# Computer Graphics Lecture Notes

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## Acknowledgement

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# 1 Geometry

### Loop Subdivision

- Subdivide each triangle into 4 sub-triangles
- Move both the old/new vertices
- Repeat (if desired)
- C2 continuity almost everywhere (except at some extraordinary vertices where its only C1)

### Cardinal Cubic Splines

4 control points $\{p_{i-1}, p_i, p_{i+1}, p_{i+2}\}$. What we want is $f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3$ such that

$$f(0) = p_i, \quad f(1) = p_{i+1}, \quad f'(0) = s(p_{i+1} - p_{i-1}) \quad \text{and} \quad f'(1) = s(p_{i+2} - p_i).$$

## 1.1 Transformation

### Rotation

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \quad R_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \text{and} \quad R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

### Scaling

$$S = \begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_1 & 0 \\ 0 & 0 & s_3 \end{pmatrix}.$$

### Homogeneous Coordinates

$$\vec{p}_H = \begin{pmatrix} x & y & z & 1 \end{pmatrix}^T.$$

Then we have

$$\begin{pmatrix} \boldsymbol{M} & \boldsymbol{t} \\ \boldsymbol{O} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{M}\boldsymbol{x} + \boldsymbol{t} \\ 1 \end{pmatrix}$$

where $\boldsymbol{M} \in \mathbb{R}^{3\times 3}$ and $\boldsymbol{O} \in \mathbb{R}^{1\times 3}$.

# 2 Optics

**Definition 2.1.** **Radiant Intensity** of a light source: $I(\omega) = \mathrm{d}\Phi/\mathrm{d}\omega$

- Total light power (exiting a light) per unit solid angle
- Measure of how strong a (point) light source is

**Definition 2.2.** **Irradiance** on a surface: $E = \mathrm{d}\Phi/\mathrm{d}A$

- Total light power (hitting a surface) per unit surface area.
- Measure of how much light is hitting a surface.
- Varies based on distance from the light and the tilting angle of the surface.

Some engineering approximations are as follows.

- BRDF (Bidirectional Reflectance Distribution Function): models how much light is reflected.
- BTDF (Bidirectional Transmittance Distribution Function): models how much light is transmitted.
- BSSRDF (Bidirectional Surface Scattering Reflectance Distribution Function): combined reflection/transmission model.

Now we define the **lighting equation**:

$$L_o(\omega_0) = \sum_{i \in \mathrm{in}} L_{o\text{ due to }i}(\omega_i, \omega_o)$$

where the BRDF gives each of $L_{o\text{ due to }i}(\omega_i, \omega_o)$. Then we have

$$L_o(\omega_0) = \int_{i \in \mathrm{in}} \mathrm{BRDF}(\omega_i, \omega_0)\,\mathrm{d}E_i(\omega_i) = \int_{i \in \mathrm{in}} \mathrm{BRDF}(\omega_i, \omega_0) L_i \cos\theta_i \,\mathrm{d}\omega_i\,.$$

Diffuse Materials: a surface reflects light equally in all directions. I.e., BRDF = Const.