FIITJEE

Solutions to JEE(Main) -2024

Test Date: 1st February 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

(1) $\frac{2}{5}$ (2) $\frac{2}{7}$

(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

Ans. (2)

Sol. A – 2 Black and 2 white balls drawn

B – Bag contains 4 white and 4 black balls

$$P\left(\frac{B}{A}\right)=\frac{P\left(\frac{A}{B}\right)P(B)}{P(A)}=\frac{{}^4C_2\,{}^4C_2}{{}^2C_2\,{}^6C_2+{}^3C_2\,{}^5C_2+{}^4C_2\,{}^4C_2+{}^5C_2\,{}^3C_2+{}^6C_2\,{}^2C_2}$$

$$=\frac{36}{15+30+36+30+15}=\frac{2}{7}$$

2. The value of the integral $\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}$ equals:

(1) $\frac{\sqrt{2}\pi^2}{8}$ (2) $\frac{\sqrt{2}\pi^2}{16}$

(3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{64}$

Ans. (3)

Sol. $I=\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4 2x+\cos^4 2x}=\frac{\pi}{8}\int_0^{\frac{\pi}{4}}\frac{dx}{1-2\sin^2 2x\cos^2 2x}=\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{dx}{2-\sin^2 4x}$

$$=\frac{\pi}{4}\left[\int_0^{\frac{\pi-}{8}}\frac{\sec^2 4x\,dx}{2+\tan^2 4x}+\int_{\frac{\pi^+}{8}}^{\frac{\pi}{4}}\frac{\sec^2 4x\,dx}{2+\tan^2 4x}\right]=\frac{\pi}{16}\left[\int_0^{\infty}\frac{dt}{2+t^2}+\int_{-\infty}^{0}\frac{dt}{2+t^2}\right]=\frac{\sqrt{2}\pi^2}{32}$$

3. If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, $C = ABA^T$ and $X = A^TC^2A$, then det X is equal to:

(1) 243 (2) 729

(3) 27 (4) 891

Ans. (2)

Sol. $|X| = |A|^6 |B|^2 = 729$

*4. If $\tan A = \frac{1}{\sqrt{x\left(x^2+x+1\right)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}$, $0 < A, B, C < \frac{\pi}{2}$, then A + B is equal to:

(1) C (2) $\pi - C$

(3) $2\pi - C$ (3) $\frac{\pi}{2} - C$

Ans. (1)

Sol. $\tan(A+B) = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \frac{1}{\sqrt{x(x^2+x+1)}}\frac{\sqrt{x}}{\sqrt{x^2+x+1}}} = \frac{(1+x)\sqrt{x^2+x+1}}{\sqrt{x}(x^2+x)}$

$= \sqrt{\frac{x^2+x+1}{x^3}} = \tan C$

*5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 47 (2) 53

(3) 51 (4) 43

Ans. (3)

Sol. Case-I: 5, 0, 0, 0 $\rightarrow$ 1 way

Case-II: 4, 1, 0, 0 $\rightarrow$ $\frac{5!}{4!}$ = 5 ways

Case-III: 3, 2, 0, 0 $\rightarrow$ $\frac{5!}{3!\,2!}$ = 10 ways

Case-IV: 3, 1, 1, 0 $\rightarrow$ $\frac{5!}{3!\,2!}$ = 10 ways

Case-V: 2, 2, 1, 0 $\rightarrow$ $\frac{5!}{2!\,2!\,2!}$ = 15 ways

Case-VI: 2, 1, 1, 1 $\rightarrow$ $\frac{5!}{2!\,3!}$ = 10 ways

51 ways

*6. Let $S = \{z \in C : |z-1| = 1 \text{ and } \left(\sqrt{2}-1\right)\left(z+\bar{z}\right) - i\left(z-\bar{z}\right) = 2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z\in S}|z|$ and $|z_2| = \max_{z\in S}|z|$. Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals:

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. $z\bar{z} - z - \bar{z} = 0 \Rightarrow \bar{z} = \dfrac{z}{z-1}$

$\Rightarrow (\sqrt{2}-1)\left(z + \dfrac{z}{z-1}\right) - i\left(z - \dfrac{z}{z-1}\right) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)z^2 - i(z^2 - 2z) = 2\sqrt{2}(z-1)$

$\Rightarrow (\sqrt{2}-1-i)z^2 + z(2i - 2\sqrt{2}) + 2\sqrt{2} = 0$

$\Rightarrow z = \dfrac{2\sqrt{2} - 2i \pm \sqrt{8 - 4 - 8\sqrt{2}i - 8\sqrt{2}(\sqrt{2}-1-i)}}{2(\sqrt{2}-1-i)}$

$= \dfrac{2\sqrt{2} - 2i \pm \sqrt{8\sqrt{2} - 12}}{2(\sqrt{2}-1-i)} = \dfrac{\sqrt{2} - i \pm (\sqrt{2}-1)i}{\sqrt{2}-1-i} = \underbrace{\dfrac{\sqrt{2} + (\sqrt{2}-2)i}{\sqrt{2}-1-i}}_{z_2}; \underbrace{\dfrac{\sqrt{2} - \sqrt{2}i}{\sqrt{2}-1-i}}_{z_1}$

$\Rightarrow \left|\sqrt{2}z_1 - z_2\right|^2 = \left|\dfrac{2 - \sqrt{2} - \sqrt{2}i}{\sqrt{2}-1-i}\right|^2 = 2$

*7. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\dfrac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is:

(1) 31 (2) 28

(3) 30 (4) 32

Ans. (BONUS)

Sol. Median is 170 $\Rightarrow$ a, b < 170

$\dfrac{205}{7} = \dfrac{60 + 40 + 20 + 170 - a + 170 - b}{7} \Rightarrow a + b = 255$

$\Rightarrow$ Mean = $\dfrac{1180}{7}$

To calculate mean deviation about mean it is required to know whether a, b are less than or greater than $\dfrac{1180}{7}$ which is not known

Incomplete information

8. Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = \left(\left(\left(\vec{a} \times \vec{b}\right) \times \hat{i}\right) \times \hat{i}\right) \times \hat{i}$. Then $\vec{c}.\left(-\hat{i} + \hat{j} + \hat{k}\right)$ is equal to:

(1) – 12 (2) – 10

(3) – 13 (4) – 15

Ans. (1)

Sol. $\left(\left(\left((\vec{a} \times \vec{b}) \times \hat{i}\right) \times \hat{i}\right) \times \hat{i}\right) \cdot \left(-\hat{i} + \hat{j} + \hat{k}\right) = \left((\vec{a} \times \vec{b}) \times \hat{i}\right) \times \hat{i} \cdot \left(\hat{k} - \hat{j}\right) = \left((\vec{a} \times \vec{b}) \times \hat{i}\right) \cdot \left(-\hat{j} - \hat{k}\right) = (\vec{a} \times \vec{b}) \cdot \left(-\hat{k} + \hat{j}\right)$

Where $\vec{a} \times \vec{b} = 2\hat{i} - 23\hat{j} - 11\hat{k}$

*9. Let S = $\left\{x \in R : \left(\sqrt{3} + \sqrt{2}\right)^x + \left(\sqrt{3} - \sqrt{2}\right)^x = 10\right\}$. Then the number of elements in S is:

(1) 4 (2) 0

(3) 2 (4) 1

Ans. (3)

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x + \frac{1}{\left(\sqrt{3}+\sqrt{2}\right)^x} = 10$. Let $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$\Rightarrow t^2 - 10t + 1 = 0 \Rightarrow t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^x = 5+2\sqrt{6} \Rightarrow x = 2$

$\left(\sqrt{3}+\sqrt{2}\right)^x = 5-2\sqrt{6} = \left(5+2\sqrt{6}\right)^{-1} \Rightarrow x = -2$

10. The area enclosed by the curves xy + 4y = 16 and x + y = 6 is equal to:
(1) $28 - 30\log_e 2$ (2) $30 - 28\log_e 2$
(3) $30 - 32\log_e 2$ (4) $32 - 30\log_e 2$

Ans. (3)

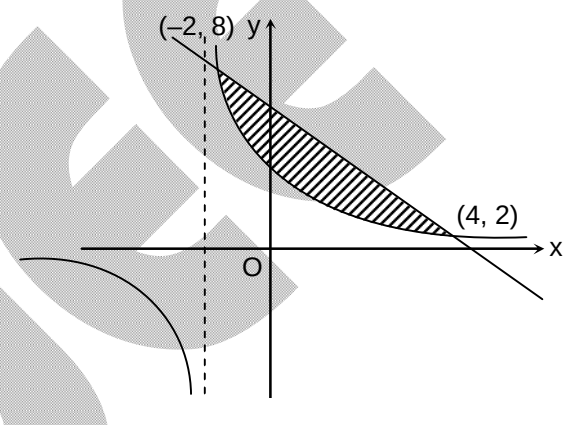
Sol. (x + 4)y = 16, x + y = 6

Area = $\int_{-2}^{4} 6 - x - \frac{16}{4+x}dx$

$= \left|6x - \frac{x^2}{2} - 16\ln(4+x)\right|_{-2}^{4}$

$= 36 - 6 - 16\ln 4$

$= 30 - 32\ln 2$



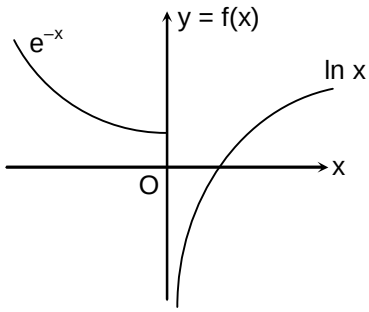
11. let $f : R \to R$ and $g : R \to R$ be defined as $f(x) = \begin{cases} \log_e x, x > 0 \\ e^{-x}, x \le 0 \end{cases}$ and $g(x) = \begin{cases} x, x \ge 0 \\ e^x, x < 0 \end{cases}$. Then,

$gof : R \to R$ is:
(1) one-one but not onto (2) neither one-one nor onto
(3) onto but not one-one (4) both one-one and onto

Ans. (2)

Sol. $gof(x) = \begin{cases} f(x) & f(x) \ge 0 \\ e^{f(x)} & f(x) < 0 \end{cases}$

$= \begin{cases} e^{-x} & x \le 0 \\ x & 0 < x < 1 \\ \ln x & x \ge 1 \end{cases}$



12. If the system of equations
$2x + 3y - z = 5$
$x + \alpha y + 3z = -4$
$3x - y + \beta z = 7$
has infinitely many solutions, then $13\alpha\beta$ is equal to _____.
(1) 1110 (2) 1120
(3) 1210 (4) 1220

Ans. (2)

Sol. $\begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix} = 0$

$\Rightarrow 14\alpha - 8 - 3(19) + 5(-1 - 3\alpha) = 0$

$\Rightarrow -70 = \alpha$

and $\begin{vmatrix} 2 & 5 & -1 \\ 1 & -4 & 3 \\ 3 & 7 & \beta \end{vmatrix} = 0$

$\Rightarrow -8\beta - 42 - 5(\beta - 9) - 1(19) = 0$

$\Rightarrow -16 = 13\beta$

$\Rightarrow -\frac{16}{13} = \beta$

$\Rightarrow 13\alpha\beta = 1120$

*13. For $0 < \theta < \frac{\pi}{2}$, if the eccentricity of the hyperbola $x^2 - y^2\text{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\text{cosec}^2\theta + y^2 = 5$, then the value of θ is:

(1) $\frac{\pi}{6}$ (2) $\frac{5\pi}{12}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (3)

Sol. $\sqrt{1+\sin^2\theta} = \sqrt{7}\sqrt{1-\sin^2\theta} \Rightarrow \sin^2\theta = \frac{6}{8} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

14. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1, y(0) = 1$.

Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals:

(1) $\frac{4}{4+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$

(3) $\frac{2}{1+\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

Ans. (4)

Sol. $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} - 1 = 2t^3x - tx = 1 \Rightarrow \int \frac{dt}{t(2t^2-1)} = \int x dx$

$\Rightarrow \frac{1}{2}\left[-2\ln t + \ln\left(t - \frac{1}{\sqrt{2}}\right) + \ln\left(t + \frac{1}{\sqrt{2}}\right)\right] = \frac{x^2}{2} + c$

$\Rightarrow \frac{t^2 - \frac{1}{2}}{t^2} = ce^{x^2} \Rightarrow 1 - ce^{x^2} = \frac{1}{2t^2} \Rightarrow (x+y)^2 = \frac{1}{2(1-ce^{x^2})}$

$y(0) = 1 \Rightarrow 1 = \frac{1}{2(1-c)} \Rightarrow \frac{1}{2} = c$

$x = \frac{1}{\sqrt{2}} \Rightarrow \left(\frac{1}{\sqrt{2}} + y\right)^2 = \frac{1}{2\left(1-\frac{\sqrt{e}}{2}\right)} = \frac{1}{2-\sqrt{e}}$

15. Let f: R $\to$ R be defined as:

$$f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}; x < 0 \\ x^2 + cx + 2; 0 \le x \le 1 \\ 2x + 1; x > 1 \end{cases}$$

If f is continuous every where in R and m is the number of points where f is NOT differential then m + a + b + c equals:

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. $$f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

'f' of continuous at x = 0 $\Rightarrow$ a = b and a.2 = 2 $\Rightarrow$ a = b = 1

'f' is continuous at x = 1 $\Rightarrow$ 3 = 3 + c $\Rightarrow$ c = 0

$$\Rightarrow f(x) = \begin{cases} \frac{1-\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

LHD at x = 0 is $\lim\limits_{h\to 0} \frac{\frac{1-\cos 2h}{h^2} - 2}{-h} = \lim\limits_{h\to 0} \frac{2(\sin^2 h - h^2)}{-h^3} = 0$

RHD at x = 0 is 0

So, f(x) is differentiable at x = 0

Similarly 'f' is differentiable at x = 1

m + a + b + c = 2

*16. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latusrectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(1) 3 (2) $\frac{7}{2}$

(3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Ans. (3)

Sol. $e_e = \frac{1}{\sqrt{2}}$

$e_e^2 = \frac{1}{2}$

$1 - \frac{b^2}{a^2} = \frac{1}{2}$

$\frac{b^2}{a^2} = \frac{1}{2}$

$\frac{b^2}{a^2} + 1 = \frac{3}{2}$

$e_H^2 = \frac{3}{2}$

*17. Let 3, a, b, c be in A.P. and 3, a – 1, b + 1, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is:

(1) – 4 (2) – 1

(3) 13 (4) 11

Ans. (4)

Sol. Let a = 3 + d, b = 3 + 2d, c = 3 + 3d

a – 1 = 2 + d, b + 1 = 4 + 2d, c + 9 = 12 + 3d

$(a - 1)^2 = 3(b + 1)$

$(2 + d)^2 = 3(4 + 2d)$

$4 + d^2 + 4d = 12 + 6d$

$d^2 - 2d - 8 = 0$

$d^2 - 4d + 2d - 8 = 0$

$(d - 4)(d + 2) = 0$

d = 4, –2 for d = –2, G.P. = 3, 0, 0, 6 which is not possible

So, d = 4

Hence, A.M. of a, b, c = $\frac{a+b+c}{3} = \frac{9+6d}{3} = \frac{33}{3} = 11$

*18. Let C : $x^2 + y^2 = 4$ and C′ : $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C′ intersect at two distinct points, is R – [a, b], then the point (8a + 12, 16b – 20) lies on the curve:

(1) $x^2 + 2y^2 - 5x + 6y = 3$ (2) $5x^2 - y = -11$

(3) $x^2 - 4y^2 = 7$ (4) $6x^2 + y^2 = 42$

Ans. (4)

Sol. $|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$\left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda| < \left|2 + \sqrt{4\lambda^2 - 9}\right|$

R.H.I

$|2\lambda| - 2 < \sqrt{4\lambda^2 - 9}$

$4\lambda^2 + 4 - 8|\lambda| < 4\lambda^2 - 9$

$\lambda > \frac{13}{8}, \lambda < -\frac{13}{8}$

$4\lambda^2 - 9 > 0$

$\lambda > \frac{3}{2}, \lambda < -\frac{3}{2}$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

L.H.I

$\left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda|$

$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$

$4\sqrt{4\lambda^2 - 9} > -5$

$\lambda \in R$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

$\lambda \in R - \left[-\frac{13}{8}, \frac{13}{8}\right]$

(8a + 12, 16b – 20) = (–1, 6) lies on $6x^2 + y^2 = 42$

19. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall\ x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in:

(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ (2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(3) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$ (4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Ans. (2)

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2$ ..... (1)

Put $x = \frac{1}{x}$ in above

$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2$ ..... (2)

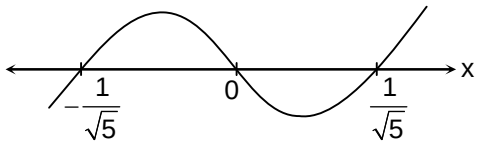
Solving equation (1) and (2), we get

$f(x) = \frac{1}{9}\left[5x^2 - \frac{4}{x^2} - 2\right]$

$y = 9x^2 f(x) = 5x^4 - 2x^2 - 4$

$y' = 20x^3 - 4x = 4x(5x^2 - 1)$

S.I. in $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$



20. If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is:

(1) 0 (2) $2\sqrt{3}$

(3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

Ans. (2)

Sol. $$\frac{\begin{vmatrix}\lambda-\sqrt{3} & 1 & -1\\ -2 & 1 & 1\\ 1 & -2 & 1\end{vmatrix}}{\begin{Vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 1\\ 1 & -2 & 1\end{Vmatrix}} \Rightarrow \frac{(\lambda-\sqrt{3})(3)-1(-3)+1(3)}{|i(3)-j(-3)+\hat{k}(3)|}=1$$

$$\Rightarrow \frac{3(\lambda-\sqrt{3})}{\sqrt{9+9+9}}=1 \Rightarrow 3(\lambda-\sqrt{3})=3\sqrt{3}$$

$$\lambda=2\sqrt{3}$$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. If x = x(t) is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)dt$, x(0) = 2, then, x(1) equals _____.

Ans. 14

Sol. $(t + 1)dx = (2x + (1 + t)^4)dt$

$$\frac{dx}{dt}-\frac{2x}{t+1}=(t+1)^3$$

I.F = $e^{\int\frac{-2}{t+1}dt}=e^{-2\ln|t+1|}=\frac{1}{(t+1)^2}$

Solution is $\frac{x}{(t+1)^2}=\int(t+1)\,dt$

$$\frac{x}{(t+1)^2}=\frac{t^2}{2}+t+c$$

At t = 0, x = 2, c = 2

$$\frac{x}{(t+1)^2}=\frac{t^2}{2}+t+2$$

x(1) = 14

*22. The number of elements in the set S = $\{(x,y,z): x,y,z \in Z; x+2y+3z=42; x,y,z \geq 0\}$ equals _____.

Ans. 169

Sol. x + 2y + 3z = 42

S.N.		
0	x + 2y = 42	22 cases
1	x + 2y = 39	20 cases

2	x + 2y = 36	19 cases
3	x + 2y = 33	17 cases
4	x + 2y = 30	16 cases
5	x + 2y = 27	14 cases
6	x + 2y = 24	13 cases
7	x + 2y = 21	11 cases
8	x + 2y = 18	10 cases
9	x + 2y = 15	08 cases
10	x + 2y = 12	07 cases
11	x + 2y = 09	05 cases
12	x + 2y = 06	04 cases
13	x + 2y = 03	02 cases
14	x + 2y = 0	01 cases
	Total Solutions: 169	

*23. If the coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6 (1 + x^2)^7(1 - x^3)^8$; $x \neq 0$ is α, then $|\alpha|$ equals _____.

Ans. 678

Sol. Required coefficient = coefficient of x^{36} in $(1 + x)^6 (1 + x^2)^7 (1 - x^3)^8$

$T_{r+1} = (-1)^{r_3}\ {}^6C_{r_1}\ {}^7C_{r_2}\ {}^8C_{r_3}\ x^{r_1+2r_2+3r_3}$ ..... (1)

$r_1 + 2r_2 + 3r_3 = 36$

r_3	r_2	r_1
6	6	6
6	7	4
7	5	5
7	6	3
7	7	1
8	3	6
8	4	4
8	5	2
8	6	0

Putting above in (1)
Required coefficient = 678

*24. Let 3, 7, 11, 15, ……., 403 and 2, 5, 8, 11, ……, 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _____.

Ans. 6699

Sol. Common terms are 11, 23, 35, terms

$t_n \le 403$

$11 + (n-1)\,12 \le 403$

$11 + 12n - 12 \le 403$

$12n \le 404$

$n \le 33 \cdot 6$

$n = 33$

Sum of common terms = 11 + 23 + 35 + ….. upto 33 terms

$= \frac{33}{2}(2\cdot 11 + (33-1)12)$

$= \frac{33}{2}(22 + 33\cdot 12 - 12)$

$= \frac{33}{2}(10 + 396)$

$= \frac{33}{2}(406) = 33 \times 203 = 6699$

25. Let {x} denote the fractional part of x and $f(x) = \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$, $x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of $f(x)$ at x = 0, then $\frac{32}{\pi^2}\left(L^2+R^2\right)$ is equal to _____.

Ans. 18

Sol. $L = \lim\limits_{x\to 0^-} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$

$= \lim\limits_{x\to 0^-} \frac{\cos^{-1}\left(1-\{x\}^2\right)}{\{x\}(1+\{x\})} = \frac{\pi}{4}$

$L = \frac{\pi}{4}$

$R = \lim\limits_{x\to 0^+} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$

$= \lim\limits_{x\to 0^+} \frac{\cos^{-1}\left(1-\{x\}^2\right)}{\{x\}} \lim\limits_{x\to 0^+} \frac{\sin^{-1}(1-\{x\})}{\left(1-\{x\}^2\right)}$

Let $\cos^{-1}(1-\{x\}^2) = \theta$

$$= \lim_{\theta\to 0^+} \frac{\theta}{\sqrt{1-\cos\theta}} = \lim_{\theta\to 0^+} \frac{\frac{\theta}{2}\cdot\pi}{\sqrt{2}\left|\sin\frac{\theta}{2}\right|} = \frac{\pi}{\sqrt{2}}$$

$$\frac{32}{\pi^2}(L^2+R^2) = \frac{32}{\pi^2}\left(\frac{\pi^2}{16}+\frac{\pi^2}{2}\right) = 32\left(\frac{1+8}{16}\right) = 18$$

*26. Let the line L : $\sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ____.

Ans. 72

Sol. $x^2 + y^2 = 3$; $x^2 = 2y$

Solving above $y = -3, 1$

For 1st quadrant

$y = 1$

$x^2 = 2$

$x = \sqrt{2}$

$p = (\sqrt{2}, 1)$

$\sqrt{2}x + y = \alpha$

$\sqrt{2}\cdot\sqrt{2} + 1 = \alpha$

$\alpha = 3$

Equation of circle is $x^2 + (y - \lambda)^2 = 12$

$$\left|\frac{\sqrt{2}\times 0 + \lambda - 3}{\sqrt{3}}\right| = 2\sqrt{3}$$

$|\lambda - 3| = 6$

$\lambda - 3 = \pm 6$

$\lambda = 9, -3$

$\Delta = \frac{1}{2}\times 12\times\sqrt{2} = 6\sqrt{2}$

$\Delta^2 = 72$

*27. Let $P = \{z \in C : |z + 2 - 3i| \le 1\}$ and $Q = \{z \in C : z(1+i) + \bar{z}(1-i) \le -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals _____.

Ans. 36

Sol. $z(1+\hat{i}) + \bar{z}(1-\hat{i}) \le -8$

$(x + \hat{i}y)(1+\hat{i}) + (x - \hat{i}y)(1-\hat{i}) \le -8$

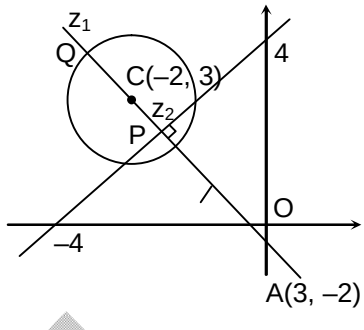
Point (x, y) lies above x – y + 4 = 0

$|z+2-3\hat{i}| \le 1$

$|z-(-2+3\hat{i})| \le 1$

z_2 = foot of perpendicular from A upon line

$=\left(-\frac{3}{2}, \frac{5}{2}\right)$



Applying parametric at C

$z_1 = \left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 2 \times 34/4 = 31 + 5\sqrt{2}$

$\alpha + \beta = 36$

28. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x dx}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)} = \alpha\pi + \beta \log_e\left(3+2\sqrt{2}\right)$, where α, β are integers, then $\alpha^2 + \beta^2$ equals _____.

Ans. 8

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\, dx}{(1+e^{\sin x})(1+\sin^4 x)}$

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\, dx}{(1+\sin^4 x)}$

(Using even odd property)

$2I = 2\int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\, dx}{(1+\sin^4 x)}$ let sin x = t

$I = 4\sqrt{2}\int_0^1 \frac{2dt}{1+t^4} = 4\sqrt{2}\int_0^1 \frac{(t^2+1)-(t^2-1)dt}{1+t^4} = 4\sqrt{2}\int_0^1 \frac{t^2+1}{1+t^4}dt$

$-4\sqrt{2}\int_0^1 \frac{t^2-1}{1+t^4}dt = 4\sqrt{2}\int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt - 4\sqrt{2}\int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}$

$= \left(4\sqrt{2}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{t-\frac{1}{t}}{\sqrt{2}} - 4\sqrt{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\frac{t-\frac{1}{\sqrt{2}}-\sqrt{2}}{t+\frac{1}{\sqrt{2}}+\sqrt{2}}\right|_0^1 \right)$

$= 2\pi + 2\ln\left(3+2\sqrt{2}\right) = a^2 + b^2 = 8$

29. Let the line of the shortest distance between the lines

$L_1 : \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$

$L_2 : \vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to _____ .

Ans. 21

Sol. $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{1} = \lambda$

$P = (\lambda + 1, -\lambda + 2, \lambda + 3)$

$L_2 : \frac{x-4}{1} = \frac{y-5}{1} = \frac{z-6}{-1} = \mu$

$Q = (\mu + 4, \mu + 5, -\mu + 6)$

$\overline{PQ} = (\lambda - \mu - 3, -\lambda - \mu - 3, \lambda + \mu - 3)$

$\bar{N} = \bar{N}_1 \times \bar{N}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{j} + 2\hat{k}$

$\overline{PQ} \parallel \bar{N}$

So, $\frac{-\lambda + \mu + 3}{0} = \frac{\lambda + \mu + 3}{2} = \frac{-\lambda - \mu + 3}{2}$

$\lambda = \frac{3}{2}$; $\mu = -\frac{3}{2}$

$P = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$; $Q = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

$(\alpha, \beta, \gamma) = \left(\frac{5}{2}, 2, 6\right)$

$2(\alpha + \beta + \gamma) = 21$

$-\lambda - \mu - 3$

$= \lambda + 4 - 3$

$P\lambda = -2\mu$, $\lambda = -\mu$

30. Let A = {1, 2, 3, ……, 20}. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

Then, number of elements in $R_1 - R_2$ is equal to _____ .

Ans. 46

Sol. R_1 = (1, 1) (1, 2) ….. (1, 20)
(2, 2), (2, 4) ….. (2, 20)
(3, 3), (3, 6) ….. (3, 18)
(4, 4), (4, 8) ….. (4, 20)
(5, 5), (5, 10) ….. (5, 20)
(6, 6), (6, 12) (6, 18)
(7, 7) (7, 14)
(8, 8) (8, 16)
(9, 9) (9, 18)
(10, 10) (10, 20)
(11, 11), (12, 12) ….. (20, 20)

$n(R_1) = 66$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2) = 66 - 20 = 46$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

*31. With rise in temperature, the Young's modulus of elasticity:
(1) Changes erratically
(2) Decreases
(3) Increases
(4) Remains unchanged

Ans. (2)

Sol. $Y = \frac{F\ell}{A\Delta\ell}$

$\Delta\ell$ increases, Y decreases.

*32. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2 m/s^2$, then the length of the second's pendulum at a height h = 2R from the surface of earth will be:
(1) $\frac{2}{9}$m
(2) $\frac{1}{9}$m
(3) $\frac{4}{9}$m
(4) $\frac{8}{9}$m

Ans. (2)

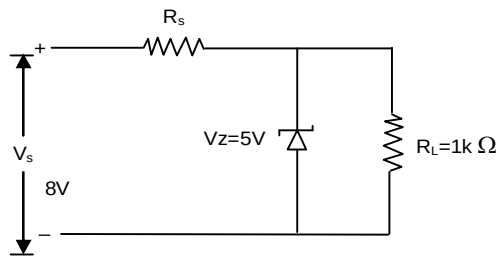
Sol. $T = 2\pi\sqrt{\frac{\ell}{g'}}$

$T = 2s$

$g' = g/9$

$\Rightarrow \ell = \frac{T^2 \times g'}{(2\pi)^2} = \frac{4 \times g/9}{4\pi^2} = \frac{1}{9}$m

33. In the given circuit if the power rating of Zener diode is 10mW, the value of series resistance R_s to regulate the input unregulated supply is:

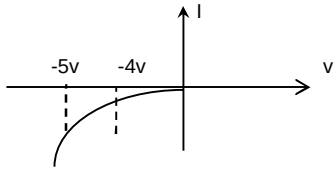


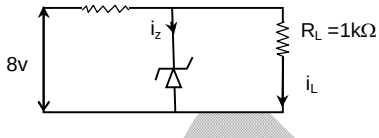
(1) 5kΩ
(2) 10Ω
(3) 1kΩ
(4) 10 kΩ

Ans. (3)

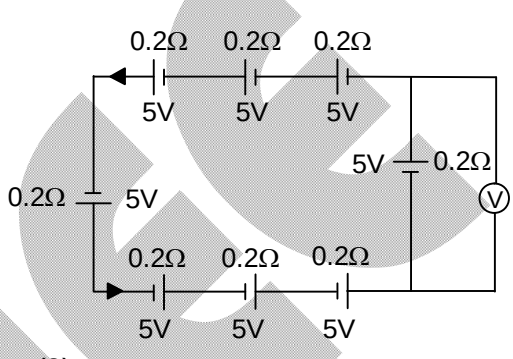
Sol. $\Delta v_Z = 4v \Rightarrow i_L = 4mA$ and $i_Z = 0$

So $R_c = \dfrac{8-4}{i_L + i_z} = 1k\Omega$





34. The reading in the ideal voltmeter (V) shown in the given circuit diagram is:



(1) 5V
(2) 10V
(3) 0V
(4) 3V

Ans. (3)

Sol. $I = \dfrac{40}{1.6}A = 25A$

$V = 5 - I \times 0.2 = 0$

35. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is:

(1) $\dfrac{1}{4}CV^2$
(2) $2CV^2$
(3) $\dfrac{1}{2}CV^2$
(4) $\dfrac{3}{4}CV^2$

Ans. (1)

Sol. $U_i = \dfrac{1}{2}CV^2 + \dfrac{1}{2}C(2V)^2 = \dfrac{5}{2}CV^2$

$U_f = \dfrac{1}{2}C\left(\dfrac{3V}{2}\right)^2 \times 2 = \dfrac{9}{4}CV^2$

$\Delta U = \dfrac{5}{2}CV^2 - \dfrac{9}{4}CV^2 = \dfrac{CV^2}{4}$

*36. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture of constant volume is:

(1) $\frac{9}{4}R$ (2) $\frac{7}{4}R$

(3) $\frac{3}{2}R$ (4) $\frac{5}{2}R$

Ans. (1)

Sol. $(C_v)_{mix} = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2} = \frac{2\times\frac{3}{2}R + 6\times\frac{5}{2}R}{2+6} = \frac{9}{4}R$

*37. A ball of mass 0.5 kg is attached to a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is:

(1) 1600 (2) 40

(3) 1000 (4) 20

Ans. (2)

Sol. $T = m\omega^2 r \Rightarrow \omega = \sqrt{\frac{T}{mr}} = 40\,rad/s$

38. A parallel plate capacitor has a capacitance $C = 200\,pF$. It is connected to $230\,V$ ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are:

(1) $1.38\,\mu A$ and $1.38\,\mu A$ (2) $14.3\,\mu A$ and $143\,\mu A$

(3) $13.8\,\mu A$ and $138\,\mu A$ (4) $13.8\,\mu A$ and $13.8\,\mu A$

Ans. (4)

Sol. $I_{rms} = \frac{V}{X_c} = 13.8\mu A = I_d$

*39. The pressure and volume of an ideal gas are related as $PV^{\frac{3}{2}} = K$ (Constant). The work done when the gas is taken from state A (P_1, V_1, T_1) and (P_2, V_2, T_2) is:

(1) $2(P_1V_1 - P_2V_2)$ (2) $2(P_2V_2 - P_1V_1)$

(3) $2\left(\sqrt{P_1}V_1 - \sqrt{P_2}V_2\right)$ (4) $2\left(P_2\sqrt{V_2} - P_1\sqrt{V_1}\right)$

Ans. (1)

Sol. $W = \frac{P_2V_2 - P_1V_1}{1-\frac{3}{2}} = 2(P_1V_1 - P_2V_2)$

40. A galvanometer has a resistance of 50Ω and it allows maximum current of $5mA$. It can be converted into voltmeter to measure upto 100V by connecting in series a resistor of resistance:

(1) 5975Ω (2) 20050Ω

(3) 19950Ω (4) 19500Ω

Ans. (3)

Sol. $V = I_g (G + R)$

$\Rightarrow 50 + R = \frac{100}{5\times10^{-3}} = 20000\Omega$

$\Rightarrow R = 19950\ \Omega$

41. The de Broglie wavelength of a proton and an α particle are λ and 2λ respectively. The ratio of the velocities of proton and α particle will be:

(1) 1 : 8 (2) 1 : 2

(3) 4 : 1 (4) 8 : 1

Ans. (1)

Sol. $\frac{V_P}{V_\alpha} = \frac{m_\alpha \lambda_\alpha}{m_p \lambda_p} = 8:1$

42. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is:

(1) $\frac{1}{2}$ (2) $\frac{10}{11}$

(3) $\frac{50}{11}$ (4) $\frac{5}{11}$

Ans. (4)

Sol. 1 V.S.D. $= \frac{10}{11}$M.S.D. $= \frac{50}{11}$units.

L.C. = 1 M.S.D. – 1 V.S.D. $= \frac{5}{11}$units

43. In series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be:

(1) Reduced by $\frac{1}{4}$L (2) Increased by $2L$

(3) Reduced by $\frac{3}{4}$L (4) Increased to 4L

Ans. (3)

Sol. $\omega_1 = \omega_2$

$\Rightarrow L' = L/4$

44. The radius (r), Length (ℓ), and resistance (R) of a metal wire was measured in the laboratory as:

$r=(0.35\pm0.05)\text{cm}$

$R=(100\pm10)\text{ohm}$

$\ell = (15\pm0.2)\text{cm}$

The percentage error in resistivity of the material of the wire is:

(1) 25.6% (2) 39.9%

(3) 37.3% (4) 35.6%

Ans. (2)

Sol. $\rho = \frac{AR}{\ell}$

$\Rightarrow \frac{\Delta\rho}{\rho} = 2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta\ell}{\ell}$

P.C. error = $\left(\frac{\Delta\rho}{\rho}\times100\right)\% = 39.9\%$

*45. The dimensional formula of angular impulse is:

(1) $\left[ML^{-2}T^{-1}\right]$ (2) $\left[ML^{2}T^{-2}\right]$

(3) $\left[MLT^{-1}\right]$ (4) $\left[ML^{2}T^{-1}\right]$

Ans. (4)

*46. A simple pendulum of length 1m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of $2\times10^{2}\,ms^{-1}$. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (Use $g = 10m/s^2$).

(1) 0.30 m (2) 0.20 m

(3) 0.35 m (4) 0.40 m

Ans. (2)

Sol. $v = \frac{mu}{(M+m)}$

$h = \frac{v^2}{2g} \approx 0.20m$

*47. A particle moving in a circle of radius r with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by its equal to 4R. The angle of projection θ is then given by:

(1) $\sin^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$ (2) $\sin^{-1}\left[\frac{\pi^2R}{2gT^2}\right]^{\frac{1}{2}}$

(3) $\cos^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$ (4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Ans. (1)

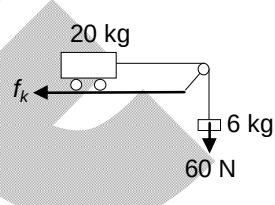
Sol. $v = \frac{2\pi R}{T}$

$4R = \frac{v^2 \sin^2\theta}{2g}$

$\sin\theta = \sqrt{\frac{8Rg}{v^2}}$

$\Rightarrow \theta = \sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{1/2}$.

*48. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is: (Consider that the string is massless and unstretchable and the pulley is also massless and frictionless):



(1) 3 (2) 4
(3) 2 (4) 1.2

Ans. (3)

Sol. $a = \frac{60 - f_k}{26} = 2\ m/s^2$

49. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly:

(1) 1.5 eV (2) 13.6 eV
(3) 1.9 eV (4) 12.1 eV

Ans. (4)

Sol. $E = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \approx 12.1 eV$

50. A monochromatic light of wavelength $6000\ \text{Å}$ is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is:

(1) 60 mm (2) 24 mm
(3) 120 mm (4) 12 mm

Ans. (2)

Sol. $W = \frac{2\lambda}{a} \times x = 24 \times 10^{-3} m = 24 mm$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. A regular polygon of 6 sides is formed by bending a wire of length 4π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon; the magnetic field at the centre of the polygon would be $x\times10^{-7}$T. The value of x is_____.

Ans. 72

Sol. $a = \frac{4\pi}{6}$

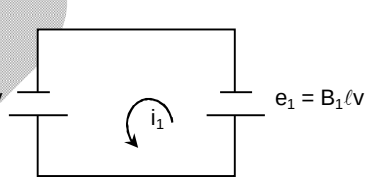
$$B = \frac{\mu_0 I}{4\pi\left(\frac{\sqrt{3}a}{2}\right)}\left(\sin(30^0)+\sin(30^0)\right)\times 6 = 72\times10^{-7}\,T.$$

52. A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively, moves with a velocity of 5 cm/s in the positive x-axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of 10^{-3} T/cm along the negative x direction and it is decreasing with time at the rate of 10^{-3} T/s. If the resistance of the loop is $6\,m\Omega$, the power dissipated by the loop as heat is _____ $\times10^{-9}$ W.

Ans. 216

Sol. $e_2 - e_1 = (B_2 - B_1)\ell v$

$= (B_1 + 10^{-3}\times 12) - B_1)\ell v = 3\times10^{-5}$ V



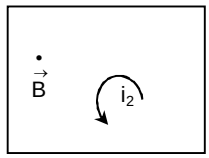
$$i_1 = \frac{e_2 - e_1}{R} = 5\times10^{-3}\,A$$

$$\varepsilon = A\frac{dB}{dt} = 60\times10^{-7}\,V$$

$$i_2 = \frac{\varepsilon}{R} = 10^{-3}\,A$$

$i = i_1 + i_2 = 6\times10^{-3}$A

$P = i^2R = 216\times10^{-9}$ W.



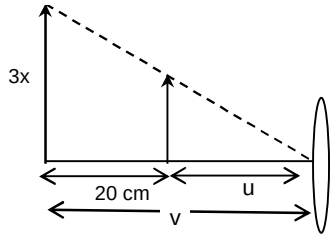
53. The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is _____ cm.

Ans. 15

Sol. $\frac{v}{u} = 3$ cm, $v - u = 20$ cm,

$u = 10$ cm, $v = 30$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow f = 15\text{cm}$



54. Two identical charged sphere are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be _____ (Take density of water = 1 g/cc)

Ans. 3

Sol. $\tan(\theta/2) = \frac{F}{mg} = \frac{F/k}{mg - \rho_w v_g}$

$\Rightarrow k = \frac{mg}{mg - m\frac{\rho_w}{\rho_s}g} = \frac{\rho_s}{\rho_s - \rho_w} = 3$

55. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is _____.

Ans. 27

Sol. $R \propto A^{1/3}$

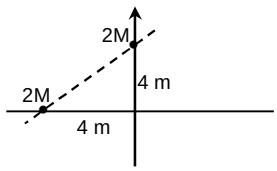
$\frac{R_1^3}{R_2^3} = \frac{A_1}{A_2} \Rightarrow A_2 = \frac{1000}{27}$

*56. The identical sphere of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of system is $\frac{4\sqrt{2}}{x}$, where the value of x is _____ .

Ans. 2

Sol. $\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = -2\hat{i} + 2\hat{j}$

$|\vec{r}_{com}| = 2\sqrt{2}m = \frac{4\sqrt{2}}{2}m$



*57. A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the fork is _____ Hz.

Ans. 6

Sol. Assuming fundamental frequency

$v = \sqrt{\frac{T}{\mu}}$ $v_1 = \sqrt{\frac{6}{\mu}}, v_2 = \sqrt{\frac{54}{\mu}} = 3v_1$

$f_1 = \frac{v_1}{2L} = \frac{v_1}{2}; f_2 = \frac{v_2}{2L} = \frac{3v_1}{2}$
$f_2 = 3f_1,\ f_2 - f_1 = 12$
$f_1 = 6$ Hz.

*58. A plane is in level flight at constant speed and each of its two wings has an area of $40\ m^2$. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is _____ kg. (Take air density to be $1\,kg\,m^{-3}$ and $g = 10\,m^{-2}$)

Ans. 9600

Sol. $\Delta P = \frac{1}{2}\rho(V_u^2 - V_\ell^2)$
$V_u = 70$ m/s, $V_l = 50$ m/s
$\Delta P = \frac{1}{2}\times 1(70^2 - 50^2) = 1200 Pa$
$Mg = \Delta P \times A$
$\Rightarrow M = \frac{1200\times 40\times 2}{10} = 9600$ kg

59. The current in a conductor is expressed as $I=3t^2+4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during $t=1s$ to t = 2s is _____ C.

Ans. 22

Sol. $I = 3t^2 + 4t^3$
$Q = \int_1^2 I dt = 22C$

*60. A particle is moving in one dimension (along x axis) under the action of a variable force, It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x= -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _____ m/s.

Ans. 52

Sol. $x = -3t^3 + 18t^2 + 16t$
$v = -9t^2 + 36t + 16$
$a = -18t + 36$
$a = 0 \Rightarrow t = 2s$
$v(2s) = 52$ m/s.

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

61. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is:
(1) CATTAGCT (2) TACGAAGT
(3) GTACTTAC (4) ATGCGACT

Ans. (2)
Sol. TAGCTTCA
TACGAAGT

62. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R)
Assertion (A): Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product
Reason (R): KCN and AgCN both are highly ionic compounds.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) (A) is correct but (R) is not correct
(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(3) (A) is not correct but (R) is correct
(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (1)
Sol. $R-X \xrightarrow{KCN} R-CN$
$R-X \xrightarrow{AgCN} R-NC$
KCN is ionic but AgCN is covalent.

*63. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction:
$Cr_2O_7^{2-} + XH^+ + Ye^{\ominus} \rightarrow 2A + ZH_2O$
X, Y, Z and A are respectively are:
(1) 8, 6, 4 and Cr_2O_3 (2) 14, 7, 6 and Cr^{3+}
(3) 8, 4, 6 and Cr_2O_3 (4) 14, 6, 7 and Cr^{3+}

Ans. (4)
Sol. $Cr_2O_7^{--} + XH^+ + Ye^- \longrightarrow 2A + ZH_2O$
$Cr_2O_7^{--} + 14H^+ + 6e^- \longrightarrow 2Cr^{+++} + 7H_2O$
X = 14, Y = 6, Z = 7, A = Cr^{+++}

*64. Which of the following reactions are disproportionation reactions?

(A) $Cu^{+} \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^{+} \longrightarrow 2MnO_4^{-} + MnO_2 + 2H_2O$

(C) $2KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^{-} + 3Mn^{2+} + 2H_2O \longrightarrow 5MnO_2 + 4H^{+}$

Choose the correct answer from the options given below:

(1) (A), (B) (2) (B), (C), (D)

(3) (A), (B), (C) (4) (A), (D)

Ans. (1)

Sol. $Cu^{+1} \longrightarrow Cu^{++} + Cu$

$(+1) \quad (+2) \quad (0)$

(Yes)

$KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

$(+7) \quad (+6) \quad (+4)$

(No)

$2MnO_4^{-} + 3Mn^{++} + 2H_2O \longrightarrow 5MnO_2 + 4H^{+}$

$(+7) \quad (+2) \quad (+4)$

(No)

$4H^{+} + MnO_4^{--} \longrightarrow MnO_4^{-} + MnO_2 + O_2$

$(+6) \quad (+7) \quad (+4)$

(Yes)

*65. In case of isoelectronic species the size of F^{-}, Ne and Na^{+} is affected by:

(1) Principal quantum number (n)

(2) None of the factors because their size is the same

(3) Electron-electron interaction in the outer orbitals

(4) Nuclear charge (z)

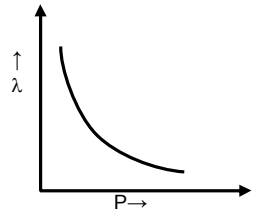
Ans. (4)

Sol. The order is

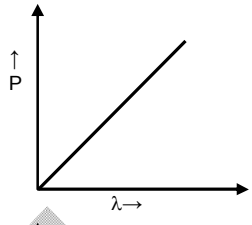
F^{-}	>	Ne	>	Na^{+}
$10e^{-}$		$10e^{-}$		$10e^{-}$
9p		10p		11p
Z_{eff} is minimum				Z_{eff} is maximum

*66. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron(λ) and momentum of electron(p)?

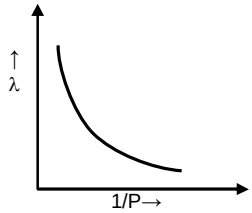
(1)



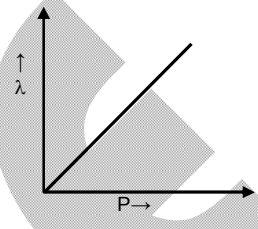
(2)



(3)



(4)

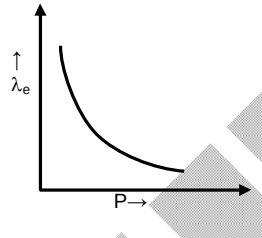


Ans. (1)

Sol. de-Broglie equation is $\lambda_e = \frac{h}{p}$

or $\lambda_e.p = h$

= Rectangular hyperbola.



67. Given below are two statements:

Statement (I): A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour

Statement (II): A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. (2)

Sol. $\left[Ni(H_2O)_6\right]^{++}$

$\Rightarrow Ni^{++} \equiv d^8 \equiv t_{2g}^6 e_g^2$

$\equiv d-d$ transition takes place.

$\equiv$ Green colour.

(I) is correct.

$\left[Ni(CN)_4\right]^{2-}$

$\Rightarrow Ni^{++} \equiv d^8$ colourless since it does not absorb radiation from visible range of electromagnetic spectrum.

(II) is correct.

*68. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): PH_3 has lower boiling point then NH_3.
Reason (R): In liquid state NH_3 molecules are associated through van der Waal's forces, but PH_3 molecules are associated through hydrogen bonding.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(2) (A) is not correct but (R) is correct
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(4) (A) is correct but (R) is not correct

Ans. (4)

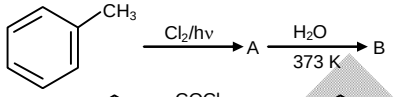
Sol. Boiling point of PH_3 < Boiling point of NH_3

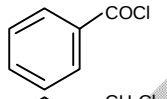
↓ ↓

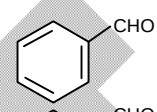
No H-bonding Intermolecular H-bonding

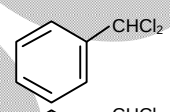
(A) is correct but (R) is incorrect.

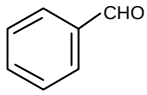
69. Identify A and B in the following sequence of reaction

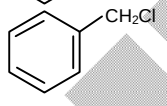


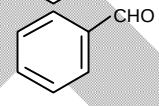
(1) A = 

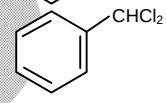
B = 

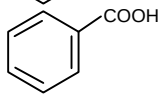
(2) A = 

B = 

(3) A = 

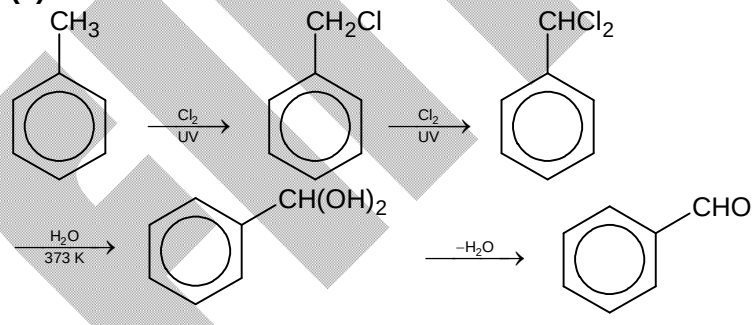
B = 

(4) A = 

B = 

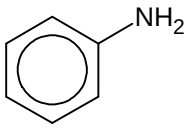
Ans. (2)

Sol.



*70. Given below are two statements:
Statement (I): Aminobenzene and aniline are same organic compounds.
Statement (II): Aminobenzene and aniline are different organic compounds.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Statement I is incorrect but Statement II is correct
(4) Both statement I and Statement II are incorrect

Ans. (2)

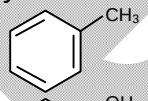
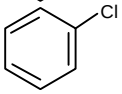
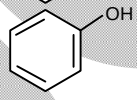
Sol.  ≡ Aniline
or Benzenamine
or Aminobenzene

Statement (I) is correct.
Statement (II) is incorrect.

71. Which of the following complex is homoleptic?
(1) $[Ni(CN)_4]^{2-}$ (2) $[Ni(NH_3)_2Cl_2]$
(3) $[Fe(NH_3)_4Cl_2]^+$ (4) $[Co(NH_3)_4Cl_2]^+$

Ans. (1)

Sol. Homoleptic complex means having only one kind of ligands in the co-ordination sphere.
So, option (1) is correct.

*72. Which of the following compound will most easily be attacked by an electrophile?
1. 
2. 
3. 
4. 

Ans. (4)

Sol. Benzene ring to which activating group is connected is readily attacked by an electrophile. So, option (4) is correct.

*73. Ionic reactions with organic compounds proceed through:
(A) homolytic bond cleavage (B) heterolytic bond cleavage
(C) free radical formation (D) primary free radical
(E) secondary free radical
Choose the correct answer from the options given below:
(1) (A) only (2) (C) only
(3) (B) only (4) (D) and (E) only

Ans. (3)

Sol. Ionic reactions with organic compounds proceed through heterolytic bond cleavage.

*74. Arrange the bonds in order of increasing ionic character in the molecules, LiF, K_2O, N_2, SO_2 and ClF_3
(1) $ClF_3 < N_2 < SO_2 < K_2O < LiF$ (2) $LiF < K_2O < ClF_3 < SO_2 < N_2$
(3) $N_2 < SO_2 < ClF_3 < K_2O < LiF$ (4) $N_2 < ClF_3 < SO_2 < K_2O < LiF$

Ans. (3)

Sol. Order of increasing ionic character
$N_2 < SO_2 < ClF_3 < K_2O < LiF$
N_2 is non-polar molecule while remaining are polar molecules.

75. We have three aqueous solutions of NaCl labelled as 'A', 'B' and 'C' with concentration 0.1 M, 0.01 M and 0.001 M, respectively. The value of van't Hoff factor(i) for these solutions will be in the order.

(1) $i_A < i_B < i_C$ (2) $i_A < i_C < i_B$

(3) $i_A = i_B = i_C$ (4) $i_A > i_B > i_C$

Ans. (1)

Sol. As the solution gets diluted, degree of dissociation of NaCl will increase and hence Van't Hoff factor will also increase.

Therefore, $i_A < i_B < i_C$

*76. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as:

(1) reducing agent (2) Catalytic agent

(3) hydrolysis agent (4) Oxidising agent

Ans. (2)

Sol. In Kjeldahl method for estimation of nitrogen, $CuSO_4$ acts as catalytic agent.

*77. Given below are two statements:

Statement (I): Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution

Statement (II): In this titration phenolphthalein can be used as indicator

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both statement I and Statement II are incorrect

Ans. (1)

Sol. Potassium hydrogenphthalate is a weak acid and NaOH is a strong base.

So in the titration of weak acid vs strong base, phenolphthalein can be used as an indicator.

78. Match List-I with List-II

List-I (Reactions)		List-II (Reagents)	
(A)	$CH_3(CH_2)_5-\underset{\underset{O}{\|}}{C}-OC_2H_5 \longrightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4, H^+$
(D)	$CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\|}}{C}(OH)CH_2COOC_2H_5$	(IV)	DIBAL-H, H_2O

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II) (2) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I) (4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (2)

Sol. $CH_3(CH_2)_5-\underset{\underset{O}{\|}}{C}-OC_2H_5 \xrightarrow[(ii)\ H_2O]{(i)\ DIBALH} CH_3(CH_2)_5CHO$

$C_6H_5COC_6H_5 \xrightarrow{Zn-Hg+conc.HCl} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow[(ii)\ H_3O^+]{(i)\ CH_3MgBr} C_6H_5CH(OH)CH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow[(ii)\ H_3O^+]{(i)\ NaBH_4} CH_3\underset{\underset{H}{|}}{C}(OH)CH_2COOC_2H_5$

So, correct matching is option (2).

*79. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following:

(1) $q=0,\Delta T\neq 0,w=0$ (2) $q=0,\Delta T<0,w\neq 0$

(3) $q\neq 0,\Delta T=0,w=0$ (4) $q=0,\Delta T=0,w=0$

Ans. (4)

Sol. For adiabatic free expansion of an ideal gas

$q=0,\quad w=0,\quad \Delta U=0\ (\text{or } \Delta T=0)$

*80. Given below are two statements:

Statement (I): The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II): Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Both statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. (1)

Sol. Statement (I) and Statement (II) both are correct.

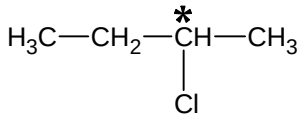
SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*81. Number of optical isomers possible for 2-chlorobutane_____ .

Ans. 2

Sol. 

2-Chlorobutane contains one chiral centre.
So, number of optical isomers = $2^1 = 2$

82. The potential for the given half cell at 298 K is (–) _____ × 10^{-2}V.
$2H^+_{(aq)} + 2e^- \longrightarrow H_2(g)$
$[H^+]$ = 1 M, $P_{H_2} = 2\,atm$
(Given: 2.303RT/F = 0.06 V, log2 = 0.3)

Ans. 1

Sol. $$E_{RP} = E^o_{RP} - \frac{0.06}{2}\log_{10}\frac{P_{H_2}}{\left[H^+\right]^2}$$

$$E_{RP} = 0.0 - \frac{0.06}{2}\log_{10}\frac{2}{(1)^2}$$

$$= -0.03 \times 0.3010 = -0.009\ V$$

$$= -0.9 \times 10^{-2}\ V$$

So, integer answer will be (1).

83. The number of white coloured salts, among the following is _____ .
(a) $SrSO_4$ (b) $Mg(NH_4)PO_4$ (c) $BaCrO_4$ (d) $Mn(OH)_2$
(e) $PbSO_4$ (f) $PbCrO_4$ (g) AgBr (h) PbI_2
(i) CaC_2O_4 (j) $[Fe(OH)_2(CH_3COO)]$

Ans. 5

Sol. White coloured salts among the following
$SrSO_4, Mg(NH_4)PO_4, PbSO_4, CaC_2O_4, Mn(OH)_2$
So, answer is (5).

84. The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is _____ years.

Ans. 17190

Sol. $$\frac{0.693}{5730} = \frac{2.303}{t}\log_{10}\frac{N_o}{\frac{N_o}{8}}$$

$$\frac{0.693}{5730} = \frac{2.303}{t}\log_{10} 8$$

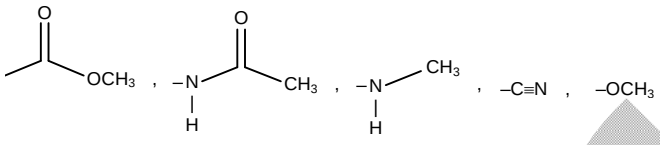
$$t = 17190\ year$$

*85. The number of molecules/ion/s having trigonal bipyramidal shape is _____ .
$PF_5, BrF_5, PCl_5, [PtCl_4]^{2-}, BF_3, Fe(CO)_5$

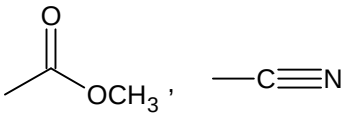
Ans. 3

Sol. Molecules/ions having trigonal bipyramidal shape is
$PF_5, PCl_5, Fe(CO)_5$

*86. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is_____ .



Ans. 2

Sol. 

Are deactivating.

*87. The lowest oxidation number of an atom in a compound A_2B is –2. The number of electrons in its valence shell is _____ .

Ans. 6

Sol. $A_2B \rightarrow (-2)$, i.e.
Group (16) compound like H_2O, H_2S etc.
Number of valence electrons = 6.

88. Among the following oxides of p-block elements, number of oxides having amphoteric nature is _____ .
$Cl_2O_7, CO, PbO_2, N_2O, NO, Al_2O_3, SiO_2, N_2O_5, SnO_2$

Ans. 3

Sol. $PbO_2, Al_2O_3, SnO_2 \equiv$ Amphoteric
$CO, NO \equiv$ Neutral
$Cl_2O_7, SiO_2, N_2O, N_2O_5 \equiv$ Acidic

*89. Consider the following reaction:
$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$
If 72 mmol of $PbCl_2$ is mixed with 50 mmol of $(NH_4)_3PO_4$, then the amount of $Pb_3(PO_4)_2$ formed is _____ mmol (nearest integer).

Ans. 24

Sol. $3PbCl_2 + 2(NH_4)_3PO_4 \longrightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

$\therefore$ Millimoles of $Pb_3(PO_4)_2$ formed $\equiv 72 \times \frac{1}{3} = 24$

*90. K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be _____ .

Ans. 7

Sol. CH_3COONH_4 is a salt of weak acid and weak base.

$$pH = \frac{1}{2}[pK_w + pK_a - pK_b]$$

$$pH = \frac{1}{2}[14 + 0] = 7 \qquad [\because pK_a = pK_b]$$

Ans. $\equiv [7]$