FIITJEE

Solutions to JEE(Main) -2024

Test Date: 4th April 2024 (Second Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours | **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q1. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}).\vec{c}$ is equal to

(A) 9 (B) 11
(C) 3 (D) 6

Ans. **B**

Sol. $\vec{d} = \lambda\left(\vec{b} + \vec{c}\right)$

$= \lambda\left((2 + x)\hat{i} + 6\hat{j} - 2\hat{k}\right)$

$\vec{a} \cdot \vec{d} = 1 \Rightarrow \lambda(x + 6) = 1$……(i)

and $|\vec{d}| = 1 \Rightarrow \lambda^2\left((x + 2)^2 + 36 + 4\right) = 1$…..(ii)

from (i) and (ii)

x = 1

$\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a} \times \vec{b} = -9\hat{i} + 7\hat{j} + 2\hat{k}$

Now, $\left(\vec{a} \times \vec{b}\right).\vec{c} = -9 + 14 + 6 = 11$

Q2. For $\lambda > 0$, let θ be the angle between the vectors $\vec{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to

(A) 50 (B) 25
(C) 40 (D) 20

Ans. **B**

Sol. $\left(\vec{a} - \vec{b}\right).\left(\vec{a} + \vec{b}\right) = 0$

$\Rightarrow |a|^2 = |\vec{b}|^2$

$\Rightarrow 1 + \lambda^2 + 9 = 9 + 1 + 4$

$\Rightarrow \lambda = 2$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos\theta = \dfrac{-5}{14}$

So, $(14\cos\theta)^2 = 25$

Q3. If the function

$$f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+!}{\sqrt{2}-\sqrt{1+\cos x}}, & x\neq 0\\ a\log_e 2\log_e 3\,, & x=0\end{cases}$$

is continuous at x = 0, then the value of a^2 is equal to

(A) 1152 (B) 968
(C) 746 (D) 1250

Ans. **A**

Sol. Continuous at x = 0

So, $f(0)=\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$

$$f(0)=\lim_{x\to 0}\frac{\left(\frac{9^x-1}{x}\right)\left(\frac{8^x-1}{x}\right)x^2}{\sqrt{2}\left(\frac{1-\cos\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\cdot\frac{x^2}{4}}$$

So, $a\log_e^2\log_e^3=\dfrac{\ell n9\cdot\ell n8}{\sqrt{2}\cdot\frac{1}{2}\cdot\frac{1}{4}}$

So, $a^2 = 1152$

Q4. Let a relation R on N × N be defined as :
$(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1\le x_2$ or $y_1\le y_2$
Consider the two statements :
(I) R is reflexive but not symmetric
(II) R is transitive
Then which one of the following is true?

(A) Neither (I) nor (II) is correct (B) Only (II) is correct
(C) Both (I) and (II) are correct (D) Only (I) is correct

Ans. **D**

Sol. Reflexive : $(x_1, y_1)R(x_1, y_1)$

$\Rightarrow x_1\le x_1$ or $y_1\le y_1$ true

Symmetric : $(x_1, y_1)R(x_2, y_2)$

$\Rightarrow x_1\le x_2$ or $y_1\le y_2$

but $x_2\le x_1$ or $y_2\le y_1$ Not always true

So, Not symmetric

Transitive : (3,5) R (4,4) and (4,4) R (2,4)
but (3,5) Not related to (2,4)
So, Not transitive.

Q5. Let y = y(x) be the solution of the differential equation
$(x^2 + 4)^2\,dy + (2x^3y + 8xy - 2)\,dx = 0$. If y(0) = 0, then y(2) is equal to

(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{32}$

(C) 2π (D) $\dfrac{\pi}{16}$

Ans. **B**

Sol. $\frac{dy}{dx}+\left(\frac{2x}{x^2+4}\right)y=\frac{2}{\left(x^2+4\right)^2}$

I.f = $e^{\int \frac{2x}{x^2+4}dx}=x^2+4$

So, $y\left(x^2+4\right)=\int \frac{2}{x^2+4}dx$

$y\left(x^2+4\right)=\tan^{-1}\frac{x}{2}+c$

$y\left(0\right)=0$

So, c = 0

So, $y\left(x^2+2\right)=\tan^{-1}\frac{x}{2}$

at x = 2 $y=\frac{\pi}{32}$

Q6. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line x + y =2 intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope-1. Then, a distance (in units) between the chord PQ and the chord MN is

(A) $3-\sqrt{2}$ B) $\sqrt{2}-1$

C) $2-\sqrt{3}$ D) $\sqrt{2}+1$

Ans. A

Sol. Distance of centre from chord $PQ=\left|\frac{0+0-2}{\sqrt{1^2+1^2}}\right|$

Let Distance of centre from chord MN = P

Then length of chord : $2=2\sqrt{10-P^2}$

$\Rightarrow P=3$

So, Distance between chord = $3-\sqrt{2}$

Q7. The value of $\frac{1\times 2^2+2\times 3^2+...+100\times(101)^2}{1^2\times 2+2^2\times 3+...+100^2\times 101}$ is

(A) $\frac{305}{301}$ (B) $\frac{306}{305}$

(C) $\frac{31}{30}$ (D) $\frac{32}{30}$

Ans. A

Sol. $S_n=\frac{\sum n\left(n+1\right)^2}{\sum n^2\left(n+1\right)}=\frac{\sum n^3+2\sum n^2+\sum n}{\sum n^3+\sum n^2}$

$$=\frac{\left(\frac{n\left(n+1\right)}{2}\right)^2+2\left(\frac{n\left(n+1\right)\left(2n+1\right)}{6}\right)+\frac{n\left(n+1\right)}{2}}{\left(\frac{n\left(n+1\right)}{2}\right)^2+\frac{n\left(n+1\right)\left(2n+1\right)}{6}}$$

$S_n=\frac{3n+5}{3n+1}$

So, $S_{100}=\frac{305}{301}$

Q8. Let $f(x)=\int_0^x\left(t+\sin\left(1-e^t\right)\right)dt,\ x\in\mathbb{R}$. Then, $\lim\limits_{x\to 0}\frac{f(x)}{x^3}$ is equal to

(A) $-\frac{1}{6}$ (B) $-\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

Ans. A

Sol. $\lim\limits_{x\to 0}\frac{f(x)}{x^3}\left\{\frac{0}{0}\text{from}\right\}$

So, $\lim\limits_{x\to 0}\frac{f'(x)}{3x^2}$

$=\lim\limits_{x\to 0}\frac{x+\sin\left(1-e^x\right)}{3x^2}$

$\frac{0}{0}$ from again

So, $\lim\limits_{x\to 0}\frac{1+\cos\left(1-e^x\right)\left(-e^x\right)}{6x}$

$\frac{0}{0}$ form again

$=\lim\limits_{x\to 0}\frac{0+\left(-e^x\right)\cos\left(1-e^x\right)+\left(-e^x\right)\left(-\left(\sin\left(1-e^x\right)\right)\right)\left(-e^x\right)}{6}$

$=-\frac{1}{6}$

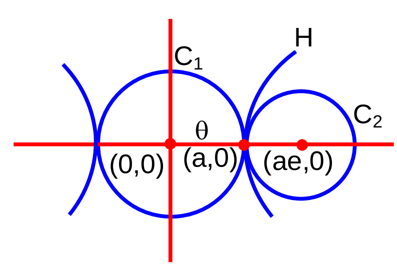
Q9. Consider a hyperbola H having centre at the origin and foci on the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq units) of C_1 and C_2 are 36π, respectively, then the length (in units) of latus rectum of H is

(A) $\frac{14}{3}$ (B) $\frac{28}{3}$

(C) $\frac{11}{3}$ (D) $\frac{10}{3}$

Ans. B

Sol. $c_1: x^2+y^2=a^2$



$c_2:\left(x-ae\right)^2+y^2=\left(ae-a\right)^2$

Given : $\pi a^2=36\pi$

$\Rightarrow a=6$

and $\pi\left(ae-a\right)^2=4\pi$

$e^2=\frac{16}{9}\Rightarrow\frac{1+b^2}{a^2}$

So, $b^2=28$

Hence, Length of L.R $=\frac{2b^2}{a}=\frac{28}{3}$

Q10. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2 + ... + (\text{adj } A)^{10}$

Then, the sum of all the elements of the matrix B is

(A) 22 (B) –110
(C) –88 (D) –124

Ans. **C**

Sol. $\text{adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$(\text{adj}(A))^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

$(\text{adj}(A))^r = \begin{bmatrix} 1 & -2r \\ 0 & 1 \end{bmatrix}$

$B = I + (\text{adj}(A)) + (\text{adj}(A))^2 + ...(\text{Adj}(A))^{10}$

$= \begin{bmatrix} 11 & \sum_{r=0}^{10}(-2r) \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix}$

Sum of element of B = –88

Q11. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$. Then, a value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Ans. **D**

Sol. By property

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

So, $I = \int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx = \int_{-1}^{1} \frac{\cos \alpha x}{1+3^{-x}} dx$

So, $I + I = \int_{-1}^{1} \frac{\cos \alpha x}{1+3x} dx + \int_{-1}^{1} \frac{\cos \alpha x}{1+3^{-x}} dx$

$2I = \int_{-1}^{1} (\cos \alpha x) dx$

$= \left. \frac{\sin \alpha x}{\alpha} \right|_{-1}^{1}$

So, $2\left(\frac{2}{\pi}\right) = \frac{\sin \alpha}{\alpha}$

So, $\alpha = \frac{\pi}{2}$

Q12. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to

(A) 38 (B) 42
(C) 24 (D) 44

Ans. **B**

Sol. $f'(x)=\frac{3}{2\sqrt{x-2}}-\frac{1}{2\sqrt{4-x}}$

$f'(0)=0\Rightarrow x=\frac{19}{5}\in[2,4]$

$f(2)=\sqrt{2}$

$f\left(\frac{19}{5}\right)=2\sqrt{5}$

$f(4)=3\sqrt{2}$

So, $\alpha=\sqrt{2},\beta=2\sqrt{5}$

So, $\alpha^2+2\beta^2=42$

Q13. If the mean of the following probability distribution of a radam variable X :

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(A) $\frac{566}{81}$
(B) $\frac{173}{27}$
(C) $\frac{581}{81}$
(D) $\frac{151}{27}$

Ans. **A**

Sol. We know, $\sum \Pr obability=1$ or $\sum P(x)=1$

$\Rightarrow 4a+6b=1$……(i)

$\bar{x}=\frac{46}{9}\Rightarrow \sum P_i X_i=\frac{46}{9}$

$\Rightarrow 4a+20b=\frac{23}{9}$…..(ii)

From (i) and (ii)

$a=\frac{1}{12}, b=\frac{1}{3}$

So variance $\sum P_i x_i^2-(\bar{x})^2=24a+280b-\left(\frac{46}{9}\right)^2$

$=\frac{566}{81}$

Q14. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is

(A) 7
(B) 28
(C) 14
(D) 21

Ans. **C**

Sol. $n_{C_4}, n_{C_5}.n_{c_6}$ are in A.P

$2\left(n_{C_5}\right)=n_{c_4}+n_{c_6}$

$\Rightarrow n^2-21n+98=0$

Maximum value of n = 14

Q15. The area (in sq. units) of the region $S=\{z\in\mathbb{C};|z-1|\le 2;(z-\bar{z})+i(z-\bar{z})\le 2, \operatorname{Im}(z)\ge 0\}$ is

(A) $\frac{17\pi}{8}$ (B) $\frac{7\pi}{3}$

(C) $\frac{3\pi}{2}$ (D) $\frac{7\pi}{4}$

Ans. C

Sol. $|z-1|\le 2$

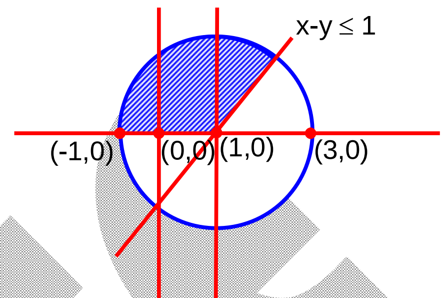
$\Rightarrow (x-1)^2+y^2=4$

$(z+\bar{z})+i(z+\bar{z})\le 2$

$\Rightarrow x-y\le 1$

$\operatorname{Im}|z|\ge 0 \Rightarrow y\ge 0$

Area of region $=\frac{1}{2}(2)^2\frac{(3\pi)}{4}=\frac{3\pi}{2}$



Q16. The area (in sq. units) of the region described by $\{(x, y): y^2 \le 2x, \text{ and } y \ge 4x-1\}$ is

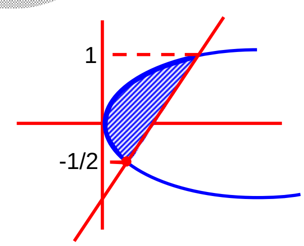
(A) $\frac{11}{32}$ (B) $\frac{11}{12}$

(C) $\frac{9}{32}$ (D) $\frac{8}{9}$

Ans. C

Sol. Area $=\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$

$=\left(\frac{y^2}{8}+\frac{y}{4}-\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}=\frac{9}{32}$



Q17. Let P be the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$. Then, the shortest distance of P from the line 4x = 2y = z is

(A) $\frac{6\sqrt{14}}{7}$ (B) $\frac{5\sqrt{14}}{7}$

(C) $\frac{3\sqrt{14}}{7}$ (D) $\frac{\sqrt{14}}{7}$

Ans. C

Sol. For point of intersection

$P(\lambda+2,5\lambda+4,\lambda+2)(2\mu+3,3\mu+2,2\mu+3)$

$\lambda+2=2\mu+3$

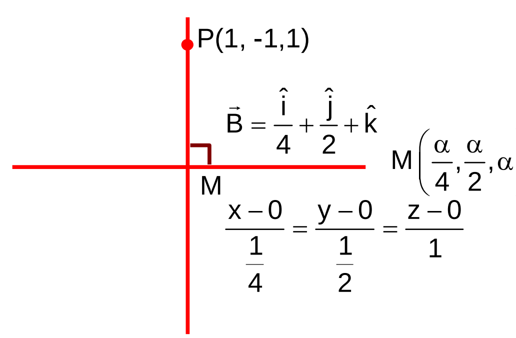
$5\lambda+4=3\mu+2$

$\lambda+2=2\mu+3$

$\lambda=-1,\ \mu=-1$

So, $P(1,-1,1)$

$\overrightarrow{PM}\cdot\vec{B}=0$



$$\frac{1}{4}\left(\frac{\alpha}{4}-1\right)+\frac{1}{2}\left(\frac{\alpha}{2}+1\right)+1(\alpha-1)=0$$

$$\Rightarrow \alpha=\frac{4}{7}$$

$$M=\left(\frac{1}{7},\frac{2}{7},\frac{4}{7}\right)$$

Shortest distance

$$PM=\sqrt{\left(\frac{1}{7}-1\right)^2+\left(\frac{2}{7}+1\right)^2+\left(\frac{4}{7}-1\right)^2}$$

$$=\frac{3\sqrt{14}}{7}$$

Q18. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [-1, 1] such that $\cos^{-1}x - \sin^{-1}y = \alpha$, $\frac{-\alpha}{2} \le \alpha \le \pi$. Then, the minimum value of $x^2 + y^2 + 2xy\sin\alpha$ is

(A) $\frac{1}{2}$
(B) –1
(C) $\frac{-1}{2}$
(D) 0

Ans. D

Sol. $\cos^{-1}x-\sin^{-1}y=\alpha$

$$\cos^{-1}x+\cos^{-1}y=\frac{\pi}{2}+\alpha$$

taking cos : $xy-\sqrt{1-x^2}\sqrt{1-y^2}=-\sin\alpha$

$$xy+\sin\alpha=\sqrt{1-x^2}\sqrt{1-y^2}$$

Now square both side

$$x^2y^2+2xy\sin\alpha+\sin^2\alpha=1-x^2-y^2+x^2y^2$$

$$x^2+y^2+2xy\sin\alpha=\cos^2\alpha$$

So, min value of $x^2+y^2+2xy\sin\alpha=0$

Q19. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4, 1). Then, which of the following point lies on the line passing through the points P and Q?

(A) (3, –3)
(B) (2, –9)
(C) $\left(\frac{1}{2},-20\right)$
(D) $\left(\frac{3}{2},-16\right)$

Ans. C

Sol. Chord whose mid point is given

T = s,

$$y(1)+(-12)\left(\frac{x+4}{2}\right)=1^2-12\times 4$$

$$y=6x-23$$

Point $\left(\frac{1}{2},-20\right)$ lies on chord

Q20. Let three real numbers a, b, c be in arithmetic progression and a + 1, b, c + 3 be in geometric progression, If a > 10 and the arithmetic mean of a, b, and c is 8, then the cube of the geometric mean of a, b and c is

(A) 316 (B) 120
(C) 128 (D) 312

Ans. **B**

Sol. Given $b^2 = (a+1)(c+3)$ ……(i)

$2b = a + c$ ………(ii)

$\frac{a+b+c}{3} = 8$ ……(iii)

Using (i), (ii) and (iii)

a = 15, b = 8, c = 1

So, $\left((abc)^{\frac{1}{3}}\right)^3 = abc = 120$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q21. Let $S = \{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots}. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha-2)^2 + (\beta-1)^2)$ equals_____.

Ans. **4**

Sol. $D \geq 0$

$\Rightarrow \sin^2 2\theta - 4(\sin^4\theta + \cos^4\theta)(\sin^6\theta + \cos^6\theta) \geq 0$

$\Rightarrow t - 4\left(1 - \frac{t}{2}\right)\left(1 - \frac{3t}{4}\right) \geq 0$ $\quad \{\text{let } \sin^2 2\theta = t\}$

$\Rightarrow 3t^2 - 12t + 8 \leq 0$

$\Rightarrow t \in \left[2 - \frac{2}{\sqrt{3}}, 2 + \frac{2}{\sqrt{3}}\right]$……(i)

But $t = \sin^2 2\theta \in [0,1]$……..(ii)

(i) $\cap$ (ii)

$t \in \left[2 - \frac{2}{\sqrt{3}}, 1\right]$

So, $\alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1$

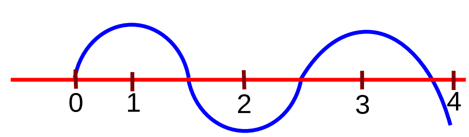
So, $3\left((\alpha-2)^2 + (\beta-1)^2\right) = 4$

Q22. Let $f : R \to R$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = –1, f(3) = 2 and f(4) = –2. Then, the minimum number of zeros of (3f'f" + f'f" + ff''') (x) is_____.

Ans. **5**

Sol. Minimum roots of $f(x) = 0$ are 4

Minimum roots of $f'(x) = 0$ are 3



Minimum roots of $f(x)\,f'(x) = 0$ are 7
Minimum roots of $(f(x)\cdot f'(x))' = 0$ are 6
Minimum roots of $(f(x)\cdot f'(x))'' = 0$ are 5
$(f(x)\cdot f'(x))'' = \left(f(f'(x))^2 + f(x)\cdot f''(x)\right)'$
$= 3f'(x)\cdot f''(x) + f'(x).f''(x) + f(x)'''f(x)$
$= 3f'(x)\cdot f''(x) + f'(x)\cdot f'''(x)$
So, Minimum number of zeros of
$(3f'f'' + ff''')(x)$ is 5

Q23. Let A be a 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2, then $\alpha + \beta$ equals _____.

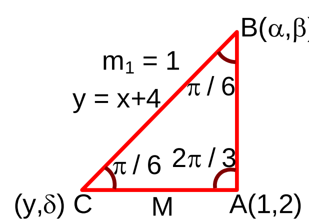
Ans. 5

Sol. $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix} \Rightarrow \begin{bmatrix}a & c\\c & b\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$
$a + c = 3,\ b + c = 7$ ....(i)
Also, $|A| = 1 \Rightarrow ab - c^2 = 1$......(i)
From (i) and (ii)
$a = 1,\ b = 5,\ c = 2$
$A = \begin{bmatrix}1 & 2\\2 & 5\end{bmatrix}$
Also, $\alpha A^2 + \beta A - I = 0$
So, $-\dfrac{\beta}{\alpha} = 6$ and $-\dfrac{1}{\alpha} = 1$
$\Rightarrow \alpha = -1,\ \beta = 6$
So, $\alpha + \beta = 5$

Q24. Consider a triangle ABC having the vertices A(1, 2). B (α, β) and c(γ, δ) and angles $\angle ABC = \dfrac{\pi}{6}$ and $\angle BAC = \dfrac{2\pi}{3}$. If the points B and C lie on the lie y = x + 4, then $\alpha^2 + \gamma^2$ is equal to_____.

Ans. 14

Sol. Equation of line passing through P(1,2) and making angle $\pi/4$ with y = x + 4 – (i)
y – 2 = m (x – 1)
For m



$\tan\dfrac{\pi}{6} = \left|\dfrac{m - m_1}{1 + mm_1}\right|$
$\pm\dfrac{1}{\sqrt{3}} = \dfrac{m-1}{1+m}$
$y - 2 = (2 \pm \sqrt{3})(x - 1)$ .....(ii)
Solve (i) and (ii)

$x+4-2=\left(2\pm\sqrt{3}\right)(x-1)$

$x=\dfrac{4+\sqrt{3}}{1+\sqrt{3}}=\alpha$

and $x=\dfrac{4-\sqrt{3}}{1-\sqrt{3}}=y$ So, $\alpha^2+\gamma^2=14$

Q25. Consider a lint L passing through the points P(1, 2, 1) and Q (2, 1, –1). If the mirror image of the point A(2, 2, 2) in the line L is (α, β, γ), then $\alpha+\beta+6\gamma$ is equal to_____.

Ans. **6**

Sol. $M(\lambda+1,-\lambda+2,-2\lambda+1)$

$\overrightarrow{PM}\cdot\vec{B}=0$

$1(\lambda-1)-1(-\lambda)-2(-2\lambda-1)=0$

$\lambda=-\dfrac{1}{6}$

$M\left(\dfrac{5}{6},\dfrac{13}{6},\dfrac{4}{3}\right)\equiv\left(\dfrac{\alpha+2}{2},\dfrac{\beta+2}{2},\dfrac{\gamma+2}{2}\right)$

$\alpha=-\dfrac{1}{3},\gamma=\dfrac{2}{3}$

$\beta=\dfrac{7}{3}$

$\alpha+\beta+6\gamma=6$

Q26. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is_____.

Ans. **5626**

Sol. $A(4M)B(4W)\rightarrow 4c_4 4_{C_4}=1$

$A(3M,W)B(1M,3W)\rightarrow\left(4_{C_3}\cdot5_{C_1}\right)\cdot\left(5_{C_1}\cdot4_{C_3}\right)=400$

$A(2M,2W)B(2M,2W)\rightarrow\left(4_{C_2}.5_{C_2}\right)\cdot\left(5_{C_2}\cdot4_{C_2}\right)=3600$

$A(1M,3W)B(3M,1W)\rightarrow\left(4_{C_1},5_{C_3}\right)\cdot\left(5_{C_3}\cdot4_{C_1}\right)=1600$

$A(4W)B(4M)\rightarrow 5_{C_4}5_{C_4}=25$

So, Total number of ways = 5626

Q27. If $\int \operatorname{cosec}^5 x\,dx=\alpha\cot x\operatorname{cosec}x\left(\operatorname{cosec}^2x+\dfrac{3}{2}\right)+\beta\log_e\left|\tan\dfrac{x}{2}\right|+C$ where $\alpha,\beta\in R$ and C is the constant of integration, then the value of $8(\alpha+\beta)$ equals_____.

Ans. **1**

Sol. $In=\int \operatorname{cosec}^n x\,dx$

Reduction formula $I_n=-\dfrac{\operatorname{cosec}^{n-2}x\cot x}{n-1}+\dfrac{n-2}{n-1}I_n-2$

$I_5=\dfrac{\operatorname{cosec}^3x\cot x}{4}+\dfrac{3}{4}\left(\dfrac{-\operatorname{cosec}x\cot x}{2}+\dfrac{1}{2}\int\operatorname{cosec}x\right)+c$

$I_5=-\dfrac{1}{4}\operatorname{cosec}x\cot x\left(\operatorname{cosec}^2x+\dfrac{3}{2}\right)+\dfrac{3}{8}\ell n\tan\left|\dfrac{x}{2}\right|+c$

$\alpha = -\frac{1}{4}, \beta = \frac{3}{8}$

and $8(\alpha + \beta) = 1$

So, 1

Q28. Consider the function $f: R \to R$ defined by $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f.\underbrace{(f \circ f \circ f \circ \ldots \circ f)}_{10\,\text{times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to_____.

Ans. **1024**

Sol.

$$f(f(x)) = \frac{2f(x)}{\sqrt{1+9f(x)^2}} = \frac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$$

$$f(f(f(x))) = \frac{\frac{2^3x}{\sqrt{1+9x^2}}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}} = \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$$

So, By observation

$$\alpha = 1 + 2^2 + 2^4 + \ldots\ldots\ldots + 2^{18}$$

$$= 1\left(\frac{(2^2)^{10}-1}{2^2-1}\right) = \frac{2^{20}-1}{3}$$

$3\alpha + 1 = 2^{20}$ So, $\sqrt{3\alpha+1} = 2^{10} = 1024$

Q29. Let y = y(x) be the solution of the differential equation $(x + y + 2)^2\,dx = dy$, y(0) = –2. Let the maximum and minimum values of the function y = y(x) in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha+\pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}, \gamma, \delta \in Z$, then $\gamma + \delta$ equals _____.

Ans. **31**

Sol.

$$\frac{dy}{dx} = (x+y+2)^2 \left\{\text{Let } x+y+2 = 6 \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}\right\}$$

$$\frac{dt}{dx} - 1 = t^2 \Rightarrow \int \frac{dt}{1+t^2} = \int dx$$

$$\tan^{-1} t = x + c \Rightarrow x + y + 2 = \tan(x+c)$$

$\because y(0) = -2$

So, c = 0

$$y = \tan x - x - 2, x \in \left[0, \frac{\pi}{3}\right]$$

$$\beta = \tan\theta - 0 - 2 = -2$$

$$\alpha = \sqrt{3} - \frac{\pi}{3} - 2 \Rightarrow 3\alpha + \pi = 3\sqrt{3} - 6$$

$$(3\alpha+\pi)^2 + \beta^2 = 67 - 36\sqrt{3}$$

So, y = 67

δ = -36 and y + δ = 31

Q30. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that them loses. If the probability P(|x – y| $\leq$ 2) is p, then 3^9p equals_____.

Ans. **8288**

Sol. $|x-y| \leq 2$ and $0 \leq x \leq 10,\ 0 \leq y \leq 10$

$$P = P(x=4,\ y=6) + P(x=5,\ y=5) + P(x=6,\ y=4)$$

$$P = \left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6 \cdot \frac{10!}{4!\,6!} + \left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5 \frac{10!}{5!\,5!} + \left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 \frac{10!}{6!4!}$$

$$P = \frac{2^5}{3^{10}}\left(420 + 3(84) + 105\right)$$

So, $3^9 P = 8288$

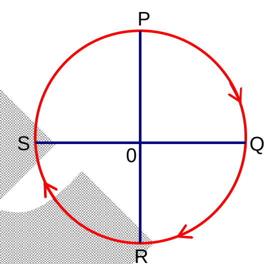
PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q31. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of a cyclist is:



(A) 6 km (B) $\sqrt{8}$ km
(C) 8 km (D) 4 km

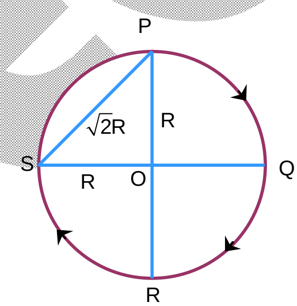
Ans. B

Sol. The displacement of cyclist is $|\overrightarrow{PS}| = \sqrt{R^2 + R^2}$

$= R\sqrt{2}$

$= 2\sqrt{2}$ km

$= \sqrt{8}$ km



Q32. The magnetic moment of a bar magnet is $0.5\ \text{Am}^2$. It is suspended in a uniform magnetic field of 8×10^{-2} T. The work done in rotating it from its most stable to most unstable position is:

(A) 4×10^{-2} J (B) 8×10^{-2} J
(C) Zero (D) 16×10^{-2} J

Ans. B

Sol. $U = -MB\cos\theta$

At most stable position $[\cos\theta = 1]$

$U_i = -MB$

At most unstable position $[\cos\theta = -1]$

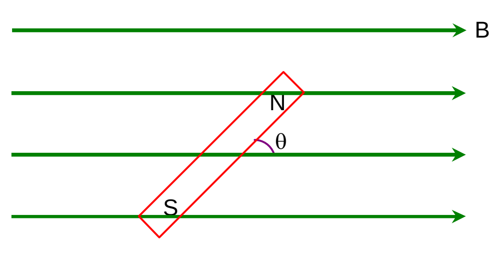
$U_f = MB$

$W_{ext} = \Delta U = U_f - U_i$

$W_{ext} = 2MB$

$= 2 \times 0.5 \times 8 \times 10^{-2}]$

$= 8 \times 10^{-2}]$



Q33. The width of one of the two slits in a Young's double slit experiment is 4 times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is:

(A) 9:1 (B) 16:1
(C) 1:1 (D) 4:1

Ans. A

Sol. (Intensity) $\propto$ (width of slit)

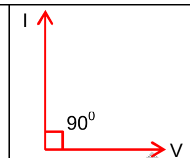
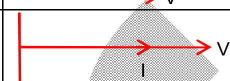
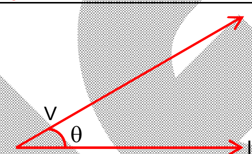
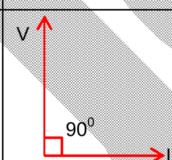
So let $I_1 = 4I$

$I_2 = I$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{4I}+\sqrt{I}}{\sqrt{4I}-\sqrt{I}}\right) = \left(\frac{3}{1}\right)^2$$

$9:1$

Q34. Match List I with List II

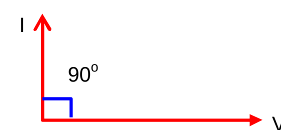
LIST I		LIST II	
a.	Purely capacitive circuit	I.	
b.	Purely inductive circuit	II.	
c.	LCR series at resonance	III.	
d.	LCR series circuit	IV.	

Choose the correct answer from the options given below:

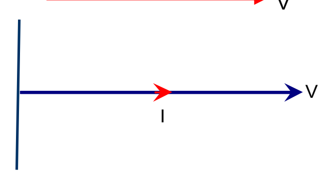
(A) a - I, b - IV, c - II, d - III
(B) a - I, b - IV, c - III, d - II
(C) a - IV, b - I, c - III, d - II
(D) a - IV, b - I, c - II, d - III

Ans. A

Sol. For purely capacitive circuit voltage lags by 90^o from current

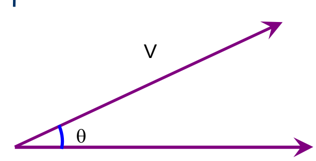


LCR series at resonance voltage and current are in phase.

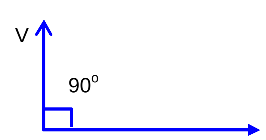


LCR series circuit there is phase difference in voltage and current

$$\cos\phi = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}}$$



For purely inductive circuit voltage leads the current by 90^o



Q35. An electric bulb rated 50 W – 200 V is connected across a 100 V supply. The power dissipation of the bulb is:

(A) 50 W
(B) 25 W
(C) 12.5 W
(D) 100 W

Ans. C

Sol. Electric bulb rated as '50W-200V'

So, resistance of bulb is

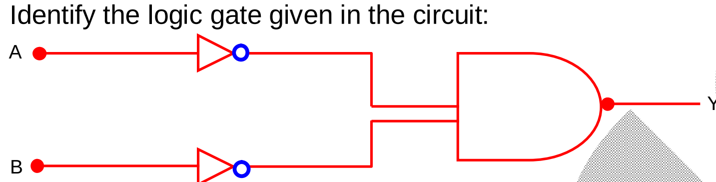
$$R_{bulb} = \frac{(200)^2}{50} = 800\Omega$$

The power dissipation of the bulb across 100 V power supply is

$$P = \frac{V^2}{R} = \frac{(100)^2}{800}$$

$$P = 12.5W$$

Q36. Identify the logic gate given in the circuit:



(A) NAND-gate
(B) AND gate
(C) NOR gate
(D) OR- gate

Ans. D

Sol.

$$Y = \overline{\overline{A} \bullet \overline{B}}$$

$$Y = \overline{\overline{A+B}} = A+B$$

Q37. A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is $\gamma = 3/2$. The work done by the gas in the process is:

$(\mu = 1\text{mole})$

(A) $RT\left[2\sqrt{2}-1\right]$
(B) $RT\left[\sqrt{2}-2\right]$
(C) $RT\left[1-2\sqrt{2}\right]$
(D) $RT\left[2-\sqrt{2}\right]$

Ans. D

Sol. For adiabatic process

$$W = \frac{nR\Delta T}{l-\gamma}$$

$$TV^{\gamma-1} = \text{const}$$

$$T_iV_i^{\gamma-1} = T_fT_f^{\gamma-1}$$

$$T.V^{\frac{3}{2}-1} = T_f(2v)^{\frac{3}{2}-1}$$

$$T_f = \frac{T}{\sqrt{2}}$$

$$W = \frac{nR(T_f - T)}{\gamma - 1}$$

$$\left(n = 1, \gamma = \frac{3}{2}\right)$$

$$W \frac{R\left(\frac{T}{\sqrt{2}}-1\right)}{\frac{1}{2}-1}$$

$$W = RT\left(2-\sqrt{2}\right)$$

Q38. A charge q is placed at the center of one of the surface of a cube. The flux linked with the cube is:

(A) $\frac{q}{2 \in_0}$ (B) Zero

(C) $\frac{q}{4 \in_0}$ (D) $\frac{q}{8 \in_0}$

Ans. A

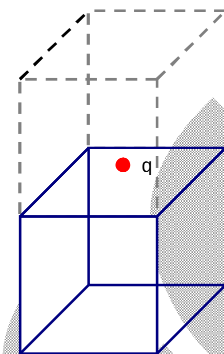
Sol. Let flux through one cube is ϕ

According to gauss' law

$$\phi_{Total} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$2\phi = \frac{q}{\varepsilon_0}$$

$$\phi = \frac{q}{2\varepsilon_0}$$



Q39. Arrange the following in the ascending order of wavelength:

(a) Gamma rays (λ_1)

(b) x – rays (λ_2)

(c) Infrared waves (λ_3)

(d) Microwaves (λ_4)

Choose the **most appropriate** answer from the options given below

(A) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ (B) $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$

(C) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$ (D) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$

Ans. A

Sol. Wavelength is inversely proportion to energy or frequency of ray

$\lambda_{microwave} > \lambda_{\text{infrared wave}} > \lambda_{x\text{-ray}} > \lambda_{\text{gamma rays}}$

$\lambda_4 > \lambda_3 > \lambda_2 > \lambda_1$

Q40. Correct formula for height of a satellite from earths surface is:

(A) $\left(\frac{T^2R^2g}{4\pi}\right)^{1/2} - R$ (B) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$

(C) $\left(\frac{T^2R^2}{4\pi^2 g}\right)^{1/3} - R$ (D) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{-1/3} + R$

Ans. B

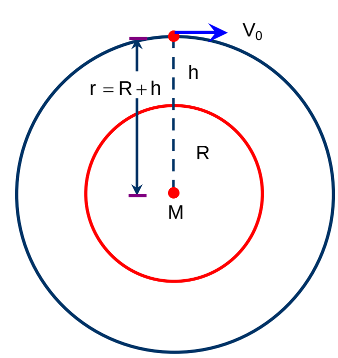
Sol.

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM.R^2}{R^2.r}}$$

$$V_0 = R\sqrt{\frac{g}{r}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$



$$T^2 = \frac{4\pi^2}{R^2 g} r^3,$$

$$r = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3}$$

$$R + h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3},$$

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

Q41. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are:
(A) $f_t = 2$ and $f_r = 2$
(B) $f_t = 3$ and $f_r = 2$
(C) $f_t = 3$ and $f_r = 3$
(D) $f_t = 2$ and $f_r = 3$

Ans. C

Sol. $CH_4 \rightarrow$ non-linear polyatomic molecule
$\rightarrow$Translation degree of freedom
$f_t = 3$
$\rightarrow$Rotational degree of freedom
$f_R = 3$

Q42. A 90 kg body placed at 2R distance from surface of earth experience gravitational pull of:
(R = Radius of earth, g = 10 ms^{-2})
(A) 300 N
(B) 120 N
(C) 225 N
(D) 100 N

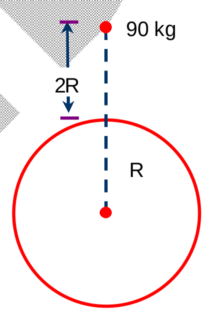
Ans. D

Sol. $F = mg'$

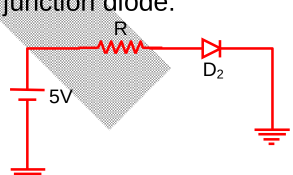
$$g' = g\left(\frac{R}{R+n}\right)^2$$

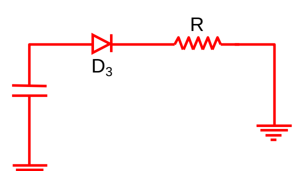
$$g' = 10\left(\frac{R}{R+2R}\right)^2 = \frac{10}{9} m/s^2$$

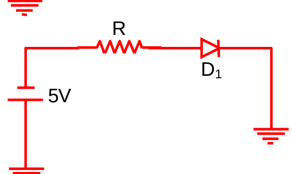
$$F = mg' = 90 \times \frac{10}{9} = 100N$$

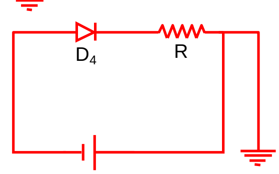


Q43. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of p-n junction diode:

(A)


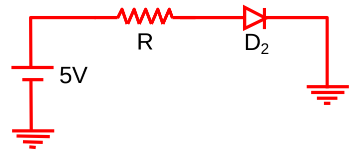
(B)


(C)


(D)


Ans. A

Sol. For the measurement of dynamic resistance of P-n junction diode should be in forward blased.



Q44. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45^0 with respect to horizontal axis. The co-efficient of static friction between their surfaces is:

(A) 1.7 (B) 0.5

(C) $\frac{1}{\sqrt{3}}$ (D) 1

Ans. D

Sol. Angle of repose θ

$\tan\theta = \mu$

$\tan 45^o = \mu$

$\Rightarrow \mu = 1$

OR

$Mg\sin 45^o = f$ ……………..(I)

$Mg\cos 45^o = N$ ……………..(II)

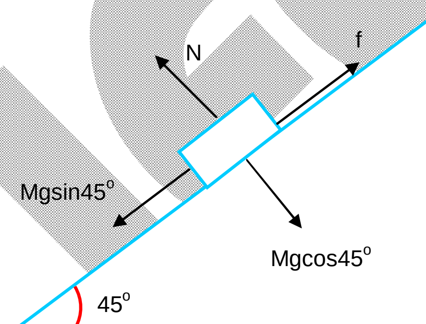
$\tan\theta = \frac{f}{N}$

The brick is begins to slide

$(f = \mu N)$

$\tan\theta = \frac{\mu N}{N} = \mu$

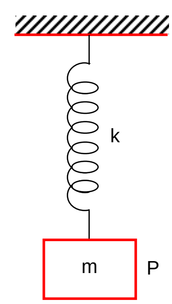
$\mu = \tan 45^o = 1$



Q45. In simple harmonic motion, the total mechanical energy of given system is E. If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is:

(A) E (B) 2E

(C) $E\sqrt{2}$ (D) $\frac{E}{\sqrt{2}}$



Ans. A

Sol. Total energy

$E = \frac{1}{2}KA^2$

For same amplitude energy will be same.

Q46. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both **Statement I** and **Statement II** are true.

(B) **Statement I** is true but **Statement II** is false.

(C) **Statement I** is false but **Statement II** is true.

(D) Both **Statement I** and **Statement II** are false.

Ans. B

Sol. The contact angle between a solid and liquid is a property of the material of the solid and liquid as well because it depends on both cohesive and adhesive force.

$h = \dfrac{2T\cos\theta}{\rho r g}$ (for sufficient length of capillary)

h depends on r.

Q47. Applying the principle of homogeneity of dimensions, determine which one is correct, where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

(A) $T^2 = 4\pi^2 r^3$

(B) $T^2 = \dfrac{4\pi^2 r}{GM^2}$

(C) $T^2 = \dfrac{4\pi^2 r^2}{GM}$

(D) $T^2 = \dfrac{4\pi^2 r^3}{GM}$

Ans. D

Sol. Time period = [T]

Gravitational const [G] = $[M^{-1}L^3T^{-2}]$

Mass = [M]

Radius of orbit = [L]

According to principle of homogentiy of dimension

Dimension of L.H.S = Dimension of RHS

By checking options

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\left[T^2\right] = \frac{\left[L^3\right]}{\left[M^{-1}L^3T^{-2}\right][M]} = \left[T^2\right]$$

Q48. According to Bohr's theory, the moment of momentum of an electron revolving in 4^{th} orbit of hydrogen atom is:

(A) $\dfrac{h}{2\pi}$

(B) $8\dfrac{h}{\pi}$

(C) $2\dfrac{h}{\pi}$

(D) $\dfrac{h}{\pi}$

Ans. C

Sol. Moment of memetum i.e angular momentum of an electron revolving in n^{th} orbit is

$$L = mvr = \frac{nh}{2\pi}$$

For $n = 4$

$$L = \frac{4h}{2\pi} = \frac{2h}{\pi}$$

Q49. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R**.

Assertion A: Number of photons increases with increase in frequency of light.

Reason R: Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

(A) **A** is correct but **R** is not correct.

(B) Both **A** and **R** are correct and **R** is **NOT** the correct explanation of **A**.

(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(D) **A** is not correct but **R** is correct.

Ans. D

Sol. There is no direct relation between number of photon and frequency of light. But if we consider intensity of light as constant then

$$I = \frac{nh\nu}{(\text{Area})}$$

$$n = \frac{I(\text{Area})}{h\nu}$$

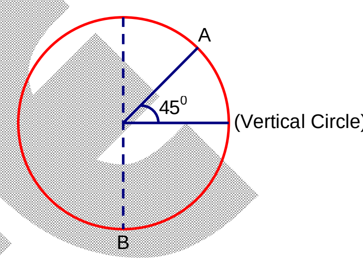
So, statement 1 is wrong

Statement 2 is correct because

$K_{max} = h\nu - \phi$

i.e KE_{max} will increase with increase in frequency

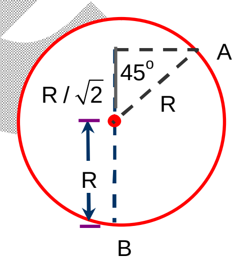
Q50. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is: (given, $R = 14\ m, g = 10\ m/s^2$ and $\sqrt{2} = 1.4$)



(A) 10.6 m/s
(B) 21.9 m/s
(C) 16.7 m/s
(D) 19.8 m/s

Ans. B

Sol. Applying work energy theorem from point A to B



$W_{gravity} = \Delta KE$

$$mg\left(R + \frac{R}{\sqrt{2}}\right) = \frac{1}{2}mv_B^2 - 0$$

$$V_B = \sqrt{2gR\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$$

$$V_B = \sqrt{2 \times 10 \times 14\left(\frac{1.4+1}{1.4}\right)}$$

$$V_B = \sqrt{480}\ m/s$$

$$V_B \approx 21.9\ m/s$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q51. The displacement of a particle executing SHM is given by $x = 10\sin\left(wt + \frac{\pi}{3}\right)m$. The time period of motion is 3.14 s. The velocity of the particle at t = 0 is………

Ans. 10

Sol. Given $T = \frac{2\pi}{\omega} = 3.14s$

$\omega = 2\ rad/s$

$$x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$v = \frac{dx}{dt} = 10\omega\cos\left(\omega t + \frac{\pi}{3}\right) m/s$$

At $t = 0$

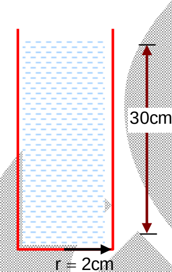
$$v = 10 \times 2\cos\left(\frac{\pi}{3}\right)$$

$$v = 10\, m/s$$

Q52. Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is………. N.

Ans. **177**

Sol. Force on the bottom of tube



F = P (Area)

$$F = \left(P_{atm} + \rho_{Hg}.g.h\right)\pi r^2$$

$$F = \left(10^5 + 1.36 \times 10^4 \times 10 \times 0.30\right) \times \frac{22}{7}(0.02)^2$$

$$F = 177\, N$$

Q53. The disintegration energy Q for the nuclear fission of $^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n$ is…….MeV. Given atomic masses of $^{235}U : 235.0439u$; $^{140}Ce : 139.9054\, u$. $^{94}Zr : 93.9063u$; $n : 1.0086\, u$, Value of $c^2 = 931\, MeV/u$

Ans. **208**

Sol. $^{235}U =^{140} Ce +^{94} Zr + n$

$$\Delta m = m_v - \left(m_{ce} + m_{zr} + m_n\right)$$

$$\Delta m = 235.0439u - (139.905u + 93.9063u + 1.0086u)$$

$$\Delta m = 0.2236u$$

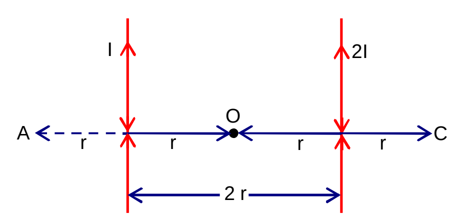
Disintegration energy

$$Q = \Delta m c^2 \quad \left(c^2 = 931\frac{mev}{u}\right)$$

$$Q = 0.2236u \times 931\frac{Mev}{u}$$

$$Q = 208.17\, MeV$$

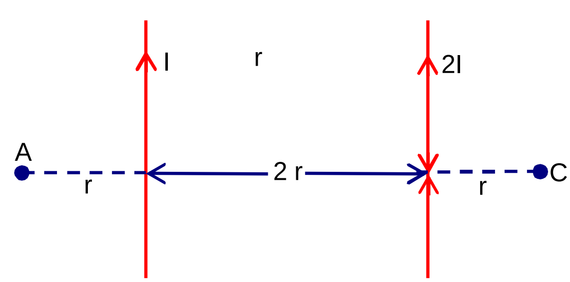
Q54. Two parallel long current carrying wire separated by a distance 2r are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{7}$. The value of x is…….



Ans. **5**

Sol.

$$B_A = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0(2I)}{2\pi(3r)}$$



$$B_A = \frac{5\mu_0 I}{6\pi r}$$

$$B_C = \frac{\mu_0 I}{2\pi(3r)} + \frac{\mu_0 2I}{2\pi r}$$

$$B_C = \frac{7\mu_0 I}{6\pi r}$$

$\frac{B_A}{B_C} = \frac{5}{7} = \frac{x}{7}$

i.e $x = 5$

Q55. In a system two particles of masses m_1 = 3 kg and m_2 = 2 kg are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2 cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance……….cm.

Ans. **3**

Sol.

Let m_2 will move towards center of mass by distance 'ℓ' cm

According to question $\Delta X_{com} = 0$

$$\Delta\vec{X}_{com} = \frac{M_1\Delta\vec{X}_1 + M_2\Delta\vec{X}_2}{M_1 + M_2}$$

$$0 = \frac{3\times 2 + 2(-\ell)}{3+2}$$

$\ell = 3\text{cm}$

Q56. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4s after applying the brakes. The distance traveled by the bus during this time (Assume the retardation is uniform) is………m.

Ans. **40**

Sol. $u = 72\text{km/h} = 20\text{m/s}$

$$s = \left(\frac{u+v}{2}\right)t$$

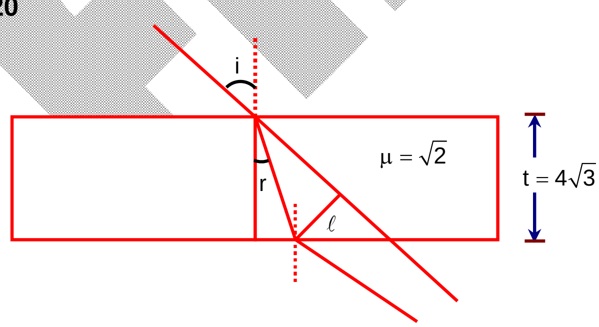
$$s = \left(\frac{20+0}{2}\right)\times 4$$

$s = 40\text{m}$

Q57. A light ray is incident on a glass slab of thickness $4\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is…….. cm.

Ans. **20**

Sol.



Lateral displacement

$$\ell = \frac{t\sin(i-r)}{\cos r} \text{ ....(I)}$$

Given angle of incidence is equal to critical angle for the glass-air interface.

$$i = \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$i = 45^o$
Also using Snell's law
$\mu_1 \sin i = \mu_2 \sin r$
$1 \sin 45^o = \sqrt{2} \sin r$
$\frac{1}{\sqrt{2}} = \sqrt{2} \sin r \Rightarrow \sin r = \frac{1}{2}$
$r = 30^0$
From equation (I)
$$\ell = \frac{t \sin(i-r)}{\cos r} = \frac{\left(4\sqrt{3}\,\text{cm}\right)\sin\left(45^0 - 30^o\right)}{\cos\left(30^0\right)}$$
$$\ell = \frac{4\sqrt{3}\sin 15^0}{\cos 30^o} = \frac{4\sqrt{3}\times 0.25}{\left(\frac{\sqrt{3}}{2}\right)}\text{cm}$$
$\ell = 2\,\text{cm}$

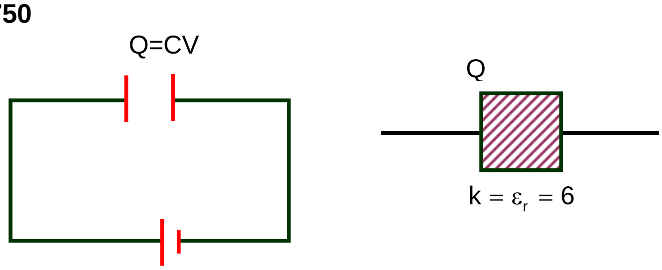
Q58. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of 2.0 mm and wire B has radius of 4.0 mm. The resistance of wire B is 2Ω. The resistance of wire A is………… Ω.
$R_A = 32\Omega$

Ans. **32**

Sol. $R = \rho\frac{\ell}{A}$
$$R = \frac{\rho(\ell A)}{A^2} = \frac{\rho v}{A^2} = \frac{\rho(m/d)}{A^2}$$
For same $\rho, m \,\&\, d$
$$R \propto \frac{1}{A^2}$$
$$\frac{R_A}{R_B} = \left(\frac{A_B}{A_A}\right)^2 = \left(\frac{r_B}{r_A}\right)^4$$
Where r_A & r_B are radius of wire A and wire B respectively.
$$\frac{R_A}{2} = \left(\frac{4}{2}\right)^4 = 16$$
$R_A = 32\Omega$

Q59. A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to potential difference of 12.0 V. The battery is now disconnected and a dielectric slab $(\in_r = 6)$ is inserted between the plates. The change in its potential energy after inserting the dielectric slab is……….. $\times 10^{-12}$ J.

Ans. **750**

Sol.


$Q = CV = 12.5pF \times 12V = 150\,pC$

$U_i = \dfrac{Q^2}{2C}$ & $U_f = \dfrac{Q^2}{2(KC)}$

$\Delta U = U_f - U_i = \dfrac{Q^2}{2C}\left(\dfrac{1}{k} - 1\right)$

$\Delta U = \dfrac{(150)^2}{2 \times 12.5}\left(\dfrac{1}{6} - 1\right) pJ$

$\Delta U = -750pJ = -750 \times 10^{-12}\,J$

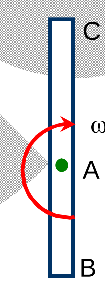
Difference in energy $|\Delta U|$

$= 750 \times 10^{-12}\,J$

Q60. A rod of length 60 cm rotates with a uniform angular velocity $20\,rads^{-1}$ about its perpendicular bisector in a uniform magnetic filed 0.5T. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is……………V.

Ans. **0**

Sol. $V_C - V_A = \dfrac{B\omega\ell^2}{2}$ …………..(I)



$V_B - V_A = \dfrac{B\omega\ell^2}{2}$ ……………..(II)

I-II

$V_C - V_B = 0$

i.e potential difference between ends of the rod is zero.

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q61. The correct order of the first ionization enthalpy is

(A) B > Al > Ga (B) Ga > Al> B
(C) Al > Ga > Tl (D) Tl > Ga > Al

Ans. D

Sol. Due to lanlhanide contraction ionization energy of $T\ell$ is more than Ga & $A\ell$
Incase of Ga there are 10 d electrons in penultimate shell which screen nuclear charge less effectively and thus outer electron held firmly by the nucleus ionisation energy of Ga is slightly greater than $A\ell$.

Q62. Given below are two statements:
Statement I : The correct order of first ionization enthalpy values of Li,Na, F and Cl is Na< Li < Cl <F.
Statement II: The correct order of negative electron gain enthalpy vaues of Li, Na, F and Cl is Na < Li < F < Cl
In the light of the above statements, choose the **correct** answer from the options given below:
(A) **Statement I** is false but **Statement II** is true.
(B) **Statement I** is true but **Statement II** is false.
(C) Both **Statement I** and **Statement II** are true.
(D) Both **Statement I** and **Statement II** are false.

Ans. C

Sol. Ionisation energy increases from left to right in period and decrease down the group so for ionization energy Na< Li < Cl < F in general the electron gain enthalpy become less negative from top to bottom in a group. In case of F & Cl electron gain enthalpy of Cl is more negative than F because in case of F we have 7 electrons in smaller second shell as result there is strong enter electronic repulsion when extra electron is added. So for electron gain enthalpy Na < Li < F < Cl

Q63. A first row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86BM. The atomic number of the metal is.

(A) 22 (B) 25
(C) 23 (D) 26

Ans. C

Sol.
$Ti^{+2}(22) \longrightarrow [Ar]3d^2$

$V^{+2}(23) \longrightarrow [Ar]3d^3$

$Mn^{+2}(25) \longrightarrow [Ar]3d^5$

$Fe^{+2}(26) \longrightarrow [Ar]3d^6$

$\mu = \sqrt{n(n+2)}\,BM$

$= \sqrt{3 \times 5}\,BM$

$= 3.86\,BM$

Q64. The adsorbent used in adsorption chromatography is / are
a. Silica gel
b. Alumina
c. Quick lime
d. Magnesia
Choose the **most appropriate** answer from the options given below:
(A) b only
(B) a only
(C) c and d only
(D) a and b only

Ans. D

Sol. Most common acidic adsorbent is silica gel and basic adsorbent is alumina.

Q65. If an iron (III) complex with the formula $[Fe(NH_3)_x(CN)_y]^-$ has no electron in its eg orbital, then the value of x +y is
(A) 3
(B) 6
(C) 4
(D) 5

Ans. B

Sol. For balancing charge

$3 - y = -1$

$y = 4$

$x = 2$

$x + y = 6$

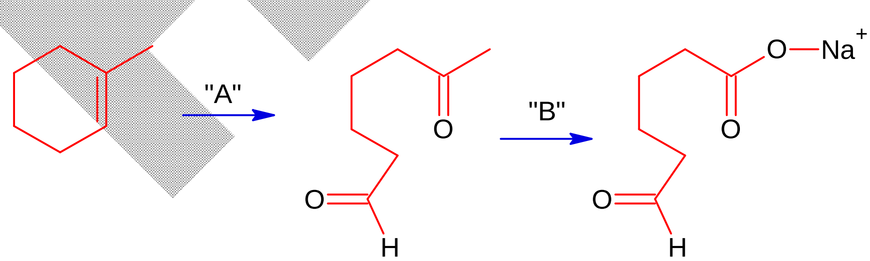
$Fe^{+3} = 3d^5 \longrightarrow t_2g^5 \quad eg^0$

Q66. The correct statement/s about Hydrogen bonding is/ are
A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom.
B. Intermolecular H bonding is present in O- nitro phenol
C. Intermolecular H bonding is present in HF.
D. The magnitude of H bonding depends on the physical state of the compound.
E. H- bonding has powerful effect on the structure and properties of compounds
Choose the **correct** answer from the options given below:
(A) A only
(B) A,B,D only
(C) A,B,C only
(D) A,D,E only

Ans. D

Sol. Hydrogen bonding exist when hydrogen is covalently bonded to the highly electronegative atom F, O and N intra molecular H bonding is present in O- nitro phenol.
Inter molecular H bonding is present in HF. Magnitude of hydrogen bonding depend upon the physical state of compound and it has powerful effect on the structure and properties of compound.

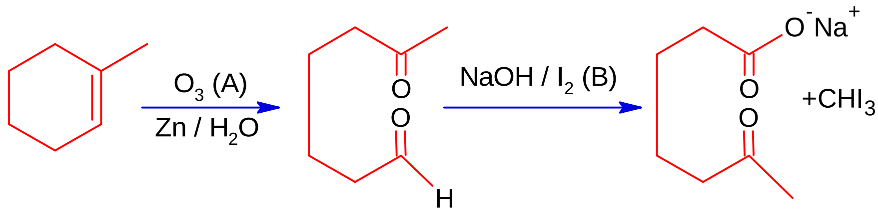
67.



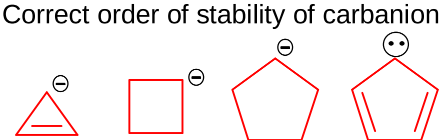
In the above chemical reaction sequence "A" and "B" respectively are
(A) H_2O, H^+ and $NaOH_{(alc)} / I_2$
(B) $O_3, Zn / H_2O$ and $NaOH_{(alc)} / I_2$
(C) $O_3, Zn / H_2O$ and $KMnO_4$
(D) H_2O, H^+ and $KMnO_4$

Ans. B

Sol.



Q68. Correct order of stability of carbanion is

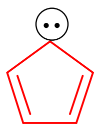


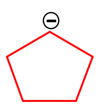
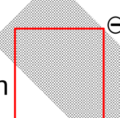
a b c d

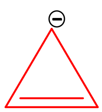
(A) a > b > c > d
(B) d > a > c > b
(C) c > b > d > a
(D) d > c > b > a

Ans. D

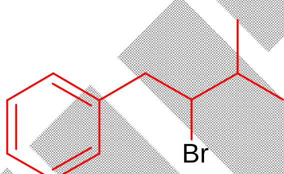
Sol.


is Aromatic


has lesser angle strain than


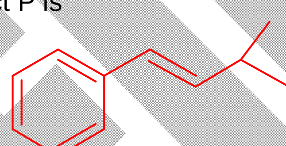

is anti aromatic

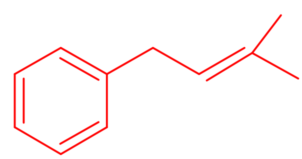
69.

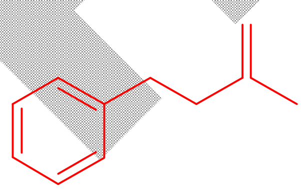


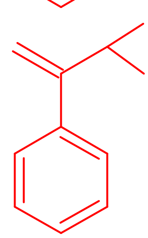
$\xrightarrow[\Delta]{\text{KOH (alc)}}$ major product "P"

Product P is

(A)


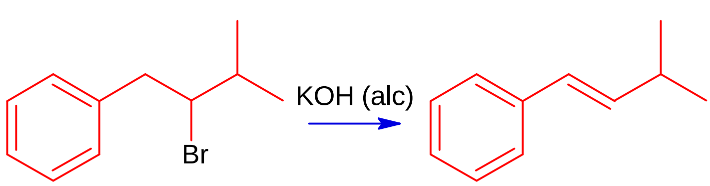
(B)


(C)


(D)


Ans. A

Sol.



More stable compound is in resonance (conjugation) with ring.

Q70. The number of species from the following that have pyramidal geometry around the central atom is__________

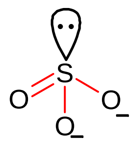
$S_2O_3^{2-}, SO_4^{2-}, S_2O_7^{2-}, SO_3^{2-}$

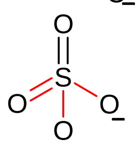
(A) 4 (B) 3

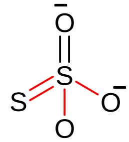
(C) 1 (D) 2

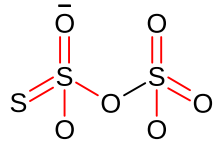
Ans. C

Sol.

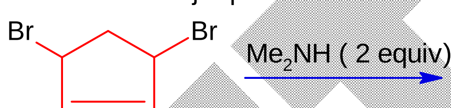
SO_3^{-2}  Pyramidal

SO_4^{-2}  tetrahedral

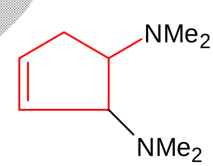
$S_2O_3^{-2}$  Tetrahedra

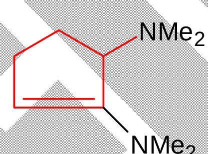
$S_2O_7^{-2}$  tetrahedral

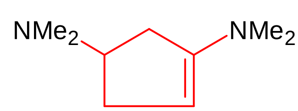
Q71. Find out the major product formed from the following reaction $[Me : -CH_3]$



(A) 

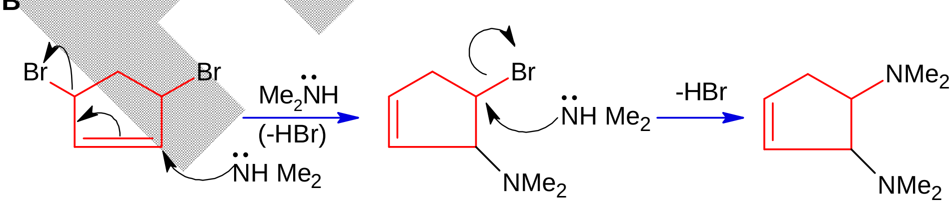
(B) 

(C) 

(D) 

Ans. B

Sol.



Q72.

$$CH_3-CH_2-CH_2-Br + NaOH \xrightarrow{C_2H_5OH} \text{Product 'A'}$$

$$\text{Product A} \xrightarrow[H^+]{H_2O} \text{Prdouct "B"}$$

$$\text{Product A} \xrightarrow[H_2O\ /\ H_2O_2\ /\ \bar{O}H]{\text{Diborane}} \text{Product "C"}$$

Consider the above reactions, identify product B and product C.

(A) B=1-Propanol C= 2-Propanol
(B) B = C = 1-Propanol
(C) B = 2-Propanol C = 1-Propanol
(D) B = C = 2-Propanol

Ans. C

Sol.

$$CH_3-CH_2-CH_2-Br + NaOH \xrightarrow{C_2H_5OH} CH_3-CH=CH_2$$

$$CH_3-CH=CH_2 + H_2O \xrightarrow{H^+} CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$$

$$3CH_3-CH=CH_2 + \frac{1}{2}B_2H_6 \longrightarrow (CH_3-CH_2-CH_2)_3B \xrightarrow{H_2O\ /\ O\bar{H}} CH_3-CH_2-CH_2-OH$$

Q73. Choose the **incorrect** Statement about Dalton's Atomic Theory

(A) Matter consists of indivisible atoms.
(B) Compounds are formed when atoms of different elements combine in any ratio.
(C) All the atoms of a given element have identical properties including identical mass.
(D) Chemical reaction involve reorganization of atoms.

Ans. B

Sol. Compounds are formed when atoms of different elements combine in a fixed ratio by mass

Q74. The equilibrium constant for the reaction

$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$ is:

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction given below is

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

(A) 4.9
(B) 49
(C) 41.6
(D) 416

Ans. D

Sol. $SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$

$K_1 = 4.9 \times 10^{-2}$

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$$K_2 = \left(\frac{1}{K_1}\right)^2 = \left(\frac{1}{4.9\times10^{-2}}\right)^2$$
$$K_2 = 416.49$$

Q75. Fuel cell, using hydrogen and oxygen as fuels.
a. has been used in spaceship
b. has as efficiency of 40% to produce electricity
c. uses aluminum as catalysts
d. is eco-friendly
e. is actually a type of Galvanic cell only.
Choose the **correct** answer from the options given below:
(A) a, d, e only
(B) a, b, c only
(C) a, b, d, e only
(D) a, b, d only

Ans. **A**

Sol. Fuel cell is used in spaceship. It is eco-friendly galvanic cell

Q76. For a strong electrolyte, a plot of molar conductivity against $(\text{concentration})^{1/2}$ is a straight line, with a negative slope, the correct unit for the slope is
(A) $S\,cm^2\,mol^{-1}\,L^{1/2}$
(B) $S\,cm^2\,mol^{-3/2}\,L^{-1/2}$
(C) $S\,cm^2\,mol^{-3/2}\,L^{1/2}$
(D) $S\,cm^2\,mol^{-3/2}\,L$

Ans. **C**

Sol. $\Lambda_m = \Lambda_m^0 - A\sqrt{C}$

Unit of $A\sqrt{C} = Scm^2mol^{-1}$

$A\times\left(mol\ell^{-1}\right)^{1/2} = Scm^2mol^{-1}$

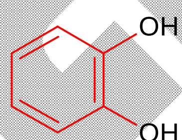
$$A = \frac{Scm^2\times mol^{-1}}{mol^{1/2}\,L^{-1/2}}$$

$S\,cm^2\,mol^{-3/2}\,L^{1/2}$

Q77. Common name of Benzene-1,2-diol is
(A) Resorcinol
(B) catechol
(C) quinol
(D) o-cresol

Ans. **B**

Sol.



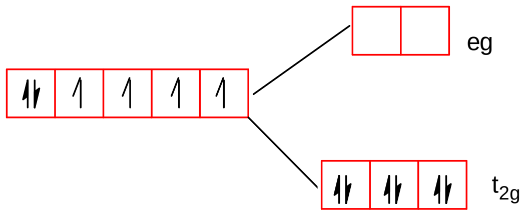
Common name of Benzene -1,2-diol is catechol

Q78. The number of unpaired d-electrons in $\left[Co(H_2O)_6\right]^{3+}$ is__________.
(A) 1
(B) 4
(C) 2
(D) 0

Ans. **D**

Sol. $\left[Co(H_2O)_6\right]^{+3}$

$Co^{+3} = 3d^6$



$t_{2g}^{6}\ eg^{0}$

Q79. Match **List I** with **List II**

	List-I		**List-II**
(a)	α - Glucose and α -Galactose	(I)	Functional isomers
(b)	α -Glucose and β - Glucose	(II)	Homologous
(c)	α -Glucose and α - Fructose	(III)	Anomers
(d)	α -Glucose and α - Ribose	(IV)	Epimers

Choose the **correct** answer from the options given below:

(A) (a – III), (b – IV), (c – I), (d – II)
(B) (a – IV), (b – III), (c – I), (d – II)
(C) (a – IV), (b – III), (c – II), (d – I)
(D) (a – III), (b – IV), (c – II), (d – I)

Ans. **B**

Sol. α **-**glucose & α -galactose are epimer they differ in configuration of one asymmetric carbon.

- α & β glucose differ at anomeric carbon thus they are anomers
- Glucose and fructose have different functional group so they are functional isomer
- α -glucose and α - ribose are homologous

Q80. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is:

(A) NH_4Cl
(B) KNO_3
(C) NaBr
(D) CaI_2

Ans. **A**

Sol. $2NH_4Cl + MnO_2 + 2H_2SO_4 \xrightarrow{\Delta} MnSO_4 + (NH_4)_2SO_4 + 2H_2O + Cl_2 \uparrow$

<u>SECTION - B</u>

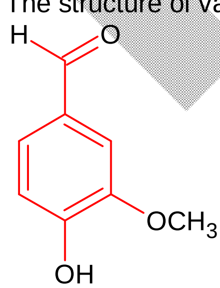
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q81. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is________________.

Ans. **110**

Sol. The structure of vanillin is



Total number of oxygen atom = 3
Total no of π bonds = 4
Total number of π-electron = 8
Sum of O atoms and π electrons = 3+8=11

Q82. 2.7 kg of each of water and acetic acid are mixed. The freezing point of the solution will be $-x^{o}C$. Consider the acetic acid does not dimerise in water, nor dissociates in water
x =______(nearest integer)
[Given: Molar mass of water = 18 g mol^{-1}, acetic acid = 60 g mol^{-1}
K_f H_2O: 1.86 K kg mol^{-1}
K_f acetic acid: 3.90 K kg mol^{-1}
Freezing point: H_2O = 273K, acetic acid = 290K]

Ans. **31**

Sol. As moles of H_2O is greater than moles of acetic acid so water is solvent

$$\Delta_T = K_f m$$

$$= 1.86 \times \frac{2700 \times 1000}{60 \times 2700}$$

$$= 31$$

Q83. The maximum number of orbitals which can be identified with n=4 and ml = 0 is______.

Ans. **4**

Sol.

n=4							
4s	√						
ml	0						
4p		√					
ml	-1	0	-1				
4d			√				
ml	-2	-1	0	1	2		
4f				√			
ml	-3	-2	-1	0	1	2	3

Q84. Number of compounds / species from the following with non- zero dipole moment is____.
$BeCl_2$, BCl_2, NF_3, XeF_4, CCl_4, H_2O, H_2S, HBr, CO_2, H_2, HCl

Ans. **95**

Sol. Molecules with zero dipole moment $BeCl_2$, BCl_3, XeF_4,CCl_4,CO_2,H_2O
Molecules with non-zero dipole moment
NF_3,H_2O,H_2S,HBr and HCl

Q85. Consider the following reaction, the rate expression of which is given below
$A + B \rightarrow C$
$\text{rate} = k[A]^{1/2}[B]^{1/2}$
The reaction is initiated by taking 1M concentration of A and B each. If the rate constant (k) is $4.6 \times 10^{-2} s^{-1}$, then the time taken for A to become 0.1 M is _________sec. (nearest integer)

Ans. **50**

Sol.

$$t = \frac{2.303}{K} \log \frac{1}{0.1}$$

$$= \frac{2.303}{4.6 \times 10^{-2}} \log 10$$

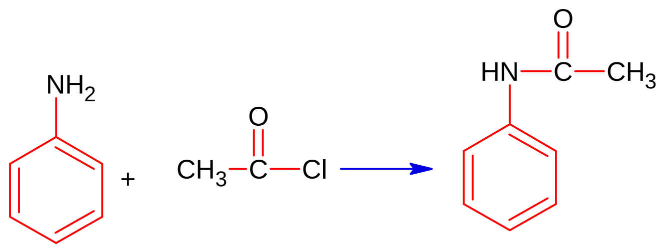
$$= \frac{2.303 \times 100 \times 1}{4.6}$$

$$= 50 \text{ sec}$$

Q86. From 6.55 g aniline, the maximum amount of acetanilide that can be prepared will be _________$\times 10^{-1}$ g.

Ans. **95**

Sol.



93 gm Aniline give $\longrightarrow$ 135 gm of acetamilide

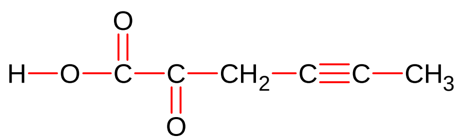
6.55gm Aniline give $\longrightarrow$ $\frac{135}{93} \times 6.55 = 9.5\text{gm}$

95×10^{-1}

Q87. The total number of 'sigma' and 'Pi' bonds in 2- oxohex-4- ynoic acid is_________.

Ans. **18**

Sol.



2-oxo hex-4- ynoic acid

No of σ bond = 14

No of π bonds = 4

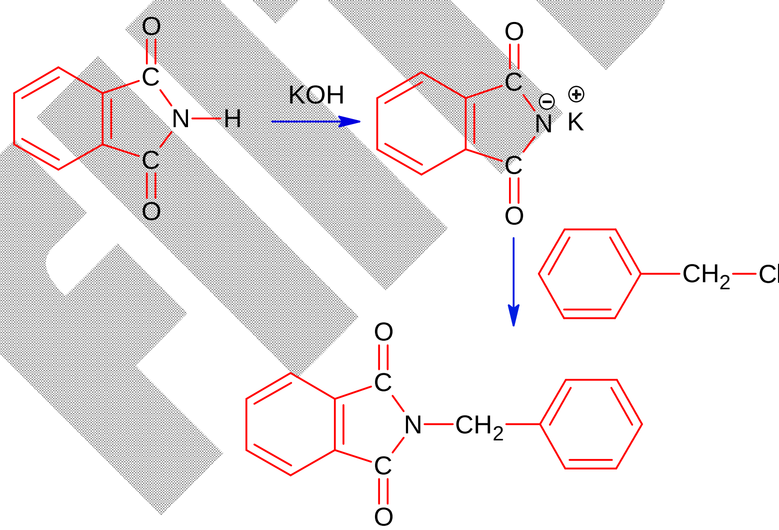
Q88. Phthalimide is made to undergo following sequence of reactions.

$$\text{Phthalimide} \xrightarrow[\text{(ii) Benzylcholride}]{\text{(i) KOH}} \text{'P'}$$

Total number of π bonds present in product 'P' is / are___________.

Ans. **8**

Sol.



Total no of π bonds 8

Q89. Three moles of an ideal gas are compressed isothermally from 60L to 20L using constant pressure of 5 atm. Heat exchange Q for the compression is ____Lit. atm.

Ans. **200**

Sol. Process is isothermal ΔU

$\Delta U = q + w$

$0 = q - P_{ext}(v_2 - v_1)$

$q = P_{ext}(v_2 - v_1)$
$= 5(60 - 20)$
$= 5 \times 40 \text{ Litre atm}$
$= 200 \text{ Litre atm}$

Q90. A first row transition metal with highest enthalpy of atomisation, upon reaction upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where n=3,4,5). The 'spin only' magnetic moment value of the amphoteric oxide from the above oxides is______ BM (near integer)
(Given atomic number : Sc:21, Ti:22,V:23,Cr:24,Mn:25,Fe:26,Co:27 Ni:28,Cu:29,Zn:30)

Ans. **0**

Sol. V has highest enthalpy of atomisation among first row transition elements. So compound is V_2O_5
V^{+5} $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$
No unpaired electron
$\mu = 0$