FIITJEE

Solutions to JEE(Main) -2024

Test Date: 27th January 2024 (Second Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*1. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b, which passes through the foci of the hyperbola. If $e_1e_2 = 1$, then the length of the cord of the ellipse parallel to the x-axis and passes through (0, 2) is:

(1) $\frac{8\sqrt{5}}{3}$ (2) $4\sqrt{5}$
(3) $3\sqrt{5}$ (4) $\frac{10\sqrt{5}}{3}$

Ans. **(4)**

Sol. $e_1=\sqrt{1+\frac{9}{16}}=\frac{5}{4} \Rightarrow e_2 = 4/5 \Rightarrow 1-\frac{b^2}{a^2} \Rightarrow \frac{b}{a}=\frac{3}{5}$

$\Rightarrow$ $Ae_1 = a \Rightarrow 4\times\frac{5}{4}=a \Rightarrow a = 5$

Solving $\frac{x^2}{25}+\frac{y^2}{9}=1$ with the line y = 2

$\Rightarrow$ $\frac{x^2}{25}=1-\frac{4}{9} \Rightarrow x=\pm\frac{5\sqrt{5}}{3}$

$\Rightarrow$ length of chord $|x_2-x_1|=\frac{10\sqrt{5}}{3}$.

*2. If α, β are the roots of the equation, $x^2 - x - 1$ and $S_n = 2023\,\alpha^n + 2024\,\beta^n$, then :

(1) $S_{12} = S_{11} + S_{10}$ (2) $2S_{11} = S_{12} + S_{10}$
(3) $2S_{12} = S_{11} + S_{10}$ (4) $S_{11} = S_{10} + S_{12}$

Ans. **(1)**

Sol. $S_n = 2023\alpha^n + 2024\beta^n$

$S_{n-1} = 2023\alpha^{n-1} + 2024\beta^{n-1}$

$S_{n-2} = 2023\alpha^{n-2} + 2024\beta^{n-2}$

$S_n = S_{n-1} + S_{n-2}$

$S_{12} = S_{11} + S_{10}$.

*3. Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is:

(1) $(-3,-1)\cup\left(-\frac{1}{3},1\right)$
(2) $(-3,0)\cup\left(\frac{1}{3},1\right)$
(3) $(-3,0)\cup\left(\frac{2}{3},1\right)$
(4) $(-3,-1)\cup\left(\frac{1}{3},1\right)$

Ans. (2)

Sol. $3a^2-(a+1)+1>0$

$3a^2 - a > 0 \Rightarrow a(3a - 1) > 0 \Rightarrow (-\infty, 0)\cup\left(0,\frac{1}{3}\right)$ … (i)

$a^2 + 3(a + 1) - 5 < 0$
$a^2 + 2a - 3 < 0$
$a^2 + 3a - a - 3 < 0$
$(a + 3)(a - 1) < 0 \Rightarrow a \in(-3, 1)$ …(ii)
From (i) and (ii)
$a \in (-3, 0) \cup (1/3, 1)$

4. Let $f:R-\left\{-\frac{1}{2}\right\}\to R$ and $g:R-\left\{\frac{-5}{2}\right\}\to R$ be defined as

$f(x)=\frac{2x+3}{2x+1}$ and $g(x)=\frac{|x|+1}{2x+5}$. Then. The domain of the function fog is:

(1) $R-\left\{-\frac{7}{4}\right\}$
(2) $R-\left\{-\frac{5}{2}\right\}$
(3) $R-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$
(4) R

Ans. (2)

Sol. $$f(g(x))=\frac{2\cdot(g(x))+3}{2(g(x))+1}=\frac{2\cdot\left(\frac{|x|+1}{2x+5}\right)+3}{2\left(\frac{|x|+1}{2x+5}\right)+1}$$

$$=\frac{2(|x|+1)+3(2x+5)}{2(|x|+1)+2x+5}=\frac{2|x|+2+6x+15}{2|x|+2+2x+5}$$

$$=\frac{2|x|+6x+17}{2|x|+2x+7}$$

$\frac{8x+17}{4x+7}$, if $x \geq 0$,
$\frac{4x+17}{7}$, if $x < 0$.

5. The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$, lie in the interval

(1) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (2) (–3, 0)
(3) (0, 3) (4) (–2, 1)

Ans. **(2)**

Sol. $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$

$C_3 \to C_3 - (\alpha C_1 + C_2)$

$\begin{vmatrix} 1 & \frac{3}{2} & 0 \\ 1 & \frac{1}{3} & 0 \\ 2\alpha+3 & 3\alpha+1 & -\left(2\alpha^2+6\alpha+1\right) \end{vmatrix} = 0$

$\Rightarrow \quad \frac{7}{6}\left(2\alpha^2+6\alpha+1\right)=0 \quad \alpha = \frac{-3+\sqrt{7}}{2}, \frac{-3-\sqrt{7}}{2}$.

6. If $\lim\limits_{x\to 0} \frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x} = \frac{1}{3}$, then $2\alpha - \beta$ is equal to :

(1) 7 (2) 1
(3) 5 (4) 2

Ans. **(3)**

Sol. $\lim\limits_{x\to 0} \frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x} = \frac{1}{3}$

$$\frac{3+\alpha\left(x-\frac{x^3}{3}\right)+\beta\left(1-\frac{x^2}{2}\right)+\left(-x+\frac{x^2}{2}\right)}{3x^2} = \frac{1}{3}.$$

$3 + \beta = 0$ $\alpha - 1 = 0$
$\beta = -3$ $\alpha = +1$

$2\alpha - \beta = 5$.

7. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for all $x \in (0, 3)$. If g is decreasing in (0, α) and increasing in (α, 3), then 8α is:

(1) 24
(2) 18
(3) 0
(4) 20

Ans. **(2)**

Sol. $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$

$g'(x) = 3f'\left(\frac{x}{3}\right) \cdot \frac{1}{3} - f'(3-x)$

$= f'\left(\frac{x}{3}\right) - f'(3-x)$

$g'(x) < 0,\ f'\left(\frac{x}{3}\right) - f'(3-x) < 0$

$f'\left(\frac{x}{3}\right) < f'(3-x)$

$\frac{x}{3} > 3 - x$

$\frac{x}{3} + x > 3 \Rightarrow \frac{4x}{3} > 3,\ x > \frac{9}{4}$

$\alpha = \frac{9}{4}$ = 18.

8. If y = y(x) is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, x > 2, $y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals:

(1) $\frac{3}{1+2\sqrt{2}}$
(2) $\frac{3}{1+(8)^{1/4}}$
(3) $\frac{3}{1-(8)^{1/4}}$
(4) $\frac{3}{1-2\sqrt{2}}$

Ans. **(2)**

Sol. $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, x > 2, $y(4) = \frac{3}{2}$

$$\int \frac{dy}{y^2 - 3y} = \int \frac{dx}{x^2 - y}$$

$$\frac{1}{3}\int \frac{y - (y-3)}{y(y-3)}dy = \int \frac{dx}{x^2 - 2^2}$$

$$\frac{1}{3}\left(\ln(y-3) - \ln y\right) = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + c$$

$$\frac{1}{3}\ln\left(\left|\frac{y-3}{y}\right|\right)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + c$$

x = 4, y = 3/2

$$\frac{1}{3}\ln\left(\left|\frac{\frac{3}{2}-3}{\frac{3}{2}}\right|\right)=\frac{1}{4}\left(\left|\frac{4-2}{4+2}\right|\right) + c$$

$$\Rightarrow \quad c = -\frac{1}{4}\ln\frac{1}{3}.$$

$$\frac{1}{3}\ln\left|\left(\frac{y-3}{y}\right)\right|=\frac{1}{4}\ln\left|\left(\frac{x-2}{x+2}\right)\right|-\frac{1}{4}\ln\frac{1}{3}$$

x = 10

$$\frac{1}{3}\ln\left(\frac{y-3}{y}\right)=\frac{1}{4}\ln\left(\frac{8}{12}\right)-\frac{1}{4}\ln\frac{1}{3}$$

$$=\frac{1}{4}\left[\ln\frac{2}{3}-\ln\frac{1}{3}\right]$$

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}[\ln 2]$$

$$\ln\left|\left(\frac{y-3}{y}\right)\right|=\frac{3}{4}\ln 2=\ln\left(2^{3/4}\right)=\ln\left(8^{1/4}\right)$$

$$\Rightarrow \quad \frac{-y+3}{y}=8^{1/4} \Rightarrow y=\frac{3}{1+8^{1/4}}.$$

*9. The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17, \frac{3}{4}, ..., -129\frac{1}{4}$ is:

(1) – 118
(2) – 110
(3) – 100
(4) – 115

Ans. **(4)**

Sol. $20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, ..., -\frac{129\times 4+1}{4} = -\frac{517}{4}$

Let $a=-\frac{517}{4}, d=\frac{3}{4}$

$$t_{20}=-\frac{517}{4}+19\times\frac{3}{4}=-\frac{460}{4}=-115.$$

10. For 0 < a < 1, the value of the integral $\int_0^{\pi}\frac{dx}{1-2a\cos x+a^2}$ is:

(1) $\frac{\pi^2}{\pi+a^2}$
(2) $\frac{\pi}{1+a^2}$
(3) $\frac{\pi}{1-a^2}$
(4) $\frac{\pi^2}{\pi-a^2}$

Ans. **(3)**

Sol. $I = \int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}$

$$I = \int_0^{\pi} \frac{dx}{1 + a^2 - 2a\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} \quad \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{\left(1+a^2\left(1+\tan^2\frac{x}{2}\right) - 2a\left(1+\tan^2\frac{x}{2}\right)\right)}dx$$

If $\tan\frac{x}{2} = t,\ \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$2\int_0^{\infty} \frac{dt}{\left(1+a^2\right)\left(1+t^2\right) - 2a\left(1-t^2\right)}$$

$$2\int_0^{\infty} \frac{dt}{t^2\left(1+a^2+2a\right) + \left(1+a^2-2a\right)}$$

$$2\int_0^{\infty} \frac{dt}{t^2\left(a+1\right)^2 + \left(1-a\right)^2}$$

$$\frac{2}{\left(a+1\right)^2}\int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1-a}{a+1}\right)^2}$$

$$\frac{2}{\left(a+1\right)^2}\cdot\frac{1}{\left(\frac{1-a}{a+1}\right)}\cdot\left(\tan^{-1}\left(\frac{t}{\left(\frac{1-a}{a+1}\right)}\right)\right)_0^{\infty}, \quad \frac{2}{\left(a+1\right)\left(1-a\right)}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{1-a^2}.$$

11. Consider the function $f:(0, 2) \to R$ defined by $f\left(x\right) = \frac{x}{2} + \frac{2}{x}$ and the function g(x) defined by $g\left(x\right) = \begin{cases} \min\left\{f\left(t\right)\right\}, & 0 < t \le x \text{ and } 0 < x \le 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases}$. Then,

(1) g is not continuous for all $x \in (0, 2)$
(2) g is continuous and differentiable for all $x \in (0, 2)$
(3) g is continuous but not differentiable at x = 1
(4) g is neither continuous nor differentiable at x = 1

Ans. **(3)**

Sol. $f\left(x\right) = \frac{x}{2} + \frac{2}{x}$

$f'\left(t\right) = \frac{1}{2} - \frac{2}{t^2} = 0$

$\frac{t^2-4}{2t^2}=0 \Rightarrow t = \pm 2,$

$\frac{(t-2)(t+2)}{2t^2} \Rightarrow$ f(t) is decreasing in (0, 2)

$g(x)=\begin{cases}\frac{x}{2}+\frac{2}{x}, & 0<x\le 1\\ \frac{3}{2}+x, & kx<2\end{cases}, \ g'(x)=\begin{cases}\frac{1}{2}-\frac{2}{x^2}, & 0<x\le 1\\ 1 & 1<x<0\end{cases}.$

*12. Let $\alpha=\frac{(4!)!}{(4!)^{3!}}$ and $\beta=\frac{(5!)!}{(5!)^{4!}}$. Then :

(1) $\alpha \in N$ and $\beta \in N$
(2) $\alpha \notin N$ and $\beta \in N$
(3) $\alpha \in N$ and $\beta \notin N$
(4) $\alpha \notin N$ and $\beta \notin N$

Ans. **(3)**

Sol. $\alpha=\frac{(4!)!}{(4!)^{3!}}, \beta=\frac{(5!)!}{(5!)^{4!}}$

$\alpha=\frac{24!}{(24)^{3!}}, \beta=\frac{120!}{(120)^{4!}} \Rightarrow \alpha \in N, \beta \in N.$

*13. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) form the point Q(–2, –3) is:

(1) 4
(2) 6
(3) 10
(4) 8

Ans. **(3)**

Sol. Total subset and A = 2^m
Total subset and B = 2^n
$2^m - 2^n = 56$
M = 6, n = 3
$= \sqrt{(6+2)^2+(3+3)^2} = \sqrt{64+36} = 10$

14. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1) $\frac{5}{256}$
(2) $\frac{3}{715}$
(3) $\frac{3}{256}$
(4) $\frac{5}{715}$

Ans. **(2)**

Sol. $\dfrac{{}^{6}C_4}{{}^{15}C_4} \times \dfrac{{}^{9}C_4}{{}^{11}C_4} = \dfrac{6!}{4! \times 2! \times \dfrac{15!}{4! \times 11!}} \times \dfrac{9!}{4! \times 5! \times \dfrac{11!}{4! \times 7!}}$

$= \dfrac{3}{715}$.

15. Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the orthocenter of the triangle on the sides AB, BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals:

(1) 1/3 (2) 1/2
(3) 1/5 (4) 1/4

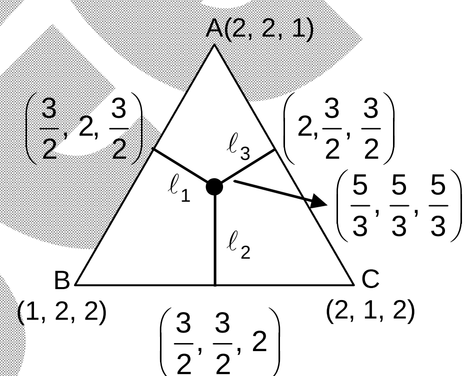
Ans. **(2)**

Sol. $\ell_1 = \sqrt{\left(\dfrac{5}{3} - \dfrac{3}{2}\right)^2 + \left(\dfrac{5}{3} - \dfrac{3}{2}\right)^2 + \left(\dfrac{5}{3} - 2\right)^2}$

$= \sqrt{\dfrac{1}{36} + \dfrac{1}{36} + \dfrac{1}{9}} = \dfrac{1}{\sqrt{6}}$

$\ell_1^2 = \dfrac{1}{6}$

$\ell_1^2 + \ell_2^2 + \ell_3^2 = 3 \cdot \dfrac{1}{6} = \dfrac{1}{2}$



16. Let the image of the point (1, 0, 7) in the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\dfrac{2\pi}{3}$ and $\dfrac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?

(1) $\left(1, -2, 1+\sqrt{2}\right)$ (2) $\left(3, 4, 3-2\sqrt{2}\right)$
(3) $\left(3, -4, 3+2\sqrt{2}\right)$ (4) $\left(1, 2, 1-\sqrt{2}\right)$

Ans. **(2)**

Sol. Image is 1, 6, 3 and point satisfying the line $\left(3, 4, 3-2\sqrt{2}\right)$

17. The integral $\displaystyle\int \frac{\left(x^8 - x^2\right)}{\left(x^{12} + 3x^6 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$ is equal to:

(1) $\log_e\left(\left|\tan^{-1}\left(x^3 + \dfrac{1}{x^3}\right)\right|\right)^{1/2} + C$ (2) $\log_e\left(\left|\tan^{-1}\left(x^3 + \dfrac{1}{x^3}\right)\right|\right)^{1/3} + C$

(3) $\log_e\left(\left|\tan^{-1}\left(x^3 + \dfrac{1}{x^3}\right)\right|\right)^{3} + C$ (4) $\log_e\left(\left|\tan^{-1}\left(x^3 + \dfrac{1}{x^3}\right)\right|\right) + C$

Ans. **(2)**

Sol. $$I = \int \frac{\left(x^8 - x^2\right)}{\left(x^{12} + 3x^6 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}dx$$

$$I = \int \frac{x^2 - \frac{1}{x^4}dx}{\left(x^6 + 3 + \frac{1}{x^6}\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$$

$$I = \int \frac{x^2 - \frac{1}{x^4}dx}{\left(\left(x^3 + \frac{1}{x^3}\right)^2 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$$

$$\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$$

$$\frac{1}{\left(x^3 + \frac{1}{x^3}\right)^2 + 1}\left(3x^2 - \frac{3}{x^4}\right)dx = dt$$

$$I = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t|$$

$$I = \ln\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|^{1/3} + C$$

*18. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in N$, then $\sum_{k=1}^{n}\frac{k}{2^k}$ is equal to:

(1) $1 - \frac{15}{2^{13}}$

(2) $\frac{1}{2^{15}}\left(2^{14} - 14\right)$

(3) $\frac{1}{2^{13}}\left(2^{14} - 15\right)$

(4) $\frac{1}{2^{14}}\left(2^{15} - 15\right)$.

Ans. **(3)**

Sol. $2\tan^2\theta - 5\sec\theta = 1$

$2\sec^2\theta - 2 - 5\sec\theta = 1$

$2\sec^2\theta - 5\sec\theta - 3 = 0$

$(\sec\theta - 3)(2\sec\theta + 1) = 0$

$\sec\theta = 3, \frac{-1}{2}$ not possible. n = 13

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots\frac{13}{2^{13}}$$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{2}{2^3} + \dots \frac{12}{2^{13}} + \frac{13}{2^{14}}$$
$$\frac{1}{2}S = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{13}}\right) - \frac{13}{2^{14}}$$
$$\frac{1}{2}S = \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^{13}}{1-\frac{1}{2}}\right) - \frac{13}{2^{14}}$$
$$\frac{1}{2}S = 1 - \frac{1}{2^{13}} - \frac{13}{2^{14}} = \frac{2^{14} - 2 - 13}{2^{14}}.$$
$$S = \frac{2^{14} - 15}{2^{13}}.$$

19. The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals:

(1) 45 (2) 50
(3) 49 (4) 42

Ans. (1)

Sol. AB = 5, AC = 5
D mid point of BC as AB = AC
$$D = \left(\frac{1}{2}, \frac{3}{2}, 3\right)$$
$$\ell = AD = \sqrt{\frac{9}{4} + \frac{81}{4}}$$
$$2\ell^2 = 2 \cdot \frac{90}{4} = 45.$$

*20. Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is:

(1) 2 (2) more than 2
(3) 0 (4) 1

Ans. (1)

Sol. $\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}$
$$\tan^{-1}\left(\frac{x + 2x}{1 - 2x^2}\right) = \frac{\pi}{4}$$
$$\frac{3x}{1 - 2x^2} = 1$$
$$3x = 1 - 2x^2$$

$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}, \frac{-3+\sqrt{17}}{2}, \frac{-3-\sqrt{17}}{2}$$
$$x = \frac{\sqrt{17}-3}{2}$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to ______

Ans. **108**

Sol. $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ any one line $(2 + 2\lambda, -2\lambda, 7 + 16\lambda)$

$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ any one line $(-3 + 4\mu, -2 + 3\mu, -2 + \mu)$

Point of intersection

(i) $2 + 2\lambda = -3 + 4\mu$

$2\lambda = -5 + 4\mu$

(ii) $-2\lambda = -2 + 3\mu$

$2\lambda = 2 - 3\mu$

From (i) & (ii) $2 - 3\mu = -5 + 4\mu$

$\mu = 1$

Point of intersection (1, 1, –1)

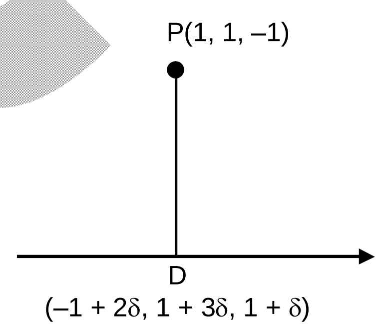
DR(PD) $= -2 + 2\delta, 3\delta, 2 + \delta$

PD perpendicular to line $2(-2 + 2\delta) + 3(3\delta) + 2(2 + \delta) = 0$

$-4 + 4\delta + 9\delta + 2 + \delta = 0$

$14\delta = 2$

$\delta = 1/7$



$\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$

$$PD = \ell = \sqrt{\left(-2+\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(2+\frac{1}{7}\right)^2}$$

$$\ell^2 = \frac{144}{49} + \frac{9}{49} + \frac{225}{49} = \frac{378}{49} = \frac{54}{7}$$

$$14\ell^2 = 108.$$

*22. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote that mean and variance of the correct observations respectively, then $15\left(\mu+\mu^2+\sigma^2\right)$ is equal to _______

Ans. **2521**

Sol. $\frac{\sum x_i}{15}=12 \Rightarrow \sum x_i = 180$

$\frac{\sum x_i^2}{15}-(12)^2=9 \Rightarrow \sum x_i^2 = 2295$

Actual : $\sum x_i = 182, \sum x_i^2 = 2339$

$\sigma^2 = \frac{2339}{15}-\mu^2$

$\sigma^2+\mu^2=\frac{2339}{15}$

$\Rightarrow \quad 15\left(\mu+\mu^2+\sigma^2\right)=15\left(\frac{182}{15}+\frac{2339}{15}\right)$ = 2521.

*23. The coefficient of x^{2012} in the expansion $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to _______

Ans. **0**

Sol. Coefficient of x^{2012} in $(1-x)^{2008}(1+x+x^2)^{2007}(1-x)(1-x^3)^{2007}$

$\Rightarrow \quad x^{2012}$ in $(1-x^3)^{2007}$ + coefficient of x^{2011} in $(1-x^3)^{2007}$

$\Rightarrow \quad 0-0=0.$

24. If the sum of squares of all real values of α, for which the lines $2x-y+3=0$, $6x+3y+1=0$ and $\alpha x+2y-2=0$ do not form a triangle is p, then the greatest integer less than or equal to p is _______

Ans. **32**

Sol. Not triangle (i) concurrent

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix}=0$$

$2(-6-2)+1(-12-\alpha)+3(12-3\alpha)=0$

$-16-12-\alpha+36-9\alpha=0$

$10\alpha=8$, $\alpha=4/5$.

(ii) $ax+2y-2=0$ parallel to $2x-y+3=0$

$-\frac{a}{2}=2$, $a=-4$

(iii) $ax+2y-2=0$ parallel to $6x+3y+1=0$

$\frac{a}{6}=\frac{2}{3}$, $a=4$

$$P = \left(\frac{4}{5}\right)^2 + (4)^2 + (-4)^2$$

$$P = 16 + 16 + \frac{16}{25} = 32 + \frac{16}{25}$$

[P] = 32.

25. Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)$ dt, where g is a continuous odd function. If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to ________

Ans. **2**

Sol. $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$

g(t) . $\ln\left(\frac{1-t}{1+t}\right)$ = odd function × odd function = even function

f(x) = Integration of even function = odd function.

$$\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \int_{-\pi/2}^{\pi/2} f(x)dx + \int_{-\pi/2}^{\pi/2}\frac{x^2\cos x}{1+e^x}dx$$

$$I = 0 + \int_{-\pi/2}^{\pi/2}\frac{x^2\cos x}{1+e^x}dx \quad \ldots(i)$$

$$I = \int_{-\pi/2}^{\pi/2}\frac{(-x)^2(\cos(-x))}{1+e^{-x}}dx = \int_{-\pi/2}^{\pi/2}\frac{e^x x^2\cos x}{1+e^x}dx \quad \ldots(ii)$$

Add (i) & (ii)

$$2I = \int_{-\pi/2}^{\pi/2}\frac{(1+e^x)x^2\cos dx}{(1+e^x)}$$

$$2I = \int_{-\pi/2}^{\pi/2} x^2\cos dx = 2\int_0^{\pi/2} x^2\cos x\, dx$$

$$I = x^2\sin x + 2x\cos x - 2\sin x\Big|_0^{\pi/2}$$

$$I = \frac{\pi^2}{4} - 2 - 0 = \frac{\pi^2}{4} - 2 = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$$

α = 2.

26. Let A be a 2 × 2 real matrix and I be the identity matrix of order 2. If the roots of the equation. |A – xI| = 0 be – 1 and 3, then the sum of the diagonal elements of the matrix A^2 is _____

Ans. **10**

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $|A - xI| = \begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix}$

$|A - xI|$ = ad – x(a+ d) + x^2 – bc

= x^2 – x(a + d) + ad – bc

Given = (x + 1)(x – 3) = x^2 – 2x – 3

a + d = 2, ad – bc = – 3

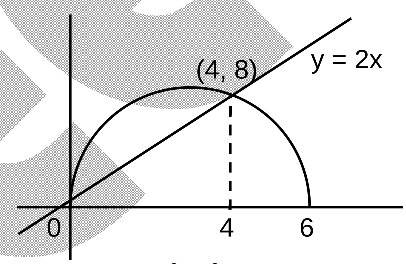
$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix}$

Sum = a^2 + bc + bc + d^2 = $(a + d)^2$ – 2(ad – bc) = 4 + 6 = 10.

27. If the area of the region $\{(x, y) : 0 \le y \le \min\{2x, 6x - x^2\}\}$ is A, then 12A is equal to ______

Ans. **304**

Sol. $0 \le y \le \min(2x, 6x - x^2)$



$2x = 6x - x^2$, $x^2 = 4x$, x = 0, 4

Area = $\frac{1}{2} \times 4 \times 8 + \int_4^6 6x - x^2 \; dx$

$A = 16 + 3x^2 - \frac{x^3}{3}\Big|_4^6$

$A = 16 + 3(36-16) - \frac{(216-64)}{3}$

$A = 16 + 60 - \frac{152}{3} = 76 - \frac{152}{3} = \frac{228-152}{3}$

$A = \frac{76}{3}$

$12A = 4 \times 76 = 304$.

*28. Let the complex numbers α and $\frac{1}{\overline{\alpha}}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of 100 $|\alpha|^2$ is __________

Ans. **20**

Sol. $|z - z_0|^2 = 4$ $z_0 = 1 + i$

$|z_0| = \sqrt{2}$

$(\alpha - 20)(\overline{\alpha} - \overline{z}_0) = 4$

$\alpha\overline{\alpha} - \alpha\overline{z}_0 - z_0\overline{\alpha} + z_0\overline{z}_0 = 4$

$|\alpha|^2 - \alpha\overline{z}_0 - z_0\overline{\alpha} + 2 = 4$

$\alpha\overline{z}_0 + z_0\overline{\alpha} = |\alpha|^2 - 2$ … (i)

$\left(\frac{1}{\overline{\alpha}} - z_0\right)\left(\overline{\left(\frac{1}{\overline{\alpha}}\right)} - \overline{z}_0\right) = 16$

$$\left(\frac{\alpha}{|\alpha|^2} - z_0\right)\left(\frac{\bar{\alpha}}{|\alpha|^2} - \bar{z}_0\right) = 16$$

$$\frac{\alpha\bar{\alpha}}{|\alpha|^4} - \frac{\alpha\bar{z}_0}{|\alpha|^2} - \frac{\bar{\alpha}z_0}{|\alpha|^2} + z_0\bar{z}_0 = 16$$

$$\frac{1}{|\alpha|^2} - \frac{\alpha\bar{z}_0}{|\alpha|^2} - \frac{\bar{\alpha}z_0}{|\alpha|^2} + 2 = 16$$

$$1 - \alpha\bar{z}_0 - \bar{\alpha}\bar{z}_0 + 2|\alpha|^2 = 6|\alpha|^2$$

$$\alpha\bar{z}_0 + \bar{\alpha}\bar{z}_0 = 1 - 14|\alpha|^2 \qquad \ldots \text{(ii)}$$

(i) & (ii)

$|\alpha|^2 - 2 = 1 - 14\,|\alpha|^2$

$15\,|\alpha|^2 = 3$, $|\alpha| = \frac{1}{5}$ $100|\alpha|^2 = 20$.

*29. Consider a circle $(x - \alpha)^2 + (y - \beta)^2 = 50$, where $\alpha, \beta > 0$. If the circle touches the line $y + x = 0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha + \beta)^2$ is equal to ______

Ans. **100**

Sol. CP⊥'e to OP

$\frac{\beta - 4}{\alpha + 4} = 1$, $\beta = \alpha + 8$

CP = $5\sqrt{2}$

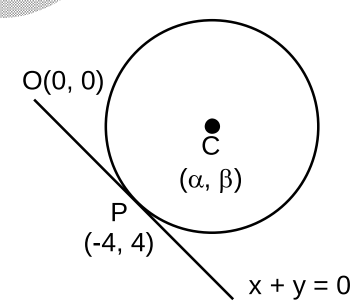
$\left|\frac{\alpha + \beta}{\sqrt{2}}\right| = 5\sqrt{2}$

$|2\alpha + 8| = 10$

$\alpha + 4 \pm 5$, $\alpha = 1$, $\alpha = -9$ (no reallow)

$\beta = 9$

$(\alpha + \beta)^2 = 100$.



$r = 5\sqrt{2}$, $OP = 4\sqrt{2}$

30. If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x + y - 2}{x - y}$ passing through the point (2, 1) is $\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|$, then $5\beta + \alpha$ is equal to :

Ans. **11**

Sol. $\frac{dy}{dx} = \frac{x + y - 2}{x - y}$

$x = X + h$, $y = Y + k$

$$\frac{dy}{dx} = \frac{X + h + Y + k - 2}{X + h - Y - k}$$

$$\frac{dy}{dx} = \frac{X+Y}{X-Y},$$

h + k – 2 = 0, h – k = 0 $\Rightarrow$ 2h = 2, h = 1, k = 1

put Y = vX

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{1+v}{1-v}$$

$$X\frac{dv}{dX} = \frac{1+v}{1-v} - v = \frac{1+v-v(1-v)}{1-v} = \frac{1+v-v+v^2}{1-v}$$

$$X\frac{dv}{dX} = \frac{1+v^2}{1-v}$$

$$\int \frac{1-v}{1+v^2}dv = \int \frac{dX}{X}$$

$$\int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dX}{X}$$

$$\tan^{-1} v - \frac{1}{2}\ln\left(1+v^2\right) = \ln X + C$$

$$\tan^{-1}\left(\frac{Y}{X}\right) - \frac{1}{2}\ln\left(1+\left(\frac{Y}{X}\right)^2\right) = \ln X + C$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1| + C$$

α = 1, β = 2, 5 $\times$ 2 + 1 = 11.

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*31. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be

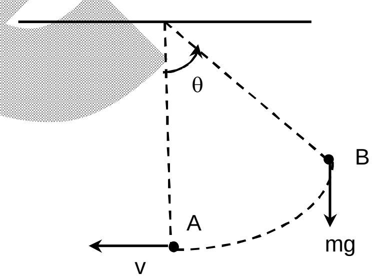
(1) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (2) $\tan^{-1}\left(\sqrt{2}\right)$

(3) $2\tan^{-1}\left(\frac{1}{2}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

Ans. **(3)**

Sol. $a_B = g\sin\theta$ . . . (i)

$a_A = v^2/\ell$ . . . (ii)



Applying Work energy theorem

$\frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$

$v^2 = 2g\ell\,(1 - \cos\theta)$ . . . (iii)

Given : $a_A = a_B$

$\frac{v^2}{\ell} = g\sin\theta$

$2(1 - \cos\theta) = \sin\theta$ (using iii)

$4\sin^2(\theta/2) = 2\sin\theta/2\cos\theta/2$

$\tan(\theta/2) = ½$

$\theta = 2\tan^{-1}(1/2)$.

*32. A bullet is fired into a fixed target looses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is:

(1) 3 (2) 2

(3) 4 (4) 5

Ans. (**None**)

Sol. Velocity becomes 2/3, $v^2 = u^2 + 2as$

$4v^2/9 = v^2 - 2a \times 4$

$5v^2/9 = 8a$

$4v^2/9 = 2aD$

$D = 3.2\text{ cm} = 32 \times 10^{-3}$ m

No options given.

*33. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): The angular speed of the moon in its orbit about the earth is more than angular speed of the earth in its orbit about the sun.

Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

Options.

(1) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanations of **(A)**
(2) **(A)** is not correct but **(R)** is correct
(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
(4) **(A)** is correct but **(R)** is not correct

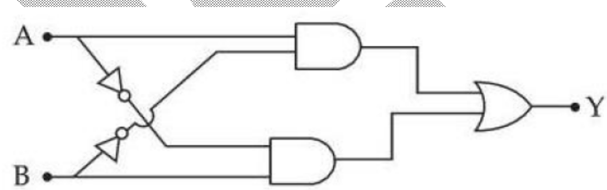
Ans. (3)

Sol. Time period of earth T_E = 365 days
Time period of moon T_m = 27 days.

$T_E > T_m$

$\omega_E < \omega_m$

34. The truth table of the given circuit diagram is



Options

(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(3)

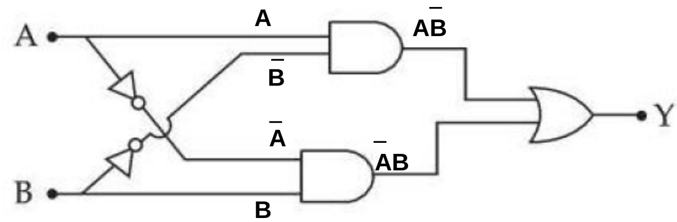
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Ans. **(3)**

Sol. $Y = A\bar{B} + \bar{A}B$
$= A \oplus B$
= XOR gate.



35. The equation of state of a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of:

(1) P
(2) R
(3) RT
(4) PV

Ans. **(1)**

Sol. Dimension of a = $[V^2P]$
Dimension of b = $[V]$
$\Rightarrow$ Dimension of a/b^2 has dimension P

*36. A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60^0 with the horizontal, the weight experienced by the man is :

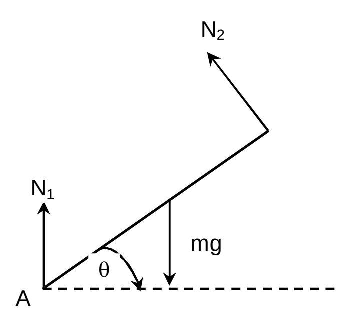
(1) 3 kg
(2) 6 kg
(3) $6\sqrt{3}$ kg
(4) 12 kg

Ans. **(1)**

Sol. $\tau_A = 0$

$$N_2 \ell = \frac{mg\ell\cos\theta}{2}$$

$$N_2 = \frac{mg}{4} = 3g$$



37. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R): The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**
(2) **(A)** is false but **(R)** is true
(3) **(A)** is true but **(R)** is false
(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

Ans. **(1)**
Sol. For positive error measured reading is more than true reading.

38. Wheatstone bridge principle is used to measure the specific resistance (S_1) of given wire, having length (L), radius (r). If X is the resistance of wire, then specific resistance is; $S_1 = X\left(\frac{\pi r^2}{L}\right)$

If the length of the wire gets doubled then the value of specific resistance will be:

(1) $\frac{S_1}{4}$
(2) S_1
(3) $\frac{S_1}{2}$
(4) $2S_1$

Ans. **(2)**
Sol. Specific resistance is the property of material.

39. The threshold frequency of a metal with work function 6.63eV is :
(1) 16×10^{15} Hz.
(2) 1.6×10^{15} Hz.
(3) 16×10^{12} Hz.
(4) 16×10^{12} Hz.

Ans. **(2)**
Sol. $h\nu_0 = \phi$ = work function

$$\nu_0 = \frac{6.63\times1.6\times10^{-19}}{6.63\times10^{-34}}Hz$$

$\nu_0 = 1.6 \times 10^{15}$ Hz.

40. Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46Ω. The power consumed in it is:
(1) 12.5 W
(2) 11.5 W
(3) 12.0 W
(4) 10.0 W

Ans. **(2)**
Sol. $\frac{V_1}{V_2} = \frac{n_1}{n_2} = 10$

$V_2 = \frac{V_1}{10} = 23V$

$P = \frac{V_2^2}{R} = \frac{23\times23}{46} = 11.5W$

*41. Given below are two statements:
Statement (I) : The limiting force of static friction depends on the area of contact and independent of materials.
Statement (II): The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below:
Options
(1) Both **Statement I** and **Statement II** are incorrect
(2) Both **Statement I** and **Statement II** are correct
(3) **Statement I** is correct but **Statement II** is incorrect
(4) **Statement I** is incorrect but **Statement II** is correct.

Ans. (4)
Sol. Both static and kinetic frictions are independent of area of contact.

42. A current of 200μA deflects the coil of a moving coil galvanometer through 60^0. The current to cause deflection through $\frac{\pi}{10}$ radian is :
(1) 120 μA (2) 180 μA
(3) 30 μA (4) 60 μA

Ans. (4)
Sol. $\theta = \left(\frac{nBA}{k}\right)I$
For I = 200 μA, $\theta = \pi/3$
$\Rightarrow \pi/3 = \left(\frac{nBA}{k}\right)200\mu A$ . . . (i)
For $\theta = \pi/10$
$I = \frac{\theta}{(nBA/k)} = \frac{\pi/10}{\pi/600}\mu A$
I = 60 μA

*43. The total kinetic energy of 1 mole of oxygen at 27^0 C is :
[Use universal gas constant (R) = 8.31J/ mole K]
(1) 6232.5 J (2) 6845.5J
(3) 5670.5 J (4) 5942.0 J
Ans. (1)
Sol. Total Kinetic energy $= \frac{f}{2}nRT$ (f = 5, T = 300 K)
TE = 6232.5 J

*44. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_P}{C_V}$ for the gas is:
(1) $\frac{7}{5}$ (2) $\frac{9}{7}$
(3) $\frac{5}{3}$ (4) $\frac{3}{2}$

Ans. **(4)**

Sol. $P \propto T^3$

$\Rightarrow P = kT^3$ (k is constant)

$\Rightarrow PV^{3/2}$ = const.

$\Rightarrow C_p/C_v = 3/2$

45. The atomic mass of ${}_6C^{12}$ is 12.000000u and that of ${}_6C^{13}$ is 13.003354u . The required energy to remove a neutron from ${}_6C^{13}$, if mass of neutron is 1.008665u , will be :

(1) 6.25 MeV
(2) 49.5 MeV
(3) 4.95 MeV
(4) 62.5 MeV

Ans. **(3)**

Sol. Mass defect = |13.003354 – (12 + 1.008665)| u

$\Delta m = 0.005311$ u

$E = \Delta mc^2 = 0.00511 \times 931.5$ MeV

E = 4.95 MeV

*46. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.

Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.

In the light of the above statements, choose the **correct** answer from the options given below :

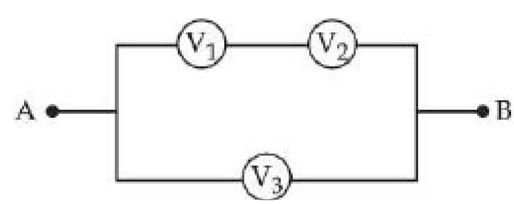
Options

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(2) **(A)** is true but **(R)** is false
(3) **(A)** is false but **(R)** is true
(4) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

Ans. **(1)**

Sol. The restoring force is dependent on the bonded interatomic and intermolecular force of solid.

47. Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B , their readings are V_1, V_2 and V_3. Choose the correct option.



(1) $V_1 \neq V_3 - V_2$
(2) $V_1 + V_2 > V_3$
(3) $V_1 + V_2 = V_3$
(4) $V_1 = V_2$

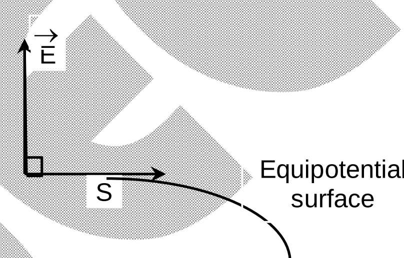
Ans. **(3)**

Sol. The parallel branches have same potential difference

$V_1 + V_2 = V_3$ (also, using KVL)

48. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.
Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces. In the light of the above statements, choose the most appropriate answer from the options given below :
Options:
(1) **(A)** is not correct but **(R)** is correct
(2) **(A)** is correct but **(R)** is not correct
(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

Ans. **(3)**
Sol. $W = Q\Delta V$
For equipotential surface $\Delta V = 0$
$\Rightarrow W = 0$
The electric field lines are perpendicular to equipotential surfaces.



49. An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity 6×10^8 W/m^2 falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space = 3×10^8m/s):
(1) 2 Nm^{-2}
(2) 6 Nm^{-2}
(3) 36 Nm^{-2}
(4) 18 Nm^{-2}

Ans. **(2)**
Sol. Radiation press $P = \dfrac{I}{(c/\mu)} = 6\text{Nm}^{-2}$

50. When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :
(1) 90^0
(2) 45^0
(3) 30^0
(4) 60^0

Ans. **(2)**
Sol. $I = I_0 \cos^2 \phi \sin^2 \phi = \dfrac{I_0}{4}\sin^2\{2\phi\}$
For maximum I, $2\phi = \pi/2$
$\phi = \pi/4$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*51. A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is ……………..m/s

Ans. 294

Sol. $\nu_1 = \dfrac{v}{4\ell_1}$

$\nu_2 = \dfrac{v}{2\ell_2}$

$\nu_1 - \nu_2 = 7$

$\Rightarrow \dfrac{v}{2}\left[\dfrac{1}{2\ell_1} - \dfrac{1}{\ell_2}\right] = 7$

v = 294 m/s.

52. The electric potential at the surface of an atomic nucleus (z = 50) of radius 9×10^{-13} cm is ……………$\times 10^6$V .

Ans. 8

Sol. $Q = 50 \times 1.6 \times 10^{-19}$ C

$\Rightarrow Q = 8 \times 10^{-18}$ C

$V = \dfrac{kQ}{R} = \dfrac{9\times10^9 \times 8\times10^{-18}}{9\times10^{-15}} = 8 \times 10^6$ V

*53. A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is ……………..m (use g = $10ms^{-2}$).

Ans. 45

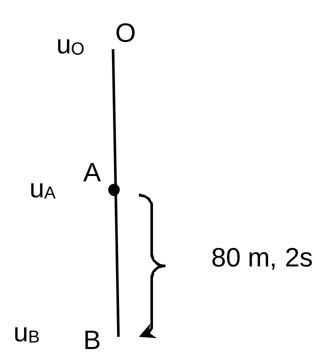
Sol. $S = ut + \dfrac{1}{2}at^2$

$S_{AB} = 80 = u_A \times 2 + 5 \times 2^2$

$\Rightarrow u_A = 30$ m/s

$\Rightarrow u_A^2 = u_0^2 + 2\times10\times S_{OA}$ ($u_0 = 0$)

$\Rightarrow S_{OA} = 45$ m



*54. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\dfrac{7}{x}$, where x is

Ans. **Data incomplete**
Sol. Masses of ring and sphere were not given.

55. A series LCR circuit with L = $\frac{100}{\pi} mH, C = \frac{10^{-3}}{\pi}$ F and R = 10Ω, is connected across an ac source of 220V, 50Hz supply. The power factor of the circuit would be

Ans. **1**

Sol. $z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\omega L = 2\pi \times \frac{100}{\pi} \times 10^{-3} \times 50 = 10, R = 10$

$\frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 10^{-3}} \Rightarrow 10$

Z = R

$\cos \phi = 1$, (p. f. = 1)

56. A parallel beam of monochromatic light of wavelength 5000Å is incident normally on a single narrow slit of width 0.001mm . The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of (degree).

Ans. **30**
Sol. $d \sin \theta = \lambda$ for first minima

$1 \times 10^{-6} \sin \theta = 5 \times 10^{-7}$

$\sin \theta = ½$

$\theta = 30^0$

57. Two charges of −4μC and +4μC are placed at the points A(1, 0, 4) m and B(2, −1, 5) m located in an electric field $\vec{E} = 0.20\hat{i}\ V / cm$. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha} \times 10^{-5}$ Nm, where α =

Ans. **2**

Sol. $\vec{r}_A = \hat{i} + 4\hat{k}$

$\vec{r}_B = 2\hat{i} - \hat{j} + 5\hat{k}$

$\vec{P} = q(\vec{r}_B - \vec{r}_A)$

$\vec{P} = 4 \times 10^{-6}(\hat{i} - \hat{j} + \hat{k})$

$\vec{E} = 20\ \hat{i}$ V/m

$\vec{\tau} = \vec{P} \times \vec{E} = 8 \times 10^{-5}(\hat{k} + \hat{i}) = 8\sqrt{2} \times 10^{-5}$ Nm.

$\alpha = 2$

58. If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$ where α =

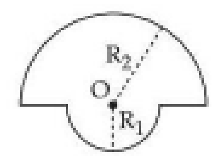
Ans. **144**

Sol. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$,

$n_1 = 3, n_2 = 4$

$\frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{16}\right] \Rightarrow \lambda = \frac{144}{7R}$, $\alpha = 144$

59. The magnetic field at the centre of a wire loop formed by two semi-circular wires of radii $R_1 = 2\pi$ m and $R_2 = 4\pi$ m, carrying current I = 4A as per figure given below is $\alpha \times 10^{-7}$T. The value of α is …………….. (Centre O is common for all segments)



Ans. **3**

Sol. $B = B_1 + B_2 = \frac{\mu_0 NI}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{4\pi \times 10^{-7} \times 4}{2}\left[\frac{1}{2\pi} + \frac{1}{4\pi}\right] = 3 \times 10^{-7}\,T$

*60. The reading of pressure metre attached with a closed pipe is 4.5×10^4 N/m^2. On opening the valve, water starts flowing and the reading of pressure metre falls to 2.0×10^4 N/m^2. The velocity of water is found to be $\sqrt{V}$ m/s. The value of V s

Ans. **50**

Sol. $\frac{1}{2}\rho v^2 + P = P_0$

$\frac{1}{2} \times 10^3 (v^2) = P_0 - P = 2.5 \times 10^4 \Rightarrow v = \sqrt{50}$, so, V = 50

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

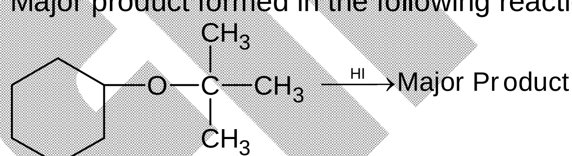
61. Given below are two statements:
Statement (I): Oxygen being the first member of group 16 exhibits only-2 oxidation state.
Statement (II): Down the group 16 stability of +4 oxidation state decreases and +6 oxidation state increases.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement I and Statement II are incorrect
(2) Statement I is correct but Statement II is incorrect
(3) Statement I is incorrect but Statement II is correct
(4) Both Statement I and Statement II are correct

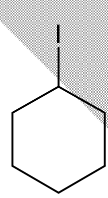
Ans. **(1)**
Sol. Both the statements are wrong.

62. Phenolic group can be identified by a positive :
(1) Phthalein dye test
(2) Carbylammine test
(3) Tollen's test
(4) Lucas test

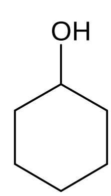
Ans. **(1)**
Sol. In pthalein dye test, phenol gives pink colour due to formation of phenolphthalein in basic medium.

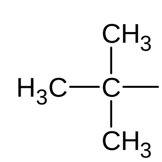
63. Major product formed in the following reaction is a mixture of :

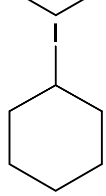


(1) 

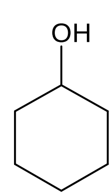
and $(CH_3)_3COH$

(2) 

and 

(3) 

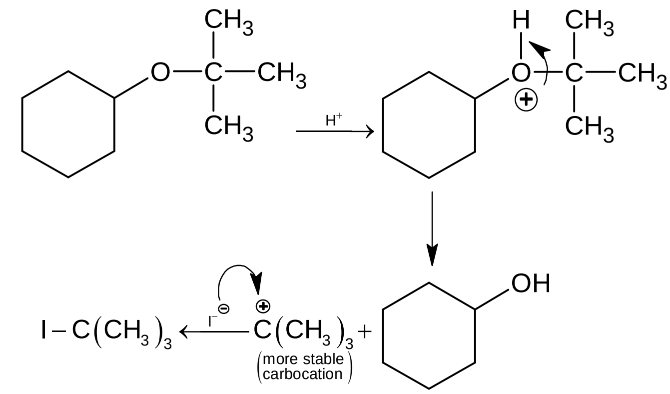
and $(CH_3)_3Cl$

(4) 

and $(CH_3)_3COH$

Ans. **(2)**

Sol.



Hence, major product is given in option (2).

64. Which structure of protein remains intact after coagulation of egg white on boiling?
(1) Tertiary (2) Secondary
(3) Quaternary (4) Primary

Ans. **(4)**
Sol. Primary structure of protein remains intact after coagulation of egg.

*65. Which of the following cannot function as an oxidizing agent?
(1) MnO_4^- (2) SO_4^{2-}
(3) N^{3-} (4) BrO_3^-

Ans. **(3)**
Sol. N^{3-} is in its lowest oxidation state. Hence, it can not undergo reduction. So can not act as an oxidisiing agent.

*66. The incorrect statement regarding conformations of ethane is :
(1) Eclipsed conformation is the most stable conformation
(2) Ethane has infinite number of conformations
(3) The conformations of ethane are inter-convertible to one-another
(4) The dihedral angle in staggered conformation is 60^0

Ans. **(1)**
Sol. Eclipsed conformation is the least stable conformation.

67. Choose the correct option having all the elements with d^{10} electronic configuration from the following
(1) ^{27}Co, ^{28}Ni, ^{26}Fe, ^{24}Cr (2) ^{46}Pd, ^{28}Ni, ^{26}Fe, ^{24}Cr
(3) ^{29}Cu, ^{30}Zn, ^{48}Cd, ^{27}Ag (4) ^{28}Ni, ^{24}Cr, ^{26}Fe, ^{29}Cu

Ans. **(3)**
Sol. Only option (3) is having all the elements in d^{10} electronic configuration.

68. Given below are two statements:
Statement (I): In the Lanthanoids the formation Ce^{+4} is favoured by its noble gas configuration
Statement (II): Ce^{+4} is a strong oxidant reverting to the common +3 state

In the light of the above statements, choose the most appropriate from the options given below:
(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

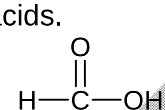
Ans. **(1)**
Sol. Both the statements are correct.

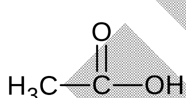
*69. The technique used for purification of steam volatile water immiscible substance is :
(1) steam distillation
(2) distillation
(3) fraction distillation under reduced pressure
(4) fractional distillation

Ans. **(1)**
Sol. Steam distillation is used for purification of steam volatile water immiscible substances.

70. The molecular formula of second homologue in the homologous series of mono carboxylic acid is
(1) $C_2H_4O_2$
(2) $C_3H_6O_2$
(3) CH_2O
(4) $C_2H_2O_2$

Ans. **(1)**
Sol. Acetic acid will be second homologue in the homologous series of mono carboxylic acids.

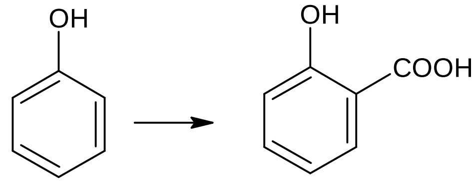
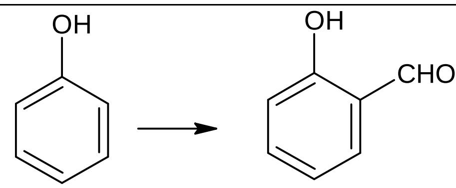
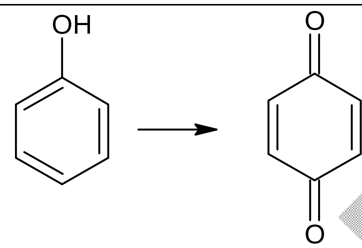
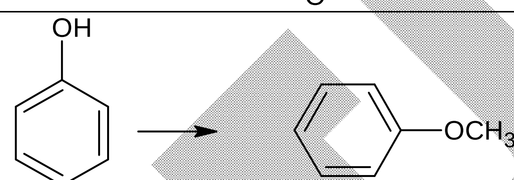

First homologue


Second homologue

71. Which of the following statements is not correct about rusting of iron?
(1) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.
(2) Dissolved acidic oxides SO_2, NO_2 in water act as catalyst in the process of rusting.
(3) When pH lies above 9 or 10, rusting of iron does not take place
(4) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

Ans. **(1)**
Sol. Tin is having lower standard oxidation potential than Fe. Hence, Sn can not prevent rusting of iron, if its coating is peeled off.

72. Match List I with List II

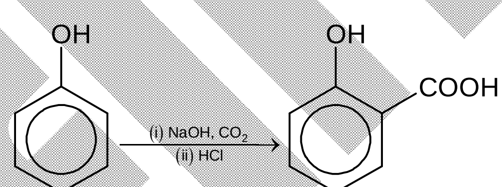
List –I (Reaction)	List-II (Reagent(s))
(A) 	(I) $Na_2Cr_2O_7, H_2SO_4$
(B) 	(II) (i) NaOH (ii) CH_3Cl
(C) 	(III) (i) NaOH, $CHCl_3$ (ii) NaOH (iii) HCl
(D) 	(IV) (i) NaOH (ii) CO_2 (iii) HCl

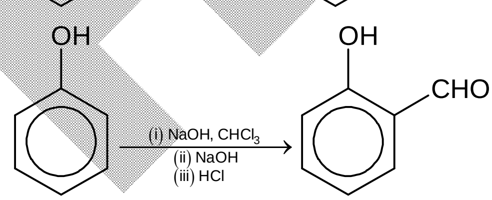
Choose the correct answer from the options given below :

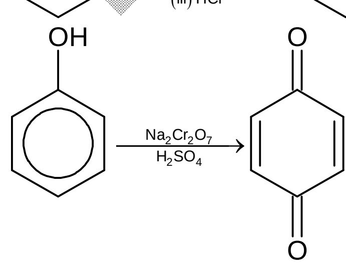
(1) (A) – (II), (B) – (I), (C) – (III), (D) – (IV) (2) (A) – (IV), (B) – (I), (C) – (III), (D) – (II)
(3) (A) – (IV), (B) – (III), (C) – (I), (D) – (II) (4) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

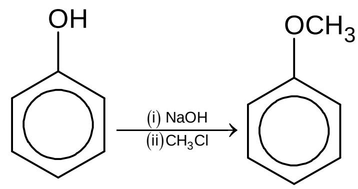
Ans. (3)

Sol.

(A) 

(B) 

(A) 

(A) 

73. Identify from the following species in which d^2sp^3 hybridization is shown by central atom :

(1) BrF_5 (2) $[Co(NH_3)_6]^{3+}$

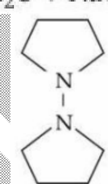
(3) $[Pt(Cl_4)]^{2-}$ (4) SF_6

Ans. (2)

Sol. $[Co(NH_3)_6]^{3+}$ will form inner orbital complex. Hence its hybridization will be d^2sp^3.

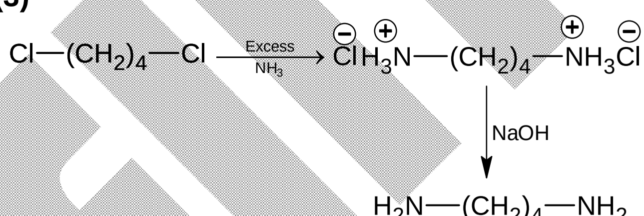
74. Identify B formed in the reaction

$$Cl-(CH_2)_4-Cl \xrightarrow{\text{excess } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$$

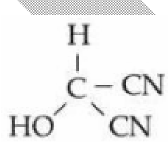
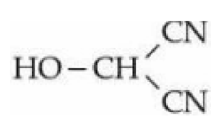
(1) $Cl\overset{+}{N}H_3-(CH_2)_4-\overset{+}{N}H_3Cl^-$ (2) 

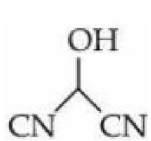
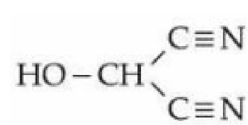
(3) $H_2N-(CH_2)_4-NH_2$ (4) 

Ans. (3)

Sol. 

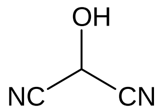
75. Bond line formula of $HOCH(CN)_2$ is

(1)  (2) 

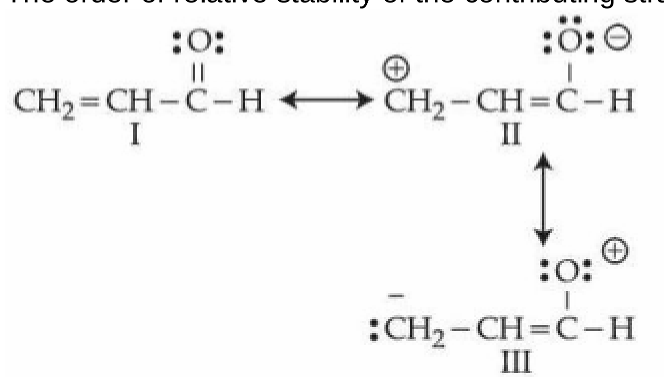
(3)  (4) 

Ans. (3)

Sol. Correct representation of bond line formula for given compound is



*76. The order of relative stability of the contributing structure is:



Choose the correct answer from the option given below :

(1) I > II > III
(2) I = II = III
(3) III > II > I
(4) II > I > III

Ans. **(1)**

Sol. (i) Neutral > charged

(ii) Positive charge on electropositive and negative on electronegative atom is more stable.

I > II > III

*77. The quantity which changes with temperature is :

(1) Mole fraction
(2) Molarity
(3) Mass percentage
(4) Molality

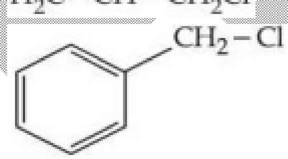
Ans. **(2)**

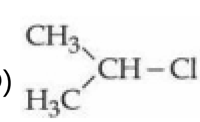
Sol. Molarity depends upon volume of solution, which changes with temperature.

78. Which among the following halide/s will not show S_N1 reaction :

(A) $H_2C=CH-CH_2Cl$

(B) $CH_3-CH=CH-Cl$

(C) 

(D) 

Choose the most appropriate answer from the options given below :

(1) (A), (B) and (D) only
(2) (B) and (C) only
(3) (B) only
(4) (A) and (B) only

Ans. **(3)**

Sol. Vinylic carbocation is highly unstable. So, it does not show S_N1 reaction.

79. Identify the incorrect pair from the following

(1) Polythene preparation $-TiCl_4, Al(CH_3)_3$

(2) Photography - AgBr

(3) Haber process –Iron

(4) Wacker process –Pt Cl_2

Ans. **(4)**

Sol. In the wacker process, $PdCl_2$ is used as catalyst.

*80. The final product A, formed in the following reaction sequence is :

$$Ph-CH=CH_2 \xrightarrow[\text{(iii) HBr} \quad \text{(iv) Mg, ether, then } HCHO/H_3O^+]{\text{(i) } BH_3 \quad \text{(ii) } H_2O_2, {}^{\ominus}OH} A$$

(1) $Ph-\underset{\displaystyle CH_2OH}{\underset{|}{CH}}-CH_3$

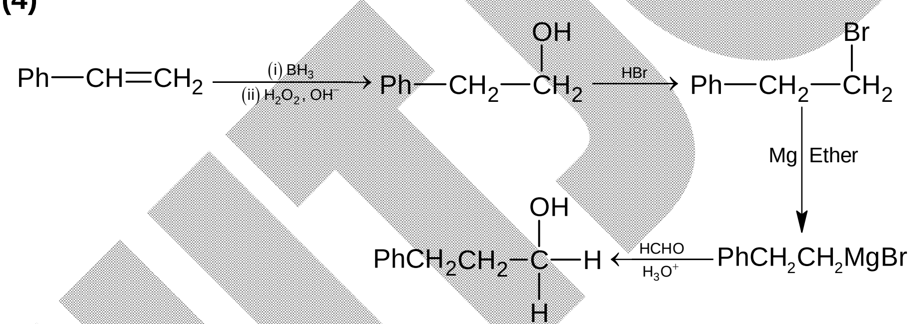
(2) $Ph-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_3$

(3) $Ph-CH_2-CH_2-CH_3$

(4) $Ph-CH_2-CH_2-CH_2-OH$

Ans. **(4)**

Sol.



SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

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81. The hydrogen electrode is dipped in a solution of pH = 3 at 25^{0}C. The potential of the electrode will be _______ $\times 10^{-2}$.

$$\left(\frac{2.303RT}{F} = 0.059\ V\right)$$

Ans. **18**

Sol. $2H^+ + 2e^- \longrightarrow H_2(g)$

$E_{H^+/H_2} = 0$

Given pH = 3

$\therefore [H^+] = 10^{-3}$

$E^0_{H^+/H_2} = 0 - \frac{0.0591}{2}\log\frac{pH_2}{\left[H^+\right]^2}$

$= -\frac{0.0591}{2}\log\frac{1}{10^{-6}}$

$= -\frac{0.0591}{2}\times 6$

$= -0.0591 \times 3$ V

$= -\ 0.1773$ V

$= 17.7 \times 10^{-2}$ V

Nearest integer is 18.

*82. Volume of 3 M NaOH (formula weight 40 g mol^{-1}) which can be prepared from 84 g of NaOH is ____________ $\times 10^{-1}$ dm^3.

Ans. **7**

Sol. Number of moles of NaOH = $\frac{84\,g}{40\ g/mol}$

= 2.1 moles

Let the volume be x dm^3

M_1V_1 = number of moles = 2.1

$\Rightarrow 3 \times x = 2.1$

x = 0.7 dm^3 = 7×10^{-1} dm^3

83. Total number of ions from the following with noble gas configuration is _________ .

$Sr^{2+}\ (z = 38)$, $Cs^+\ (z = 55)$, $La^{2+}\ (z = 57)$, $Pb^{2+}\ (z = 82)$, $Yb^{2+}\ (z = 70)$ and $Fe^{2+}\ (z = 26)$

Ans. **2**

Sol. Sr^{2+} and Cs^+ are having noble gas configuration.

84. The spin only magnetic moment value of square planar complex $\left[Pt(NH_3)_2Cl(NH_2CH_3)\right]Cl$ is

___________ B.M. (Nearest integer)

(Given atomic number for Pt = 78)

Ans. **0**

Sol. $\left[Pt(NH_3)_2Cl(NH_2CH_3)\right]Cl \Rightarrow Pt^{2+} \rightarrow d^8$ (square planar)

$\therefore$ Number of unpaired electron = 0

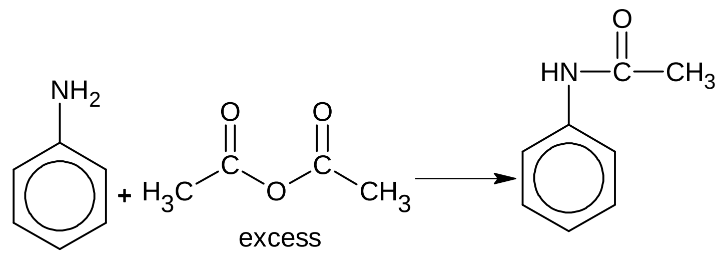
$\mu_{spin} = 0$

*85. 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _______ $\times 10^{-1}$ g.

(Given molar mass in g mol^{-1} N : 14, O : 16, C : 12, H = 1)

Ans. **135**

Sol.



Mass 9.3 g

moles $\frac{9.3}{93} = 0.1$ 0.1

Mass of product formed will be
$0.1 \times$ molar mass of acetanilide = 0.1×135 g
= 13.5 g = 135×10^{-1} g

*86. For a certain thermochemical reaction
$M \rightarrow N$ at $T = 400$ K, $\Delta H^{\circ} = 77.2$ kJ mol^{-1}, $\Delta S = 122$ JK^{-1}
log equilibrium constant (log K) is __________ $\times 10^{-1}$

Ans. **37**

Sol.
$\Delta G^0 = \Delta H^0 - T\Delta S^0$
$\Delta G = \Delta G^0 + RT \ln K_{eq}$
$\Delta G^0 = -RT \ln K_{eq}$
$\Delta G^0 = 77.2$ kJ/mol $- 400 \times 0.122$ kJ/mol
= 77.2 kJ/mol – 48.8 kJ/mol = 28.4 kJ/mol
28.4 kJ/mol = $-8.314 \times 400 \ln K$

$$\Rightarrow \ln K = -\frac{28400}{8.314 \times 400}$$

$$\Rightarrow 2.303 \log K = -\frac{284}{8.314 \times 4}$$

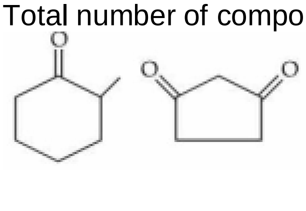
$\Rightarrow \log K = -3.708$
$= -37.08 \times 10^{-1}$

*87. 1 mole of PbS is oxidized by 'X' moles of O_3 to get 'Y' moles of O_2, X + Y = __________

Ans. **3**

Sol. The balanced chemical eqation is
$PbS + 2O_3 \longrightarrow PbSO_4 + O_2$
X is 2 and Y is 1.
So, X + Y = 2 + 1 = 3

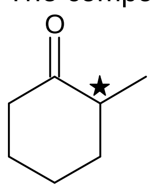
*88. Total number of compounds with Chiral carbon atoms from following is __________ .



$CH_3-CH_2-CH(NO_2)-COOH$
$CH_3-CH_2-CHBr-CH_2-CH_3$
$CH_3-CH(I)-CH_2-NO_2$
$CH_3-CH_2-CH(OH)-CH_2OH$
$CH_3-\underset{\displaystyle I}{\underset{|}{C}H}-CH(I)-C_2H_5$

Ans. 5

Sol. The compounds with chiral atoms are:



$H_3C-CH_2-\overset{*}{C}H(NO_2)-COOH$

$CH_3\overset{*}{C}H(I)-CH_2-NO_2$

$CH_3CH_2\overset{*}{C}H(OH)CH_2OH$

$H_3C-\underset{\displaystyle I}{\underset{|}{\overset{*}{C}H}}-\overset{*}{C}H(I)-C_2H_5$

89. Time required for completion of 99.9 % of a first order reaction is ________ times of half life.
($t_{1/2}$) of the reaction

Ans. 10

Sol. $kt = 2.303\log\frac{a_o}{a_t}$

$t = \frac{2.303}{\frac{\ln 2}{t_{1/2}}}\log\frac{a_o}{a_t}$

$\Rightarrow t_{99.9\%} = \frac{2.303}{0.693}t_{1/2}\log\frac{100}{0.1}$

$= \frac{2.303}{0.693}.3.t_{1/2} \simeq 10$

*90. The number of non-polar molecules from the following is __________ .
$HF, H_2O, SO_2, H_2, CO_2, CH_4, NH_3, HCl, CHCl_3, BF_3$

Ans. 4

Sol. The non polar molecules are H_2, CO_2, CH_4 and BF_3.