HW5

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February 26, 2017

4.8 Each of the regression coefficients show the relationship between that particular independent variable and the dependent variable, holding all other independent variables constant (partial derivative). However, it is possible that there is a negative relationship between x1 and x2, and that as x1 increases, x2 decreases. Thus, if there is a regression with just x1 as the independent variable and no x2, then the coefficient does not account for holding x2 constant. If for every unit increase in x1, there is more than 2 unit decrease in x2 (since in the original regression, the coefficient of 4 in front of x1 is double the coefficient of 2 in front of x2), then as x1 increases, y would actually decrease because x2 is not included in this model and, rather than being held constant, would be decreasing.

4.10

$Given : \mu_x, \sigma_x^2, \beta_0, \beta_1, \sigma^2$

$\beta_0 = \mu_y - \beta_1\mu_x \ \mu_y = \beta_0 + \beta_1\mu_x$

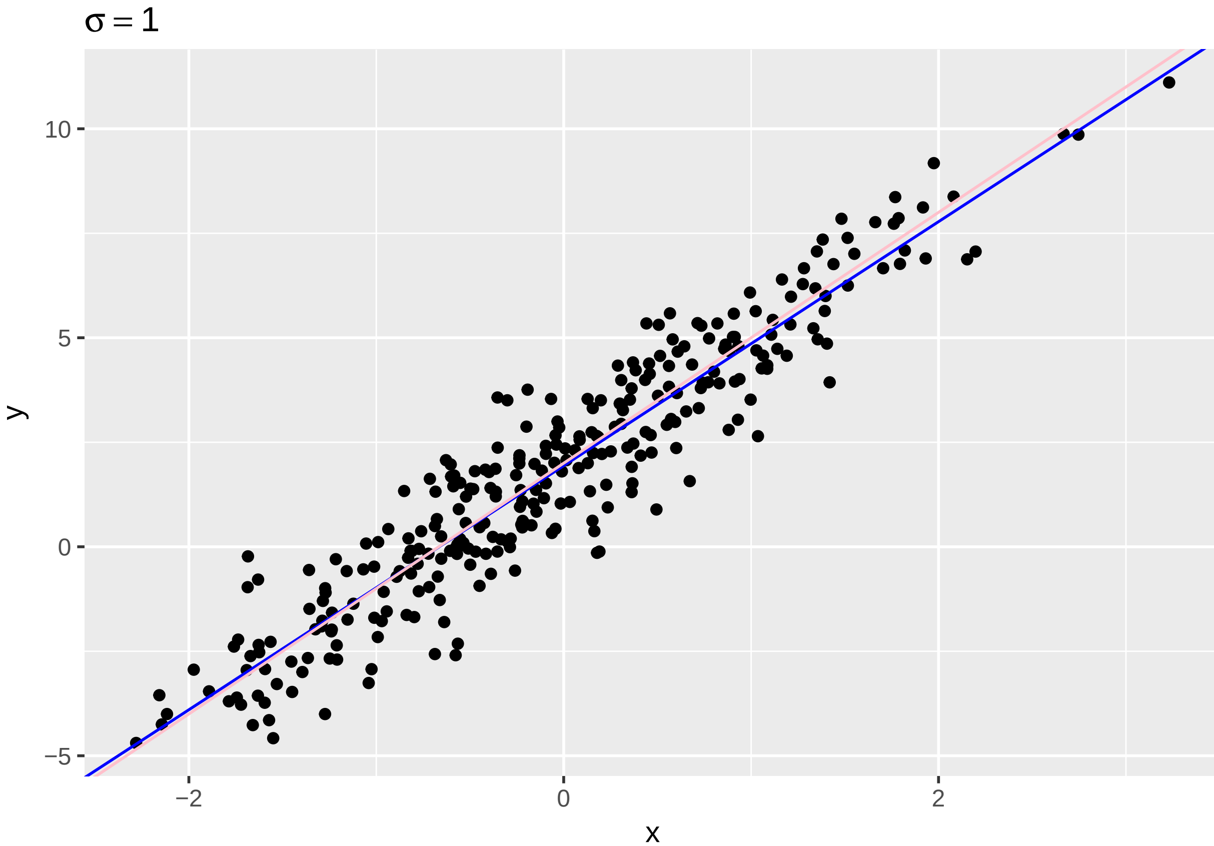
$\beta_0 = \rho_{xy}\frac{\sigma_y}{\sigma_x} \ \rho_{xy} = \frac{\beta_1\sigma_x}{\sigma_y}$

$\sigma^2 = \sigma_y^2(1-\rho_{xy}^2) \ \sigma^2 = \sigma_y^2(1-(\frac{\beta_1\sigma_x}{\sigma_y})^2) \ \sigma^2 = \sigma_y^2(1-\frac{\beta_1^2\sigma_x^2}{\sigma_y^2}) \ \sigma^2 = \sigma_y^2 - \beta_1^2\sigma_x^2 \ \sigma_y^2 = \sigma^2 + \beta_1^2\sigma_x^2$

$\rho_{xy} = \frac{\beta_1\sigma_x}{\sigma_y} \ \rho_{xy} = \frac{\beta_1\sigma_x}{\sqrt{\sigma^2+\beta_1^2\sigma_x^2}}$

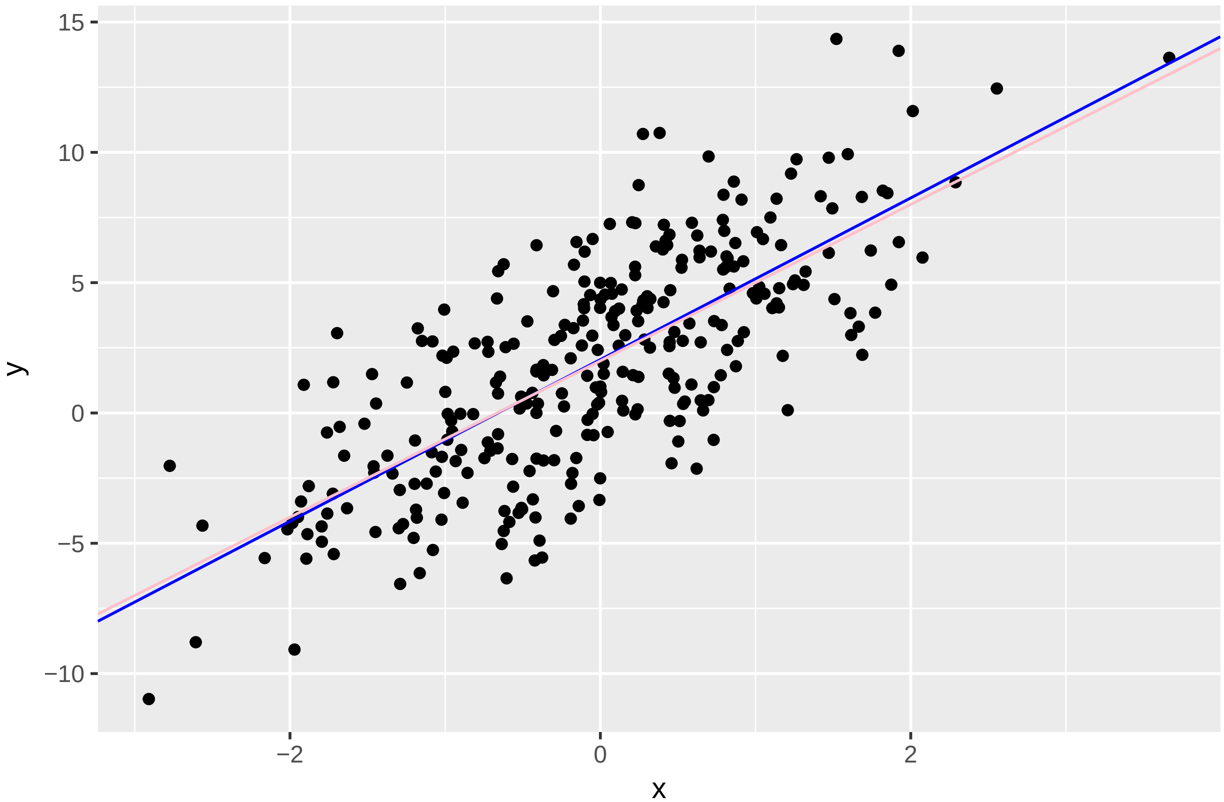
$(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho_{xy}) = (\mu_x, \beta_0 + \beta_1\mu_x, \sigma_x^2, \sigma^2 + \beta_1^2\sigma_x^2, \frac{\beta_1\sigma_x}{\sqrt{\sigma^2+\beta_1^2\sigma_x^2}})$

4.12.1. The scatterplot is shown below. The true regression line is shown in pink and the OLS regression line is shown in blue. The scatterplot does appear to be approximately elliptical, with the regression line going approximately through the major axis of the ellipse.

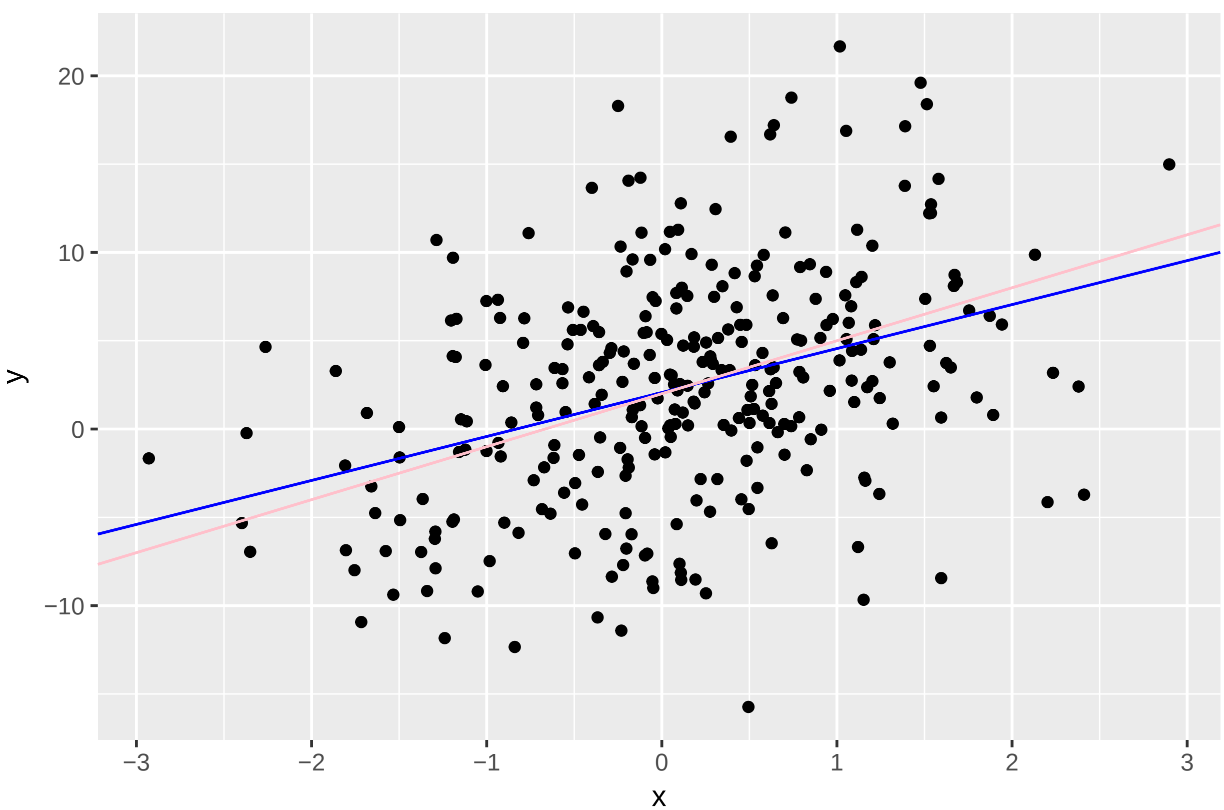


4.12.2. The scatterplots for both are shown below, where $\sigma = 3$ first and $\sigma = 6$ right below that. The true regression line is shown in pink and the OLS regression line is shown in blue. As σ increases, the points becomes more spread out.

$\sigma = 3$



$\sigma = 6$



4.12.1. The scatterplot where e has a Cauchy distribution is shown below. The true regression line is shown in pink and the OLS regression line is shown in blue.