1 Notation

- t_n: date step, $n = 1, \cdots, N$
- m: item (e.g. contract number), $m = 1, \cdots, M$
- $s(m, t_n)$: a series of marks.
- If t_n is a roll date,
  - $s(m, t_{n-1})$ becomes $s(m-1, t_n)$
  - $s(1, t_{n-1})$ rolls off, i.e. disappears
  - $s(M, t_n)$ appears as a new item
- $q(m, t_n)$: adjusted marks
- $\alpha(m, t)$: drift slope calculated for the m-th item as of t
- $T(m, t)$: the roll-off (i.e. maturity) date for the m-th item as of t
- $\phi(m, j)$: last drift slope tracked for the m-th item over the j-th rolling period $[r_j, r_{j+1})$
- $t' - t$: year-fraction from t to t' calculated using the Act 365 convention.

2 Drift Adjustment

2.1 Remarking Criteria

t_n is a remarked date for the m-th item if

- if t_n is not a roll date:

$$\left| \frac{s(m, t_n) - s(m, t_{n-1})}{s(m, t_{n-1})} \right| > \epsilon \tag{1}$$

- if t_n is a roll date:

$$\left| \frac{s(m, t_n) - s(m+1, t_{n-1})}{s(m+1, t_{n-1})} \right| > \epsilon \tag{2}$$

for some $\epsilon > 0$.

2.2 Slope Function

Given $\{s(m,t)\}_m$, the m-th slope $\alpha(m,t)$ is calculated by

$$\alpha(1,t) = \frac{s(1,t)}{T(1,t)-t} \tag{3}$$

$$\alpha(m,t) = \frac{s(m,t)-s(m-1,t)}{T(m,t)-T(m-1,t)} \text{ for } m = 2,\cdots,M \tag{4}$$

Note: For standardised products like Futures, we may assume $T(m,t) - T(m-1,t) = 90/365$ for simplicity.

Motivation: Consider two marks: $s(m,t)$ and $s(m-1,t)$. After time is elapsed by $\Delta := (T(m,t) - T(m-1,t))$, the time-to-roll-off of the first item becomes $(T(m-1,t) - t)$, which is the current time-to-roll-off of the second item.

Therefore, the natural adjustment slope is the one makes

$$q(m, t+\Delta) = s(m-1,t)$$

if no remarks occurs for all future time steps.

2.3 Algorithm

Consider a rolling period $I_j = \{t_n | t_n \in [r_j, r_{j+1}]\}$ where r_j and r_{j+1} are two adjacent rolling dates. Assume that $t_0 \in I_0$ and $t_N \in I_J$.

For each rolling period j starting from 0 to J:

1. Initialisation: Let t^j be the first date in the period, i.e. $t^j := \max\{t_0, r_j\}$
   - If $j = 0$ or t^j is a remark date, simply set
     - $q(m,t^j) = s(m,t^j)$
     - $\phi(m,j) = \alpha(m,t^j)$

     for $m = 1,\cdots,M$.
   - Otherwise, $j > 0$ and t^j is not a remark date.

     For $m = 1,\cdots M-1$, set

  - $\phi(m, j) = \phi(m+1, j-1)$, i.e. carry over the slope from the previous period
  - $q(m, t^j) = q(m+1, p(t^j)) - \phi(m, j)(t^j - p(t^j))$

  where $p(t)$ is the previous time step in $\{t_n\}$.

  For $m = M$, borrowing from $m = M - 1$, set

  - $\phi(M, j) = \phi(M-1, j)$, i.e. the slope is carried over.
  - $q(M, t^j) = s(M, t^j) + [q(M-1, t^j) - s(M-1, t^j)]$

2. For each t_n with $t^j < t_n < r_{j+1}$:

   - If t_n is a remarked date, set
     - $q(m, t_n) = s(m, t_n)$
     - $\phi(m, j) = \alpha(m, t_n)$

     for $m = 1, \cdots, M$
   - Otherwise i.e. not remarked
     - $q(m, t_n) = q(m, t_{n-1}) - \phi(m, j)(t_n - t_{n-1})$
     - $\phi(m, j)$: no update

     for $m = 1, \cdots, M$