PSet 5 - LaTex Equations

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Question 1
(a)

$$\mathscr{L}(\lambda|y) = f(y_1) \times f(y_2) \times ... \times f(y_n)$$

$$\mathscr{L}(\lambda|y) = \prod_{i=1}^{n} f(y_i)$$

$$f(y_i) = \frac{y_i}{\lambda^2} exp(-\frac{y_i}{\lambda})$$

$$\mathscr{L}(\lambda|y) = \prod_{i=1}^{n} \frac{y_i}{\lambda_i^2} exp(-\frac{y_i}{\lambda_i})$$

$$ln(\mathscr{L}(\lambda|y)) = ln(\prod_{i=1}^{n} \frac{y_i}{\lambda_i^2} exp(-\frac{y_i}{\lambda_i}))$$

$$Q = \sum_{i=1}^{n} [ln(y_i) - ln(\lambda_i^2) + ln(exp(-\frac{y_i}{\lambda_i}))]$$

$$Q = \sum_{i=1}^{n} [ln(y_i) - 2 \times ln(\lambda_i) - y_i \times \lambda_i^{-1}]$$

$$\lambda_i = \frac{exp(x_i'\beta)}{2}$$

$$Q = \sum_{i=1}^{n} [ln(y_i) - 2 \times ln(\frac{exp(x_i'\beta)}{2}) - y_i \times (\frac{exp(x_i'\beta)}{2})^{-1}]$$

$$Q = \sum_{i=1}^{n} [ln(y_i) - 2 \times [ln(exp(x_i'\beta)) - ln(2)] - 2y_i \times exp(-x_i'\beta)]$$

$$Q = \sum_{i=1}^{n} [ln(y_i) - 2x_i'\beta + 2ln(2) - 2y_i \times exp(-x_i'\beta)]$$

Then it needs to be scaled by N^{-1}

$$Q = \frac{1}{N}\sum_{i=1}^{n}[ln(y_i) - 2x_i'\beta + 2ln(2) - 2y_i \times exp(-x_i'\beta)]$$

(b)

$$\frac{\partial Q_N(\beta)}{\partial\beta} = \frac{1}{N}\sum_i[0 - 2x_i + 0 - 2y_i(-x_i)exp(-x_i'\beta)]$$

$$\frac{\partial Q_N(\beta)}{\partial\beta} = \frac{1}{N}\sum_i[-2x_i^+ \frac{2y_i(x_i)}{exp(x_i'\beta)}]$$

$$\frac{\partial Q_N(\beta)}{\partial\beta} = \frac{1}{N}\sum_i 2[-1 + \frac{y_i}{exp(x_i'\beta)}]x_i$$

$$\frac{\partial Q_N(\beta)}{\partial\beta} = \frac{1}{N}\sum_i 2[-\frac{exp(x_i'\beta)}{exp(x_i'\beta)} + \frac{y_i}{exp(x_i'\beta)}]x_i$$

$$\frac{\partial Q_N(\beta)}{\partial\beta} = \frac{1}{N}\sum_i 2[\frac{y_i - exp(x_i'\beta)}{exp(x_i'\beta)}]x_i$$

(c)
To ensure consistency, it needs to be true that $E_0[g_i(\theta_0)] = 0$, which thus means that $E_0[y_i]$ must be equal to $exp(x_i'\beta)$.

Question 2
(a)
Let U_{nb} be the utility function, n is the individual, b is the brand, and V_{ij} is the deterministic component of the utility function.

$$U_{nb} = V_{nb} + \epsilon_{nb}$$

$$V_n = \beta_0 + \beta_1 Age + \beta_2 Female$$

$$y_n = \begin{cases} 1, & \text{if } U_{n1} \geq U_{n2} \text{ and } U_{n1} \geq U_{n3} \\ 2, & \text{if } U_{n2} \geq U_{n1} \text{ and } U_{n2} \geq U_{n3} \\ 3, & \text{otherwise} \end{cases} \tag{1}$$

(b)

$$ln\mathscr{L} = \sum_{1=n}^{N}[1(y* = 1)\times lnPr(y* = 1) + 1(y* = 2)\times lnPr(y* = 2) + 1(y* = 3)\times lnPr(y* = 3)]$$

(c)

$$P_{nb} = \frac{exp(x_n'\beta_b)}{\sum_c exp(x_n'\beta_c)} \ \forall \ c \neq b$$

$$ln\mathscr{L} = \sum_{1=n}^{N}[1(y* = 1)\times ln(\frac{exp(x'_n\beta_b)}{\sum_c exp(x'_n\beta_c)}) + 1(y* = 2)\times ln(\frac{exp(x'_n\beta_b)}{\sum_c exp(x'_n\beta_c)}) + 1(y* = 3)\times ln(\frac{exp(x'_n\beta_b)}{\sum_c exp(x'_n\beta_c)})]$$

For parameter estimates and marginal effects, see other pages.

(d)

$$P_{nb} = \Psi(x\beta)$$

$$ln\mathscr{L} = \sum_{1=n}^{N}[1(y* = 1) \times ln(\Psi(x\beta)) + 1(y* = 2) \times ln(\Psi(x\beta)) + 1(y* = 3) \times ln(\Psi(x\beta))]$$

For parameter estimates and marginal effects, see other pages.

(e)

See other pages

Question 3

See other pages