2 cm

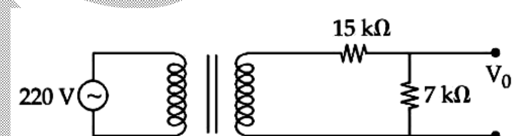
(3) 0.02 mm (4) 0.2 mm

Ans. (1)

Sol. D = 100 cm, b = 0.2 mm and λ = 400 nm

$\therefore$ width $= D\frac{\lambda}{b}$

41. Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transformer are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistances shown in figure. The output voltage (V_0) is :



(1) 7 V (2) 15 V

(3) 44 V (4) 22 V

Ans. (1)

Sol. $\frac{E_S}{E_P} = \frac{N_S}{N_P}$

$E_s = \frac{10}{100} \times 200 = 22\ V$

$V_0 = \frac{22}{(15+7)\times 10^3} \times 7 \times 10^3$

$V_0 = 7V$

*42. The gravitational potential at a point above the surface of earth is $-5.12 \times 10^7\ J/kg$ and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is:

(1) 1600 km (2) 540 km

(3) 1200 km (4) 1000 km

Ans. (1)

Sol. $\frac{-GM}{R+h} = -5.12 \times 10^7$

$\frac{GM}{(R+h)^2} = 6.4$

$(R+h) = \frac{5.12 \times 10^7}{6.4}$

$6.4 \times 10^3 + h = 8 \times 10^6$

h = 1600 km

43. An electric toaster has resistance of 60 Ω at room temperature (27°C). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around: (if $\alpha = 2 \times 10^{-4}$/°C)

(1) 694°C (2) 1235°C
(3) 1694°C (4) 1667°C

Ans. **(3)**

Sol. $R = \frac{220}{2.75} = 80\Omega$

$80 = 60\,(1 + \alpha\Delta T)$

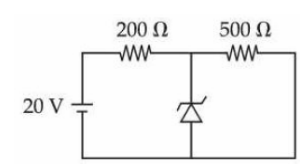
$\alpha(\Delta T) = 1/3$

$\Delta T = \frac{1}{3 \times 2 \times 10^{-4}}$

$T - 27 = 1666.66$

$T = 1693.66$°C

44. A Zener diode of breakdown voltage 10 V is used as a voltage regulator as shown in the figure. The current through the Zener diode is:

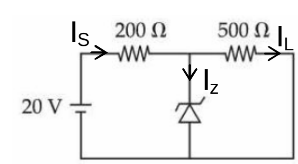


(1) 50 mA (2) 0
(3) 30 mA (4) 20 mA

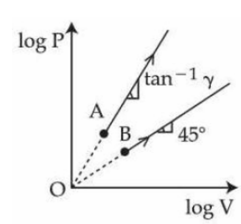
Ans. **(3)**

Sol. $I_z = I_S - I_L$

$= \frac{10}{200} - \frac{10}{500} = 30$ mA



*45. Two thermodynamical processes are shown in the figure. The molar heat capacity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement.



(1) $C_B = \infty$, $C_A = 0$ (2) $C_A = 0$ and $C_B = \infty$
(3) $C_P > C_V > C_A = C_B$ (4) $C_A > C_P > C_V$

Ans. **No answer matching**

Sol. For A, log P = Y log V

$P = V^{\gamma}$
$PV^{-\gamma}$ = constant
$C_A = C_V + \frac{R}{1+\gamma}$
For B,
Log P = log V
PV^{-1} = constant
$C_B = C_V + \frac{R}{2}$

46. The electrostatic potential due to an electric dipole at a distance 'r' varies as:
(1) r
(2) $1/r^2$
(3) $1/r^3$
(4) 1/r

Ans. (2)

Sol. $V = \frac{KP\cos\theta}{r^2}$
Where P is the dipole moment

*47. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5 m. The impulse of force imparted by the ground to the body is given by: (given, g = 9.8 m/s^2)
(1) 4.32 $kgms^{-1}$
(2) 43.2 $kgms^{-1}$
(3) 23.9 $kgms^{-1}$
(4) 2.39 $kgms^{-1}$

Ans. (4)

Sol. Impulse = $|\vec{P}_F - \vec{P}_i|$
$= |m\sqrt{2g\cdot 5} - (-m\sqrt{2g\cdot 10})|$
$= |m\sqrt{2(9.8)\cdot 5} - (-m\sqrt{2(9.8)\cdot 10})|$

*48. A particle of mass m is projected with a velocity 'u' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is:
(1) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$
(2) $\frac{\sqrt{3}}{2}\frac{mu^2}{g}$
(3) $\frac{mu^3}{\sqrt{2}g}$
(4) zero

Ans. (1)

Sol. L = h(mu cos 30°)
$= \frac{u^2\sin^2\theta}{2g}\cdot mu\frac{\sqrt{3}}{2} = \frac{mu^3\sqrt{3}}{16g}$

*49. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(1) 80 K
(2) – 73 K
(3) 4 K
(4) 20 K

Ans. **(4)**

Sol. $\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(273+47)}{32}}$

T/2 = 10

T = 20 K

50. A series L.R circuit connected with an ac source E=(25 sin 1000 t) V has a power factor of 1/√2. If the source of emf is changed to E=(20 sin 2000 t) V, the new power factor of the circuit will be:

(1) $\frac{1}{\sqrt{2}}$
(2) $\frac{1}{\sqrt{3}}$
(3) $\frac{1}{\sqrt{5}}$
(4) $\frac{1}{\sqrt{7}}$

Ans. **(3)**

Sol. $\tan 45° = \frac{\omega L}{R}, \omega L = R$

$\tan\phi = \frac{2\omega L}{R} = \frac{2}{1}$

$\cos\phi = \frac{1}{\sqrt{5}}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. The horizontal component of earth's magnetic field at a place is 3.5×10^{-5} T. A very long straight conductor carrying current of √2 A in the direction from South east to North West is placed. The force per unit length experienced by the conductor is __________ $\times 10^{-6}$ N / m .

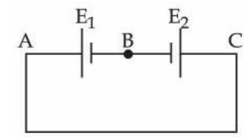
Ans. **35**

Sol. Only if we are finding the force on current carrying wire due to horizontal component of magnetic field. Then

$\frac{dF}{d\ell} = i(B)\sin 45°$

$= \sqrt{2}(3.5 \times 10^{-5}) \times \frac{1}{\sqrt{2}} = 35 \times 10^{-6}$ N/m

52. Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and 2 Ω internal resistance; the cell E_2 is of 2 V emf and 4 Ω internal resistance. The terminal potential difference of cell E_2 is_________ V.



Ans. 6

Sol. $i = \frac{8-2}{2+4} = 1$

$V_C - 2 - 4 \times 1 = V_B$

Terminal voltage = $V_C - V_B$

53. A electron of hydrogen atom on an excited state is having energy $E_n = -0.85$ eV. The maximum number of allowed transitions to lower energy level is_________.

Ans. 6

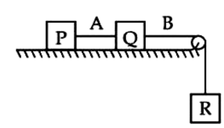
Sol. $0.85 = \frac{13.6}{n^2}$

$n^2 = 16$

$n = 4$

allowed transition = $\frac{4(4-1)}{2} = 6$

*54. Each of three blocks P, Q and R shown in figure has a mass of 3 kg. Each of the wires A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2×10^{11} Nm^{-2}. Neglecting friction, the longitudinal strain on wire B is _________ $\times 10^{-4}$. (Take g = 10 m/s^2)



Ans. 2

Sol. $a = \frac{g}{3}$, Strain = $\frac{T_B}{A} \times \frac{1}{Y}$

$T_B = 6 \times \left(\frac{g}{3}\right) = 20$ N

55. The distance between object and its two times magnified real image as produced by a convex lens is 45cm. The focal length of the lens used is _________cm.

Ans. 10

Sol. |u| + |v| = 45 cm

$\frac{|v|}{|u|} = 2$

v = 2u, 3u = 45, u = 15 cm

$\frac{1}{2u}+\frac{1}{u}=\frac{1}{F}$

$\frac{3}{30}=\frac{1}{F}$

F = 10 cm

*56. The displacement and the increase in the velocity of a moving particle in the time interval of t to (t+1)s are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t+2)^{th}$ s is _________m.

Ans. **175**

Sol. Let velocity at t sec = u
and acceleration a

$u \times 1+\frac{1}{2}a(1)^2=125$ (from t to t+1)

a = 50

$\therefore$ u = 100 m/s

$v_{t+1}=150$ m/s

$S_{t+2}=150(1)+\frac{1}{2}(50)1^2=175\,m$

57. A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance 2 C and potential 2 V. Then the loss of energy is $\frac{x}{3}E$, where x is_________

Ans. **2**

Sol. Initial energy = $\frac{1}{2}CV^2+4CV^2=9E$

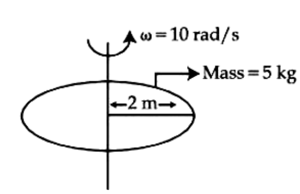
Final common potential = $\frac{5CV}{3C}=\frac{5V}{3}$

Final energy = $\frac{1}{2}C\left(\frac{5V}{3}\right)^2+\frac{1}{2}(2C)\left(\frac{5V}{3}\right)^2=\frac{75}{9}E$

Loss = $9E-\frac{75}{9}E=\frac{6E}{9}=\frac{x}{3}E$

$\therefore$ x = 2

*58. Consider a Disc of mass 5 kg, radius 2m rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _________J.



Ans. **250**

Sol. $I\omega_0 = 2I\omega$ (conservation of angular momentum)

$$\omega = \frac{\omega_0}{2}$$

$$\text{loss} = \frac{1}{2}I\omega_0^2 - \frac{1}{2}2I\frac{\omega_0^2}{4} = \frac{I\omega_0^2}{4}$$

*59. In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2, the amount of water poured in the organ tube is _________g. (Take speed of sound in air is 330 m/s)

Ans. 400

Sol. $\frac{V}{4L} = 30, L = \frac{11}{4}$ m

$\frac{V}{4(L-x)} = 110$, x = 2 m

$\therefore$ amount of water poured = $xA\rho_w$

60. A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is 30°. The emf induced across the blades is N $\pi \times 10^{-5}$ V, The value of N is_________.

Ans. 32

Sol. $$\varepsilon = \frac{B\omega R^2}{2}\sin 30^\circ$$

$$\varepsilon = \frac{1}{2}\left[\frac{0.5\times 10^{-4}}{2}\times\frac{1200\times 2\pi}{60}\times(0.8)^2\right]$$

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

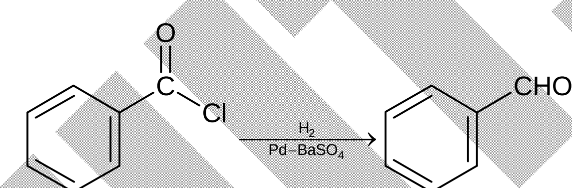
This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

61. Given below are two statments:
Statement-I: The gas liberated on warming a salt with dilute H_2SO_4 turns a piece of paper dipped in lead acetate into black, it is a conformatory test for sulphide ion.
Statement-II: in statement-I the colour of paper turns black because of formation of lead sulphite
In the light of the above statements, choose the ***most appropriate answer*** from the options given below:
Option:
(1) Both Statement I and Statement II are false.
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Ans. (3)

Sol. Lead acetate turns into lead sulphide (black ppt.)

62.



This reduction reaction is known as:
(1) Rosenmund reduction (2) Wolf – Kishner reduction
(3) Stephen reduction (4) Etard reduction

Ans. (1)

Sol. Rosenmund reduction converts acid halide to aldehyde.

63. Sugar which does not give redish brown precipitate with Fehling's reagent is
(1) Sucrose (2) Lactose
(3) Glucose (4) Maltose

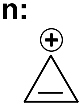
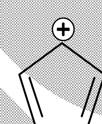
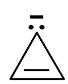
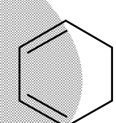
Ans. (1)

Sol. Sucrose is a non – reducing sugar.

64. Given below are two statments: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : There is a considerable increase in colvalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.
Reason (R): Covalent and ionic radii in a particular oxidaiton state increase down the group.
In the light of the above statements, choose the ***most appropriate answer*** from the options given below:
Option:
(1) (A) is false but (R) is true.
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. (2)

Sol. Smaller increase in lower elements of group is due to lanthanoid contraction.

*65. Which of the following molecule/species is most stable?
Option:
(1) 
(2) 
(3) 
(4) 

Ans. (1)

Sol.  is aromatic in nature.

66. Diamagnetic Lanthanoid ions are
Option:
(1) Nd^{3+} and Eu^{3+}
(2) La^{3+} and Ce^{4+}
(3) Nd^{3+} and Ce^{4+}
(4) Lu^{3+} and Eu^{3+}

Ans. (2)

Sol. $La^{3+} \Rightarrow [Xe]^{54}$
$Ce^{4+} \Rightarrow [Xe]^{54}$

*67. Aluminium chloride in acidified aqueous solution forms an ion having geometry
Option:
(1) Octahedral
(2) Square planar
(3) Tetrahedral
(4) Trigonal bipyramidal

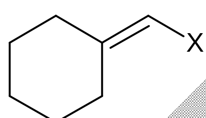
Ans. (1)

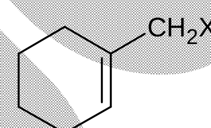
Sol. $AlCl_3 \xrightarrow{H_2O/H^+} \underset{\text{(octahedral)}}{\left[Al(H_2O)_6\right]^{3+}} + HCl + Al(OH)_3$

*68. Given below are two statments:
Statement-I: The orbitals having same energy are called as degenerate orbitals.
Statement-II: In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.
In the light of the above statements, choose the ***most appropriate*** answer from the options given below:
Option:
(1) Statement I is true but Statement II is false.
(2) Both Statement I and Statement II are true
(3) Statement I and Statement II are false
(4) Statement I is false but Statement II is true

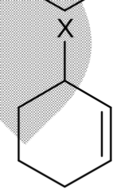
Ans. **(1)**

Sol. In hydrogen atom 3s, 3p and 3d all are degenerate orbitals.

69. Example of vinylic halide is
Option:
(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

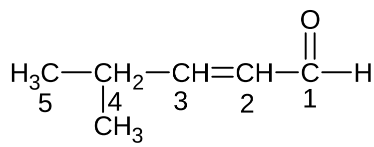
Sol. 

vinylic halide is halogen on double bond.

*70. Structure of 4 – Methylpent-2-enal is
Option:
(1) $H_2C{=}\underset{H}{C}{-}\overset{CH_3}{\underset{H}{C}}{-}CH_2{-}\overset{O}{\overset{\|}{C}}{-}H$
(2) $H_3C{-}CH_2{-}\underset{CH_3}{C}{=}CH{-}\overset{O}{\overset{\|}{C}}{-}H$
(3) $H_3C{-}CH_2{-}CH{=}\underset{CH_3}{CH}{-}\overset{O}{\overset{\|}{C}}{-}H$
(4) $H_3C{-}\underset{CH_3}{CH_2}{-}CH{=}CH{-}\overset{O}{\overset{\|}{C}}{-}H$

Ans. **(4)**

Sol.



*71. Math List-I with List-II:

List – I (Molecule)		List – II (Shape)	
(A)	BrF_5	(I)	T-shape
(B)	H_2O	(II)	See saw
(C)	ClF_3	(III)	Bent
(D)	SF_4	(IV)	Sqaure pyramidal

Choose the correct answer from the options given below:

Option:

(1) (A) → (I), (B) → (II), (C) → (IV), (D) → (III)
(2) (A) → (II), (B) → (I), (C) → (III), (D) → (IV)
(3) (A) → (III), (B) → (IV), (C) → (I), (D) → (II)
(4) (A) → (IV), (B) → (III), (C) → (I), (D) → (II)

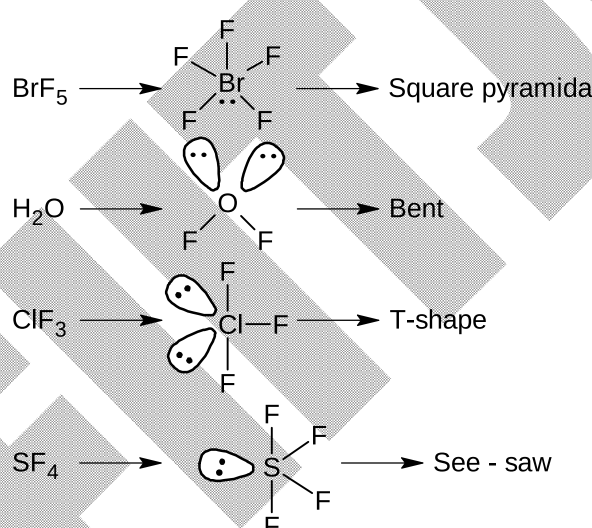
Ans. (4)

Sol. (A)

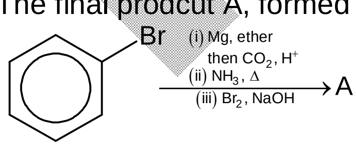
(B)

(C)

(D)

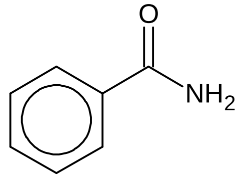


72. The final prodcut A, formed in the following multistep reaction sequence is

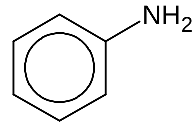


Option:

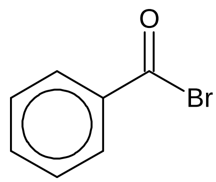
(1)



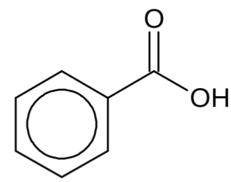
(2)



(3)

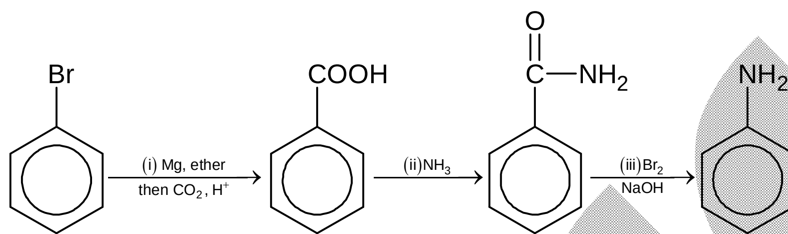


(4)

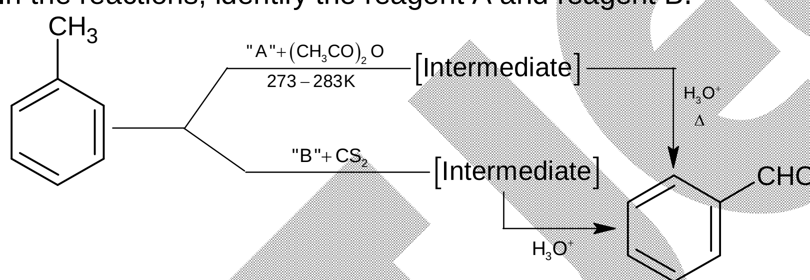


Ans. (2)

Sol.



73. In the reactions, identify the reagent A and reagent B.

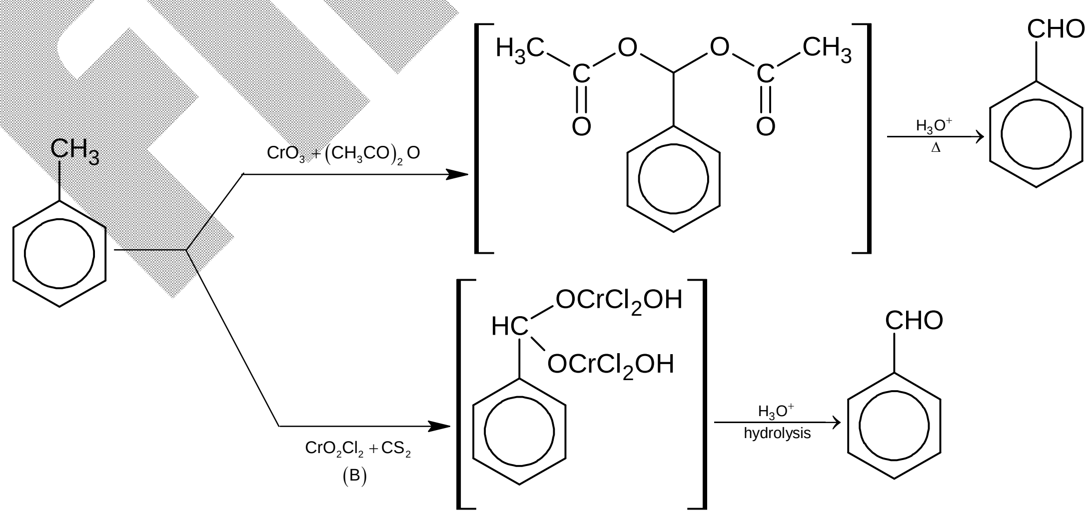


Option:

(1) A – CrO_3 B – CrO_3
(2) A – CrO_3 B – CrO_2Cl_2
(3) A – CrO_2Cl_2 B – CrO_2Cl_2
(4) A – CrO_2Cl_2 B – CrO_3

Ans. (2)

Sol.



74. Given below are two statments: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide.
Reason (R): Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.
In the light of the above statements, choose the ***most appropriate answer*** from the options given below:
Option:
(1) (A) is true but (R) is true.
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(3) (A) is false but (R) is true
(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. (1)

Sol. Allylic halides have halogen atom connected to sp^3 – carbon, which is next to double bond.

75. What happens to freezing point of benzene when small quantity of napthalene is added to benzene?
Option:
(1) Increases
(2) Remains unchanged
(3) First decreases and then increases
(4) Decreases

Ans. (4)

Sol. Addition of non – volatile solute causes depression in freezing point.

76. Math List-I with List-II:

List – I (Species)		**List – II (Electronic distribution)**	
(A)	Cr^{2+}	(I)	$3d^8$
(B)	Mn^+	(II)	$3d^34s^1$
(C)	Ni^{2+}	(III)	$3d^4$
(D)	V^+	(IV)	$3d^54s^1$

Choosef the correct answer from the options given below:
Option:
(1) (A) $\rightarrow$ (I), (B) $\rightarrow$ (II), (C) $\rightarrow$ (III), (D) $\rightarrow$ (IV)
(2) (A) $\rightarrow$ (III), (B) $\rightarrow$ (IV), (C) $\rightarrow$ (I), (D) $\rightarrow$ (II)
(3) (A) $\rightarrow$ (IV), (B) $\rightarrow$ (III), (C) $\rightarrow$ (I), (D) $\rightarrow$ (II)
(4) (A) $\rightarrow$ (II), (B) $\rightarrow$ (I), (C) $\rightarrow$ (IV), (D) $\rightarrow$ (III)

Ans. (4)

Sol. (A) $Cr^{2+} = 3d^4$
(B) $Mn^+ = 3d^54s^1$
(C) $Ni^{2+} = 3d^8$
(D) $V^+ = 3d^34s^1$

*77. Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$$H_3C-C\equiv CH + Na \longrightarrow A \xrightarrow{B} \underset{(C)}{H_3C-C\equiv C-CH_2-\underset{\substack{|\\CH_3}}{CH_2}} + NaBr$$

Option:

(1) $A = CH_3 - C\equiv \overset{-}{C}\overset{+}{Na}, B = CH_3 - CH_2 - CH_2 - Br$

(2) $A = CH_3 - CH = CH_2, B = CH_3 - CH_2 - CH_2 - Br$

(3) $A = CH_3 - CH_2 - CH_3, B = CH_3 - C\equiv CH$

(4) $A = CH_3 - C\equiv \overset{-}{C}\overset{+}{Na}, B = CH_3 - CH_2 - CH_3$

Ans. (1)

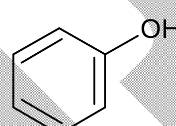
Sol. $H_3C-C\equiv CH + Na \longrightarrow \underset{(A)}{H_3C-C\equiv C^{\ominus}\ Na^{\oplus}} \xrightarrow[(B)]{CH_3-CH_2-CH_2-Br} H_3C-C\equiv C-CH_2-CH_2-CH_3 + NaBr$

78. Following is a confiromatroy test for aromatic primary amines. Identify reagent (A) and (B):

Option:

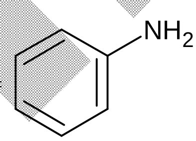
(1)

$A = HNO_3 / H_2SO_4$ B =



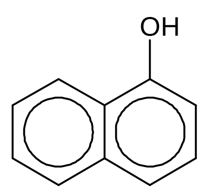
(2)

$A = NaNO_2 + HCl, 0-5^0C;$ B =



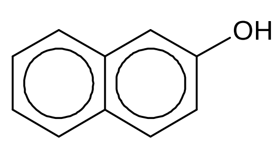
(3)

$A = NaNO_2 + HCl, 0-5^0C;$ B =



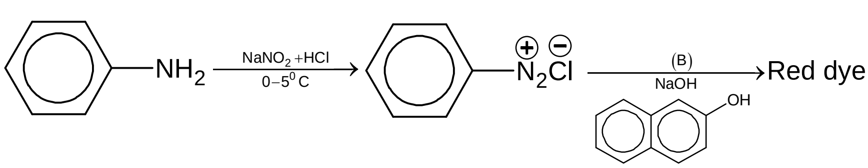
(4)

$A = NaNO_2 + HCl, 0-5^0C;$ B =



Ans. (4)

Sol.



*79. The Lassiagne's extract is boiled with dilute HNO_3 before testing for halogen because

Option:

(1) AgCN is soluble in HNO_3
(2) Silver halides are soluble in HNO_3
(3) Ag_2S is soluble in HNO_3
(4) Na_2S and NaCN are decomposed by HNO_3

Ans. **(4)**

Sol. Na_2S and NaCN are decomposed by HNO_3, also they will react with $AgNO_3$ and interfere in test.

80. Choose the correct statements from the following:

(A) Ethane-1,2-diamine is a chelating ligand.
(B) Metallic aluminium is prodcued by electrolysis of aluminium oxide in presence of cryolite
(C) Cyanide ion is used as ligand for leaching of silver
(D) Phosphine acts as a ligand in Wilkinson catalyst
(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes

Choose the correct answer from the options given below:

Option:

(1) (B), (C), (E) only
(2) (C), (D), (E) only
(3) (A), (B), (C) only
(4) (A), (D), (E) only

Ans. **(3)**

Sol. Wilkinson catalsyt has triphenyl phosphine and the stability constant of Ca^{2+} is more than Mg^{2+}.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

---

81. The rate of First reaction is 0.04 mol $L^{-1}s^{-1}$ at 10 minutes and 0.03 mol $L^{-1}s^{-1}$ at 20 minutes after intiation. Half life of the reaction is ___________ minutes (Given log 2 = 0.3010, log 3 = 0.4771)

Ans. **24**

Sol. Rate = k[concentration]

$$\therefore\ k\left[t_2 - t_1\right] = 2.303\ \log\left(\frac{C_1}{C_2}\right)$$

$$k = \frac{2.303}{\left[20-10\right]}\log\left(\frac{4}{3}\right)$$

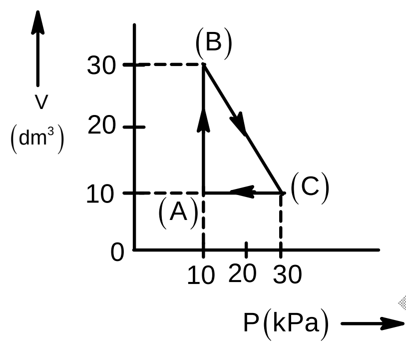
$$t_{1/2} = \frac{0.693}{k} = 24.14\ \text{min}$$

*82. The pH at which $Mg(OH)_2$ [$K_{sp} = 1 \times 10^{-11}$] begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is ______________.

Ans. 9

Sol. $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2HO^-$
$K_{sp} = 10^{-11} = [Mg^{2+}] [HO^-]^2$
$[OH^-] = 10^{-5} \Rightarrow pOH = 5$
pH = 9

*83.



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ A as shown in the diagram above. The total work done in the process is ______________J

Ans. 200

Sol. Total work done = $W_{AB} + W_{BC} + W_{CA}$
$= 10[30 - 10] + [30 \times 10 - 10 \times 20] +$ zero
= 200 J

*84. If IUPAC name of an element is "Unununnium" then the element belongs to nth group of periodic table. The value of n is _____________.

Ans. 11

Sol. Unununnium has atomic number of 111.

*85. The total number of molecular orbitals from 2s and 2p atomic orbitals of a diatomic molecule is ________.

Ans. 8

Sol. $2s - 2s \rightarrow 2$
$2p_x - 2p_x \rightarrow 2$
$2p_y - 2p_y \rightarrow 2$
$2p_z - 2p_z \rightarrow 2$

*86. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retradation factor of the organic compound is ___________ $\times 10^{-1}$.

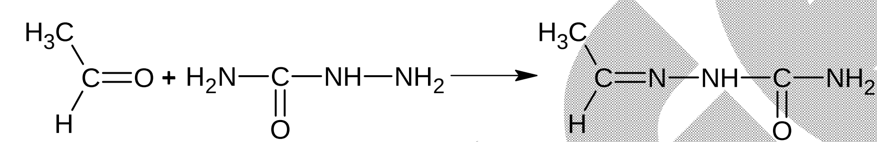
Ans. 7

Sol. $R_f = \frac{3.5}{5.0} = 0.7 = 7 \times 10^{-1}$

87. The compound formed by the reaction of ethanal with semicarbazide contains __________ number of nitrogen atoms.

Ans. 3

Sol.



*88. 0.05 cm thick coating of silver is deposited on a plate of 0.05 m^2 area. The number of silver atoms deposited on plate are ___________ $\times 10^{23}$. (Atomic mass Ag = 108, d = 7.9 g cm^{-3})

Ans. 11

Sol. Volume = 0.05 cm $\times$ 0.05 $\times$ 10^4 cm^2 = 25 cm^3

Mass = d $\times$ v = $\frac{7.9\,g}{cm^3} \times 25\ cm^3 = 197.5\ g$

$n = \frac{197.5}{108}$

$N = \frac{197.5}{108} \times 6.023 \times 10^{23}$

= 11.01 $\times 10^{23}$

*89. $2MnO_4^- + bI^- + cH_2O \longrightarrow xI_2 + yMnO_2 + z\bar{O}H$

If the above equation is balanced with integer coefficient, the value of z is __________.

Ans. 8

Sol. $2MnO_4^- + 6I^- + 4H_2O \longrightarrow 3I_2 + 2MnO_2 + 8HO^-$

90. The mass of sodium acetate (CH_3COONa) required to prepare 250 ml of 0.35 M aqueous solution is _____________ g. (Molar mass of CH_3COONa is 82.02 g mol^{-1})

Ans. 7

Sol. $0.35\ M = \frac{W \times 1000}{82.02 \times 250}$

W = 7.17 g