Homework 1

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Proposition 1. *If* $Z = UDV^T$*, then*

$$Z^T Z = (UDV^T)^T (UDV^T) = VD^T U^T UDV^T = VD^2 V^T.$$

Proposition 2. *If* $A = VDV^T$*, then*

$$A^{-1} = VD^* V^T$$

where $D = \mathrm{diag}(d_1, d_2, \cdots, d_n)$*,* $D^* = \mathrm{diag}(d_1^{-1}, d_2^{-1}, \cdots, d_n^{-1})$ *and* V *is orthogonal.*

Proof.

$$VD^* V^T VDV^T = VD^* DV^T = VV^T = I.$$

□

Thus

$$\begin{aligned}(Z^T Z + \lambda I)^{-1} Z^T y &= (VD^2 V^T + V\lambda I V^T)^{-1} VDU^T y \\ &= \left[V\left(D^2 + \lambda I\right)V^T\right]^{-1} VDU^T y \\ &= V\left[\mathrm{diag}_i\left(\frac{1}{d_i^2 + \lambda}\right)\right] V^T VDU^T y \\ &= V\left[\mathrm{diag}_i\left(\frac{1}{d_i^2 + \lambda}\right)\right] DU^T y \\ &= V\left[\mathrm{diag}_i\left(\frac{d_i}{d_i^2 + \lambda}\right)\right] U^T y\end{aligned}.$$