$$\boxed{L_H(v) = Hv}$$

$$\boxed{L_{I-H}(v) = (I-H)v}$$

$$X(X^TX)^{-1}X^TY = HY = \hat{Y}$$

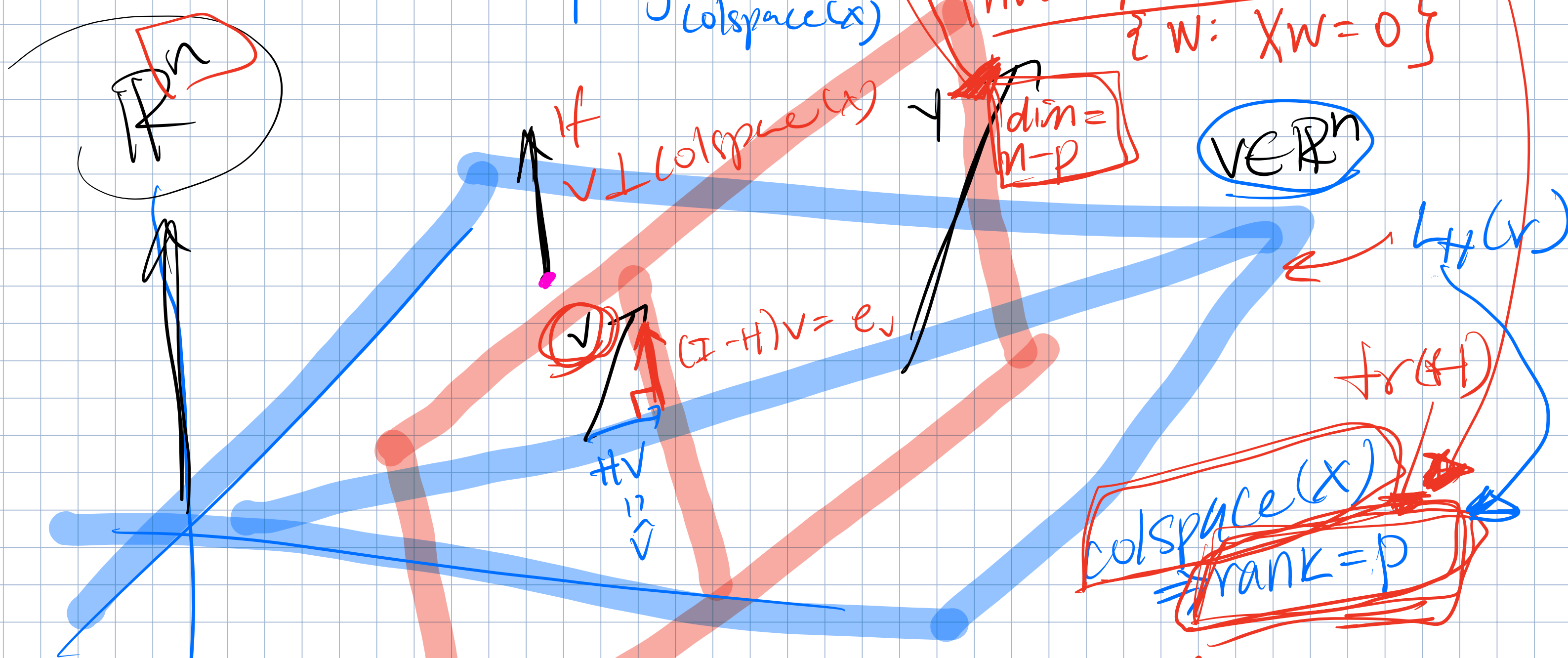
$tr(I-H)$

$$H = \underline{X(X^TX)^{-1}X^T}$$

$$v \longrightarrow Hv$$

$$= proj_{colspace(X)}(v)$$

$\boxed{nullspace(X)}$ $\{W: XW = 0\}$



$$v = \hat{v} + e \Rightarrow \hat{v} \perp e$$

$$nullspace(X) \perp colspace(X)$$

$$= span\{1_n, X_1, X_2, \dots, X_{p-1}\}$$

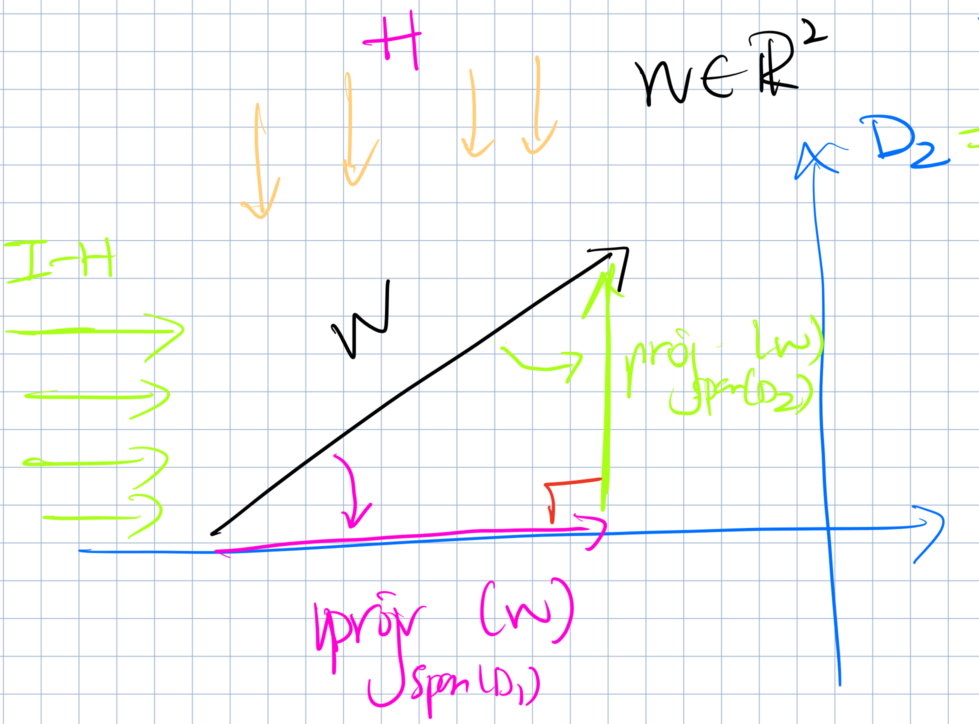
if $(I-H)v = Iv - Hv$

$$= v - \hat{v}$$

$$tr(H) = tr(X(X^TX)^{-1}X^T)$$

$$= tr(\cancel{(X^TX)^{-1}}\cancel{X^TX})$$

$= tr(I_p) = p$ ($p\times n\ n\times p$)



$X_2 \sim X_1$

$res_2 = X_2 - \hat{X}_2$

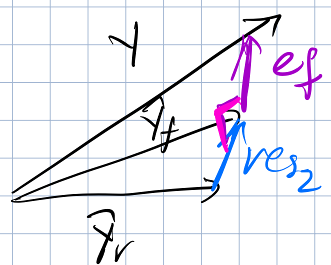
$= \underbrace{X_2 - (X_1(X_1^T X_1)^{-1} X_1^T X_2)}$

$X_2 - H_{X_1} X_2$

$\langle (I-H)Y \,,\, res2 \rangle$

$\langle a, b \rangle =_{1\times 1}$

$a^T b$ ($1\times n \ n\times 1$)



$((I-H)Y)^T res_2$

$\underline{Y}^T \underbrace{(I-H)\, res_2} = 0$

$Y^T (I-H)(X_2 - H_{X_1} X_2)$

$= Y^T (I-H)(I - H_{X_1}) X_2$

$H H_{X_1} =$

$\underline{H} X_1 (X_1^T X_1)^{-1} X_1^T$

$= X_1 (X_1^T X_1)^{-1} X_1^T = H_{X_1}$

$$= Y^T(I - H - H_{X_1} + HH_{X_1})X_2$$

$$= Y^T(I - H - \cancel{H_{X_1} + H_{X_1}})X_2$$

$$= Y^T(X_2 - H\underline{X_2}) = Y^T(X_2 - X_2) = 0$$

Heteroskedasticity

→ Transformation

→ Robust SEs

→ $\boxed{\text{WLS}}$

β unknown

Σ known

$$Y = X\beta + \varepsilon \qquad \varepsilon \sim N(0, \Sigma)$$

$\boxed{\text{not } \sigma^2 I}$

potentially hetero...

If Σ is know

$$\Sigma^{-1/2} Y = \Sigma^{-1/2} X\beta + \Sigma^{-1/2}\varepsilon$$

$$\tilde{Y} = \tilde{X}\beta + \tilde{\varepsilon}$$

$$Var(\tilde{\varepsilon}) = Var(\Sigma^{-1/2}\varepsilon)$$

$$= \Sigma^{-1/2} Var(\varepsilon) (\Sigma^{-1/2})^T$$

$$= \Sigma^{-1/2}\, \Sigma\, \Sigma^{-1/2}$$

$$= \underbrace{\Sigma^{-1/2} \Sigma^{1/2}}\, \underbrace{\Sigma^{1/2} \Sigma^{-1/2}} = I_n$$

RL: β unknown

Σ unknown

$$\tilde{Y} = \hat{\Sigma}^{-1/2} Y$$

$$\tilde{X} = \hat{\Sigma}^{-1/2} X$$

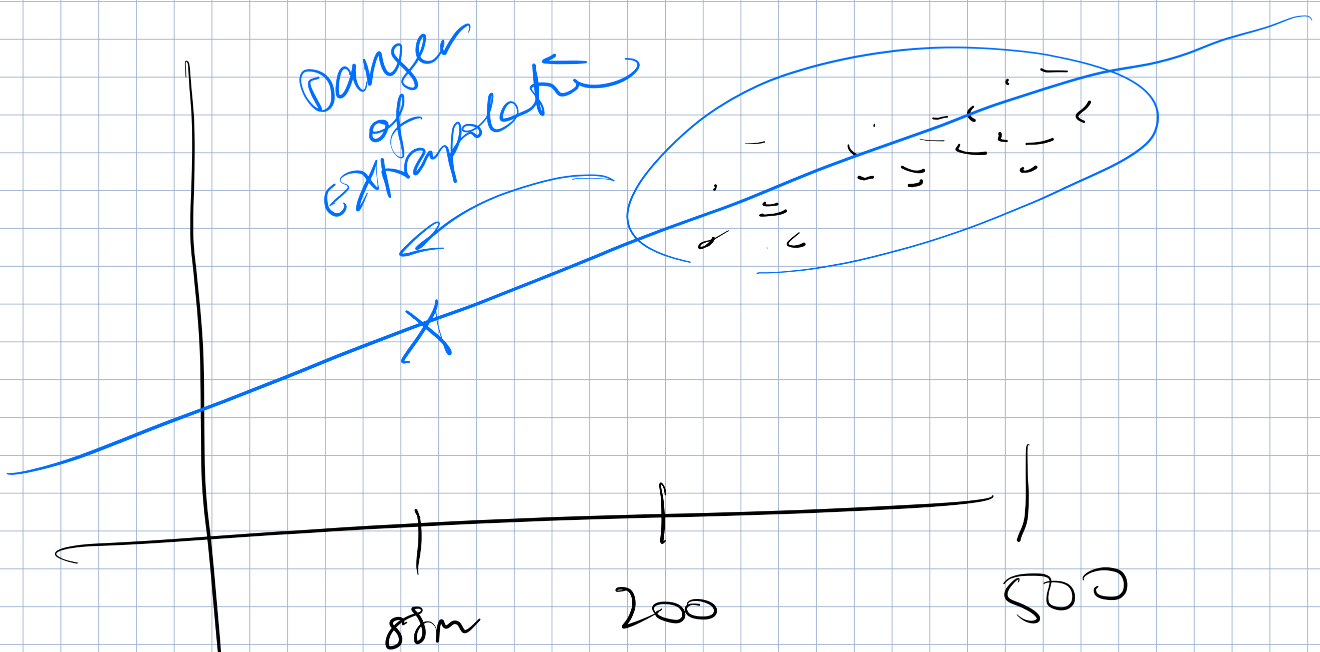
$$\tilde{\Sigma} = \hat{\Sigma}^{-1/2} \Sigma$$

How to estimate Σ?

- MLE
- if diagonal

$\epsilon_i^2 \sim X_1 + \cdots + X_p$

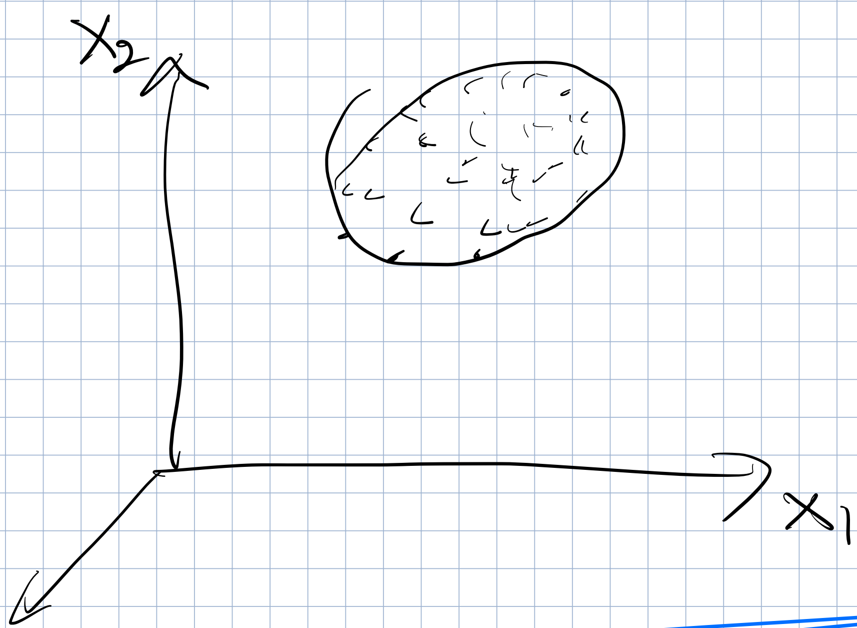
↳ $\hat{\sigma}_1^2 \cdots \hat{\sigma}_n^2$



$X = (1_n \; x_1 \; x_2)$

if x_1 & x_2 are not lin. dep.

then



For a single β

F-test:

↳ Global

→ Partial (type = 2)

$\boxed{\text{→ Sequential (type = 1)}}$

general partial F-test

multiple $H_0: \beta_j\text{'s} = 0$

$$SSE = \|Y - \hat{Y}\|^2$$

$$SSE = \underline{\underline{\sum_{i=1}^{n} e_i^2}}$$ length^2 of e

$$\underline{\|V\|^2} = \langle v, v \rangle$$

$$= \underset{1\times n \; n\times 1}{v^T v}$$

$$SSE = \|e\|^2 = \langle e, e \rangle$$

$$= e^T e = (e_1^2 + e_2^2 + e_3^2 + \cdots + e_n^2)$$

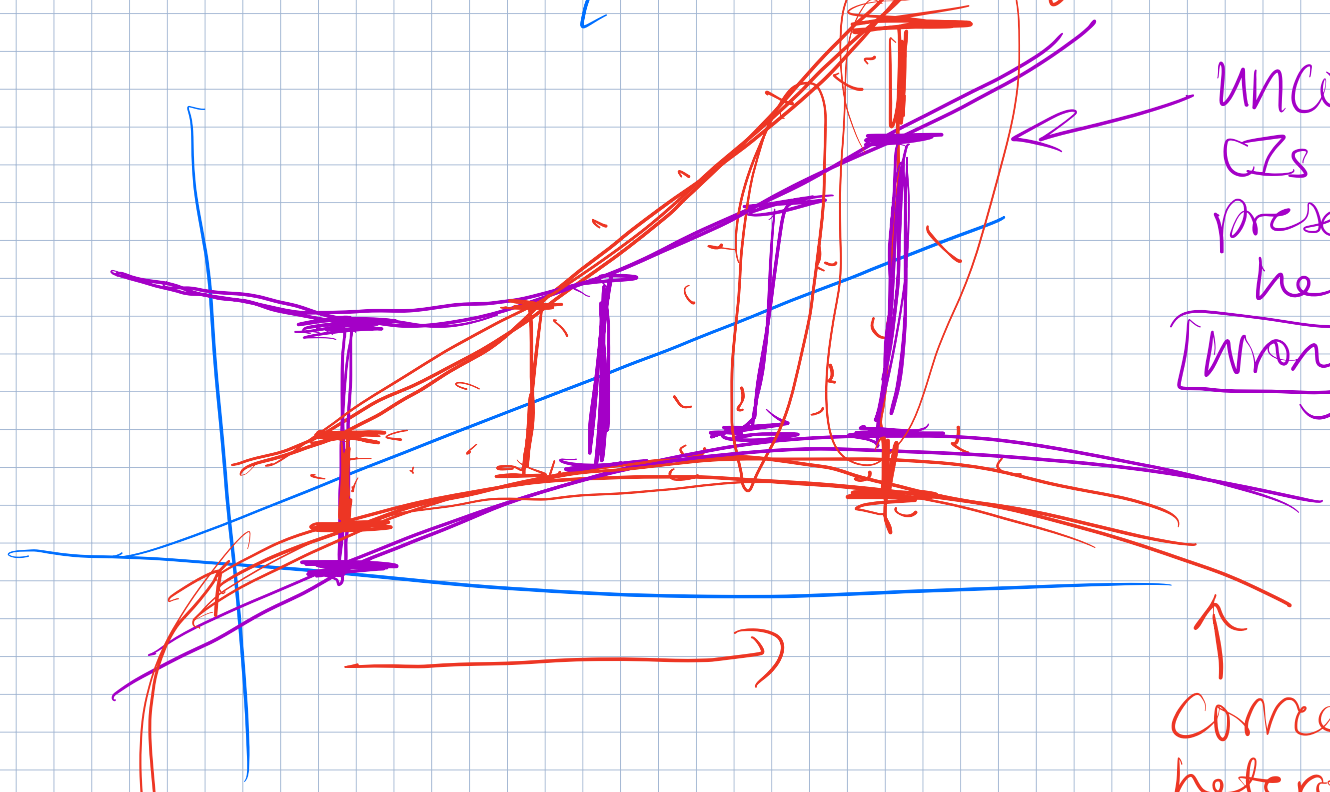
$$SST = SSR + SSE$$

$$\|Y - \underline{\bar{Y}}\|^2 = \|\hat{Y} - \bar{Y}\|^2 + \|Y - \hat{Y}\|^2$$

Dummy Notation

$$X = \begin{cases} 1 & \text{vegetarian} \quad (\text{"veg"}) \\ 0 & \text{not vegetarian} \quad (\text{"not"}) \end{cases}$$

$$\mathbb{1}(x_i = \text{"veg"}) = \begin{cases} 1 & \text{if } x_i = \text{"veg"} \\ 0 & \text{if not} \end{cases}$$



$\mathrm{tr}(H) = \mathrm{rank}(H)$ if H is projection

$\mathrm{tr}(ABC) = \mathrm{tr}(BCA)$

High Leverage	outlier in X
High Discrepancy	outlier in Y
High Influence	combination of both

Cooks D

ext. stand resid = high discrepancy