Attractors in artificial neural networks

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Abstract

This short project aims to have a glance at stability and arising attractors in back-propagation artificial neural networks, and to find the relevance with recognition processes in biological neurons with an example of olfactory neurons. The simulation of neuronal network largely follows the logic and methods of [1], but additionally examples of chaotic attractors are found.

1 Introduction

Since the early occurence of powerful computers and the first developments in neural networks, memory-like applications using artificial neural networks started to appear. They were largely supported with the biological findings at the same time about how the neurons connect with each other and the more they are fired at the same time, the more they are associated, and the more detailed models of how the neurons are activated [4] which helped to develop a similar neural network implication [2] and to determine the threshold functional for neuronal activation. The first networks were barely networks and consisted of only one neuron, "perceptron" [3], but eventually developed into a variety of neural networks: feedforward (connections and "signals" propagating only forward) and backpropagation (connections back and forth), single-layered and multi-layered, with linear and nonlinear activation function, supervised and unsupervised. In the modern networks, usually many layers are incorporated, acting like "feature recognizers", and the state-of-the art-networks are mostly convolutional deep neural networks, a kind of feed-forward network.

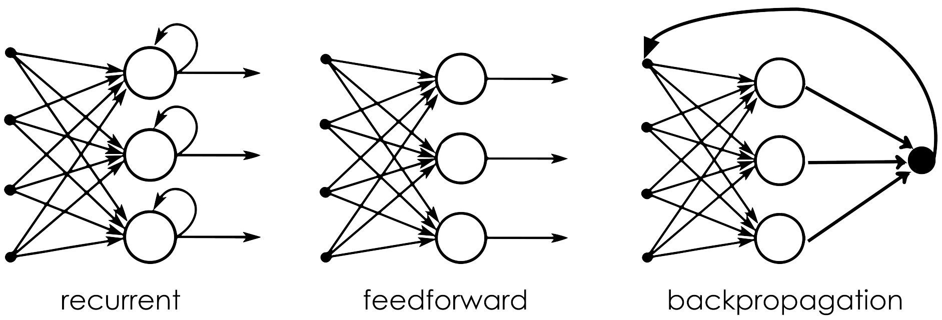


Figure 1: Neural network types: feedforward (not connected), recurrent and backpropagation (connected).

The similarity of the recognizing networks to the memory and vice versa was drawn during the development of the recognition applications of neural networks. The networks possess the properties of associative memory, for instance, which was shown in the pattern completion tasks [6, 7], see Fig. 1. Furthermore, it was immediately recognzed that these systems behave like a nonlinear dynamical system, and, for instance, memories in the artificial neural networks might be represented as attractors: fixed points or continuous attractors [8]. This is not the only connection to attractors in neuronal theory, for example, neural firing process represents a limit cycle – see, for instance, Hodgkin-Huxley model, voltage vs gating parameter phase space [5]. But here, we will focus on a different remarkable property of the artificial neural networks – attractors in the space of neuronal weights.

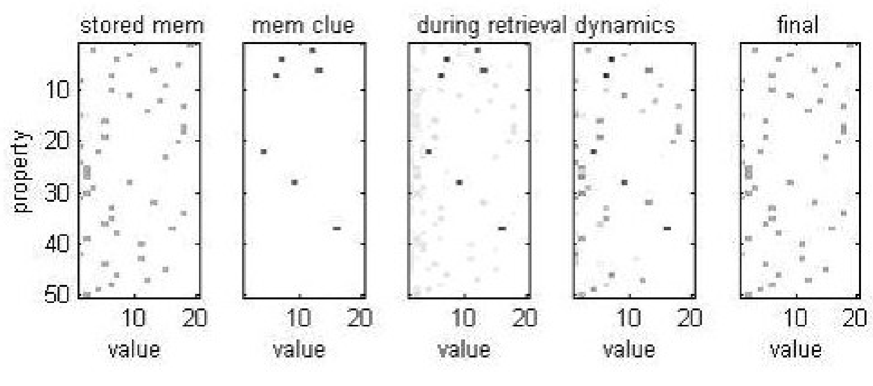


Figure 2: Memory retrieval using Hopfield network, picture from [10].

It has been hypothesized [11] that chaotic attractors can describe certain recognition processes in brain with an example of the olfactory bulb and odor recognition, and this process was shown to rest on a chaotic attractor. They suggest that being chaotic helps system to converge to recognition (limit cycle) faster. As authors name it, the "chaotic well" helps neurons not to converge, and, therefore, to avoid the previously learnt patterns to produce a new pattern. We will focus on this kind of behavior in backpropagation networks, which represents it the best.

2 Network description

In order to form a dynamic system with possible chaotic attractors, the neural network must be connected with itself, therefore representing a map (and Hopf bifurcation leading to chaos for maps was proven by Neimark and Sacker, see, for ex., [13]), i.e. should be self-learning.

The network consists of a layer of D input values $(y_1, y_2, ...y_D)$, a hidden layer of N neuron units $(x_1, x_2, ..., x_N)$, and output Y (all the values $\in R$):

$$x_i = tanh \sum_{j=1}^{D} w_{ij} y_i, \tag{1}$$

$$Y = s \sum_{i=1}^{N} \beta_i x_i \tag{2}$$

where $w_{ij}, \beta_i \in R$ are the weight matrices elements, and s is a scaling factor of the weights. $tanh$ here plays a role of "squashing function", i.e. mapping everything into the [-1,1] interval.

This kind of neural networks is used in image recognition and classification, for instance, classifying a set of given pictures of animals and telling if it's a cat (0) or a dog (1) and similar tasks.

Now, we can take the weights of the system as random and fixed, and investigate the case of backpropagation network: we will be feeding the output vector to one of the input vectors at a time, so that $y_0 = Y(t = 0), y_1 = Y(t = 1), ...y_D = Y(t = D-1)$ and so on, with y_i changing along the time with $1/D$ rate. This method of back-propagation is used to adjust the weights of the connections in the network to minimize a measure of the difference between the actual output vector and the desired output vector [9].

It was shown theoretically that back-propagation neural networks can be chaotic in the case of simplest, input-output networks [1]. In order to investigate how prone the system is to chaos, we will calculate the Lyapunov exponents for different sets of parameters. Lyapunov exponent is a convenient way of measuring how unstable the system is and is just an exponent factor

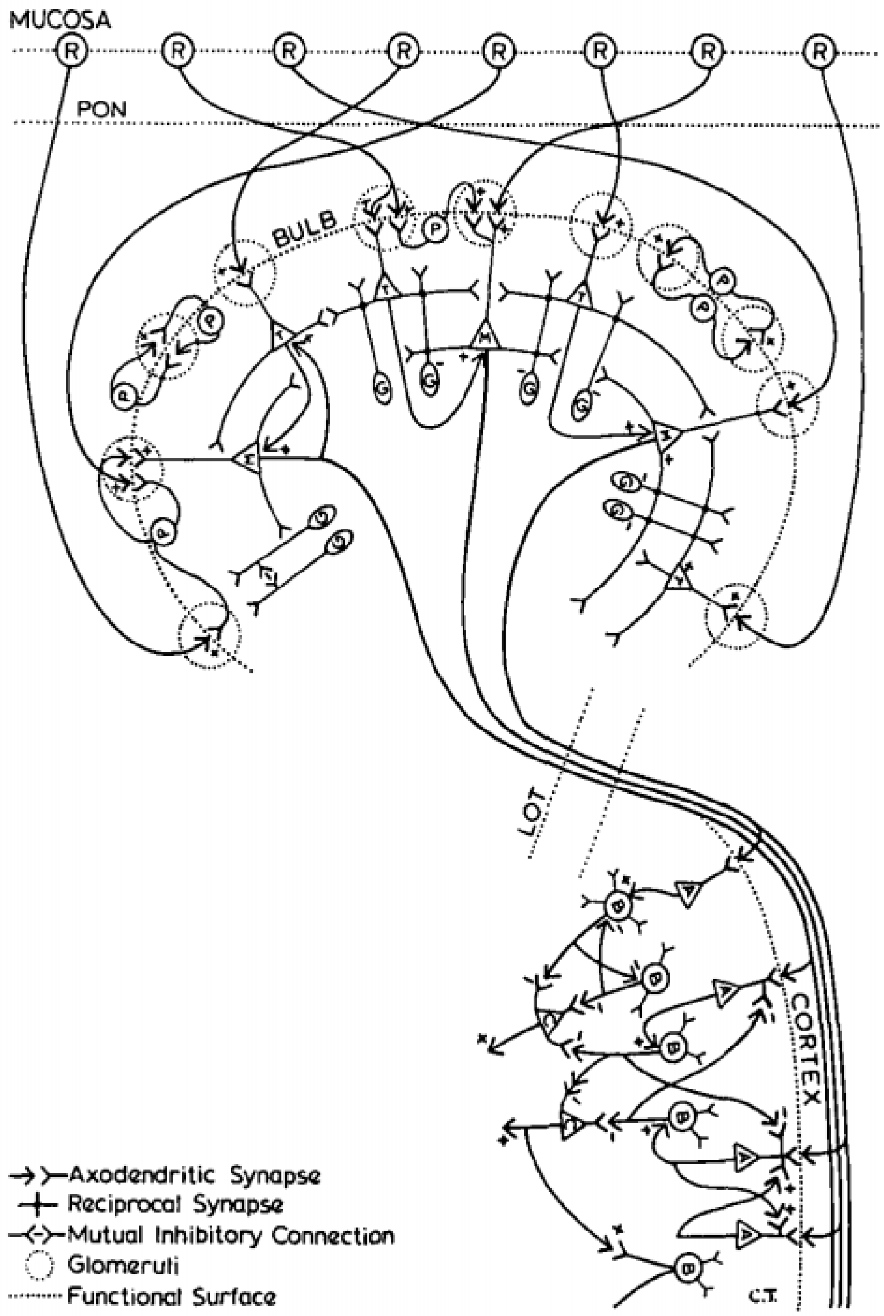


Figure 3: Olfactory bulb scheme analogous to self-connected network, picture from [11].

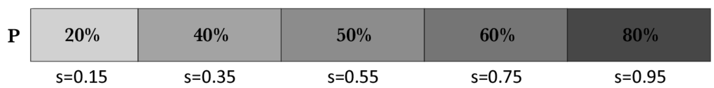
in an approximate equation $|\delta Z(t)| \approx e^{\lambda t} |\delta(Z(0))|$. In our case, we need to apply a method of calculation for discrete systems [14]. The resulting *lambda* will be:

$$\lambda = n_t \sum_{k=0}^{n_t - 1} ln \frac{|\Delta x_{k+1}|}{|\Delta x_k|}, \tag{3}$$

where n_t is a number of time steps, Δx_k is a distance between the states $x(y_i)$ and $x(y_i + \epsilon)$ at a point of time. For stable systems converging to a fixed point this parameter is less than 0, for unstable (diverging, and, therefore, chaotic systems) it is greater than 0.

We'll start with $D = 16, N = 4$ and vary s, then performing 100 000 timesteps to calculate the Lyapunov factor. As is observed in practice during this computation, usually it takes not more than 10 000 steps to approach the attractor, so the first 10 000 timesteps results might be discarded.

The results for $s = 0.15, 0.35, 0.55, 0.75, 0.95$ are shown on Fig. 2. The percentage (variable P) represents a chance of chaotic behaviour in a system, derived from 10 simulations with the same s parameter. So, for instance, for $P = 40\%$ 4 out of 10 Lyapunov's factors were positive.



If we will take a look at the dynamic behavior of the system, it will be clear that neurons dynamic behavior indeed resemble regular neuronal spikes in the case of low scaling factors s and start to be chaotic (like odor recognition spikes in [11]) with higher s, so this type of network represents these processes of odor recognition described in [11] indeed, see Fig. 4.

The typical chaotic attractors of the system, projected into space of two neurons x_1, x_2 are shown in Fig. 5 for different s.

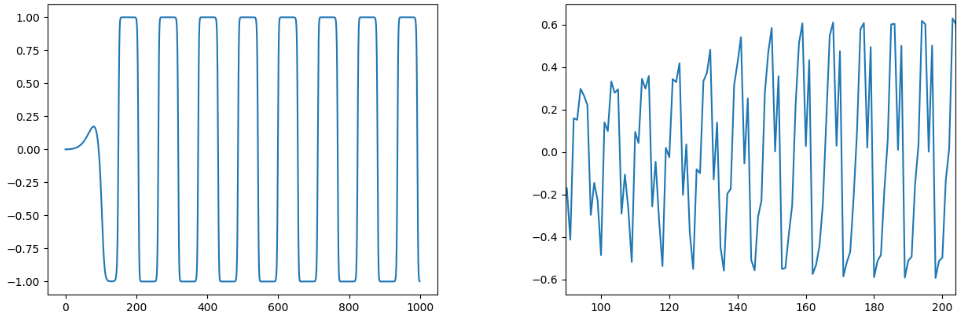


Figure 4: Left: $x - t$ dynamics with $s = 0.25$ resembling regular neuronal activity, right: $x - t$ dynamics with $s = 0.75$ exhibiting chaotic recognition activity. $D = 16$.

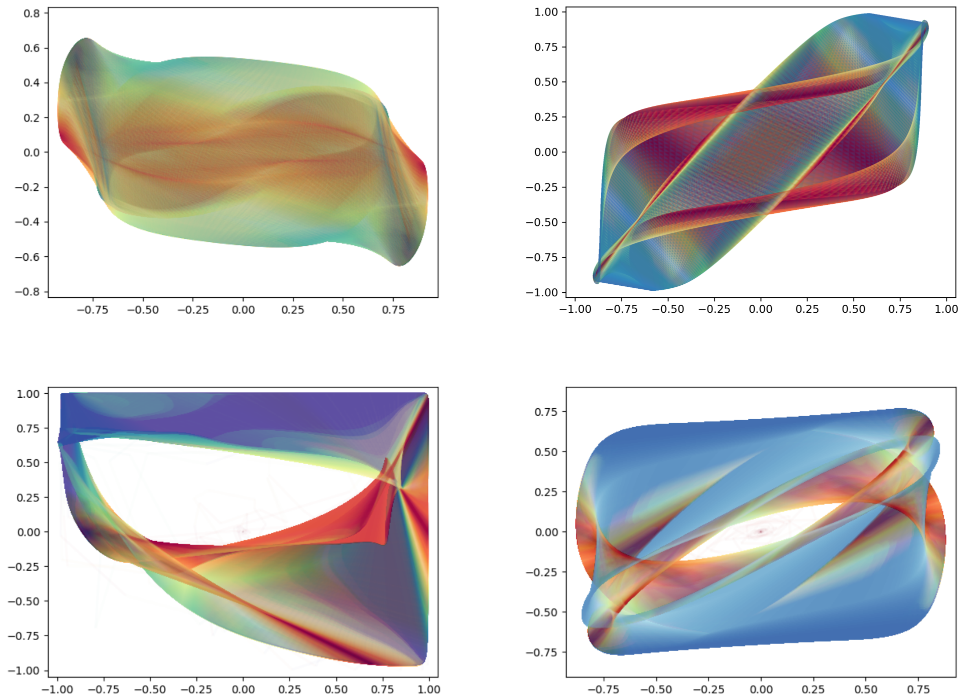


Figure 5: Top: tori chaotic attractors in a back-propagation network with $s = 0.35$ parameter, bottom: two chaotic attractors for $s = 0.55$ and $s = 0.65$, left and right respectively. $D = 16$.

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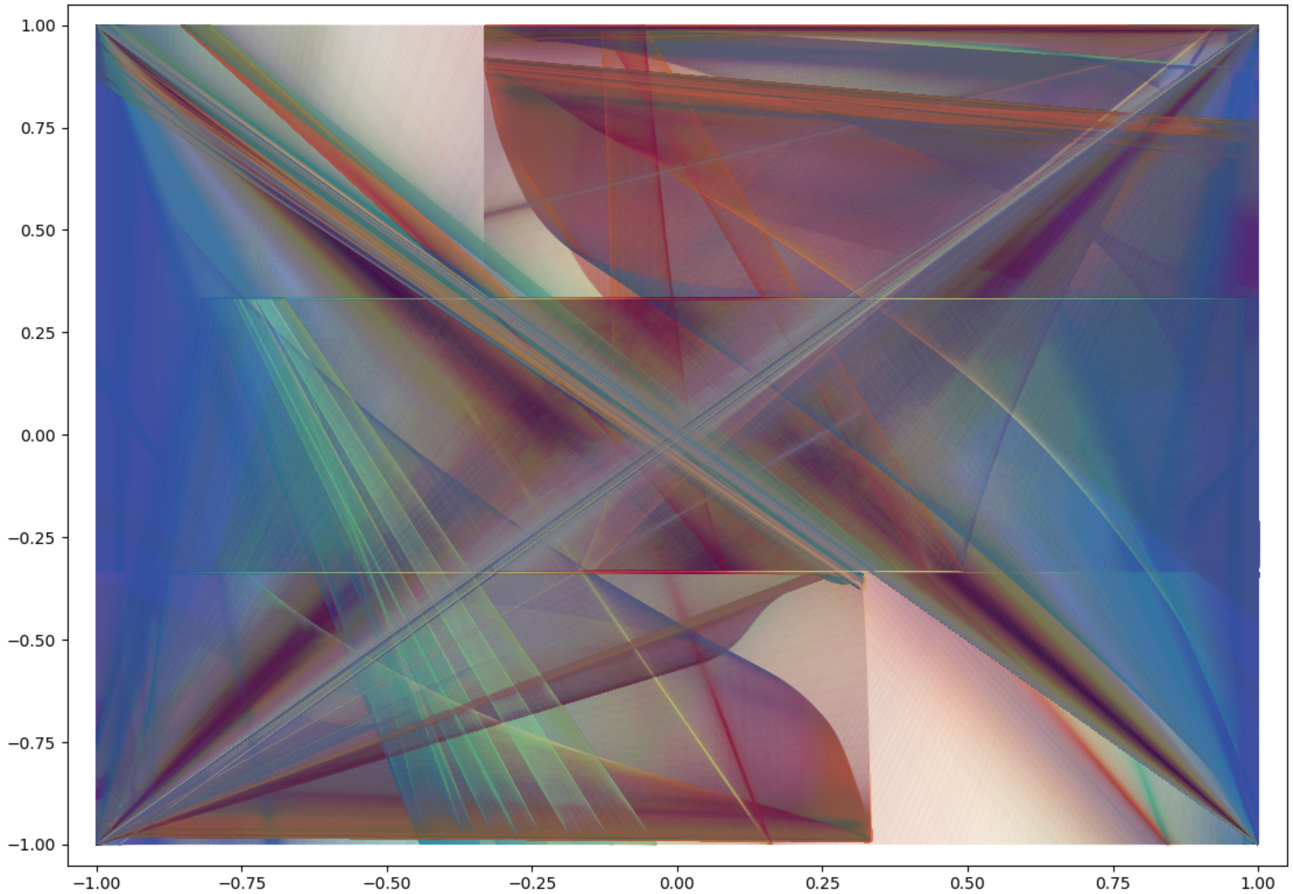

Figure 6: D = 32. Quadratic limit cycle in a back-propagation network with $s = 0.25$ parameter

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