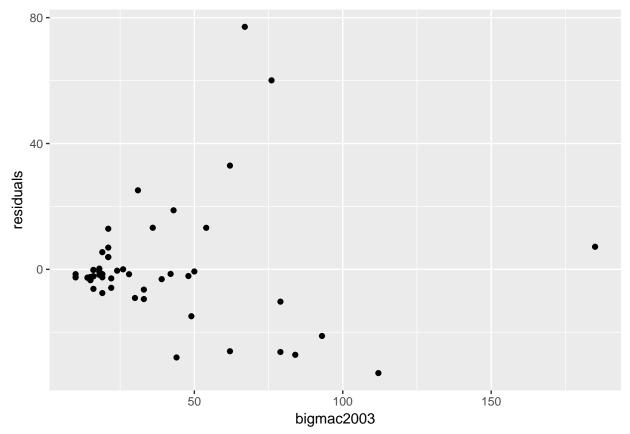
HW3

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February 12, 2017

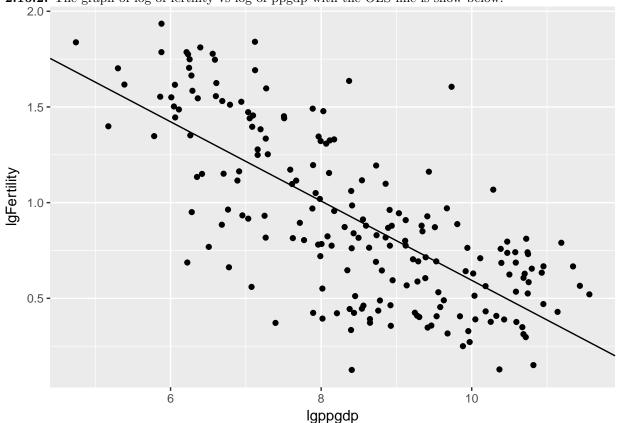
- **2.2.1.** The points above the line x=y represent those cities which had a higher price in 2009 compared to 2003. The points on the line x=y represent those cities with the same price in 2009 compared to 2003. The points below the line x=y represent those cities with a lower price in 2009 compared to 2003.
- **2.2.2.** The city with the smallest increase in price is Mumbai, the city with the largest increase in price is Vilnius.
- **2.2.3.** Since the OLS line slope is less than 1, this does not suggest that prices are lower in 2009 than in 2003, but rather that for every increase in 2003 price by 1 unit, there is a β_1 increase in the 2009 price. It represents the amount of variation in 2009 explained by changes in 2003 price, and this is less than a 1 to 1 ratio, so a 1 dollar increase in 2003 price corresponds to a less than 1 dollar increase in 2009 price.
- **2.2.4.** A simple linear regression is not appropriate for this model for a number of reasons. One of these reasons is that the Give two reasons why fitting simple linear regression to the figure in this problem is not likely to be appropriate. First of all, the residual plot is shown below. The variance appears to fan out with higher 2003 prices, and is not constant. In addition to this, this particular model is probably influenced by many other variables. A simple linear model is enough without other explanatory variables.



2.16.1. The simple linear regression model corresponding to the graph in problem 1.1.3 where the dependent variable (y) is the log of fertility and the dependent variable (x) is the log of the per person GDP is represented by the following equation: y = 2.67 - 0.21x.

```
##
## Call:
## lm(formula = lgFertility ~ lgppgdp, data = UNdata)
##
## Coefficients:
## (Intercept) lgppgdp
## 2.6655 -0.2071
```

2.16.2. The graph of log of fertility vs log of ppgdp with the OLS line is show below.

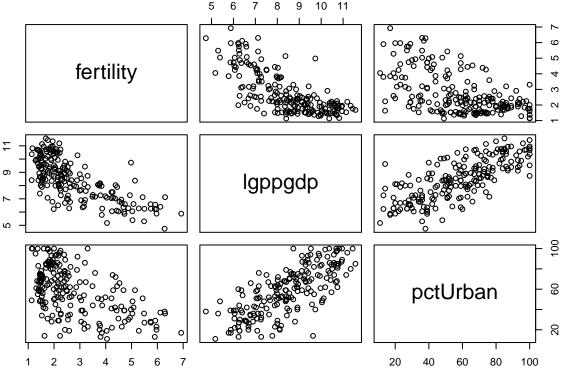


2.16.3 The summary of the model is shown below. The probability given for the slope is for that of a two-tailed t-test, for null hypothesis of the parameter equaling 0. However, we are only interested in a 1-tailed t-test, which means we only need the probability that, given the true slope is 0, it is at least as extreme as the estimate of -0.20715. Thus we can actually devide that probability in half. However, the probability is less than 2e-16, which means that the chance of getting this value if the true parameter value were actually 0 is next to nothing. Thus at almost any significance level (we can use $\alpha = 0.01$), this is statistically significant, and so we can reject the null hypothesis that the slope is 0, and conclude that the alternative hypothesis of the slope being negative is true.

```
##
## Call:
  lm(formula = lgFertility ~ lgppgdp, data = UNdata)
##
##
##
   Residuals:
##
        Min
                   1Q
                        Median
                                      3Q
                                               Max
##
   -0.79828 -0.21639
                       0.02669
                                 0.23424
                                          0.95596
##
##
  Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)
                           0.12057
                2.66551
                                     22.11
                                    -14.79
               -0.20715
                           0.01401
                                              <2e-16
##
  lgppgdp
                         ' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 0.3071 on 197 degrees of freedom
## Multiple R-squared: 0.526, Adjusted R-squared: 0.5236
## F-statistic: 218.6 on 1 and 197 DF, p-value: < 2.2e-16
```

- **2.16.4** The coefficient of determination, or R^2 , is 0.526. This means that 52.6% of the variation in fertility is explained by this model.
- **2.16.5** The point prediction for log(fertility) is 1.234567. The 95% prediction interval for log(fertility) is between 0.6258791 and 1.843218. The point prediction for fertility is 3.436891, and the 95% prediction interval for fertility is between 1.869889 and 6.316831.
- 2.16.6 The locality with the highest value of fertility is Niger. The locality with the lowest value of fertility is Bosnia and Herzegovina. The two localities with the largest positive residuals from the regression when both variables are in the log scale are Equatorial Guinea and Angola. The two localities with the smallest positive residuals from the regression when both variables are in the log scales are Bosnia and Herzegovina, and Moldova.
- **3.2.1** The scatterplot matrix is hown below. Fertility appears to have a medium negative correlation with both log of ppgdp, and with pctUrban. Hard to tell if they are linear or slightly curved. The log of ppgdp has a strong positive linear correlation with pctUrban.

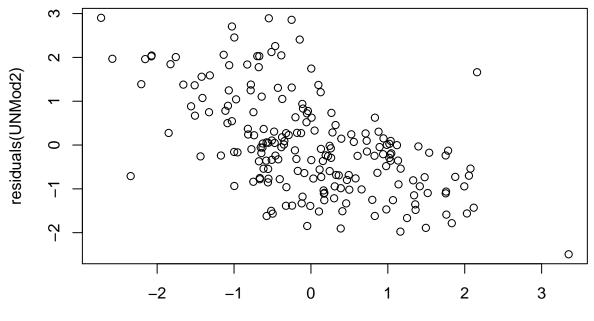


3.2.2. The summaries of each model are shown below. The slope p-values in each model are almost 0, and thus each is statistically significant even at $\alpha = 1$. Thus, the null hypothesis of the parameter being equal to 0 can be rejected for each one, and we can conclude that the slopes are not 0.

```
##
## Call:
## lm(formula = fertility ~ lgppgdp, data = UNdata)
##
```

```
## Residuals:
##
       Min
                  1Q
                       Median
                                    30
                                            Max
##
  -2.16313 -0.64507 -0.06586 0.62479
                                        3.00517
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                                     21.93
## (Intercept)
               8.00967
                           0.36529
                                             <2e-16 ***
## lgppgdp
               -0.62009
                           0.04245
                                   -14.61
                                             <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9305 on 197 degrees of freedom
## Multiple R-squared:
                        0.52, Adjusted R-squared: 0.5175
## F-statistic: 213.4 on 1 and 197 DF, p-value: < 2.2e-16
##
## Call:
## lm(formula = fertility ~ pctUrban, data = UNdata)
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
  -2.4932 -0.7795 -0.1475 0.6517
                                    2.9029
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                4.559823
                           0.213681
                                     21.339
                                              <2e-16 ***
## pctUrban
               -0.031045
                           0.003421
                                     -9.076
                                              <2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.128 on 197 degrees of freedom
## Multiple R-squared: 0.2948, Adjusted R-squared: 0.2913
## F-statistic: 82.37 on 1 and 197 DF, p-value: < 2.2e-16
```

3.2.3. The added variable plots are both shown below, followed by the linear model using both variables. No, it is not useful having both variables there. Though each is statistically significant when regressed with fertility on their own, on a model that has both variables, only one is statistically significant. In fact, pctUrban is not even close to being statistically significant at any level. It is a problem of auto-correlation: each variable explains the same variation in the dependent variable.



residuals(UNMod.AVP.IgPPGDP)

```
##
## Call:
## lm(formula = fertility ~ lgppgdp + pctUrban, data = UNdata)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
   -2.15114 -0.64929 -0.06604 0.63253
                                        2.99102
##
##
  Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                7.9932699
                           0.3993367
                                      20.016
                                                <2e-16 ***
               -0.6151425
                           0.0641565
                                      -9.588
                                                <2e-16 ***
## lgppgdp
## pctUrban
               -0.0004393
                           0.0042656
                                       -0.103
                                                 0.918
##
## Signif. codes:
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9328 on 196 degrees of freedom
## Multiple R-squared:
                        0.52, Adjusted R-squared: 0.5151
## F-statistic: 106.2 on 2 and 196 DF, p-value: < 2.2e-16
```

3.2.4. The slope of the added variable plot of each variable is the same as their coefficient in the model that used both: -0.6151425 for PPGDP and -0.0004393 for pctUrban.

```
##
## Call:
## lm(formula = residuals(UNMod2) ~ residuals(UNMod.AVP.lgPPGDP))
##
## Residuals:
## Min 1Q Median 3Q Max
## -2.15114 -0.64929 -0.06604 0.63253 2.99102
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)
                                -1.986e-16 6.596e-02
                                                       0.000
## residuals(UNMod.AVP.lgPPGDP) -6.151e-01 6.399e-02 -9.613
                                                                <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9305 on 197 degrees of freedom
## Multiple R-squared: 0.3193, Adjusted R-squared: 0.3158
## F-statistic: 92.4 on 1 and 197 DF, p-value: < 2.2e-16
##
## Call:
## lm(formula = residuals(UNMod1) ~ residuals(UNMod.AVP.pctUrban))
##
## Residuals:
       Min
                  1Q
                      Median
                                    3Q
                                            Max
## -2.15114 -0.64929 -0.06604 0.63253 2.99102
## Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                  6.313e-17 6.596e-02
                                                         0.000
                                                                  1.000
## residuals(UNMod.AVP.pctUrban) -4.393e-04 4.255e-03 -0.103
                                                                  0.918
##
## Residual standard error: 0.9305 on 197 degrees of freedom
## Multiple R-squared: 5.411e-05, Adjusted R-squared: -0.005022
## F-statistic: 0.01066 on 1 and 197 DF, p-value: 0.9179
## Call:
## lm(formula = fertility ~ lgppgdp + pctUrban, data = UNdata)
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
## -2.15114 -0.64929 -0.06604 0.63253 2.99102
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 7.9932699 0.3993367 20.016
                                               <2e-16 ***
## lgppgdp
               -0.6151425 0.0641565 -9.588
                                               <2e-16 ***
## pctUrban
               -0.0004393 0.0042656 -0.103
                                                0 918
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9328 on 196 degrees of freedom
## Multiple R-squared: 0.52, Adjusted R-squared: 0.5151
## F-statistic: 106.2 on 2 and 196 DF, p-value: < 2.2e-16
3.2.5. The residuals for all of them are shown below, and all are equal. I could not think of another way to
show their equality, sorry.
resid(AVP.lgPPGDP)
                           2
                                                                  5
##
                                        3
                                                     4
   1.806471381 -1.394725716 -0.651064642
                                           2.317282243 -0.087771575
##
              6
                           7
                                        8
                                                     9
  -0.168576339 -1.298836348 -0.127579044
                                           0.732413953 -0.024338567
##
##
             11
                          12
                                       13
                                                    14
```

```
## -0.509310405 0.084087945 0.509329590 -1.820260779 -0.504380849
##
                                 18
                                      19
           16
                     17
  -1.161219859 0.458814918 -0.117086980 1.168124662 0.845618678
                      22
           21
                                 23
                                             24
##
  -1.029908512 -0.066035443 -1.666325646 0.131655421 -0.446865194
           26
                     27
                                 28
                                            29
##
   0.417614948 -1.027864058 1.615231349 -0.754760019 -1.452652523
##
                      32
                                 33
                                             34
##
   0.684428961 0.342532633 -0.713876902 0.387480913 0.206012253
##
                      37
                                 38
                                             39
   1.809496787 -0.350132615 -1.258997870 -0.293454577 0.822228188
##
          41
                     42
                                 43
                                            44
##
   ##
           46
                     47
                                 48
                                            49
  -1.188960997 -0.197095105 -0.404501402
                                    0.768606850 0.653497770
##
           51
                     52
                                 53
                                             54
   1.972654240 -0.457312014
                     57
                                 58
                                             59
           56
  -0.488336672 -0.787052560 2.991020553 -0.011802803 -0.381949755
##
          61
                     62
                               63
                                            64
##
  -0.580683208 -0.340566018 0.503112558 0.542924816 0.283255316
                     67
                               68
   1.040907389 \quad 0.634861124 \quad -1.586247829 \quad 0.012475281 \quad 0.443730204
##
##
          71
                      72
                          73
                                             74
  -0.160781847 0.702051731 -0.321843761 0.769130498 0.781049116
          76
                     77
                                 78
                                            79
   0.766435985 -0.866294823 -0.862641086 -0.290783431 -0.434565424
##
##
          81
                     82
                                 83
                                            84
  ##
           86
                      87
                                88
                                           89
   0.747253639 \quad 0.738317934 \quad 1.283338436 \quad -0.070719466 \quad -0.481693226
##
##
           91
                      92
                                 93
                                             94
   96
                      97
                                 98
                                            99
##
   0.897528398 -1.196568603 -1.157475511 -0.752238503 -0.571101506
                    102
##
         101
                               103
                                           104
  -0.692704752 0.379737823 0.192977908 -0.745908955 0.839729774
          106
##
                     107
                                108
                                           109
  -0.130757434 0.231741470 1.599849442 0.167221705 -1.108066404
##
          111
                     112
                                113
                                            114
   2.073775022 -0.587935243 1.361949371 0.711213245 -0.897074486
          116
                     117
                                118
                                           119
  -0.124301792 0.179091646 -1.973971615 -0.772406925 -0.934868470
          121
                     122
                                123
                                           124
  ##
          126
                     127
                                128
                                            129
## -1.533779547 0.049436520 0.453650216 0.564658133 0.567794773
          131
                     132
                                133
                                            134
  -1.142323366 2.556047736 1.841611902 -2.151142213 0.969005743
          136
                    137
                                138
                                            139
   0.300713368 -0.525236893 -0.242116500 0.926691147 -0.053338086
##
                    142
                                143
                                           144
##
   0.280026457 - 0.231800461 - 0.261598459 - 0.204458857 - 0.760302329
##
          146
                     147
                                148
                                            149
```

```
## -0.519718513 0.071477309 1.136303651 -0.444213869 -1.049251161
##
                     152
                                 153
          151
                                            154
## -0.745286020 1.158453172 -0.656755425 0.770220661 -0.074903677
          156
                     157
                                158
                                            159
## 0.630200073 0.899666023 -1.152409627 0.120326630 0.358306062
##
          161
                     162
                                163
                                           164
## -0.008312461 -0.643242240 -0.313786278 0.414142702 1.224161239
          166
##
                     167
                                168
                                            169
## -0.114775684 -0.102501044 -0.970691992 -0.607508799 0.869017880
##
          171
                     172
                                173
                                            174
176
                     177
                                178
                                            179
##
## -0.695678635 1.359836182 -1.404896447 -1.260629400 -0.257528196
                                183
          181
                     182
   0.827682927 -0.431675602 -0.919135501 -0.269026475 -0.468984115
##
                     187
                          188
                                     189
  0.691424937 1.747284197 -1.547767798 0.263270784 0.368059693
##
                     192
                           193
                                      194
## 0.731860991 -0.134036602 -1.245344021 0.685677941 0.289423958
          196
                    197
                                198
## -1.877177720 1.431150867 2.703029588 -0.960331139
```

resid(AVP.pctUrban)

##	1	2	3	4	5
##	1.806471381	-1.394725716	-0.651064642	2.317282243	-0.087771575
##	6	7	8	9	10
##	-0.168576339	-1.298836348	-0.127579044	0.732413953	-0.024338567
##	11	12	13	14	15
##	-0.509310405	0.084087945	0.509329590	-1.820260779	-0.504380849
##	16	17	18	19	20
##	-1.161219859	0.458814918	-0.117086980	1.168124662	0.845618678
##	21	22	23	24	25
##	-1.029908512	-0.066035443	-1.666325646	0.131655421	-0.446865194
##	26	27	28	29	30
##	0.417614948	-1.027864058	1.615231349	-0.754760019	-1.452652523
##	31	32	33	34	35
##	0.684428961	0.342532633	-0.713876902	0.387480913	0.206012253
##	36	37	38	39	40
##	1.809496787	-0.350132615	-1.258997870	-0.293454577	0.822228188
##	41	42	43	44	45
##	1.328646556	0.359524527	-0.647518298	0.590549704	-0.602124340
##	46	47	48	49	50
##	-1.188960997	-0.197095105	-0.404501402	0.768606850	0.653497770
##	51	52	53	54	55
##	0.031471320	0.484252561	-0.209651184	1.972654240	-0.457312014
##	56	57	58	59	60
##	-0.488336672	-0.787052560	2.991020553	-0.011802803	-0.381949755
##	61	62	63	64	65
##	-0.580683208	-0.340566018	0.503112558	0.542924816	0.283255316
##	66	67	68	69	70
##	1.040907389	0.634861124	-1.586247829	0.012475281	0.443730204
##	71	72	73	74	75
##	-0.160781847	0.702051731	-0.321843761	0.769130498	0.781049116
##	76	77	78	79	80

```
-0.711849595 0.653257645 -0.983053847 -1.003930375 -1.108469986
                          88
##
         86
                 87
                                    89
##
  0.747253639 0.738317934 1.283338436 -0.070719466 -0.481693226
             92
                      93
##
        91
                               94
  98
                              99
##
        96
            97
##
  0.897528398 - 1.196568603 - 1.157475511 - 0.752238503 - 0.571101506
        101 102 103 104
##
  106
##
            107
                      108
                               109
##
  -0.130757434 0.231741470 1.599849442 0.167221705 -1.108066404
       111 112
                         113
  2.073775022 -0.587935243 1.361949371 0.711213245 -0.897074486
            117
                     118
                              119
##
        116
  -0.124301792 0.179091646 -1.973971615 -0.772406925 -0.934868470
           122
                     123
        121
                              124
  -0.887619293 0.433893414 -1.871378548 0.333302706 0.721286386
             127
        126
                     128
                               129
##
 -1.533779547 0.049436520 0.453650216 0.564658133 0.567794773
       131 132 133
                              134
  -1.142323366 2.556047736 1.841611902 -2.151142213 0.969005743
            137
                     138
##
       136
                              139
  0.300713368 -0.525236893 -0.242116500 0.926691147 -0.053338086
       141
            142
                     143
                              144
  0.280026457 -0.231800461 -0.261598459 -0.204458857 -0.760302329
##
##
        146
            147
                     148
                               149
 -0.519718513 0.071477309 1.136303651 -0.444213869 -1.049251161
        151 152 153
                                   154
  -0.745286020 1.158453172 -0.656755425 0.770220661 -0.074903677
##
        156
            157
                          158
                                    159
  161
            162
                     163
                                   164
  -0.008312461 -0.643242240 -0.313786278 0.414142702 1.224161239
            167
##
       166
                     168
                               169
  -0.114775684 -0.102501044 -0.970691992 -0.607508799 0.869017880
           172 173
                              174
##
       171
  -0.248674979 0.175725241 0.611167728 0.428000168 -0.285822140
                     178
                              179
##
        176
                 177
  -0.695678635 1.359836182 -1.404896447 -1.260629400 -0.257528196
                     183
        181
            182
                                   184
##
  0.827682927 -0.431675602 -0.919135501 -0.269026475 -0.468984115
       186 187 188 189
  0.691424937 \quad 1.747284197 \quad -1.547767798 \quad 0.263270784 \quad 0.368059693
                     193
##
        191
           192
                              194
  0.731860991 -0.134036602 -1.245344021 0.685677941 0.289423958
           197
       196
                          198
## -1.877177720 1.431150867 2.703029588 -0.960331139
resid(UNMod)
                   2
                            3
  1.806471381 -1.394725716 -0.651064642 2.317282243 -0.087771575
##
             7
                        8
                                 9
```

0.766435985 -0.866294823 -0.862641086 -0.290783431 -0.434565424

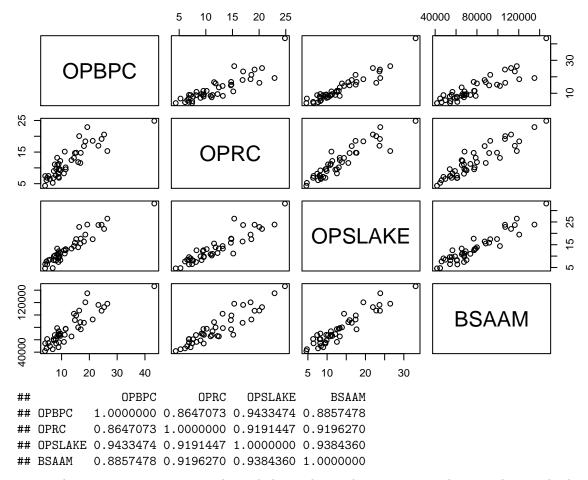
##

82 83 84

```
## -0.168576339 -1.298836348 -0.127579044 0.732413953 -0.024338567
##
                   12
                       13
                                         14
          11
  -0.509310405 0.084087945 0.509329590 -1.820260779 -0.504380849
          16
                    17
                              18
                                         19
##
  -1.161219859 0.458814918 -0.117086980 1.168124662 0.845618678
                    22
##
          21
                             23
                                         24
  -1.029908512 -0.066035443 -1.666325646 0.131655421 -0.446865194
##
          26
                    27
                               28
                                         29
##
   0.417614948 - 1.027864058 1.615231349 - 0.754760019 - 1.452652523
##
          31
                    32
                               33
                                         34
   0.684428961  0.342532633  -0.713876902  0.387480913  0.206012253
##
          36
                    37
                              38
                                         39
   1.809496787 -0.350132615 -1.258997870 -0.293454577 0.822228188
##
##
          41
                    42
                              43
                                         44
   ##
                    47
                              48
                                         49
  -1.188960997 -0.197095105 -0.404501402
                                 0.768606850 0.653497770
                   52
                               53
                                         54
   0.031471320 0.484252561 -0.209651184 1.972654240 -0.457312014
##
          56
                   57
                             58
                                         59
##
  -0.488336672 -0.787052560 2.991020553 -0.011802803 -0.381949755
          61
                   62
                             63
  -0.580683208 -0.340566018  0.503112558  0.542924816  0.283255316
##
          66
                    67
                              68
                                         69
##
   1.040907389 0.634861124 -1.586247829 0.012475281 0.443730204
          71
                   72
                              73
                                         74
  ##
##
          76
                    77
                               78
                                         79
   0.766435985 -0.866294823 -0.862641086 -0.290783431 -0.434565424
##
          81
                    82
                             83
                                         84
  -0.711849595 0.653257645 -0.983053847 -1.003930375 -1.108469986
##
          86
                    87
                              88
                                         89
   91
                    92
                               93
                                         94
##
   0.019101738 0.097385464 0.125796935 0.753204606 0.011549594
                   97
                              98
##
         96
                                         99
   0.897528398 -1.196568603 -1.157475511 -0.752238503 -0.571101506
##
         101
                   102
                              103
                                       104
   ##
         106
                    107
                              108
                                        109
  -0.130757434 0.231741470 1.599849442 0.167221705 -1.108066404
         111
                    112
                              113
                                        114
   2.073775022 -0.587935243 1.361949371 0.711213245 -0.897074486
##
         116
                    117
                              118
                                        119
  123
##
         121
                    122
                                         124
126
                    127
                              128
                                         129
## -1.533779547 0.049436520 0.453650216 0.564658133 0.567794773
         131
                    132
                              133
                                         134
## -1.142323366 2.556047736 1.841611902 -2.151142213 0.969005743
                   137
                             138
   0.300713368 -0.525236893 -0.242116500 0.926691147 -0.053338086
##
         141
                    142
                              143
                                         144
```

```
0.280026457 -0.231800461 -0.261598459 -0.204458857 -0.760302329
##
             146
                           147
                                         148
                                                        149
                                                                      150
   -0.519718513
                                1.136303651
##
                  0.071477309
                                              -0.444213869
                                                           -1.049251161
##
             151
                           152
                                                        154
                                                                      155
                                         153
##
   -0.745286020
                  1.158453172
                               -0.656755425
                                               0.770220661
                                                            -0.074903677
             156
##
                           157
                                         158
                                                        159
                                                                      160
    0.630200073
                  0.899666023 -1.152409627
                                               0.120326630
##
                                                             0.358306062
##
             161
                           162
                                         163
                                                        164
                                                                      165
##
   -0.008312461
                 -0.643242240
                               -0.313786278
                                               0.414142702
                                                             1.224161239
##
             166
                           167
                                         168
                                                        169
                                                                      170
                 -0.102501044
##
   -0.114775684
                               -0.970691992
                                              -0.607508799
                                                             0.869017880
             171
                           172
                                                        174
##
                                         173
                                                                      175
##
   -0.248674979
                  0.175725241
                                0.611167728
                                               0.428000168
                                                            -0.285822140
##
             176
                           177
                                         178
                                                        179
                                                                      180
##
   -0.695678635
                  1.359836182 -1.404896447
                                              -1.260629400
                                                            -0.257528196
##
             181
                           182
                                         183
                                                        184
                                                                      185
##
    0.827682927
                 -0.431675602
                               -0.919135501
                                              -0.269026475
                                                            -0.468984115
##
             186
                           187
                                         188
                                                        189
                                                                      190
    0.691424937
                                                             0.368059693
                  1.747284197
                               -1.547767798
                                               0.263270784
##
##
             191
                           192
                                         193
                                                        194
                                                                      195
##
    0.731860991
                 -0.134036602
                               -1.245344021
                                               0.685677941
                                                             0.289423958
##
             196
                                         198
                                                        199
                           197
                  1.431150867
                                2.703029588 -0.960331139
   -1.877177720
```

- **3.2.6.** The t-test values for each of the added variable plots and the model with both variables are shown above. The t-vale for the added variable plot for lgppgdp is -9.613, but the t-value for lgppgdp in the model with both variables is -9.588. They are slightly different because the number of variables is different, meaning the degrees of freedom is different, and thus the standard error calculation is slightly different, and so the t-value will be slightly different.
- **3.6.1.** The scatterplot matrix appears to show that each of these variables have a strong positive linear correlation with each other. The correlation matrix should show that all of these have postive correlations close to 1. The scatterplot matrix and correlation matrix are both shown below, and the correlation matrix does indeed match what was expected.



3.6.2 The regression summary is shown below. The t-values represent where on the standardized t-curve these parameter estimates (the intercept and the coefficients) would be if the true parameter value were actually 0, and thus allow a t-test to be performed for a null hypothesis of whether or not the parameter actually equals 0 or not.

```
##
  Call:
##
   lm(formula = BSAAM ~ OPBPC + OPRC + OPSLAKE, data = Waterdata)
##
##
   Residuals:
##
        Min
                   1Q
                        Median
                                     3Q
                                              Max
##
   -15964.1
             -6491.8
                        -404.4
                                 4741.9
                                          19921.2
##
##
   Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
   (Intercept) 22991.85
                            3545.32
                                      6.485
                                              1.1e-07
##
##
  OPBPC
                  40.61
                             502.40
                                      0.081
                                              0.93599
  OPRC
                1867.46
                             647.04
                                      2.886
                                              0.00633 **
##
   OPSLAKE
                2353.96
                             771.71
                                      3.050
                                              0.00410 **
##
## Signif. codes:
                     '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8304 on 39 degrees of freedom
## Multiple R-squared: 0.9017, Adjusted R-squared: 0.8941
## F-statistic: 119.2 on 3 and 39 DF, p-value: < 2.2e-16
```