

## Homework 2

Haoyu Zhen

May 2, 2022

Firtly we update the  $m_i$ :

$$m_i^* = \frac{n_i m_i - \hat{x}}{n_i - 1} = m_i - \frac{\hat{x} - m_i}{n_i - 1}.$$

Then by defination,  $J_i^*$  reads:

$$\begin{aligned} J_i^* &= \sum_{x \in H_i} \|x - m_i^*\|^2 - \|\hat{x} - m_i^*\|^2 \\ &= \sum_{x \in H_i} \left\| x - m_i - \frac{\hat{x} - m_i}{n_i - 1} \right\|^2 - \left\| \hat{x} - m_i - \frac{\hat{x} - m_i}{n_i - 1} \right\|^2 \\ &= \sum_{x \in H_i} \|x - m_i\|^2 + \frac{2}{n_i - 1} \sum_{x \in H_i} (x - m_i)^T (\hat{x} - m_i) + \frac{n_i}{(n_i - 1)^2} \|\hat{x} - m_i\|^2 - \frac{n_i^2}{(n_i - 1)^2} \|\hat{x} - m_i\|^2. \\ &= J_i - \frac{n_i^2 - n_i}{(n_i - 1)^2} \|\hat{x} - m_i\|^2 \\ &= J_i - \frac{n_i}{n_i - 1} \|\hat{x} - m_i\|^2 \end{aligned}$$