Routes to chaos in recognizing neural networks

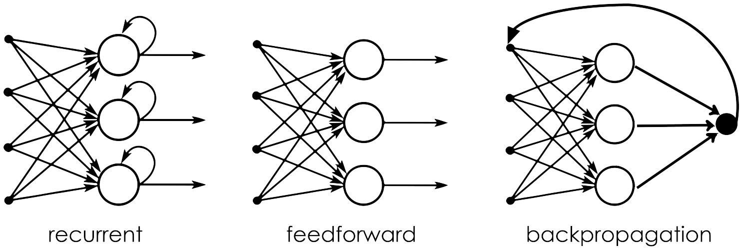
Anna Korsakova
Nanyang Technological University
PAP721 Nonlinear Dynamics
project 2018

November 15, 2018

Outline

Background to neural networks

types

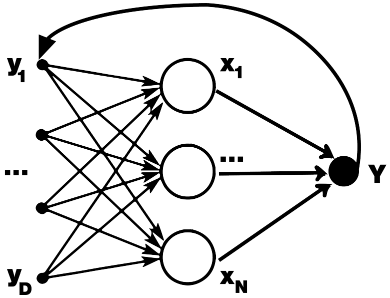


Neural network types: feedforward (not connected), recurrent and backpropagation (connected); also single-layered and multi-layered, with linear and nonlinear activation function, supervised and unsupervised.

Background to neural networks

backpropagation network definition

backpropagation network consists of a layer of D input values, a hidden layer of N neuron units, and output Y:



$$x_i = tanh \sum_{j=1}^{D} w_{ij} y_i,$$

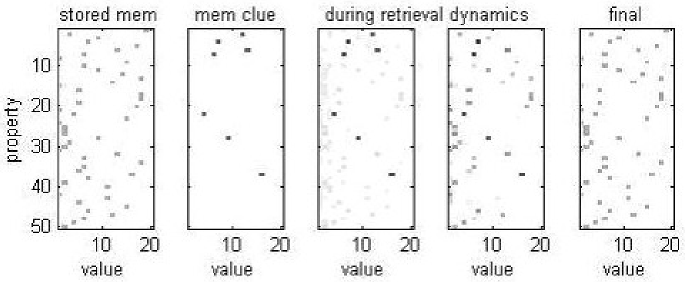
$$Y = s \sum_{i=1}^{N} \beta_i x_i$$

w_{ij}, β_i – weights, s – scaling factor; $tanh$ – ”squashing function”

Backpropagation neural networks and brain

memory retrieval

property of associative memory[1] of backpropagation networks: usage in recognition



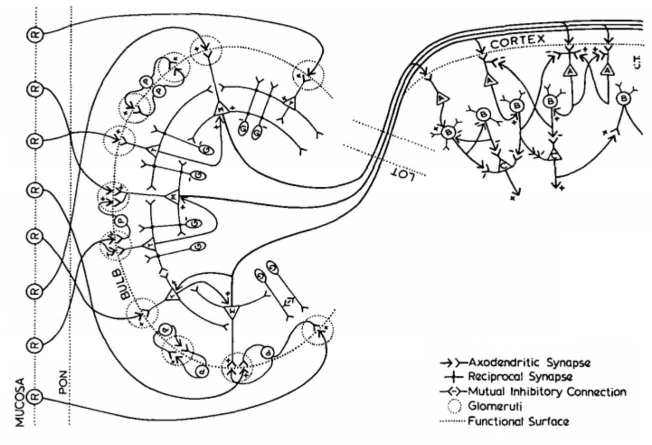
memory retrieval using Hopfield network, picture from[2].

[1] J.J. Hopfield, Proc. Nat. Acad. Sci. USA (79), pp2554-2558, 1982.

[2] J.J. Hopfield, Neural Comput. 20(5), pp1119-64, 2008.

Backpropagation neural networks and brain

odor sensing



Olfactory bulb scheme analogous to self-connected network, picture from[3].

[3] C.A. Skarda, W. J. Freeman, Behavioral and Brain Sciences 10, pp161-195, 1987.

Backpropagation neural networks and brain

odor sensing

hypothesis[4]: chaotic attractors describe recognition processes

example: olfactory bulb and odor recognition

chaos helps neurons not to converge → avoid previously learnt patterns & produce a new pattern

we'll demonstrate this behavior for backpropagation network

[4] C.A. Skarda, W. J. Freeman, Behavioral and Brain Sciences 10, pp161-195, 1987.

Chaotic routes in backpropagation neural networks

stability

self-connected $\rightarrow$ similar to dynamic map $\rightarrow$ prone to chaos;

we can calculate Lyapunov exponents for different s – excitability:

$$\lambda = n_t \sum_{k=0}^{n_t-1} ln \frac{|\Delta x_{k+1}|}{|\Delta x_k|},$$

n_t – number of time steps, Δx_k – distance between the states $x(y_i)$ and $x(y_i + \epsilon)$

Chaotic routes in backpropagation neural networks

stability

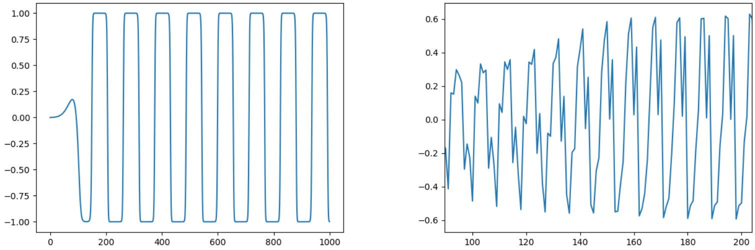
$D = 16$, $N = 4$, 60 000 timesteps; first 10000 discarded

P	20%	40%	50%	60%	80%
	s=0.15	s=0.35	s=0.55	s=0.75	s=0.95

Probability of positive Lyapunov exponent for different s

Chaotic routes in backpropagation neural networks

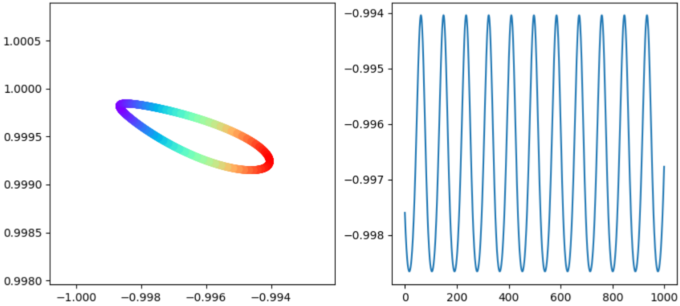
dynamics



left: $x - t$ dynamics with $s = 0.25$ resembling regular neuronal activity, right: $x - t$ dynamics with $s = 0.75$ exhibiting chaotic recognition activity; $D = 16$

Chaotic routes in backpropagation neural networks

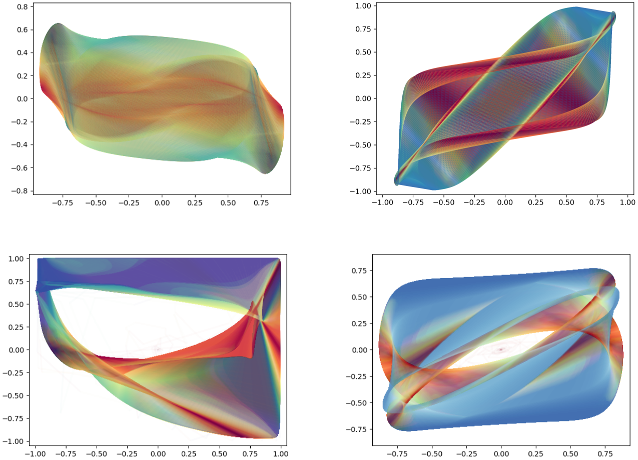
dynamics



left: limit cycle x_1/x_2; right: x_1 dynamics; $s = 0.25$ and $D = 16$

Chaotic routes in backpropagation neural networks

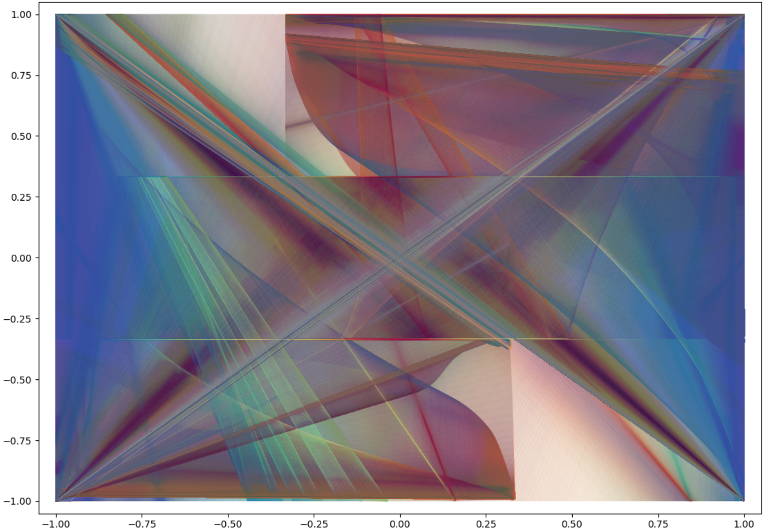
attractors



Top: tori chaotic attractors in a back-propagation network with $s = 0.35$ parameter, bottom: two chaotic attractors for $s = 0.55$ and $s = 0.65$, left and right respectively. $D = 16$.

Chaotic routes in backpropagation neural networks

attractors



$s = 0.25$, $D = 32$

Conclusions

- we demonstrated chaotic behavior of the beckpropagation network, which mimics some types neuronal activity
- with higher excitability system does not converge to limit cycles; instead, it reaches chaotic attractors
- chaotic attractors correspond to different states of recognition.

THANK YOU ☺