Derivation of ray optics analytic solution for transverse acceleration

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Applying transverse acceleration

Numeric implementation

In the code, the wavefront tilt caused by transverse acceleration of the cavity frame is implemented as follows:

The change in velocity between the current step and the last step is

$$dv = adt$$

where a is an instantaneous value fed into the code at the given time.

Each step the incremental phase shift is

$$d\theta = \frac{dv}{c}.$$

The wavefront tilt due to transverse acceleration of the cavity frame is implemented as a phase screen

$$B = e^{-ik_0 d\theta X}$$

which gets multiplied with the current beam profile.

Analytic solution

The transverse velocity of the frame as a function of time is:

$$v_\perp(t) = v_0 + at. \tag{1}$$

The wavefront tilt due to this motion is

$$\theta(t) = \frac{v_\perp(t)}{c} = \frac{v_0}{c} + \frac{a}{c}t. \tag{2}$$

Mapping time to space, we have

$$t(z) = \frac{z - z_0}{c}. \tag{3}$$

And wavefront tilt as a function of propagation distance z becomes

$$\theta(z) = \frac{v_0}{c} + \frac{a}{c^2}(z - z_0). \tag{4}$$

For small angles,

$$\frac{dx}{dz} \approx \theta(z). \tag{5}$$

So

$$\frac{dx}{dz} = \frac{v_0}{c} + \frac{a}{c^2}(z - z_0). \tag{6}$$

Now we integrate from z_0 to z to get the trajectory $x(z)$.

$$\int_{x_0}^{x} dx = \int_{z_0}^{z} \frac{v_0}{c} + \frac{a}{c^2}(\zeta - z_0) d\zeta \tag{7}$$

$$x(z) = \frac{v_0}{c}(z - z_0) + \frac{a}{2c^2}(z - z_0)^2 \tag{8}$$