Asymptotics of Bernoulli Line Ensembles

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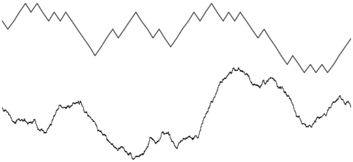
The Gaussian universality class

Let $\{X_i\}$ be a sequence of i.i.d. random variables, s.t. $\mathbb{E}[X_1] = \mu$, $Var(X_1^2) = \sigma^2$. Let $S_n = \sum_{i=1}^n X_i$:

- **Law of Large Numbers:** $\frac{S_n}{n} \to \mu$ as $n \to \infty$ almost surely
- **Central Limit Theorem:** $\frac{S_n - n\mu}{\sqrt{n}} \to N(0, \sigma^2)$ as $n \to \infty$
- **Donsker's Theorem:** Let $S(x) = S_k$ if $x = k$ and linearly interpolate for $x \in [0, n]$ Let $\mu = 0$ and $\sigma = 1$. Then $\frac{S(n\cdot)}{\sqrt{n}} \in C([0,1])$ and $\frac{S(n\cdot)}{\sqrt{n}} \to B(\cdot)$, where B denotes a standard Brownian Motion.



Increase the number of walkers (avoiding Bernoulli random walks and Dyson BM)

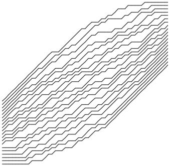
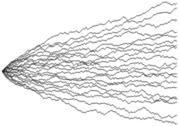


Figure: Multiple Bernoulli Random Walks



As $N \to \infty$, the rescaled walks converge in distribution, uniformly over compact sets of $\mathbb{N} \times \mathbb{R}$, to the Airy line ensemble, $\mathcal{A}$, and the top curve converges to Airy process, $\mathcal{A}_1$.

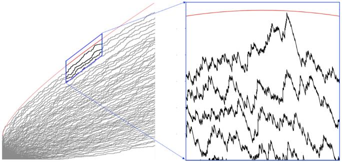


Figure: Multiple Dyson Brownian walks

Increasing the number of paths pushes us outside of the Gaussian universality class and into Kardar-Parisi-Zhang (KPZ) universality class.

Open Question

Show that any random walks with generic initial conditions convergence to the Airy line ensemble.

Convergence to the Airy Line Ensemble

Two sufficient conditions:

1. Finite dimensional distribution convergence
2. Tightness, or the existence of weak subsequential limits.

We focused on tightness, which requires a maximum, minimum, and conditions on the Modulus of Continuity

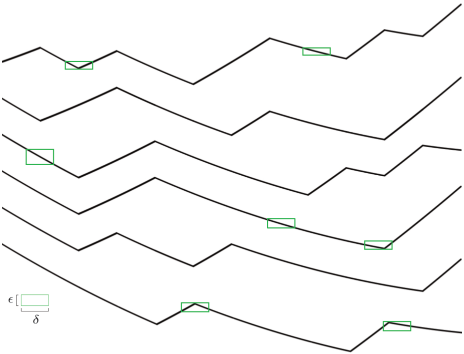


Figure: The Modulus of Continuity

Our Result

Theorem

For $n \in \mathbb{Z}$, there exists a subsequence N_T such that

$$\lim_{T\to\infty} P(L_1^{N_T}(nN^{\alpha}) - nN_T^{\alpha}p + \lambda n^2 N_T^{\alpha/2} \leq N_T^{\alpha/2}x) \to F_{TW}(x)$$

If the one-point marginal probabilities at integer times weakly converge to the Tracy Widom distribution then the Line Ensemble is tight.

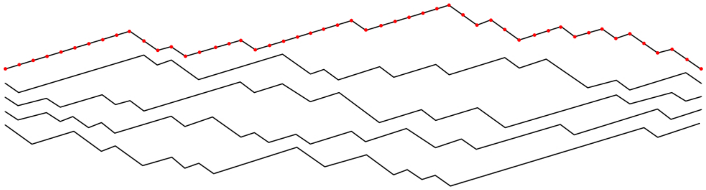


Figure: Integer time points of top line

Improvements

[Duavergne, Nica, & Virag, 2019] - tightness assuming finite dimensional convergence to the Airy Line Ensemble.
We achieve the same result with much less restrictive assumptions

[Unsure of Image Choice]

Arguments are inspired by [Corwin-Hammond '14, '15] (continuous setting) [Corwin-Dimitrov '17] (discrete setting) Description of the problem (min, max and modulus of continuity) 2 min $P(\inf_{s\in[-r,r]} L_1^N(sN^{2/3}) - psN^{2/3} < -RN^{1/3}) < \epsilon$ if R, N are large enough

Proof (mention monotone coupling lemmas somewhere) - say MC with picture 2min

Proof (mention strong coupling somewhere) - say SC with picture L = Bernoulli bridge B is a Brownian bridge with variance. There is a probability space such that $P(sup|L - B| \geq k(\log N)^2) < \epsilon$. This is a comparison that allows for example to compare the modulus of continuity of the two. [Dimitrov-Wu '19] 2 min

Controlling the minimum: pinning the bottom curve

Lemma (——)

For any $r, \epsilon > 0$, there exists $R > r$ and a constant $A > 0$ so that for large N,

$$\mathbb{P}\Big(\max_{x \in [r,R]} \big(L_k^N(xN^{2/3}) - pxN^{2/3}\big) \leq -AN^{1/3} \Big) < \epsilon.$$

The same is true of the maximum on $[-R, -r]$.

- Recall our assumption:

$$\mathbb{P}\left(L_1^N(nN^{2/3}) - pnN^{2/3} + \lambda n^2 N^{1/3} \leq xN^{1/3}\right) \xrightarrow[N\to\infty]{} F_{TW}(x).$$