Homework 6

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(b)

Let f whose inputs are 2 graphs H and G denote the subgraph problem.

- $f \in \mathrm{NP}$

  Let $\mathcal{M}$ be a injective mapping from V_H to V_G. Then the string y representing $\mathcal{M}$ is the certificate in a Turning machine $\mathcal{A}$. If for all $u, v \in V_H$ such that $\{u, v\} \in E_H \implies \{\mathcal{M}(u), \mathcal{M}(v)\} \in E_G$ and $\{u, v\} \notin E_H \implies \{\mathcal{M}(u), \mathcal{M}(v)\} \notin E_G$, then $\mathcal{A}(x, y) = 1$ (note taht $\mathcal{A}(x, y) = 0$ otherwise). This could be done in $\mathcal{O}(|E|)$ time.

- Clique $\leq_k f$

  Obviously, a loop containing k vetices is a subgraph G **iff** G has a k-clique.

Finally, we obtain that f is NP-complete.

(e)

Here we face the "SubsetSum0" problem. In the lecture we've already proved that SubsetSum+ is NP-complete.

- Obviously, SubsetSum0 $\in$ NP because addition operation is petty.

- SubsetSum+ $\leq_k$ SubsetSum0 (Indeed, trivial proposition)

  `Proof:` Let S denote the set of positive integers. We want to decide whether $\exists A \subseteq S$ such that $\sum_{a \in A} = k$. Now $S^\dagger \leftarrow S \cup \{-k\}$ and feed S into SubsetSum0 algorithm. We could get a set B such that $\sum_{b \in B} b = 0$. Note that $-k \in B$ and we have $\sum_{a \in B - \{-k\}} a = k$.

Thus, "SubsetSum0" is NP-complete.

(f)

f denotes the decision problem whose inputs are a colored graph G and a number k.

- $f \in$NP

  For any given G which is colored: we could check whether all vertices will eventually become black after updates in $\mathcal{O}(|V||E|)$ time.

- $g \triangleq k$-Vertex-Cover $\leq_k f$

  `Proof:` Assume that (G, k) is input of g. I.e., we want to decide whether there exists a vertex cover whose size is k in G. Here we colour all vertices in G **white** and feed it into f with k.

Lemma 1. *u, v are white and $(u, v) \in E \implies u, v$ will be white forever. (This is a stable structure.)*

Naturally, lemma 1 entails the following lemma.

Lemma 2. *If a coloured G will end with a black one, then $\forall (u, v) \in E$, u is black or v is black, which means that G is **covered** by balck vertices.*

Thus, $g(G, k) = f(G_{\text{white}}, k)$.

Hence we could deduce that f is NP-complete.

Misc

5-6 hours. Difficulty 2. No collaborator.