Question 1

(a). By defination,

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-2jk\pi n/N}, \quad \tilde{X}_3[k] = \sum_{n=0}^{3N-1} \tilde{x}[n] e^{-2jk\pi n/3N}.$$

Thus

$$\begin{aligned}\tilde{X}_3[k] &= \sum_{n=0}^{N-1} \tilde{x}[n] e^{-2jk\pi n/3N} \left(1 + e^{-2jk\pi/3} + e^{-4jk\pi/3}\right) \\ &= \begin{cases} 3\tilde{X}[k/3] & \text{if } k \equiv 0 \pmod 3 \\ 0 & \text{otherwise} \end{cases}\end{aligned}.$$

(b).

$$\tilde{X}[k] = \begin{cases} 3 & \text{if } k \equiv 0 \pmod 2 \\ -1 & \text{otherwise} \end{cases} \text{ and } \tilde{X}_3[k] = \begin{cases} 9 & \text{if } k \equiv 0 \pmod 6 \\ -3 & \text{if } k \equiv 3 \pmod 6 \\ 0 & \text{otherwise} \end{cases}.$$

Question 2

(a). Apparently, $|\alpha| < 1$. Then

$$X(e^{j\omega}) = \sum_{n=0}^{+\infty} \alpha^n e^{j\omega n} = \frac{1}{1 - \alpha e^{j\omega n}}.$$

(b). Firstly,

$$\tilde{x}[n] = \alpha^n \sum_{r=0}^{+\infty} \alpha^{rN} = \frac{\alpha^n}{1 - \alpha^N}.$$

Then

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \frac{\alpha^n}{1-\alpha^N} e^{-2jk\pi n/N} = \frac{1 - \alpha^N e^{2j\pi k}}{(1-\alpha^N)\left(1 - \alpha e^{-2j\pi k/N}\right)} = \frac{1}{1 - \alpha e^{-2j\pi k/N}}.$$

(c). $\tilde{X}[k]$ is the sampling of $X(e^{j\omega})$:

$$\omega \to \frac{2k\pi}{N}.$$