Bernoulli Line Ensembles and the Airy 2 Process

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Gaussian universality (CLT, Donsker Theorem)

Increase the number of walkers (avoiding Bernoulli random walks and Dyson BM)

Airy Line Ensemble

What happens as N (number of walkers) goes to infinity -¿ new type of limit occurs Airy line ensemble, top curve is the Airy process. Increasing the number of paths pushes us outside of the Gaussian universality class and into what is called the "KPZ universality class"

Big open problem: Show that for "generic random walks" with "generic" initial conditions we have convergence to Airy LE. This problem is open even for Bernoulli random walks (only known if all are started from 0)

Finite dimensional convergence (very hard, algebraic need good formulas)
Show tightness (i.e. existence of weak subsequent limits) – easier,
qualitative/analysis. You need to control min, max, modulus of continuity

Our Result

Main result here: if top line 1 point marginals at integer times go to Tracy-Widom then the full LE is tight.

$P(L_1(nN^{2/3}) - nN^{2/3}p + \lambda n^2 N^{1/3} \leq N^{1/3}x) \to F_{TW}(x)$

Compare to Virag+Duavergne+Nica '19 (they assume fd convergence to Airy Line ensemble vs we assume only 1 point convergence of the top line to TW)

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Introduction
(5-6 min)

Convergence to
Airy Line
Ensemble (6-7
min)

Section of
Paper (7-9
min)

Arguments are inspired by [Corwin-Hammond '14, '15] (continuous setting) [Corwin-Dimitrov '17] (discrete setting)

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Description of the problem (min, max and modulus of continuity) 2 min
$P(\max_{[-r,r]} L_1(sN^{2/3}) - psN^{2/3} > RN^{1/3}) < \epsilon$ if R is large enough

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Proof (mention monotone coupling lemmas somewhere) - say MC with picture 2min

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Proof (mention strong coupling somewhere) - say SC with picture L = Bernoulli bridge B is a Brownian bridge with variance. There is a probability space such that $P(sup|L - B| \geq k(\log N)^2) < \epsilon$. This is a comparison that allows for example to compare the modulus of continuity of the two. [Dimitrov-Wu '19] 2 min

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Remind: 1. Max on [r, R] and [-R, r] does not deteriorate. 2. Monotone coupling lemmas show that min [-r, r] does not dip to -infty – this controls min. 2 min

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min)

(if you have time) In the last (-1 min) we can say a few words about why
the max on [r,R] does not deteriorate – utilizes built in convexity of the
problem. Focus on 2 lines
$P(L_1(nN^{2/3}) - nN^{2/3}p + \lambda n^2 N^{1/3} \leq N^{1/3}x) -> F_{TW}(x)$ - emphasize the
parabolic shift