**Системы аналитических вычислений.**

**Лабораторная работа №7.** Вариант 5.

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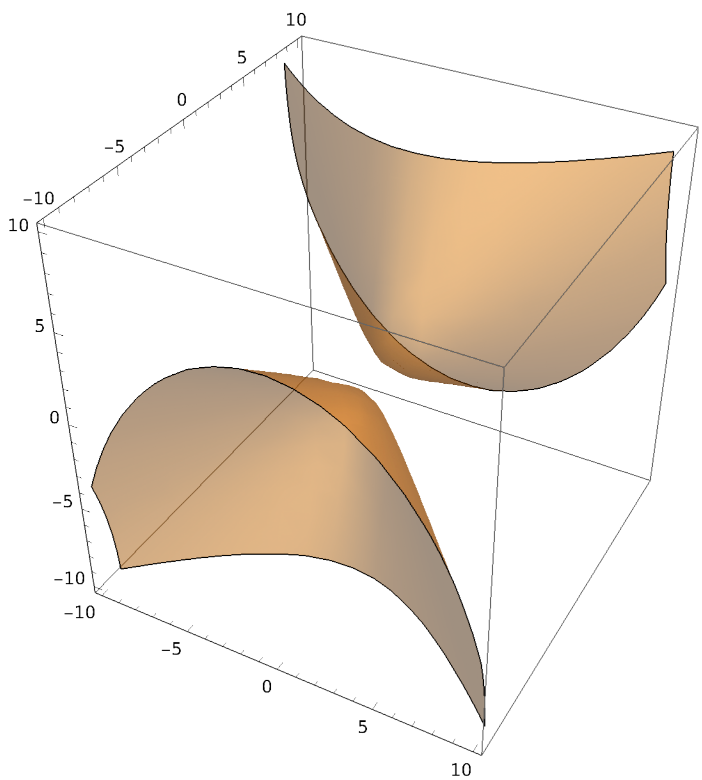
```
Clear[f]
f[x_, y_, z_] = -9*x^2 + 7*y^2 + 8*y*z - 3*z^2 - 4*x + 9*y - 10;
```

In[279]:=
```
xmin = -10;
xmax = 10;
ymin = -10;
ymax = 10;
zmin = -10;
zmax = 10;
ContourPlot3D[f[x, y, z] == 0, {x, xmin, xmax}, {y, ymin, ymax},
 {z, zmin, zmax}, ContourStyle → Opacity[0.5], Mesh → False]
```

Out[285]=



```
(*матрица кв. формы*)
A = {
   {-9, 0, 0},
   {0, 7, 4},
   {0, 4, -3}
  };
```

```
A // MatrixForm
```

Out[134]//MatrixForm=

$$\begin{pmatrix} -9 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 4 & -3 \end{pmatrix}$$

```
(*хар. многочлен*)
In[290]:= myCharPoly = Det[A - IdentityMatrix[3] * l];
          myCharPoly // TraditionalForm
```

Out[291]//TraditionalForm=

$$-l^3 - 5\, l^2 + 73\, l + 333$$

```
In[137]:= autoCharPoly = CharacteristicPolynomial[A, l]
```

Out[137]= $333 + 73\, l - 5\, l^2 - l^3$

```
In[138]:= SameQ[myCharPoly, autoCharPoly]
Out[138]= True
```

```
(*поиск собственных чисел*)
In[139]:= sols = Solve[myCharPoly == 0, l]
```

Out[139]= $\{\{l \to -9\}, \{l \to 2 - \sqrt{41}\}, \{l \to 2 + \sqrt{41}\}\}$

```
In[140]:= myEigenValues = l /. sols
```

Out[140]= $\{-9, 2 - \sqrt{41}, 2 + \sqrt{41}\}$

```
In[141]:= autoEigenValues = Eigenvalues[A]
```

Out[141]= $\{-9, 2 + \sqrt{41}, 2 - \sqrt{41}\}$

```
In[142]:= myEigenValues = Sort[myEigenValues];
          autoEigenValues = Sort[autoEigenValues];
In[144]:= SameQ[myEigenValues, autoEigenValues]
Out[144]= True
```

```
(* поиск СВ *)

In[145]:= Clear[x, y, z]

X = {x, y, z};

In[147]:= A1 = A - IdentityMatrix[3] * l /. l → myEigenValues[[1]];
A1X = A1.X;

In[224]:= myEigenVector1 = Solve[A1X == 0 /. x → 1];
myEigenVector1 = {1, y, z} /. myEigenVector1[[1]]

Out[225]= {1, 0, 0}

In[238]:= A2 = A - IdentityMatrix[3] * l /. l → myEigenValues[[2]];
A2X = A2.X;

myEigenVector2 = Solve[A2X == 0 /. z → 1];
myEigenVector2 = {x, y, 1} /. myEigenVector2[[1]]
```

Out[241]= $\left\{0, \frac{1}{4} \times \left(5 - \sqrt{41}\right), 1\right\}$

```
In[234]:= A3 = A - IdentityMatrix[3] * l /. l → myEigenValues[[3]];
A3X = A3.X;

myEigenVector3 = Solve[A3X == 0 /. z → 1];
myEigenVector3 = {x, y, 1} /. myEigenVector3[[1]]
```

Out[237]= $\left\{0, \frac{1}{4} \times \left(5 + \sqrt{41}\right), 1\right\}$

```
In[173]:= autoEigenSystem = Eigensystem[A]
```

Out[173]= $\left\{\left\{-9, 2 + \sqrt{41}, 2 - \sqrt{41}\right\}, \{1, 0, 0\}, \left\{0, \frac{1}{4} \times \left(5 + \sqrt{41}\right), 1\right\}, \left\{0, \frac{1}{4} \times \left(5 - \sqrt{41}\right), 1\right\}\right\}$

```
In[245]:= SameQ[autoEigenSystem[[2, 1]], myEigenVector1]
SameQ[N[autoEigenSystem[[2, 3]]], N[myEigenVector2]]
SameQ[N[autoEigenSystem[[2, 2]]], N[myEigenVector3]]

Out[245]= True

Out[246]= True

Out[247]= True

(* матрица из СВ, предварительно нормализованных *)
```

In[248]:=
```
S = Transpose[{
  Normalize[myEigenVector1],
  Normalize[myEigenVector2],
  Normalize[myEigenVector3]
}];
```

```
N[S] // MatrixForm
```

Out[249]//MatrixForm=

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & -0.331007 & 0.943628 \\ 0. & 0.943628 & 0.331007 \end{pmatrix}$$

In[250]:=
```
a = {-2, 4.5, 0};
a // MatrixForm
```

Out[251]//MatrixForm=

$$\begin{pmatrix} -2 \\ 4.5 \\ 0 \end{pmatrix}$$

In[252]:=
```
a1 = Transpose[S].a;
N[a1] // MatrixForm
```

Out[253]//MatrixForm=

$$\begin{pmatrix} -2. \\ -1.48953 \\ 4.24633 \end{pmatrix}$$

In[254]:=
```
a0 = -10;
(*канонический вид уравнения*)
fCanonical = myEigenValues[[1]] * x1^2 + myEigenValues[[2]] * y1^2 +
  myEigenValues[[3]] * z1^2 + 2 * a1[[1]] * x1 + 2 * a1[[2]] * y1 + 2 * a1[[3]] * z1 + a0
```

Out[255]= $-10 - 4.\ x1 - 9\ x1^2 - 2.97906\ y1 + \left(2 - \sqrt{41}\right) y1^2 + 8.49265\ z1 + \left(2 + \sqrt{41}\right) z1^2$

In[256]:=
```
ExpandAll[FullSimplify[fCanonical]]
```

Out[256]= $-10. - 4.\ x1 - 9.\ x1^2 - 2.97906\ y1 - 4.40312\ y1^2 + 8.49265\ z1 + 8.40312\ z1^2$

```
fCanonical = fCanonical /. (myEigenValues[[2]] * y1^2 + 2 * a1[[2]] * y1) →
    (myEigenValues[[2]] * (y1 + a1[[2]] / myEigenValues[[2]])^2 -
      myEigenValues[[2]] * (a1[[2]] / myEigenValues[[2]])^2);
fCanonical = fCanonical /. (myEigenValues[[3]] * z1^2 + 2 * a1[[3]] * z1) →
    (myEigenValues[[3]] * (z1 + a1[[3]] / myEigenValues[[3]])^2 -
      myEigenValues[[3]] * (a1[[3]] / myEigenValues[[3]])^2);

fCanonical = fCanonical /. (y1 + a1[[2]] / myEigenValues[[2]]) → y1;
fCanonical = fCanonical /. (z1 + a1[[3]] / myEigenValues[[3]]) → z1;
fCanonical
```

Out[261]= $-11.6419 - 4.\ x1 - 9\ x1^2 + \left(2 - \sqrt{41}\right) y1^2 + \left(2 + \sqrt{41}\right) z1^2$

```
In[262]:= ExpandAll[FullSimplify[fCanonical]]
```

Out[262]= $-11.6419 - 4.\ x1 - 9.\ x1^2 - 4.40312\ y1^2 + 8.40312\ z1^2$

```
In[263]:= newa0 = fCanonical /. {x1 → 0, y1 → 0, z1 → 0}
```

Out[263]= $-11.6419$

```
fCanonical =
  fCanonical /. (2 * a1[[1]] * x1 + newa0) → (2 * a1[[1]] * (x1 + newa0 / (2 * a1[[1]])));
fCanonical = fCanonical /. (x1 + newa0 / (2 * a1[[1]])) → x1;
fCanonical
```

Out[266]= $-4.\ x1 - 9\ x1^2 + \left(2 - \sqrt{41}\right) y1^2 + \left(2 + \sqrt{41}\right) z1^2$

```
fCanonical = ExpandAll[FullSimplify[fCanonical]]
```

Out[267]= $-4.\ x1 - 9.\ x1^2 - 4.40312\ y1^2 + 8.40312\ z1^2$

```
In[268]:= fCanonical = fCanonical /. {x1 → z1, z1 → x1};
          fCanonical = fCanonical /. {x1 → y1, y1 → x1}
```

Out[269]= $-4.40312\ x1^2 + 8.40312\ y1^2 - 4.\ z1 - 9.\ z1^2$

```
In[270]:= canonical[a_, b_, c_] := fCanonical /. {x1 → a, y1 → b, z1 → c}
```

In[287]:=
```
ContourPlot3D[canonical[x, y, z] == 0, {x, -10, 10},
 {y, -10, 10}, {z, -10, 10}, ContourStyle → Opacity[0.5], Mesh → False]
```

Out[287]=