

Algorithm Design and Analysis

Assignment 6

Due: Sunday, June 11, 2022

1. (100 points) Choose *any three* of the following questions. Each question carries $\frac{100}{3}$ points. (You are encouraged to solve as many the remaining questions as possible “in your mind”.)
 - (a) Given an undirected graph $G = (V, E)$ with $n = |V|$, decide if G contains a clique with size exactly $n/2$. Prove that this problem is NP-complete.
 - (b) Given two undirected graphs G and H , decide if H is a subgraph of G . Prove that this problem is NP-complete.
 - (c) Given an undirected graph $G = (V, E)$ and an integer k , decide if G has a spanning tree with maximum degree at most k . Prove that this problem is NP-complete.
 - (d) Given a ground set $U = \{1, \dots, n\}$, a collection of its subsets $\mathcal{S} = \{S_1, \dots, S_m\}$, and a positive integer k , the *set cover* problem asks if we can find a subcollection $\mathcal{T} \subseteq \mathcal{S}$ such that $\bigcup_{S \in \mathcal{T}} S = U$ and $|\mathcal{T}| = k$. Prove that set cover is NP-complete.
 - (e) Given a collection of integers (can be negative), decide if there is a subcollection with sum exactly 0. Prove that this problem is NP-complete.
 - (f) In an undirected graph $G = (V, E)$, each vertex can be colored either black or white. After an initial color configuration, a vertex will become black if all its neighbors are black, and the updates go on and on until no more update is possible. (Notice that once a vertex is black, it will be black forever.) Now, you are given an initial configuration where all vertices are white, and you need to change k vertices from white to black such that all vertices will eventually become black after updates. Prove that it is NP-complete to decide if this is possible.
 - (g) Consider the decision version of *Knapsack*. Given a set of n items with weights $w_1, \dots, w_n \in \mathbb{Z}^+$ and values $v_1, \dots, v_n \in \mathbb{Z}^+$, a capacity constraint $C \in \mathbb{Z}^+$, and a positive integer $V \in \mathbb{Z}^+$, decide if there exists a subset of items with total weight at most C and total value at least V . Prove that this decision version of Knapsack is NP-complete.

2. (5 Bonus points) Choose *one* of the following two questions.
- (a) Given an undirected graph $G = (V, E)$, the *3-coloring* problem asks if there is a way to color all the vertices by using three colors, say, red, blue and green, such that every two adjacent vertices have different colors. Prove that 3-coloring is NP-complete.
 - (b) Given a ground set $U = \{1, \dots, n\}$ and a collection of its subsets $\mathcal{S} = \{S_1, \dots, S_m\}$, the *exact cover* problem asks if we can find a subcollection $\mathcal{T} \subseteq \mathcal{S}$ such that $\bigcup_{S \in \mathcal{T}} S = U$ and $S_i \cap S_j = \emptyset$ for any $S_i, S_j \in \mathcal{T}$. Prove that exact cover is NP-complete.
3. How long does it take you to finish the assignment (including thinking and discussion)? Give a score (1,2,3,4,5) to the difficulty. Do you have any collaborators? Please write down their names here.