Department of Mathematics
II Semester 2017-2018
$MTL776$ Graph Algorithms
Tutorial Sheet 1

1 If the maximum degree in a tree T is k, then prove that T has at least k pendant vertices (vertices of degree 1). Is the converse true?

2 Let T_1 and T_2 be two spanning trees of a connected graph G. If edge e is in T_1 but not in T_2, prove that there exists another edge f in T_2 but not in T_1 such that $(T_1 \setminus \{e\}) \cup \{f\}$ and $(T_2 \setminus \{f\}) \cup \{e\}$ are also spanning trees of G.

3. Prove that in a tree every vertex of degree greater than one is a cut vertex.

4. Prove that a pendant edge in a connected graph G is contained in every spanning tree of G.

5. Prove that an edge e of a connected graph G is a cut edge if and only if e belongs to every spanning tree of G.

6. Let T be a tree of order m, and let G be a graph with $\delta(G) = m - 1$. Then prove that T is isomorphic to some sub graph of G.

7. Suppose T is a tree of order n that contains only vertices of degree 1 and 3. Prove that T contains $(n-2)/2$ vertices of degree 3.

8. Prove or disprove: if $d_1, d_2, \ldots, d_n$ is the degree sequence of a tree, then $1, d_1{+}1, d_2, d_3, \ldots, d_n$ is the degree sequence of a tree.

9. Let G be a connected weighted graph whose edges have distinct weights. Show that G has a unique minimum spanning tree.

10. Let T be a tree of order n and size m having n_i vertices of degree $i (i = 1, 2)$.show that $n_1 = n_3 + 2n_4 + 3n_5 + 4n_6 + \ldots + 2$.

11. Prove or disprove: if n_i denotes the number of vertices of degree i in a tree T, then $\sum in_i$ depend only on the number of vertices in T.

12. Let T be an n vertex tree having one vertex of each degree $i, 2 \leq i \leq k$, the remaining $n - k + 1$ vertices are leaves. Determine n in terms of k.

13. Draw a weighted connected graph G on 11 vertices having 10 different MSTs.

14. Let e be a minimum cost edge of a weighted connected graph G. Show that e belongs to some MST of G.

15. If e be the only minimum cost edge of G, then e belongs to every MST of G.

16. Describe five applications of MST.

17. Design algorithms for a Tree for each of the following: (i) To find a maximum independent set, (ii) To 2-color all the vertices of G, (iii) To find a path from x to y.

18. Suppose $n \geq 2$ and $d_1, d_2, \ldots, d_n, d_{n+1}$ are $n+1$ positive integers such that their sum equals $2n$. Prove that there exists an index i such that $d_i = 1$ and there is an index j such that $d_j > 1$.

19. Use 18. and Mathematical induction to show that if $n \geq 2$ is an integer and $d_1, d_2, \ldots, d_n$ are positive integers such that $\sum_{i=1}^{n} d_i = 2n - 2$, then there is a tree T_n with n vertices whose degrees are $d_1, d_2, \ldots, d_n$.

20. Characterize all connected graphs with same number of vertices and edges.