Homework 8

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Problem 1

(a) The output is:

```
Newton's method
number of iterations:  5
solution:  [-3.46573590e-01 -1.41533671e-10]
value:  2.5592666966582156
```

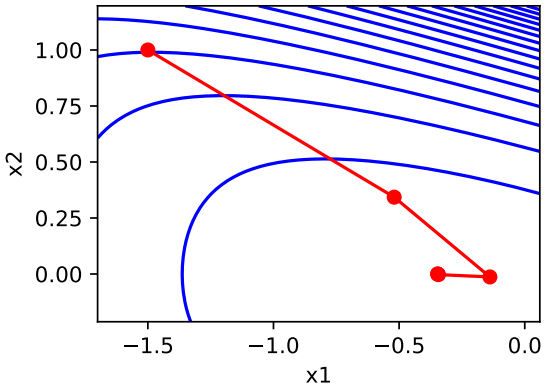

Figure 1: the trajectory of $\boldsymbol{x}_k$

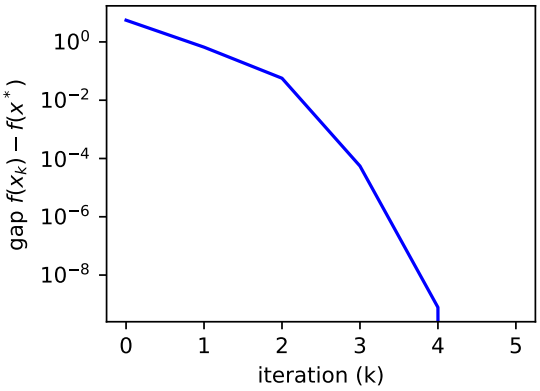

Figure 2: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

(b) The output is:

```
Newton's method
number of iterations:  8
solution:  [-3.46573573e-01 1.13424622e-08]
value:  2.559266696658217
```

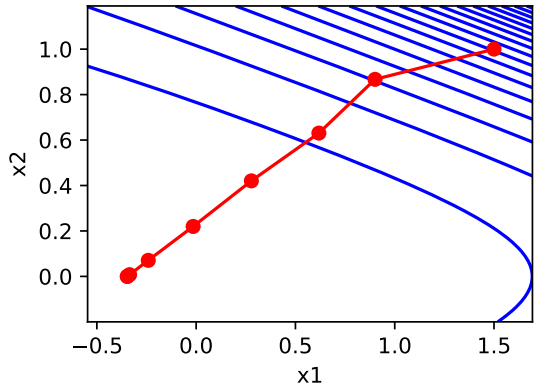

Figure 3: the trajectory of $\boldsymbol{x}_k$

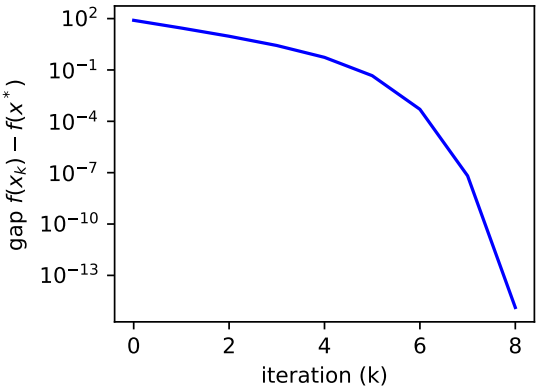

Figure 4: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

Problem 2

$$\nabla f(\boldsymbol{\omega}) = -\sum_{i=1}^{m} \left[1 - \sigma(y_i \boldsymbol{x}_i^{\mathcal{T}} \boldsymbol{\omega})\right] y_i \boldsymbol{x}_i$$

(a)

$$\begin{aligned}\nabla^2 f(\boldsymbol{\omega}) = &= -\sum_{i=1}^{m} \nabla\left\{\left[1 - \sigma(y_i \boldsymbol{x}_i^{\mathcal{T}} \boldsymbol{\omega})\right] y_i \boldsymbol{x}_i\right\} \\ &= -\sum_{i=1}^{m} -y_i^2 \sigma'(y_i \boldsymbol{x}_i^{\mathcal{T}} \boldsymbol{\omega}) \boldsymbol{x}_i \boldsymbol{x}_i^{\mathcal{T}} \\ &= \sum_{i=1}^{m} \sigma'(y_i \boldsymbol{x}_i^{\mathcal{T}} \boldsymbol{\omega}) \boldsymbol{x}_i \boldsymbol{x}_i^{\mathcal{T}}\end{aligned}$$

(b) The output is:

```
Damped Newton's method
number of iterations in outer loop:  20
total number of iterations in inner loop:  10
solution:  [-1.47021646 4.44403104 -4.37591078]
value:  2.8766810999687493
```

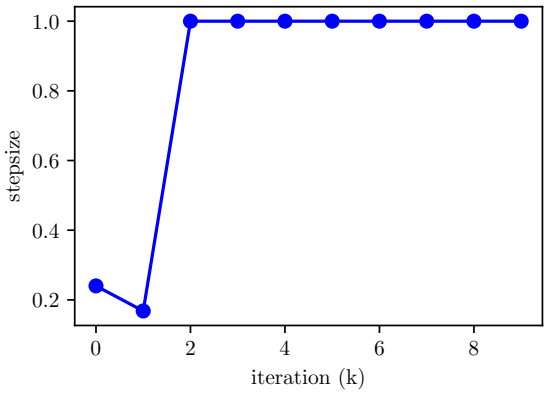

Figure 5: the step sizes t_k

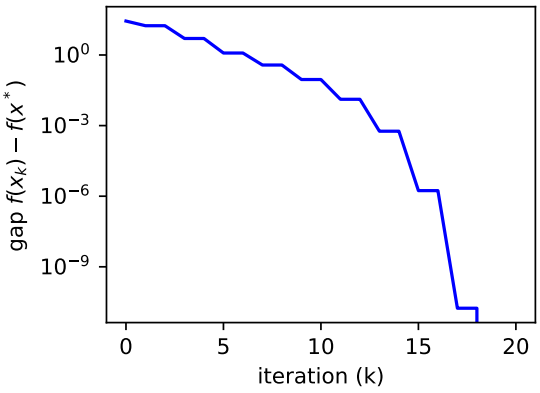

Figure 6: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

(c) It will cause a "gradient ascent":

$$\begin{bmatrix}1 & 1 & 0\end{bmatrix}^{\mathcal{T}} \to \begin{bmatrix}-28.5 & -4.8 & 44.9\end{bmatrix}^{\mathcal{T}} \to \begin{bmatrix}34330.7 & 5659.0 & -54500.0\end{bmatrix}^{\mathcal{T}} \to \cdots$$

Problem 3

(a) By the fact that $f'(x) = 4(x-a)^3$, $f''(x) = 12(x-a)^2$,

$$\Delta x = \frac{f'(x)}{f''(x)} = \frac{1}{3}(x-a)$$

(b)

$$\begin{aligned}x_{k+1} &= x_k - \frac{1}{3}(x_k - a) \\ \Longrightarrow x_{k+1} - a &= \frac{2}{3}(x_k - a) \\ \Longrightarrow y_{k+1} &= \frac{2}{3} y_k\end{aligned}$$

(c) When $k \to 0$,

$$|x_k - a| = \left(\frac{2}{3}\right)^k |x_0 - a| = \exp\left[-\ln\left(\frac{3}{2}\right)k\right] |x_0 - a| \longrightarrow 0$$

Problem 4

(a) The output is:

```
lambda = 2
number of iterations:  169
solution:  [1.00000000e+00 9.24600449e-09]
value:  6.5
```

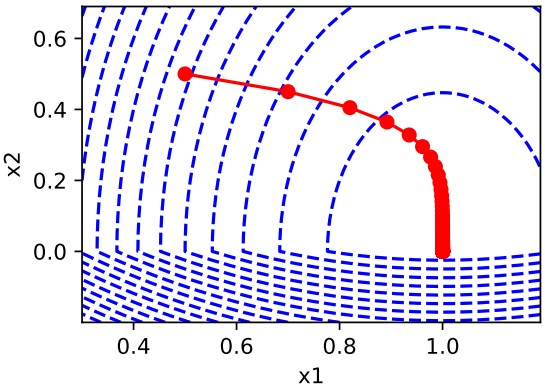

Figure 7: the trajectory of $\boldsymbol{x}_k$

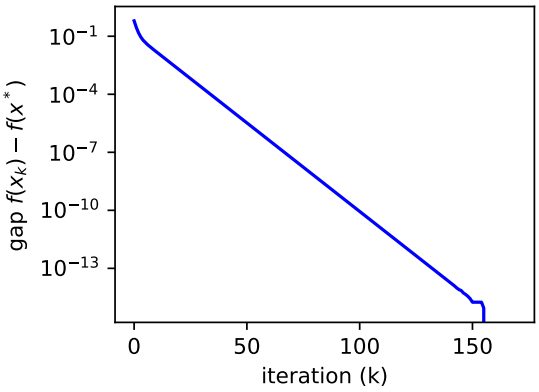

Figure 8: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

(b) The output is:

```
lambda = 1
number of iterations:  169
solution:  [1.25 0.99999999]
value:  4.875
```

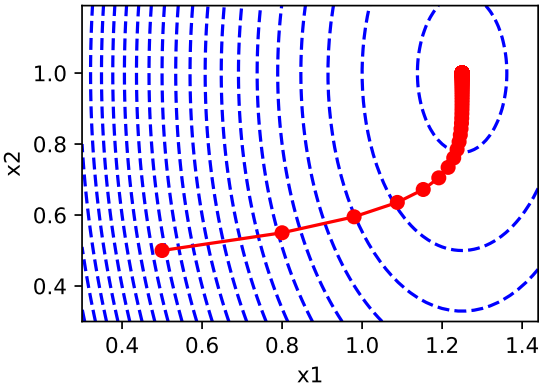

Figure 9: the trajectory of $\boldsymbol{x}_k$

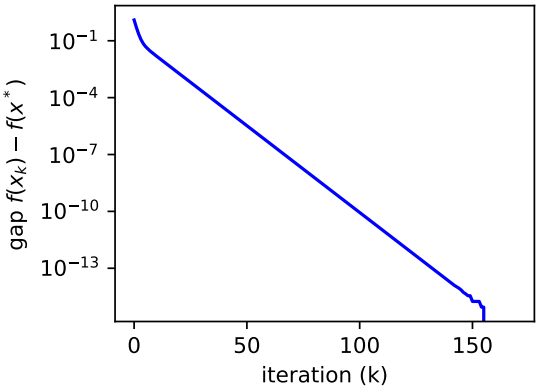

Figure 10: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

No zero in ω^*.

(c) The output is:

```
lambda = 6
number of iterations:  38
solution:  [ 1.85659632e-09 -0.00000000e+00]
value:  8.500000000000002
```

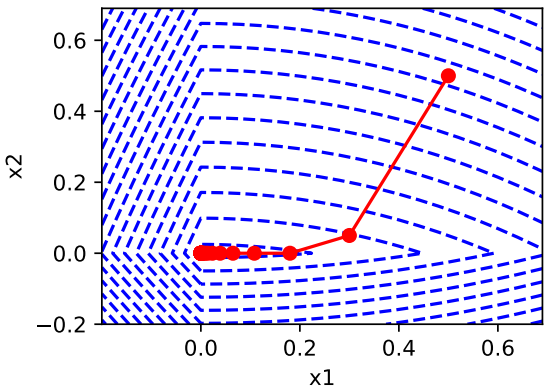

Figure 11: the trajectory of $\boldsymbol{x}_k$

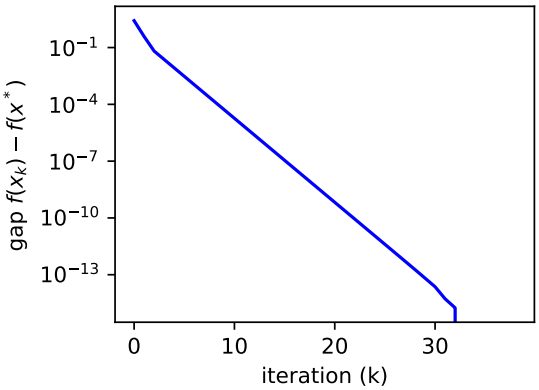

Figure 12: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

There are 2 zeros in $\boldsymbol{\omega}^*$.