Note on Deal Contingent Trades

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1 Structure

The firm enters a derivative trade such as FX forwards or FX options with a counterparty where the trade is contingent on an agreed deal such as merger, regulatory approval, etc.

Consider such a deal-contingent trade (DCT) and its hedge trade, both of which are booked in the trading book. Figure 1 illustrates two situations:

- Deal succeeds where DCT is well hedged throughout its life.
- Deal fails where there is a potential sudden P&L due to (i) the mark-to-market is *released* from DCT and the PV changes from the *naked* hedge.

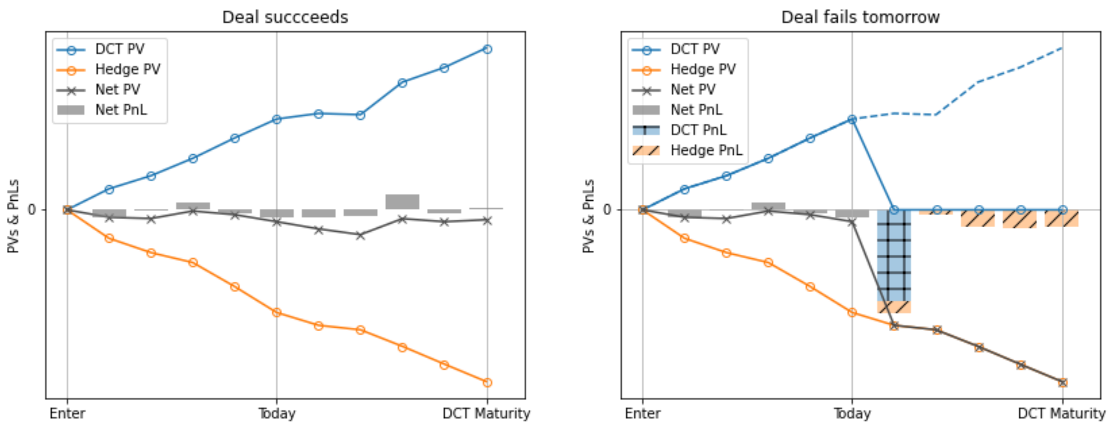

Figure 1: left: The deal succeeds. right: The deal fails tomorrow. There are two sources of P&Ls: (i) blue bar with '+': The mark-to-market of DCT drops to zero. (ii) orange bar with '/': The mark-to-market of the hedge trade moves.

2 Risk-Not-In-VaR

2.1 Risk-Not-In-VaR P&L

For typical VaR models, it is not easy to incorporate the risk of deal failures.

To express the P&L of this missing risk, let

- V: mark-to-market of DCT
- $\bar{V}$: today's mark-to-market
- ΔV: random variable to represent the total P&L in V
- ΔV^{iv}: the part of ΔV included in VaR.
  For example, if DCT is a simple FX forward on the spot rate z,
$$\Delta V^{\mathrm{iv}} = \delta_z \cdot \Delta Z \tag{1}$$
  where δ_z is the first-order sensitivity and ΔZ is the random variable representing the change in z.
- ΔV^{niv}: the part of ΔV not included in VaR
- F is the random variable indicating the deal failure:
$$F = \begin{cases} 1, & \text{if the deal has failed.} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Then, we have
$$\Delta V = -\bar{V} \cdot F + \Delta V^{\mathrm{iv}} \cdot (1 - F) \tag{3}$$
and
$$\begin{aligned} \Delta V^{\mathrm{niv}} &= \Delta V - \Delta V^{\mathrm{iv}} && (4) \\ &= -\bar{V} \cdot F - \Delta V^{\mathrm{iv}} \cdot F && (5) \end{aligned}$$

With multiple deals, each of which denoted by i,
$$\Delta V^{\mathrm{niv}} = \sum_{i=1}^{I} \left[-\bar{V}_i \cdot F_i - \Delta V_i^{\mathrm{iv}} \cdot F_i \right] \tag{6}$$

2.2 Model

To move Eq (6), we use a simple multi-variate normal distribution.

2.2.1 Failure Indicator

To model F_i, we can use a standard framework used for credit default modelling.

Let P_i be the probability of the deal failure. To simulate the deal failure events, let X_i be an $N(0,1)$ random variable, indicating the deal quality and set
$$F_i := \begin{cases} 1, & \text{if} \quad \Phi(X_i) < P_i, \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

For joint simulations of $\{F_i\}_{i=1}^{I}$, the correlation should be specified:
$$\rho_{i,j}^{F} := \mathrm{corr}(F_i, F_j). \tag{8}$$

2.2.2 VaR P&Ls

To model ΔV_i^{iv}, without loss of generality, assume that it can be written as a function of $\Delta\boldsymbol{Z}$, the return distribution of a set of risk factors $\boldsymbol{z} = [z_1, \cdots, z_K]$ included in VaR:

$$\Delta V_i^{\text{iv}} := \Delta V_i^{\text{iv}}(\Delta\boldsymbol{Z}) \tag{9}$$

As an example, see Eq (1).

Remark 2.1: VaR P&L functions

ΔV_i^{iv} is the same P&L (approximation) function used in VaR.

We assume that ΔZ follows a multi-variate normal distribution with

$$\Delta Z_k \sim N(\mu_k, \sigma_k^2) \quad \text{and} \quad \text{corr}(\Delta Z_k, \Delta Z_l) = \rho_{k,l}^Z. \tag{10}$$

2.2.3 Correlation: Deal Failures and Risk Factors

Finally, the correlations among deal quality indices X_i's and VaR risk factors Z_k's should be specified:

$$\rho_{i,k}^{F,Z} = \text{corr}(X_i, Z_k) \tag{11}$$

2.3 Model Parameter Estimations

Postulations: The following parameters are postulated by appropriate *experts*:

- Failure Probability and Correlations: $\{P_i\}$ and $\{\rho_{i,j}^F\}$ where $i, j = 1, \cdots, I$
- Deal vs market correlations: $\{\rho_{i,k}^{F,Z}\}$ for $i = 1, \cdots, I$ and $k = 1, \cdots, K$.
  Due to the nature of typical deal contingent trades, they would be set to zero.

Calibrations: The following risk factor model parameters

$$\{\mu_k\}, \{\sigma_k\} \text{ and } \{\rho_{k,l}^Z\} \text{ where } k, l = 1, \cdots, I:$$

would be calibrated to simulated returns of the relevant historical VaR models.

2.4 Quantification

1. Run standard Monte Carlo simulations on
$$\{X_i\}_{i=1}^I \quad \text{and} \quad \{Z_k\}_{k=1}^K. \tag{12}$$
2. Generate scenarios of the risk-not-in-VaR P&L ΔV^{niv} using Eq (6).
3. Calculate the 99th tail measure.