Homework 11

Zhen

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Problem 1

```python
def proj(x):
  if np.linalg.norm(x,1)<1:
    return x
  k = x / np.abs(x)
  c = (t - k.T @ x) / (np.linalg.norm(k)**2)
  x = c * k + x
  return x
```

The implementation of the projection onto l_1 ball is above. Let the stepsize= 0.1. The output is:

```
t = 1
number of iterations:  122
solution:  [9.99999975e-01 2.51943348e-08]
value:  4.500000000000001
```

Some useful plots:

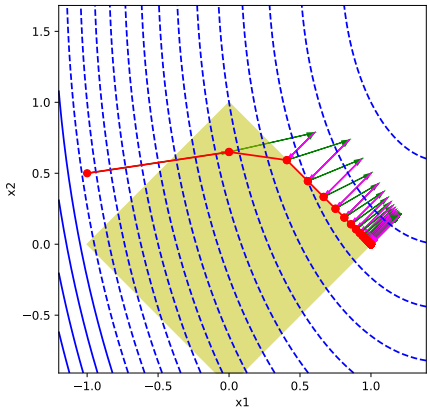

Figure 1: the trajectory of $\boldsymbol{x}_k$

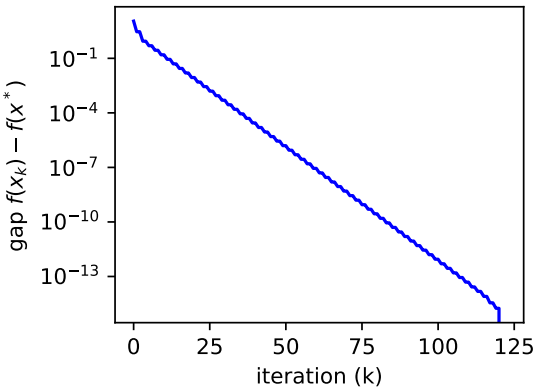

Figure 2: the gap $f(\boldsymbol{x}_k) - f(\boldsymbol{x}^*)$

Problem 2

(a) The Lagrangian condition is:

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ e^{x_1} + \lambda &= 0 \\ 2e^{2x_2} + \lambda &= 0 \\ 2e^{2x_3} + \lambda &= 0 \end{aligned}$$

Then we have:

$$x_1 = \ln(-\lambda), \quad x_2 = x_3 = \frac{1}{2}\ln\left(-\frac{\lambda}{2}\right)$$

Thus we have: $\lambda^* = -\sqrt{2e}, \boldsymbol{x}^* = \left[\ln\left(\sqrt{2e}\right) \quad \frac{1}{2}\ln\left(\sqrt{2e}/2\right) \quad \frac{1}{2}\ln\left(\sqrt{2e}/2\right)\right]$ and $f^* = -2\lambda = 2\sqrt{2e}$.

(b) Similarly, let the stepsize= 0.1. The output is:

```
number of iterations:  92
solution:  [0.84657357 0.07671322 0.07671322]
value:  4.66328796319425
```

which is in accord with theoretical value.