FIITJEE

Solutions to JEE(Main) -2024

Test Date: 6th April 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**.** **Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q1. Let y = y(x) be the solution of the differential equation

$(2x\log_e x)\dfrac{dy}{dx} + 2y = \dfrac{3}{x}\log_e x,\ x > 0$ and $y(e^{-1}) = 0$. Then, y(e) is equal to

(A) $-\dfrac{3}{2e}$

(B) $-\dfrac{2}{3e}$

(C) $-\dfrac{3}{e}$

(D) $-\dfrac{2}{e}$

Ans. C

Sol. $2x\ln x\dfrac{dy}{dx} + 2y = \dfrac{3}{x}\ln x$

$\Rightarrow \quad \dfrac{dy}{dx} + \dfrac{y}{x\ln x} = \dfrac{3}{2x^2} \quad .....(1)$

I.F. $= e^{\int \frac{dx}{x\ln x}} = e^{\ln(\ln x)} = \ln x$

$y\ln x = \dfrac{3}{2}\int \dfrac{\ln x}{x^2}dx$

Put, $\ln x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$\Rightarrow y\ln x = \dfrac{3}{2}\int \dfrac{te^t dt}{e^{2t}} = \dfrac{3}{2}\int te^{-t}dt$

$y\ln x = \dfrac{3}{2}\left[-te^{-t} + \int e^{-t}dt\right]$

$= \dfrac{3}{2}\left[-te^{-t} - e^{-t}\right] + c$

$\therefore y\ln x = \dfrac{3}{2}\left[-\dfrac{\ln x}{x} - \dfrac{1}{x}\right] + c$

$y\left(\dfrac{1}{e}\right) = 0$

$\Rightarrow \quad \dfrac{3}{2}\left[-\dfrac{\ln\left(\frac{1}{e}\right)}{\left(\frac{1}{e}\right)} - e\right] + c = 0 \Rightarrow c = 0$

$y\ln x = -\dfrac{3}{2}\left(\dfrac{\ln x + 1}{x}\right)$

For x = e

$y = -\dfrac{3}{e}$

Q2. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}\,|x|$. Then, which one of the following points lies on the circle C?

(A) (2, 4) (B) (1, 2)
(C) (1, 1) (D) (2, 2)

Ans. A

Sol. Eqn. of parabola

$y = 6 - x^2$ .....(1)

Eqn. of circle

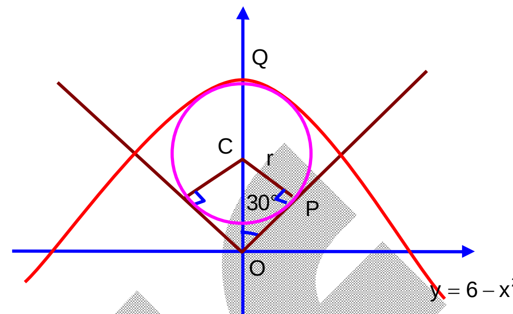
$X^2 + (y - 2r)^2 = r^2$ .....(2)

$\frac{r}{OC} = \sin 30^\circ = 1/2 \Rightarrow OC = 2r = \beta$

Again OQ = 2r + r = 6 ⇒ r = 2

∴ Eqn. of circle is $x^2 + (y - 4)^2 = 4$

(2, 4) lies on the circle



Q3. A company has to plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is

(A) 64 (B) 56
(C) 66 (D) 54

Ans. D

Sol. E : Event that a motorcycle is standard quality

Given $P\left(\frac{E}{A}\right) = \frac{80}{100} = \frac{8}{10}$ $\quad P\left(\frac{E}{B}\right) = \frac{90}{100} = \frac{9}{10}$

$P(A) = \frac{60}{100} = \frac{6}{10}, P(B) = \frac{40}{100} = \frac{4}{10}$

$P\left(\frac{B}{E}\right) = \frac{P(B) \times P(E/B)}{P(A) \times P(E/A) + P(B) \times P(E/B)}$

$$\Rightarrow \quad p = \frac{\frac{4}{10} \times \frac{9}{10}}{\frac{6}{10} \times \frac{8}{10} + \frac{4}{10} \times \frac{9}{10}} = \frac{36}{48 + 36} = \frac{3}{7}$$

$$\therefore \quad 126p = 126 \times \frac{3}{7} = 54$$

Q4. Let $A = \{n \in [100, 700] \cap N : n$ is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is

(A) 310 (B) 290
(C) 280 (D) 300

Ans. D

Sol. $n(3) = \frac{600}{3} = 200$

$n(4) = \frac{680}{4} = 150$

$n(12) = \frac{600}{12} = 50$

$n(3 \cup 4) = n(3) + n(4) - n(12)$

$= 200 + 150 - 50 = 350$

$\therefore$ N (A) = 300

Q5. Let a variable line of slope m > 0 passing through the point (4, –9) intersect the coordinate axes at the points A and B. The minimum value of the sum of the distances of A and B from the origin is

(A) 30 (B) 10
(C) 25 (D) 15

Ans. C

Sol. Equation of line

y + 9 = m (x – 4)

Cuts x axis at $A\left(4+\frac{9}{m}, 0\right)$

Cuts y axis at B (0, –4m – 9)

$\therefore\ |OA| + |OB| = 4 + \frac{9}{m} + 4m + 9 \quad$ as $m > 0$

$$= 13 + \underbrace{4m + \frac{9}{m}}_{(AM)} \geq 13 + 2\underbrace{\sqrt{4m \times \frac{9}{m}}}_{(GM)} = 25$$

Q6. If $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$, then

(A) $f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$ (B) f''(0) = 0

(C) f''(0) = 1 (D) $f''\left(\frac{2}{\pi}\right) = \frac{12-\pi^2}{2\pi}$

Ans. A

Sol. $f(x) = \begin{cases} x^3 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$f'(x) = 3x^2 \sin\frac{1}{x} - x\cos\frac{1}{x}$

$f''(x) = 6x \sin\frac{1}{x} - 4\cos\frac{1}{x} - \frac{1}{x}\sin\frac{1}{x}$

$f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24-\pi^2}{2\pi}$

Q7. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then m + n^2 is equal to

(A) 312 (B) 408
(C) 396 (D) 414

Ans. C

Sol. area of $\Delta = \frac{\sqrt{3}}{4}(12)^2$

$= 36\sqrt{3}$

$2s = 36 \Rightarrow s = 18$

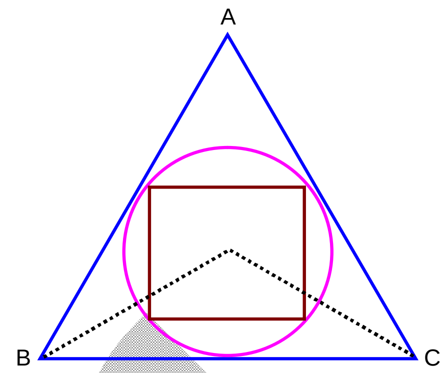
$r = \frac{\Delta}{s} = \frac{36\sqrt{3}}{18}$

$r = 2\sqrt{3}$

Side length of square $a \Rightarrow \sqrt{2}a = 2r = 4\sqrt{3}$

$\Rightarrow a = 2\sqrt{6}$ perimeter $4a = 8\sqrt{6}$

Area = 24 $m^2 + n^2 = 408$



Q8. Let y = y(x) be the solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$, y(1) = 0. Then y (0) is

(A) $\frac{1}{4}\left(e^{\pi/2} - 1\right)$ (B) $\frac{1}{2}\left(1 - e^{\pi/2}\right)$

(C) $\frac{1}{2}\left(e^{\pi/2} - 1\right)$ (D) $\frac{1}{4}\left(1 - e^{\pi/2}\right)$

Ans. B

Sol. $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. $= e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1}x}$

Sol. $ye^{\tan^{-1}x} = \int \frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}dx$

$\Rightarrow ye^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + c$

$y(1) = 0 \Rightarrow 0 = \frac{e^{\pi/2}}{2} + c \Rightarrow c = -\frac{e^{\pi/2}}{2}$

$ye^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} - \frac{1}{2}e^{\pi/2}$

$y(0) \Rightarrow ye^0 = \frac{e^0}{2} - \frac{1}{2}e^{\pi/2}$

Q9. For $\alpha, \beta \in R$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is

(A) $2\alpha + 4\beta$ (B) 2n

(C) 0 (D) $4\alpha + 2\beta$

Ans. D

Sol. $2A_{10} - A_8 = \begin{vmatrix} 20-8 & 1 & \frac{n^2}{2}+\alpha \\ 40-16 & 2 & n^2-\beta \\ 56-22 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$= \begin{vmatrix} 12 & 1 & \frac{n^2}{2}+\alpha \\ 24 & 2 & n^2-\beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} = \begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$-2\left[n^2 - \beta - n^2 - 2\alpha\right] = 4\alpha + 2\beta$

Q10. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(A) $\sqrt{3.86}$ (B) 1.94
(C) $\sqrt{3.96}$ (D) 1.8

Ans. **C**

Sol. $\bar{x} = \frac{\sum x_i}{20} = 10$

$\therefore \sum x_i = 200$

Correct $\sum x_i = 204$

Correct mean $\bar{x} = \frac{204}{20} = 10.2$ .....(1)

$\frac{1}{20}\sum x_i^2 - (10)^2 = 4$

$\Rightarrow \sum x_i^2 = 104 \times 20$

Correct $\sum x_i^2 = 104 \times 20 + (144 - 64)$

= 104 × 10 + 80
= 16 (26 × 5 + 5)
= 16 × 135

$\therefore$ New variance

$\sigma_{new}^2 = \frac{1}{20}(16 \times 135) - \left(\frac{204}{20}\right)^2$

$= \frac{1}{20}\left[16 \times 135 - \frac{204 \times 20}{20}\right]$

$= \frac{1}{5}\left[4 \times 135 - \frac{51 \times 51}{5}\right] = \frac{1}{25}[135 \times 20 - 2601] = \frac{99}{25}$

$\therefore \sigma_{new} = \sqrt{3.96}$

Q11. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x + \sin^3 x\right)^2} dx$ is equal to

(A) 1/3 (B) 1/9
(C) 1/6 (D) 1/12

Ans. **C**

Sol. $I=\int_0^{\frac{\pi}{4}}\frac{\tan^2 x\cdot\sec^2 x\,dx}{\left(1+\tan^3 x\right)^2}$

Put $1+\tan^3 x=t\Rightarrow 3\tan^2 x\sec^2 x dx=dt$

x	0	π/4
t	1	2

$I=\frac{1}{3}\int_1^2\frac{dt}{t^2}=-\frac{1}{3}\times\frac{1}{t}\Big]_1^2$

$=-\frac{1}{3}\left[\frac{1}{2}-1\right]=\frac{1}{6}$

Q12. Let the area of the region enclosed by the curves y = 3x, 2y = 27 – 3x and y = 3x – $x\sqrt{x}$ be A. Then 10A is equal to

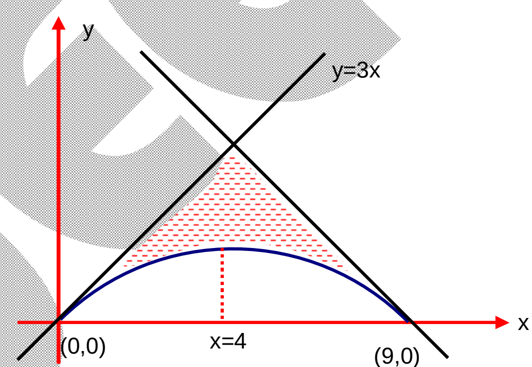
(A) 184 (B) 172
(C) 154 (D) 162

Ans. **D**

Sol. Required area

$A=\int_0^3[3x-(3x-x\sqrt{x})]dx+\int_3^9\left[\frac{27-3x}{2}-(-x\sqrt{x})\right]dx$

$\Rightarrow\quad A=\frac{81}{5}\qquad\therefore 10A=162$



Q13. Let the relations R_1 and R_2 on the set X = {1, 2, 3,…..20} be given by R_1 = {(x, y) : 2x – 3y =2} and R_2 = {(x, y): –5x + 4y = 0}. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M+N equals

(A) 12 (B) 8
(C) 16 (D) 10

Ans. **D**

Sol. X = { 1, 2, 3, ….., 20}

R_1 = {(x, y) : 2x – 3y = 2}
= {(4, 2). (7, 4), (10, 6) (13, 8), (16, 10), (19, 20)}

R_2 = {(x, y) : –5x + 4y = 0}
= {(4, 5), (8, 10), (12, 15), (16, 20)}

to make R_1 symmetric it need 6 elements
to make R_2 symmetric it need 4 elements
$\therefore$ m + n = 10

Q14. The interval in which the function f(x) = x^x, x > 0, is strictly increasing is

(A) $\left(0,\frac{1}{e}\right]$ (B) $\left[\frac{1}{e^2},1\right)$
(C) $\left[\frac{1}{e},\infty\right)$ (D) $(0,\infty)$

Ans. **C**

Sol. $f(x)=x^x=e^{x\ln x}$

$f'(x) = e^{x\ln x}(\ln x + 1) = x^x(1 + \ln x)$

$f'(x) > 0 \Rightarrow 1 + \ln x > 0 \Rightarrow \ln x > 1$

$\Rightarrow x > 1/e$

Q15. The function $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 4x + 9}, x \in R$ is

(A) neither one-one nor onto
(B) one-one but not onto
(C) both one-one and onto
(D) onto but not one-one

Ans. **A**

Sol. $f(x) = \dfrac{(x+5)(x-3)}{x^2 - 4x + 9}$

f(–5) = 0, f(3) = 0 $\therefore$ f(x) is many one

$y = \dfrac{x^2 + 2x - 15}{x^2 - 4x + 9}$

$\Rightarrow y\left(x^2 - 4x + 9\right) = x^2 + 2x - 15$

$\Rightarrow (y-1)x^2 - 2x(1+2y) + 9y + 15 = 0$

$x \in R \Rightarrow D \geq 0$

$\Rightarrow \quad (1+2y)^2 - (y-1)(9y+15) \leq 0$

$\Rightarrow \quad -5y^2 - 2y + 16 \leq 0$

$\Rightarrow \quad 5y^2 + 2y - 16 \geq 0$

$\Rightarrow \quad (5y-8)(y+2) \geq 0$

$y \in [2, 8/5]$

$\therefore$ f(x) is neither one-one nor onto

Q16. The shortest distance between the line $\dfrac{x-3}{2} = \dfrac{y+15}{-7} = \dfrac{z-9}{5}$ and $\dfrac{x+1}{2} = \dfrac{y-1}{1} = \dfrac{z-9}{-3}$ is

(A) $6\sqrt{3}$
(B) $8\sqrt{3}$
(C) $5\sqrt{3}$
(D) $4\sqrt{3}$

Ans. **D**

Sol. $\dfrac{x-3}{2} = \dfrac{y+15}{-7} = \dfrac{z-9}{5} \Rightarrow \vec{r} = (3i - 15j + 9k) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\dfrac{x+1}{2} = \dfrac{y-1}{1} = \dfrac{z-9}{-3} \Rightarrow \vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

$\vec{b} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\vec{d} = 2\hat{i} + \hat{j} - 3\hat{k}$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= 16(\hat{i} + \hat{j} + \hat{k})$

$\overrightarrow{AC} = -4\hat{i} + 16\hat{j}$

$$\Rightarrow \quad \left|\overrightarrow{AC} \cdot \frac{\vec{b} \times \vec{d}}{|\vec{b} \times \vec{d}|}\right| = \left|\frac{-64 + 256}{16\sqrt{3}}\right| = \left|\frac{-4 + 16}{\sqrt{3}}\right| = 4\sqrt{3}$$

Q17. Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $t \in R$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\dfrac{a_{2023} + a_{2025}}{a_{2024}}$ is

(A) 1/4 (B) 1/2
(C) –1/4 (D) –1/2

Ans. D

Sol. $\alpha + \beta = t^2 - 5t + 6 \quad \alpha\beta = 1$

$$\frac{a_{2023} + a_{2025}}{a_{2024}} = \frac{\alpha^{2023} + \beta^{2023} + \alpha^{2025} + \beta^{2025}}{\alpha^{2024} + \beta^{2024}}$$

$$= \frac{\alpha^{2023}\left(\alpha^2 + 1\right) + \beta^{2023}\left(\beta^2 + 1\right)}{\alpha^{2024} + \beta^{2024}}$$

$$= \frac{\alpha^{2023}\left(t^2 - 5t + 6\right)\alpha + \beta^{2023}\left(t^2 - 5t + 6\right)\beta}{\alpha^{2024} + \beta^{2024}}$$

$$= t^2 - 5t + 6 \geq -\frac{25 - 24}{2} = -1/2$$

Q18. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(A) 48 (B) 56
(C) 24 (D) 16

Ans. D

Sol. no. of vertices = 8
one vertex can be selected in ${}^8C_1$ ways
rest two vertices can be selected in ${}^4C_2$ ways

Required no. of triangles = $\dfrac{{}^8C_1 \times {}^4C_1}{\dfrac{3!}{2!}} = \dfrac{8 \times 6}{3} = 16$

Q19. Let $f : (-\infty, \infty) - \{0\} \to R$ be a differentiable function such that $f'(1) = \lim\limits_{a\to\infty} a^2 f\left(\dfrac{1}{a}\right)$. Then $\lim\limits_{a\to\infty} \dfrac{a(a+1)}{2}\tan^{-1}\left(\dfrac{1}{a}\right) + a^2 - 2\log_e a$ is equal to

(A) $\dfrac{3}{8} + \dfrac{\pi}{4}$ (B) $\dfrac{3}{2} + \dfrac{\pi}{4}$

(C) $\dfrac{5}{2} + \dfrac{\pi}{8}$ (D) $\dfrac{3}{4} + \dfrac{\pi}{8}$

Ans. C

Sol. $\lim\limits_{a\to\infty} \dfrac{a(a+1)}{2}\tan^{-1}\dfrac{1}{a} + a^2 - 2\log_e^a$

$$= \lim_{a\to\infty} a^2\left[\frac{1}{2}\left(1 + \frac{1}{a}\right)\tan^{-1}\frac{1}{a} + 1 + \frac{2}{a^2}\log\frac{1}{a}\right]$$

$$\therefore\ f(x) = \frac{1}{2}(1+x)\tan^{-1}x + 1 + 2x^2 \ln x$$

$$f'(x) = \frac{1}{2}\left[\tan^{-1}x + \frac{1+x}{1+x^2}\right] + 4x\ln x + 2x$$

$$f'(1) = \frac{1}{2}\left[\frac{\pi}{4} + 1\right] + 2 = \frac{\pi}{8} + \frac{5}{2}$$

Q20. If A(3, 1, –1). $B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, C(2, 2, 1) and $D\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is

(A) $\frac{4\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$

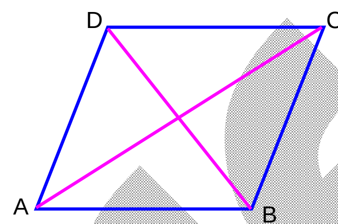
(C) $2\sqrt{2}$ (D) $\frac{5\sqrt{2}}{3}$

Ans. **A**

Sol. $\overrightarrow{AC} = -\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{BD} = \frac{5}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{2}{3}\hat{k}$

$\text{Area} = \frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}| = \frac{4\sqrt{2}}{3}$



SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q21. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r$, r = 2, 3, …..,n. If the sum of the first n terms of this series is $\frac{1}{5}(n^2-12n+39)(4.6^n-5.3^n+1)$, then n is equal to_____.

Ans. **6**

Sol. $T_1 = 6$

$T_2 = 3.6 + 6^2$

$T_3 = 3[3.6 + 6^2] + 6^3 = 3^2.6 + 3.6^2 + 6^3$

$T_4 = 3[3^2.6 + 3.6^2 + 6^3] + 6^4 = 3^3.6 + 3^2 6^2 + 3.6^2 + 6^4$……..

$T_n = 3^{n-1}\, 6 + 3^{n-2}\, 6^2 + …..+3.6^{n-2} + 6^{n-1} + 6^n$

$= 3^{n-1}.6\,[1 + 2 + 2^2 + …..2^{n-2} + 2^{n-1}]$

$= 3^{n-1}.6\,(2^n - 1) = 2(6^n - 3^n)$

$$T_n = 2\left[6\left(\frac{6^n-1}{5}\right)-3\left(\frac{3^n-1}{2}\right)\right]$$

$$=\frac{3}{5}\times 4\left(6^n-1\right)-3\left(3^n-1\right)$$

$$=\frac{3}{5}\left[4.6^n-4-5\left(3^n-1\right)\right]$$

$$=\frac{3}{5}\left[4.6^n-5.3^n+1\right]$$

$\therefore n^2-12n+39=3$

$\Rightarrow n^2-12n+36=0$

$\Rightarrow n=6$

Q22. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$. respectively, then $6(n^3 + x^2 + y)$ is equal to_____.

Ans. **806**

Sol. $^nC_1\, x^{n-1}\, y = 135$ …(1)
$^nC_2\, x^{n-2}\, y^2 = 30$ … (2)
$^nC_3\, x^{n-3}\, y^3 = \frac{10}{3}$ …(3)

(1) ÷ (2) $\dfrac{nx}{\frac{n(n-1)}{2}y} = \dfrac{135}{30}$

$\Rightarrow \dfrac{2x}{(n-1)y} = \dfrac{27}{6}$ …(4)

(2) ÷ (3) $\dfrac{\frac{n(n-1)}{2}x}{\frac{n(n-1)(n-2)}{6}y} = \dfrac{30}{10/3}$

$\Rightarrow \dfrac{3x}{(n-2)y} = 9$ …(5)

Solving we get n = 5 x = 3 y = 1/3
$\therefore$ 6 ($n^3 + x^2 + y$) = 6 × 125 + 6 × 9 + 2
= 806

Q23. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right) = \frac{125}{16}m$. Then the value of m is_____.

Ans. **221**

Sol. $y = 4 + x^2$
$\Rightarrow x = \sqrt{y-4}$ is a solution of
$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$
$\Rightarrow 4(y-4)^2 + 8(y-4)^{3/2} - 17(y-4) - 12(y-4)^{1/2} + 9 = 0$
$\Rightarrow 4\left(y^2 - 8y + 16\right) - 17(y-4) + 92(y-4)^{1/2}[12 - 8(y-4)]$
$\Rightarrow 4y^2 - 49y + 141 = (y-4)^{1/2}[44 - 8y]$
Requiring
$\Rightarrow \left[4y^2 - 49y + 141\right]^2 = (y-4)(8y-44)^2$
$\therefore \left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4\right)^2$
$= \dfrac{\text{constant term}}{\text{coefficient of } y^4} = \dfrac{(141)^2 + (44)^2 \times 4}{16}$
$= \dfrac{125m}{16}$
$\Rightarrow m = 221$
$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$
Put x = 2i and x = –2i then multiply.

Q24. Let L_1, L_2 be the lines passing through the point P(0, 1) and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2, then $16\left(m_1^2 + m_2^2\right)$ is equal to_____.

Ans. **68**

Sol. $9x^2 + 12x + 4 = -18(y-1)$

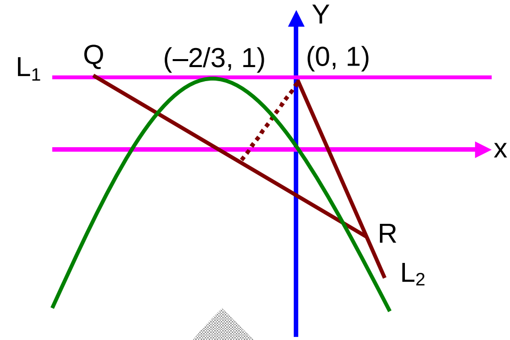
$\Rightarrow \left(x+\frac{2}{3}\right)^2 = -2(y-1)$

Eqn. of tangents to $(x+2/3)^2 = -2(y-1)$

$\left(x+\frac{2}{3}\right) = m(y-1) - \frac{1}{2m}$

Passes $(0,1) \Rightarrow \frac{2}{3} = -\frac{1}{2m} \Rightarrow m = -\frac{3}{4}$

Scope of $L_2 = \frac{1}{m} = -\frac{4}{3}$



Eqn. L_1 : y = 1 ...(1)

Eqn. L_2 : $y - 1 = -\frac{1}{3}x \Rightarrow 4x + 3y = 3$ ...(2)

Eqn. of angle bisectors between (1) & (2)

$\frac{y-1}{1} = \pm\frac{(4x+3y-3)}{\sqrt{16+9}}$

$5y - 1 = \pm(4x+3y-3)$

$m_1 = 2, m_2 = 1/2 \quad \therefore 76\left(m_1^2 + m_2^2\right) = 16\left(\frac{2}{4}+4\right) = 68$

Q25. Let $r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$, $k \in N$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$ is equal to _____.

Ans. 65

Sol. $I_k = \int_0^1 (1-x^7)^k dx$

$= x(1-x^7)^k \Big|_0^1 - \int_0^1 k(1-x^7)^{k-1}(-7x^6)x\,dx$

$= 7k\int_0^1 (1-x^7)^{k-1} x^7 dx$

$= 7k\int_0^1 \{1-(1-x^7)\}(1-x^7)^{k-1} dx$

$= 7kI_{k-1} - 7kI_k$

$\Rightarrow (1+7k)I_k = 7kI_{k-1}$

$\Rightarrow I_k = \frac{7k}{1+7k} I_{k-1}$

$\Rightarrow I_{k+1} = \frac{7(k+1)}{1+7(k+1)} I_k$

$\therefore \frac{I_k}{I_{k+1}} = \frac{7k+8}{7(k+1)} = r_k$

$\Rightarrow r_{k-1} = \frac{1}{7(k+1)} \Rightarrow \frac{1}{7(r_k - 1)} = k+1$

$\Rightarrow \sum_{k=1}^{10} \frac{1}{7(\gamma_k - 1)} = \sum_{k=1}^{10} (k+1) = \frac{11\times 12}{2} - 1 = 65$

Q26. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a}x\left(\vec{b} + \vec{c}\right) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} . \vec{c} = 13$, then $(24 - \vec{b}.\vec{c})$ is equal to_____.

Ans. **46**

Sol. $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 33\hat{k}$

$\Rightarrow \vec{a} \times \vec{b} + (\vec{a} + \vec{b}) \times \vec{c} = (i + 83 + 13k)$

Taking cross product with $\vec{a}$ both side

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times (\vec{a} \times \vec{c}) + \vec{a} \times (\vec{b} \times \vec{c})$

$= (2\vec{i} - 3j + 4k) \times (i + 8\hat{j} + 13\hat{k})$

$\Rightarrow (\bar{a}.\bar{b})\ \bar{a} - |\,\bar{a}\,|^2\ \bar{b} + (\bar{a}.\bar{c})\bar{a} = |\,\bar{a}\,|^2\ \bar{c}\ \ + (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{c})\bar{c} = -71\,\hat{i} - 2^2\hat{j} + 19\hat{k}$

$-13\bar{a} + 16\bar{b} - 3\bar{c} = -71\,\hat{i} - 22\hat{j} + 19\hat{k}$

$13\bar{a} - 16\bar{b} + 3\bar{c} = 71\,\hat{i} + 2\hat{j} - 19\hat{k}$

Taking dot product with $\vec{b}$ both side

$\Rightarrow\ 13\bar{a}.\bar{b} + 16\bar{b}^2 + 3\bar{c}.\bar{b} = (71\,\hat{i} + 22\hat{j} - 18\hat{k}).(3\hat{i} + 4\hat{j} - 5\hat{k})$

$\Rightarrow$ (13) (–26) + 16 (50) + 3 $(\bar{b}.\bar{c})$ = 213 + 88 + 95

$\Rightarrow$ 462 + 3 $(\bar{b}.\bar{c})$ = 396

$\Rightarrow\ \bar{b}.\bar{c}\ = -22$

$\therefore 24 - (\bar{b}.c) = 46$

Q27. For $n \in N$, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$, then n is equal to_____.

Ans. **47**

Sol. $\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{4}}{1 - \frac{1}{12}}\right) + \tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{n}}{1 - \frac{\pi}{5n}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{7}{11}\right) + \tan^{-1}\left(\frac{n+5}{5n-1}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{n+5}{5n-1}\right) = \frac{\pi}{4} - \tan^{-1}\frac{7}{11}$$

$$\Rightarrow \frac{n+5}{5n-1} = \tan\left(\frac{\pi}{4} - \tan^{-1}\frac{7}{11}\right) = \frac{1 - \frac{7}{11}}{1 + \frac{7}{11}}$$

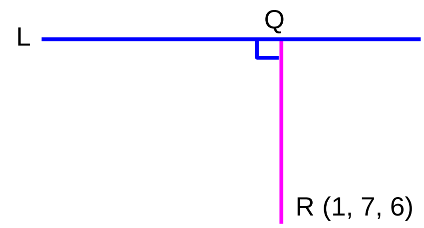
$$\Rightarrow \frac{n+5}{5n-1} = \frac{4}{18} = \frac{2}{9}$$

$\Rightarrow 9n + 45 = 10n - 2$

n = 47

Q28. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to_____.

Ans. **13**



Sol. Equation of line joining
(2, –5, 11) and (–6, 7, –5) is

$$\frac{x-2}{8}=\frac{y+5}{-12}=\frac{z-11}{16}$$

or, $\frac{x-2}{2}=\frac{y+5}{-3}=\frac{z-11}{4}=r$

Let the point Q is (2r+2, –3r – 5, 4r + 11)

$\therefore$ dir ratio of line RQ is 2r + 1, –3r – 12, 4r + 5

RQ is $\perp$ to L

$\therefore$ 2 (2r + 1) – 3 (–3r – 12) + 4 (4r + 5) = 0

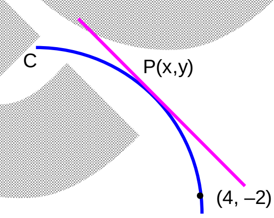
$\Rightarrow$ 29r + 58 = 0 $\Rightarrow$ r = –2

$\therefore$ Q(–2, 1, 3) & P(10, –2, –1)

$\therefore PQ=\sqrt{144+9+16}=13$

Q29. Let a conic C pass through the point (4, –2) and P(x, y), $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, –5). If the focal distance of the point (7, 1) on C is d, then 12d equals_____.

Ans. 75



Sol. $\frac{dy}{dx}=\frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow 2\frac{dy}{y+5}=\frac{dx}{x-3} \Rightarrow\ 2\ln(y+5)$

$=\ln(x-3)+\ln C$

$\Rightarrow\ (y+5)^2=c(x-3)$

It passes through (4, –2)

$\Rightarrow\ c=9$

$\therefore\ (y+5)^2=9(x-3)$

focus $\left(3+\frac{9}{4},-5\right)=\left(\frac{21}{4},-5\right)$

$d=\sqrt{\left(7-\frac{21}{4}\right)^2+(1+5)^2}=\sqrt{\frac{49}{16}+36}=\frac{25}{4}.$

$\therefore 12d = 75$

Q30. Let $\alpha\beta\gamma=45:\alpha,\beta,\gamma\in R$. If x ($\alpha$, 1, 2) + y(1, β, 2) + z(2, 3, γ) = (0, 0, 0) for some $x,y,z\in R$, $xyz\neq 0$, then $6\alpha+4\beta+\gamma$ is equal to_____.

Ans. 55

Sol. $\alpha x+y+2z=0$

$x+\beta y+3z=0$

$2x+2y+\gamma z=0$

as $xyz\neq 0$ $\therefore$ the system has infinite solution

$$\Rightarrow \Delta=\begin{vmatrix}\alpha & 1 & 2\\ 1 & \beta & 3\\ 2 & 2 & \gamma\end{vmatrix}=0$$

$\Rightarrow \alpha(\beta\gamma-6)-1(\gamma-6)+2(2-2\beta)=0$

$\Rightarrow \alpha\beta\gamma-6\alpha-\gamma+6+4-4\beta=0 \Rightarrow 45+10=6\alpha+4\beta+\gamma$

$\Rightarrow 6\alpha+4\beta+\gamma=55$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q31. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After sometime, the ball falls with constant velocity. The viscous force on the ball is :

(A) $mg(1-\rho\rho_0)$ (B) $mg\left(\frac{\rho_0}{\rho}-1\right)$

(C) $mg\left(1-\frac{\rho_0}{\rho}\right)$ (D) $mg\left(1+\frac{\rho}{\rho_0}\right)$

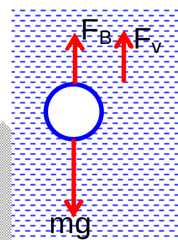
Ans. C

Sol. $F_V = mg - F_B$

$F_V = mg - V\rho_0 g$

$F_V = mg - \frac{m}{\rho}\rho_0 g$

$F_B = mg\left(1-\frac{\rho_0}{\rho}\right)$



Q32. Electromagnetic waves travel in a medium with speed of $1.5\times10^8\,ms^{-1}$. The relative permeability of the medium is 2.0. The relative permittivity will be :

(A) 5 (B) 1

(C) 4 (D) 2

Ans. D

Sol. $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ -(i)

$V = \frac{1}{\sqrt{\mu_m\varepsilon_m}}$ -(ii)

(i) ÷ (ii)

$\frac{\varepsilon_m}{\varepsilon_0}\times\frac{\mu_m}{\mu_0} = \frac{c^2}{v^2}$

$\varepsilon_r \times \mu_r = \frac{c^2}{v^2}$

$\varepsilon_r \times 2 = \frac{(3\times10^8)^2}{(1.5\times10^8)^2}$

$\varepsilon_r = 2$

Q33. A bullet of mass 50g is fired with a speed 100m/s on a plywood and emerges with 40m/s. The percentage loss of kinetic energy is :

(A) 84% (B) 16%

(C) 32% (D) 44%

Ans. A

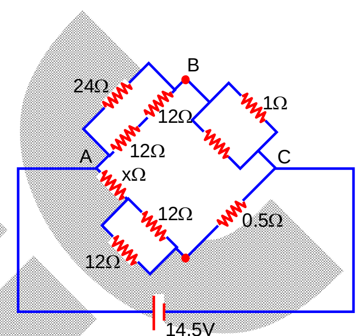
Sol. $kE = \frac{1}{2}mv^2$

$\frac{k^2}{k_1} = \left(\frac{v_2}{v_1}\right)$

$\frac{k_2}{k_1} = \left(\frac{40}{100}\right)^2 = 0.16$

$\left(\frac{k_2}{k} - 1\right)\% = (0.16 - 1) \times 100 = -84\%$

Q34. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown is :



(A) 9Ω
(B) 6Ω
(C) 3Ω
(D) 42Ω

Ans. B

Sol. $\frac{12}{x+6} = \frac{0.5}{0.5}$

$\Rightarrow x = 6\Omega$

Q35. To find the spring constant (k) of a spring experimentally, a students commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of k is :

(A) 1%
(B) 5%
(C) 4%
(D) 3%

Ans. B

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 \alpha \frac{m}{k}$

$\frac{2\Delta T}{T} = \frac{\Delta m}{m} - \frac{\Delta k}{k}$

$\Rightarrow \frac{\Delta k}{k} = \frac{\Delta m}{m} - \frac{2\Delta T}{T}$

$\Rightarrow \frac{\Delta k}{k}\% = (-1) - 2 \times 2 = -5\%$

Q36. Four particles A,B,C,D of mass $\frac{m}{2}$, m, 2m, 4m, have same momentum, respectively. The particle with maximum kinetic energy is :

(A) B
(B) D
(C) C
(D) A

Ans. D

Sol. $kE = \frac{P^2}{2m}$

$kE \; \alpha \; \frac{1}{m}$
A particle has maximum k.E.

Q37. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is :

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{32}$

(C) $\frac{1}{4}$ (D) $\frac{2\sqrt{2}}{1}$

Ans. **D**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_{He}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{He}}} = \sqrt{\frac{32}{4}} = 2\sqrt{2}$

Q38. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$mm. The value of x is :

(A) 142 (B) 21
(C) 42 (D) 71

Ans. **D**

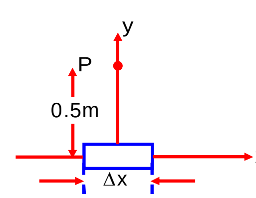
Sol. $L \cdot C = \frac{1}{100} = 0 \cdot 01 mm$

$\text{Diameter} = MSR + L \cdot C \times C \cdot S \cdot D$

$D = 1 + 0 \cdot 01 \times 42$
$D = 1 \cdot 42 mm$

$\frac{x}{50} = 1 \cdot 42$
$x = 71$

Q39. An element $\Delta\ell = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10A. The magnetic field on the y-axis at a distance of 0.5m from the elements Δx of 1cm length is :



(A) 4×10^{-8}T (B) 12×10^{-8}T
(C) 10×10^{-8}T (D) 8×10^{-8}T

Ans. **A**

Sol. $\overrightarrow{dB} = \frac{\mu_0 i\left(\overrightarrow{d\ell} \times \vec{r}\right)}{4\pi r^3}$

$\overrightarrow{dB} = \frac{10^{-7} \times 10 \times \frac{1}{100} \times 0.5}{(0.5)^3}\hat{k}$

$\overrightarrow{dB} = 4 \times 10^{-8}T\;\hat{k}$

Q40. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time 3t. The average speed of the train for this duration of journey will be (in km/h) :

(A) 80 (B) 40

(C) 70 (D) 30

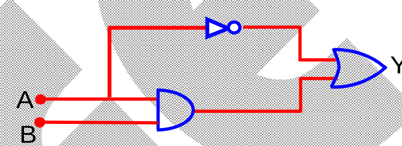
Ans. **C**

Sol. $\langle V \rangle = \dfrac{\text{Total distance}}{\text{Total time}}$

$$\langle V \rangle = \frac{\frac{80 \times t}{2} + 80 \times 3t}{4t}$$

$$\langle V \rangle = \frac{280t}{40t} = 70\,\text{km/h}$$

Q41. The correct truth for the following logic circuit is :



(A)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(B)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(C)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(D)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Ans. **B**

Sol.

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

Q42. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is :

(A) 0.5 eV (B) 1.68 eV

(C) 1.98 eV (D) 2.48 eV

Ans. **C**

Sol. $hf = \phi + ev$

$\phi = hf - ev$

$\phi = 2\cdot 48 - 0\cdot 5$

$\phi = 1\cdot 98\,ev$

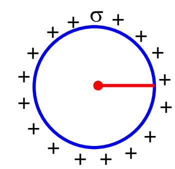
Q43. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is :

(A) σ / ϵ_0 (B) $\sigma / \epsilon_0 R$

(C) $\sigma / 2\epsilon_0$ (D) $\sigma / 4\epsilon_0$

Ans. **A**

Sol. $Q = \sigma \times 4\pi R^2$



$$E = \frac{kQ}{R^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{\sigma \times 4\pi R^2}{R^2}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Q44. Given below are two statements :
Statement I : In an LCR series circuit. Current is maximum at resonance.
Statement II : Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.
In the light of the above statements, choose the correct from the options given below :
(A) Both Statement I and Statement II are true
(B) Statement I is false but Statement II is true
(C) Statement I is true but Statement II is false
(D) Both Statement I and Statement II are false

Ans. **A**

Sol. At resonance

$X_L = X_C$

$Z = R$

$$i_{max} = \frac{V}{R}$$

In purely resistive circuit

$$i = \frac{V}{R}$$

Q45. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is :

(A) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$
(B) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$
(C) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$
(D) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

Ans. **D**

Sol. $$\frac{g}{\sqrt{2}} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$$

$$m_2 + m_1 = \sqrt{2}(m_2 - m_1)$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

Q46. The ratio of the wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is :
(A) 4 : 1
(B) 1 : 4
(C) 1 : 2
(D) 2 : 1

Ans. **A**

Sol. $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_B} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\frac{1}{\lambda_L} = RZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$
$$\frac{\lambda_B}{\lambda_L} = \frac{4}{1}$$

Q47. Match List I with List II

	LIST - I		LIST - II
a.	Torque	I.	$\left[M^1L^1T^{-2}A^{-2}\right]$
b.	Magnetic field	II.	$\left[L^2A^1\right]$
c.	Magnetic moment	III.	$\left[M^1T^{-2}A^{-1}\right]$
d.	Permeability of free space	IV.	$\left[M^1L^2T^{-2}\right]$

Choose the correct answer from the options given below :
(A) a - IV, b - II, c - III, d - I
(B) a - IV, b - III, c - II, d - I
(C) a - I, b - III, c - II, d - IV
(D) a - III, b - I, c - II, d - IV

Ans. B

Sol. $[\tau] = [r \times F] = \left[ML^2T^{-2}\right]$

$$[B] = \left[\frac{F}{qv}\right] = \left[MT^{-2}A^{-1}\right]$$
$$[M] = [iA] = \left[AL^2\right]$$
$$B = \frac{\mu_0 i d\ell}{4\pi r^2}$$
$$[\mu_0] = \left[\frac{Br^2}{id\ell}\right] = \left[MLT^{-2}A^{-2}\right]$$

Q48. Which of the following phenomena does not explain by wave nature of light.
a. reflection
b. diffraction
c. photoelectric effect
d. interference
e. polarization
Choose the **most appropriate** answer from the options given below :
(A) a, c only
(B) c only
(C) e only
(D) b, d only

Ans. B

Sol. Photoelectric effect explain by particle nature of light.

Q49. The specific heat at constant pressure of a real gas obeying $PV^2 = RT$ equation is :

(A) R
(B) $C_v + \frac{R}{2V}$
(C) $C_V + R$
(D) $\frac{R}{3} + C_v$

Ans. B

Sol. $dQ = du + dw$

$CdT = C_v dT + Pdv$ -(i)

$Pv^2 = RT$

$P2vdv = RdT$

$$Pdv = \frac{RdT}{2v} \qquad \text{-(ii)}$$

$$CdT = C_v dT + \frac{RdT}{2v}$$

$$C = C_v + \frac{R}{2v}$$

Q50. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E, g = acceleration due to gravity on the surface of earth):
(A) $1/2mgR_E$ (B) $2mgR_E$
(C) $4mgR_E$ (D) mgR_E

Ans. **D**

Sol. $g = \frac{GM}{R_E^2}$

$$\frac{1}{2}mv^2 = \frac{GMm}{R_E}$$

$$k \cdot E \cdot = mgR_E$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q51. A particle is doing simple harmonic motion of amplitude 0.06m and time period 3.14s. The maximum velocity of the particle is _______cm/s.

Ans. **12**

Sol. Maximum velocity at mean position

$$V_{max} = A\omega = A \times \frac{2\pi}{T}$$

$$V_{max} = 0.06 \times \frac{2 \times 3.14}{3.14}$$

$$V_{max} = 0.12\,m/s = 12\,cm/s$$

Q52. For three vectors $\vec{A} = \left(-x\hat{i} - 6\hat{j} - 2\hat{k}\right)$. $\vec{B} = \left(-\hat{i} + 4\hat{j} + 3\hat{k}\right)$ and $\vec{C} = \left(-8\hat{i} - \hat{j} + 3\hat{k}\right)$, if $\vec{A}.\left(\vec{B} \times \vec{C}\right) = 0$, then value of x is ________.

Ans. **4**

Sol. $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = 0$

$$\Rightarrow \left(-x\hat{i} - 6\hat{j} - 2\hat{k}\right) \cdot \left\{\left(-\hat{i} + 4\hat{j} + 3\hat{k}\right) \times \left(-8\hat{i} - \hat{j} + 3\hat{k}\right)\right\} = 0$$

$$\Rightarrow \left(-x\hat{i} - 6\hat{j} - 2\hat{k}\right) \cdot \left(15\hat{i} - 21\hat{j} + 33\hat{k}\right) = 0$$

$$\Rightarrow -15x + 126 - 66 = 0$$

$$\Rightarrow x = 4$$

Q53. A wire of resistance R and radius r is stretched till its radius became r/2. If new resistance of the stretched wire is x R, then value of x is _______.

Ans. **16**

Sol. $R = \rho\frac{\ell}{A}$

$V_i = V_f$

$\Rightarrow \pi r^2 \ell = \pi\frac{r^2}{4}\times \ell_f$

$\Rightarrow \ell_f = 4\ell$

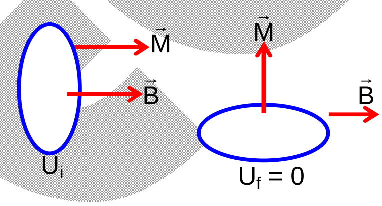
$R_f = \rho\frac{4\ell}{\pi\frac{r^2}{4}} = \rho\frac{\ell}{A}\times 16 = 16\times R$

x = 16

Q54. A circular coil having 200 turns, $2.5\times10^{-4}m^2$ area and carrying 100 μA current is placed in a uniform magnetic field of 1T. Initially the magnetic dipole moment $(\vec{M})$ was directed along $\vec{B}$. Amount of work, required to rotate the coil through 90° from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$. is ________ μJ.

Ans. **5**

Sol. $\Delta U = U_f - U_i = U - U_i = -U_i$



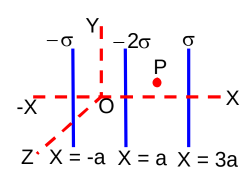
$\Delta w = -\Delta U = U_i = \vec{M}\cdot\vec{B}$

$\Delta w = N(iA)B$

$\Delta w = 200\times100\times10^{-6}\times2.5\times10^{-4}\times1$

$\Delta w = 5\mu J$

Q55. Three infinitely long charge thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is ________(all quantities are measured in SI units).



Ans. **2**

Sol. $\vec{E}_P = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)(-\hat{i})$

$\vec{E}_P = -\frac{4\sigma}{2\varepsilon_0}\hat{i} = \frac{-2\sigma}{\varepsilon_0}\hat{i} = \frac{x\sigma}{\varepsilon_0}$

x = 2

Q56. The refractive index of prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is ______ o.

Ans. **60**

Sol. $\frac{S_{min}}{A} = 1$

$S_{min} = A$

$\Rightarrow 2i - A = A$

$i = \frac{A}{2}$

$1\sin i = \mu \sin r$

$\sin A = \mu \sin\left(\frac{A}{2}\right)$

$$\Rightarrow 2\times\sin\frac{A}{2}\times\cos\frac{A}{2}=\mu\times\sin\frac{A}{2}$$
$$\Rightarrow 2\cos\frac{A}{2}=\sqrt{3}$$
$$\Rightarrow \frac{A}{2}=30^{\circ}$$
$$A=60^{\circ}$$

Q57. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$. The value of x is ______.

Ans. **1**

Sol. Surface energy $=4\pi r^2 T$

$$\frac{S_{1000}}{S_1}=\frac{1000\times 4\pi r^2\times T}{1\times 4\pi R^2\times T}$$
$$1000\times\frac{4}{3}\pi r^3=1\times\frac{4}{3}\pi R^3$$
R = 10r
$$\frac{S_{1000}}{S_1}=\frac{1000r^2}{100r^2}=10=\frac{10}{x}$$
x = 1

Q58. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be _______hours 30 minutes.

Ans. **13**

Sol. $I_1\omega_1=I_2\omega_2$

$$\Rightarrow \frac{2}{5}MR^2\times\frac{2\pi}{T_1}=\frac{2}{5}M\left(\frac{3}{4}R\right)^2\times\frac{2\pi}{T_2}$$
$$\Rightarrow \frac{1}{T_1}=\frac{9}{16T_2}$$
$$\Rightarrow T_2=\frac{9}{16}\times T_1=\frac{9}{16}\times 24=\frac{27}{2}\text{hr}$$
$$\Rightarrow T_2=13\text{hr }30\text{mint}$$

Q59. Radius of a certain orbit of hydrogen atom is 8.48 Å. If energy of electron in this orbit is E/x, then x = _______.

(Given $a_0=0.529$ Å, $E=$ energy of electron in ground state).

Ans. **16**

Sol. $r=0.529\frac{n^2}{z}$

$$8.48=0.529\frac{n^2}{1}$$
n = 4
$$E_4=\frac{E_1}{n^2}=\frac{E}{16}=\frac{E}{x}$$
x = 16

Q60. When a dc voltage of 100V is applied to an inductor, a dc current of 5A flows through it. When an ac voltage of 200V peak value is connected to inductor, its inductive reactance is found to be $20\sqrt{3}\Omega$. The power dissipated in the circuit is _______W.

Ans. **250**

Sol0. $x_L = 20\sqrt{3}\Omega$

$$R = \frac{V}{i} = \frac{100}{5} = 20\Omega$$

$$z = \sqrt{x_L^2 + R^2} = \sqrt{\left(20\sqrt{3}\right)^2 + 20^2}$$

z = 40

$$\text{Power} = i_{rms}^2 \times R = \left(\frac{V_{rms}}{z}\right)^2 \times R$$

$$P = \left(\frac{\frac{200}{\sqrt{2}}}{40}\right)^2 \times 20 = 250\text{w}$$

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q61. Consider the following complexes.

$\left[CoCl(NH_3)_5\right]^{2+}, \left[Co(CN)_6\right]^{3-}, \left[Co(NH_3)_5(H_2O)\right]^{3+}, \left[Cu(H_2O)_4\right]^{2+}$

(A) (B) (C) (D)

The correct order of A,B,C and D in terms of wave number of light absorbed is:

(A) B < C < A < D
(B) A < C < B < D
(C) D < A < C < B
(D) C < D < A < B

Ans. C

Sol. As value of Δ [Crystal field split energy] increases. The light of more energy [Lower wave length or more wave number of radiation] will get absorbed order of $\Delta = D < A < C < B$ wave number of light $\Rightarrow D < A < C < B$

Q62. Given below are two statements:

Statement I : Gallium is used in the manufacturing of thermometers

Statement II: A thermometer containing gallium is useful for measuring the freezing point (256K) of brine solution

In the light of the above statements, choose the **correct** answer from the options given below:

(A) **Statement I** is false but **Statement II** is true.
(B) Both **Statement I** and **Statement II** are false
(C) **Statement I** is true but **Statement II** is false.
(D) Both **Statement I** and **Statement II** are true.

Ans. C

Sol. Melting point of Ga = 303 K
Boiling point of Ga = 2676 K
Statement (I) – True
Statement (II) – False

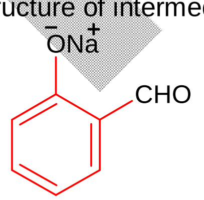
Q63. Which of the following material is not a semiconductor:

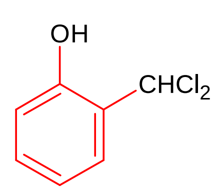
(A) Silicon
(B) Copper oxide
(C) Graphite
(D) Germanium

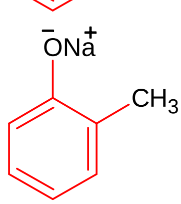
Ans. C

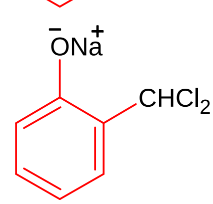
Sol. Graphite is conductor.

Q64. In Reimer-Tiemann reaction phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is_____________.

(A) 

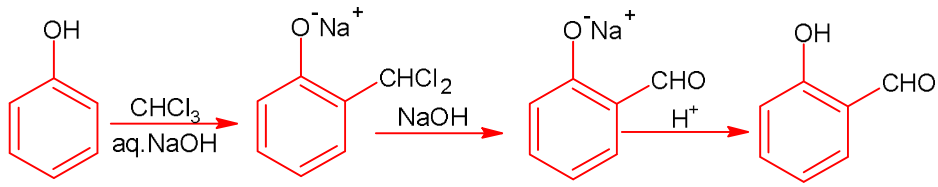
(B) 

(C) 

(D) 

Ans. D

Sol.



Q65. Match the **List I** with **List II**

List-I (Compound)		List-II (Uses)	
(a)	Iodoform	(I)	Fire extinguisher
(b)	Carbon tetrachloride	(II)	Insecticide
(c)	CFC	(III)	Antiseptic
(d)	DDT	(IV)	Refrigerants

Choose the correct answer from the options given below:

(A) a - III, b - I, c - IV, d - II
(B) a - III, b - II, c - IV, d - I
(C) a - II, b - IV, c - IV, d - I
(D) a - I, b - II, c - III, d - IV

Ans. A

Sol.

Compound	**Use**
Iodofrorm	Antiseptic
CCl_4	Fire extinguisher
CFC	Refrigerants
DDT	Insecticide

Q66. Given below are two statements:

Statement I : Picric acid is 2,4,6- trinitrotoluene.

Statement II: Phenol-2,4- disulphonic acid is treated with Conc. HNO_3 to get picric acid.

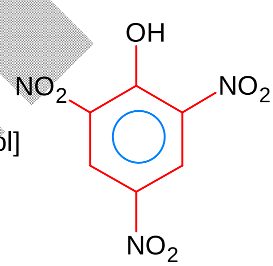
In the light of the above statements, choose the **most appropriate** answer from the options given below:

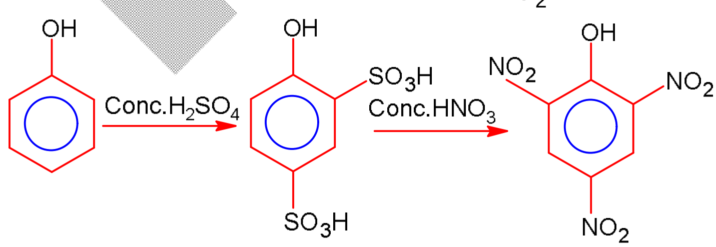
(A) Both **Statement I** and **Statement II** are correct.
(B) **Statement I** is correct but **Statement II** is incorrect
(C) **Statement I** is incorrect but **Statement II** is correct.
(D) Both **Statement I** and **Statement II** are incorrect..

Ans. C

Sol.

Pecric Acid [2,4,6 trinitrophenol]





Q67. At -20°C and 1 atm pressure, a cylinder is filled with equal number of H_2, I_2 and HI molecules for the reaction.
$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$, the K_p for the process is $x \times 10^{-1}$.
X = ______________.
[Given: R= 0.082 L atm K^{-1}]
(A) 10 (B) 0.01
(C) 1 (D) 2

Ans. **A**

Sol. As $\Delta n_g = 0 \Rightarrow k_P = k_C$, $n_{H_2} = n_{I_2} = n_{HI} = a$ moles, Vol $= V\ell$ $k_C = \dfrac{[HI]^2}{[H_2][I_2]} = \dfrac{(a/v)^2}{(a/v)\times\left(\dfrac{a}{v}\right)} = 1$

$k_P = 1 = 10 \times 10^{-1} \quad \Rightarrow x = 10$

Q68. Match the **List I** with **List II**

List-I (Hybridization)		**List-II (Orientation in Space)**	
(a)	sp^3	(I)	Trigonal bipyramidal
(b)	dsp^2	(II)	Octahedral
(c)	sp^3d	(III)	Tetrahedral
(d)	sp^3d^2	(IV)	Square planar

Choose the **correct** answer from the options given below:
(A) a - IV, b - III, c - I, d - II (B) a - II, b - I, c - IV, d - III
(C) a - III, b - I, c - IV, d - II (D) a - III, b - IV, c - I, d - II

Ans. **D**

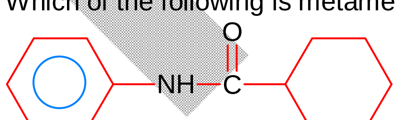
Sol. sp^3 – Tetrahedral
dsp^2 – Square Planar
sp^3d – Trigonal Bipyramid
sp^3d^2 – Octahedral

Q69. The density of 'x' M solution ('x' molar) of NaOH is 1.12 g mL^{-1}, while in molality, the concentration of the solution is 3m (3 molal). Then x is
(Given: Molar mass of NaOH is 40 g/ mol)
(A) 2.8 (B) 3.0
(C) 3.5 (D) 3.8

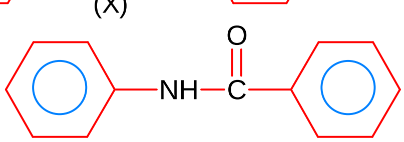
Ans. **B**

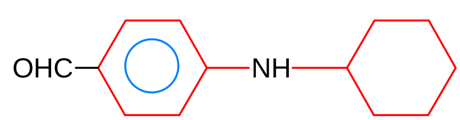
Sol. $d = \left(\dfrac{M}{m}\right) + \left(\dfrac{M \times (MM)_{solute}}{1000}\right) \Rightarrow 1.12 = \left(\dfrac{x}{3}\right) + \left(\dfrac{x \times 40}{1000}\right) \Rightarrow x = 3\text{molar}$

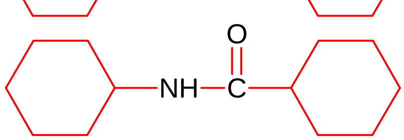
Q70. Which of the following is metamer of the given compound (X)?

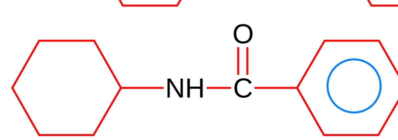


(X)

(A) 

(B) 

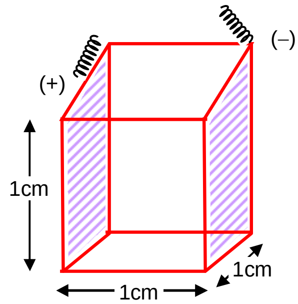
(C) 

(D) 

Ans. **D**

Sol. Metamers $\Rightarrow$ Structural isomers of bivalent functional groups.
Where different alkyl /aryl groups are attached on either side of functional group.

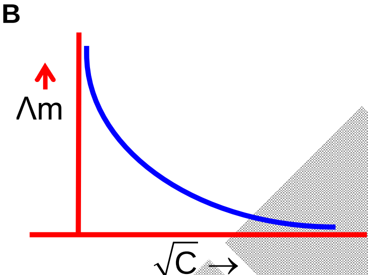
Q71. A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is double by adding more water at constant temperature, the molar conductivity of the cell will-



(A) increase sharply
(B) remain same or can not be measured accurately
(C) decrease sharply
(D) depend upon type of electrolyte

Ans. **B**

Sol.



(Graph for weak electrolyte)

Addition of water to infinite dilute solution of weak electrolyte will not change molar conductivity

Q72. Which among the following aldeydes is most reactive towards nucleophile addition reactions?

(A) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-H$ (B) $C_3H_7-\overset{\overset{\displaystyle O}{||}}{C}-H$

(C) $H-\overset{\overset{\displaystyle O}{||}}{C}-H$ (D) $C_2H_5-\overset{\overset{\displaystyle O}{||}}{C}-H$

Ans. **C**

Sol.

$H-\overset{\overset{\displaystyle O}{||}}{C}-H \Longrightarrow$ Less steric hinderance at carbonyl carbon for attack of nucleophile
$\Longrightarrow$ High partial positive charge on carbonyl carbon

Q73. Match the **List I** with **List II**

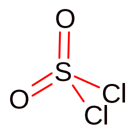
List-I (Molecule / Species)		List-II (Property / Shape)	
(a)	SO_2Cl_2	(I)	Paramagnetic
(b)	NO	(II)	Diamagnetic
(c)	NO_2^-	(III)	Tetrahedral
(d)	I_3^-	(IV)	Linear

Choose the **correct** answer from the options given below:

(A) a - IV, b - I, c - III, d - II
(B) a - II, b - III, c - I, d - IV
(C) a – III, b - IV, c - II, d - I
(D) a - III, b - I, c - II, d - IV

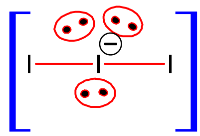
Ans. **C**

Sol.

$SO_2Cl_2 \Rightarrow$  Hybridization of S - sp^3 (Tetrahedral shape)

NO $\Rightarrow$ Paramagnetic [odd election species]

NO_2^- $\Rightarrow$ Diamagnetic

$I_3^{\ominus}$ =  $(2bp + 3\ell p)$.

Hybridization of central I $\Rightarrow$ sp^3d

[Linear shape]

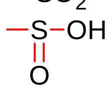
Q74. The number of element from the following that do not belong to lanthanoids is
Eu, Cm, Er, Tb, Yb and Lu

(A) 4 (B) 3
(C) 1 (D) 5

Ans. **C**

Sol4. Cm is actinide wheras others are Lanthanoids.

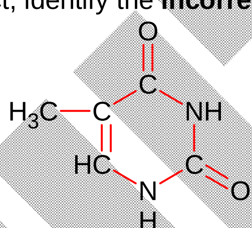
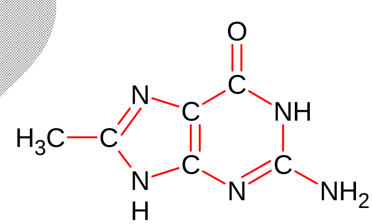
Q75. Functional group present in sulphonic acids is:

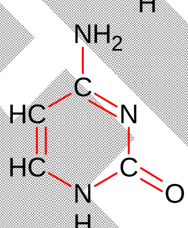
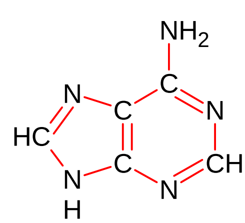
(A) $-SO_2$ (B) $-SO_3H$
(C)  (D) $-SO_4H$

Ans. **B**

Sol. Functional group in sulphonic acid = SO_3H

Q76. DNA molecule contains 4 bases whose structure are shown below. One of the structure is not correct, identify the **incorrect** base structure.

(A)  (B) 

(C)  (D) 

Ans. **B**

Sol. A. Structure of Thymine
C. Cytosine
D. Adenine
DNA containl four bases : Adenine, cytosine, Guanine and thymine

Q77. Match the **List I** with **List II**

	List-I (Molecule / Species)		**List-II (Property / Shape)**
(a)	$NH_4Cl + NH_4OH$	(I)	Mn^{2+}
(b)	$NH_4OH + Na_2CO_3$	(II)	Pb^{2+}
(c)	$NH_4OH + NH_4Cl + H_2S$ gas	(III)	Al^{3+}
(d)	Dilute HCl	(IV)	Sr^{2+}

Choose the **correct** answer from the options given below:

(A) a - III, b - IV, c - I, d - II
(B) a - IV, b - III, c -II, d - I
(C) a - IV, b - III, c - I, d - II
(D) a - III, b - IV, c - II, d - I

Ans. A

Sol. Mn^{2+} - Group (IV) basic radical
Pb^{2+} - Group (I) basic radical
Al^{3+} - Group (III) basic radical
Sr^{2+} - Group (V) basic radical

Q78. Match the **List I** with **List II**

	List-I (Compound / Species)		**List-II (Shape / Geometry)**
(a)	SF_4	(I)	Tetrahedral
(b)	BrF_3	(II)	Pyramidal
(c)	BrO_3^-	(III)	See saw
(d)	NH_4^+	(IV)	Bent T-Shape

Choose the **correct** answer from the options given below:

(A) a - III, b - II, c - IV, d - I
(B) a - II, b - III, c - I, d - IV
(C) a - III, b - IV, c - II, d - I
(D) a - II, b - IV, c - III, d - I

Ans. C

Sol. $SF_4 \Rightarrow (4bp + 1\ell p) \Rightarrow$ See – saw

$BrF_3 \Rightarrow (3bp + 2\ell p) \Rightarrow$ Bent T- shape

$BrO_3^- \Rightarrow (3bp + 1\ell p) \Rightarrow$ Pyramidal

$\overset{\oplus}{N}H_4 \Rightarrow (4bp + 0\ell p) \Rightarrow$ Tetrahedral

Q79. Which of the following statements are correct?

a. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.
b. Aniline can be purified by steam distillation as aniline is miscible in water.
c. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.
d. An organic compound is pure if mixed M.P, is remained same.

Choose the **most appropriate** answer from the options given below:

(A) a, b, c only
(B) b, c, d only
(C) a, c, d only
(D) a, b, d only

Ans. C

Sol. Aniline is immiscible in water.

Q80. The electron affinity value are negative for

a. $Be \rightarrow Be^-$
b. $N \rightarrow N^-$
c. $O \rightarrow O^{2-}$
d. $Na \rightarrow Na^-$
e. $Al \rightarrow Al^-$

Choose the **most appropriate** answer from the options given below:

(A) a, b and c only
(B) d and e only
(C) a and d only
(D) a, b, d and e only

Ans. A

Sol.

$Be \longrightarrow Be^{-}$
$N \longrightarrow N^{-}$ Electron affinity is -ve
$O^{-} \longrightarrow O^{2-}$ (As energy needs to be supplied)

$O \longrightarrow O^{-}$
$Na \longrightarrow Na^{-}$ Electron affinity is +ve
$Al \longrightarrow Al^{-}$ (As energy will be released).

SECTION - B

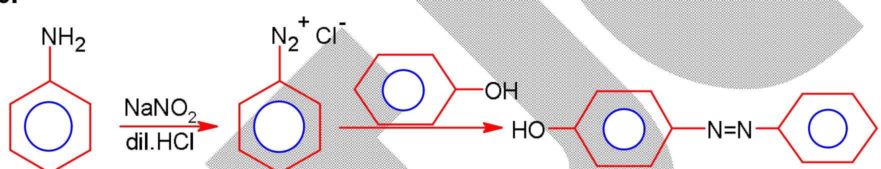
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q81. 9.3 g of pure aniline upon diazotization followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield / conversion) is ________g. (nearest integer)

Ans. **20.**

Sol.



1 mole of Aniline given 1 mole of orange dye.
No. of moles of orange dye = no. of moles of Aniline

$$= \frac{9.3\,gm}{93\,gm/mole} = 0.1\,mole$$

Mass of orange dye $= 199 \times 0.1 = 19.9\,gm \sim 20\,gm$

Q82. Consider the dissociation of weak acid HX as given below

$HX(aq) \rightleftharpoons H^{+} + X^{-}(aq), Ka = 1.2 \times 10^{-5}$

[K_a : dissociation constant]

The osmotic pressure of 0.03 M aqueous solution of HX at 300 K is_____ $\times 10^{-2}$ bar (nearest integer)

[Given: R= 0.083 L bar mol^{-1} K^{-1}]

Ans. 76

Sol.

	HX	$\rightleftharpoons$	H^+	+	X^-
t = 0	0.03M		O		O
	-x		+x		+x
Eq.	0.03-x		x		x

$$K_a(HX) = \frac{[H^+]_{eq} \times [X^-]_{eq}}{[HX]_{eq}} = \frac{x.x}{(0.03 - x)} = 1.2 \times 10^{-5}$$

As value of K_a is very less so value of x is less.

$$(0.03 - x)M \longrightarrow 0.03M$$

$$\frac{x^2}{0.03} = 1.2\times10^{-5} \Rightarrow x = 6\times10^{-4}\,M$$

$$[\quad]_{Solution} = (0.03 - x) + x + x = (0.03 + x)$$

$$= (0.03 + 6\times10^{-4})M$$

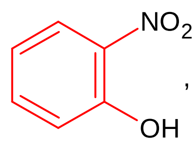
$$\pi = [\quad]_{solution} \times S \times T$$

$$= (0.03 + 6\times10^{-4}) \times 0.083 \times 300$$

$$= 76.19\times10^{-2}$$

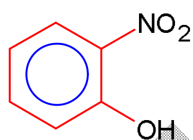
$$= 76\times10^{-2}\,bar$$

Q83. Number of molecules from the following which can exhibit hydrogen bonding is________ (nearest integer)

$CH_3OH, H_2O, C_2H_6, C_6H_6$  , HF, NH_3

Ans. 5

Sol.

CH_3OH, H_2O,  , HF, NH_3 $\rightarrow$ Show H-bonding

C_2H_6 & C_6H_6 $\rightarrow$ Do not show H- Bonding

Q84. The difference in the 'spin-only' magnetic moment values of $KMnO_4$ and the manganese product formed during titration of $KMnO_4$ against oxalic acid in acidic medium is ________B.M (nearest integer)

Ans. 6

Sol. $MnO_4^- + C_2O_4^{2-} \longrightarrow CO_2 + Mn^{2+}$

$\overset{+7}{Mn}O_4^-$ $\Rightarrow$ no. of unpaired e = 0 $\quad [Ar]4s^0\,3d^0$

Mn^{2+} $\Rightarrow$ no. of unpaired e = 5 $\quad [Ar]4s^0\,3d^5$

Spin only, magnetic moment is $MnO_4^- = 0$

Spin only, magnetic moment in $Mn^{2+} = \sqrt{5(5+2)}\,BM$

$= \sqrt{35}\,BM$

= 5.9BM

= 6 BM

Q85. An ideal gas $\bar{C}_v = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm until it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm respectively then the final temperature is _________K (nearest integer)

Ans. 274

Sol. $\Delta U = q + w$

$\Rightarrow \Delta U = w$ (Adiabatic process)

$\Rightarrow n\,C_V\,\Delta T = -P_{ext}(V_2 - V_1)$

$$\Rightarrow n\times\frac{5}{2}R(T_2 - T_1) = -1\left[\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1}\right]$$

$V_2 = 2V_1$

$$\frac{nRT_2}{P_2} = 2\times\frac{nRT_1}{P_1} \quad \Rightarrow \quad P_2 = \frac{5T_2}{2\times 298}$$

$T_1 = 298\,K$, $\quad P_1 = 5\,atm$

$$n\times\frac{5}{2}R\left(T_2 - 298\right) = -1\left[\frac{nRT_2}{-\left(\frac{5T_2}{2\times 298}\right)} - \frac{nR\times 298}{5}\right] \Rightarrow T_2 = 274.16k$$

= 274 k

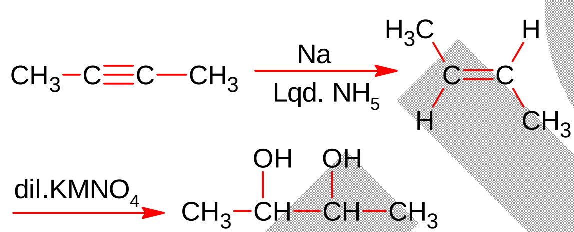
Q86. The major product of the following reaction is P.

$$CH_3C\equiv C-CH_3 \xrightarrow[\text{(ii) dil. } KMnO_4 \text{ 273 K}]{\text{(i) Na / liq. } NH_3} \text{'P'}$$

Number of oxygen atoms present in product __________(nearest integer)

Ans. **2**

Sol.



Q87. Among CrO, Cr_2O_3 and CrO_3, the sum of spin-only magnetic moment values of basic and amphoteric oxides is __________10^{-2}BM (nearest integer)
(Given atomic number of Cr is 24)

Ans. **877**

Sol. CrO – Basic oxide
Cr_2O_3 – Amphoteric oxide
CrO_3 – Acidic oxide

$\overset{+2}{Cr}O \quad Cr^{2+} - [Ar]4s^{\circ}\,3d^{4}$

No. of unpaired electrons = 4

$\mu = \sqrt{4(4+2)}\,BM = \sqrt{24}\,BM = 4.9BM$

$\overset{+3}{Cr_2}O_3 \quad Cr^{3+} - [Ar]4s^{\circ}3d^{3}$

no. of unpaired electrons = 3

$\mu = \sqrt{3(3+2)} = \sqrt{15}\,BM = 3.87\,BM$

Sum = 4.90+3.87= 8.77 = 877×10^{-2}

Q88. Time required for 99.9% completion of a first order reaction is________times the time required for completion of 90% reaction (nearest integer)

Ans. **3**

Sol. $t_{99.9} = \frac{1}{k}\ell n\frac{1}{0.001}$

$t_{90} = \frac{1}{k}\ell n\frac{1}{0.1}$

$\frac{t_{99.9}}{t_{90}} = 3$

Q89. Frequency of the de-Broglie wave of electron in Bohr's first orbit of hydrogen atom is_________ $\times 10^{13}$ Hz (nearest integer)

[Give: R_H (Rydberg constant) = 2.18×10^{-18} J, h (Plank's constant 6.6×10^{-34} J.s] $= 661\times10^{13}$ J

Ans. **661**

Sol. $\lambda = \frac{h}{mv} = \frac{hv}{mv^2} \Rightarrow \left(\frac{mv^2}{h}\right) = \frac{v}{\lambda} = v$ (frequency)

$mv^2 = 2\times2.18\times10^{-18}$ J

$K\cdot E\cdot \text{of } e = \frac{1}{2}mv^2 = -E_1, H \sim R_H \left(\text{in J}\right)$

$h = 6.6\times10^{-34}$ J – sec

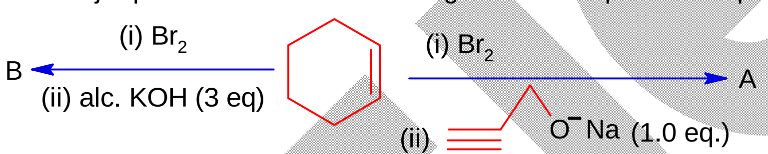
$v = \frac{mv^2}{h} = \frac{2\times2.18\times10^{-18}}{6.6\times10^{-34}} = \frac{2\times2.18\times10^{16}}{6.6}$

$= \frac{2\times2.18\times1000}{6.6}\times10^{13}$

$= 660.6\times10^{13}$ J

$= 661\times10^{13}$ J

Q90. The major products from the following reaction sequence are product A and product B



The total sum of π electrons in product A and product B are _________(nearest integer)

Ans. **8**

Sol.