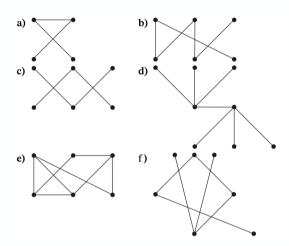
## Exercise Sheet 15

Discrete Mathematics, 2020.11.17

1. ([R], Page 755, Exercise 2(a)(b)(c)(d)(e)(f)) Which of these graphs are trees?



- 2. Suppose G = (V, E) is an undirected graph with  $|V| = n \ge 1$  and |E| = n 1. Prove that if G is connected, then G is a tree.
- 3. Suppose G = (V, E) is an undirected graph with  $|V| = n \ge 1$  and |E| = n 1. Prove that if G has no simple circuit, then G is a tree.

If it has no simple desired for the first the state of the first t	1. ([R], Page 755, Exercise $2(a)(b)(c)(d)(e)(f)$ ) Which of these graphs are trees?	3. Suppose G = (V, E) is an undirected graph with  V  = n ≥ 1 and  E  = n − 1. Prove that if G has no simple circuit, then G is a tree.
Assert that G's connected conjuments are to G. G.,, S. where Gir (No. E.) A kel  Then we have Yes [1,, No.,, No., with no simple climits.  \$\frac{1}{2}\text{Is a 146C}\$.  Then  M  =  E  =  - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - -     V  = \frac{1}{2} E  =  - - - - - - -     V  = \frac{1}{2} E  =  - - - - - - - -     V  = \frac{1}{2} E  =  - - - - - - - - -     V  = \frac{1}{2} E  =  - - - - - - - - - - - - - - - - - -	a) •	
Then we have Vie 1: "** }, Gi is consected and undirected with no singly cinalit.  \$\to \text{Gi is a tree.}\$  Than \( \text{No. } \text{Im} \) \( \text{No. } \text{No. } \text{No. } \text{No. } \text{No. } \\ \text{No. } \) \( \text{No. } \text{No. } \text{No. } \text{No. } \\ \text{No. } \text{No. } \\ \text{No. } \text{No. } \\ \tex		
## Gi is a tree.    Than    v		
Then   W   -   E   - 1        -	0 0 0	·
V  = \$\frac{1}{2}   \text{Pic}   \text{ K}   \text{Eq}   \text{ K}   \text{ Eq}   \text{ Connected Component } \text{ (d) Yes is an unadirected graph with \$ V  = n \geq 1 and \$ E  = n - 1\$. Prove that if  2. Suppose \$G = V(E)\$ is an unadirected graph with \$ V  = n \geq 1 and \$ E  = n - 1\$. Prove that if  3. For isomercial, then \$G\$ is a tree.  If \$G\$ is connected, then \$G\$ is a tree.  If \$G\$ is connected to \$ G\$ is a tree.  If \$\text{ Notice that \$G\$ is a tree.}    And \$V\$ is \$ A  \text{ Apple \$ A  \text{ Eq}   \tex		⇒ Gi is a tree.
(A) Yes (b) Yes (c) No. Note connected components.  (d) Yes (e) No. Have circuits (f) Yes  2 Suppose G = (V, E) is an undirected graph with  V  = n ≥ 1 and  E  = n - 1. Prove that if  3 G is connected.  2 Suppose G = (V, E) is an undirected graph with  V  = n ≥ 1 and  E  = n - 1. Prove that if  3 G is a tree.  3 G is a tree.  3 G is a tree.  4 If G is connected.  4 In  V   Notherwheel Lebestion:  An  V   Notherwheel   L		
(d) Yes (e) No. Howe circuits if Yes  ⇒ G is connected  ⇒ G is connected and undirected and undirected with no simple circuits.  ⇒ G is a tree.  G is a tree.  G is a tree.  If G is connected Ladweign:  hard for inches a tree.  Lit. when n=k, M=k h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h =k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h M=k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h M=k+1 h G is connected → G is a tree  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected of a.  Let n=k+1. M=k+1 h G is connected or a tree of a tre		$ V  = \sum_{k=1}^{K}  V_k  = \sum_{k=1}^{K}  E_k  + K =  E  + K =  E  + 1$
First by Methanetted Loductions    First by Methanetted Loductions   Part by Methanetted Loduction	(a) Yes (b) Yes (c) No. Not connected	⇒ K=1, which means G only has one connected component
2. Suppose G = (V, E) is an undirected graph with  V  = n ≥ 1 and  E  = n - 1. Prove that if  G is connected, then G is a tree.  If G is consected.  Int by Mathematical Induction:  Not! Obstituting G is a tree.  I.H.: when Nock,  M  + N  E  + k   N  G is connected → G is a tree.  I.H.: when Nock,  M  + N  E  + k   N  G is connected → G is a tree.  If nockel:  M  + k   N  E  + k   N  G is connected → G is a tree.  If nockel:  M  + k   N  E  + k   N  G is connected of o.  (If not item Vuav, deglin) ≥ 2. Then Zinglin) = 2 P  = 2 x-1  > 2 l. Contradiction!)  Debth we note o. Then we get a new graph G'=(V:E) N V + V  V  J   N  E - E   V  J    →  M  + k  B  + k   N  G   is connected  →  G   k   a tree.  G   k   a tree  G   k   a tree  G   k   a so simple cirmit  Then we have G has so simple cirmit  (If not, simple cirmit does not pass though w ⇒  G   k   a   a   simple cirmit . Contradiction!)  So  G   is connected and undirected with no simple circuit.	(d) Yes (e) No. Have circuits if Yes	⇒ G is connected
2. Suppose G = ((i, E) is an undirected graph with  V  = n ≥ 1 and  E  = n − 1. Prove that if G is connected, then G is a tree.  If G is Connected Laduction:  N=1: Obsignably G is a tree.  I.H.: when back, N=n-N=k=h=h=h G is connected → G is a tree  I.H.: when back, N=n-N=k=h=h=h G is connected → G is a tree  I.t.: when back, N=n-N=k=h=h=h G is connected → G is a tree  I.t.: when back, N=n-n-k=h=h=h G is connected → G is a tree  I.t.: when back, N=n-n-k=h=h=h G is connected  □ a=V, day n>1. Assume that is is the codynate of a.  (If n=t, then V=n, day n>2. Then Zipday n=2 k =2 k-1  ≥ 2h. contradiction!)  Dobble to enable. Then we get a new graph G=(V:E) N=N=N(si) ∧ E=E\{si}.  →  V =n-k E =h-h G'is connected  → G'is a tree  → G'has no simple circuit  Then we have G has no simple circuit.  (If n=t, simple circuit does not pass though to ⇒ G'has a simple circuit.)  So G is connected and undirected with no simple directit.		⇒ G is connected and undirected with no simple circuit.
If G is connected, then G is a tree.  Prof by Methoretical Induction:  N=1: Obstractly G is a tree.  I.H.: when N=k, N=k,N=k,N=k,A G is connected → G is a tree  Let n=k=1: N=k+1AE=k+1 A G is connected → G is a tree  Let n=k=1: N=k+1AE=k+1 A G is connected  = u=v, day(s)=1. Assume that u is the adopted of a.  (If net. then VusV, day(s) ≥ 2. Then Zyday(s) = 2 F =21=0 > 2n contradiction!)  Dolds us and a. Then we get a new graph G*a(v.E) AV=V(is) AE*aE\is).  — N*1=kA E=k+1 A G' is connected  — G' is a tree  — G' has no simple circuit  Then we have G has so simple circuit.  (If net, stapk circuit does not pass though u ⇒ G' has a simple circuit.  (If net, stapk circuit does not pass though u ⇒ G' has a simple circuit.		⇒ G is a tree.
Proof by Motherwitch Inductions:  N=1: Obstractly G is a tree.  I.H.: when N=K, N=KA E=K+A G is connected \rightarrow G is a tree  Let N=KA E=K+A G is connected  = n=V, deg(n)=1. Assume that u is the codpoint of a.  (If not. than V=V, deg(u) ≥ 2. Then Ziday(u)=2 E =2 x-1) > 2h. contradiction!)  Didds us and a. Then we get a new graph G=(V:E) AV=V\D] AE=E\infty  \rightarrow G' is a tree  \rightarrow G' has no simple circuit  Then we have G has no simple circuit.  (If not, simple circuit does not pass-through u \rightarrow G' has a simple circuit.  (If not, simple circuit does not pass-through u \rightarrow G' has a simple circuit.	2. Suppose $G=(V,E)$ is an undirected graph with $ V =n\geq 1$ and $ E =n-1$ . Prove that if $G$ is connected, then $G$ is a tree.	
N=  : Obsignately Gi is a tree.  I.H.: when n=k.   N =k= A = =k=A G is connected \rightarrow G is a tree.  Let n=k=  :   N =k= A = =k=A G is connected \rightarrow G is a tree.  Let n=k=  :   N =k= A = =k=A G is connected \rightarrow G is a tree.  I no V, dag(u)=1. Assume that u is the outpoint of a.  (If not, then VuaV, dag(u)=2. Then Zipdy(u)=2 F =2 x=1) > 2h. contradiction!)  Delete us and a. Then we get a new graph G=(V.E) AV=V[si] AE=E\set].  \rightarrow  V =k= A E =k=1 A G' is connected  \rightarrow G' is a tree.  \rightarrow G' is a tree.  (If not, simple circuit does not pass through \( \mu \rightarrow G' \) has a simple circuit.  (If not, simple circuit does not pass through \( \mu \rightarrow G' \) has a simple circuit.  So G is connected and unclineted with no simple circuit.	If G is connected.	
I.H.: when $h=k$ , $ V =k \wedge  E =k \wedge G$ is connected $\rightarrow G$ is a tree  Let $h=k+1$ : $ V =k \wedge  A = =k \wedge G$ is connected $\exists u \in V$ , $deg(u)=1$ . Assume that $u$ is the codepoint of $g$ .  (If not: then $\forall u \in V$ , $deg(u)\ge 2$ . Then $\sum_{i \in V} deg(u)=2 E =2 n-1>>2k$ contradiction!)  Delete $u$ and $g$ . Then we get a new graph $G=(V:E) \wedge V=V \wedge G$ $A \in E \wedge G$ . $\rightarrow  V =k \wedge  E =k \wedge G$ is connected $\rightarrow G'$ is a tree $\rightarrow G'$ has no simple circuit  Then we have $G$ has no simple circuit.  (If not: simple circuit does not pass though $u \Rightarrow G'$ has $a$ simple circuit. Contradiction!)  So $G$ is connected and undirected with no simple circuit.	Proof by Mothematical Induction:	
Let n=k+1: N=k+1 ke   A E =kA G is connected  ∃ neV, deg n =1. Assume that u is the endpoint of a.  (If not. thun VueV, deg n >2. Then Zigdeg u =2 E =2(n-1)>2th contradiction!)  Delete us and a. Thun we get a new graph G'=(V'.E) AV=V[ir] AE'=E\isi3.  →  V =kA E =k+1 A G' is connected  → G' is a tree  → G' has no simple circuit  Then we have G has no simple circuit.  (If not, simple circuit does not pass through u ⇒ G' has a simple circuit. Contradiction!)  So G is connected and undirected with no simple circuit.	N=1: Obiquely G is a tree.	
∃ ueV, deg(n)=1. Assume that u is the endpoint of e.  (If not. then VueV, deg(u)≥2. Then Zipdeg(u)=2 E =2(n·1) > 2h contradiction!)  Delois u and o. Then we get a new graph G'=(v'.E') ∧ V'=V\sij ∧ E'=E\sej.  →  v' =k∧ E =k·1 ∧ G'is connected  → G'is a tree  → G'has no simple circuit  Then we have G has no simple circuit.  (If not, simple circuit does not pass through u ⇒ G'has a simple circuit. contradiction!)  So G is connected and undirected with no simple circuit.	I.H.: where to=k, $ V =k\Lambda E =k+1\Lambda$ G is connected $\rightarrow$ G is a tree	
(If not, than VueV, deg(u) ≥ 2. Then Z deg(u) = 2 E  = 2 n-1) ≥ 2h contradiction!)  Debte u and e. Then we get a new graph G=(v:E) ∧ V=V\inj ∧ E'=E\ieij.  →  v =k∧ E =k-1 ∧ G'io connected  → G'is a tree  → G' has no simple circuit  Then we have G has no simple circuit.  (If not, simple circuit does not pass-though u ⇒ G' has a simple circuit. Cordradiction!)  So G is connected and undirected with no simple circuit.	Let n=k+1:  V =k+   A  E =k  A  G  Tilde Communication	
Delta wande. Then we get a new graph $G'=(V',E') \wedge V'=V \wedge Sif \wedge E'=E \wedge Sif$ . $\Rightarrow  V' =k \wedge  E' =k + 1 \wedge G'$ is connected. $\Rightarrow G'$ is a tree. $\Rightarrow G'$ has no simple circuit.  Then we have $G$ has no simple circuit.  (If not, simple circuit closes not pass-through $u \Rightarrow G'$ has a simple circuit. Contradiction!)  So $G$ is connected and undirected with no simple circuit.	$\exists$ $u \in V$ , $\deg(u) = 1$ . Assume that $u$ is the endpoint of $e$ .	
<ul> <li>→  v' =k-  A G'is connected</li> <li>→ G'is a tree</li> <li>→ G'has no simple circuit</li> <li>Then we have G has no simple circuit.</li> <li>(If not, simple circuit does not pass-though u ⇒ G'has a simple circuit. Contradiction!)</li> <li>So G is connected and undirected with no simple circuit.</li> </ul>	(If not. thun VueV, deglu) >2. Then Eddglu) = 2 E =2 n-1) > 2h contradiction!)	
<ul> <li>→  v' =k-  A G'is connected</li> <li>→ G'is a tree</li> <li>→ G'has no simple circuit</li> <li>Then we have G has no simple circuit.</li> <li>(If not, simple circuit does not pass-though u ⇒ G'has a simple circuit. Contradiction!)</li> <li>So G is connected and undirected with no simple circuit.</li> </ul>	Dolote u and o. Then we got a new graph G'=(V.E) AV'=V\\$ij AE'=E\\$ej.	
<ul> <li>→ G' is a tree</li> <li>→ G' has no simple circuit</li> <li>Then we have G has no simple circuit.</li> <li>(If net, simple circuit does not pass-through u ⇒ G' has a simple circuit. Contradiction!)</li> <li>So G is connected and undirected with no simple circuit.</li> </ul>		
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