Computer Graphics Lecture Notes

Haoyu Zhen

July 16, 2022

Acknowledgement

These Notes contain material developed and copyright by CS148: Introduction to Computer Graphics and Imaging, Stanford University. To get more information, you could visit the course website: https://web.stanford.edu/class/cs148/index.html.

1 Geometry

Loop Subdivision

- Subdivide each triangle into 4 sub-triangles
- Move both the old/new vertices
- Repeat (if desired)
- C2 continuity almost everywhere (except at some extraordinary vertices where its only C1)

Cardinal Cubic Splines

4 control points $\{p_{i-1}, p_i, p_{i+1}, p_{i+2}\}$. What we want is $f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3$ such that

$$f(0) = p_i, \quad f(1) = p_{i+1}, \quad f'(0) = s(p_{i+1} - p_{i-1}) \quad \text{and} \quad f'(1) = s(p_{i+2} - p_i).$$

1.1 Transformation

Rotation

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \quad R_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \text{and} \quad R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Scaling

$$S = \begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_1 & 0 \\ 0 & 0 & s_3 \end{pmatrix}.$$

Homogeneous Coordinates

$$\vec{p}_H = \begin{pmatrix} x & y & z & 1 \end{pmatrix}^T.$$

Then we have

$$\begin{pmatrix} \boldsymbol{M} & \boldsymbol{t} \\ \boldsymbol{O} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{M}\boldsymbol{x} + \boldsymbol{t} \\ 1 \end{pmatrix}$$

where $\boldsymbol{M} \in \mathbb{R}^{3\times 3}$ and $\boldsymbol{O} \in \mathbb{R}^{1\times 3}$.

2 Color and Image

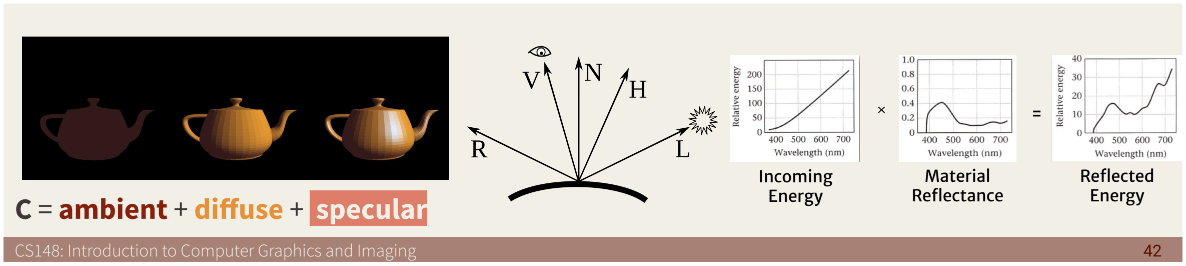
2.1 Rasterization

For each pixel, if the center of the pixel is inside the triangle, consider it part of the triangle (and color it with the triangles color.)

2.2 Phong Reflection Model

$$
\begin{aligned}
C &= \text{ambient} + \text{diffuse} + \text{specular} \\
&= k_a + k_d \max(0, \hat{N} \cdot \hat{L}) + k_s \left(\max(0, \hat{V} \cdot \hat{R}) \right)^{\alpha}
\end{aligned}
$$

where $\hat{N}, \hat{L}, \hat{V}$ and $\hat{R}$ are defined as follows.



- **Ambient**: "Base color". Light bounces arounf the environment.
- **Diffuse**: "Rough marterial". Given the same light source, objects look brighter when hit "perpendicularly".
- **Specular**: "Shiny material". Bright highlights when light reflects into our eyes.

3 Optics

Definition 3.1. **Radiant Intensity** of a light source: $I(\omega) = \mathrm{d}\Phi / \mathrm{d}\omega$

- Total light power (exiting a light) per unit solid angle
- Measure of how strong a (point) light source is

Definition 3.2. **Irradiance** on a surface: $E = \mathrm{d}\Phi / \mathrm{d}A$

- Total light power (hitting a surface) per unit surface area.
- Measure of how much light is hitting a surface.
- Varies based on distance from the light and the tilting angle of the surface.

Some engineering approximations are as follows.

- BRDF (Bidirectional Reflectance Distribution Function): models how much light is reflected.
- BTDF (Bidirectional Transmittance Distribution Function): models how much light is transmitted.
- BSSRDF (Bidirectional Surface Scattering Reflectance Distribution Function): combined reflection/transmission model.

Now we define the **lighting equation**:

$$
L_o(\omega_0) = \sum_{i \in \text{in}} L_{o \text{ due to } i}(\omega_i, \omega_o)
$$

where the BRDF gives each of $L_{o \text{ due to } i}(\omega_i, \omega_o)$. Then we have

$$
L_o(\omega_0) = \int_{i \in \text{in}} \text{BRDF}(\omega_i, \omega_0) \, \mathrm{d}E_i(\omega_i) = \int_{i \in \text{in}} \text{BRDF}(\omega_i, \omega_0) L_i \cos\theta_i \, \mathrm{d}\omega_i \, .
$$

Diffuse Materials: a surface reflects light equally in all directions. I.e., BRDF = Const.