

Learning to Learn in the Context of Spiking Neural Networks

Seminar on the Human Brain Project

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Abstract—Artificial Neural Networks (ANNs) today demonstrate impressive capabilities on a variety of tasks, often challenging performances of humans in those specific areas. Yet these networks seem to lack the unrivaled ability of the human brain to generalize over an almost arbitrary range of situations. In Neuroscience this ability is believed to be linked to a nested system of learning mechanisms, here referred to as Learning to Learn (L2L). Although originally inspired by the workings of the human brain, an insufficient understanding of biological learning processes on a low level and the prioritization of technical performances have led to diverging developments in AI and Neuroscience. However, recent work has delivered promising results while employing biologically more plausible Networks of Spiking Neurons (SNNs). The, to this date, unparalleled performances of Long Short-Term Memory(LSTM)-based methods are being approached by Long Short-Term Memory Networks of Spiking Neurons(LSNN) and therefore give new perspectives for applications of L2L in spatio-temporal problems. LSNNs employ an adaptive version of Leaky-Integrate and Fire neurons (adaptive LIF) and demonstrate a well working internal memory, which is considered crucial for encoding learned knowledge on different levels of abstraction. The proposed architectures include an outer-loop optimizer, which accounts for a slow learning process over a wide family of tasks F , while an inner-loop learner utilizes the optimized structure in order to enhance performances on any specific subtask of F . We further explore the applicability of these approaches in high-dimensional problems, as they are often faced in robotics, and infer future potential as well as challenges.

I. INTRODUCTION

Manifold new developments and theories have originated from the ever-more intertwined fields of Neuroscience and Machine Learning. One apparent disparity between the human brain and ANNs is the ability to extract abstract knowledge from widely varying tasks and utilize such conceptual information to learn on radically less training data than any ANN to this date. In order to generate a learning system capable of grasping knowledge on different levels of abstraction, one has to account for a network architecture, that allows multiple learning processes, which interact in a way that creates a mechanism described as Learning to Learn (L2L). The pioneering meta-RL approach by Wang et al. [65] builds on a nested system in which a Deep Reinforcement Learning (DRL) algorithm purposes the optimization of a LSTM based learner. Comparisons of Meta-RL to an RNN-less baseline version reveals a need for well-working short-term memory in order to store immediate task specific knowledge. We will derive the mathematical and conceptual

framework necessary to achieve such short-term memory capabilities in the first chapter. Recurrent Neural Networks (RNNs) enable spatio-temporal information-transfer through the network but often lack capability when facing long range dependencies. A milestone in the development of RNNs was then first described by Hochreiter and Schmidhuber [34], who implemented a context sensitive read-and-write-like behaviour in an RNN and improved performances on long-range temporal problems greatly, thus the name Long Short-Term Memory(LSTM). Furthermore, we will give an introduction to previous work on Networks of Spiking Neurons (SNNs) in chapter 2 and describe the adoption of RNN architectures in SNNs. We deem these concepts necessary to discuss the implementation of L2L in both ANNs and SNNs in chapter 3. Groundwork for L2L was laid by the RL-based algorithms meta-RL[65] and RL[23]. Moreover Bellec et al. [6] [7] and Bohnstingl et al. developed this framework in SNNs, while maintaining more biological plausibility. Long Short-Term Memory Networks of Spiking Neurons (LSNN) resort to adaptive Leaky-Integrate and Fire neurons for fast inner-loop learning and implement L2L by attributing a BPTT-optimized meta-learner with updating all synaptic weights of the networks. In more recent work, Bellec et al. lay focus on the promotion of sophisticated learning signals and the use of eligibility traces to assign credit for network performances. Chapter 4 will inspect performances of the proposed techniques to evaluate applicability to high-dimensional spaces, as found plenty in fields like Robotics. Finally we will discuss the implications proposed by these methods for Neuroscience and Machine Learning by elaborating opportunities and challenges of L2L.

II. RECURRENT NEURAL NETWORKS (RNNs)

Artificial Neural Networks (ANNs) are networks of biologically inspired computational units, neurons, that learn according to certain learning rules thus improving their ability to perform a desired task, often classification or regression problems. Accordingly, the performance demonstrated by ANNs is based on knowledge inferred from data [59]. Many of the data driven problems in engineering involve the recognition of time dependent patterns (e.g. speech recognition, machine translation, machine vision in real-time video material), requiring special network architectures such as the Time-Delay Neural Networks (TDNN) [63] or Recurrent Neural Networks (RNNs). The latter are based on the idea of including a loop in the network's neurons, allowing information to be passed from past network states to subsequent time steps.

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A. Training RNNs

Unfolding an RNN over time results in a more approachable representation of this model as seen in fig. 1. Training a model of this structure can now be achieved in a similar way to the well-known backpropagation algorithm with the additional ability to encode information on the past[66].

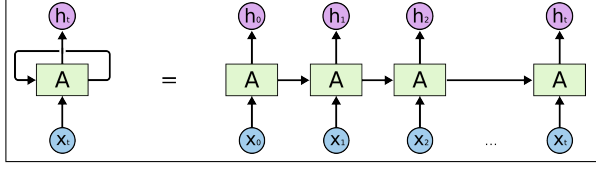


Fig. 1. Unrolled RNN [2]

The notation of symbols in the following is according to elaborations of Jian Guo [31]. The inputs are denoted by a vector \mathbf{x} with components indexed by i . We similarly define the current hidden layer state as vector \mathbf{s} with components indexed by j , the previous hidden layer state as vector $\mathbf{s}(t-1)$ with components indexed by h and the output layer as vector \mathbf{y} with components indexed by k . The weight matrices are written as bold uppercase letters where \mathbf{V} maps \mathbf{x} to the current state $\mathbf{s}(t)$, \mathbf{U} maps the previous hidden state $\mathbf{s}(t-1)$ and \mathbf{W} transforms the current state $\mathbf{s}(t)$ to the output layer \mathbf{y} . For clearness, the previously defined symbols are demonstrated in fig. 2. Furthermore we describe the relation between outputs and net input function net_j or net_k of a layer as the activation functions $f(net_j(t))$ and $g(net_k(t))$ respectively for the hidden and output layer. Including biases b_j and b_k for the net input functions, we conclude, that the hidden state becomes (1) and finally the output is described by (3).

$$s_j(t) = f(net_j(t)) \quad (1)$$

$$net_j(t) = \sum_i^l x_i(t)v_{ji} + \sum_h^m s_h(t-1)u_{jh} + b_j \quad (2)$$

$$y_k(t) = g(net_k(t)) \quad (3)$$

$$net_k(t) = \sum_j^m s_j(t)w_{kj} + b_k \quad (4)$$

In order to minimize the loss of our network, we define the widely used summed squared error (5) as a loss function with desired outputs d , total number of training samples n and the number of output units o . For computational reasons, the stochastic version of gradient descent utilizes only subsets of the total training collection, up to the extremum where $p = n$.

$$E = \frac{1}{2} \sum_p^n \sum_k^o (d_{pk} - y_{pk})^2 \quad (5)$$

By propagating this error backwards throughout the network, we can construct the loss function's derivative with respect to each weight in order to obtain the weight update

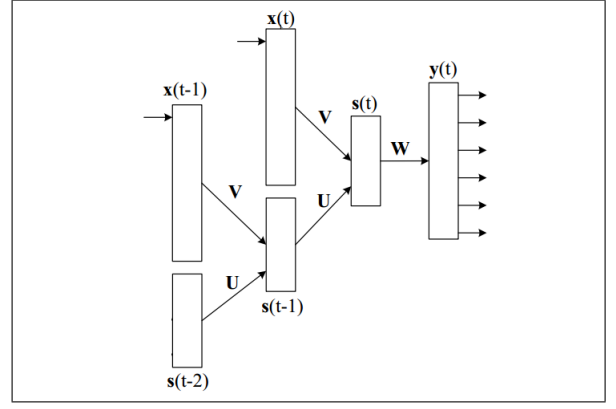


Fig. 2. An Unrolled RNN with shared weight Matrices \mathbf{U} and \mathbf{V} between temporal states [31]

of a specific weight for the next time step. After application of the chain rule, the weight updates for connections between hidden and output layer Δw_{kj} result in (6) and for connections between input and hidden layer Δw_{ji} become (7):

$$\Delta w_{kj} = \eta \sum_p^n \delta_{pk} s_{pj} = \eta \sum_p^n (d_{pk} - y_{pk}) g'(net_{pk}) s_{pj} \quad (6)$$

$$\Delta w_{ji} = \eta \sum_p^n \delta_{pj} x_{pi} = \eta \sum_p^n \sum_k^o \delta_{pk} w_{kj} f'(net_{pj}) x_{pi} \quad (7)$$

The terms δ_{pk} and δ_{pj} are referred to as the components of the specific error vectors per layer. The net function of our hidden layer net_{pj} however still depends on the previous state s_{t-1} which in turn depends on previous states itself. Unfolding the network up to a temporal depth τ with the same weights throughout the time steps (fig. 2) allows us to define an error vector for previous time steps as seen in (8).

$$\delta_{pj}(t-1) = \sum_h^m \delta_{ph}(t) u_{hj} f'(s_{pj}(t-1)) \quad (8)$$

More detailed reflections on the construction of the equations above can be found in [31]. The procedure previously used to obtain $\delta_{pj}(t-1)$ allows for theoretically arbitrary depths τ to be recursively calculated. In practice, large values for τ and the subsequently generated long-term dependencies tend to be difficult to handle with gradient descent. This is due to the fact, that deriving error terms through multiple layers of an unfolded RNN decreases or increases the magnitude of the resulting derivative exponentially, thus creating phenomena known as vanishing and exploding gradient.

B. Long Short Term Memory (LSTM)

These problems occur only when facing long range dependencies in RNNs and can be successfully tackled by adopting the Long Short Term Memory Network(LSTM) by Hochreiter and Schmidhuber [34]. As shown by Hochreiter, a key idea towards avoiding the explosion or vanishing of gradients through a large number of time steps is to enforce

a constant error flow through a single unit j by requiring the net activation derivative of j to follow equation 9 with the weight w_{jj} of a single connection to itself.

$$f'_j(\text{net}_j(t))w_{jj} = 1.0 \quad (9)$$

Simple integration over time gives us $f_j(\text{net}_j(t)) = \frac{\text{net}_j}{w_{jj}}$, a linear function. Therefore our recurrent definition of $\text{net}_j(t+1) = w_{jj}y^j(t)$ leads us to the conclusion, that the unit j 's activation has to remain constant (10) [34]:

$$y_j(t+1) = f_j(\text{net}(t+1)) = f_j(w_{jj}y^j(t)) = y^j(t) \quad (10)$$

This approach alone however leads to a problem, that becomes evident during learning in networks with more than one neuron connected to the so far single unit j . The input weights w_{ji} from neuron i and the outgoing weights w_{kj} to neuron k now are both responsible for regulating the desired protection from yet unneeded information of previous time steps as well as conceiving said information when deemed necessary. Take for instance a language processing network trying to estimate the importance of the subject "You" regarding the following word sequence "are a tall, friendly, level-headed, smart person". The same weights, that have just been trained on this sentence would receive conflicting weight updates in the next learning step, when confronted with the new sentence "You are a person". To overcome this shortcoming, Hochreiter and Schmidhuber propose a more context sensitive approach, using separate gates for forgetting, input and output regulation within a single cell as seen in fig. 3.

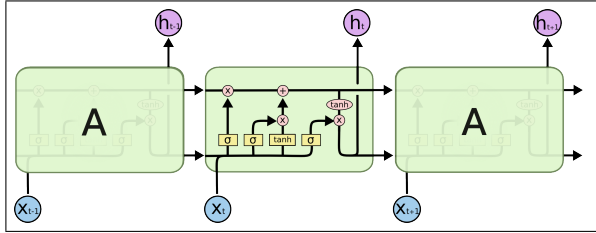


Fig. 3. Inner composition of an LSTM cell [2]

The *forget gate* consists of a sigmoid-activated layer f_t , essentially determining, how much of the previous output h_{t-1} should be kept within the LSTM Cell State C_t , based on its own set of weights W_f and biases b_f :

$$f_t = \sigma(W_f \times [h_{t-1}, x_t] + b_f) \quad (11)$$

The *input gate* is composed of the sigmoid-activated layer i_t and the hyperbolic-tangent-activated layer \tilde{C}_t , creating candidates for replacing the current (possibly forgotten) cell state C_{t-1} with $i_t \times \tilde{C}_t$. Each of these layers, again, have their own set of weights and biases W_i, b_i, W_c and b_c , leaving us with the respective layer outputs (12) and (13) [2]:

$$i_t = \sigma(W_i \times [h_{t-1}, x_t] + b_i) \quad (12)$$

$$\tilde{C}_t = \tanh(W_c \times [h_{t-1}, x_t] + b_c) \quad (13)$$

In order to determine whether to inhibit possibly disturbing signals to following cell states, the output h_t is filtered by

applying $\tanh(x)$ and multiplying with the outcome of the *output gate layer* o_t . The output layer comprises weights W_o and biases b_o and therefore results in the total output h_t :

$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o) \quad (14)$$

$$h_t = o_t \times \tanh(C_t) = o_t \times \tanh(f_t \times C_{t-1} + i_t \times \tilde{C}_t) \quad (15)$$

This design allows for separate read and write capabilities and therefore enables the capture of error signals within the cell for longer periods of time than previously possible with standard RNNs [34]. Many applications with record-breaking performances followed, thus constituting the leading role of LSTMs in temporal problems. Graves and Jaitly [28] provide an overview over variations of the vanilla LSTM, such as the introduction of peepholes by Gers and Schmidhuber [25] or the lighter Gated Recurrent Unit (GRU) by Cho et al.[15]. Furthermore, attention-based models as proposed by Graves et al. [29][27] and Neelakantan et al.[50] are promising more recent developments in the field of RNNs.

III. SPIKING NEURAL NETWORKS (SNN)

The above described ANN models differ from Feedforward Networks in regards of activation, connection design, learning rules and more, yet implicitly share a common-ground in how they encode information in their neurons. These second generation neural networks, as commonly described in the literature, activate on continuous functions and make use of their differentiability in backpropagation. Maass et al. [42] provide a setup, that integrates a different, biologically more accurate take on modeling neuron activation. These Spiking Neural Networks (SNN) employ integrate-and-fire neurons [41] which encode information with an additional temporal factor in its activation pulses, increasing the density of encoded information.

A. Neurons - Activation and Signal Processing

The fundamental idea behind the computational units of an SNN builds on integrating a temporal factor in the representation of information. Various models of these spiking neurons, such as the integrate-and-fire model [4], the Hodgkin-Huxley model [35], the model by Izhikevich [36] and the Spike Response Model by Gerstner [26] exist and vary in their attempt to trade off biological accuracy and computational complexity [30]. The Leaky-Integrate-and-Fire model is arguably the most widespread approach due to its simplicity and computational advantages. The representation of the activation process of the neuron is modeled by an electrical circuit in which the membrane potential, threshold voltage, resting potential and leak rate are realized through a capacitor, gate, battery and resistance [4][55].

B. Spike-based Neural Codes

Whilst encoding and decoding of the desired information is rather straight forward in second generation neural network models, this challenge proves harder for the time-dependent neurons in an SNN, due to the arbitrary number of theoretically possible ways of encoding information in the neurons.

In fact, the biological process of information decoding is still being researched, whereas various methods have been discussed in Neuroscience and Machine Learning:

- Rate Coding is an approach aiming at recording spike rates during fixed time frames. This implementation of spike encoding can be seen as an analog way of interpreting spike trains in SNNs.
- Latency Coding encodes spikes based on their timing rather than their multiplicity. This encoding has for example been used in unsupervised learning, and supervised learning methods like SpikeProp [11]
- Fully temporal codes are a more general term, which includes the above mentioned approaches. It encodes information based on the precise timing of each spike in a spike train[30]
- Gaussian Coding applies a gaussian distribution over recorded spikes of each neuron and encodes information based on their stochastic occurrence.

C. Learning in Spiking Neural Networks - Synaptic Plasticity

While conventional neural networks employ a stochastic version of gradient descent to backpropagate errors throughout the network, the same approach is difficult to apply in the realm of SNNs due to their temporal dependencies and the non-differentiability of spike trains. Whereas multiple learning rules addressing SNNs exist (such as Hebbian Rule, Binarization of ANNs, Conversion from ANNs and Variations of backpropagation [54]), a state-of-the-art algorithm such as backpropagation has yet to emerge. We will first focus on a more biologically motivated training rule called spike-timing-dependant plasticity (STDP). The key feature of this approach is to adjust weights between a pre- and post-synaptic neuron according to their relative spike times within an interval of roughly tens of milliseconds in length [11]. If a postsynaptic neuron fires shortly after its presynaptic neuron, the connecting weights is strengthened whereas presynaptic neurons firing after the postsynaptic neuron will lead to weakening of the weights. The experimentally refined and commonly used formula according to dan et al. [17] for the exact weight updates is given in (16)

$$\Delta w = \begin{cases} Ae^{\frac{-(t_{pre} - t_{post})}{\tau}} & t_{pre} - t_{post} \leq 0, A > 0 \\ Be^{\frac{-(t_{post} - t_{pre})}{\tau}} & t_{pre} - t_{post} > 0, B < 0 \end{cases} \quad (16)$$

where δw is the update of the weight w with adjustable learning rates A, B and pre/postsynaptic fire times t_{pre}/t_{post} . A related rule to STDP is the more general hebbian learning rule, which in contrast to STDP claims, that synaptic efficacy arises from the general temporal proximity of these signals independent from the order of occurrence. This rule is often referred to as "fire-together-wire-together". Both mentioned approaches are illustrated in fig. 4.

The above mentioned learning rules share the common characteristic in which they don't require information other than what is available in a local neighborhood of neurons. Therefore they do not require the biologically unrealistic transport of weight information throughout numerous layers

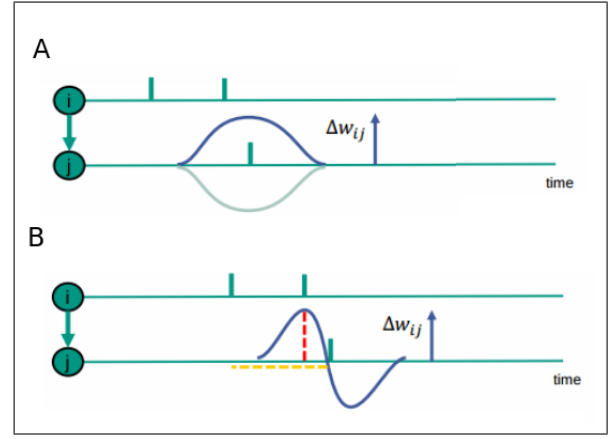


Fig. 4. **A:** illustration of weight-updates Δw_{ij} with presynaptic neuron i and postsynaptic neuron j according to STDP **B:** symmetric hebbian learning rule with weight update Δw_{ij} , presynaptic neuron i and postsynaptic neuron j

of neurons [57] [14] [16][19]. Consequently, these learning rules are suited for unsupervised learning tasks and pose interesting insights from a neuroscientific standpoint. Guyonnet et al. [46] [62] showed, that STDP-equipped SNNs are capable of learning input patterns and decrease latencies between in- and output throughout training. Furthermore, T. Masquelier [46] and S. R. Kheradpisheh [62] point out, that the increased density of encoded information in SNNs allows even a single neuron to learn spatio-temporal patterns. However practical usability of these learning rules raises the need for proper supervised learning algorithms, often clashing with mathematical feasibility, numerical efficiency or biological plausibility. In this, we encounter problems, such as the non differentiability of spikes and the biological absence of outer error signals in deeper layers. These challenges remain generally unsolved, however various promising advances have been proposed in tackling them. In a rather biological aspect, Markov et al.[45] propose the existence of feedback connections, designed to project information within hierarchically organized networks. Approaches such as SpikeProp by Bohte and Kok deal with discontinuous nature of spiking neurons by linearizing the relationship between post-synaptic input and the resulting spiking time, consequently circumventing the discontinuity of the thresholding function [11]. Further techniques aiming at surrogating the gradient of discontinuous activations have been researched and implemented by Neftci et al. [51].

D. Eligibility Traces

When performing on tasks, that last between few seconds to multiple minutes until an output of an ANN can be evaluated, it can be difficult to assign blame or reward for the achieved outcome to particular spikes, which usually happen within the range of few milliseconds. Accordingly, the field of Reinforcement Learning and Neuroscience provide an approach to close this temporal gap by introducing a dynamic variable in synapses henceforth called eligibility trace [60]. Eligibility traces record recent synaptic activity and

enable better assignment of error signals to culprit neurons. Neuroscience suggests, that eligibility traces may be a key feature to neuromodulatory reward based learning such as the midbrain dopaminergic system [53]. Updates to the eligibility trace are described by the eligibility function $f_c(t)$ in close accordance to STDP spike patterns of pre-/postsynaptic and post-/presynaptic spike pairs. Respectively, a pair of spikes will increase or decrease the eligibility trace $c_{ji}(t)$ of the synapse over time (fig. 5 A). Reward signals $d(t)$ received after a time delay can now be assigned to synapses using a learning rule, that changes synaptic weights proportionally to the product of $c_{ji}(t)$ and $d(t)$, (17).

$$\frac{d}{dt} w_{ji}(t) = c_{ji}(t) d(t) \quad (17)$$

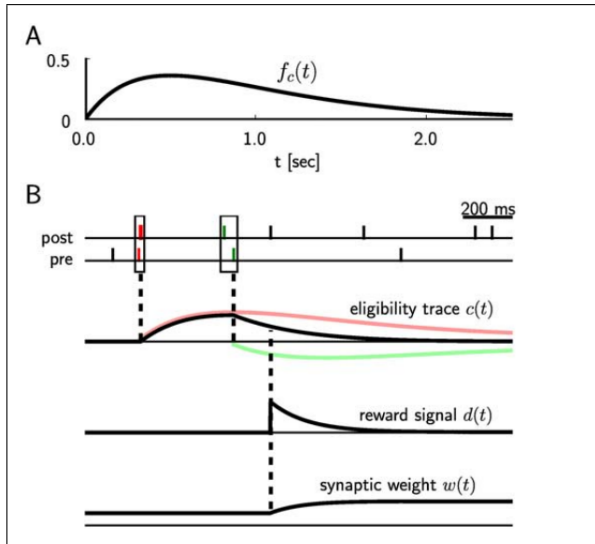


Fig. 5. **A:** Eligibility function $f_c(t)$. Illustrated are values for a pre-/postsynaptic spike pattern. Negative for post-/presynaptic spike patterns. **B:** Contributions of spike patterns (red and green) to an eligibility trace $c(t)$ and the according weight update $w(t)$ upon reception of a reward $d(t)$. [40]

IV. LEARNING TO LEARN (L2L)

The field of reinforcement learning (RL) has recently celebrated great success at reaching human-like and even surpassing human abilities on complex environments such as Atari [48] and Go [61] with the implementation of Deep Neural Networks to account for non-linear function approximation over high-dimensional action and state spaces. However Artificial Intelligence in general [39] and Reinforcement Learning in particular [23] currently suffer from two major drawbacks, that are limiting their application and design [65]:

- Firstly the immense volume of required training data and the relatively expensive generation of this data in often simulated environments.
- Secondly RL-algorithms often have to be heavily tailored to a specific range of tasks and various algorithms, each of which depending on numerous hyperparameters and thus requiring immense computational efforts.

Botvinick et al. [13] explain these weak spots in AI with a need for low learning rates and the bias-variance trade-off. Low learning rates are necessary to prevent both catastrophic interference (discarding previously reached successful configurations) and overfitting [32]. The bias-variance trade-off describes the contrary working directions of efficiency-driving biases or priors and performing on a wider range of tasks.

Learning to Learn (L2L) addresses these issues, by mimicking human learning behaviour to extract abstract information on wide families of tasks, thus reducing the required training data for any more specific subtask. Landsell and Kording [39] argue, that these L2L approaches can be categorized into either Learning to Optimize or Structure Learning. Learning to Optimize focusses on the general adaption of network parameters to achieve efficient learning rules on arbitrary Task classes without hand-selection. Similarly to the way gradient descent applies small changes of the weights in an NN in order to minimize loss functions, the design of AI systems can be viewed as an optimization problem itself, that requires parameter optimization to ensure a well performing algorithm. Structure Learning on the other hand makes use of structural similarities within a finite family of tasks to reach higher data efficiency due to its prior adaptedness to the given family of tasks [39].

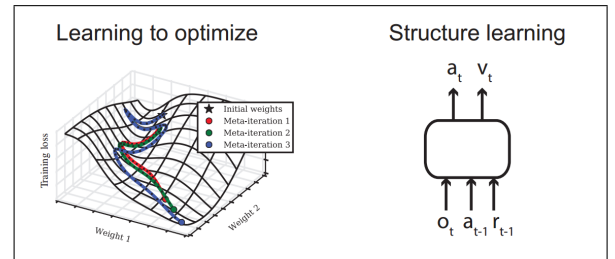


Fig. 6. Types of learning-to-learn in AI. Learning-to-learn can be roughly divided into learning to optimize and structure learning. In AI, hyperparameter optimization is an example of learning to optimize [43], while a recurrent neural network taking rewards, actions and observations can often be used to perform structure learning [65] [39].

A. Learning to Reinforcement Learn (meta-RL) and RL

A high level architecture consisting of a learner (performing on the task itself) and a meta-learner (adjusting the learner) is inherent to most implementations of L2L [39] and has been refined in various ways to create new L2L Systems, as will be explained in the following section.

Wang et al. [65] as well as Duan et al. [23] introduced frameworks, that can be thought of as generating an RL algorithm of their own and provide agents, who are given a predesigned prior to efficiently learn any task $T \in \mathcal{F}$ (in the original papers denoted as a Markov Decision Process (MDP) $m \in \mathcal{M}$) from a family of interrelated tasks \mathcal{F} (i.e. a set of MDPs \mathcal{M}).

In their attempt to design an algorithm, capable of performing well on a set \mathcal{M} of Markov Decision Processes

(MDPs), Duan et al. implement a nested system in which learning an RL algorithm is regarded as a reinforcement learning problem itself, hence the name RL [23]. The agent performing on a randomly drawn separate MDP $m \in M$ from the distribution $\rho_M : M \rightarrow R_+$ is represented as an RNN, which outputs the probability distribution over the tasks action-space π (policy) based on a function $\phi(s, a, r, d)$ of the tuple (state, action, reward, termination flag) (Duan et al.). On a higher abstraction layer, this RNN is being optimized by an implementation of Trust Region Policy Optimization (TRPO), a state-of-the-art DRL algorithm [58] with several advantages regarding stability and hyperparameter dependence.

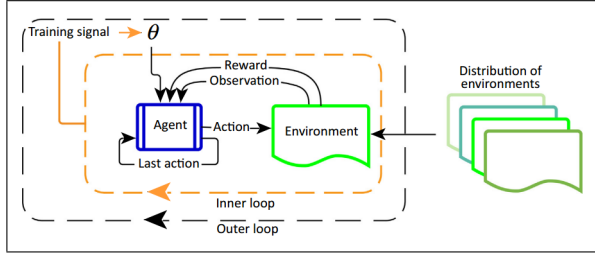


Fig. 7. Schematic of Meta-reinforcement Learning, Illustrating the Inner and Outer Loops of Training. The outer loop trains the parameter weights θ , which determine the inner-loop learner (Agent, instantiated by a recurrent neural network), that interacts with an environment for the duration of the episode. For every cycle of the outer loop, a new environment is sampled from a distribution of environments, which share some common structure [13].

Wang et al.[65] define a similar setup in which a RL-algorithm is responsible for learning the weights of a nested RNN. Both, inner and outer loop in this framework draw their learning experience from the reward information generated by the actions of the RNN, where the RNN holds information on the previously chosen action and the subsequent rewards. However the process of learning in each of these loops is realized differently and results in specializations of different scopes. While the wrapping RL-algorithm used to optimize the weights of the RNN operates over the entire set of episodes, that is to say all MDPs M , learning of the nested RNN within a single task m is based on the inner recurrent dynamics of the network. Notably, the RNN is able to encode learned experience on a specific task using only its inner memory variables, leaving us with the observation, that a well working short-term memory is a key factor to learning on different levels of abstraction. The policy outputs π of this network can then be viewed as an RL-algorithm on its own, resulting in the name meta-RL. For the implementation of this framework Wang et al. [65] used an LSTM according to Hochreiter and Schmidhuber [34] to account for the inner RNN, while both synchronous asynchronous advantage actor critics (A2C and A3C) (Mnih et al.) were employed to learn inter-cell weights. The observation vectors of experiment environments were either directly fed to the LSTM one-hot-encoded or passed through an additional deep encoder model[65]. Experiments on a

series of bandit problem and two MDP-centered problems with implementation architectures as described above showed, that meta-RL delivers competitive results compared to problem-specific algorithms (Thompson sampling, UCB, Gittins) while operating on a wider set of tasks.

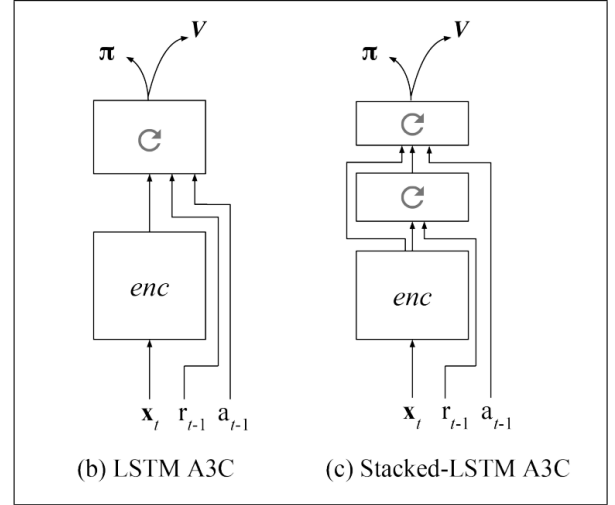


Fig. 8. **b:** Convolutional-LSTM architecture with autoencoder and A3C **c:** Stacked-LSTM architecture with additional recurrent network. Rewards and actions are given to different stack RNNs [65]

A notable characteristic of both previously described setups is, that the learning rate of the nested RNN is chosen lower compared to the outer optimization loop, consequently preventing the agent from overfitting to a single task m , yet gathering knowledge from the entire MDP space M [13].

B. L2L in the Context of Spiking Neural Networks

When applied to Networks of Spiking Neurons, the previously described framework of L2L offers a promising alternative to common approaches, which embed a series of biologically implausible ingredients. One major biological implausibility in learning algorithms employing error feedback is suggested by evidence, that real neurons almost certainly lack weight transport when unrolling the chain rule of backpropagation on the backward pass[57] [14][16]. In order to calculate gradients of an error at the outputs of a network, these neurons would have to maintain information about the strength of synapses outside their local neighborhood, which seems unrealistic from a biological point of view. This evidence suggests the existence of a more sophisticated system of learning signals, which don't require information on global synaptic weights. Furthermore, oversimplistic models of real neurons, such as the Leaky integrate-and-fire neuron, lack key characteristics of real neurons and are often even further modified by engaging derivative surrogate functions in order to deal with the non-differentiable nature of spiking neurons. Among these mismatches between biological neurons and their artificial models is the ability of biological neurons, to adapt to previously experienced inputs and weights [57]. The ability to adapt to previous spikes of presynaptic neurons

however not merely represents a visual difference but can be interpreted as the ability to store temporal information. By deploying adaptive LIF neurons in a Network of Spiking Neurons, Bellec et al.[6] were able to overcome not only a discrepancy between model and reality but also achieved a well working long short-term memory, thus creating a new type of RNNs, the LSNN. Here, the adaptivity of neurons is realized by increasing the firing threshold $B_j(t)$ of a neuron j by a fixed amount $\frac{\beta}{\tau_{a,j}}$ for incoming spike trains $z_j(t)$ and decaying it exponentially to a baseline b_j^0 value in spike-free intervals with time constant $\tau_{a,j}$. The time constant can be chosen to fit the desired range of the created short-term memory. For discrete timesteps of $\delta t = 1ms$ the dynamics of the adaptive threshold becomes (18) with the recursive update rule (19):

$$B_j(t) = b_j^0 + \beta b_j(t) \quad (18)$$

$$b_j(t + \delta t) = e^{-\frac{\delta t}{\tau_{a,j}}} b_j(t) + (1 - e^{-\frac{\delta t}{\tau_{a,j}}}) z_j(t) \quad (19)$$

As pointed out in the meta-RL, a well-working short-term memory is crucial to the ability to store learned experience on different levels of abstraction and now allows for the application of L2L to a network of spiking neurons. On a side note, the increase of the firing threshold in the adaptive neurons clearly leads to an overall decrease of spikes, thus operating more energy efficiently when deployed on neuromorphic hardware. The network architecture comprises additional sets of non-adaptive excitatory and inhibitory LIF neurons [6](fig. 9). Similarly to Wang et al.[65], the synaptic weights of this network are subject to optimization of an outer loop algorithm, in this case a combination of BPTT and a biologically inspired rewiring method called DEEP R [5](fig. 9). Since BPTT requires calculation of error gradients, the authors utilized a pseudo-derivate to surrogate the non-existing derivative of neuronal spike trains with a simple damped triangular function. The synaptic weights are updated only after completed runs of tasks, while in-task optimization is realized through internal memory dynamics according to (17) and (18). Further details on the network setup and implementation can be reviewed in [6].

The results of this model might already offer interesting perspectives from a biological and neuroscientific view, however one is tempted to wonder if this framework can be applied to more biologically plausible models. Recall, that the previously described setup including BPTT still requires the transport of global weight information as well as a transmission of error signals through time, which are widely considered implausible in real RNNs. Bellec et al.[8] consequently developed an approach designed to tackle these shortcomings. The chapter *eligibility traces* already hinted at the possibility of avoiding the need for weight transport in credit assignment by conducting error propagation through dynamic eligibility traces. Bellec et al. take up the learning rule (17) with a different motivation, namely the approximation of error gradients. The fundamental role of eligibility traces in this algorithm led to the name e-Prop. The underlying

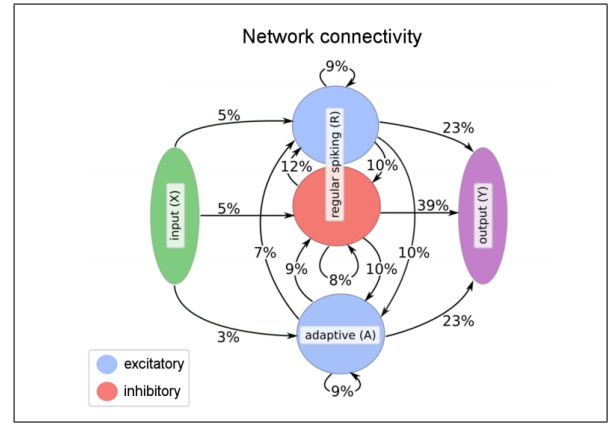


Fig. 9. Basic architecture of LSNN. Percentages on synaptic connections represent connectivity of the LSNN after applying DEEP R in conjunction with BPTT. Populations X,Y and R of regular spiking LIF neurons coupled with a population A of adaptive LIF neurons. [6]

basis for e-Prop is the assumption, that error gradients w.r.t. a synaptic weight θ_{ji} can be factorized into a learning signal L_j^t and its eligibility trace e_{ji}^t , summed up over time[8]:

$$\frac{dE}{d\theta_{ji}} = \sum_t L_j^t e_{ji}^t \quad (20)$$

Bellec et al.'s claim for biological plausibility is coupled to obtaining the learning signal L_j^t in a way, that does not require a propagation of error gradients through time or numerous layers (fig. 10). In the following we will focus on two different approximations for these learning signals \hat{L}_{j1}^t and \hat{L}_{j2}^t constituting two versions e-Prop 1 and e-Prop 2.

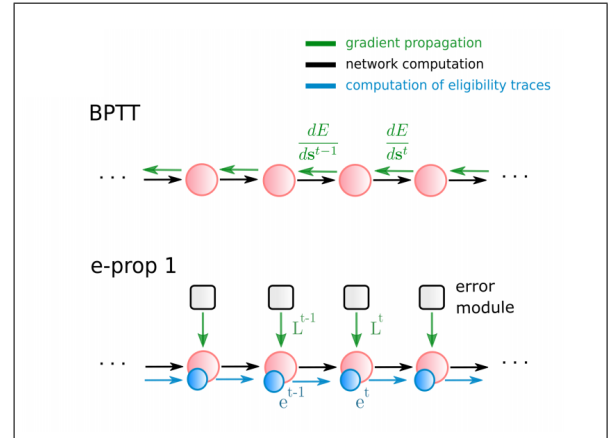


Fig. 10. Illustration of the error signal flow through network layers. Evidently, e-prop 1 does not require information transport in a backwards pass. (edited from [8])

E-Prop 1 uses the findings described in *feedback alignment* to broadcast error signals directly to the affected synaptic weights while adopting a random weight matrix instead of the actual weights. Assuming an error at the output layer k of leaky readout neurons y_k^t of $((y_k^{*,t} - y_k^t))$ our Learning signal to a neuron

θ_{ji}^{rec} thus becomes $\sum_k B_{jk}^{random}(y_k^{*,t} - y_k^t)$ instead of $\sum_k \theta_{kj}^{out}(y_k^{*,t} - y_k^t)$. Since this network is an RSNN, these learning signals are broadcasted to all synaptic weights in different time slices of an unrolled recurrent network. The authors however reached better results when choosing the random weight projection matrix to be constant over all time-slices.

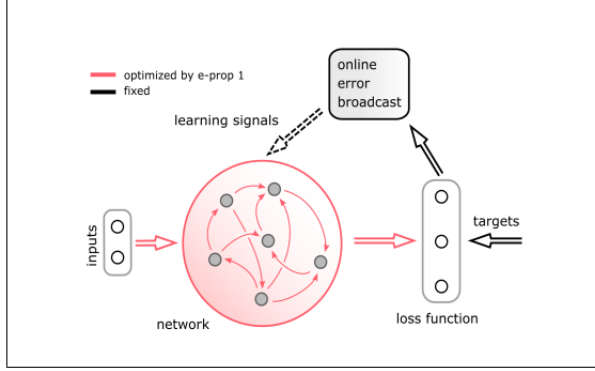


Fig. 11. Basic architecture of e-prop 1. Error broadcasts are calculated online at the output layer and sent directly to deeper layers of the RSNN, avoiding the necessity for weight transport on the backwards pass.(edited from [8])

This approach is applicable to architectures with short-term memory, such as LSNNs, thus providing a basis for L2L frameworks, where learning on different levels could be allocated to the outer optimization through e-prop 1 and inner adaptive memory variables.

However Bellec et al. developed a second version of e-Prop, specifically dedicated to the utilization of L2L, called e-Prop 2. Neuroscience has long considered the existence of a sophisticated system of error feedback generation, where a sudden sharp negative potential in the brain called error-related negativity (ERN) is suspected to play a role in reflecting such error signals. ERN accompanies motoric misbehaviour responses and typically peaks within 80ms - 120ms after beginning of an error signal [24][20]. However further research remarkably points out that ERN can be observed even before motoric feedback becomes evident through sensory feedback, implying the existence of additional error response- and prediction systems embedded in biological neural networks, [44] which may well be a result of evolutionary development. Motivated by this finding, Bellec et al.[8] engaged an additional RSNN solely designed to generate optimized learning signals in e-Prop 2's architecture. These error-modules now take over the role of an outer-loop meta learner as mentioned in the chapter "meta-RL". Learning of the error module is again accomplished by training on a whole family of tasks F with BPTT. Conceivably, the application of BPTT contradicts the claim for biological plausibility, howbeit the authors argue, that the origin of well-developed systems of error-prediction and signal-generation are not fully understood but do not require biologically plausible optimization algorithms if interpreted as priors formed by a long process of evolution. The error module contains weights Ψ and receives the inputs

\mathbf{x}^t and a target vector $\mathbf{y}^{*,t}$, which may be the target output of the network or a desired target state (e.g. a target vector may be the position of a robotic manipulator at time t , while the network output are joint velocities) [8]. A distinct difference between this setup and meta-RL or L2L LSNNs lies within the split network neurons of outer meta-learner and inner learner. Since the outer error modules now possess their own set of neurons with weights Ψ , the inner learner can store learned experience in its own set of synaptic weights θ , implicitly alleviating the categorical need for short-term memory.

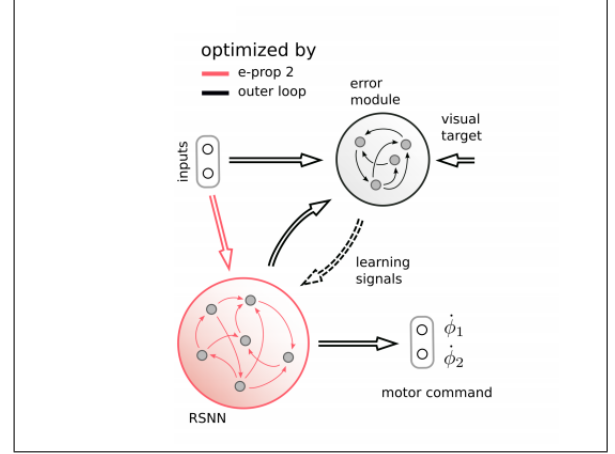


Fig. 12. Basic architecture of e-prop 2. An Error module network receives network inputs, a target state and the network state to produce learning signals optimized to improve learning in a nested RSNN (edited from [8]).

Training of this model is split into a training phase and a testing phase both consisting of exactly one trial. The initial output of the inner learner considering a specific task C therefore depends on its initial set of weights θ_{init} . As e-Prop 2 optimizes θ_{init} after a single training trial using its own initial weights Ψ_{init} , the output on the test trial with the same specific task C is now a result of the improved priors $\theta_{test,C}$. A cost function for BPTT in the error module is created from the results of the test run $L_C(\theta_{test,C})$. The complete learning dynamics of e-Prop 2 are then described by the minimization problem (21) and the update rule (22):

$$\min\{\mathbf{E}_{C \sim \mathcal{C}}[L_C(\theta_{test,C})]\} \quad (21)$$

$$(\theta_{test,C})_{ji} = (\theta_{init})_{ji} - \eta \sum_t \hat{L}_j^t e_{ji}^t \quad (22)$$

Duan et al.[21] identified the outer-loop optimization algorithm as an imminent bottleneck to this approach, hence it comes as little surprise, that further approaches such as the work by Bohnstingl et al.[10] have aimed at designing better outer-loop algorithms. The unsolved status of L2L architectures inspired the authors to implement a modular setup in which the optimization algorithms for the outer loop as well as the inner learner can easily be exchanged, allowing a fast evaluation of various possible algorithms. In particular, gradient-free optimization algorithms were employed to generate well-performing hyperparameters of a

learner concerning a family of tasks. Here, Cross-Entropy-Method (CE) [12], Evolution Strategies(ES) [9] and Simulated Annealing(SA)[38] were applied to mimic evolutionary processes involved in the development of human learning. CE and ES are metaheuristic global optimization algorithms designed to regions of well-performing parameters, while SA generates a single set of parameters. Given their probabilistic nature, these algorithms tend to be computationally expensive and were thus deployed to neuromorphic hardware to ensure feasibility [10]. In comparison to e-Prop, Bohnstingl et al. arranged a setup in which a new learning rule itself is represented by a multilayer perceptron (MLP) and fitted by adjusting its weights and biases (fig. 13).

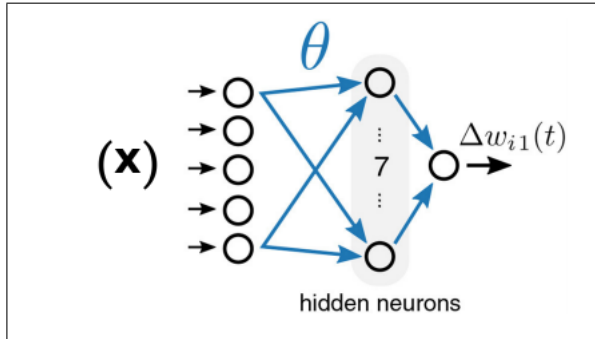


Fig. 13. MLP architecture of meta-plasticity with inputs (\mathbf{x}) , one hidden layer of seven neurons and parameters θ . The network outputs the weight updates for the agent-network. (edited from [10]).

The resulting learning rule $f_{ANN}(\mathbf{x}_{ji}(t); \theta)$ is called meta-plasticity and offers a new perspective on how synaptic plasticity in SNNs may be adjusted to boost performance on whole task families. Meta-plasticity, polished over various tasks of a family F , may enable an SNN-based learner to better perform on each of these tasks by adhering to the simple weight update rule (23):

$$\Delta w_{ji} = f_{ANN}(\mathbf{x}_{ji}(t); \theta) \quad (23)$$

Bohnstingl et al. conclude, that the algorithm applied in the outer loop of an L2L framework is crucial to its function and emphasize the stability offered by CE and ES due the generation of regions of well-working parameters as opposed to single values in SA. The perspective of declaring even plasticity rules subject to L2L optimization might offer new paths for research and eventually provide important insights into learning in biological networks of neurons.

V. APPLICATIONS OF L2L IN ROBOTICS

Robotics has undergone many successful developments in the recent past with advances being pushed from numerous fields of engineering, including that of machine learning. Yet the design process is still a tedious and highly tailored one, usually requiring domain expert knowledge. Many of the underlying algorithms in control, motion planning and sensoric interpretation require suitable setups of the environment with little room for variation. For example industrial manipulator robots can perform outstandingly when placed

in a fixed production line, yet recognizing and grasping everyday objects in a kitchen or workshop poses a much higher challenge, as it requires the skill to make sense of broad environments with numerous imaginable tasks. Furthermore the dominating problems of applying RL in robotics can be summarized by the following problem classes [56]:

- Sample efficiency
- Sim2Real
- Reward function optimization
- Safety

The previous sections revealed a high potential in L2L frameworks in terms of sample efficiency and generalization, thus constituting feasible answers to expensive data generation or overfitting to simulator-specific features. However this further implies a reflection on the scalability of said L2L approaches in order to evaluate their applicability in the often very high-dimensional task spaces faced in robotics.

Wang et al.[65] examine meta-RL's ability to detect abstract task structures in large scale problems by adapting a well-known behavioural experiment described by Harlow [33] to a visual fixation task. In Harlows experiment, monkeys were presented two unfamiliar objects, with one hiding a bowl filled with food and while the other holds an empty bowl. The monkeys were allowed to choose one of the objects and received the reward, if present. Despite switching the objects for new unknown objects in each episode, upon replaying several trials in several episodes of this game, the animals showed a general understanding of the underlying structure of the problem. After beginning a new episode with new objects, the monkeys would, inevitably, take one random guess but managed to succeed in the following trials of the episode [13].

A. Tasks in high-dimensional spaces

Motion and path planning are fundamental problems in robotics, whether it be within a space of rich visual input, sensory data or configuration/joint-spaces. Similiar to the problem described by Harlow, the navigational task in the I-maze environment as described by Mirowski et al.[47] and Jaderberg et al.[37] requires an understanding of the general structure of the problem in order to learn sample-efficiently on the specific task. In this case the same maze spawns a goal location on random position within the maze where the agent has to learn a motion path to the goal in as few trials as possible. The results of Wang et al.[65] show, that an architecture of stacked LSTM is able to solve the task after having conducted one exploration run (finishing the episode in 100 timesteps) notably faster (30 timesteps) within few explotation runs. The reference baseline, a feedforward architecture A3C learner, is not able to solve the problem at all.

Duan et al.[23] take a similiar approach in their evaluation of the feasibility of RL in high-dimensional state spaces. Again, a randomly generated maze with a randomly placed

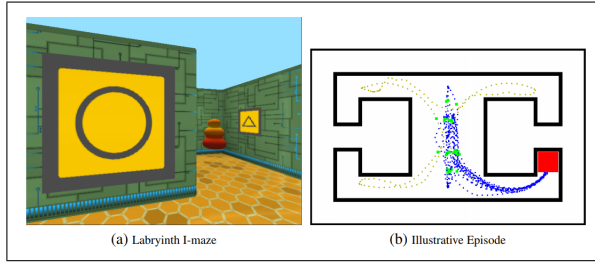


Fig. 14. **a:** I-maze labyrinth view with goal **b:** following initial exploration (light trajectories), agent repeatedly goes to goal (blue trajectories). [65]

target is chosen as the problem to solve for the agent. During one test run, the agent is given a number of episodes during which the maze structure and target position remain fixed. In contrast to an earlier approach to this RL-Task shown by Oh et al. [52] RL bases its actions within a more granular action space. The environments sparse reward payout design (+1 for target, -0.001 for wall hits, -0.04 per time frame) poses additional challenges to the agents learning and requires well-developed exploration strategies in the first episode in order to gain information on the problems ground structure. Cross-validation with a small and a larger version of the maze environment show a significant reduction in solving trajectory lengths between the first to episodes and indicate, that the RL algorithm managed to utilize previously gained information to come to good solutions more quickly. However the shown results are not yet optimal as the agent still forgets, though rarely, initially explored target positions and explores further paths in the second episode. Duan et al. indicate, that further improvements might come with improved RL-algorithms as the outer-loop optimizer (fig. 15).

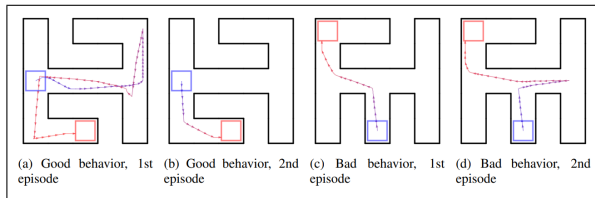


Fig. 15. Visualization of the agents behavior. In each scenario, the agent starts at the center of the blue block, and the goal is to reach anywhere in the red block [23].

Bellec et al.[6] tested an implementation of LSNN on a navigational task in a 2D arena in reinforcement learning to demonstrate applicability in complex environments. The LSNN-based agent was placed in a circular arena in which a specific task M would be to reach a goal within this arena on a fixed position, while the whole family F of tasks would include various goal positions within this arena. By setting these goal positions to be close to the arena boundaries, the agent is challenged to develop an abstract understanding of the commonalities between all tasks, which he can exploit when facing one of the tasks M . For each of these specific tasks, the objective is to reach the goal (at a fixed position throughout one task M) as many times as possible within a fixed time frame after being placed

randomly upon reaching the goal. The according sparse reward function was chosen to award goal attainment with a score of 1 while hitting the arena boundaries will lead to -0.02 punishment. According to the applied L2L framework, outer-loop optimization through BPTT and DEEP-R was performed on the synaptic weights over the whole task family F , while specific task optimization was accomplished by adapting the short-term memory, i.e. the thresholds of the adaptive LIF neurons. Remarkably this model was able to generate an abstract understanding of the characteristic of the task, exhibiting human-like strategies to first explore the boundaries of the given environment to find the goal position and utilizing this knowledge to efficiently reach the goal in subsequent runs(fig. 16).

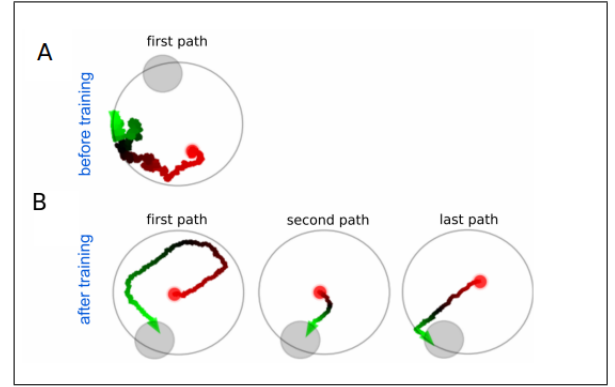


Fig. 16. **A:** The untrained model does not show any strategic or efficient way to find the goal. (Bellec et al.)

B: The trained model performs initial exploration runs alongside environment boundaries, exhibiting the abstract understanding, that all tasks of family F share the characteristic of goals close to the arena boundaries. Efficient path-planning of the agent in the subsequent runs prove, that the adaptive LIF neurons were capable of storing the position of the goal in short-term memory. [6]

B. Few-Shot Learning

One promising aspect of L2L is the capability of generalizing more than any conventional neural network. A model agile enough to understand underlying abstract knowledge about robotic motion does not only foreshadow better, more robust motoric control but especially a reduction in training data. Bellec et al.[8] evaluated e-Prop 2 on a one-shot learning task, where a trajectory of a 2-joint robotic arm should be learned by the agent. That is, the error modules of e-Prop 2 had previously been trained sufficiently on a family of robotic kinematic tasks to generate fitting learning signals \hat{L}_j^t and RSNN priors θ_{init} . The learning phase of the agent was restricted to a single training run to gain a strict evaluation on the capabilities of the L2L system. Interestingly, the agent was not given an inverse kinematic model, thus requiring the model to understand the space mismatch between the endeffector pose (network target) in euclidean space and the joint space (network output). A family of tasks F was generated with each task C representing a randomly generated target trajectory of the arm endeffector. The inner RSNN is a network composed of 400 recurrent LIF neurons which receives inputs \mathbf{x}_t and outputs

the required joint velocities to influencing the endeffector pose. An outer loop error module, consisting of 300 LIF neurons was previously trained with the inputs \mathbf{x}_t , spiking activity of the RSNN \mathbf{z}^t and the target trajectory $\mathbf{y}^{*,t}$ in Euclidean space. The optimization was conducted with BPTT over batches of 200 different tasks to minimize expectation of the loss sum across the family of tasks.

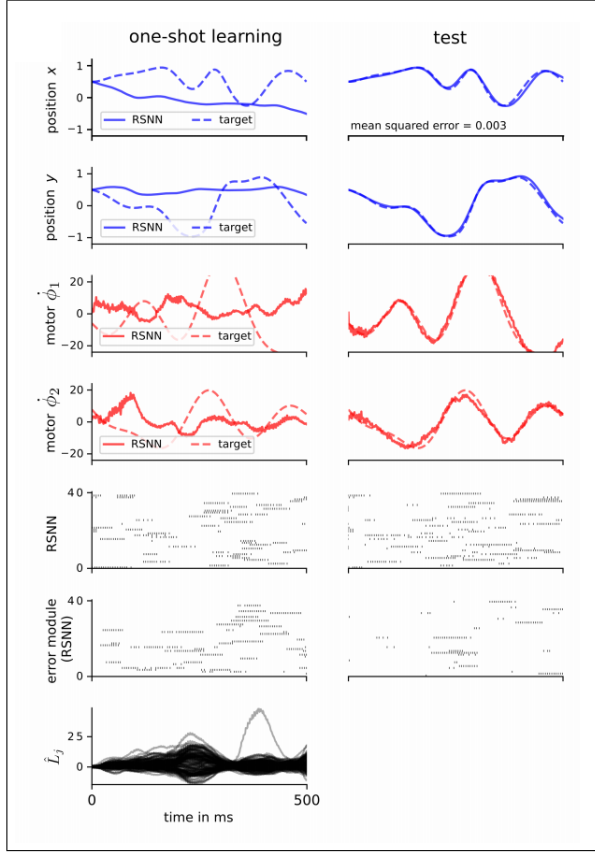


Fig. 17. Results of one-shot learning tasks before (left) and after (right) a single training trial. After a training run, the RSNN synaptic weights were updated using learning signal \hat{L}_j^t (bottom). In the fifth row, spikes in the error module have become sparse after the training run, which can be explained by the low error-rate generated by a learned RSNN [8]

Fig. 17 shows the result of the motor outputs by the RSNN after a single training trial, provided the error modules experienced sufficient training. In plots 1-4 it becomes apparent, that the first and only training trial was inevitably unsuccessful but generally sufficed to generate trajectories with a MSE of 0.003 after a single update of weights with Learning signals from an outer error module. We can conclude, that the network has achieved one-shot learning capability for the infinitely large family of random trajectories in a 2-joint robotic arm.

VI. DISCUSSION

The heretofore described framework of L2L and its various applications poses promising perspectives for future work in the fields of machine learning, robotics but also neuroscience and cognitive sciences. Historically, these fields have been intertwined and underwent phases

of more and phases of less interaction but nonetheless generated implications, that could be of scientific use for the other. Cognitive sciences provide ideas and inspirations for ever further sophisticated algorithms, while Artificial Neural Networks can offer evaluations of plausibility for biological models or in turn stimulate new implications for neuroscience on the basis of well-working algorithmic models. L2L is particularly interesting when considering the gap in learning efficiency between human brains and ANNs. The idea of certain preshaped priors innate to human brains is certainly not new to cognitive sciences and seem quite obvious when taking into account the remarkable learning efficiency all humans share. However conclusions on the origin and development of such preengineered priors remain vague and manifold. Wang et al.[64] propose, that midbrain dopamine-driven synaptic plasticity in the brain may well be an outer-loop optimization system shaping learning in the prefrontal cortex (PFC), which in turn acts as a hierarchically lower learner. The dopamine neurons of the midbrain are thought to employ reward prediction error signals, which shape synaptic connectivity in such a way, that animals are prone to improve performance [49]. Despite this, it is up to debate, to which extent these signals contain information, based on a environment-dependent model (model-based) or occur model-free. For example, Daw et al.[18] discovered evidence, that signals strongly correlated to dopaminergic prediction errors contain robust model-based information [13][18]. In this regard, Wang et al.[64] point out, that a meta-RL system trained with a model-free RL algorithm can generate model-based like behaviour in the inner loop algorithm. They suggest, that very similarly, human brains may actually operate in a grey area between model-free and model-based RL and do so in coordinance with the uniformity of environmental structures.

Bellec et al.[6] yield a different interpretation of the nested nature of L2L and argue, that sophisticated priors and learning algorithms have been shaped by long evolutionary processes. In fact this perspective alleviates the imminent need for biologically implausible processes such as the backpropagation of error-gradient learning signals through multiple layers and even time. By optimizing directly broadcasted learning signals coupled with eligibility traces of spiking neurons, Bellec et al. were the first to reproduce meta-learning in networks of spiking neurons and provide promising results, albeit not yet surpassing the performance of BPTT-based ANNs.

On another note, the implementation and performance of SNNs is strongly linked to the development of fitting neuromorphic hardware and appropriate algorithms. Taking into consideration the analogy of evolution for L2L, it should come with little surprise, that the outer loop optimization of L2L is considered a crucial bottleneck in terms of capability and computational efforts [22]. In the light of energy and computation efficiency, LSNNs exhibit the remarkable characteristic of sparse firing activity and

imply further capabilities in activity-silent memory, much like human brains.

Applications of L2L such as the experiments described in previous chapters point at the opportunities of this approach but to this date remain proof-of-concepts. Duan et al.[23] hold upscaling of L2L for broader distributions of tasks to be the most important next step in this approach. It is yet to be determined, how L2L models could be combined to generate a single system with dedicated learning modules to tackle applications which require vast ranges of skills. Large-scale projects such as the *Scalable Deep Reinforcement Learning for Robotic Manipulation* by Google Brain[1] are evidence of the imminent need for data efficient, generalizing learning algorithms, especially in the light of the immense efforts currently needed to apply deep learning algorithms in robotics. Simulation based projects like *Learning Dexterity* by OpenAI [3] are challenged by sim2real and again invest immense efforts in domain randomization approaches. A network architecture, that promises greater generalization ability could hold answers for better results in terms of stability and safety as well. Especially when deployed in large machines (e.g. industrial robotics, autonomous driving), Deep Learning is required to adhere to high safety standards with nearly impossible-to-get training data. L2L might therefore be part of some of the largest upcoming challenges faced in this field.

REFERENCES

- [1] Scalable Deep Reinforcement Learning for Robotic Manipulation.
- [2] Understanding LSTM Networks – colah’s blog. <http://colah.github.io/posts/2015-08-Understanding-LSTMs/>.
- [3] Learning Dexterity. <https://openai.com/blog/learning-dexterity/>, July 2018.
- [4] L. F Abbott. Lapique’s introduction of the integrate-and-fire model neuron (1907). *Brain Research Bulletin*, 50(5):303–304, November 1999.
- [5] Guillaume Bellec, David Kappel, Wolfgang Maass, and Robert Legenstein. Deep Rewiring: Training very sparse deep networks. *arXiv:1711.05136 [cs, stat]*, November 2017.
- [6] Guillaume Bellec, Darjan Salaj, Anand Subramoney, Robert Legenstein, and Wolfgang Maass. Long short-term memory and Learning-to-learn in networks of spiking neurons. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, *Advances in Neural Information Processing Systems 31*, pages 787–797. Curran Associates, Inc., 2018.
- [7] Guillaume Bellec, Franz Scherr, Elias Hajek, Darjan Salaj, Robert Legenstein, and Wolfgang Maass. Biologically inspired alternatives to backpropagation through time for learning in recurrent neural nets. page 37.
- [8] Guillaume Bellec, Franz Scherr, Elias Hajek, Darjan Salaj, Robert Legenstein, and Wolfgang Maass. Biologically inspired alternatives to backpropagation through time for learning in recurrent neural nets. *arXiv:1901.09049 [cs]*, January 2019.
- [9] Hans-Georg Beyer and Hans-Paul Schwefel. Evolution strategies - A comprehensive introduction. *Natural Computing*, 1:3–52, March 2002.
- [10] Thomas Bohnstingl, Franz Scherr, Christian Pehle, Karlheinz Meier, and Wolfgang Maass. Neuromorphic Hardware Learns to Learn. *Frontiers in Neuroscience*, 13, May 2019.
- [11] Sander M Bohte and Joost N Kok. SpikeProp: Backpropagation for Networks of Spiking Neurons. page 7.
- [12] Zdravko I. Botev, Dirk P. Kroese, Reuven Y. Rubinstein, and Pierre L’Ecuyer. The Cross-Entropy Method for Optimization. In *Handbook of Statistics*, volume 31, pages 35–59. Elsevier, 2013.
- [13] Matthew Botvinick, Sam Ritter, Jane X. Wang, Zeb Kurth-Nelson, Charles Blundell, and Demis Hassabis. Reinforcement Learning, Fast and Slow. *Trends in Cognitive Sciences*, 23(5):408–422, May 2019.
- [14] Lakshminarayan V. Chintala and Douglas Blair Tweed. Adaptive Optimal Control Without Weight Transport. *Neural Computation*, 24:1487–1518, 2012.
- [15] Kyunghyun Cho, Bart van Merriënboer, Caglar Gulcehre, Dzmitry Bahdanau, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation. *arXiv:1406.1078 [cs, stat]*, June 2014.
- [16] Francis Crick. The recent excitement about neural networks. *Nature*, 337(6203):129, January 1989.
- [17] Yang Dan and Mu-Ming Poo. Spike timing-dependent plasticity: From synapse to perception. *Physiological Reviews*, 86(3):1033–1048, July 2006.
- [18] Nathaniel D. Daw, Samuel J. Gershman, Ben Seymour, Peter Dayan, and Raymond J. Dolan. Model-based influences on humans’ choices and striatal prediction errors. *Neuron*, 69(6):1204–1215, March 2011.
- [19] Gustavo Deco and Edmund T. Rolls. A Neurodynamical cortical model of visual attention and invariant object recognition. *Vision Research*, 44(6):621–642, March 2004.
- [20] Ziya V. Dikman and John J. B. Allen. Error monitoring during reward and avoidance learning in high- and low-socialized individuals. *Psychophysiology*, 37(1):43–54, 2000.
- [21] Yan Duan, Marcin Andrychowicz, Bradly Stadie, OpenAI Jonathan Ho, Jonas Schneider, Ilya Sutskever, Pieter Abbeel, and Wojciech Zaremba. One-Shot Imitation Learning. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, *Advances in Neural Information Processing Systems 30*, pages 1087–1098. Curran Associates, Inc., 2017.
- [22] Yan Duan, Xi Chen, Rein Houthoofd, John Schulman, and Pieter Abbeel. Benchmarking Deep Reinforcement Learning for Continuous Control. page 10.
- [23] Yan Duan, John Schulman, Xi Chen, Peter L. Bartlett, Ilya Sutskever, and Pieter Abbeel. RL2S: Fast Reinforcement Learning via Slow Reinforcement Learning. *arXiv:1611.02779 [cs, stat]*, November 2016.
- [24] William J. Gehring, Brian Goss, Michael G. H. Coles, David E. Meyer, and Emanuel Donchin. The Error-Related Negativity. *Perspectives on Psychological Science*, 13(2):200–204, March 2018.
- [25] F. A. Gers and J. Schmidhuber. Recurrent nets that time and count. In *Proceedings of the IEEE-INNS-ENNS International Joint Conference on Neural Networks. IJCNN 2000. Neural Computing: New Challenges and Perspectives for the New Millennium*, volume 3, pages 189–194 vol.3, July 2000.
- [26] Wulfram Gerstner. Spike-response model. *Scholarpedia*, 3(12):1343, December 2008.
- [27] Alex Graves. Adaptive Computation Time for Recurrent Neural Networks. *arXiv:1603.08983 [cs]*, March 2016.
- [28] Alex Graves and Navdeep Jaitly. Towards End-to-End Speech Recognition with Recurrent Neural Networks. page 9.
- [29] Alex Graves, Greg Wayne, and Ivo Danihelka. Neural Turing Machines. *arXiv:1410.5401 [cs]*, October 2014.
- [30] Andre Gruning and Sander M Bohte. Spiking Neural Networks: Principles and Challenges. *Computational Intelligence*, page 10, 2014.
- [31] Jiang Guo. BackPropagation Through Time. 2013.
- [32] Moritz Hardt, Benjamin Recht, and Yoram Singer. Train faster, generalize better: Stability of stochastic gradient descent. *arXiv:1509.01240 [cs, math, stat]*, September 2015.
- [33] Harry F. Harlow. The formation of learning sets. *Psychological Review*, 56(1):51–65, 1949.
- [34] Sepp Hochreiter and Jürgen Schmidhuber. Long Short-Term Memory. *Neural Comput.*, 9(8):1735–1780, November 1997.
- [35] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544, August 1952.
- [36] E. M. Izhikevich. Simple model of spiking neurons. *IEEE Transactions on Neural Networks*, 14(6):1569–1572, November 2003.
- [37] Max Jaderberg, Volodymyr Mnih, Wojciech Marian Czarnecki, Tom Schaul, Joel Z. Leibo, David Silver, and Koray Kavukcuoglu. Reinforcement Learning with Unsupervised Auxiliary Tasks. *arXiv:1611.05397 [cs]*, November 2016.
- [38] Scott Kirkpatrick, Charles Daniel Gelatt, and Mario P. Vecchi. Optimization by simulated annealing. *Science*, 220(4598):671–680, 1983.

- [39] Benjamin James Lansdell and Konrad Paul Kording. Towards learning-to-learn. *arXiv:1811.00231 [cs, q-bio]*, November 2018.
- [40] Robert Legenstein, Dejan Pecevski, and Wolfgang Maass. A Learning Theory for Reward-Modulated Spike-Timing-Dependent Plasticity with Application to Biofeedback. *PLoS computational biology*, 4:e1000180, November 2008.
- [41] Wolfgang Maass. Networks of Spiking Neurons Learn to Learn and Remember. page 27.
- [42] Wolfgang Maass. Networks of spiking neurons: The third generation of neural network models. *Neural Networks*, 10(9):1659–1671, December 1997.
- [43] Dougal Maclaurin, David Duvenaud, and Ryan P Adams. Gradient-based Hyperparameter Optimization through Reversible Learning. page 10.
- [44] Stephane J. MacLean, Cameron D. Hassall, Yoko Ishigami, Olav E. Krigolson, and Gail A. Eskes. Using brain potentials to understand prism adaptation: The error-related negativity and the P300. *Frontiers in Human Neuroscience*, 9:335, 2015.
- [45] Nikola T. Markov, Julien Vezoli, Pascal Chameau, Arnaud Falchier, René Quilodran, Cyril Huisoud, Camille Lamy, Pierre Misery, Pascale Giroud, Shimon Ullman, Pascal Barone, Colette Dehay, Kenneth Knoblauch, and Henry Kennedy. Anatomy of hierarchy: Feedforward and feedback pathways in macaque visual cortex. *The Journal of Comparative Neurology*, 522(1):225–259, January 2014.
- [46] Timothée Masquelier, Rudy Guyonneau, and Simon J. Thorpe. Spike timing dependent plasticity finds the start of repeating patterns in continuous spike trains. *PLoS One*, 3(1):e1377, January 2008.
- [47] Piotr Mirowski, Razvan Pascanu, Fabio Viola, Hubert Soyer, Andrew J. Ballard, Andrea Banino, Misha Denil, Ross Goroshin, Laurent Sifre, Koray Kavukcuoglu, Dhharshan Kumaran, and Raia Hadsell. Learning to Navigate in Complex Environments. *arXiv:1611.03673 [cs]*, November 2016.
- [48] Volodymyr Mnih, Adrià Puigdomènech Badia, Mehdi Mirza, Alex Graves, Timothy P. Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous Methods for Deep Reinforcement Learning. *arXiv:1602.01783 [cs]*, February 2016.
- [49] P. R. Montague, P. Dayan, and T. J. Sejnowski. A framework for mesencephalic dopamine systems based on predictive Hebbian learning. *Journal of Neuroscience*, 16(5):1936–1947, March 1996.
- [50] Arvind Neelakantan, Jeevan Shankar, Alexandre Passos, and Andrew McCallum. Efficient Non-parametric Estimation of Multiple Embeddings per Word in Vector Space. *arXiv:1504.06654 [cs, stat]*, April 2015.
- [51] Emre O. Neftci, Hesham Mostafa, and Friedemann Zenke. Surrogate Gradient Learning in Spiking Neural Networks. *arXiv:1901.09948 [cs, q-bio]*, January 2019.
- [52] Junhyuk Oh, Satinder Singh, Honglak Lee, and Pushmeet Kohli. Zero-Shot Task Generalization with Multi-Task Deep Reinforcement Learning. *arXiv:1706.05064 [cs]*, June 2017.
- [53] Wei-Xing Pan, Robert Schmidt, Jeffery R. Wickens, and Brian I. Hyland. Dopamine Cells Respond to Predicted Events during Classical Conditioning: Evidence for Eligibility Traces in the Reward-Learning Network. *Journal of Neuroscience*, 25(26):6235–6242, June 2005.
- [54] Michael Pfeiffer and Thomas Pfeil. Deep Learning With Spiking Neurons: Opportunities and Challenges. *Frontiers in Neuroscience*, 12, October 2018.
- [55] Filip Ponulak and Andrzej Kasiński. *Introduction to Spiking Neural Networks: Information Processing, Learning and Applications*, volume 71. January 2011.
- [56] Or Rivlin. Reinforcement Learning for Real-World Robotics. <https://towardsdatascience.com/reinforcement-learning-for-real-world-robotics-148c81dbdcff>, May 2019.
- [57] Arash Samadi, Timothy P. Lillicrap, and Douglas B. Tweed. Deep Learning with Dynamic Spiking Neurons and Fixed Feedback Weights. *Neural Computation*, 29(3):578–602, March 2017.
- [58] John Schulman, Sergey Levine, Philipp Moritz, Michael I. Jordan, and Pieter Abbeel. Trust Region Policy Optimization. *arXiv:1502.05477 [cs]*, February 2015.
- [59] M. Schuster and K. K. Paliwal. Bidirectional recurrent neural networks. *IEEE Transactions on Signal Processing*, 45(11):2673–2681, November 1997.
- [60] H. Sebastian Seung. Learning in Spiking Neural Networks by Reinforcement of Stochastic Synaptic Transmission. *Neuron*, 40(6):1063–1073, December 2003.
- [61] David Silver, Aja Huang, Chris J. Maddison, Arthur Guez, Laurent Sifre, George van den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, Sander Dieleman, Dominik Grewe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel, and Demis Hassabis. Mastering the game of Go with deep neural networks and tree search. *Nature*, 529(7587):484–489, January 2016.
- [62] Amirhossein Tavanaei, Masoud Ghodrati, Saeed Reza Kheradpisheh, Timothée Masquelier, and Anthony Maida. Deep learning in spiking neural networks. *Neural Networks*, 111:47–63, March 2019.
- [63] A. Waibel, T. Hanazawa, G. Hinton, K. Shikano, and K. J. Lang. Phoneme recognition using time-delay neural networks. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 37(3):328–339, March 1989.
- [64] Jane X. Wang, Zeb Kurth-Nelson, Dhharshan Kumaran, Dhruva Tirumala, Hubert Soyer, Joel Z. Leibo, Demis Hassabis, and Matthew Botvinick. Prefrontal cortex as a meta-reinforcement learning system. *Nature Neuroscience*, 21(6):860–868, June 2018.
- [65] Jane X. Wang, Zeb Kurth-Nelson, Dhruva Tirumala, Hubert Soyer, Joel Z. Leibo, Remi Munos, Charles Blundell, Dhharshan Kumaran, and Matt Botvinick. Learning to reinforcement learn. *arXiv:1611.05763 [cs, stat]*, November 2016.
- [66] P. J. Werbos. Backpropagation through time: What it does and how to do it. *Proceedings of the IEEE*, 78(10):1550–1560, October 1990.