FIITJEE

Solutions to JEE(Main) -2024

Test Date: 8th April 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**.** **Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q1. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = γ; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ. Where O is origin. Then the distance of P from the x-axis is

(A) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$
(B) $\gamma\sqrt{1+\sin^2\phi\sin^2\theta}$
(C) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$
(D) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$

Ans. **D**

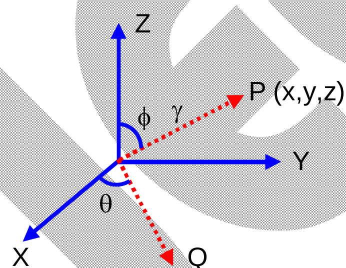
Sol. $z = \gamma\cos\phi$

$OQ = \gamma\sin\phi$

$x = (\gamma\sin\phi)\cos\theta$

$y = (\gamma\sin\phi)\sin\theta$

distance from x-axis = $\sqrt{y^2+z^2}$



Q2. If the shortest distance between the lines

$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}, \quad \lambda \in \mathbb{R}$

$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k}, \quad \mu \in \mathbb{R}$

is $\frac{m}{\sqrt{n}}$, where gcd(m, n) = 1, then the value of m + n equals

(A) 387
(B) 390
(C) 377
(D) 384

Ans. **A**

Sol. rewrite the lines

$L_1 : \vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 4\hat{k})$

$L_2 : \vec{r} = (2\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 1\hat{k})$

Shortest distance = $\left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$\vec{a}_2 \times \vec{b}_2 = \langle 0, 2, 2 \rangle$

$$\vec{a}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 4 \\ 2 & 3 & 1 \end{vmatrix} = -15\hat{i} + 7\hat{j} + 9\hat{k}$$

shortest distance = $\left|\frac{0(-15) + 2.7 + 2.9}{\sqrt{355}}\right|$

$$\frac{m}{\sqrt{n}} = \frac{32}{\sqrt{355}} \Rightarrow m + n = 387$$

Q3. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval (0, 2π) is

(A) 2
(B) 4
(C) 1
(D) 3

Ans. **A**

Sol. $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$

$f'(x) = \sin 2x\,(6\cos x - 3\sqrt{3})$

critical pts in $(0, 2\pi)$ are $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

$f''(x) = -6\sin x.\sin 2x + \cos 2x\,(12\cos x - 6\sqrt{3})$

$f''\left(\frac{\pi}{2}\right) = +ve$

$f''\left(\frac{3\pi}{2}\right) = +ve$

$f''\left(\frac{\pi}{6}\right) = -ve$

$f''\left(\frac{11\pi}{6}\right) = -ve$

So two points of local maxima

Q4. Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If I(0) = 3, then $I\left(\frac{\pi}{12}\right)$ is equal to

(A) $\sqrt{3}$
(B) $3\sqrt{3}$
(C) $2\sqrt{3}$
(D) $6\sqrt{3}$

Ans. **B**

Sol. $I_{(x)} = \int \frac{6dx}{\sin^2 x(1-\cot x)^2}$

1 – cot x = 5

diff. w.r.t. x

$\text{cosec}^2 x\,dx = dt$

$= 6\int \frac{dt}{t^2}$

$= \frac{-6}{t} + c$

$I(x) = \frac{-6}{1-\cot x} + c = \frac{-6\sin x}{\sin x - \cos x} + c$

x = 0, I = 3 ⇒ c = 3

$x = \frac{\pi}{12} \Rightarrow I = 3\sqrt{3}$

Q5. The value of $k \in N$ for which the integral $I_n = \int_0^1 (1-x^k)^n dx, n \in N$, satisfies $147\, I_{20} = 147\, I_{21}$ is

(A) 7
(B) 8
(C) 10
(D) 14

Ans. **A**

Sol. $I_n = \int_0^1 (1-x^k)^n dx$

$= x(1-x^k)^n\Big|_0^1 - \int nx(1-x^k)^{n-1}(-kx^{k-1})dx$

$= x(1-x^k)^n\Big|_0^1 - nk\int(1-x^k)^{n-1}(-x^k)dx$

$I_n = x(1-x^k)^n\Big|_0^1 - nk\int(1-x^k)^{n-1}(1-x^k-1)dx$

$I_n = x(1-x^k)^n\Big|_0^1 - nk\,I_n + nk\,I_{n-1}$

$I_n = \left.\dfrac{x(1-x^k)^n}{1+nk}\right|_0^1 + \dfrac{nk}{1+nk}I_{n-1}$

$I_n = 0 + \dfrac{nk}{1+nk}.I_{n-1}$

n = 21

$I_{21} = \dfrac{+21k}{1+21k}.\,I_{20}$

$\dfrac{147}{148} = \dfrac{21k}{1+21k} \Rightarrow k = 7$

Q6. The set of all α, for which the vectors $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in R$, is

(A) $\left(-\dfrac{4}{3}, 1\right)$
(B) $\left(-\dfrac{4}{3}, 0\right)$
(C) [0, 1)
(D) (–2, 0]

Ans. **B**

Sol. $\cos\theta = \dfrac{\vec{a}.\vec{a}}{|a||b|}$

$\dfrac{\pi}{2} < \theta < \pi \Rightarrow \vec{a}\,.\,\vec{b} = -Ve$

$\vec{a}.\vec{b} = \alpha t^2 + 6\alpha.t - 12 < 0$

since $t \in R$, (assuming $\alpha \neq 0$)

So $\alpha < 0$ & $D < 0$

& $36\alpha^2 + 48\alpha < 0$

$12\alpha(3\alpha + 4) < 0$

$\Rightarrow \alpha \in \left(\dfrac{-4}{3}, 0\right)$

Check for $\alpha = 0$, $\vec{a}.\vec{b} = -ve$, So possible

so $\alpha \in \left[\dfrac{-4}{3}, 0\right]$

Q7. For the function f(x) = (cos x) – x + 1, $x \in R$, between the following two statements

(S1) f(x) = 0 for only one value of x in [0, π]

(S2) f(x) is decreasing in $\left[0, \dfrac{\pi}{2}\right]$ and increasing in $\left[\dfrac{\pi}{2}, \pi\right]$

(A) Both (S1) and (S2) are correct
(B) Only (S1) is correct
(C) Both (S1) and (S2) are incorrect
(D) Only (S2) is correct

Ans. **B**

Sol. f(x) = cos x – x + 1

f'(x) = –sin x – 1

for $x \in [0, \pi]$, $f'(x) = -ve \Rightarrow f(x)$ is $\downarrow$
$f(0) = 2$
$f(\pi) = -\pi$
sign is changing so exactly one-root in $[0, \pi]$

Q8. Let f(x) be a positive function such that the area bounded by y = f(x), y = 0 from x = 0 to x = a > 0 is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is

(A) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
(B) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
(C) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
(D) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Ans. A

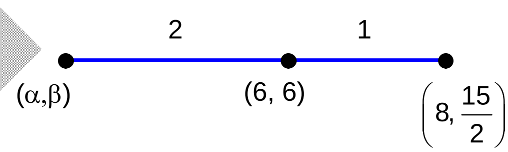
Sol. Given $\int_0^a f(x)\,dx = e^{-a} + 4a^2 + a - 1$
apply Leibneitz - rule
$f(a) = -e^{-a} + 8a + 1$
$\Rightarrow f(x) = -e^{-x} + 8x + 1$
$\Rightarrow f'(x) = e^{-x} + 8 \;\Rightarrow\; \frac{dy}{dx} = c_1(e^{-x} + 8)$
$\Rightarrow f''(x) = -e^{-x} \;\Rightarrow \frac{d^2y}{dx^2} = -c_1e^{-x}$
option (A) satisfies

Q9. Let the circle $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1. then $(\alpha + \beta) + 4(r_1^2 + r_2^2)$ equals

(A) 110
(B) 145
(C) 130
(D) 125

Ans. C

Sol. $6 = \frac{16+\alpha}{3}$ & $6 = \frac{15+\beta}{3}$



$\Rightarrow (\alpha, \beta) = (2, 3)$
$r_1 = 5$, $r_2 = 5/2$
A/Q = $(\alpha + \beta) + 4\,(r_1^2 + r_2^2)$

Q10. Let y = y(x) be the solution of the differential equation
$(1 + y^2)\, e^{\tan x}\, dx + \cos^2 x\,(1 + e^{2\tan x})\, dy = 0$, y(0) = 1. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(A) $\frac{2}{e}$
(B) $\frac{1}{e}$
(C) $\frac{2}{e^2}$
(D) $\frac{1}{e^2}$

Ans. B

Sol. Re-write the equation
$\Rightarrow \frac{e^{\tan x}.\sec^2 x}{1 + (e^{\tan x})^2}\,dx = \frac{-dy}{1+y^2}$

Integrate

$\Rightarrow \tan^{-1}(e^{\tan x}) = -\tan^{-1} y + c$

$x = 0,\ y = 1 \ \Rightarrow\ c = \frac{\pi}{2}$

$x = \frac{\pi}{4} \Rightarrow \tan^{-1}(e) = -\tan^{-1} y + \frac{\pi}{2}$

$\Rightarrow \tan^{-1} e = \cot^{-1} y = \tan^{-1}\frac{1}{y}$

$\Rightarrow\ y = \frac{1}{e}$

Q11. The equations of two sides AB and AC of a triangle ABC are 4x + y = 14 and 3x – 2y = 5, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2 : 1, the equation of the side BC is

(A) x + 6y + 6 = 0
(B) x + 3y + 2 = 0
(C) x – 6y – 10 = 0
(D) x – 3y – 6 = 0

Ans. B

Sol. $\frac{2x_2 + x_1}{3} = 3,$

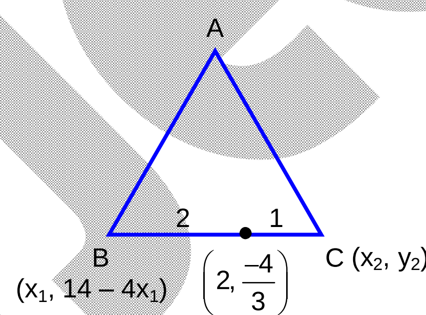
$\frac{2y_2 + 14 - 4x_1}{3} = \frac{-4}{3}$

$\Rightarrow x_2 = \frac{6 - x_1}{2}$ & $y_2 = 2x_1 - 9$

now (x_2, y_2) lie on AC

$\Rightarrow \frac{3(6 - x_1)}{2} - 2(2x_1 - 9) = 5$

$\Rightarrow x_1 = 4 \Rightarrow (x_2, y_2) = (1, -1)$



Equation of BC $\Rightarrow \frac{y+1}{x-1} = \frac{\frac{-4}{3}+1}{2-1}$

x + 3y + 2 = 0

Q12. Let $H: \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6), \alpha > 0$ lies on H. If β is the product of the focal distances of the point (α, 6) then $\alpha^2 + \beta$ is equal to

(A) 170
(B) 172
(C) 169
(D) 171

Ans. D

Sol. $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$e^2 = 1 + \frac{a^2}{b^2} \Rightarrow 3 = 1 + \frac{a^2}{b^2} \Rightarrow \frac{a^2}{b^2} = 2$

Length of latus rectum $\Rightarrow \frac{2a^2}{b} = 4\sqrt{3}$

$\Rightarrow b = \sqrt{3}$

$\Rightarrow a = \sqrt{6}$

(α, 6) lies on Hyperbola

$\frac{\alpha^2}{6} - \frac{36}{3} = -1 \Rightarrow \alpha^2 = 66 \Rightarrow \alpha = \sqrt{66}$

Focus is (0, be) & (0, –be)

Focal distance,

d_1 = distance b/w (0, 3) & $(\sqrt{66}, 6)$

$d_1 = \sqrt{66 + 3^2} = \sqrt{75} = 5\sqrt{3}$

$d_2 = \sqrt{66 + 9^2} = \sqrt{147} = 7\sqrt{3}$

$\beta = 5\sqrt{3}.7\sqrt{3} = 105$

A/Q $\alpha^2 + \beta = 66 + 105$

= 171

Q13. The number of critical points of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ is

(A) 2
(B) 3
(C) 1
(D) 0

Ans. **A**

Sol. $f(x) = (x - 2)^{2/3}(2x + 1)$

$f'(x) = \frac{7x - 10}{3(x-2)^{1/3}}$

Critical pt, at which f'(x) = 0 or does not defined in the domain of f(x).

So, here $x = 2, \frac{10}{7}$

Q14. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

(A) 18
(B) 108
(C) 109
(D) 19

Ans. **C**

Sol. Since $x \in \left(\pi, \frac{3\pi}{2}\right)$

$\sin x = \frac{-3}{5}, \cos x = \frac{-4}{5}$

A/Q $80(\tan^2 x - \cos x) = 109$

Q15. Let z be a complex number such that $|z + 2| = 1$ and $1m\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $|Rc\left(\overline{z+2}\right)$ is

(A) $\frac{\sqrt{6}}{5}$
(B) $\frac{24}{5}$
(C) $\frac{2\sqrt{6}}{5}$
(D) $\frac{1+\sqrt{6}}{5}$

Ans. **C**

Sol. Given $|z + 2| = 1$

$\frac{z+1}{z+2} = \frac{(z+1)(\bar{z}+2)}{(z+2)(\bar{z}+2)} = (z+1)(\bar{z}+2)$

$= z\bar{z} + 2 + 2z + \bar{z}$

let z = x + iy = $x^2 + y^2 + 2 + 3x + yi$

A/Q $I_m\left(\frac{z+1}{z+2}\right) = \frac{1}{5} \Rightarrow y = \frac{1}{5}$

$|z+2|^2 = (x+2)^2 + y^2 = 1$

$\Rightarrow (x+2)^2 = 1 - \frac{1}{25} = \frac{24}{25}$

$x + 2 = \pm\frac{\sqrt{24}}{5}$

A/Q $|\text{Re}(\overline{z+2})| = |x+2| = \frac{\sqrt{24}}{5}$

Q16. If the set $R = \{(a,b); a + 5b = 42, a, b \in N\}$ has m elements and $\sum_{n=1}^{m}(I - i^{n!}) = x + iy$, where $i = \sqrt{-1}$, then the value of m + x + y is

(A) 4
(B) 8
(C) 12
(D) 5

Ans. C

Sol. R = {(a, b)| a + 5b = 42, a, b $\in$ N}
R = {(2, 8), (7, 7), (12, 6), (17, 5), (22, 4), (27, 3) (32, 2), (37, 1)}
m = 8

$\sum_{n=1}^{8}(1 - (i)^{n!}) = x + iy$

$\Rightarrow 8 - i^1 - i^2 - i^6 - i^{24} - i^{120} - i^{720} - i^{7!} - i^{8!}$

$\Rightarrow 8 - i + 1 + 1 - 1 - 1 - 1 - 1 - 1 = x + iy$

$\Rightarrow 5 - I = x + iy$

$\Rightarrow x = 5, y = -1$

Q17. Let [t] be the greatest integer less than of equal to t. Let A be the set of all prime factors of 2310 and f : A→Z be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$. The number of one-to-one functions from A to the range of f is

(A) 120
(B) 25
(C) 24
(D) 20

Ans. A

Sol. 2310 = 2 × 3 × 5 × 7 × 11
So, A = {1, 3, 5, 7, 11}

$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{3}\right]\right)\right]$

So range {f(2), f(3), f(5), f(7), f(11)}
now total one-one f"s from set A to Range = 5!

Q18. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3 × 3, then 2a + 3b is equal to

(A) –9
(B) –13
(C) –10
(D) –12

Ans. B

Sol. use cayley Hamitton equation

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 3-\lambda & 1 \\ 0 & 5 & b-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - (5 + b)\lambda^2 + (11 + a + 5b)\lambda + ab + 10 - 6b = 0$

It satisfies 'A' also

$\Rightarrow A^3 = (5 + b)A^2 - (11 + a + 5b)A - (ab + 10 - 6b)I$

compare with $A^3 = 4A^2 - A - 21I$

$\Rightarrow b = -1, a = -5$

Q19. The sum of all the solution of the equation $(8)^{2x} - 16(8)^x + 48 = 0$

(A) $\log_8(6)$
(B) $\log_8(4)$
(C) $1 + \log_8(6)$
(D) $1 + \log_6(8)$

Ans. **C**

Sol. $8^{2x} - 16.8^x + 48 = 0$

$(8^x)^2 - 16.8^x + 48 = 0$

$(8^x - 4)(8^{-x} - 12) = 0$

$\Rightarrow 8^x = 4, 12$

$x = \log_8^4, \log_8^{12}$

$= \log_8^{48} = \log_8^8 + \log_8^6$

$= 1 + \log_8^6$

Q20. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest possible product, is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m equals

(A) 9
(B) 11
(C) 10
(D) 8

Ans. **C**

Sol. $AM \geq GM$

$\frac{x+y}{2}\sqrt{xy}$

$\Rightarrow xy \leq 12^2$

Required is xy should be $\geq \frac{3}{4}(144)$

≥ 108

Total pairs where x + y = 24 are

{(1, 23), (2, 22),…..,(6, 18), (7, 17)….., (18, 6), (19, 8)…..(23, 1)}

So favourable cases = 13

Sample space = 23

Probability = $\frac{13}{23} = \frac{m}{n}$

n – m = 10

repetition of digits is not allowed, and which are not divisible by 3, is equal to_____.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q21. The number of 3-digit numbers formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to_____.

Ans. **36**

Sol. Required ways = total – numbers divisible by 3

total number $\underset{5}{\dagger}\ \underset{4}{\dagger}\ \underset{3}{\dagger} = 5.4.3 = 60$

divisible by 3

Drop	Use	ways
2, 4	3, 5, 7	3!
2, 7	3, 4, 5	3!
4, 5	2, 3, 7	3!
5, 7	2, 3, 4	3!
		24

So required is = 60 – 24

= 36

Q22. Let the positive integers be written in the form
If the kth row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is_____.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \\ \vdots & & & & & & \vdots \end{array}$$

Ans. **103**

Sol. Starting element of nth row = $\frac{n(n-1)}{2}+1$

& end element $\frac{n(n+1)}{2}$

Row	Starting number	End number
100	4951	
101	5050	
102	5151	
103	5253	5356

Number 5310 lies in row 103

Q23. Let $\alpha = \sum_{r=0}^{n}(4r^2 + 2r + 1)\,{}^nC_r$ and $\beta = \left(\sum_{r=0}^{n} \frac{{}^nC_r}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is_____.

Ans. **5**

Sol. Use $\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{n}{r}$

$(4r^2 + 2r + 1)^nC_r = 4r.\ (r.\ {}^nC_r) + 2.r.{}^nCr + {}^nC_r$

$= 4r.n.{}^{n-1}C_{r-1} + 2.n.{}^{n-1}C_{r-1} + {}^nC_r$

$= 4n\ (r - 1 + 1)^{n-1}C_{r-1} + 2.n.{}^{n-1}C_{r-1} + {}^nC_r$

use $\frac{{}^{n-1}C_{r-1}}{{}^{n-2}C_{r-2}} = \frac{n-1}{r-1}$

$= 4n(n - 1).\ {}^{n-2}C_{r-2} + 4n.\ {}^{n-1}C_{r-1} + 2n.\ {}^{n-1}C_{r-1} + {}^nC_r$

A/Q $\alpha = \sum(4r^2 + 2r + 1)\,{}^nC_r = 4n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + 6n.\sum_{r=1}^{n} {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r$

$= 4n.(n - 1).2^{n-2} + 6n.2^{n-1} + 2^n$

$= 2^n\ (n^2 - n + 3n + 1)$

$= 2^n\ (n^2 + 2n + 1) = 2^n\ (n + 1)^2$

use $\frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{n+1}{r+1}$

then $\frac{{}^nC_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}$

$\beta = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1} + \frac{1}{n+1} = \sum_{r=0}^{n} \frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1}$

$\beta = \frac{2^{n+1} - 1}{n+1} + \frac{1}{n+1} = \frac{2^{n+1}}{n+1}$

$\frac{\alpha}{\beta} = \frac{2^n(n+1)^2(n+1)}{2^n.2} = \frac{(n+1)^3}{2}$

A/Q $140 < \frac{2\alpha}{\beta} < 281$

$140 < (n + 1)^3 < 281$

Since $n \in N$, $(n+1)^3$ can be 216

$\Rightarrow n = 5$

Q24. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}, \vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ -and $\vec{r}.(\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} - +67\vec{a}|^{-2}}{(593)^2}$ is equal to_____.

Ans. **569**

Sol. Given $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$

$(\vec{r} - (\vec{b} + \vec{c})) \times \vec{a} = 0$

$\Rightarrow\ \vec{r} - (\vec{b} + \vec{c}) = \lambda\vec{a} \Rightarrow \vec{r} = \vec{b} + \vec{c} + \lambda\vec{a}$

Given $\vec{r}\ .\ \vec{b} = \vec{r}.\vec{c}$

$\Rightarrow \vec{b}.\vec{b} + \vec{c}.\vec{b} + \lambda\vec{a}.\vec{b} = \vec{b}.\vec{c} + \vec{c}.\vec{c} + \lambda\vec{a}.\vec{c}$

$227 + \lambda(-389) = 294 + \lambda(204)$

$67 = -593\lambda \Rightarrow \lambda = \frac{-67}{593}$

$\vec{r} = \vec{b} + \vec{c} - \frac{67}{593}\vec{a}$

$593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$

$|593\vec{r} + 67\vec{a}| = 593\sqrt{569}$

A/Q $|593\vec{r} + 67\vec{a}|^2 = 593^2(569)$

Q25. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}, \theta \in R$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to_____.

Ans. **96**

Sol. $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta} = 1 + \frac{2\cos^2\theta}{\cos^2\theta + \sin^4\theta}$

$= 1 + \frac{2\cos^2\theta}{(1-\cos^2\theta)^2 + \cos^2\theta}$

$= 1 + \frac{2\cos^2\theta}{\cos^4\theta - \cos^2\theta + 1}$

$= 1 + \frac{2}{\cos^2\theta + \sec^2\theta - 1}$ (Assuming $\cos\theta \neq 0$)

By AM – GM $\quad \cos^2\theta + \sec^2\theta \geq 2$

$\cos^2\theta + \sec^2\theta - 1 \geq 1$

$0 < \frac{1}{\cos^2\theta + \sec^2\theta - 1} \leq 1$

$1 < 1 + \frac{2}{\cos^2\theta + \sec^2\theta - 1} \leq 2 + 1$

$f(\theta) \in (1, 3]$

now check for $\cos\theta = 0$

so $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta} = 1$

So overall range $f(\theta) \in [1, 3]$

$\Rightarrow \alpha = 1, \ \beta = 3$

Now infinite GP sum $= \frac{a}{1-r} = \frac{64}{1-\frac{1}{3}}$

= 96

Q26. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to_____.

Ans. **7**

Sol. $A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} = 3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$A^4 = 9\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = 9\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$A^8 = 81\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = 81\begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$

$$A^{12} = 3^6\begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix} = 3^6\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = 3^6 I$$

$A^{13} = 3^6 A$

Sum of diagonal elements $3^6\,(2 + 1) = 3^7$

$n = 7$

Q27. The value of $\lim\limits_{x\to 0} 2\left(\dfrac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}.....\sqrt[10]{\cos 10x}}{x^2}\right)$ is_____.

Ans. **55**

Sol. $L = \lim\limits_{x\to 0}\dfrac{2(1-\cos x.\sqrt{\cos 2x}.\,\sqrt[3]{\cos 3x}.....\sqrt[10]{\cos 10x})}{x^2}$

$\dfrac{0}{0}$ from, apply L- Hospital rule.

$$L = \lim_{x\to 0}\frac{2\left(0-\dfrac{d}{dx}\left(\cos x\sqrt{\cos 2x}.\sqrt[3]{\cos 3x}.....\sqrt[10]{\cos 10x}\right)\right)}{2x}$$

Let $\mu = \cos x\sqrt{\cos 2x}.\sqrt[3]{\cos 3x}..........$

take log on both sides

$$\ell n\mu = \ell n\cos x + \frac{1}{2}\ell n\cos 2x + \frac{1}{3}\ell n\cos 3x + ... + \frac{1}{10}\ell n\cos 10x$$

differentiate on both sides

$$\frac{1}{\mu}\frac{d\mu}{dx} = \tan x - \tan 2x - \tan 3x..... - \tan 10x$$

$$\lim_{x\to 0}\frac{\frac{1}{\mu}\frac{d\mu}{dx}}{x} = -\lim_{x\to 0}\left(\frac{\tan x}{x} + \frac{\tan 2x}{x} + \frac{\tan 3x}{x} + ... + \frac{\tan 10x}{x}\right)$$

$= -(1 + 2 + 3\ ..... + 10)$

$$\lim_{x\to 0}\frac{\frac{d\mu}{dx}}{x} = -55 \qquad (\text{at } x = 0,\ \mu = 1)$$

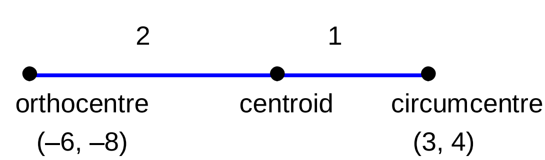
A/Q $\quad L = \dfrac{-\frac{d\mu}{dx}}{x} = 55$

Q28. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are (3, 4) and (–6, –8), then the value of $|a - b|$ is_____.

Ans. **16**

Sol. By euler's line

So (centroid = (0,0), i.e ortho centre of Δ by the lines.



Slope of AD = –1

So slope of BC = 1

$\dfrac{-a}{b} = 1 \Rightarrow b = -a$

Now solve AB $(x + 2y - 1 = 0)$

and BC

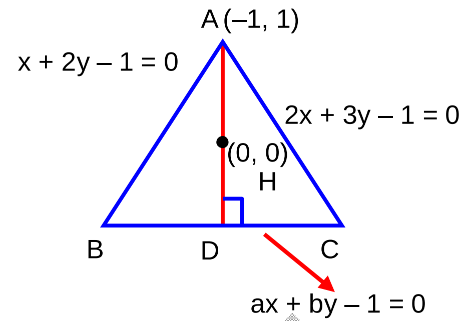
$(ax - ay - 1) = 0$

$B\left(\frac{a+2}{3a}, \frac{a-1}{3a}\right)$

Solpe of BH $= \frac{(a-1)}{a+2} = \frac{3}{2}$

$\Rightarrow a = -8$

A/Q $|a - b| = |-8 - 8| = 16$



Q29. Let the area of the region enclosed by the curve y = min{sinx.cosx} and the x-axis between x = –π to x = π be A. Then A^2 is equal to _____.

Ans. **16**

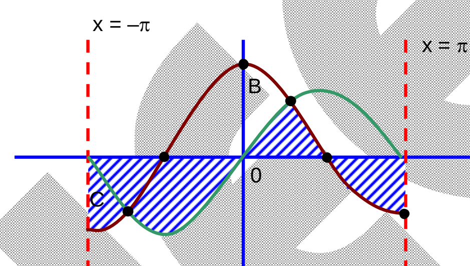
Sol. Observe that portion B & C will be of same area.

So required area

$-\int_{-\pi}^{0} \sin x \, . \, dx + 2\int_{0}^{\pi/2} \cos x . dx$

$= \cos x \Big|_{-\pi}^{0} + 2\sin x \Big|_{0}^{\pi/2}$

= 2 + 2,



Q30. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\bar{x}$ and $\bar{y}$ are the means of X and Y respectively. Then $7\bar{X} + 4\bar{Y}$ is equal to_____.

Ans. **17**

Sol.

X	P(X)	X.P(X)	Y	P(Y)	Y.P(Y)
0	$\frac{^4C_3}{^9C_3}$	0	0	$\frac{^5C_3}{^9C_3}$	0
1	$\frac{^5C_1 . ^4C_2}{^9C_3}$	$\frac{30}{84}$	1	$\frac{^5C_2 . ^4C_1}{^9C_3}$	$\frac{40}{84}$
2	$\frac{^5C_2 . ^4C_1}{^9C_3}$	$\frac{80}{84}$	2	$\frac{^5C_1 . ^4C_2}{^9C_3}$	$\frac{60}{84}$
3	$\frac{^5C_3}{^9C_3}$	$\frac{30}{84}$	3	$\frac{^4C_3}{^9C_3}$	$\frac{12}{84}$

$E(X) = \bar{X} = \sum X_i P(X_i)$

$= \frac{140}{84} = \frac{5}{3}$

$E(Y) = \bar{Y} = \sum (y_i) . P(y_i)$

$= \frac{112}{84} = \frac{4}{3}$

A/Q $7\bar{X} + 4\bar{Y} = 17$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q31. Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4}N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is :

(A) 20 m^2 (B) 0.2 m^2
(C) 0.1 m^2 (D) 0.02 m^2

Ans. D

Sol. $W_{CA}=0, W_{BC}=-10\times20=-20$ Joule

$$W_{AB}=\int_2^4 PdV = RT\int_2^4 V^{-3}dV$$

$$=RT\left[\frac{-1}{2V^2}\right]_2^4=\frac{RT}{2}\left[\frac{-1}{16}+\frac{1}{4}\right]$$

$$=\frac{RT}{4}\left[\frac{1}{2}-\frac{1}{8}\right]=\frac{3RT}{32}=\frac{3}{8}\times\frac{8}{4}\times300$$

$$=225\text{ J}$$

Q32. In an expression $a\times10^b$:

(A) b is order of magnitude for $a\ge5$ (B) b is order of magnitude for $a\le5$
(C) a is order of magnitude for $b\le5$ (D) b is order of magnitude for $5<a\le10$

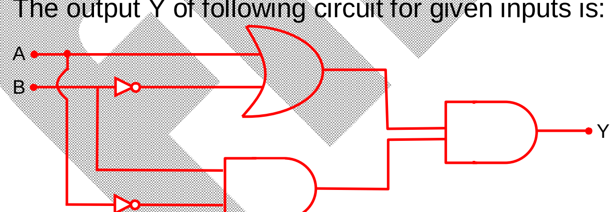
Ans. B

Sol. $a\times10^b$

If $a\le5$ order is b

$a>5$ is b+1

Q33. The output Y of following circuit for given inputs is:



(A) 0 (B) $\bar{A}\cdot B$
(C) $A\cdot B$ (D) $A\cdot B(A+B)$

Ans. A

Sol. By truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

Q34. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is 3L, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is :

(A) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$

(B) $\left(\frac{r_1}{r_2}\right)^3$

(C) $27\left(\frac{m_1}{m_2}\right)^3$

(D) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$

Ans. A

Sol. $L = mvr$

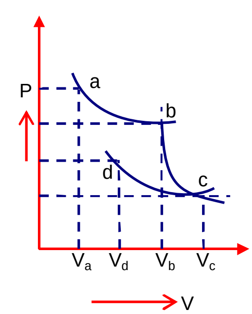
$L = m_1v_1r_1 \Rightarrow v_1 = \frac{L}{m_1r_1}$

$v_2 = \frac{L}{m_2r_2}$

$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{v_2}{v_1} \times \frac{r_1}{r_2} = \frac{\frac{3L}{m_2r_2}}{\frac{L}{m_1r_1}} \times \frac{r_1}{r_2}$

$= \frac{3m_1r_1}{m_2r_2} \times \frac{r_1}{r_2}$

Q35. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is :



(A) $\frac{V_a}{V_b} \neq \frac{V_b}{V_c}$

(B) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$

(C) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

(D) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

Ans. D

Sol. For adiabatic process

$TV^{\gamma-1} = \text{constant}$

$T_aV_a^{\gamma-1} = T_dV_d^{\gamma-1}$

$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_d}{T_a}$

$T_bV_b^{\gamma-1} = T_CV_C^{\gamma-1}$

$\left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_c}{T_b}$

$\frac{V_a}{V_d} = \frac{V_b}{V_c}$

Q36. A player caught a cricket hall of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:
(A) 3 N
(B) 300 N
(C) 150 N
(D) 30 N

Ans. **D**

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{150 \times 10^{-3} \times 20}{10^{-1}} = 3000 \times 10^{-2} = 30\,N$

Q37. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27^0C). The ratio of specific heat of gases at constant volume respectively is:
(A) $\frac{3}{5}$
(B) $\frac{3}{2}$
(C) $\frac{5}{3}$
(D) $\frac{7}{5}$

Ans. **A**

Sol. $\frac{C_{V_1}}{C_{V_2}} = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$

Q38. Correct Bernoulli's equation is (symbols have their usual meaning) :
(A) $P + \rho gh + \rho v^2 = \text{constant}$
(B) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
(C) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
(D) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$

Ans. **C**

Sol. $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Q39. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:
(A) $0.2\mu g$
(B) $20\mu g$
(C) $2\mu g$
(D) $10\mu g$

Ans. **B**

Sol. $E = \Delta mc^2$

$\Rightarrow \Delta m = \frac{18 \times 10^8}{9 \times 10^{16}} = 2 \times 10^{-8}\,kg = 2 \times 10^{-5}\,g$

$= 20\mu g$

Q40. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take $\pi = 3.14$) :
(A) 118.9
(B) 139.4
(C) 140.5
(D) 220.0

Ans. **B**

Sol. Distance travel by minute hand $= \pi(60)$

Distance travel by second hand $= 2\pi(75) \times 60$

$\Delta x = 2\pi \times 75 \times 60 - \pi \times 60$
= 139.4m

Q41. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies ($K_B : K_A$) is:

(A) $m_B : m_A$ (B) $1 : 1$
(C) $v_B : v_A$ (D) $m_B v_B : m_A v_A$

Ans. **C**

Sol. $m_A v_A = m_B v_B$

$$\frac{k_A}{k_B} = \frac{m_A}{m_B}\left(\frac{v_A}{v_B}\right)^2 = \frac{K_A}{B_B} = \frac{V_B}{V_A} \times \left(\frac{V_A}{V_B}\right)^2$$

$$= \frac{V_A}{V_B}$$

Q42. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:
(Assume $h = 6.63 \times 10^{-34}$ J s. $m_e = 9.0 \times 10^{-31}$kg and $m_p = 1836$ times m_e)

(A) $1 : \sqrt{1836}$ (B) $1 : 1836$
(C) $1 : \frac{1}{1836}$ (D) $1 : \frac{1}{\sqrt{1836}}$

Ans. **B**

Sol2. $\lambda = \frac{h}{\sqrt{2km}}$

$\Rightarrow 2k_1m_1 = 2k_2m_2$

$\Rightarrow \frac{k_1}{k_2} = \frac{m_2}{m_1} = \frac{m_e}{m_P} = \frac{1}{1836}$

Q43. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

(A) $\frac{b}{a}$ (B) $\sqrt{ab}$
(C) ab (D) $\frac{a}{b}$

Ans. **D**

Sol. $V_1 = V_2$

$\Rightarrow \frac{kq_1}{q_2} = \frac{kq_2}{b} \Rightarrow \frac{q_1}{q_2} = \frac{a}{b}$

Q44. Critical angle of incidence for a pair of optical media is 45^0. The refractive indices of first and second media are in the ratio:

(A) $\sqrt{2} : 1$ (B) $2 : 1$
(C) $1 : \sqrt{2}$ (D) $1 : 2$

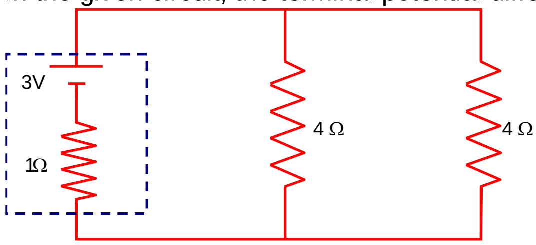
Ans. **A**

Sol. $\mu_1 \sin\theta_C = \mu_2 \sin 90^\circ$

$\Rightarrow \mu_1 \sin 45^\circ = \mu_2$

$\Rightarrow \frac{\mu_1}{\mu_2} = \sqrt{2}$

Q45. In the given circuit, the terminal potential difference of the cell is:



(A) 1.5 V
(B) 4 V
(C) 3 V
(D) 2 V

Ans. **D**

Sol. $i = \frac{3}{3} = 1A$

$V = 2V$

Q46. Paramagnetic substances:
a. align themselves along the directions of external magnetic field.
b. attract strongly towards external magnetic field.
c. has susceptibility little more than zero.
d. move from a region of strong magnetic field to weak magnetic field.
Choose the **most appropriate** answer from the options given below:
(A) a, b, c only
(B) a, b, c, d
(C) b, d only
(D) a, c only

Ans. **D**

Sol. Basic fact

Q47. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:
(A) halved
(B) Zero
(C) same
(D) double

Ans. **D**

Sol. $i = \frac{V}{R}$

$i' = \frac{V}{\frac{R}{2}} = 2i$

Q48. Three bodies A, B, and C have equal kinetic energies and their masses are 400 g. 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is:
(A) $1:\sqrt{3}:\sqrt{2}$
(B) $\sqrt{3}:\sqrt{2}:1$
(C) $1:\sqrt{3}:2$
(D) $\sqrt{2}:\sqrt{3}:1$

Ans. **C**

Sol. $P = \sqrt{2km}$

$P_1 : P_2 : P_3 = 20 : 20\sqrt{3} : 40 = 1 : \sqrt{3} : 2$

Q49. Young's modulus is determined by the equation given by $Y = 49000\frac{m}{\ell}\frac{\text{dyne}}{\text{cm}^2}$ where M is the mass and ℓ is the extension of wire used in the experiment. Now error in Young modules(Y) is estimated by taking data from $M-\ell$ plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and ℓ are 500 g and 2 cm respectively then percentage error of Y is :

(A) 2% (B) 0.2%
(C) 0.02% (D) 0.5%

Ans. A

Sol. $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta l}{l}$

$= \frac{5}{500} + \frac{0.02}{2}$

$= 0.01 + 0.01$

$\frac{\Delta Y}{Y} = 0.02$

$\frac{\Delta Y}{Y} = 2\%$

Q50. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is:

(A) 1.7 g/cm^3 (B) 2.0 g/cm^3
(C) 2.2 g/cm^3 (D) 2.5 g/cm^3

Ans. B

Sol. $9MSD = 10VSD$

$1VSD = \frac{9}{10}MSD$

$LC = 1MSD - \frac{9}{10}MSD = \frac{1}{10}MSD$

$= \frac{1}{10} \times 1mm$

= 0.1mm

$R = 20mm + 2 \times 0.1mm$

$= 20.2mm$

= 2.02cm

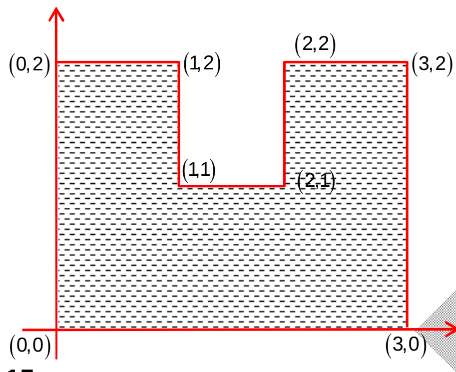
$S = \frac{8.635}{\frac{4}{3}\pi(1.01)^3} = 2g/cm^3$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q51. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is………



Ans. **15**

Sol. Mass of whole plate = 6m

Mass of one plate with cavity = -m

$$x_{cm} = \frac{6m\times\frac{3}{2} - m\times\frac{3}{2}}{6m-m} = \frac{\frac{15}{2}m}{5m} = \frac{3}{2}$$

$$y_{cm} = \frac{6m\times 1 - m\times\frac{3}{2}}{6-m} = \frac{\frac{9m}{2}}{5m} = \frac{9}{10}$$

$$\frac{x_{cm}}{y_{cm}} = \frac{\frac{3}{2}}{\frac{9}{10}} = \frac{15}{9}$$

Q52. An electric field, $\vec{E} = \frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}$ passes through the surface of 4 m^2 area having unit vector $\hat{n} = \left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is……….. V m.

Ans. **12**

Sol. $d = \vec{E}\cdot\vec{A} = A\left[\vec{E}\cdot\hat{n}\right]$

$$= 4\left[\left(\frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}\right)\cdot\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)\right]$$

$$= 4\left[\frac{4}{6}+\frac{6}{6}+\frac{8}{6}\right] = 4\times\frac{18}{6} = 12$$

Q53. Resistance of a wire at 0^0C, 100^0C and t^0C is found to be $10\Omega, 10.2\Omega$ and 10.95Ω respectively. The temperature t in Kelvin scale is………..

Ans. **748**

Sol. $R = R_0\left(1+\alpha\Delta T\right)$

$10.2 = 10\left[1+\alpha 100\right]$

$10.95 = 10\left[1+\alpha t\right]$

$\Rightarrow \frac{10.95}{10} = 1+\left(\frac{10.2}{10}-1\right)\times\frac{1}{100}\times t$

$\Rightarrow \frac{0.95}{10} = \frac{0.2}{10}\times\frac{1}{100}\times t$

$\Rightarrow t = \frac{0.95\times 100}{0.2} = \frac{95}{0.2} = \frac{950}{2} = 475$

$\Rightarrow t = 475+273 = 748k$

Q54. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5×10^{-14}m. If target nucleus has atomic number 80, then maximum velocity of α – particle is……….$\times 10^5$ m/s approximately.

($\frac{1}{4\pi\in_0} = 9\times 10^9$ SI unit, mass of α particle = 6.72×10^{-27}kg)

Ans. **156**

Sol. $V = \sqrt{\frac{4KZe^2}{mr_{min}}}$

$\Rightarrow \sqrt{\frac{4\times 9\times 10^9\times 80}{6.72\times 10^{-27}\times 4.5\times 10^{-14}}}\times 1.6\times^{-19}$

$= 156\times 10^5 m/s$

Q55. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of $3\mu T$ perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that electron moves along the same path, is……….NC^{-1}.

(Given, mass of electron = 9×10^{-31}kg, electric charge = 1.6×10^{-19}C)

Ans. **4**

Sol. For given condition of undeflection net force =0

$qE = qVB$

$E = VB$

$\Rightarrow \sqrt{\frac{2\times KE}{m}}\times B$

$\Rightarrow \sqrt{\frac{2\times 5\times 1.6\times 10^{-19}}{9\times 10^{-31}}}\times 3\times 10^{-6}$

$\Rightarrow$ 4N/C

Q56. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be……..$\times 10^{-3}$ rad.

Ans. **6**

Sol. $\sin\theta \cong \theta = \frac{2\lambda}{b}$

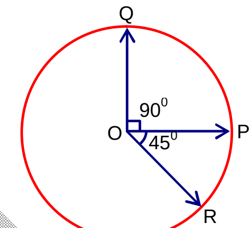
$= \frac{2\times600\times10^{-9}}{4\times10^{-4}} = 3\times10^{3}\text{rad}$

Total divergence

$= 3 + 3\times10^{-3}$

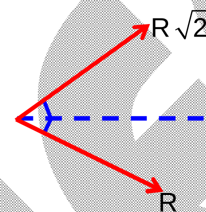
$\Rightarrow 6\times10^{-3}\text{rad}$

Q57. Three vectors $\vec{OP}, \vec{OQ}$ and $\vec{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is…………..

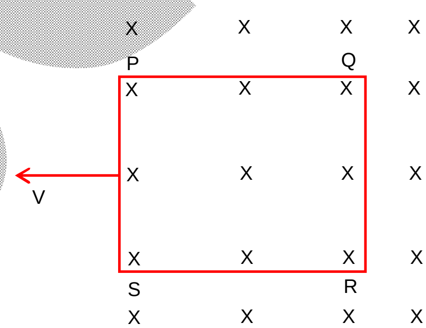


Ans. 3

Sol. $R' = \sqrt{A^2 + 2A^2} = A\sqrt{3}$



Q58. A square loop PQRS having 10 turns, area $3.6\times10^{-3}\text{m}^2$ and resistance 100Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude B = 0.5 T as shown. Work done in pulling the loop out of the field in 1.0 s is……… $\times10^{-6}$ J.



Ans. 3

Sol. $A = 36\times10^{-4}\text{m}^2$

$a = 6\times10^{-2}\text{m}$

$v = \frac{a}{t} = \frac{6\times10^{-2}}{1} = 6\times10^{-2}\text{m/s}$

$\omega = \Delta v = \mu BN = iABN = \left(\frac{Bv\ell}{R}\right)ABN$

$= \frac{0.5\times6\times10^{-2}\times6\times10^{-2}}{100}\times3.6\times10^{-3}\times0.5\times10$

$= \frac{18\times10^{-4}}{100}\times1.8\times10^{-3}\times10$

$= 18^2\times10^{-10}\times10$

$\omega_T = 10\times324\times10^{-10}\times10 = 3.24\times10^{-6}\text{J}$

Q59. A liquid column of height 0.04 cm balances excess pressure of a soap bubble of certain radius. If density of liquid is $8\times10^{3}\text{ kg m}^{-3}$ and surface tension of soap solution is 0.28 Nm^{-1}, then diameter of the soap bubble is………cm.

(if $g = 10\text{ m s}^{-2}$)

Ans. **7**

Ans. $\frac{4T}{r} = \Delta P = \rho gh$

$$r = \frac{4T}{\rho gh} = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 16 - 4}$$

$$= \frac{4 \times 28 \times 10^{-2}}{\frac{32}{8}} = \frac{7}{2} \times 10^{-2}$$

$$D = 7 \times 10^{-2} m = 7cm$$

Q60. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is…………

Ans. **16**

Sol. For close organ pipe

$$f_c = (2 \times 7 + 1)\frac{v}{4\ell} f_2 = \frac{15v}{4\ell}$$

For open organ pipe

$$f_0 = (n+1)\frac{v}{2\ell} = \frac{8v}{2\ell}$$

$$\frac{f_c}{f_0} = \frac{15}{16} = \frac{a-1}{a}$$

$$a = 16$$

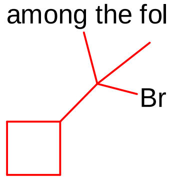
PART – C (CHEMISTRY)

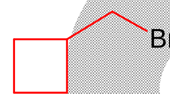
SECTION - A

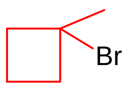
(One Options Correct Type)

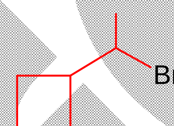
This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q61. Which among the following compounds will undergo fastest S_N2 reaction

(A) 

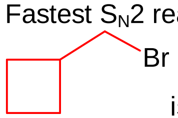
(B) 

(C) 

(D) 

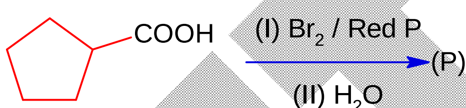
Ans. B

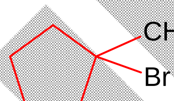
Sol. Fastest S_N2 reaction will take place at least hindered carbon atom.

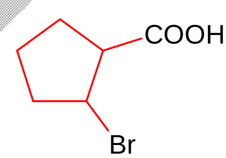


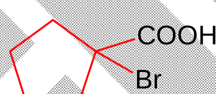
is 1° alkyl halide and will give fastest S_N2 reaction.

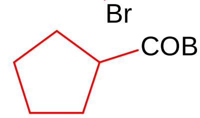
Q62. Identify the product (P) in the following reaction



(A) 

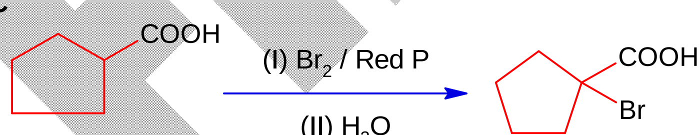
(B) 

(C) 

(D) 

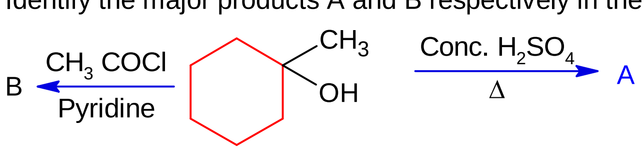
Ans. C

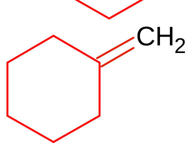
Sol.

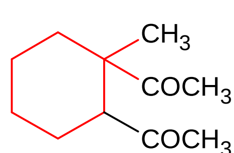


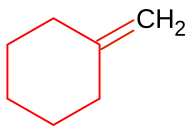
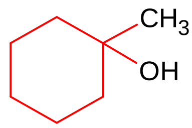
HVZ reaction.

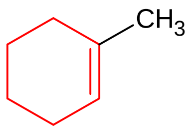
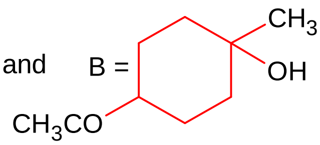
Q63. Identify the major products A and B respectively in the following set of reactions.

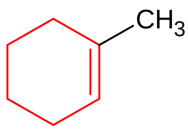
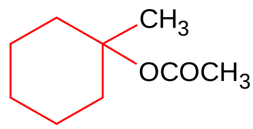


(A) A = 

and B = 

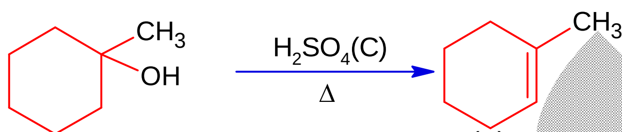
(B) A =  and B = 

(C) A =  and B = 

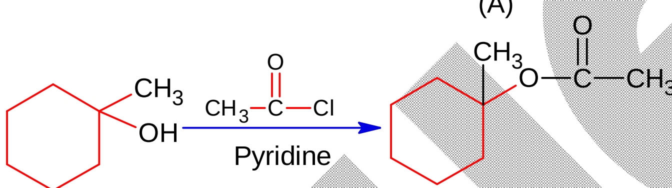
(D) A =  and B = 

Ans. D

Sol.

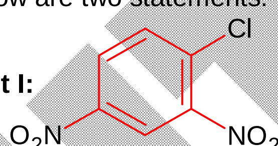


(A)



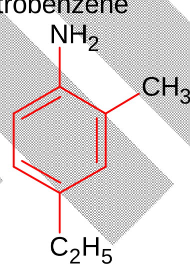
(B)

Q64. Given below are two statements:

Statement I:  IUPAC name of Compound A is

Compound A

4- chloro-1,3-dinitrobenzene

Statement II:  IUPAC name of Compound B is

Compound B

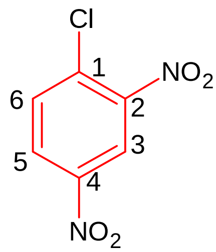
4-ethyl-2-methylaniline.

In the light of the above statements choose the most appropriate answer from the option given below:

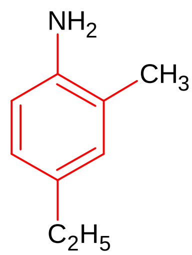
(A) Both Statement I and Statement II are correct.
(B) Statement I is incorrect but Statement II is correct.
(C) Statement I is correct but Statement II is incorrect.
(D) Both Statement I and Statement II are incorrect.

Ans. B

Sol.



Correct name is 1-chloro-2,4-dinitro benzene statement I is incorrect



4-Ethyl -2- methyl aniline
Statement II is correct.

Q65. Match **List I** with **List II**

List I (Element)		List II (Properties in their respective groups)	
(a)	Cl,S	(I)	Elements with highest electro negativity
(b)	Ge, As	(II)	Elements with largest atomic size
(c)	Fr, Ra	(III)	Elements which show properties of both metals and non-metals
(d)	F,O	(IV)	Elements with highest negative electron gain enthalpy.

Choose the **correct** answer from the options given below:
(A) a-III, b-II, c-I, d-IV
(B) a-II, b-I, c-IV, d-III
(C) a-IV, b-III, c-II, d-I
(D) a-II, b-III, c-IV, d-I

Ans. C

Sol.
- Elements with highest electro negativity $\rightarrow$ F,O
- Elements with largest size $\rightarrow$ Fe, Ra
- Elements which shows properties of both metal and non-metals (Metalloids $\rightarrow$ Ge, As)
- Elements with highest negative electron gain enthalpy $\rightarrow$ Cl, S

Q66. Thiosulphate reacts differently with iodine and bromine in the reactions given below:

$$2s_2o_{3+}^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$$

$$S_2O_2^{3-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$$

Which of the following statement justifies the above dual behaviour of thiosulphate?
(A) Bromine is a stronger oxidant than iodine
(B) Bromine undergoes oxidation and iodine undergoes reduction in these reactions
(C) Bromine is a weaker oxidant than iodine
(D) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reactions.

Ans. A

Sol. I_2 oxidise $S_2O_3^{-2}$ and oxidation state of sulphur changes +2 to 2.5, Br_2 oxidise $S_2O_3^{-2}$ and its oxidation number changes from +2 to +6, so Br_2 stronger oxidant than I_2.

Q67. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water: The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:
[Molar mass of glucose in g mol^{-1}=180]
(A) 800
(B) 32
(C) 960
(D) 480

Ans. **C**

Sol. $C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$

Moles of glucose $= \frac{900}{180} = 5$

Moles of $O_2 = 6 \times 5 = 30$

Moles $O_2 = 30 \times 32$

$= 960\,gm$

Q68. An octahedral complex with the formula $CoCl_3.nNH_3$ upon reaction with excess of $AgNO_3$ solution gives 2 moles of AgCl. Consider the oxidation state of Co in the complex is 'x'. The value of "x+n" is__________

(A) 3
(B) 8
(C) 5
(D) 6

Ans. **B**

Sol. $\left[Co(NH_3)_5\,Cl\right]Cl_2 + AgNO_3 \longrightarrow 2AgCl$

$x + 0 - 1 = +2$

$x = +3$

$n = 5$

$x + n = 3 + 5 = 8$

Q69. Among the following halogens
F_2, Cl_2, Br_2 and I_2
Which can undergo disproportionation reactions?

(A) Cl_2, Br_2 and I_2
(B) Only I_2
(C) F_2, Cl_2 and Br_2
(D) F_2 and Cl_2

Ans. **A**

Sol. F_2 is most electronegative element so fluorine do not exist in positive oxidation state but Cl_2, Br_2 and I_2 undergo disproportionation.

Q70. Give below are two statements: One is labeled as **Assertion A** and the other is labeled as **Reason R:**

Assertion A: The stability order of +1 oxidation state of Ga. In and Tl is Ga<In < Tl.

Reason R: The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both A and R are true but R is NOT the correct explanation of A.
(B) Both A and R are true and R is the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans. **B**

Sol. Stability of +1 oxidation state progressively increases in group due to inert pair effect so stability order is

$A\ell^+ < Ga^+ < In^+ T\ell^+$

Q71. Match **List I** with **List II**

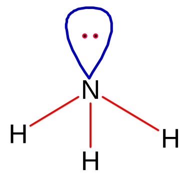
LIST I (Molecule)		**LIST I (Shape)**	
(a)	NH_3	(I)	Square pyramid
(b)	BrF_5	(II)	Tetrahedral
(c)	PCl_5	(III)	Trigonal pyramidal
(d)	CH_4	(IV)	Trigonal bipyramidal

Choose the **correct** answer from the options given below:

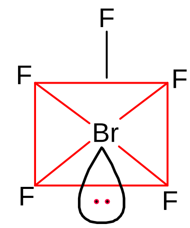
(A) a-IV, b-III, c-I, d-II
(B) a-III, b-IV, c-I, d-II
(C) a-III, b-I, c-IV, d-II
(D) a-II, b-IV, c-I, d-III

Ans. C

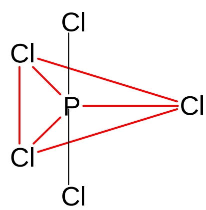
Sol.



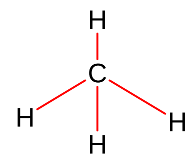
Trigonal pyramidal



Square pyramidal

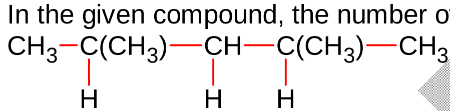


Trigonal bipyramidal



Tetrahedral

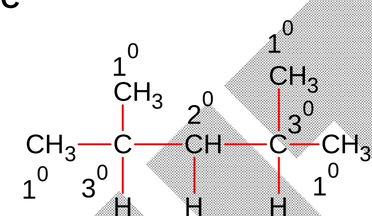
Q72. In the given compound, the number of 2^o carbon atom/s is____________.



(A) Four
(B) Three
(C) One
(D) Two

Ans. C

Sol.



Q73. Match **List I** with **List II**

	LIST I (Compound)		LIST I (Colour)
(a)	$Fe_4\left[Fe(CN)_6\right]_3.xH_2O$	(I)	Violet
(b)	$\left[Fe(CN)_5NOS\right]^{4+}$	(II)	Blood Red
(c)	$\left[Fe(SCN)\right]^{2+}$	(III)	Prussian Blue
(d)	$(NH_4)_3PO_4.12MoO_3$	(IV)	Yellow

Choose the **correct** answer from the options given below:

(A) a-III, b-I, c-II, d-IV
(B) a-IV, b-I, c-II, d-III
(C) a-II, b-III, c-IV, d-I
(D) a-I, b-II, c-III, d-IV

Ans. A

Sol.

$$Fe_4\left[Fe(CN)_6\right]_3 \longrightarrow \text{Prussian blue}$$

$$Fe\left[(CN)_5NOS\right]^{-4} \longrightarrow \text{Violet}$$

$$\left[Fe(SCN)\right]^{+2} \longrightarrow \text{Blood red}$$

$$(NH_4)_3PO_4.12MoO_3 \longrightarrow \text{yellow}$$

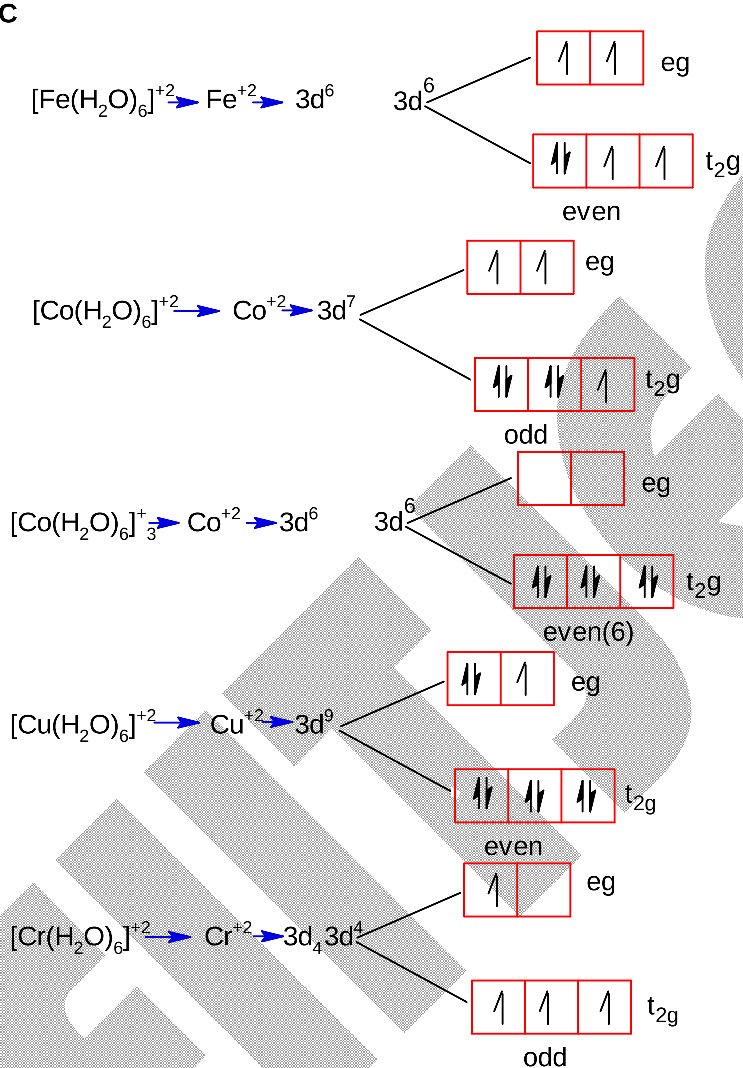
Q74. Number of complexes with even number of electrons in t_{2g} orbitals is

$\left[Fe(H_2O)_6\right]^{2+}, \left[Co(H_2O)_6\right]^{2+}, \left[Co(H_2O)_6\right]^{3-}, \left[Cu(H_2O)_6\right]^{2+}, \left[Cr(H_2O)_6\right]^{2+}$

(A) 5 (B) 1
(C) 3 (D) 2

Ans. C

Sol.



3 complexes with even number of electrons in t_2g

Q75. Give below are two statements:
Statement I : $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes
Statement II: As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.
In the light of the above statements, choose the correct answer from the options given below:
(A) Statement I is incorrect but Statement II is correct.
(B) Both Statement I and Statement II are incorrect.
(C) Both Statement I and Statement II are correct.
(D) Statement I is correct but Statement II is incorrect.

Ans. D

Sol. Both $N(CH_3)_3$ & $P(CH_3)_3$ at as lewis base and act as ligand $P(CH_3)_3$ has π acceptor character

Q76. For the given hpyothetical reactions, the equilibrium contants are as follows

$X \rightleftharpoons Y; K_1 = 1.0$

$Y \rightleftharpoons Z; K_2 = 2.0$

$Z \rightleftharpoons W; \quad K_3 = 4.0$

The equilibrium constant for the reaction $X \rightleftharpoons W$ is

(A) 12.0 (B) 8.0
(C) 7.0 (D) 6.0

Ans. **B**

Sol. $x \rightleftharpoons y \quad k_1 = 1$

$y \rightleftharpoons z \quad k_2 = 2$

$\underline{z \rightleftharpoons w \quad k_3 = 4}$

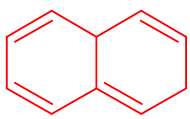
$x \rightleftharpoons w$

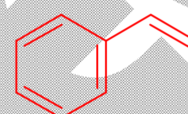
$k = k_1 \times k_2 \times k_3$

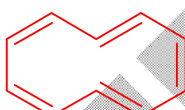
$k = 1 \times 2 \times 4$

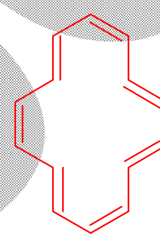
$k = 8$

Q77. Which of the following are aromatic?

(A) 

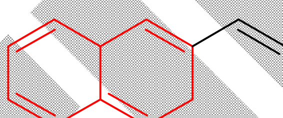
(B) 

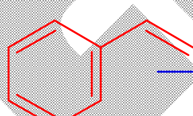
(C) 

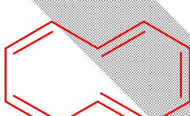
(D) 

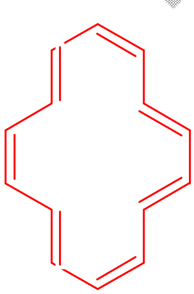
Ans. **B**

Sol.

 → Non aromatic

 → Aromatiac

 → Non aromatic

 → Aromatic

Q78. Match **List I** with **List II**

LIST I (Name of the test)		**LIST I (Reaction sequence involved) [M is metal]**
(a) Borax bead test	(I)	$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO.MO$
(b) Charcoal cavity test	(II)	$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
(c) Cobalt nitrate test	(III)	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
(d) Flame test	(IV)	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the **correct** answer from the options given below:

(A) a-III, b-I, c-II, d-IV
(B) a-III, b-II, c-IV, d-I
(C) a-III, b-IV, c-I, d-II
(D) a-III, b-I, c-IV, d-II

Ans. **C**

Sol. Cobaltnitrate test

$MCO_3 \rightarrow MO \xrightarrow[\Delta]{Co(NO_3)_2} CoO.M$

Flame test

$MCO_3 \xrightarrow{HCl} MCl_2 \longrightarrow M^{+2}$

Charcoal cavity test

$MSO_4 \xrightarrow{Na_2CO_3} MCO_3 \longrightarrow MO \rightarrow M$

Borax bead test

$MSO_4 \xrightarrow{Na_2B_4O_7} M(BO_2)_2 \longrightarrow MBO_2 \longrightarrow M$

Q79. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

a. Fe^{3+} oxidises the iodide ion
b. Fe^{3+} oxidizes the persulphate ion
c. Fe^{2+} reduce the iodide ion
d. Fe^{2+} reduces the persulphate ion

Choose the most appropriate answer from the options given below:

(A) b and c only
(B) a and d only
(C) a only
(D) b only

It self oxidized to Fe^{+3}

Ans. **B**

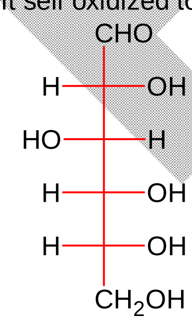
Sol. Fe^{+3} oxidize I^- to I_2 and itself reduced to Fe^{+2}

$2Fe^{+3} + 2I^- \longrightarrow 2Fe^{+2} + I_2$

Fe^{+2} reduces $S_2O_8^{-2}$ to SO_4^{-2} and

It self oxidized to Fe^{+3}

Sol.

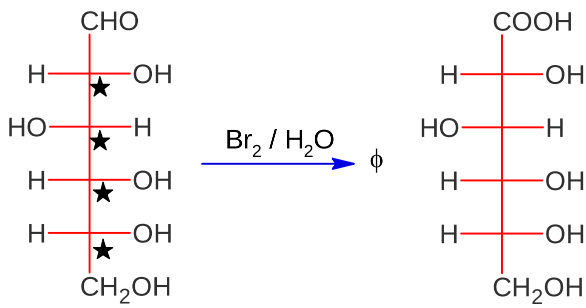


The **incorrect** statement regarding the given structure is

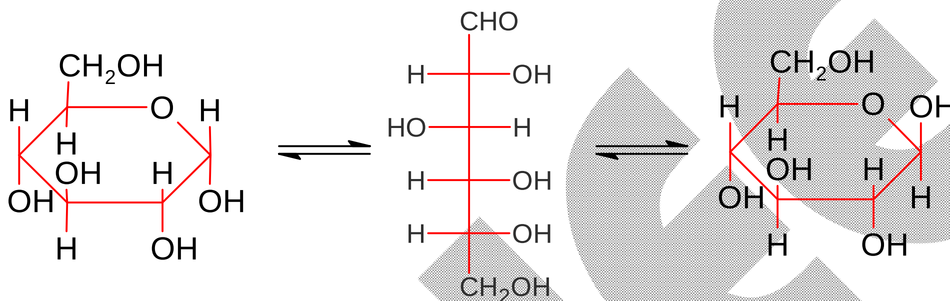
(a) Has 4 asymmetric carbon atom
(b) Despite the presence of –CHO does not give Schiffs test
(c) Can be oxidized to a dicarboxylic acid with Br_2 water
(d) Will coexist in equilibrium with 2 other cyclic structure.

Ans. C

Sol.



Compound has 4 asymmetirc carbon atom it is oxidized to mono carboxylic acid it does not give schiff's test



α – Dglucose B – D – glucose

SECTION - B

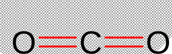
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

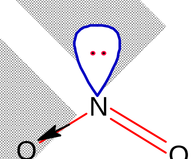
Q81. Number of molecules from the following which are exceptions to octet rule is______.
CO_2, NO_2, H_2SO_4, BF_3, CH_4, SiF_4, ClO_2,PCl_5, BeF_2,C_2H_6,$CHCl_3$,CBr_4
Given……

Ans. 6

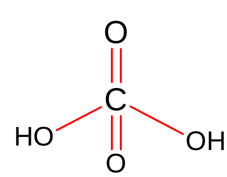
Sol.



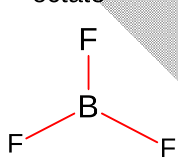
Complete octate



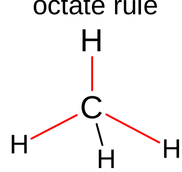
Exception to octate rule



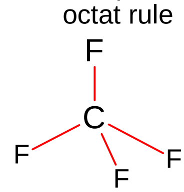
Exception to octat rule



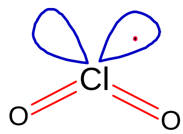
Exception to octat rule



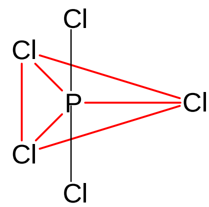
Complete octate



Complete octate



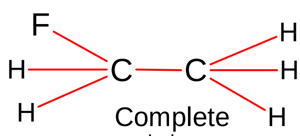
Exception to octat rule



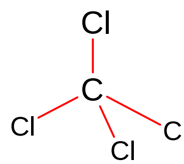
Exception to octat rule

F—Be—F

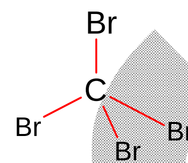
Exception to octat rule



Complete octate



Complete octate



Complete octate

Q82. The 'spin only' magnetic moment value of MO_4^{2-} is________ BM. (Where M is a metal having least metallic radii, among Sc, Ti, V, Cr, Mn and Zn)
(Given atomic number : Sc =21, Ti =22, V=23, Cr=24, Mn=25 and Zn=30)

Ans. **0**

Sol. Metal having least metallic raddi among
Si, Ti, V, Cr, Mn and Zn is Cr
Ion is CrO_4^{-2}
$Cr^{+6} = 3d^0$
Spin only magnetic moment $\mu = 0$

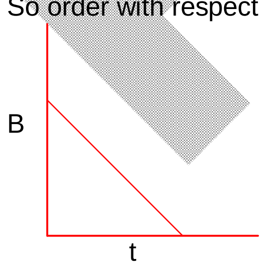
Q83. Consider the following reaction
A + B $\rightarrow$C
The time taken for A to become 1/4th of its initial concentration is twice the time taken to become ½ of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and positive intercept on the concentration axis. The overall order of the reaction is______.

Ans. **1**

Sol. $A \xrightarrow{t = x} \frac{A}{2}$

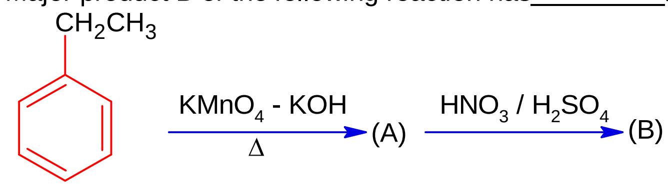
$A \xrightarrow{t = 2x} \frac{A}{4}$

So order with respect to A = 0



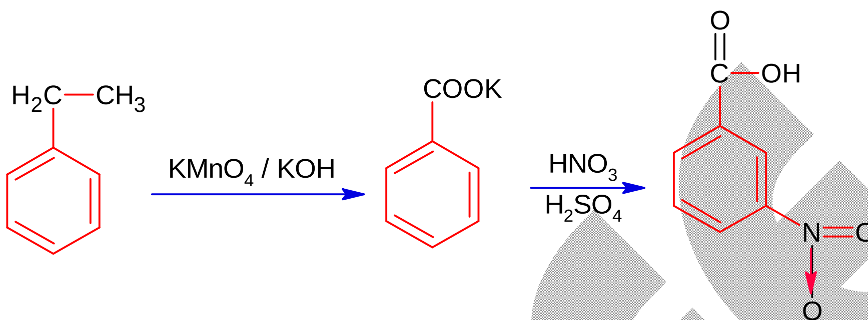
Order with respect to B = 0 order with respect to.

Q84. Major product B of the following reaction has________π-bond



Ans. **5**

Sol.

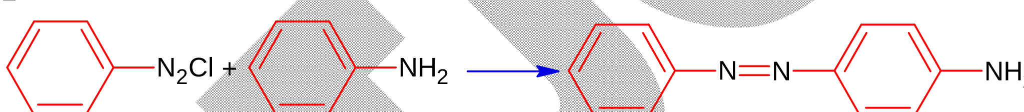


Total number of π bonds 5

Q85. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be____ g. (nearest integer)
(consider complete conversion)

Ans. **591**

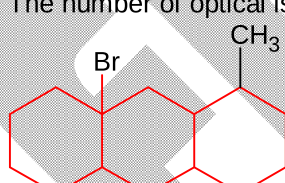
Sol.



Moles of aniline $\frac{279}{93} = 3$

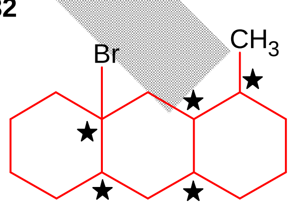
Moles of aniline yellow $= 3$
Mass of aniline yellow $= 3 \times 197$
$= 591$

Q86. The number of optical isomers in following compound is:__________



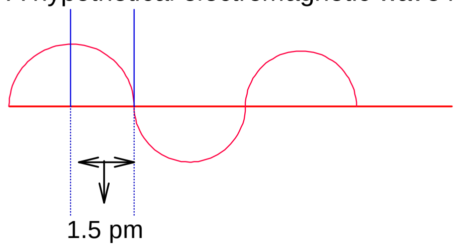
Ans. **32**

Sol.



Total number of steriogenic centre =5
Total number of optical isomer = 2^5
=32

Q87. A hypothetical electromagnetic wave is show below:



The frequency of the wave is $x \times 10^{19}$ Hz.

X = ______________(nearest integer)

Ans. **5**

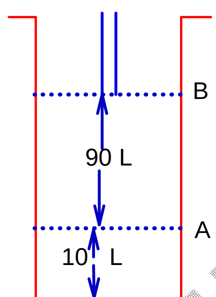
Sol. $\lambda = 1.5 \times 4 = 6\,\text{Pm}$

$$v = \frac{c}{\lambda} = \frac{3\times10^{8}}{6\times10^{-12}}$$

$$= 0.5\times10^{20}$$

$$= 5\times10^{19}$$

Q88.



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18^{o}C. If the piston is moved to position B, keeping the temperature unchanged then 'x' L atm work is done in this reversible process.

X = ______________ L atm. (nearest integer)

[Given: absolute temperature = ^{o}C+273.15, R = 0.08206 L atm mol^{-1}K^{-1}]

Ans. **55**

Sol. $v_1 = 100\text{L}$

$v_2 = 10\text{L}$

$$w = -nRT\ell n\frac{v_2}{v_1}$$

$$= -2.303\times1\times0.08206\times291.15\log\frac{10}{100}$$

$$= 55 \text{ litre atm}$$

Q89. A solution containing 10 g of an electrolyte AB_2 in 100 g of water boils at 100.52^{o}C. The degree of ionization of the electrolyte (α) is ____________ $\times10^{-1}$ (nearest integer)

[Given: Molar mass of AB_2= 200 g mol^{-1}, K_b (molal boiling point elevation const. of water) = 0.52 K kg mol^{-1}, boiling point of water = 100^{o}C; AB_2 ionises as $AB_2 \rightarrow A^{2+} + 2B^{-}$]

Ans. **5**

Sol.

$$\begin{array}{lll} AB_2 & \rightarrow A^{+2} & +\ 2B^{-} \\ 1 & 0 & 0 \\ 1-\alpha & \alpha & 2\alpha \end{array}$$

$$i = 1-\alpha+\alpha+2\alpha$$

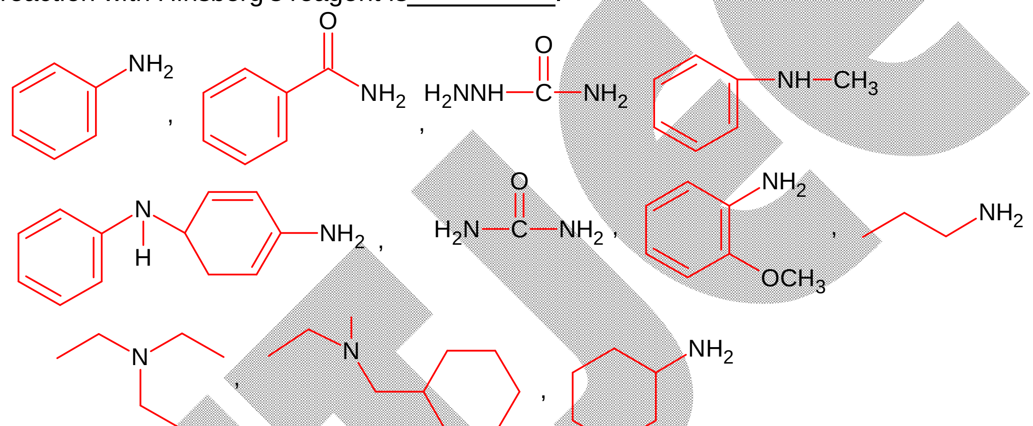
$i = 1 + 2\alpha$
$\Delta T = ik_b m$

$$0.52 = 0.52(1+2\alpha)\frac{\frac{10}{200}}{\frac{100}{1000}}$$

$$1 = (1+2\alpha)\frac{10}{20}$$

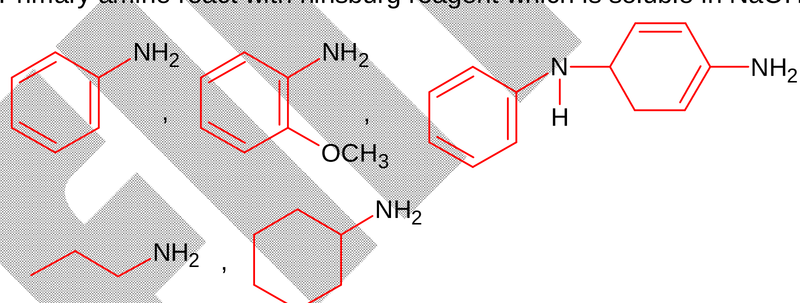
$2 = 1 + 2\alpha$
$\alpha = 0.5$
$\alpha = 5\times10^{-1}$

Q90. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is__________.



Ans. 5

Sol. Primary amine react with hinsburg reagent which is soluble in NaOH



5 primary amines are present