

Homework 11

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Problem 1

```
1 def proj(x):
2     if np.linalg.norm(x,1)<1:
3         return x
4     k = x / np.abs(x)
5     c = (t - k.T @ x) / (np.linalg.norm(k)**2)
6     x = c * k + x
7     return x
```

The implementation of the projection onto l_1 ball is above. Let the stepsize= 0.1. The output is:

t = 1

number of iterations: 122

solution: [9.99999975e-01 2.51943348e-08]

value: 4.5000000000000001

Some useful plots:

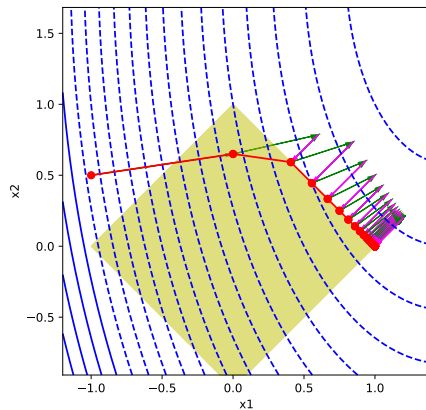


Figure 1: the trajectory of \mathbf{x}_k

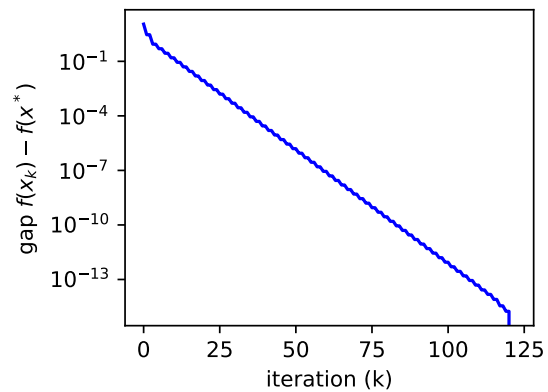


Figure 2: the gap $f(\mathbf{x}_k) - f(\mathbf{x}^*)$

Problem 2

(a) The Lagrangian condition is:

$$x_1 + x_2 + x_3 = 1$$

$$e^{x_1} + \lambda = 0$$

$$2e^{2x_2} + \lambda = 0$$

$$2e^{2x_3} + \lambda = 0$$

Then we have:

$$x_1 = \ln(-\lambda), \quad x_2 = x_3 = \frac{1}{2} \ln\left(-\frac{\lambda}{2}\right)$$

Thus we have: $\lambda^* = -\sqrt{2e}$, $\mathbf{x}^* = \left[\ln(\sqrt{2e}) \quad \frac{1}{2} \ln(\sqrt{2e}/2) \quad \frac{1}{2} \ln(\sqrt{2e}/2) \right]$ and $f^* = -2\lambda = 2\sqrt{2e}$.

(b) Similarly, let the stepsize= 0.1. The output is:

number of iterations: 92

solution: [0.84657357 0.07671322 0.07671322]

value: 4.66328796319425

which is in accord with theoretical value.