

Algorithmic Foundations 2 - Tutorial Sheet 1

Propositional Logic and Logical Equivalences

1. Which of the following are propositions and what are the truth values of those that are propositions?

- (a) Do not pass go
- (b) What time is it?
- (c) Glasgow is the largest city in Scotland
- (d) $4 + x = 5$
- (e) $4 + 1 = 6$

2. Let p , q and r be the propositions:

- p : you have the flu;
- q : you miss the final exam;
- r : you pass the course.

Express each of the following propositions as an English sentence

- (a) $p \rightarrow q$
- (b) $(p \wedge q) \rightarrow r$
- (c) $q \rightarrow (p \vee r)$
- (d) $p \vee q \vee r$
- (e) $(p \rightarrow r) \vee (q \rightarrow r)$
- (f) $(p \wedge q) \rightarrow (q \wedge r)$

3. Let p , q and r be the propositions:

- p : you get an A in the final exam;
- q : you do every tutorial exercise;
- r : you get an A for this module.

Write the following propositions using p , q , r and logical connectives.

- (a) You get an A for this module, but you do not do every tutorial exercise.
- (b) You get an A in the final exam, you do every tutorial exercise and you get an A for this module.
- (c) To get an A for this module, it is necessary for you to get an A in the final exam.
- (d) You get an A in the final exam, but you do not do every tutorial exercise; nevertheless, you get an A for this module.
- (e) Getting an A in the final exam and doing every tutorial exercise is sufficient for getting an A for this module.
- (f) You get an A for this module if and only if you either do every tutorial exercise or you get an A in the final exam.

4. Construct a truth table for each of the following compound propositions.

- (a) $(p \rightarrow q) \rightarrow q$
- (b) $(p \vee q) \wedge (p \rightarrow r)$
- (c) $(p \rightarrow q) \rightarrow r$

$$(d) (p \wedge q) \rightarrow (p \wedge r)$$

5. Show that each of the following implications is a tautology by using truth tables.

$$(a) (p \wedge q) \rightarrow q$$

$$(b) (\neg p \wedge (p \rightarrow q)) \rightarrow q$$

$$(c) (p \wedge (p \rightarrow q)) \rightarrow q$$

6. Show that each implication in Exercise 6 is a tautology without using truth tables.

$$(a) (p \wedge q) \rightarrow q$$

$$(b) (\neg p \wedge (p \rightarrow q)) \rightarrow q$$

$$(c) (p \wedge (p \rightarrow q)) \rightarrow q$$

7. State the converse and contrapositive of each of the following implications:

(a) If it snows tonight, then I will stay at home.

(b) I go to the beach whenever it is a sunny summer day.

(c) When I stay up late, it is necessary that I sleep until noon.

Di cult/challenging questions.

8. Suppose that a truth table in n Boolean variables is specified. Show that a compound proposition with this truth table can be formed by taking the disjunction of conjunctions of the variables or their negations. (Hint: there should be one conjunction included for each combination of values for which the proposition is true.) Note that the resulting compound proposition is said to be in disjunctive normal form.

9. The proposition $p \text{ NOR } q$ is true when both p and q are false and is denoted by the formula $p \# q$. Write the truth table for the logical operator $\#$ and then find a logical proposition equivalent to $p \rightarrow q$ using only the operator $\#$.

If you want a further exercise express all logical connectives using just $\#$ without using the logical equivalences you have been given.

Hint. First consider constructing the formula $\neg(p \rightarrow q)$, second consider how to define negation using only $\#$ and finally combine these results.