FIITJEE

Solutions to JEE(Main) -2024

Test Date: 27th January 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. The distance, of the point (7, – 2, 11) from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$, is :

(1) 18 (2) 14
(3) 12 (4) 21

Ans. **(2)**

Sol. General point on the given line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ = k is Q(k + 6, 4, 3k + 8)

Direction ratios of PQ = k + 6 – 7, 4 – (–2), 3k + 8 – 11 : : 2 : – 3 : 6

$\Rightarrow$ k = – 3 $\Rightarrow$ Q = [3, 4, –1]

$\Rightarrow$ $PQ=\sqrt{4^2+6^2+12^2}=\sqrt{196}$ = 14

2. Consider the matrix $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$. Given below are two statements:

Statement I: f(–x) is the inverse of the matrix f(x)
Statement II: f(x) f(y) = f(x + y).
In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both statement I and Statement II are false
(4) Statement I is false but Statement II is true.

Ans. **(2)**

Sol. We have $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$$f^{-1}(x)=\frac{\text{Adj } f(x)}{|f(x)|}=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix} = f(-x)$$

Now f(x) f(y) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y)$

*3. The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid point is $\left(1,\frac{2}{5}\right)$, is equal to :

(1) $\frac{\sqrt{1691}}{5}$ (2) $\frac{\sqrt{1541}}{5}$

(3) $\frac{\sqrt{2009}}{5}$ (4) $\frac{\sqrt{1741}}{5}$

Ans. **(1)**

Sol. Chord with middle point $\left(1,\frac{2}{5}\right)$ is 8x + 5y – 10 = 0

Solving with ellipse $16x^2 + 25y^2 - 400 = 0$

$\Rightarrow$ $20x^2 - 40x - 75 = 0 \langle^{x_1}_{x_2}$

$\Rightarrow$ length of chord = $\sqrt{\left[(x_1+x_2)^2 - 4x_1x_2\right](1+m^2)}$

$= \sqrt{\left[(2)^2 + 4\times\frac{15}{4}\right]\left[1+\frac{64}{25}\right]}$

$= \frac{\sqrt{1691}}{5}$.

*4. If $S = \{z \in C : |z-i| = |z+i| = |z-1|\}$, then n(S) is:

(1) 2 (2) 3

(3) 0 (4) 1

Ans. **(4)**

Sol. As |z – i| = |z + i| = |z – 1|

$\Rightarrow$ z is equidistant from three points (1, 0), (0, 1) & (0, –1)

$\Rightarrow$ z is circumcentre of triangle formed by three points $\Rightarrow$ z = 0

$\Rightarrow$ n(S) = 1.

5. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt}+ax=0$ and $\frac{dy}{dt}+by=0$ respectively, a, b $\in$ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is:

(1) $\log_4 3$ (2) $\log_{\frac{2}{3}} 2$
(3) $\log_{\frac{4}{3}} 2$ (4) $\log_3 4$

Ans. (3)

Sol. Given $\frac{dx}{dt} = -ax \Rightarrow \frac{dx}{x} = -adt \Rightarrow \ln x = at + c$

$\Rightarrow x = e^{-at+c} = c'e^{-at}$

$\Rightarrow 2 = c'$; putting t = 0

$\Rightarrow x = 2e^{-at}$

Similarly $y = e^{-bt}$

$x(t) = y(t) \Rightarrow e^{(a-b)t} = 2 \Rightarrow t = \frac{\ln 2}{a-b}$

As 3y(1) = 2x(1)

$\Rightarrow 3e^{-b} = 2 \times 2e^{-a}$

$\Rightarrow e^{a-b} = \frac{4}{3} \Rightarrow a - b = \ln\frac{4}{3}$

$\Rightarrow t = \log^{2}_{4/3}$

*6. If $a = \lim\limits_{x\to 0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is:

(1) 32 (2) 30
(3) 25 (4) 36

Ans. (1)

Sol. Required limit $a = \lim\limits_{x\to 0}\frac{1+\sqrt{1+x^4}-2}{x^4\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}}$

$= \lim\limits_{x\to 0}\frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)} = \frac{1}{4\sqrt{2}}$

$b = \lim\limits_{x\to 0}\frac{\sin^2 x\left[\sqrt{2}+\sqrt{1+\cos x}\right]}{2-(1+\cos x)} = \lim\limits_{x\to 0}\frac{(1-\cos^2 x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{(1-\cos x)} = 4\sqrt{2}$

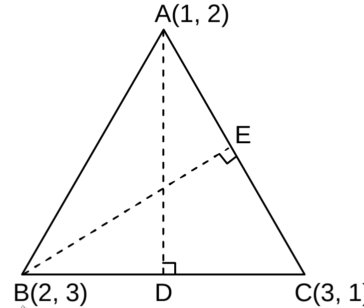
$\Rightarrow ab^3 = 32$

7. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2) (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin\left(4x-x^2\right)dx$, $I_2 = \int_a^b \sin\left(4x-x^2\right)dx$, then $36\frac{I_1}{I_2}$ is equal to:

(1) 72 (2) 88
(3) 80 (4) 66

Ans. (1)

Sol. AD equation = 2y = x + 3
BE equation = y = 2x – 1
Orthocentre: $\left(\frac{5}{3}, \frac{7}{3}\right)$



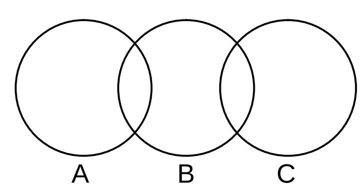
$$I_1 = \int_a^b x \sin\left(4x - x^2\right) dx$$
$$-2I_1 = \int_a^b -2x \sin\left(4x - x^2\right) dx$$
$$-2I_1 = \int_a^b (4-2x)\sin\left(4x - x^2\right) dx - 4\int_a^b \sin\left(4x - x^2\right) dx$$
$$-2I_1 = -\cos\left(4x - x^2\right)\Big|_{5/3}^{7/3} - 4I_2$$
$$-2I_1 = -\cos\left(\frac{35}{9}\right) + c_4\left(\frac{35}{9}\right) - 4I_2$$
$I_1 = 2I_2 \Rightarrow \frac{36I_1}{I_2} = 72$

*8. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :
(1) 5/13 (2) 2/13
(3) 3/13 (4) 1/13

Ans. (1)
Sol. (2k, 3k) (1, 0) (0, 1) (0, 0)
Circle passes through (1, 0) (0, 1) (0, 0)
$x^2 + y^2 - x - y = 0$
$(2k, 3k) \to 4k^2 + 9k^2 - 2k - 3k = 0$
$k = \frac{5}{13}$

*9. Let S = {1, 2, 3, …, 10}. Suppose M is the set of all the subsets of S, then the relation $R = \{(A, B) : A \cap B \neq \phi;\ A, B \in M\}$ is:
(1) reflexive only (2) symmetric and transitive only
(3) symmetric and reflexive only (4) symmetric only

Ans. (4)
Sol. S = {1, 2, 3, ….., 10}
$R = \{(A, B) ; A \cap B \neq \phi, A, B \in M\}$
For reflexive $A = \phi$, $B = \phi$, $A \cap B = \phi$; not possible
For symmetric
$A \cap B \neq \phi$, $A \cap B = B \cap A$ so it is symmetric
For transitive
$A \cap B \neq \phi$, $B \cap C \neq \phi$, $A \cap C \neq \phi$ not confirm
Only symmetric



$A \cap C = \phi$
No transitive

*10. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to:
(1) 20 (2) 24
(3) 36 (4) 16

Ans. **(1)**

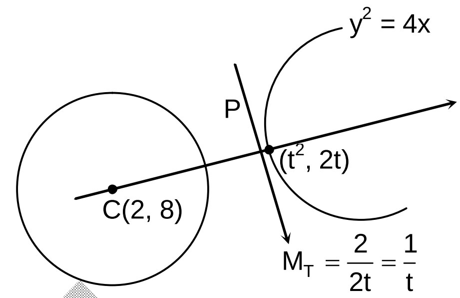
Sol. $M_{normal} = -t$

Normal $y - 2t = -t(x - t^2)$

Passes through centre $8 - 2t = -2t + t^3$

$\Rightarrow t = 2$

$P(4, 4)$ $(CP)^2 = (4 - 2)^2 + (8 - 4)^2 = 20$



*11. The number of common terms in the progressions 4, 9, 14, 19, …, upto 25^{th} term and 3, 6, 9, 12, …, up to 37^{th} term is:

(1) 7 (2) 5

(3) 9 (4) 8

Ans. **(1)**

Sol. S_1 = 4, 9, 14, 19, ….., 124 (25^{th} term)

S_2 = 3, 6, 9, 12, ….., 111 (37^{th} term)

Common term: 9, 24, 39, 54, 69, 84, 99

7 terms

12. The function $f : N - \{1\} \to N$; defined by f(n) = the highest prime factor of n, is:

(1) neither one-one nor onto (2) one-one only

(3) both one-one and onto (4) onto only

Ans. **(1)**

Sol. $f : N - \{1\} = N$ f(n) = highest prime factor

$f(2) = 2$

$\left.\begin{array}{l} f(3) = 3 \\ f(4) = 2 \\ f(5) = 5 \\ f(6) = 3 \end{array}\right\}$ many one and all natural number in out put not possible because output is prime number only.

"neither one one not onto"

*13. If A denotes the sum of all coefficients in the expansion of $(1 - 3x + 10x^2)$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$ then :

(1) $A = B^3$ (2) $3A = B$

(3) $A = 3B$ (4) $B = A^3$

Ans. **(1)**

Sol. A = sum of coefficients $(1 - 3x + 10x^2)^n$ put x = 1

$A = 8^n$

B = sum of coefficients $(1 + x^2)^n$ put x = 1

$B = 2^n$

$(B)^3 = 8^n = A$

14. Let $\bar{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to:

(1) 24 (2) 32
(3) 20 (4) 36

Ans. **(1)**

Sol. $a = \hat{i} + 2\hat{j} + \hat{k}$

$b = 3\hat{i} - 3\hat{j} + 3\hat{k}$

$c = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \times \vec{c} = \vec{b}$

$(2z - y)\hat{i} + (x - z)\hat{j} + (y - 2x)\hat{k} = 3\hat{i} - 3\hat{j} + 3\hat{k}$

$2z - y = 3$, $x - y = -3$, $y - 2x = 3$

$\vec{a} \cdot \vec{c} = 3$ $x + 2y + z = 3$

By solving $x = -1$, $y = 1$, $z = 2$

$\vec{c} = -\hat{i} + \hat{j} + 2\hat{k}$

$\vec{c} \times \vec{b} = 9\hat{i} + 9\hat{j}$

$(\vec{c} \times \vec{b} - \vec{b} - \hat{c}) = 7\hat{i} + 11\hat{j} - 5\hat{k}$

$\vec{a} \cdot (\vec{c} \times \vec{b} - \vec{b} - \hat{c}) = 7 + 22 - 5 = 24$

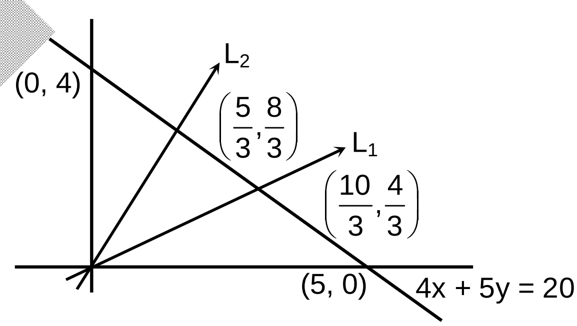
*15. The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is:

(1) 8/5 (2) 2/5
(3) 30/41 (4) 25/41

Ans. **(3)**

Sol. $M_{L_1} = \dfrac{2}{5}$, $M_{L_2} = \dfrac{8}{5}$

$$\tan\theta = \left|\frac{M_{L_2} - M_{L_1}}{1 + M_{L_2} M_{L_1}}\right| = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{8}{5} \times \frac{2}{5}}\right| = \frac{30}{41}$$

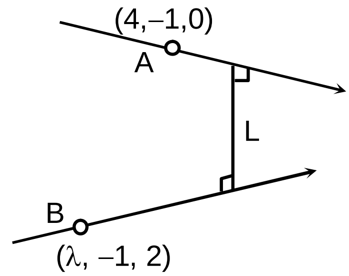


16. If the shortest distance between the lines $\dfrac{x-4}{1} = \dfrac{y+1}{2} = \dfrac{z}{-3}$ and $\dfrac{x-\lambda}{2} = \dfrac{y+1}{4} = \dfrac{z-2}{-5}$ is $\dfrac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 5 (2) 10
(3) 8 (4) 7

Ans. **(3)**

Sol. $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$



$\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$

AB = $(\lambda-4)\hat{i}+2\hat{k}$

L(DR) = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}=2\hat{i}-\hat{j}$

shortest distance $\left((\lambda-4)\hat{i}+2\hat{k}\right)\frac{\left(2\hat{i}-\hat{j}\right)}{\sqrt{5}}=\left|\frac{2\lambda-8-0}{\sqrt{5}}\right|=\frac{6}{\sqrt{5}}$

$|\lambda-4|=3$

$\lambda=7, 1$

sum = 8

17. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx=a+b\sqrt{2}+c\sqrt{3}$, where a, b, c are rational numbers, then 2a + 3b – 4c is equal to:

(1) 7
(2) 10
(3) 4
(4) 8

Ans. **(4)**

Sol. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx=a+b\sqrt{2}+c\sqrt{3}$

$\int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2}dx$

$\frac{1}{2}\left[\frac{(x+3)^{3/2}}{3/2}-\frac{(1+x)^{3/2}}{3/2}\right]_0^1$

$\frac{1}{3}\left[4^{3/2}-2^{3/2}-3^{3/2}+1\right]$

$\frac{1}{3}\left[9-2\sqrt{2}-3\sqrt{3}\right]=a+b\sqrt{2}+c\sqrt{3}$

a = 3, b = $-\frac{2}{3}$, c = –1

2a + 3b – 4c = 6 – 2 + 4 = 8

*18. ${}^{n-1}C_r=\left(k^2-8\right){}^nC_{r+1}$ if and only if:

(1) $2\sqrt{3}<k<3\sqrt{3}$
(2) $2\sqrt{2}<k\le 3$
(3) $2\sqrt{2}<k<2\sqrt{3}$
(4) $2\sqrt{3}<k\le 3\sqrt{2}$

Ans. **(2)**

Sol. ${}^{n-1}C_r=(k^2-8)\,{}^nC_{r+1}$

$k^2-8=\frac{(n-1)}{r!(n-r-1)!}\frac{1}{\frac{n!}{(r+1)!(n-r-1)}}=\frac{r+1}{n}$

$\lim\limits_{n\to\infty}\frac{r+1}{n} < \frac{r+1}{n} < \lim\limits_{n\to\infty}\frac{n+1}{n}$

$0 < \frac{r+1}{n} < 1$

$0 < k^2 - 8 < 1$

$8 < k^2 < 9$

$2\sqrt{2} < k < 3$

*19. Let $a_1, a_2, \ldots, a_{10}$ be 10 observations such that $\sum\limits_{k=1}^{10} a_k = 50$ and $\sum\limits_{\forall\, k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(1) 10 (2) 5

(3) $\sqrt{115}$ (4) $\sqrt{5}$

Ans. **(4)**

Sol. $\sum\limits_{k=1}^{10} a_k = 50$, $\sum\limits_{k<j} a_k a_j = 1100$

$$\sum_{k<j} a_k a_j = \frac{1}{2}\left[\sum_{k=1}^{10}\sum_{j=1}^{10} a_k a_j - \sum_{k=1}^{10}(a_k)^2\right]$$

$$1100 \times 2 = 50 \times 50 - \sum_{k=1}^{10}(a_k)^2$$

$$\sum_{k=1}^{10}(a_k)^2 = 2500 - 2200 = 300$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum a_k^2}{10} - \left(\frac{\sum a_k}{10}\right)^2}$$

$$\text{Standard Deviation} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2} = \sqrt{5}$$

*20. Consider the function. $f(x) = \begin{cases} \dfrac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x < 3 \\ \dfrac{\sin(x-3)}{2^{x-[x]}}, & x > 3 \\ b, & x = 3 \end{cases}$, where [x] denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is:

(1) 1 (2) 4

(3) 2 (4) Infinitely many

Ans. **(1)**

Sol. $f(x) = \begin{cases} \dfrac{a\left(7x-12-x^2\right)}{b\left|x^2-7x+12\right|} & ; \quad x<3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} & ; \quad x>3 \\ b & ; \quad x=3 \end{cases}$

LHL = $\dfrac{a(3-x)(x-4)}{b|(x-3)(x-4)|} = \dfrac{-a}{b}$

RHL = $2^{\frac{\sin(x-3)}{x-[x]}} = 2^{\frac{\sin(3+h-3)}{3+h-3}} = 2$

f(3) = b

b = 2, a = –4 one set is possible

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

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21. Let for a differentiable function f : (0, ∞) → R, $f(x) - f(y) \geq \log_e\left(\dfrac{x}{y}\right) + x - y$, $\forall$ x, $y \in (0, \infty)$.

Then $\displaystyle\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to ________

Ans. **2890**

Sol. f(x) – f(y) ≥ ln(x) – ln y + (x – y)

x > y ; $\dfrac{f(x)-f(y)}{x-y} \geq \dfrac{\ln(x)-\ln(y)}{x-y}$

y → x ; $f'(x) \geq \dfrac{1}{x}+1$ LHD

f(x) – f(y) ≥ ln (x) – ln (y) + x – y

y > x ; $\dfrac{f(x)-f(y)}{x-y} \leq \dfrac{\ln x - \ln y}{x-y}+1$

LHD = RHD

$f'(x) = \dfrac{1}{x}+1$

$\displaystyle\sum_{x=1}^{20} f'\left(\frac{1}{x^2}\right) = \sum_{x=1}^{20} x^2 + 1$ = $1^2 + 2^2 + 3^2 + \ldots.. + 20^2$ + (1 + 1 + 1 + ….. + 1)

= $\dfrac{20(21)(41)}{6} + 20 = 287 + 20 = 2890$

*22. If $8 = 3 + \dfrac{1}{4}(3+p) + \dfrac{1}{4^2}(3+2p) + \dfrac{1}{4^3}(3+3p) + \ldots, \infty$, then the value of p is ________

Ans. **9**

Sol. $8 = 3 + \frac{1}{4}(3+P) + \frac{1}{4^2}(3+2P) + \frac{1}{4^3}(3+3P) + ...\infty$ …. (1)

$\frac{8}{4} = \frac{3}{4} + \frac{(3+P)}{4^2} + \frac{(3+2P)}{4^3} .....\infty$ ….. (2)

Subtracting equation (2) from (1), we get

$6 = 3 + \frac{P}{4} + \frac{P}{4^2} + \frac{P}{4^3} + ...\infty$

$3 = \frac{P/4}{1-\frac{1}{4}} = \frac{P}{3}$ $\therefore$ P = 9

23. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrics, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. If α = |B| and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to ________

Ans. 28

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ B_1 & B_2 & B_3 \end{array}$

$AB_1 = \begin{bmatrix} 2a_1 + a_3 \\ a_1 + a_2 \\ a_1 + a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow a_1 = 1, a_2 = -1, a_3 = -1$

$AB_2 = \begin{bmatrix} 2b_1 + b_3 \\ b_1 + b_2 \\ b_1 + b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \Rightarrow b_1 = 2, b_2 = 1, b_3 = -2$

$AB_3 = \begin{bmatrix} 2c_1 + c_3 \\ c_1 + c_2 \\ c_1 + c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow c_1 = 2, c_2 = 0, c_3 = -1$

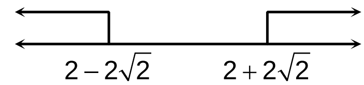
$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix} \Rightarrow \beta = 1, \alpha = |B| = 3, \alpha^3 + \beta^3 = 28$

24. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is ______

Ans. 5

Sol. For acute angle c, $\theta = \dfrac{(\alpha\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k})}{|\alpha\hat{i} - 2\hat{j} + 2\hat{k}||\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}|} > 0$

$\alpha^2 - 4\alpha - 4 > 0$
$(\alpha - 2)^2 > 8$



Least positive number is 5

25. If the solution of the differential equation (2x + 3y – 2)dx + (4x + 6y – 7)dy = 0, y(0) = 3, is $\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to _______

Ans. **29**

Sol. (2x + 3y – 2)dx + (4x + 6y – 7)dy = 0

2x + 3y – 2 = t

$2 + \dfrac{3dy}{dx} = \dfrac{dt}{dx}$; $\dfrac{dy}{dx} = \dfrac{1}{3}\left[\dfrac{dt}{dx} - 2\right]$

x = 0, y = 3, t = 7

$t + (2t-3)\dfrac{dy}{dx} = 0$

$t + \dfrac{(2t-3)}{3}\left(\dfrac{dt}{dx} - 2\right) = 0$

$3t + (2t-3)\dfrac{dt}{dx} = 4t - 6$

$(2t-3)\dfrac{dt}{dx} = t - 6$

$\int \dfrac{2(t-6)+9}{t-6}dt = \int dx$

2t + 9 ln |t – 6| = x + c

x = 0, t = 7

14 = c

2t + 9 ln |t – 6| = x + 14

2(2x + 3y – 2) + 9 ln|2x + 3y – 2 – 6| = x + 14

3x + 6y + 9 ln |2x + 3y – 8| = 18

x + 2y + 3 ln |2x + 3y – 8| = 6

$\alpha = 1, \beta = 2, \gamma = 8$

$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

*26. Let the set of all $a \in R$ such that the equation cos2x + a sinx = 2a – 7 has a solution be [p, q] and $r = \tan 9^o - \tan 27^o - \dfrac{1}{\cot 63^o} + \tan 81^o$, then pqr is equal to ________

Ans. **48**

Sol. r = tan9° – tan27° – tan63° + tan81°

= (tan9° + tan81°) – (tan27° + tan63°)

$= \dfrac{2}{\sin 18^\circ} - \dfrac{2}{\sin 54^\circ}$

$= \dfrac{2(\sin 54^\circ - \sin 18^\circ)}{\sin 18^\circ \sin 54^\circ}$

$= \dfrac{2 \times 2\cos 36^\circ \sin 18^\circ}{\sin 18^\circ \sin 54^\circ} = 4$

Given cos2x + asinx = 2a − 7

$\Rightarrow\ 2\sin^2 x - a\sin x + 2a - 8 = 0$

$\Rightarrow\ \sin x = \dfrac{a \pm \sqrt{a^2 - 8(2a-8)}}{4} = \dfrac{a \pm \sqrt{a^2 - 16a + 64}}{4} \Rightarrow \dfrac{a \pm (a-8)}{4}$

$\Rightarrow\ \sin x = \dfrac{a-4}{2} \Rightarrow \left|\dfrac{a-4}{2}\right| \le 1 \Rightarrow -2 \le a - 4 \le 2 \Rightarrow 2 \le a \le 6$

$\Rightarrow\ pqr = 2 \times 6 \times 4 = 48$

27. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6 | X > 3). Then $\dfrac{b+c}{a}$ is equal to ______

Ans. **12**

Sol. $a = p(x = 3) = \dfrac{5}{6}\dfrac{5}{6}\dfrac{1}{6} = \dfrac{25}{216}$

$b = p(x \ge 3) = \dfrac{5}{6}\cdot\dfrac{5}{6}\cdot\dfrac{1}{6} + \dfrac{5}{6}\cdot\dfrac{5}{6}\cdot\dfrac{5}{6}\cdot\dfrac{1}{6} + ...\infty = \dfrac{5^2/6^3}{1-\dfrac{5}{6}} = \dfrac{5^2}{6^2}$

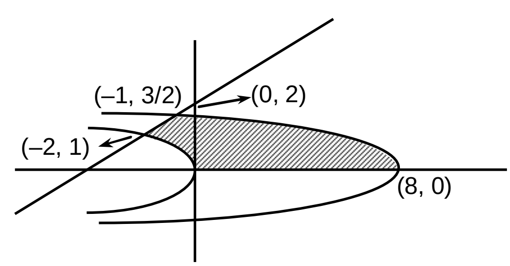
$c = p(x \ge 6 / x > 3) = \dfrac{\dfrac{5^5/6^6}{1-\dfrac{5}{6}}}{\dfrac{5^3/6^4}{1-\dfrac{5}{6}}} = \dfrac{5^5}{6^6 \cdot \dfrac{5^3}{6^3}} = \dfrac{25}{36}$

$\dfrac{b+c}{a} = \dfrac{\dfrac{25}{36} + \dfrac{25}{36}}{\dfrac{25}{216}} = 12$

28. Let the area of the region {(x, y) : x − 2y + 4 ≥ 0, x + 2y^2 ≥ 0, x + 4y^2 ≤ 8, y ≥ 0} be $\dfrac{m}{n}$, where m and n are coprime number. Then m + n is equal to ______

Ans. **119**

Sol. Required area



$= \displaystyle\int_0^{3/2} \left(8 - 4y^2 - 2y + 4\right)dy - \int_0^{1}\left(-2y^2 - 2y + 4\right)dy$

$= \left(12y - \dfrac{4}{3}y^3 - y^2\right)\Big|_0^{3/2} + \left(\dfrac{2}{3}y^3 + y^2 - 4y\right)\Big|_0^{1}$

$= \left(18 - \dfrac{4}{3}\times\dfrac{27}{8} - \dfrac{9}{4}\right) + \left(\dfrac{2}{3} + 1 - 4\right)$

$= 18 - \dfrac{27}{4} + \dfrac{2}{3} - 3$

$= 15 - \dfrac{73}{12} = \dfrac{107}{12}.$

29. Let f(x) = x^3 + x^2 f′(1) + x f″(2) + f‴(3), x ∈ R. Then f′(10) is equal to _______

Ans. **202**

Sol. $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$a = f'(1)\ \ b = f''(2)\ \ c = f'''(3)$

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$f'(1) = 3 + 2a + b = a$

$a + b + 3 = 0$ ..... (1)

$f''(x) = 6x + 2a$

$f''(2) = 12 + 2a$

$b = 12 + 2a$ ..... (2)

$f'''(x) = 6$

$c = 6$ ..... (3)

$a = -5,\ b = 2$

$f(x) = x^3 - 5x^2 + 2x + 6$

$f'(x) = 3x^2 - 10x + 2$

$f'(10) = 300 - 100 + 2 = 202$

30. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then $5(3A - 2B - C)$ is equal to ________

Ans. **5**

Sol. As α satisfies $x^2 + x + 1 = 0 \Rightarrow \alpha^2 + \alpha + 1 = 0 \Rightarrow \alpha = \omega$ or ω^2

$$\Rightarrow\quad (1+\alpha)^7 = (1+\omega)^7 = \left(-\omega^2\right)^7 = -\omega^2 = A + B\omega + C\omega^2$$

$$\Rightarrow\quad -\left(\frac{-1-\sqrt{3}i}{2}\right) = A + B\left(\frac{-1+\sqrt{3}i}{2}\right) + C\left(\frac{-1-\sqrt{3}i}{2}\right)$$

$$\Rightarrow\quad A - \frac{B}{2} - \frac{C}{2} = \frac{1}{2}\ \&\ (B-C)\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \Rightarrow B - C = 1\ \&\ 2A - B - C = 1$$

$$\Rightarrow\quad B = 1 + C\ \&\ A = 1 + C$$

Now $5(3A - 2B - C) = = 5\left[3(1+C) - 2(1+C) - C\right] = 5$.

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

31. The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be :

(1) $\frac{9}{16}R$
(2) $\frac{16}{9}R$
(3) $\frac{4}{3}R$
(4) $\frac{3}{4}R$

Ans. (2)

Sol. Bohr's radius $\propto n^2$

$R_3 = R = 9R_0$

$R_4 = 16R_0 = \frac{16R}{9}$

32. Identify the physical quantity that cannot be measured using spherometer:

(1) Thickness of thin plates
(2) Radius of curvature of concave surface
(3) Radius of curvature of convex surface
(4) Specific rotation of liquids

Ans. (4)

Sol. Specific rotation can't be measured using spherometer

33. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have:

(A) E = 0. B = 0
(B) E = 0. B $\neq$ 0
(C) E $\neq$ 0, B = 0
(D) E = 0, B $\neq$ 0

Choose the **most appropriate** answer from the options given below :

(1) (A), (B) and (C) only
(2) (A), (C) and (D) only
(3) (A), (B) and (D) only
(4) (B), (C) and (D) only

Ans. (3)

Sol. $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{F} = 0 \Rightarrow \vec{E} + \vec{v} \times \vec{B} = 0$

$\vec{F} = 0$ can be zero for (A), (B) and (D) only

34. An electric charge $10^{-6}\ \mu C$ is placed at origin $(0, 0)m$ of X-Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3}, \sqrt{3})m$ and $(\sqrt{6}, 0)m$ respectively. The potential difference between the points P and Q will be:

(1) $\sqrt{6}$ V (2) $\sqrt{3}$ V
(3) 3 V (4) 0 V

Ans. **(4)**

Sol. Distance of P from origin = $r_P = \sqrt{6}$

Distance of Q from origin = $r_Q = \sqrt{6}$

$$\Delta V = \frac{kq}{r_P} - \frac{kq}{r_Q} = 0$$

*35. The average kinetic energy of a monoatomic molecule is $0.414 eV$ at temperature :
(Use $K_8 = 1.38 \times 10^{-23}$ J/mol –K)

(1) 1600 K (2) 1500 K
(3) 3200 K (4) 3000 K

Ans. **(3)**

Sol. $(K.E.)_{average} = \frac{f}{2}kT = 0.414\ eV$ $\{f = 3\}$

$$T = \frac{2 \times 0.414 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 3200\ K$$

*36. The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be:

(1) 4 g (2) $\frac{g}{4}$
(3) $\frac{g}{2}$ (4) 2 g

Ans. **(1)**

Sol. $g = \frac{GM}{R^2}$

$D' = \frac{D}{2} \Rightarrow R' = \frac{R}{2}$

$\Rightarrow g' = 4g$

*37. 0.08 kg air is heated at constant volume through $5°C$. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/ cal. The change in its internal energy is approximately.

(1) 284 J (2) 298 J
(3) 318 J (4) 142 J

Ans. **(1)**

Sol. $\Delta U = mC_V\Delta T$

$= 0.08 \times 0.17 \times 4180 \times 5$

≈ 284 J

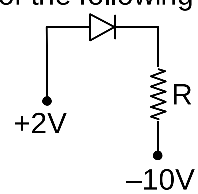
38. A convex lens of focal length 40 cm forms an image of an extended source of light on a photoelectric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is :

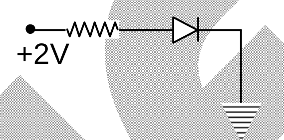
(1) I
(2) 4 I
(3) $\frac{I}{2}$
(4) 2 I

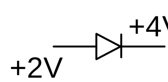
Ans. (1)

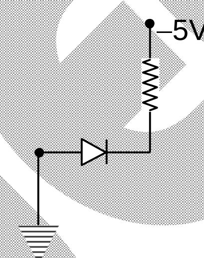
Sol. Number, of photons striking on lens per second remains same.
So, photoelectric current remains same

39. Which of the following circuits is reverse - biased ?

(1)


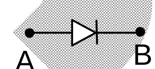
(2)


(3)


(4)


Ans. (3)

Sol. For reverse bias:
$V_B > V_A$



40. Given below are two statements:
Statement (I) : Planck's constant and angular momentum have same dimensions.
Statement (II) : Linear momentum and moment of force have same dimensions.
In the light of the above statements, choose the correct answer from the options given below:

(1) **Statement I** is true but **Statement II** is false
(2) **Statement I** is false but **Statement II** is true
(3) Both **Statement I** and **Statement II** are true
(4) Both **Statement I** and **Statement II** are false

Ans. (1)

Sol. $E = h\nu = \frac{1}{2}L\omega$
$[h] = [L]$
Dimension of h = Dimension of L

41. A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 s. The average emf induced in the loop during this time is:

(1) –1 V
(2) –2 V
(3) +1 V
(4) +2 V

Ans. **(3)**

Sol. emf = $-\dfrac{d\phi}{dt} = -\dfrac{\Delta\phi}{\Delta t} = -\left(\dfrac{0 - BA\cos\theta}{\Delta t}\right)$

emf = $\dfrac{4 \times 5 \times 0.5}{10} = 1V$

*42. Position of an ant (S in metres) moving in $Y - Z$ plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1s will be:

(1) 4 m/s in y -direction
(2) 4 m/s in x -direction
(3) 16 m/s in y -direction
(4) 9 m/s in z -direction

Ans. **(1)**

Sol. $\vec{v} = \dfrac{d\vec{S}}{dt} = 4t\,\hat{j}$

at t = 1

$\vec{v} = 4\ m/s\,\hat{j}$

*43. A body of mass 1000 kg is moving horizontally with a velocity 6m/s. If 200 kg extra mass is added, the final velocity (in m/s) is :

(1) 6
(2) 5
(3) 2
(4) 3

Ans. **(2)**

Sol. Using conservation of momentum,

$P_i = P_f$

$1000 \times 6 = 1200 \times v$

v = 5 m/s

44. A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

(1) 25 R
(2) 5 R
(3) $\dfrac{1}{25}R$
(4) $\dfrac{1}{5}R$

Ans. **(3)**

Sol. The each part has resistance $\dfrac{R}{5}$ and has length L′ = L/5

The resistances are connected in parallel

Hence, resultant resistance = R/25

*45. A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve of radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, g = $10m/s^2$):

(1) 4.8 cm
(2) 4.2 cm
(3) 5.4 cm
(4) 6.0 cm

Ans. **(3)**

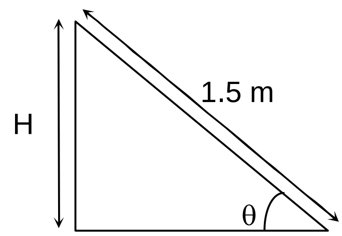
Sol. $v = \sqrt{Rg\tan\theta}$

$\tan\theta = \frac{144}{400 \times 10} = 36 \times 10^{-3}$

as tan θ is very small

$\Rightarrow \tan\theta \approx \sin\theta = \frac{H}{1.5}$

$H = 1.5 \times 36 \times 10^{-3} = 5.4$ cm



*46. Given below are two statements :

Statement (I) : Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect
(2) **Statement I** is incorrect but **Statement II** is correct
(3) Both **Statement I** and **Statement II** are correct
(4) **Statement I** is correct but **Statement II** is incorrect

Ans. **(2)**

Sol. Viscosity of gases is lesser than that of liquid

47. A plane electromagnetic wave propagating in x -direction is described by $E_y = (200\ Vm^{-1})\sin[1.5 \times 10^7 t - 0.05x]$; The intensity of the wave is :

(Use $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$)

(1) 35.4 Wm^{-2}
(2) 106.2 Wm^{-2}
(3) 26.6Wm^{-2}
(4) 53.1Wm^{-2}

Ans. **(4)**

Sol. $I = \frac{1}{2}c\varepsilon_0 E_0^2$

$= \frac{1}{2} \times 3 \times 10^8 \times 8.85 \times 10^{-12} \times 4 \times 10^4$

$I = 53.1\ Wm^{-2}$

48. A wire of length 10cm and radius $\sqrt{7} \times 10^{-4}$ m is connected across the right gap of a meter bridge. When a resistance of 4.5Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60cm from the left end. If the resistivity of the wire is $R \times 10^{-7}$ Ωm, then value of R is :

(1) 35
(2) 63
(3) 66
(4) 70

Ans. **(3)**

Sol. $\frac{4.5}{60} = \frac{R'}{40} \Rightarrow R' = 3\ \frac{\rho\ell}{A}$

$\rho = 66 \times 10^{-7}\ \Omega m$

$\Rightarrow R = 66$

*49. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

(1) 5 : 4
(2) 3 : 5
(3) 2 : 5
(4) 4 : 5

Ans. (3)

Sol. $K.E. = \frac{P^2}{2m}$

$\frac{P_1^2}{P_2^2} = \frac{m_1}{m_2} \Rightarrow \frac{P_1}{P_2} = \frac{2}{5}$

50. If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be :

(1) $\pi - 2A$
(2) $\frac{\pi}{2} - 2A$
(3) $\frac{\pi}{2} - A$
(4) $\pi - A$

Ans. (1)

Sol. $\delta = i + e - A$

For minimum deviation

$i = e,\ r_1 = r_2 = r = A/2$

$\delta_{min} = 2i - A$

Using Snell's law

$\sin i = \mu \sin r = \mu \sin\left(\frac{A}{2}\right)$

$\sin i = \cos\left(\frac{A}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right)$

$\Rightarrow i = \frac{\pi}{2} - \frac{A}{2}$

$\Rightarrow \delta = 2\left(\frac{\pi}{2} - \frac{A}{2}\right) - A = \pi - 2A$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*51. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s . The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm , where α = _______.

Ans. 12

Sol. $v = \omega\sqrt{A^2 - x^2}$

At mean position x = 0

$\Rightarrow$ v = ωA
$\Rightarrow$ 10 = $\omega \times 4$
$\Rightarrow$ ω = 5/2
At distance x, velocity becomes 5 cm/s
$\Rightarrow 5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x = \sqrt{12}$
$\Rightarrow$ α = 12

52. In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides $(A \approx 118)$, having binding energy of 8.6 MeV/Nucleon . The energy released in the process would be _______MeV .

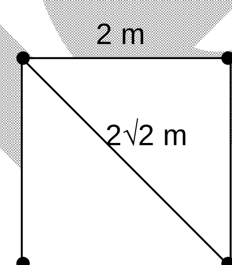
Ans. **236**

Sol. $E = (\Delta m)c^2 = [2(118) \times 8.6 - 236 \times 7.6] = 236$

*53. Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _______kgm^2.

Ans. **16**

Sol. $I = 2 \times mr^2 + m\left[\left(\sqrt{2}\right)r\right]^2$
$I = 4\ mr^2 = 16$



54. A thin metallic wire having cross sectional area of $10^{-4}\ m^2$ is used to make a ring of radius 30 cm . A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30p C is kept at the centre of the ring. The tension in the ring is_______ N; provided that the ring does not get deformed (neglect the influence of gravity).
$\left(\text{given, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ SI units}\right)$

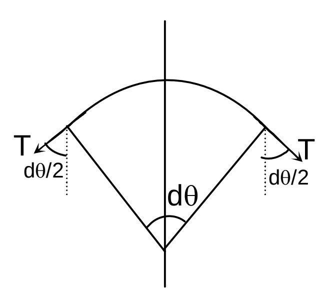
Ans. **3**

Sol. Taking $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$ for small $d\theta$

$2T\left(\frac{d\theta}{2}\right) = \frac{k\lambda R d\theta}{R^2}$

$\lambda = \frac{Q}{2\pi RA}$

$T = \frac{kQq}{2\pi R^2} = 3\ N$



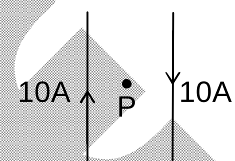
55. Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $I = i_0 \sin \omega t$, where $i_0 = 5$ A and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$V. The value of α is_______

Ans. 2

Sol. $\varepsilon = -\frac{Mdi}{dt} = -0.002 \times i_0 \omega \cos \omega t$

$\varepsilon_{max} = \frac{\pi}{2}$

56. Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is_______ μT. (Given : $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})

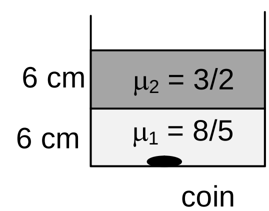


Ans. 160

Sol. $B = \frac{\mu_0 i}{2\pi d}$

$B_{net} = 2 \times \frac{\mu_0 i}{2\pi d} = 160\mu T$

57. Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$cm. The value of α is_______



Ans. 31

Sol. Apparent depth = $\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{8/5} + \frac{6}{3/2} = \frac{15}{4} + 4 = \frac{31}{4}$

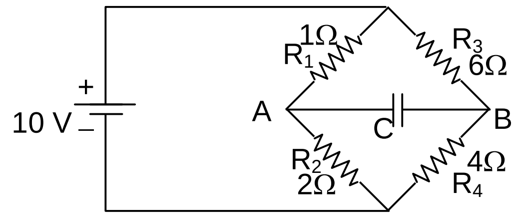
*58. If average depth of an ocean is 4000 m and the bulk modulus of water is 2×10^9 Nm^{-2}, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is _______ (Given, g = 10 ms^{-2}, ρ = 1000 kgm^{-3})

Ans. 2

Sol. $\beta = -V\frac{\Delta P}{\Delta V}$

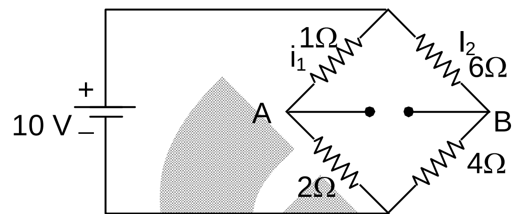
$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{\rho g h}{\beta} = \frac{10^4 \times 4 \times 10^3}{2 \times 10^9} = 2 \times 10^{-2}$

59. The charge accumulated on the capacitor connected in the following circuit is_______ μC. (Given C=150 μF)



Ans. **400**

Sol. At steady state



$R_{eq} = \frac{30}{13}\Omega$

$i = \frac{V}{R_{eq}} = \frac{13}{3}A$

$i_1 = \frac{10}{3}A$, i_2 = 1A

$V_B - V_A = 6 - \frac{10}{3} = \frac{8}{3} = \frac{Q}{C}$

$\Rightarrow$ Q = 400 μC

*60. A particle starts from origin at t = 0 with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})$ m/s^2. If the x-coordinate of the particle at that instant is 84m , then the speed of the particle at this time is $\sqrt{\alpha}$ m/s . The value of α is _______

Ans. **673**

Sol. $S_x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow 84 = 5t + \frac{1}{2}3t^2$

$\Rightarrow$ t = 6 sec

For t = 6 sec

v_x = 5 + 18 = 23

v_y = 12 $\Rightarrow v = \sqrt{673}$ m/s

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*61. Given below are two statements:
Statement (I) : Aqueous solution of ammonium carbonate is basic.
Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(1) **Statement I** is incorrect but **Statement II** is correct
(2) **Statement I** is correct but **Statement II** is incorrect
(3) Both **Statement I** and **Statement II** are correct
(4) Both **Statement I** and **Statement II** are incorrect

Ans. **(3)**

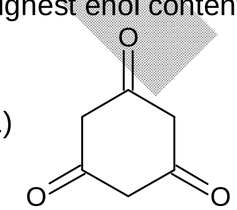
Sol. pK_a of carbonic acid 6.36 & 10.33.
pK_b of ammonium hydroxide = 4.74
So, pH $= 7 + \frac{1}{2}(pK_a - pK_b)$
pH > 7 $\rightarrow$ (Basic)

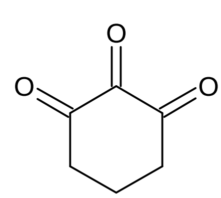
62. Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a:
(1) Neutral complex with coordination number four
(2) Dianionic complex with coordination number six
(3) Dianionic complex with coordination number four
(4) Tetraanionic complex with coordination number six

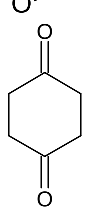
Ans. **(3)**

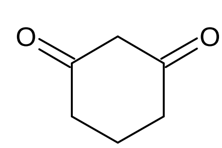
Sol.
$$PbCrO_4 + 4NaOH \longrightarrow Na_2CrO_4 + \underset{\text{(Lead complex)}}{Na_2\left[Pb(OH)_4\right]}$$
Dianionic with CN = 4.

*63. Highest enol content will be shown by:

(1) 

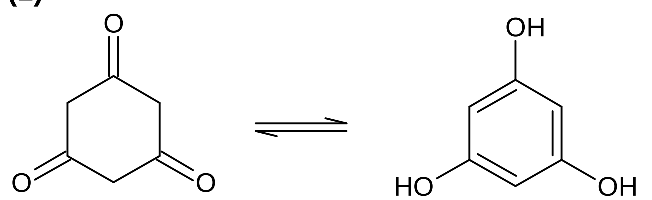
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



Aromatic
So, more stable

64. The electronic configuration for Neodymium is:
[Atomic Number for Neodymium 60]
(1) $[Xe]4f^4 6s^2$
(2) $[Xe]4f^1 5d^1 6s^2$
(3) $[Xe]4f^6 6s^2$
(4) $[Xe]5f^7 7s^2$

Ans. **(1)**
Sol. $Nd \equiv [Xe]4f^4 6s^2$

65. A solution of two miscible liquids showing negative deviation from Raoult's law will have:
(1) decreased vapour pressure, increased boiling point
(2) increased vapour pressure, decreased boiling point
(3) decreased vapour pressure, decreased boiling point
(4) increased vapour pressure, increased boiling point

Ans. **(1)**
Sol. $P_s < x_A P_A^o + x_B P_B^o$
So, vapour pressure is decreased and boiling point is increased.

66. The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is:
(1) Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.
(2) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.
(3) Racemisation occurs in both S_N1 and S_N2 reactions.
(4) Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Ans. **(2)**
Sol. $\left.\begin{array}{l}\text{In } S_N1 \text{ mech.} \longrightarrow \text{Racemisation occurs} \\ \text{In } S_N2 \text{ mech.} \longrightarrow \text{Inversion takes place}\end{array}\right\}$ if chiral alkyl halide is used.

*67. Choose the polar molecule from the following:
(1) CCl_4
(2) $CH_2 = CH_2$
(3) CO_2
(4) $CHCl_3$

Ans. **(4)**
Sol. $CCl_4 \longrightarrow \mu = 0,$ non polar
$CH_2 = CH_2 \longrightarrow \mu = 0,$ non polar
$CO_2 \longrightarrow \mu = 0,$ non polar
$CHCl_3 \longrightarrow \mu \neq 0,$ polar

68. Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is:

(1) R < P < Q
(2) Q < P < R
(3) Q < R < P
(4) R < Q < P

Ans. (3)

Sol. $P = [FeF_6]^{3-}$

$Fe^{3+}; F \rightarrow WFL$

$\downarrow$

$d^5 \rightarrow 5$ unpaired electron

$Q = [V(H_2O)_6]^{2+}$

$V^{2+}; H_2O$

$\downarrow$

$Fe^{2+} \rightarrow d^6 \rightarrow 4$ unpaired electron

So, order of spin only magnetic moment

P > R > Q

*69. Which of the following electronic configuration would be associated with the highest magnetic moment?

(1) $[Ar]3d^7$
(2) $[Ar]3d^3$
(3) $[Ar]3d^8$
(4) $[Ar]3d^6$

Ans. (4)

Sol.
1. $[Ar]3d^7 \rightarrow 3$ unpaired electron
2. $[Ar]3d^3 \rightarrow 3$ unpaird electron
3. $[Ar]3d^8 \rightarrow 2$ unpaired electron
4. $[Ar]3d^6 \rightarrow 4$ unpaired electron

*70. Given below are two statements:

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below:

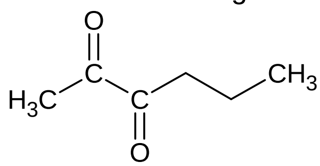
(1) **Statement I** is true but **Statement II** is false
(2) Both **Statement I** and **Statement II** are false
(3) Both **Statement I** and **Statement II** are true
(4) **Statement I** is false but **Statement II** is true

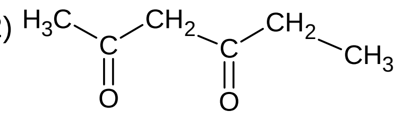
Ans. (1)

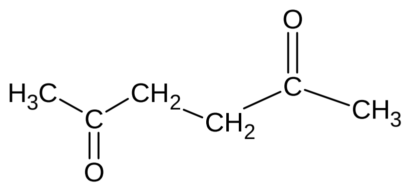
Sol. **Statement (I)** is correct.

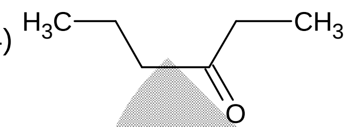
C_2H_5OH is 1^o alcohol so does not give turbidity.

*71. Which of the following has highly acidic hydrogen?

(1) 

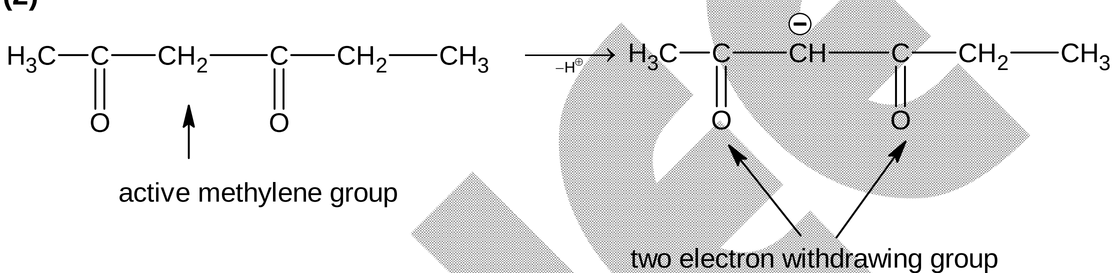
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



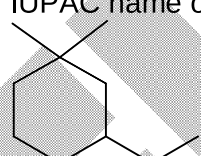
72. Element not showing variable oxidation state is:

(1) Chlorine (2) Fluorine
(3) Bromine (4) Iodine

Ans. **(2)**

Sol. Fluorine does not show variable oxidation state.

*73. IUPAC name of following compound (P) is:

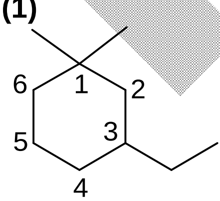


(P)

(1) 3-Ethyl-1,1-dimethylcyclohexane (2) 1-Ethyl-3,3-dimethylcyclohexane
(3) 1,1-Dimethyl-3-ethylcyclohexane (4) 1-Ethyl-5,5-dimethylcyclohexane

Ans. **(1)**

Sol.



IUPAC name 3-ethyl-1,1-dimethylcyclohexane

*74. Given below are two statements:

Statement (I) : The 4*f* and 5*f*-series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) : s-block elements can be found in pure form in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:
(1) **Statement I** is true but **Statement II** is false
(2) **Statement I** is false but **Statement II** is true
(3) Both **Statement I** and **Statement II** are true
(4) Both **Statement I** and **Statement II** are false

Ans. **(1)**

Sol. **Statement I** is true but **Statement II** is false

75. NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are:
(1) $CrO_2Cl_2, Na_2Cr_2O_7$
(2) CrO_2Cl_2, Na_2CrO_4
(3) $CrO_2Cl_2, KHSO_4$
(4) Na_2CrO_4, CrO_2Cl_2

Ans. **(2)**

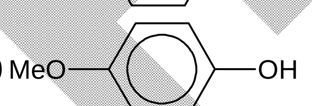
Sol.
$$NaCl + \text{conc. } H_2SO_4 + K_2Cr_2O_7 \longrightarrow \underset{\substack{\text{(reddish fumes)} \\ (B)}}{CrO_2Cl_2} \xrightarrow{NaOH} \underset{\substack{\text{(Yellow)} \\ (C)}}{Na_2CrO_4}$$

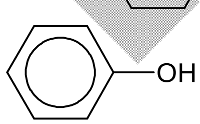
*76. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**
Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.
Reason (R) : Solid Boron has very strong crystalline lattice.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A).**
(2) **(A)** is false but **(R)** is true.
(3) **(A)** is true but **(R)** is false.
(4) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A).**

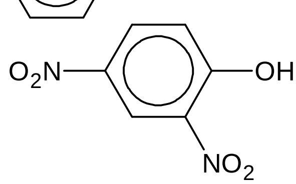
Ans. **(1)**

Sol. Assertion is correct, Reason is also correct and correct explanation as well.

*77. The ascending order of acidity of – OH group in the following compounds is:
(A) Bu – OH
(B) 

(C) 

(D) 

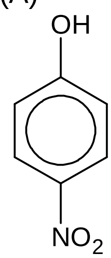
(E) 

Choose the correct answer from the options given below:
Options:

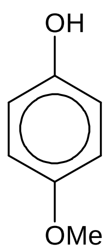
(1) (C) < (A) < (D) < (B) < (E)
(2) (C) < (D) < (B) < (A) < (E)
(3) (A) < (D) < (C) < (B) < (E)
(4) (A) < (C) < (D) < (B) < (E)

Ans. **(4)**

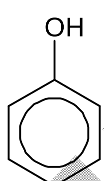
Sol. Bu – OH
(A)



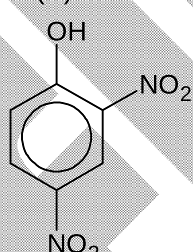
(B)



(C)

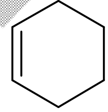


(D)



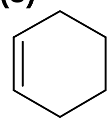
(E)

Order of A < C < D < B < E

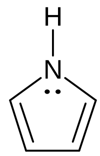
*78. Cyclohexene  is___________type of an organic compound.

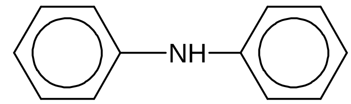
(1) Benzenoid aromatic
(2) Acyclic
(3) Alicyclic
(4) Benzenoid non-aromatic

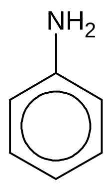
Ans. **(3)**

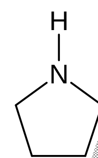
Sol.  is Alicyclic

*79. Which of the following is strongest Bronsted base?

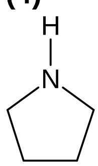
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol.  is strongest base as its lone pair electron are localised

80. Two nucleotides are joined together by a linkage known as:

(1) Phosphodiester linkage
(2) Disulphide linkage
(3) Peptide linkage
(4) Glycosidic linkage

Ans. (1)

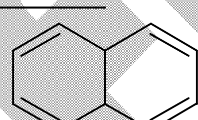
Sol. Nucleotides are joined together by phosphodiester linkage between $5'$ and $3'$ carbon atoms of pentose sugar.

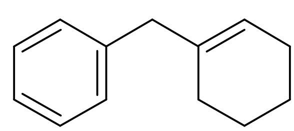
SECTION - B

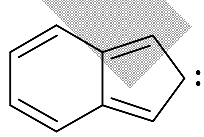
(Numerical Answer Type)

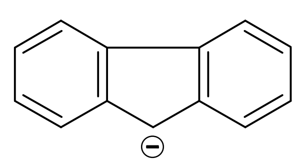
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*81. Among the given organic compounds, the total number of aromatic compounds is________

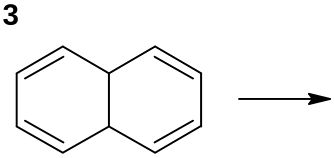
(1) 

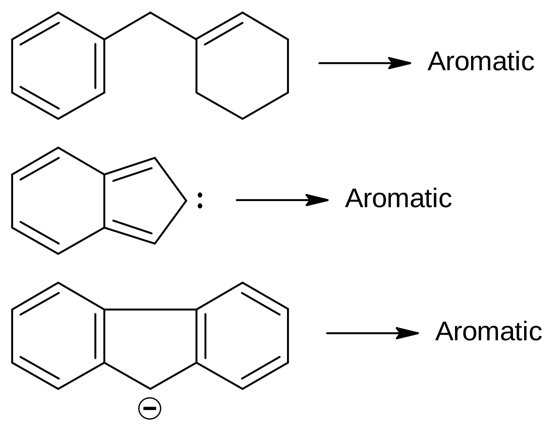
(2) 

(3) 

(4) 

Ans. 3

Sol.  Non-Aromatic



*82. From the given list, the number of compounds with +4 oxidation state of Sulphur is________
SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Ans. **3**

Sol. $SO_3 \rightarrow +6$
$H_2SO_3 \rightarrow +4$
$SOCl_2 \rightarrow +4$
$SF_4 \rightarrow +4$
$BaSO_4 \rightarrow +6$
$H_2S_2O_7 \rightarrow +6$

*83. If three moles of an ideal gas at 300 K expand isothermally from $30\ dm^3$ to $45\ dm^3$ against a constant opposing pressure of 80 kPa, then the amount of heat transferred is________J.

Ans. **1200**

Sol. $W = -P_{ext}\Delta V$
$= -80 \times 10^3 \times 15 \times 10^{-3}\ N/m^2 \times m^3$
$= -1200\ J$
$T \rightarrow \text{constant} \Rightarrow \Delta U = 0$
$q = -W \Rightarrow q = 1200\ J$

*84. Sum of bond order of CO and NO^+ is_________.

Ans. **6**

Sol. Bond order of $CO \equiv 3$
Bond order of $NO^+ \equiv 3$

*85. Mass of methane required to produce 22 g of CO_2 after complete combustion is_________g.
[Given Molar mass in $g\ mol^{-1}$ C = 12.0, H = 1.0, O = 16.0]

Ans. **8**

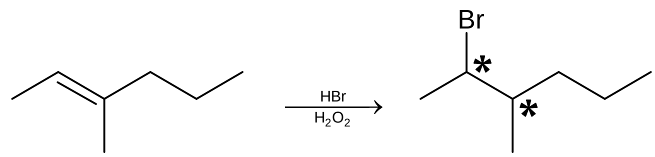
Sol. $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

Mass of CH_4 required $= \frac{22}{44} \times 16 = 8$ g.

*86. 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is________.

Ans. 4

Sol.



2 Chiral centre

87. The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S. T. P. will be__________g.

Ans. 108

Sol. $\left(\frac{W}{E}\right)_{Ag} = \left(\frac{W}{E}\right)_{O_2}$; $\quad W_{O_2} = \frac{5600}{22400} \times 32 = 8$ g

$\frac{W}{108} = \frac{8}{8}$

$W = 108$ g

88. Consider the following data for the given reaction

$2HI(g) \longrightarrow H_2(g) + I_2(g)$

	1	2	3
$HI\left(mol\ L^{-1}\right)$	0.005	0.01	0.02
$Rate\left(mol\ L^{-1}\ s^{-1}\right)$	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is________.

Ans. 2

Sol. $r = k[HI]^x$

$r_1 = k[HI]_1^x; \quad 7.5 \times 10^{-4} = k[0.005]^x \quad \ldots (1)$

$r_2 = k[HI]_2^x; \quad 3.0 \times 10^{-3} = k[0.01]^x \quad \ldots (2)$

$\frac{(1)}{(2)} \Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)^x \Rightarrow x = 2$

*89. The number of electrons present in all the completely filled subshells having $n = 4$ and $s = +\frac{1}{2}$ is______

(where n = principal quantum number and s = spin quantum number)

Ans. 16

Sol. For n = 4 possible subshells are 4s, 4p, 4d and 4f. So, number of electron with $s = +\frac{1}{2};\ = 16$

*90. Among the following, total number of meta directing functional groups is________. (Integer based)

$-OCH_3, -NO_2, -CN, -CH_3, -NHCOCH_3, -COR, -OH, -COOH, -Cl$

Ans. **4**

Sol. $-NO_2, -CN, -COR, -COOH$ are meta directing.