FIITJEE

Solutions to JEE(Main) -2023

Test Date: 6th April 2023 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

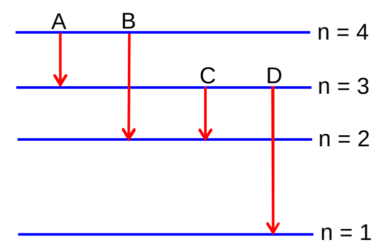
SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. For the plane electromagnetic wave given by $E = E_o \sin(\omega t - kx)$ and $B = B_o \sin(\omega t - kx)$. the ratio of average electric energy density to average magnetic energy density is
(A) 4
(B) 2
(C) 1/2
(D) 1

Q2. The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is
(A) C
(B) A
(C) B
(D) D

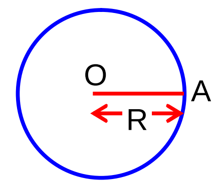


Q3. The number of air molecules per cm^3 increased from 3×10^{19} to 12×10^{19}. The ratio of collision frequency of air molecules before and after the increase in number respectively is:
(A) 0.25
(B) 0.50
(C) 0.75
(D) 1.25

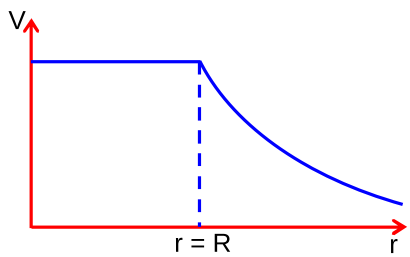
Q4. The induced emf can be produced in a coil by
A. moving the coil with uniform speed inside uniform magnetic field
B. moving the coil with non uniform speed inside uniform magnetic field
C. rotating the coil inside the uniform magnetic field
D. changing the area of the coil inside the uniform magnetic field
Choose the correct answer from the options given below:
(A) C and D only
(B) B and D only
(C) B and C only
(D) A and C only

Q5. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A : When a body is projected at an angle 45^o, it's range is maximum.
Reason R : For maximum range, the value of $\sin 2\theta$ should be equal to one.
In the light of the above statements, choose the correct answer from the options given below :
(A) **A** is false but **R** is true
(B) **A** is true but **R** is false
(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(D) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

Q6. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the centre (O) of shell can be graphically represented as -



(A)



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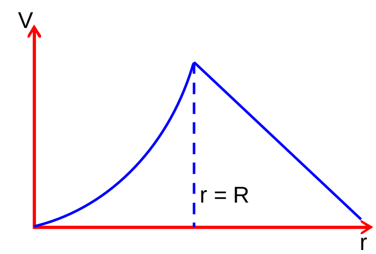

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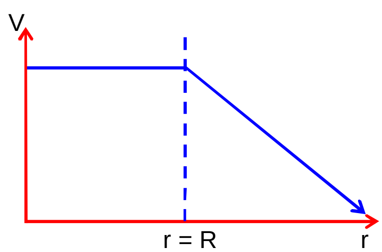

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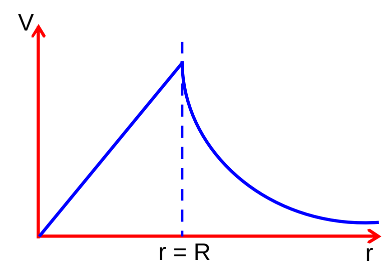


Q7. A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weight on that planet:

(A) 2W
(B) $2^{2/3}$ W
(C) $2^{1/3}$ W
(D) W

Q8. Given below are statements : One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Earth has atmosphere whereas moon doesn't have any atmosphere.

Reason R : The escape velocity on moon is very small as compared to that on earth.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(B) **A** is true but **R** is false
(C) **A** is false but **R** is true
(D) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Q9. A small ball of mass M and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with a constant velocity. What is the viscous force on the ball?

(A) $F = Mg\left(1+\frac{\rho_0}{\rho}\right)$
(B) $F = Mg\left(1-\frac{\rho_0}{\rho}\right)$
(C) $F = Mg\left(1+\frac{\rho}{\rho_0}\right)$
(D) $F = Mg\left(1\pm\rho\rho_0\right)$

Q10. Two resistances are given as $R_1 = (10\pm0.5)\Omega$ and $R_2 = (15\pm0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is –

(A) 2.33
(B) 5.33
(C) 6.33
(D) 4.33

Q11. A monochromatic light wave with wavelength λ_1 and frequency v_1 in air enters another medium. If the angle of incidence and angle of refraction at the interface are $45°$ and $30°$ respectively, then the wavelength λ_2 and frequency v_2 of the refracted wave are :

(A) $\lambda_2 = \lambda_1, v_2 = \sqrt{2}v_1$
(B) $\lambda_2 = \lambda_1, v_2 = \frac{1}{\sqrt{2}}v_1$
(C) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1, v_2 = v_1$
(D) $\lambda_2 = \sqrt{2}\lambda_1, v_2 = v_1$

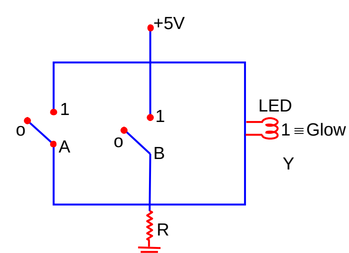
Q12. A particle is moving with constant speed in a circular path. When the particle turns by an angle $90°$, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be.

(A) 7
(B) 1
(C) 2
(D) 5

Q13. A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across the cross-section. The magnetic field is

(A) Inversely proportional to r in the region r < a and uniform through in the region r > a
(B) uniform in the region r < a and inversely proportional to distance r from the axis, in the region r > a
(C) zero in the region r < a and inversely proportional to r in the region r > a
(D) directly proportional to r in the region r < a and inversely proportional to r in the region r > a

Q14. Name the logic gate equivalent to the diagram attached

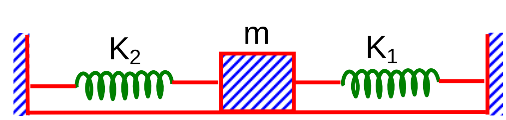


(A) AND
(B) OR
(C) NAND
(D) NOR

Q15. A small block of mass 100g is tied to a spring of spring constant 7.5N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A. then tension in the spring is -

(A) 1.5 N
(B) 0.50 N
(C) 0.75 N
(D) 0.25 N

Q16. A mass m is attached to two strings as shown in figure. The spring constants of two springs are K_1 and K_2 . For the frictionless surface, the time period of oscillation of mass m is



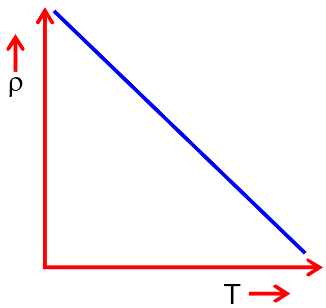
(A) $2\pi\sqrt{\frac{m}{K_1+K_2}}$
(B) $\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$
(C) $2\pi\sqrt{\frac{m}{K_1-K_2}}$
(D) $\frac{1}{2\pi}\sqrt{\frac{K_1-K_2}{m}}$

Q17. The kinetic energy of an electron, $\alpha -$ particle and a proton are given as 4K, 2K and K are respectively. The de-Broglie wavelength associated with electron (λe), α -particle $((\lambda\alpha)$ and the proton (λp) are as follows :

(A) $\lambda\alpha = \lambda p > \lambda e$
(B) $\lambda\alpha < \lambda p < \lambda e$
(C) $\lambda\alpha = \lambda p < \lambda e$
(D) $\lambda\alpha > \lambda p > \lambda e$

Q18. A source supplies heat to a system at the rate of 100W. If the system performs work at a rate of 200W. The rate at which internal energy of the system increases is
(A) 600W (B) 500W
(C) 1200W (D) 800W

Q19. The receptivity (ρ) of semiconductor varies with temperature. Which of the following curve represents the correct behaviour

(A)


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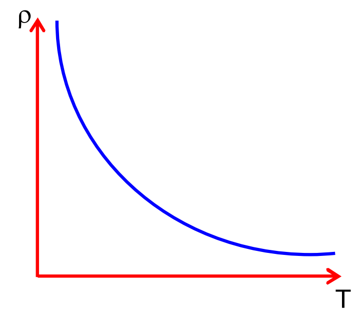

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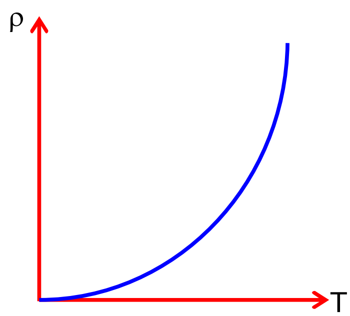

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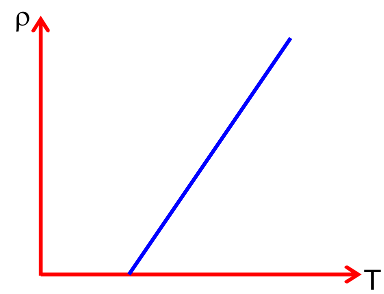


Q20. By what percentage will the transmission range of a TV tower be affected when the height of the tower in increased by 21% ?
(A) 15% (B) 10%
(C) 12% (D) 14%

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. A steel rod has a radius of 20mm and a length of 2.0m. A force of 62.8kN stretches it along its length. Young's modulus of steel is $2.0\times10^{11}N/m^2$. The longitudinal strain produced in the wire is ________ $\times10^{-5}$

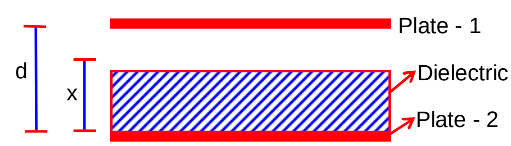
Q2. A particle of mass 10g moves in a straight line with retardation 2x, where x is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be ______

Q3. A pole is vertically submerged in swimming pool, such that it gives a length of shadow 2.15m within water when sunlight is incident at an angle of 30° with the surface of water. If swimming pool is filled to a height of 1.5m, then the height of the pole above the water surface in centimetres is $(n_w = 4/3)$ _________

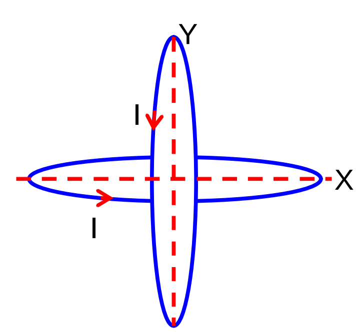
Q4. Two identical solid sphere each of mass 2kg and radii 10cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is _________ $\times10^{-3}kg-m^2$

Q5. An ideal transformer with purely resistive load operates at 12kV on the primary side. It supplies electrical energy to a number of nearby houses at 120V. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (Rs) required in the secondary circuit will be ________ $m\Omega$.

Q6. A parallel plate capacitor with plate area A and plate separation d is filled with a dielectric material of dielectric constant K = 4. The thickness of the dielectric material is x, where x < d. Let C_1 and C_2 be the capacitance of the system for $x=\frac{1}{3}d$ and $x=\frac{2d}{3}$. respectively. If $C_1 = 2\mu F$ the value of C_2 is ________ μF



Q7. Two identical circular wires of radius 20cm and carrying current $\sqrt{2}$ A are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is ________ $\times10^{-8}$ T.



Q8. The radius of fifth orbit of the Li^{++} is ________ $\times 10^{-12}$ m. Take : radius of hydrogen atom = 0.51 $\overset{o}{A}$

Q9. The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is ________

Q10. A person driving car at a constant speed of 15m/s is approaching a vertical wall. The person notices a change of 40Hz in the frequency of his horn upon reflection from the wall. The frequency of horn is _________Hz.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below: are two statements one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Loss of electron from hydrogen atom results of $\sim 1.5\times10^{-3}$ pm size.
Reason R: Proton (H^+) always exists in combined form.
In the light of the above statements, choose the most appropriate answer from the options given below.
(A) Both A and R are correct but R is NOT the correct explanation of A
(B) Both A and R are correct and R is the correct explanation of A
(C) A is not correct but R is correct
(D) A is correct but R is not correct

Q2. The possibility of photochemical smog formation is more at
(A) Marshy lands
(B) Industrial areas
(C) Himalayan villages in winter
(D) The places with healthy vegetation

Q3. For a concentrated of a weak electrolyte (K_{eq}=equilibrium constant) A_2B_2 of concentration 'c', the degree of dissociation 'a' is
(A) $\left(\frac{K_{eq}}{6c^5}\right)^{\frac{1}{5}}$
(B) $\left(\frac{K_{eq}}{25c^2}\right)^{\frac{1}{5}}$
(C) $\left(\frac{K_{eq}}{5c^4}\right)^{\frac{1}{5}}$
(D) $\left(\frac{K_{eq}}{180c^4}\right)^{\frac{1}{5}}$

Q4. A compound is formed by two elements X and Y. The element Y forms cubic close packed arrangement and those of element X occupy one third of the tetrahedral voids. What is the formula of the compound?
(A) XY_3
(B) X_3Y_2
(C) X_3Y
(D) X_2Y_3

Q5. For the reaction

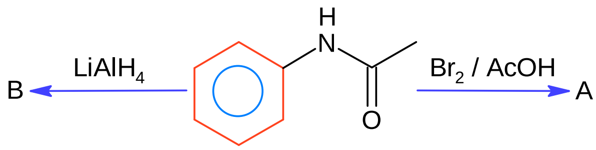
$$RCH_2Br + I^- \xrightarrow{\text{Acetone}} \underset{\text{major}}{RCH_2I} + Br^-$$

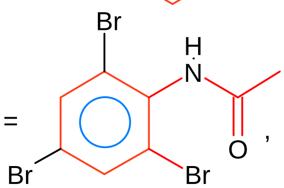
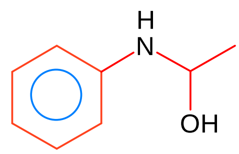
The correct statement is
(A) The reaction can occur in acetic acid also.
(B) The transition state formed in the above reaction is less polar than the localised anion.
(C) The solvent used in the reaction solvates the ions in rate determining step.
(D) Br^- can act as competing nucleophile.

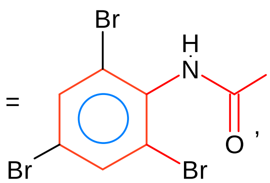
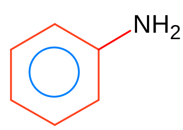
Q6. Polymer used in orlon is:
(A) Polycarbonate
(B) Polyamide
(C) Polyacrylonitrile
(D) Polyethene

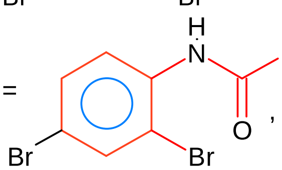
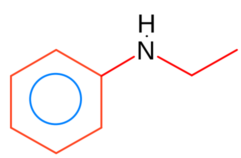
Q7. The difference between electron gain enthalpies will be maximum between:
(A) Ne and Cl
(B) Ar and Cl
(C) Ne and F
(D) Ar and F

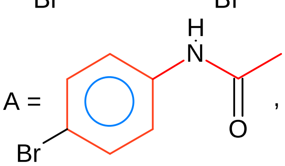
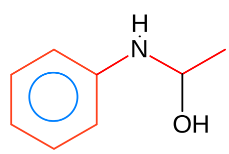
Q8. The major products A and B from the following reactions are:



(A) A =  , B = 

(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

Q9. The standard electrode potential of M^+ / M in aqueous solution does not depend on
(A) Ionisation of a gaseous metal atom
(B) Hydration of a gaseous metal ion
(C) Ionisation of a solid metal atom
(D) Sublimation of a solid metal

Q10. Match List I with List II

	List I- Enzymatic reaction		List II Enzyme
A.	Sucrose $\rightarrow$ Glucose and Fructose	I.	Zymase
B.	Glucose $\rightarrow$ ethyl alcohol and CO_2	II.	Pepsin
C.	Starch $\rightarrow$ Maltose	III.	Invertase
D.	Proteins $\rightarrow$ Amino acids	IV.	Diastase

Choose the corect answer from the option given below :
(A) A-I, B- IV, C-III, D-II
(B) A-I, B-II, C-IV, D-III
(C) A-III, B-I, C-II, D-IV
(D) A-III, B-I, C-IV, D-II

Q11. Match List I with List II

	List I- Oxide		List II Type of bond
A.	N_2O_4	I.	1N = O bond
B.	NO_2	II.	1N –O–N bond
C.	N_2O_5	III.	1N–N bond
D.	N_2O	IV.	1N=N / N≡N bond

Choose the corect answer from the option given below :

(A) A-II, B- IV, C-III, D-I
(B) A-III, B-I, C-IV, D-II
(C) A-II, B-I, C-III, D-IV
(D) A-III, B-I, C-II, D-IV

Q12. Match List I with List II

List I- Vitamin		List II Deficiency disease	
A.	Vitamin A	I.	Beri-Beri
B.	Thiamine	II.	Cheilosis
C.	Ascorbic acid	III.	Xeropthalmia
D.	Riboflavin	IV.	Scurvy

Choose the corect answer from the option given below :

(A) A-IV, B- I, C-III, D-II
(B) A-III, B-II, C-IV, D-I
(C) A-IV, B-II, C-III, D-I
(D) A-III, B-I, C-IV, D-II

Q13. The setting time of Cement is increased by adding

(A) Silica
(B) Gypsum
(C) Limestone
(D) Clay

Q14. Which of the following options are correct for the reaction

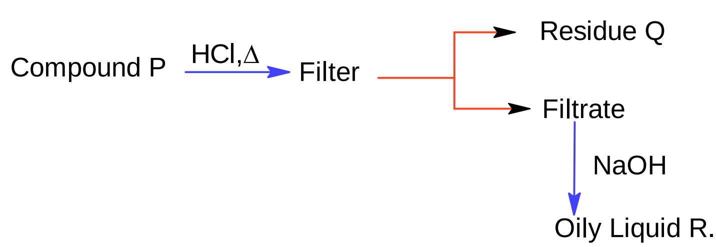
$$2\left[Au\left(CN_2\right)\right]^- (aq) + Zn(s) \rightarrow 2Au(s) + \left[Zn(CN)_4\right]^{2-} (aq)$$

A. Redox reaction
B. Displacement reaction
C. Decomposition reaction
D. Commination reaction

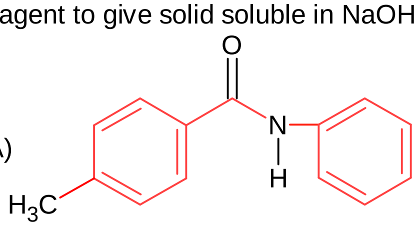
Choose the correct answer from the options given below :

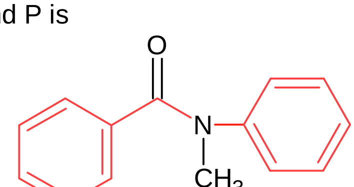
(A) A only
(B) A and B only
(C) A and D only
(D) C and D only

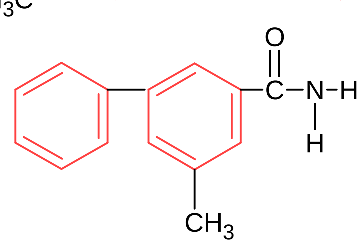
Q15.



Compound P is neutral, Q gives effervescence with $NaHCO_3$ while R reacts with Hinsbergs reagent to give solid soluble in NaOH. Compound P is

(A)


(B)


(C)


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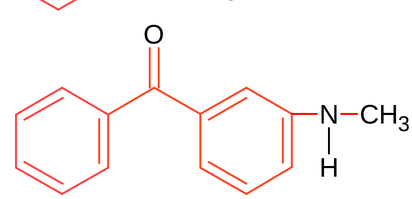


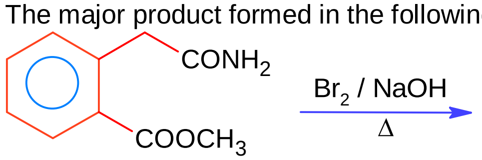
Q16. Match List I with List II

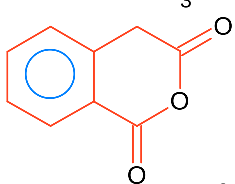
List I- Element detected		**List II Reagent used / Product formed**	
A.	Nitrogen	I.	$Na_2[Fe(CN)_5NO]$
B.	Sulphur	II.	$AgNO_3$
C.	Phosphorous	III.	$Fe_4[Fe(CN)_6]_3$
D.	Halogen	IV.	$(NH_4)_2MoO_4$

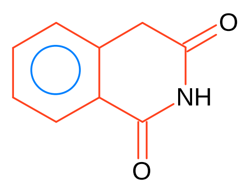
Choose the corect answer from the option given below :
(A) A-II, B- I, C-IV, D-III
(B) A-IV, B-II, C-I, D-III
(C) A-III, B-I, C-IV, D-II
(D) A-II, B-IV, C-I, D-III

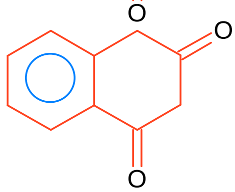
Q17. Strong reducing and oxidizing agents among the following respectively, are
(A) Eu^{2+} and Ce^{4+}
(B) Ce^{4+} and Tb^{4+}
(C) Ce^{4+} and Eu^{2+}
(D) Ce^{3+} and Ce^{4+}

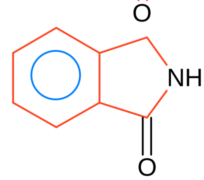
Q18. The major product formed in the following reaction is



(A)


(B)


(C)


(D)


Q19. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: The spin only magnetic moment value for $[Fe(CN)_6]^{3-}$ is 1.74 BM, whereas for $\left[Fe(H_2O)_6]^{3+}\right]$ is 5.92 BM,

Reason R: In both complexes, Fe is present in +3 oxidation state.
In the light of the above statements, choose the correct answer from the options given below:
(A) Both A and R true and R is the correct explanation of A
(B) A is false but R is true
(C) Both A and R are true but R is NOT the correct explanation of A
(D) A is true but R is false.

Q20. Match List I with List II

List I- Name of reaction		**List II Reagent used**	
A.	Hell-Volhrd-Zelinsky reaction	I.	$NaOH+I_2$
B.	Iodoform reaction	II.	(i) CrO_2Cl_2, CS_2 (ii) H_2O
C.	Etard reaction	III.	(i) Br_2 / red phosphorus (ii) H_2O
D.	Gatterman-Koch reaction	IV.	CO, HCl anhyd. $AlCl_3$

Choose the corect answer from the option given below :
(A) A-III, B- I, C-IV, D-II
(B) A-I, B-II, C-III, D-IV
(C) A-III, B-I, C-II, D-IV
(D) A-III, B-II, C-I, D-IV

SECTION - B

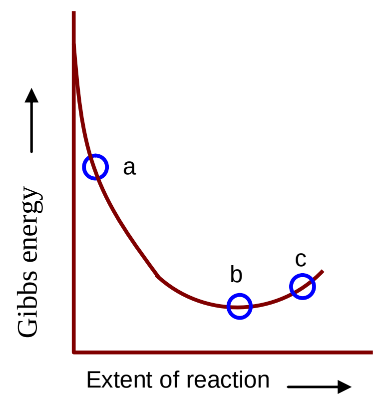
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Mass of Urea (NH_2CONH_2) required to be dissolved in 1000g of water in order reduce the vapour pressure of water by 25% is _______g. (Nearest integer)
Given: Molar mass of N,C,O and and H are 14,12,16 and 1 g mol^{-1}respectively.

Q2. Consider the graph of Gibbs free energy G vs Extent of reaction. The number of statement/s from the following which are true with respect to points (a), (b) and (c) is_____.

A. Reaction is spontaneous at (a) and (b)
B. Reaction is at equilibrium at point (b) and non-spontaneous at point (c)
C. Reaction is spontaneous at (a) and non-spontaneous at (c)
D. Reaction is non-spontaneous at (a) and (b)



Q3. Number of ambidentate ligands in a respective metal complex $[M(en)(SCN)_4]$ is_______.
[En = ethylenediamine]

Q4. The value of log K for the reaction $A \rightleftharpoons B$ at 298 K is__________. (Nearest integer)
Given: $\Delta H^o = -54.07$ kJ mol^{-1}
$\Delta S^o = 10$ J K^{-1} mol^{-1}
(Take $2.303 \times 8.314 \times 298 = 5705$)

Q5. For the adsorption of hydrogen on platinum, the activation energy is 30 kJ mol^{-1} and for the adsorption of hydrogen on nickel, the activation is 41.4kJ mol^{-1}. The logarithm of the ratio of the rates of chemisorptions on equal areas of the metals at 300K is _________(Nearest integer)
Given: ln 10 = 2.3
R = 8.3 JK^{-1} mol^{-1}

Q6. The number of species from the following which have square pyramidal structure is_____
$PF_5, BrF_4^-, IF_5, BrF_5, XeOF_4, ICl_4^-$

Q7. In ammonium – phosphomolybdate, the oxidation state of Mo is+______________.

Q8. The wavelength of an electron of kinetic energy 4.50×10^{-5}m, (Nearest integer)
Given: mass of electron is 9×10^{-31}kg, h= 6.6×10^{-34}Js.

Q9. Number of bromo derivatives obtained on treating ethane with excess of Br_2 in diffused sunlight is______________.

Q10. If 5 moles of $BaCl_2$ is mixed with 2 moles of Na_3PO_4, the maximum number of moles of $Ba_3(PO_4)_2$formedis_________(Nearest integer)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set $S=\{\lambda\in\mathbb{R}:$ the points A, B, C and D are coplanar$\}$. Then $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to

(A) 41 (B) 25

(C) $\frac{37}{2}$ (D) 13

Q2. If $2x^y+3y^x=20$, then $\frac{dy}{dx}$ at $(2,2)$ is equal to :

(A) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$ (B) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

(C) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$ (D) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

Q3. One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex $(3,4,5)$. Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :

(A) $\frac{12}{\sqrt{5}}$ (B) $12\sqrt{5}$

(C) $\frac{12}{5\sqrt{5}}$ (D) $\frac{12}{5}$

Q4. The straight lines ℓ_1 and ℓ_2 pass through the origin and trisect the line segment of the line $L: 9x+5y=45$ between the axes. If m_1 and m_2 are the slopes of the lines ℓ_1 and ℓ_2, then the point of intersection of the line $y=(m_1+m_2)x$ with L lies on

(A) $y-x=5$ (B) $y-2x=5$

(C) $6x-y=15$ (D) $6x+y=10$

Q5. If the equation of the plane passing through the line of intersection of the planes $2x-y+z=3$, $4x-3y+5z+9=0$ and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is $ax+by+cz+6=0$, then $a+b+c$ is equal to

(A) 13 (B) 15

(C) 14 (D) 12

Q6. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a} \cdot \vec{d} = 18$, then $\left|\vec{a} \times \vec{d}\right|^2$ is equal to

(A) 640 (B) 760
(C) 720 (D) 680

Q7. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to

(A) 164 (B) 75
(C) 123 (D) 82

Q8. Let $A = \{x \in R : [x+3] + [x+4] \le 3\}$, $B = \left\{x \in R : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, where $[t]$ denotes greatest integer function. Then,

(A) A = B (B) $A \subset B, A \ne B$
(C) $B \subset C, A \ne B$ (D) $A \cap B = \phi$

Q9. Let $I(x) = \int \frac{x^2 \left(x\sec^2 x + \tan x\right)}{\left(x\tan x + 1\right)^2} dx$. If $I(0) = 0$, then $I\left(\frac{\pi}{4}\right)$ is equal to

(A) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$ (B) $\log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$

(C) $\log_e \frac{(\pi+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$ (D) $\log_e \frac{(\pi+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

Q10. The sum of all the roots of the equation $\left|x^2 - 8x + 15\right| - 2x + 7 = 0$ is :

(A) $9 + \sqrt{3}$ (B) $9 - \sqrt{3}$
(C) $11 + \sqrt{3}$ (D) $11 - \sqrt{3}$

Q11. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom of Q of a vertical tower PQ are 15^0 and 60^0 respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to

(A) $300\left(\sqrt{3} - 1\right)$ (B) $200\left(3 - \sqrt{3}\right)$
(C) $600\left(\sqrt{3} - 1\right)$ (D) $300\left(\sqrt{3} + 1\right)$

Q12. If the system of equations

$x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$

has infinitely many solutions, then $2a + 3b$ is equal to

(A) 20 (B) 23
(C) 25 (D) 28

Q13. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to

(A) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$
(B) $(P \vee R) \Rightarrow Q$
(C) $(P \Rightarrow R) \vee (Q \Rightarrow R)$
(D) $(P \wedge R) \Rightarrow Q$

Q14. Let $a_1, a_2, a_3, ...a_n$ be n positive consecutive terms of an arithmetic progression. If d > 0 is its common difference, then

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+.....+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right) \text{ is}$$

(A) $\sqrt{d}$
(B) 0
(C) 1
(D) $\frac{1}{\sqrt{d}}$

Q15. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to

(A) 9
(B) 12
(C) 11
(D) 10

Q16. The sum of the first 20 terms of the series $5+11+19+29+41+...$ is

(A) 3450
(B) 3520
(C) 3420
(D) 3250

Q17. Let $A = \left[a_{ij}\right]_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $3a^2 + 4b^2$ is equal to

(A) 7
(B) 14
(C) 4
(D) 3

Q18. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)dx$ is equal to :

(A) $10\log_e 2 - 6$
(B) $5\log_e 2 - 3$
(C) $5\log_e 2 + 3$
(D) $10\log_e 2 + 6$

Q19. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$, then the third term from the beginning is :

(A) $30\sqrt{2}$
(B) $60\sqrt{2}$
(C) $60\sqrt{3}$
(D) $30\sqrt{3}$

Q20. If ${}^{2n}C_3 : {}^{n}C_3 = 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is

(A) 27:11
(B) 65:37
(C) 35:16
(D) 2:1

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Let $y = y(x)$ be a solution of the differential equation $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to…….

Q2. Let $A = \{1, 2, 3, 4, ..., 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \left\{(a,b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\right\}$ is………..

Q3. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is………..

Q4. Let the point $(p, p+1)$ lie inside the region $E = \left\{(x,y) : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to……….

Q5. Let the image of the point $P(1, 2, 3)$ in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are $(6, 10, 7)$, then the square of the area of the triangle PQR is………..

Q6. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $P(1, 3)$ on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to…………

Q7. If the area of the region $S = \left\{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\right\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to………..

Q8. Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\le t$. Then the number of points, where the function $f(x) = [a + 13\sin x]$, $x \in (0, \pi)$ is not differentiable, is……….

Q9. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is………..

Q10. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is…………..

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KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	D	2.	D	3.	**A**	4.	A
5.	C	6.	A	7.	**C**	8.	D
9.	B	10.	D	11.	**C**	12.	C
13.	D	14.	D	15.	**C**	16.	A
17.	B	18.	D	19.	B	20.	B

SECTION – B

1.	25	2.	2	3.	50	4.	176
5.	240	6.	3	7.	628	8.	425
9.	25	10.	420				

PART – B (CHEMISTRY)

SECTION - A

1.	A	2.	B	3.	D	4.	D
5.	B	6.	C	7.	A	8.	C
9.	C	10.	D	11.	D	12.	D
13.	B	14.	B	15.	A	16.	C
17.	A	18	C	19.	C	20.	B

SECTION - B

1.	1111	2.	2	3.	4	4.	10
5.	2	6.	3	7.	6	8.	7
9.	9	10.	1				

PART – C (MATHEMATICS)

SECTION - A

1.	A	2.	A	3.	D	4.	A
5.	C	6.	C	7.	C	8.	A
9.	A	10.	A	11.	C	12.	B
13.	B	14.	C	15.	D	16.	B
17.	C	18.	A	19.	C	20.	D

SECTION - B

1.	2	2.	18	3.	121	4.	3
5.	594	6.	292	7.	5	8.	25
9.	3483638676	10.	5005				

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Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. $\frac{U_E}{U_B} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{B^2}{2\mu_0}} = (\varepsilon_0\mu_0)\left(\frac{E}{B}\right)^2 = \frac{1}{c^2} \times c^2$

Sol2. As we know that

$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda \propto \frac{1}{\Delta E}$

Now, for the shortest wavelength, the energy gap must be max.
Thus, $n = 3 \to n = 1$, is the correct option

Sol3. Collision frequency,

$f = \frac{v}{\lambda}$

or $f = \frac{v}{\left(\frac{1}{\sqrt{2}\pi d^2 n_v}\right)} = \sqrt{2}\pi d^2 v n_v$

$\therefore f \propto n_v$, where n_v = no. density

or $\frac{f_1}{f_2} = \frac{n_{v_1}}{n_{v_2}} \Rightarrow \frac{f_1}{f_2} = \frac{3 \times 10^{19}}{12 \times 10^{19}} = \frac{1}{4} = 0.25$

Sol4. In a coil, the induced emf is produced only if the flux through it changes w.r.t time in a magnetic field. Thus, for uniform field, flux can be changed either by rotating the coil or by changing the area of the coil.

Sol5. For a projectile, range (R) is given as

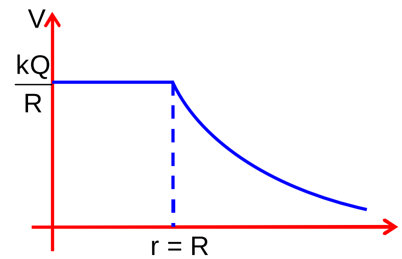
$R = \frac{u^2 \sin 2\theta}{g}$

Also, Range is max. when $\sin 2\theta$ is max ie 1
So, $\sin 2\theta = 1$
or $2\theta = 90^\circ$
or $\theta = 45^\circ$
So, both Assertion & Reason are true. Also reason is the correct explanation of assertion.

Sol6. For spherical shell (or hollow sphere), electric
potential is given by

$V = \frac{kQ}{R}$; $r \le R$

$V = \frac{kQ}{r}$; $r > R$



Sol7. $M_P = 2Me$; $\rho_P = \rho_e$

$(wt)_{\text{on earth}} = w = mg_e$

(wt) on planet $= w' = mg_P$

$\therefore \frac{w'}{w} = \frac{mg_P}{mg_e} = \frac{\frac{4}{3}\pi\rho_P GR_P}{\frac{4}{3}\pi\,\rho_e GR_e}$

$\Rightarrow \frac{w'}{w} = \frac{\rho_P}{\rho_e} \times \frac{R_P}{R_e} = 1 \times 2^{1/3}$

$w' = 2^{1/3} w$

$M_P = 2M_e = \rho_P . \frac{4}{3}\pi R_P^3$ & $Me = \rho_e \frac{4}{3}\pi R_e^{\,3}$

$\because M_P = 2Me$

$\rho_P . \frac{4}{3}\pi R_P^{\,3} = 2\rho_e \frac{4}{3}\pi R_e^{\,3}$

$R_P = 2^{1/3} Re$

Sol8. At moon, due to the low escape velocity, the rms velocity of molecules is greater than escape velocity. Hence, molecules escape and there is no atmosphere at moon. Thus both Assertion and Reason are correct and reason is correct explanation of Assertion.

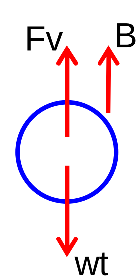
Sol9. When ball is falling with constant velocity : Fv + B = wt

$\Rightarrow Fv = wt - B$

$= Mg - \rho_0 vg$

$= Mg - \rho_0 \left(\frac{M}{\rho}\right) g$

$F_v = Mg\left(1 - \frac{\rho_0}{\rho}\right)$



Sol10. $R_1 = (10 \pm 0.5)\Omega$; $R_2 = (15 \pm 0.5)\Omega$

As we know that the equivalent resistance in parallel combination

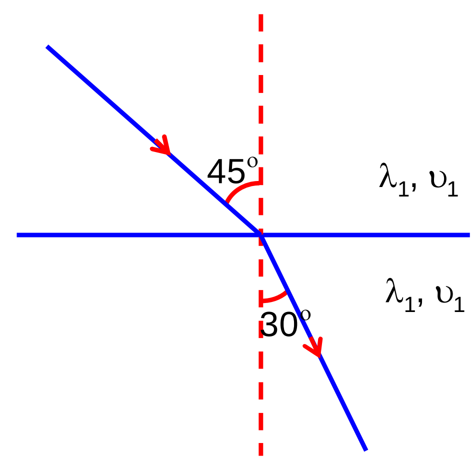
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating both sides, we get

$\frac{-\Delta R}{R^2} = \frac{-\Delta R_1}{R_1^{\,2}} - \frac{\Delta R_2}{R_2^{\,2}}$

$$\frac{\Delta R}{R} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)R$$
$$= \left(\frac{0.5}{100} + \frac{0.5}{225}\right) \times 6$$
$$= \frac{13}{300}$$
$$\therefore \frac{\Delta R}{R}\% \Rightarrow \frac{13}{300} \times 100$$
$$= \frac{13}{3}\%$$
$$= 4.33\%$$

Sol11. Using snell's law



$$\mu_1 \sin 45^o = \mu_2 \sin 30^o$$
$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \times \frac{1}{2}$$
$$\sqrt{2}\mu_1 = \mu_2$$
$$\Rightarrow \sqrt{2} \times \frac{c}{v_1} = \frac{c}{v_2}$$
$$\Rightarrow \frac{v_1}{v_2} = \frac{\sqrt{2}}{1}$$

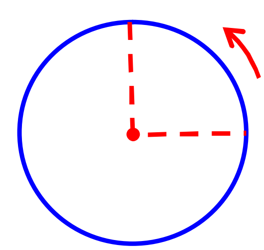
$\upsilon_2 = \upsilon_1$ (as frequency remains unchanged)

Also, $v = \upsilon\lambda$

or $v \propto \lambda$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{2}}{1}$$
$$\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Sol12. $\dfrac{v_{inst}}{\langle \upsilon \rangle} = \dfrac{v}{\left(\dfrac{\sqrt{2}R}{\pi R/2v}\right)}$



$$= \frac{\pi}{2\sqrt{2}} = \frac{\pi}{x\sqrt{2}}$$
$$\therefore x = 2$$

Sol13. For solid infinite current carrying wire :

Using ampere circuital law

$$B = \frac{\mu_0 ir}{2\pi R^2} \; ; \; r \le R$$
$$= \frac{\mu_0 i}{2r}, \; r > R$$

Sol14. The truth table for the given circuit is :
NOR Gate

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

Sol15. $kx = m\omega^2 r$

Where $r = (\ell + x)$

So, $kx = m\omega^2(\ell + x)$

$\Rightarrow 7.5x = 0.1 \times 5^2(0.2 + x)$

$\Rightarrow 7.5x = 0.5 + 2.5x$

$\Rightarrow 5x = 0.5$

$\Rightarrow x = 0.1m$

Thus, tension in the spring is T = kx = 7.5 × 0.1 = 0.75N

Sol16. $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

Where $k_{eq} = k_1 + k_2$

$$T = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

Sol17. According to De-broglie,

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mkE}}$$

$$\lambda_e = \frac{h}{\sqrt{2m_e kE_e}} = \frac{h}{\sqrt{2m_e \times 4k}} = \frac{h}{\sqrt{8m_e k}}$$

$$\lambda_P = \frac{h}{\sqrt{2m_P kE_P}} = \frac{h}{\sqrt{2m_P k}}$$

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha kE_\alpha}} = \frac{h}{\sqrt{2m_\alpha .4k}} = \frac{h}{\sqrt{8m_P K}}$$

Thus, $\lambda_\alpha < \lambda_P < \lambda_e$

Sol18. From 1st law of thermodynamics

$dQ = dU + dw$

Also, $\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dw}{dt}$

$\Rightarrow 1000w = \frac{d\upsilon}{dt} + 200w$

$\Rightarrow \frac{dU}{dt} = 800\,w$

Sol19. A semiconductor starts conduction more as the temperature increases. It means resistance decreases with increase in temperature. So, if temperature increases, its resistivity decreases.

Also, $\rho = \frac{m}{ne^2\tau}$

As temperature increases, τ decreases but n increases and n is dominant over τ.

Sol20. Range of a TV tower is given as

$R_1 = \sqrt{2Rh}$

New range, $R_2 = \sqrt{2g(h + 0.21h)} = \sqrt{2gh \times 1.21} = 1.1R_1$

It means new range increases by 10%.

SECTION - B

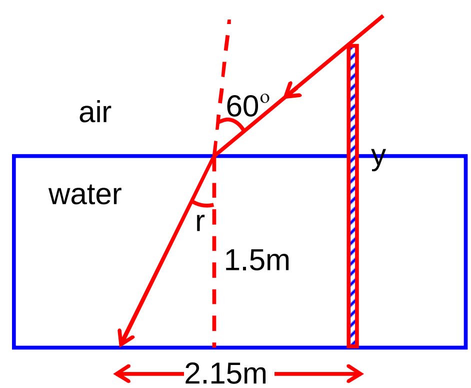
Sol1. $y = \frac{\text{stress}}{\text{strain}}$

$\Rightarrow \text{strain} = \frac{\text{stress}}{y} = \frac{F}{Ay}$

$= \frac{62.8 \times 1000}{\pi r^2 \times 2 \times 10^{11}} = \frac{62.8 \times 1000}{3.14 \times 400 \times 10^{-6} \times 2 \times 10^{11}}$

$= \frac{200}{8} \times 10^{-5} = 25 \times 10^{-5}$

Sol2. $1 \times \sin 60^o = \frac{4}{3} \times \sin r$



$\Rightarrow \sin r = \frac{3}{4} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$ - (i)

$\therefore \cos r = \sqrt{1 - \frac{27}{64}} = \frac{\sqrt{37}}{8} = 0.75$

$\therefore \tan r = \sqrt{\frac{27}{37}} = 0.85$

or $\frac{x}{1.5} = 0.85$

$\Rightarrow x = 1.275\text{m}$

$\tan 30^o = \frac{y}{2.15 - 1.275} = \frac{y}{0.875}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{0.875} \Rightarrow y = \frac{0.875}{1.732} = 0.50$

So, length of pole above water surface is 0.50m or 50cm.

Sol3. $I_{system} = 2 \times \left[I_{com} + md^2\right]$

$= 2 \times \left[\frac{2}{5}mr^2 + md^2\right]$

$= 2 \times \left[\frac{2}{5} \times 2 \times 0.01 + 2 \times 0.04\right]$

$= 2 \times [0.008 + 0.08] = 176 \times 10^{-3}$

Sol4. $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$\Rightarrow \frac{120}{12000} = \frac{N_S}{N_P}$

$\Rightarrow \frac{1}{100} = \frac{N_S}{N_P}$ - (i)

For an ideal transformer, $P_{input} = P_{output}$

$\therefore\ i_P v_P = i_S v_S = 60000\,w$

$\therefore\ i_P = \frac{60000}{12000} = 5$

Now $R_P = \frac{v_P}{i_P} = \frac{12000}{5} = 2400\,\Omega$

& $R_S = \frac{v_S}{i_S} = \frac{120}{60000/120} = 120 \times \frac{120}{60000} = 240\,m\Omega$

Sol5. $C_1 = \dfrac{\frac{A\varepsilon_0}{\left(\frac{2d}{3}\right)} \times \frac{A(4\varepsilon_0)}{\left(\frac{d}{3}\right)}}{\frac{A\varepsilon_0}{\left(\frac{2d}{3}\right)} + \frac{A(4\varepsilon_0)}{\left(\frac{d}{3}\right)}} = \frac{4}{3}\frac{A\varepsilon_0}{d}$

According to question, $\frac{4}{3}\frac{A\varepsilon_0}{d} = 2$

$\Rightarrow \frac{A\varepsilon_0}{d} = \frac{3}{2}$ - (i)

Now, $C_2 = \dfrac{\frac{A\varepsilon_0}{\left(\frac{d}{3}\right)} \times \frac{A(4\varepsilon_0)}{\left(\frac{2d}{3}\right)}}{\frac{A\varepsilon_0}{\left(\frac{d}{3}\right)} + \frac{A(4\varepsilon_0)}{\left(\frac{2d}{3}\right)}} = 2 \times \frac{A\varepsilon_0}{d} = 2 \times \frac{3}{2} = 3$

Sol6. $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2$

$\vec{B}_{net} = \frac{\mu_0 i}{2r}(\hat{i}) + \frac{\mu_0 i}{2r}(\hat{j})$

or $B_{net} = \sqrt{2}\left(\frac{\mu_0 i}{2r}\right)$

$= \sqrt{2} \times \frac{(4\pi \times 10^{-7}) \times \sqrt{2}}{2 \times 0.2} = 628 \times 10^{-8}\,T$

Sol7. $r_n \propto \frac{n^2}{z}$

or $r_n = \frac{0.51\,n^2}{z}\,\mathring{A}$

For Li^{++}, $z = 3$

So, $r_5 = \dfrac{0.51\times 5^2}{3}\times 10^{-10}\,m = 17\times 25\times 10^{-12}\,m = 425\times 10^{-12}\,m$

Sol8. $R = \dfrac{\rho\ell}{A}$

$R' = \dfrac{\rho(1.21\ell)}{(0.96)A} = \dfrac{10}{8}\times R = 1.25R$

Thus, resistance increases by 25%

Sol9. $f_{app} = f_{actual} + 40$ $\quad$ [Given]

$\Rightarrow f_{app} = f_0\left(\dfrac{330+15}{330-15}\right) = f_0 + 40$

$\Rightarrow f_0 \times \dfrac{345}{315} = f_0 + 40$

$\Rightarrow \dfrac{30}{315}\,f_0 = 40$

$\Rightarrow f_0 = \dfrac{4}{3}\times 315 = 420\,H_2$

Sol10. $a = -2x$

$\dfrac{vdv}{dx} = -2x$

On integrating, both sides, we get

$$\int_{v_0}^{0} vdv = -2\int_{0}^{x} xdx$$

$$\left[\frac{v^2}{2}\right]_{v_0}^{0} = -2\left[\frac{x^2}{2}\right]_{0}^{x}$$

$$0 - \frac{{v_0}^2}{2} = -x^2$$

$$v_0^2 = 2x^2$$

Thus, loss in $KE = KE_i - KE_f$

$$= \frac{1}{2}mv_0^2 - 0$$

$$= \frac{1}{2}\times\frac{10}{1000}\times\left(2x^2\right)$$

$$= \frac{x^2}{100}$$

$$= \left(\frac{x}{10}\right)^2$$

$$= \left(\frac{10}{x}\right)^{-2}$$

Thus, n = 2

PART – B (CHEMISTRY)

SECTION – A

Sol1. Size of nucleus is of order 1.5×10^{-3}pm. and H^+ always exists in combined form & there is no relation between these two statements.

Sol2. Photochemical smog is a mixture of pollutants that are formed when nitrogen oxides and volatile organic compounds react in the presence of sunlight, creating a brown haze above cities.

Sol3. $A_2B_{3(aq)} \rightleftharpoons 2A^{3+}_{(a1)} + 3B^{2-}_{(aq)}$

$t=0$	C	0	0
At equilibrium	$C - C\alpha$	$2C\alpha$	$3C\alpha$

$$Keq = \frac{\left[A^{3+}\right]^2\left[B^{2-}\right]^3}{\left[A_2B_3\right]} = \frac{4C^2 \propto^2 \times 27C^3 \propto^3}{C(1-\propto)}$$

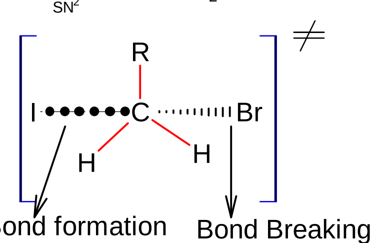
$$Keq = \frac{108C^5 \propto^5}{c} \Rightarrow \alpha = \left(\frac{Keq}{108C^4}\right)^{\frac{1}{5}}$$

Sol4. 'y' form ccp lattice so number of 'y' atom in one unit cell = 4

atom 'x' occupy one third of tetrahedral void, so number of 'x' atom in one unit cell = $\frac{8}{3}$.

Therefore formula of compound = X_8Y_4 or X_2Y_3

Sol5. $R-CH_2-Br+I^- \xrightarrow[SN^2]{acetone} R-CH_2-I + Br^-$

transition state 

From the above transition state we can say that for S_N2 reaction, transition state formed is less polar than the localized anion.

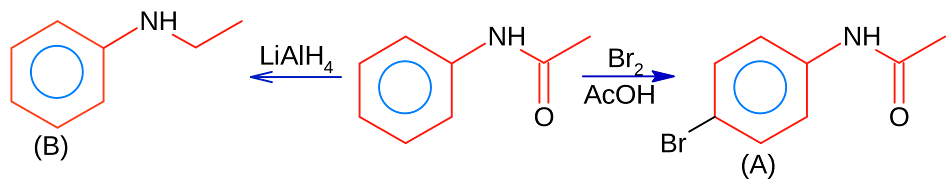
Sol6.

$$n\,CH_2{=}\underset{\displaystyle CN}{\underset{|}{CH}} \longrightarrow \left[CH_2-\underset{\displaystyle CN}{\underset{|}{CH}} \right]_n$$

Acrylonitrile — Polyacrylonitrile (PAN) or orlon

Sol7. Out of 'F' & 'Cl' , 'Cl' have more negative value of electrongain enthalpy. And out of Ne & Ar, Ne have more positive value of electrongain enthalpy.

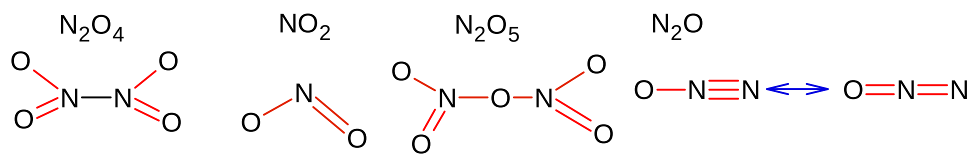
Sol8.



Sol9. Because standard electrode potential is define for 1M concentration.

Sol10. Sucrose $\xrightarrow{\text{Invertase}}$ glucose & fructose
Glucose $\xrightarrow{\text{Zymase}}$ ethyl alcohol & CO_2
Starch $\xrightarrow{\text{Diastase}}$ maltose
Proteins $\xrightarrow{\text{Pepsin}}$ Amino acids.

Sol11.

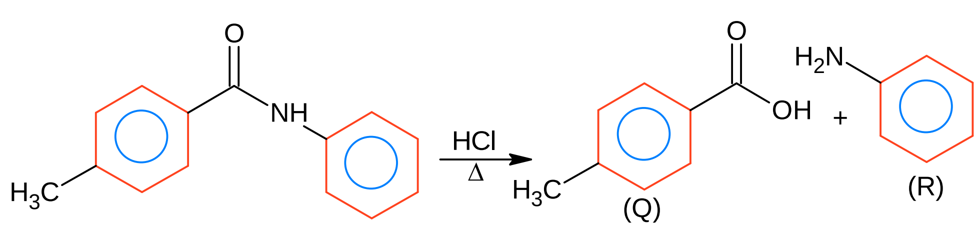


Sol12. Vitamine A : Xerophthalmia
Thiamine : Beri-Beri
Ascorbic acid : Scurvy
Riboflavin : Cheilosis

Sol13. Calcium sulphate ($CaSO_4$) is in the form of Gypsum and its function is to increase the initial setting time of cement.

Sol14. (A) Reduction of Au & oxidation of Zn takes place so it is a redox reaction.
(B) Zn is displacing Au, so it is a displacement reaction.

Sol15.

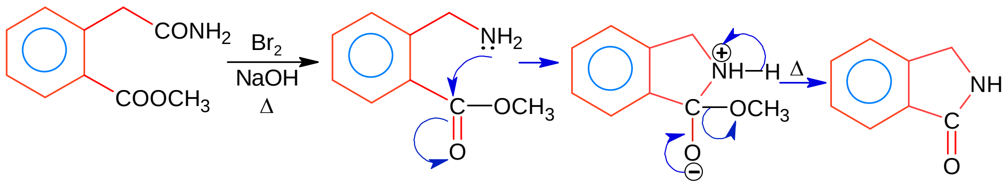


Q$\rightarrow$ Carboxylic acid, gives effervescence with $NaHCO_3$
R$\rightarrow$1^o amine, react with Hinsbergs reagent to give ppt (soluble in NaOH)

Sol16. Nitrogen detected by lassaigne's method. Sulphur is detected by sodium nitropursside Phosphorous is detected by ammonium molybdate & Halogens are defected by $AgNO_3$.

Sol17. +4 oxidation state: oxidation agent.
+2 oxidation state: Reducing agent.

Sol18.



Sol19. Explanation:-

$\left[Fe(CN)_6\right]^{3-}$ | $\left[Fe(H_2O)_6\right]^{3+}$

O.S. of $Fe = +3$ | O.S. of $Fe = +3$

$Fe^{3+} = [Ar]3d^5 4S^0$ | $Fe^{3+} = [Ar]3d^5 4S^0$

d^5 ↑ ↑ ↑ ↑ ↑ | d^5 ↑ ↑ ↑ ↑ ↑

$\because$ field is strong | $\because$ field is weak

$\therefore$ Pairing of e^- takes place | $\therefore$ Pairing of e^- not occurs.

d^5 ⇅ ⇅ ↑ | d^5 ↑ ↑ ↑ ↑ ↑

$\mu = \sqrt{n(n+2)}$ B.M. | $\mu = \sqrt{n(n+2)}$ B.M.

Put n =1 (unpaired e^-) | put n=5

$\mu = 1.74$ B.M. | $\mu = 5.92$ B.M.

So both A & R are true but R is not the correct explanation of A.

Sol20. HVZ reaction $\longrightarrow$ Br_2 / Red P

Iodoform reaction $\longrightarrow$ NaOH+I_2

Etard reaction $\longrightarrow$ CrO_2Cl_2 / CS_2

Gatterman-koch reaction $\longrightarrow$ CO, HCl & Anh. $AlCl_3$.

SECTION – B

Sol1. As we know that

$$X_B = \frac{P_0 - P_s}{P_s}$$

For very dilute solution

$$\frac{n_{solute}}{n_{solvent}} = \frac{P_0 - P_s}{P_s}$$

Let the mass of urea required = x

$$\frac{x}{60} \times \frac{18}{1000} = \frac{P_0 - 0.75P_0}{0.75P_0}$$

$$\boxed{x = 1111g}$$

Sol2. at point a Slope = -ve

dG = -ve $\Rightarrow$ reaction is spontaneous.

at point b Slope = 0

dG = 0$\Rightarrow$ reaction at equilibrium

at point c Slope = +ve

dG = +ve $\Rightarrow$ reaction is non-spontaneous.

Sol3. $SCN^{\ominus}$ is a ambidentate ligand. So no of ambidentate ligand = 4

Sol4. As we know that

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$\Delta G^0 = -54070 - 298\times 10 = -57050\,Jmol^{-1}$

Also $\Delta G^0 = -2.303RT\log k$

$-57050 = -2.303RT\log k$

$$\log k = \frac{57050}{2.303\times 8.314\times 298} = \frac{57050}{5705} = 10$$

Sol5. $$\log\frac{k_2}{k_1} = \frac{(E_a)_1 - (E_a)_2}{2.303RT}$$

$$= \frac{(41.4-30)\times 1000}{2.3\times 8.3\times 300} = 1.99$$

$= 2$

Sol6. IF_5, BrF_5 & $XeOF_4$ have square pyramidal structure.

Sol7. Ammonium-phosphomolybdate

$(NH_4)_3PO_4.12M_oO_3$

Let the O.S of M_o is x

x–6=0

x =+6

Sol8. $$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.6\times 10^{-34}}{\sqrt{2\times 9\times 10^{-31}\times 4.50\times 10^{-29}}}$$

After solving above expression we get the value of $\lambda = 7.3\times 10^{-5}$.

Sol9. $CH_3-CH_3 + \underset{(excess)}{Br_2} \xrightarrow{hv}$ Brominated product

Types of Bromination	No of different possible structure
Mono bromination	CH_3-CH_2-Br
Dibromination	CH_3-CHBr_2 & $BrCH_2-CH_2-Br$
Tribromination	CH_3-CBr_3 & Br_2CH-CH_2-Br
Tetrabromintion	$BrCH_2-CBr_3$ & $Br_2CH-CHBr_2$
Pentabromination	$Br_2CH-CBr_3$
Hexabromination	Br_3C-CBr_3

So total no of Bromo derivatives = 9

Sol10. $\underset{5}{3BaCl_2} + \underset{2}{2Na_3PO_4} \longrightarrow Ba_3(PO_4)_2 + 6NaCl$

$Na_3PO_4 \longrightarrow$ limiting reagent

So 2 mole of Na_3PO_4 will produce 1 mole of $Ba_3(PO_4)_2$

PART – C (MATHEMATICS)

SECTION – A

Sol1. $A\left(5\hat{i}+5\hat{j}+2\lambda\hat{k}\right)$, $B\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$C\left(-2\hat{i}+\lambda\hat{j}+4\hat{k}\right)$, $D\left(-\hat{i}+5\hat{j}+6\hat{k}\right)$

$\overrightarrow{AB}=-4\hat{i}-3\hat{j}+(3-2\lambda)\hat{k}$

$\overrightarrow{AC}=-7\hat{i}+(\lambda-5)\hat{j}+(4-2\lambda)\hat{k}$

$\overrightarrow{AD}=-6\hat{i}+(6-2\lambda)\hat{k}$

$\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ are coplanar.

$$\Rightarrow \begin{vmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{vmatrix}=0 \Rightarrow \lambda^2-5\lambda+6=0$$

$\lambda=2, 3$

$\therefore \Sigma(\lambda+2)^2=16+25=41$

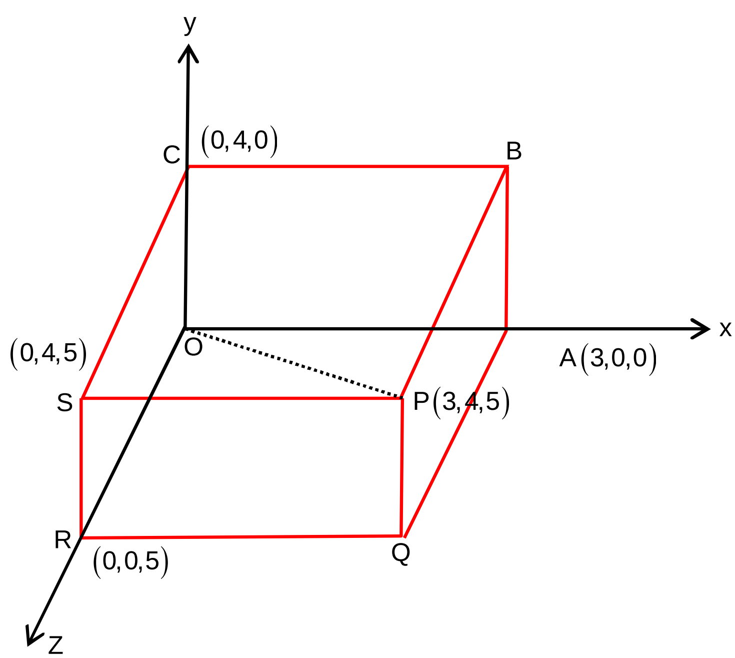
$\lambda \in S$

Sol2. $2x^y+3y^x=20$

$$\frac{dy}{dx}=-\frac{2yx^{y-1}+3y^x\ell ny}{2x^y\ell nx+3xy^{x-1}}$$

$$\left.\frac{dy}{dx}\right|_{(2,2)}=-\frac{2^3+3\cdot 2^2\ell n2}{2^3\ell n2+3\cdot 2^2}=-\frac{2+3\ell n2}{2\ell n2+3}$$

Sol3.



Equation of line OP

$\vec{r} = \lambda\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$ ……….(i)

side parallel to z axis not passing O & P is CS

Equation of CS

$\vec{r} = 4\hat{j} + \mu\left(5\hat{k}\right)$ ………..(ii)

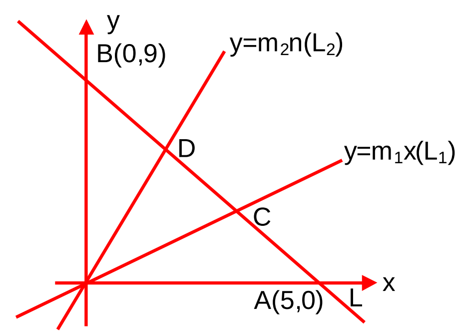
$\therefore$ shortest distance between lines OP and CS

$$= \left|4\hat{j} \cdot \frac{\left\{\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) \times 5\hat{k}\right\}}{\left|\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) \times 5\hat{k}\right|}\right|$$

$$= \left|4\hat{j} \cdot \frac{\left(-15\hat{j} + 20\hat{i}\right)}{\sqrt{(15)^2 + (20)^2}}\right|$$

$$= \frac{60}{25} = \frac{12}{5}$$

Sol4.



$L : \frac{x}{5} + \frac{y}{9} = 1$

$AC : CB \equiv 1 : 2$

$\Rightarrow C\left(\frac{10}{3}, 3\right)$

and $AD : DB \equiv 2 : 1$

$D\left(\frac{5}{3}, 6\right)$

$\Rightarrow m_1 = \frac{9}{10}, m_2 = \frac{18}{5}$

$\therefore y = \left(\frac{9}{10} + \frac{18}{5}\right)x$

$\Rightarrow y = \frac{9}{2}x$

It cut the line $L : 9x + 5y = 45$ at $\left(\frac{10}{7}, \frac{45}{7}\right)$

Sol5.

$P_1 = 2x - y + z = 3$

$P_2 = 4x - 3y + 5z + 9 = 0$

$P_1 = \lambda P_2 = 0$

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$P_3 = (2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z - (3 - 9\lambda) = 0$

P_3 is parallel to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$

$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$

$-3 + 5\lambda = 0$

$\Rightarrow \lambda = \frac{3}{5}$

$$P_3 : \frac{22x}{5} - \frac{14y}{5} + \frac{20z}{5} + \frac{12}{5} = 0$$
$$P_3 = 11x - 7y + 10z + 6 = 0$$
$$\therefore a = 11$$
$$b = -7$$
$$c = 10$$
$$\therefore a + b + c = 11 - 7 + 10 = 14$$

Sol6. $\vec{d} = \lambda\left(\vec{b}\times\vec{c}\right) = \lambda\begin{vmatrix} i & j & k \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = \lambda\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$$\vec{a}\cdot\vec{d} = 18$$
$$\Rightarrow \lambda(4 - 3 + 8) = 18 \Rightarrow \lambda = 2$$
$$\vec{d} = 4\hat{i} - 2\hat{j} + 4\hat{k}$$
$$\Rightarrow \left|\vec{a}\times\vec{d}\right|^2 = \left\|\begin{matrix} i & j & k \\ 2 & 3 & 4 \\ 4 & -2 & 4 \end{matrix}\right|^2\Bigg| \Rightarrow \left|20i + 83 - 16k\right|^2 = 16\left|5\hat{i} + 2\hat{j} - 4\hat{k}\right|^2 = (16)(45) = 720$$

Sol7. 5 can occur in $(4,1)(3,2)(2,3)(1,4)$ ways

$\therefore p = \frac{4}{36} = \frac{1}{9}$. $\qquad q = 1 - p = \frac{8}{9}$.

$$P(x \ge 4) = P(x = 4) + P(x = 5)$$
$$= {}^5C_4\left(\frac{1}{9}\right)^4\frac{8}{9} + {}^5C_5\left(\frac{1}{9}\right)^5$$
$$= \frac{5\times 8}{9^5} + \frac{1}{9^5} = \frac{41}{9^5} = \frac{41}{3^{10}} = \frac{123}{3^{11}}$$
$$\therefore k = 123$$

Sol8. $[x + 3] + [x + 4] \le 3$

$$\Rightarrow 2[x] + 7 \le 3$$
$$\Rightarrow [x] \le -2$$
$$\Rightarrow x < -1 \Rightarrow A = (-\infty, -1) \text{ ....(i)}$$

(B) $3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}$

$$\Rightarrow 3^x \times \left(\frac{\frac{3}{10}}{1 - \frac{1}{10}}\right)^{x-3} < 3^{-3x}$$
$$\Rightarrow 3^x \times 3^{-x+3} < 3^{-3x}$$
$$\Rightarrow 3^3 < 3^{-3x} \Rightarrow 3 < -3x$$
$$\Rightarrow -x > 1 \Rightarrow x < -1$$
$$B = (-\infty, -1) \text{ ....(ii)}$$
$$\therefore A = B$$

Sol9. $I(x)=\int_x^{x^2}\frac{(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$

$=-\frac{x^2}{(x\tan x+1)}+\int\frac{2x}{x\tan x+1}dx$

$-\frac{x^2}{(x\tan x+1)}+2\int\frac{x\cos x\,dx}{x\sin x+\cos x}$

$I(x)=-\frac{x^2}{(x\tan x+1)}+2\ell n|x\sin x+\cos x|+C$

$I(0)=0\Rightarrow c=0$

$I\left(\frac{\pi}{4}\right)=\frac{-\pi^2}{4(\pi+4)}+2\ell n\left|\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}\right|$

$=\ell n\frac{(\pi+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$

Sol10. $|x^2-8x+15|-2x+7=0$

$\Rightarrow |x^2-8x+15|=2x-7\ge 0\Rightarrow x\ge 7/2$ ....(i)

$x^2-8x+15=(x-3)(x-5)$

Case I $x\in(-\infty,3]\cup[5,\infty)$ ....(ii)

$x^2-8x+15=2x-7$

$\Rightarrow x^2-10x+22=0$

$x=\frac{10\pm\sqrt{100-88}}{2}=5\pm\sqrt{3}$

$\Rightarrow x=5+\sqrt{3}$

Case II $x\in(3,5)$

$x^2-8x+15=7-2x$

$x^2-6x+8=0$

$x=4,2\Rightarrow x=4$

$\therefore$ sum of roots $(9+\sqrt{3})$

Sol11. $\frac{30}{x}=\sqrt{3}$;

$\frac{30-h}{x}=2-\sqrt{3}$

$\Rightarrow 30-h=20\sqrt{3}-30$

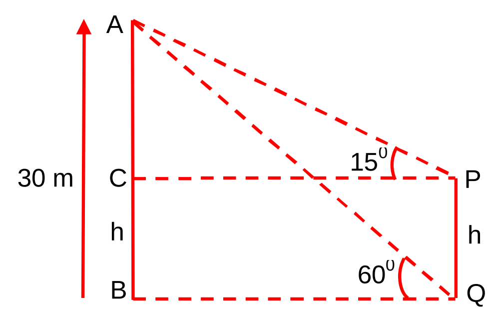
$\Rightarrow x=10\sqrt{3}$

$\Rightarrow h=60-20\sqrt{3}$

$\therefore$ area = hx

$=(60-20\sqrt{3})10\sqrt{3}$

$=200\sqrt{3}(3-\sqrt{3})$



$= 600\left(\sqrt{3}-1\right)$

$\therefore (3)$ is correct

Sol12. $x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$

System of equations can be written as

$$\begin{pmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b \\ 6 \\ 3 \end{pmatrix}$$

$R_1 - R_3, R_2 - 2R_3$

$$\begin{pmatrix} 0 & -1 & a-3 \\ 0 & 1 & -4 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b-3 \\ 0 \\ 3 \end{pmatrix}$$

$$R_1 + R_2 \Rightarrow \begin{pmatrix} 0 & 0 & a-7 \\ 0 & 1 & -4 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b-3 \\ 0 \\ 3 \end{pmatrix}$$

For infinite solution a = 7 & b = 3

$\therefore 2a + 3b = 23$

Sol13. $\left(P \to Q\right) \wedge \left(R \to Q\right)$

$\equiv \left(\sim P \vee Q\right) \wedge \left(\sim R \vee Q\right)$

$\equiv Q \vee \left(\sim P \wedge \sim R\right)$

$\equiv \sim \left(P \vee R\right) \vee Q$

$\equiv \left(P \vee R\right) \Rightarrow Q$

Sol14.

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots.. + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_2}}\right)$$

$$= \lim_{n\to\infty}\sqrt{\frac{d}{n}}\left[\frac{\sqrt{a_2}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\sqrt{a_2}}{d} + \ldots. + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}\right]$$

$$= \lim_{n\to\infty}\sqrt{\frac{d}{n}}\left[\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right]$$

$$= \lim_{n\to\infty}\frac{1}{\sqrt{d}}\left[\sqrt{\frac{a_1}{n} + \frac{(n-1)}{n}d} - \sqrt{\frac{a_1}{n}}\right]$$

$$= \frac{\sqrt{d}}{\sqrt{d}} = 1$$

Sol15. $\Sigma x_i = 15 \times 12$ and $\dfrac{\Sigma x_i^2}{15} - 12^2 = 14$

And $\Sigma y_i = 15 \times 14$ and $\frac{\Sigma y_i^2}{15} - 14^2 = \sigma^2$

Now, $13 = \frac{(14+144)\times 15 + (\sigma^2 + 196)\times 15}{30} - 13^2$

$\therefore \sigma^2 = 10$

Sol16. $S = 5 + 11 + 19 + 29 + ... + T_{n-1} + T_n$

$S = \quad 5 + 11 + 19 + ... + ... + T_{n-1} + T_n$

$0 = 5 + 6 + 8 + 10 + ... + t_{n-1} - T_n$

$T_n = 5 + \frac{n-1}{2}\left[12 + (n-2)\cdot 2\right]$

$= n^2 + 3n + 1$

$S_n = \Sigma T_n = \Sigma n^2 + 2\Sigma n + n$

$= \frac{n(n+1)(2n+1)}{6} + 3 \times \frac{n(n+1)}{2} + n$

$S_{20} = \frac{(10)(7)(41)}{6} + 3 \times \frac{20 \times 21}{2} + 20$

$= 3520$

Sol17. $A = \left[a_{ij}\right]_{2\times 2}\ a_{ij} \neq 0\ \forall\ i, j$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = I$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a^2 + bc = 1\quad (a+d)c = 0$

$ab + bd = 0\quad bc + d^2 = 1$

Again $|A| = 1 \Rightarrow ad - bc = 1$

Solving $a = 0, b = 1 \Rightarrow 3a^2 + 4b^2 = 4$

Sol18. $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ ……….(i)

Replace $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$ …………(ii)

Solving (i) and (ii) we get

$9f(x) = \frac{5}{x} - 4x + 3$

$\int_1^2 f(x)dx = \frac{1}{9}\int_1^2\left(\frac{5}{x} - 4x + 3\right)dx$

$= \frac{1}{9}\left[5\log x - 2x^2 + 3x\right]_1^2$

$= \frac{1}{9}\left[5\log x - 3\right]$

$\therefore 18\int_1^2 f(x)dx = 10\log_e 2 - 6$

Sol19. $E = \left(2^{1/4} +^{-1/4}\right)^n$

$$t_5 = {}^nC_4\left(2^{1/4}\right)^{n-4}\left(3^{-1/4}\right)^4 = {}^nC_4\,\frac{2^{\frac{n-4}{4}}}{3} \quad \text{..........(i)}$$

5^{th} term from end

$$t'_5 = {}^nC_4\left(3^{-\frac{1}{4}}\right)^{n-4}\left(2^{1/4}\right)^4 = {}^nC_4 3^{\frac{-n-4}{4}}\cdot 2 \quad \text{...........(ii)}$$

Given $\dfrac{t_5}{t'_5} = \dfrac{\sqrt{6}}{1}$

$$\Rightarrow \frac{2^{\frac{n-4}{4}}}{3}\times\frac{3^{\frac{n-4}{4}}}{2} = \sqrt{6} \Rightarrow 6^{\frac{n-4}{4}} = 6^{3/2}$$

$$\Rightarrow \frac{n-4}{4} = \frac{3}{2} \Rightarrow n = 10$$

$$\therefore t_3 = {}^{10}C_2\left(2^{1/4}\right)^8\left(3^{-1/4}\right)^2 = 45\times4\times3^{-\frac{1}{2}} = 60\sqrt{3}$$

Sol20. ${}^{2n}C_3 : {}^nC_3 = 10:1$

$$\frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!} = 10$$

$$\Rightarrow 4(2n-1) = 10n-20$$

$$\Rightarrow n = 8$$

Now $\dfrac{\left(n^2+3n\right)}{\left(n^2-3n+4\right)} = \dfrac{64+24}{64-24+4} = \dfrac{88}{44} = 2$

SECTION – B

Sol1. $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$

$0 < x < \dfrac{\pi}{2}$.

$$\Rightarrow x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$$

$$\Rightarrow \frac{dy}{dx} + y\left(\tan x + \frac{1}{x}\right) = \frac{1}{x\cos x}$$

$$IF = e^{\int\left(\tan x + \frac{1}{x}\right)dx} = e^{(\ell n\sec x + \ell n x)} = x\sec x$$

$\therefore$ solution $y\,x\sec x = \int \sec^2 x\,dx$

$$\Rightarrow yx\sec x = \tan x + C$$

$$\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3} \Rightarrow C = \sqrt{3}$$

$$yx\sec x = \tan x + \sqrt{3}$$

$$y = \frac{\sin x}{x} + \sqrt{3}\,\frac{\cos x}{x}$$

$$xy = 2\sin\left(x+\frac{\pi}{3}\right)$$

$$\Rightarrow y + x\frac{dy}{dx} = 2\cos\left(x+\frac{\pi}{3}\right)$$

$$\Rightarrow x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2\sin\left(x+\frac{\pi}{3}\right)$$

$$\Rightarrow \left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$$

Sol2. $A = \{1, 2, 3, ..., 10\}$ $\quad$ $B = \{0, 1, 2, 4\}$

$(a, b) \in A \times A$ such that

$2(a-b)^2 + 3(a-b) - k = 0$

where $k \in \{0, 1, 2, 3, 4\}$

we should have

$9 - 4 \times 2(-k)$ a perfect square for any possible (a, b)

i.e., 9 + 8k is perfect square

$\Rightarrow$ k = 0 or k = 2

for $k = 0$, $2(a-b)^2 + 3(a-b) = 0$

$\Rightarrow a - b = 0 \Rightarrow (a, b) \in \{(1, 1), (2, 2) .... (10, 10)\}$.

$\Rightarrow$ Total 10 elements belonging to R.

$a - b = -\frac{3}{2}$ is not possible

for k = 0 $\;2(a-b) + 3(a-b) - 2 = 0$

$\Rightarrow a - b = -2$ or $a - b = \frac{1}{2}$ (not possible)

$\Rightarrow (a, b) \in \{(1, 3), (2, 4), ... (8, 10)\}$

$\Rightarrow$ 8 element belonging to R

Total = 18

Sol3. Equation of circle is $(x-a)^2 + (y-a) = a^2$

Passes $P(\alpha, \beta)$

$(\alpha - a)^2 + (\beta - a)^2 = a^2$ ....(i)

Equation of AB $x + y = a$ ....(ii)

Let $Q(x_1, y_1) \therefore x_1 + y_1 = a$ ....(iii)

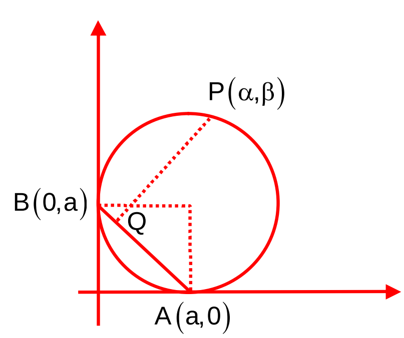
$\therefore$ slope of PQ = 1

Equation of PQ

$$\frac{x - x_1}{1/\sqrt{2}} = \frac{y - y_1}{1/\sqrt{2}} = r$$

For co-ordinate of $Q(\alpha, \beta)$ $r = 11$

$$\frac{\alpha - x_1}{1/\sqrt{2}} = \frac{\beta - y_1}{1/\sqrt{2}} = 11$$



$\alpha = \frac{11}{\sqrt{2}} + x_1$ & $\beta = \frac{11}{\sqrt{2}} + y_1$

$\alpha + \beta = \frac{22}{\sqrt{2}} + x_1 + y_1 = 11\sqrt{2} + a$

(i) $\alpha^2 + \beta^2 - 2a\alpha - 2ab + a^2 = 0$

$(\alpha + \beta)^2 - 2\alpha\beta - 2a(\alpha + \beta) + a^2 = 0$

$\Rightarrow (1/\sqrt{2} + a)^2 - 2\alpha\beta - 2a(1/\sqrt{2} + a) + a^2 = 0$

$\Rightarrow (11\sqrt{2})^2 = 2\alpha\beta \qquad \Rightarrow \alpha\beta = 11^2 = 121$

Sol4. $3 - x \le y \le \sqrt{9 - x^2}$ **;** $0 \le x \le 3$

$(P, P+1)$ lies on $y - x = 1$ ....(i)

Solving with $x + y = 3$

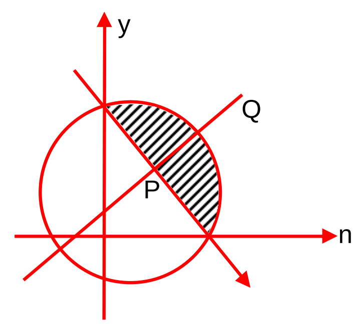
$P(1, 2)$

Again solving $y = x + 1$ & $x^2 + y^2 = 9$

$\Rightarrow x^2 + x - 4 = 0 \Rightarrow x = \frac{\sqrt{17} - 1}{2}$

$\therefore p \in \left(1, \frac{\sqrt{17} - 1}{2}\right) \qquad \therefore a = 1 \quad b^2 + b = 4$

$\therefore b^2 + b - a^2 = 3$



Sol5. Let $Q(\alpha, \beta, \gamma)$ be the image of P, about the plane

$2x - y + z = 9$

$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$

$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$

Then area of triangle PQR is $= \frac{1}{2}\left|\overline{PQ} \times \overline{PR}\right|$

$= \left|-12\hat{i} - 3\hat{j} + 21\hat{k}\right| = \sqrt{144 + 9 + 441} = \sqrt{594}$

Square of area = 594

Sol6. Equation of tangent at $P(1, 3)$ to the curve

$x^2 + 2x - 4y + 9 = 0$ is $y - x = 2$

Then the point A is $(0, 2)$

Equation of line passing through P and parallel to the line $x - 3y = 6$.

The possible coordinate of B are $(4, 4)$ or $(16, 8)$

But $(4, 4)$ does not satisfy $2x - 3y = 8$

Thus the point B is $(16, 8)$

Then $(AB)^2 = 292$

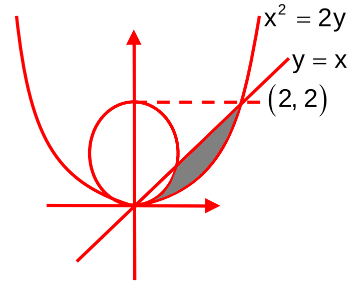
Sol7. $2y - y^2 \le x^2 \le 2y$

(i) $x^2 + y^2 - 2y \ge 0$

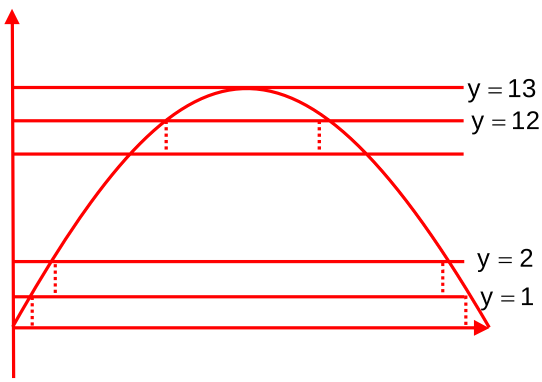
$\Rightarrow x^2 + (y-1)^2 \ge 1$

(ii) $x^2 \le 2y$

Required area = $\int_1^2 \left(\sqrt{2y} - \sqrt{2y - y^2}\right) dy = \frac{7}{6} - \frac{\pi}{4}$



Sol8.



at every line f(x) is discontinuous two times in y = 12 line it is discontinuous only once.
$\therefore$ total number of discontinuous points = 25

Sol9. 20 different oranges can be given to 3 children so that each gets at least once is $3^{20} - {}^3C_1\, 2^{20} + {}^3C_2\, 1^{20}$

Sol10. $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Gen. term = ${}^{15}C_r \left(x^4\right)^{15-r} \left(-\frac{1}{x^3}\right)^r$

$= (-1)^r + {}^{15}C_r x^{60-7r}$

$60 - 7r = 18 \Rightarrow 7r = 42 \Rightarrow r = 6$

$\therefore$ coefficient of $x^{18} = {}^{15}C_6 = 91 \times 55 = 5005$