Lecture 8 - Significance tests

Wednesday, October 03, 2012
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CHAPTER 6

Key strategy for hypothesis testing:

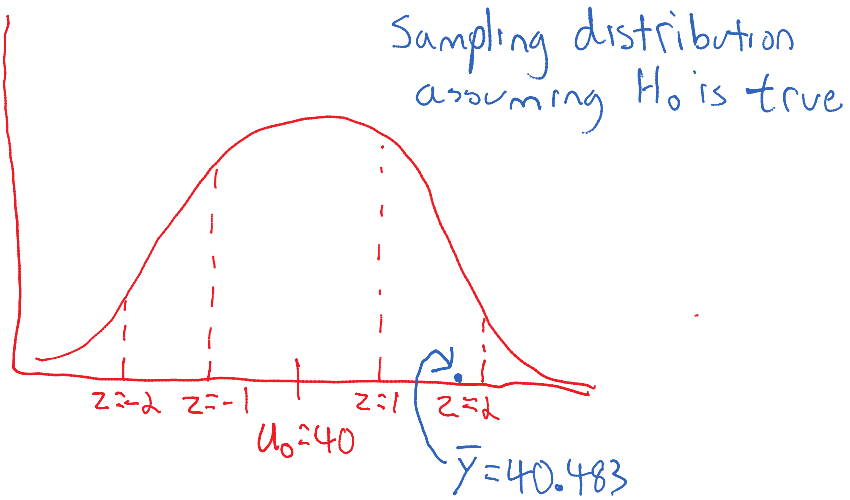
- Assuming that the null hypothesis, H_0, is true (i.e., $\mu = \mu_0$), how unlikely would it be to observe the sample mean, y, that we observed?
- What would the sampling distribution look like if the null hypothesis is true?
- Imagine this sampling distribution is true; we randomly pick sample means from the sampling distribution;
  - Z-score tables; what is the probability of a sample mean being at least 1.96 std dev higher than the population mean? What is the probability of a sample mean being within 1.96 std dev of the population mean? What is the probability of sample mean being greater than 1.96 away from null mean in *either direction* (i.e., Pr(z>1.96) + pr(z<-1.96))?

Research question: Is the population mean number of hours worked equal to 40?

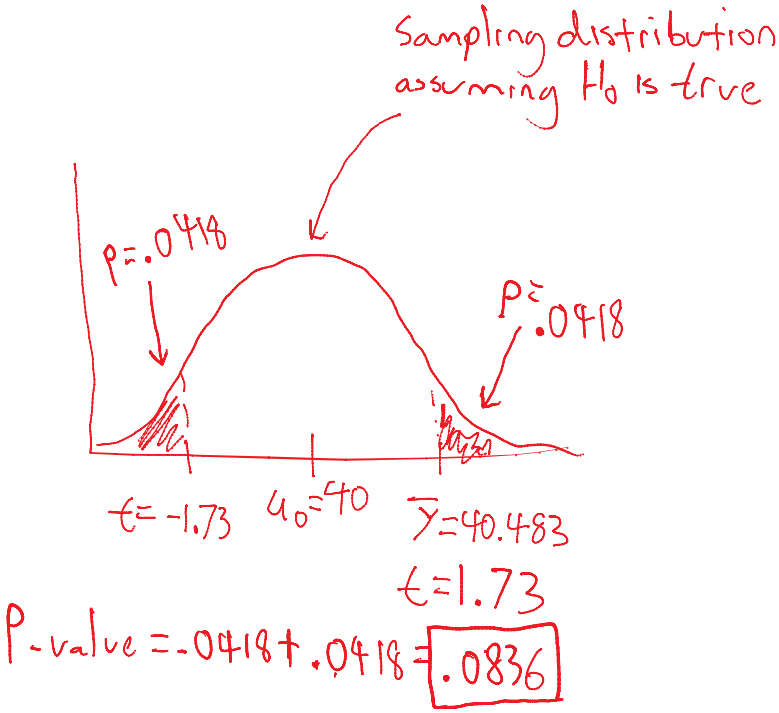
- Null hypothesis (H_o)
  - H_o: $\mu = \mu_o = 40$
- Two sided alternative hypothesis (H_a)
  - Two-sided: H_a: $\mu \neq 40$

Visualizing the test statistic

Draw sampling distribution under the assumption that the null hypothesis is true



(4) P-value



- Question: What is the probability of getting a result at least that many standard deviations away from the mean (in either direction)?

Example 2

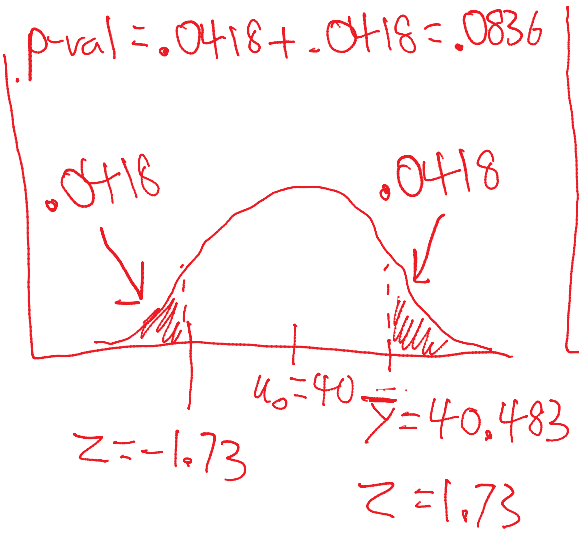
Research question: Is the population mean number of hours worked equal to 40?

- Null hypothesis (H_o)
  - H_o: $\mu = \mu_o = 40$
- One sided alternative hypothesis (H_a)
  - One-sided: H_a: $\mu > 40$

(4) P-value

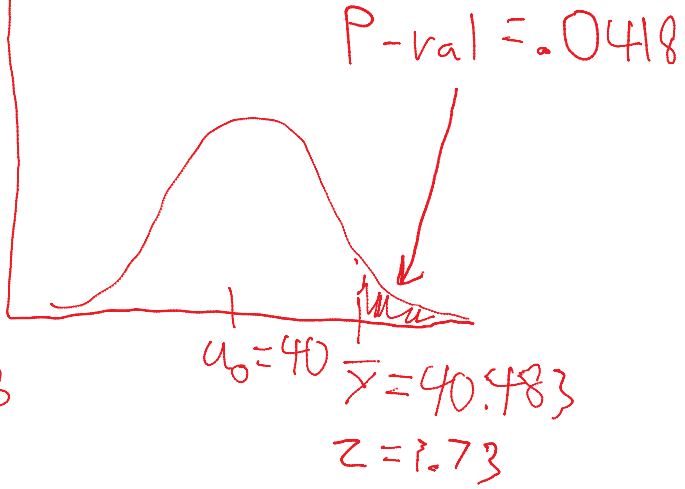
Example 1 $H_a: \mu \neq 40$

$p\text{-val} = Pr(t > 1.73) + Pr(t < -1.73)$



Example 2 $H_a: \mu > 40$

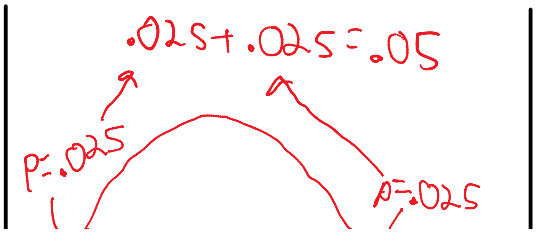
$Pvalue = Pr(t > 1.73)$



Rejection region:
This is something we decide *before* we test hypotheses

Two sided hypothesis

α = rejection region = .05



One sided hypothesis

$H_a: \mu > \mu_o$

reject H_0 if $t > 1.96$ or $t < -1.96$

reject H_0 if $t > 1.645$

Significance testing for Proportions

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. tab msamesex
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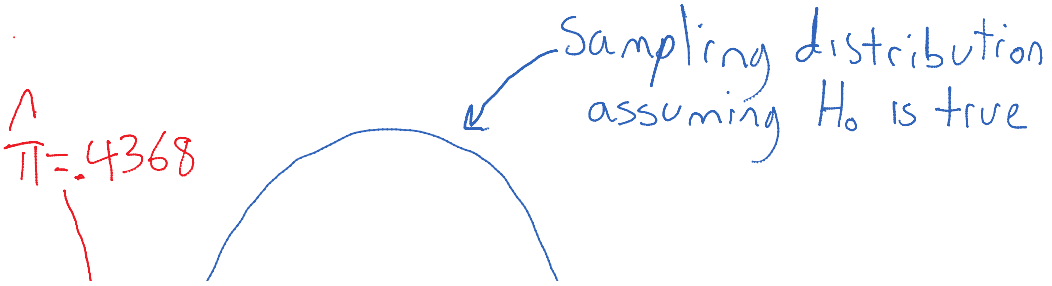
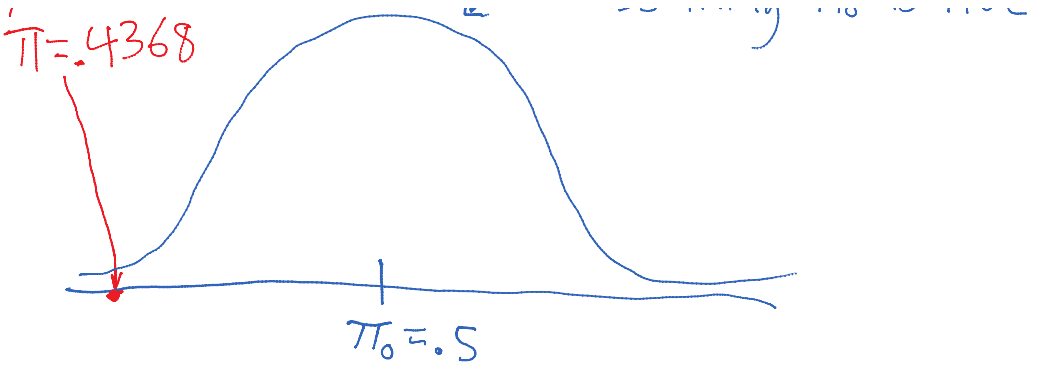
should same sex couples be allowed to marry?	Freq.	Percent	Cum.
0. no	1,792	56.32	56.32
1. yes	1,390	43.68	100.00
Total	3,182	100.00	

(3) Test statistic

Remember, test statistic is based on the assumption that the null hypothesis is true

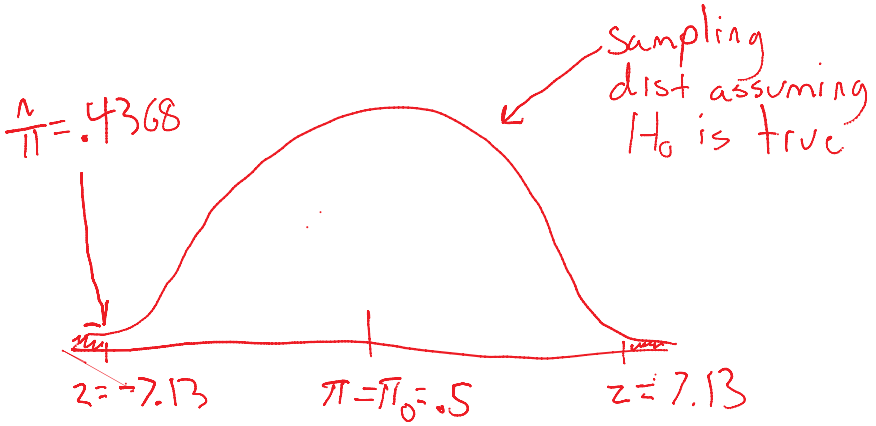
- What does the sampling distribution look like if the null hypothesis is true?
- If the null hypothesis is true (i.e., $\pi = .5$), how unlikely would it be to randomly choose a sample mean from the sampling distribution and have it be as far away from the population mean as the one we picked ($\pi = .4368$)?





P-value

How unlikely would it be to pick a sample mean 7.13 standard deviations away from the population mean (in either direction)



P-value = Pr(z > 7.13) + Pr(z < -7.13)

P-value ≈ .0000 + .0000 = .0000

Example 2

Step 4: p-value

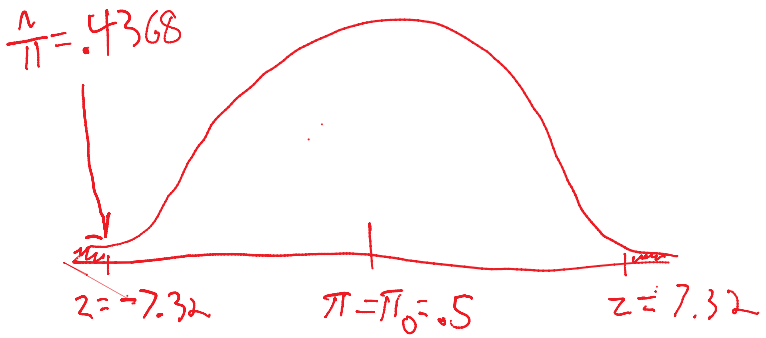
Two Sided

One Sided

Two Sided

Example 1: $H_a: \pi \neq .5$

P-value = $pr(z > 7.13) + pr(z < -7.13)$



P-value = $Pr(z > 7.13) + Pr(z < -7.13)$

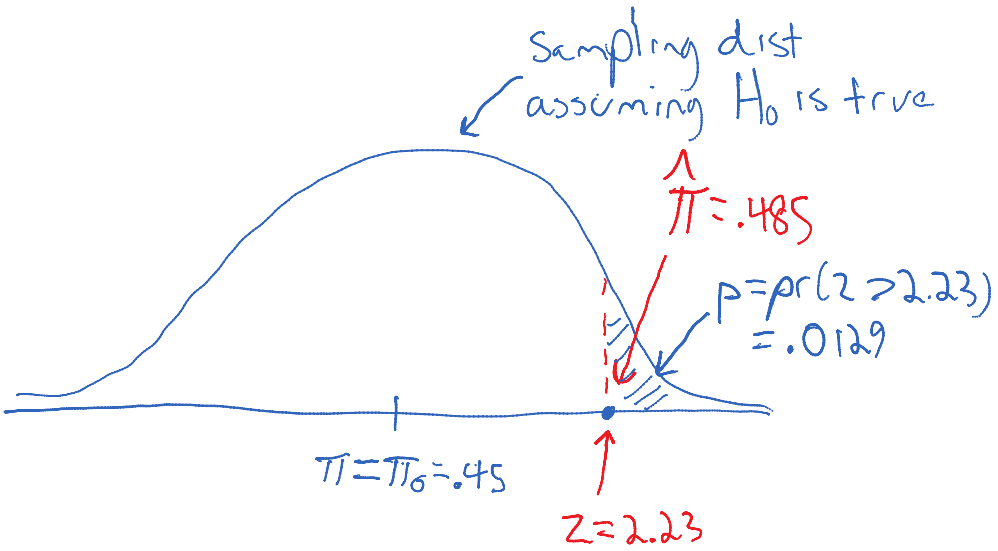
P-value $\approx .0000 + .0000 = .0000$

One Sided

Example 2: $H_a: \pi < .5$

P-value = $Pr(z < -7.13) = .0000$

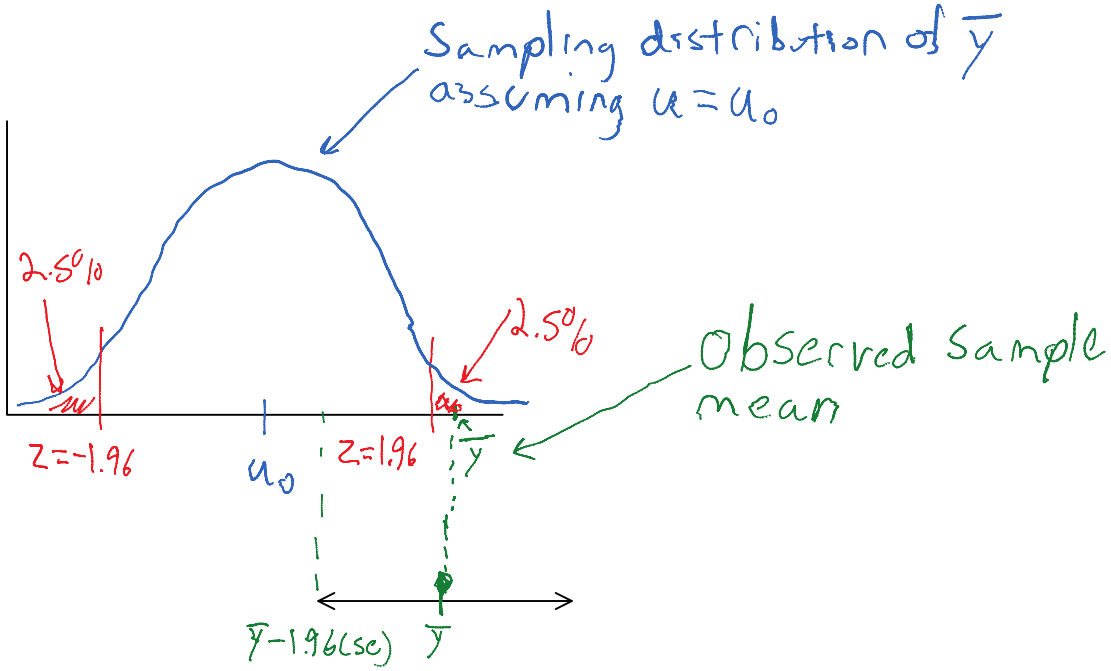
Sampling dist for in-class exercise



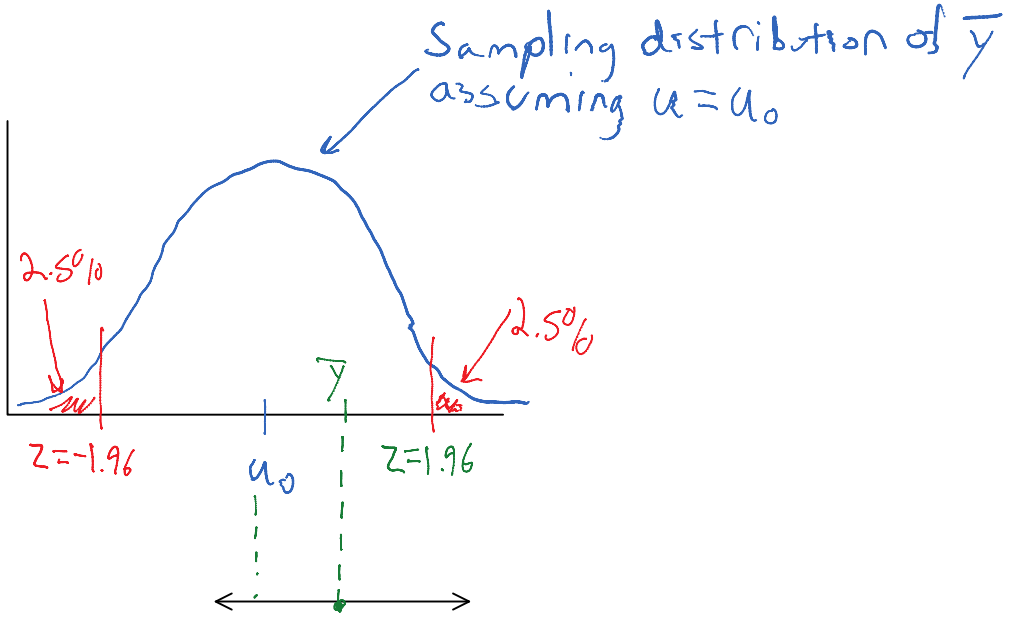
Equivalence between 95% CI and two-sided significance test

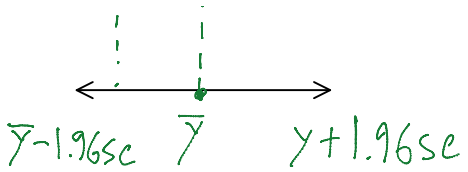
Imagine we have two-sided alternative hypothesis,
using an alpha level of .05
So, we reject H_0 if t>1.96 or t<-1.96

- If p-value<=.05 (i.e., reject H_0)
  - If p-value<=.05 in a two-sided test, a 95% CI for μ does not contain μ_0
  - **Equivalently, if 95% CI for μ does not contains μ_0 then we reject H_0**



- If p-value>.05 (i.e., do not reject H_0)
  - When p-value>.05 in a two-sided test, the 95% CI for μ contains μ_0 (associated with null hypothesis, H_0)
  - **Equivalently, If 95% CI for μ contains μ_0 then we do not reject H_0**





Significance testing in Stata

Significance test for mean

Is population mean number of hours worked equal to 40? H_0: $\mu = 40$

Null hypothesis
H_0: $\mu = 40$

```
. ttest hrs1==40

One-sample t test

------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    hrs1 |    2820     40.48298    .279642    14.85001    39.93466     41.0313
------------------------------------------------------------------------------
    mean = mean(hrs1)                                             t =   1.7271
Ho: mean = 40                                     degrees of freedom =     2819

    Ha: mean < 40                 Ha: mean != 40                  Ha: mean > 40
 Pr(T < t) = 0.9579         Pr(|T| > |t|) = 0.0843          Pr(T > t) = 0.0421
```

$\bar{Y}$ se Std dev 95% CI

H_a: $\mu \neq 40$ H_a: $\mu > 40$

p-value p-value

Not worth testing
H_a: $\mu < 40$ because
$\bar{Y} = 40.483 > 40$

Significance test for proportion

Is population proportion of people who agree with same sex marriage equal to .5? H_0: $\pi = .5$

```
. prtest msamesex==.5

One-sample test of proportion                msamesex: Number of obs =     3182
```

Variable	Mean	Std. Err.	[95% Conf.	Interval]
msamesex	.4368322	.0087928	.4195987	.4540657

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    p = proportion(msamesex)                                  z =  -7.1265
Ho: p = 0.5

     Ha: p < 0.5                 Ha: p != 0.5                 Ha: p > 0.5
 Pr(Z < z) = 0.0000         Pr(|Z| > |z|) = 0.0000          Pr(Z > z) = 1.0000
```