Code is available at "PA2.m". Also I superimpose the curves of spectrum for each process in Figure 4, 5 and 6.

Process 1

Let $t_s = 0.4$. Then $x[n] = 1$ if $n \in \{0, 1, \cdots, 25\}$ otherwise $x[n] = 0$.

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n} = \sum_{n=0}^{25} e^{-j\omega n} = \frac{1 - e^{26j\omega}}{1 - e^{j\omega}}.$$

Then $|X|$ reads:

$$|X(\omega)| = \left|\frac{\sin 13\omega}{\sin(\omega/2)}\right|.$$

Figure 1 and 4 are the diagrams of procces 1. I use $f[n] = 1$ if $n \in \{0, 1, \cdots, 101\}$ to represent rectangle function $x(t)$ approximately and use `f(1:sample:end)` to sample.

Process 2

After shiftted, $x[n] = 1$ if $n \in \{1, 2, \cdots, 25\}$ otherwise $x[n] = 0$. Similarly,

$$|X(\omega)| = \left|\frac{\sin 12.5\omega}{\sin(\omega/2)}\right|.$$

Figure 2 and 4 are the diagrams of procces 2. There are subtle differences between 1 and 2: $x[0] = 1 \to x[0] = 0$.

Process 3

In MATLAB, I use "`[b,a] = butter(order, Wn); filt = filtfilt(b,a,shiftted);`" to filter the rectangle function where order $= 9$ and Wn $= 0.1$. Then $x(t) = \mathcal{F}^{-1}[2\sin(5\omega)/\omega \cdot \text{Butter}(\omega_0)]$ and

$$X(\omega) = \mathcal{F}\left\{\mathcal{F}^{-1}\left[\frac{2\sin(5\omega)}{\omega} \cdot \text{Butter}(\omega_0)\right] \cdot \sum_{k=-\infty}^{+\infty} \delta(t - kT_s)\right\} \tag{1}$$

This entails

$$X(\omega) = \left[\frac{2\sin(5\omega)}{\omega} \cdot \text{Butter}(\omega_0)\right] * \left[\frac{2\pi}{T_s} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)\right] \tag{2}$$

It is noteworthy that the Eq.2 has no simple analytic form and $X(\omega)$ is a periodic function. However, if $\omega_0 < 2\pi/T_s$, then $X(\omega) = 2\sin(5\omega)/(T_s\omega) \cdot \text{Butter}(\omega_0)$ approximately. Figure 3 and 4 are the diagrams of procces 3.

Appendix

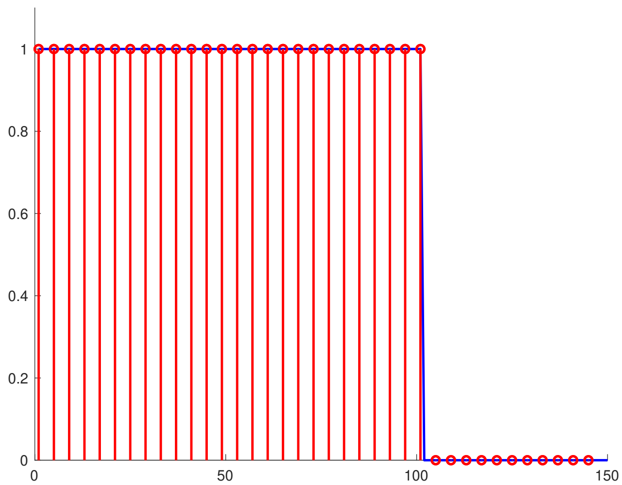

Figure 1: Original

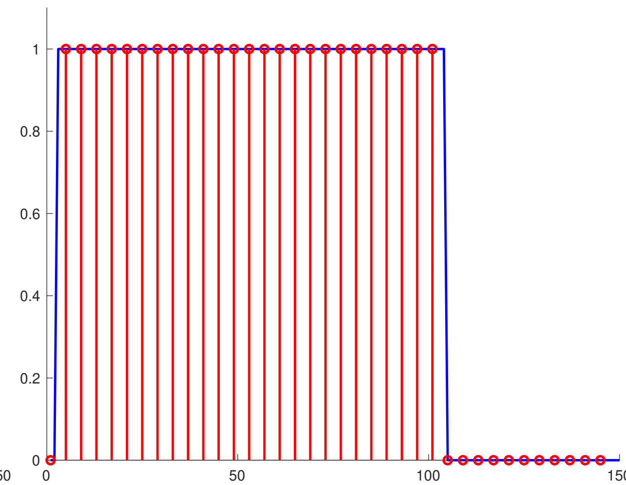

Figure 2: Shiftted

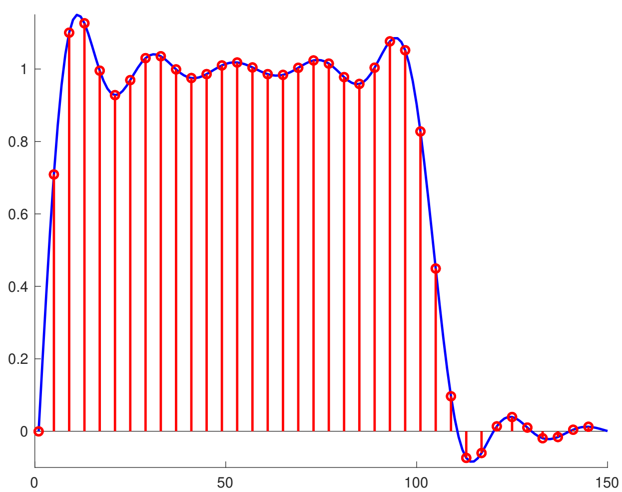

Figure 3: Filtered

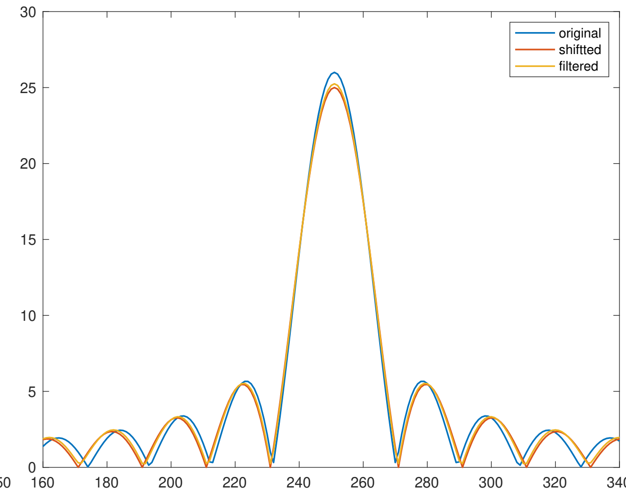

Figure 4: Spectrum

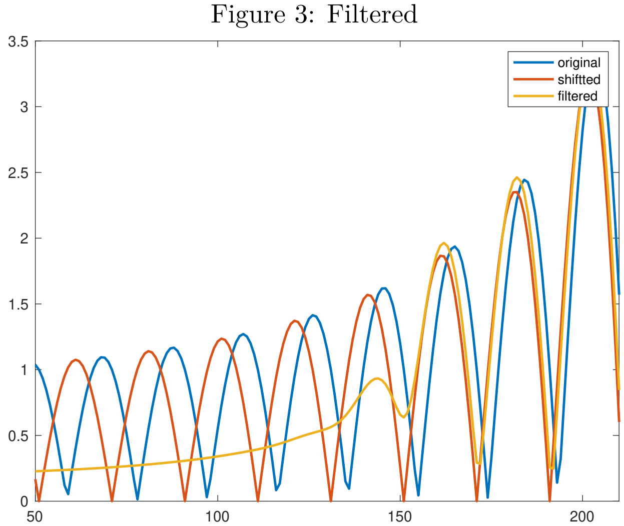

Figure 5: Spectrum (Left)

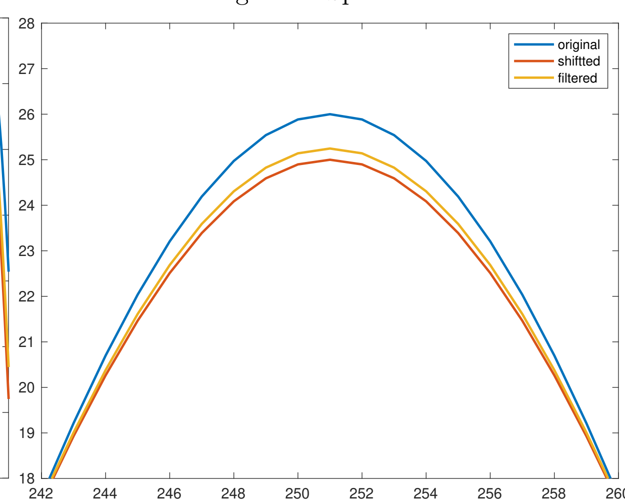

Figure 6: Spectrum (The highest peak)