Spatial Prisoner's Dilemma

CMII Project

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The aim of this project was to replicate and extend on the results presented in the 1992 Nature article "**Evolutionary Games and Spatial Chaos**" by Martin A. Nowak & Robert M. May.

To achieve this, a Python program was developed to simulate an iterated game of **Spatial Prisoner's Dilemma**.

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What is Evolutionary Game Theory?

- **Classical Game Theory**: Studies conflict and cooperation between rational players aiming to maximize their gain in one-time interactions.
- **Evolutionary Game Theory**: It's the study of frequency-dependent selection. Involves repeated interactions where strategies evolve based on success.
- **Keywords**:
  - **Payoff** (Fitness): Strategies (Phenotypes) that perform better, spread faster in the population
  - **Spatial Games**: Explore how strategies evolve in a spatial setting, simulating population dynamics.

Prisoner's Dilemma

Two defendants accused of the same crime are given the options to either **cooperate (C)** by staying silent or **betray (D)** each other by confessing. The outcomes based on their choices are:

- If both cooperate (C, C), they receive light sentences (R).
- If one defects (betrays) while the other cooperates (D, C), the betrayer goes free (T), and the cooperator gets a heavy sentence (S).
- If both betray (D, D), they receive moderate sentences (P).

The **payoff matrix**:

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

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Spatial PD game setup

- We consider only **two kinds of players**:
  1. **C**: Players who always cooperate
  2. **D**: Players who always defect
- We place them in a $n \times n$ **fixed boundary lattice** with each site occupied either by a C or a D
- In each round every player plays the **PD game** with their **8 nearest neighbors** (9, if self-interaction is added)
- The **payoff matrix** of the game:

$$\textbf{Payoff} = \begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix}$$

Spatial PD game setup

- The **total round score** of a player is the **sum** of the individual game payoffs from the interaction with their neighbors
- At the start of the next round, each lattice site is occupied by the player with the **highest score** among the previous owner and their neighbors.
- **4 types of possible transitions**:
  1. C $\longrightarrow$ C
  2. C $\longrightarrow$ D
  3. D $\longrightarrow$ C
  4. D $\longrightarrow$ D
- The game continues until all rounds are completed

Payoff b vs Dynamical behavior

The **dynamical behavior of the system is critically dependent on the value of payoff b**. Small changes in b can lead to totally different dynamical regimes.

8-Neighbor + self-interaction:

1. $b > 1.8$: large D clusters continue to grow
2. $b < 1.8$: D clusters shrink
3. $b < 2$: large C clusters continue to grow
4. $b > 2$: C clusters do not grow
5. $1.8 < b < 2$ (**interesting region**): Both C and D clusters can keep growing.
   - Dynamic Chaos
   - Defector Invasion case: $f_C = 0.318$ (dynamic equilibrium).

Payoff b vs Dynamical behavior

8-Neighbor interaction (no self)

1. $b > 1.667$: Only D clusters can keep growing
2. $b < 1.6$: Only C clusters can keep growing
3. $1.6 < b < 1.667$ (**interesting region**): Both C and D clusters can keep growing.
   - Spatial Chaos
   - Defector Invasion case: $f_C = 0.299$ (dynamic equilibrium).

Code Implementation

- **Function**:

```python
def spatial_chaos_8_nn(b, p, self_interaction, gens, n, condition,
    evolution_plot, fc_plot, theoretical_limit):
```

Different **values of b** and initial proportion of defectors **p**, led to **different dynamical states**. During each simulation:

- **Player payoff** for each round was calculated
- **Strategy transitions** were updated

Transition Type	Color
C $\rightarrow$ C	**Blue**
D $\rightarrow$ D	**Red**
D $\rightarrow$ C	**Green**
C $\rightarrow$ D	**Yellow**

Code Implementation

- **Frequency of cooperators,** f_C was monitored
- **Evolution plot** and animation were created
- f_C **plot** was displayed

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3 Results

Random Initial Configuration: $1.75 < b < 1.8$

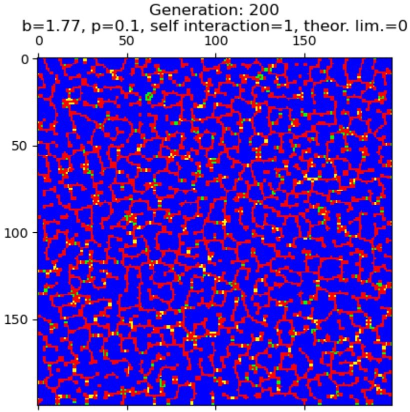


Figure: $1.75 < b < 1.8$, 10 10% D, 90% C randomly populated. Click here for Evolution gif

Random Initial Configuration: $1.75 < b < 1.8$

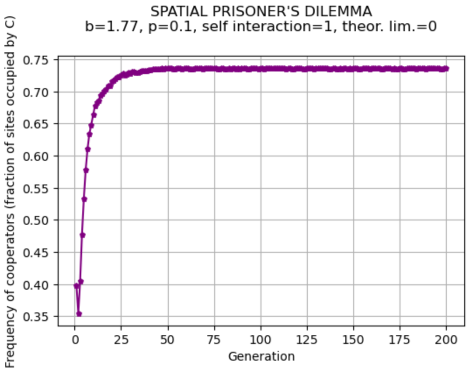


Figure: f_C plot: $1.75 < b < 1.8$, 10% D, 90% C randomly populated.

Random Initial Configuration: $1.8 < b < 2$

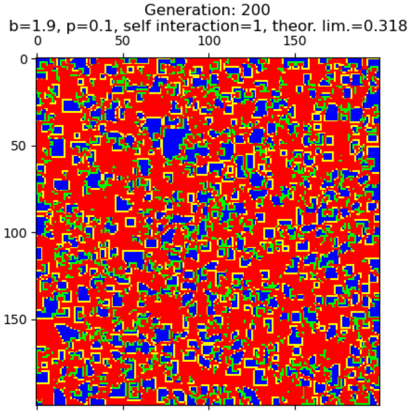


Figure: $1.8 < b < 2$, 10% D, 90% C randomly populated. Click here for Evolution gif.

Random Initial Configuration: $1.8 < b < 2$

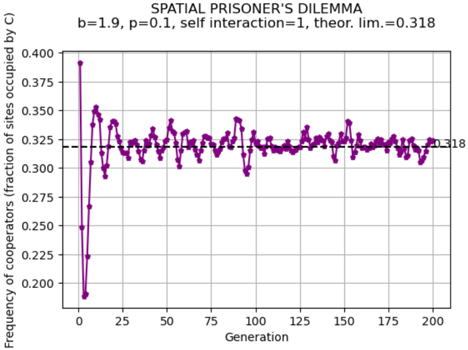
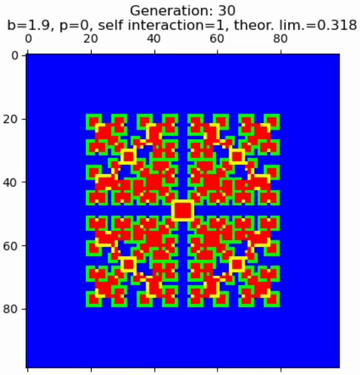
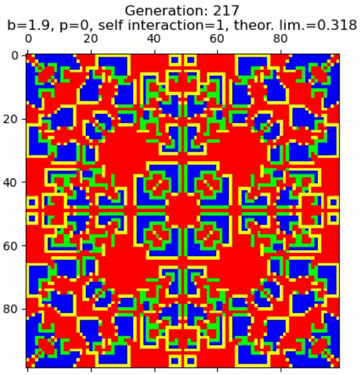


Figure: f_C plot: $1.8 < b < 2$, 10% D, 90% C randomly populated.

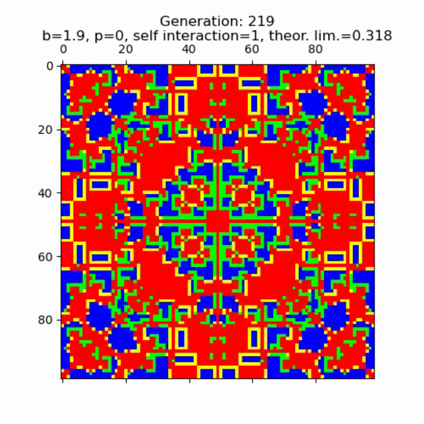
Defector Invasion: Self-Interaction

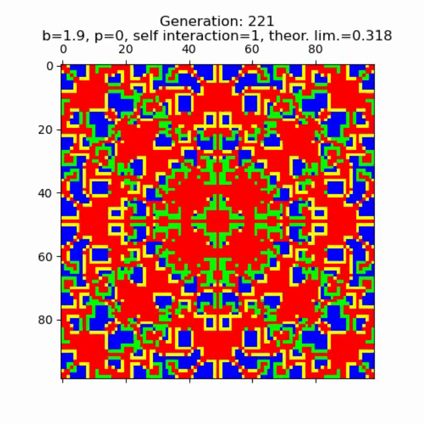
3 Results





Defector Invasion: Self-Interaction (cont.)





Evolution gif

Defector Invasion: Self-Interaction

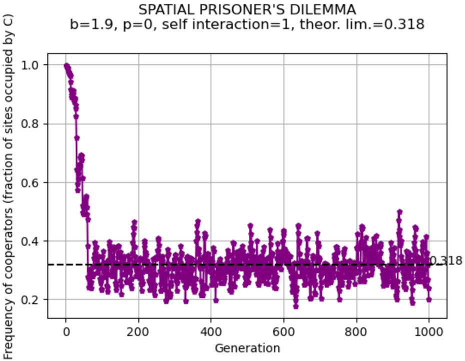
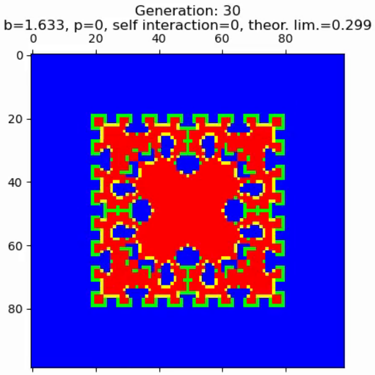
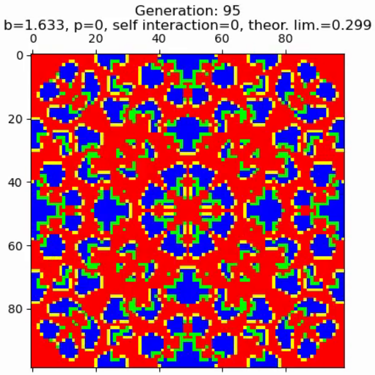


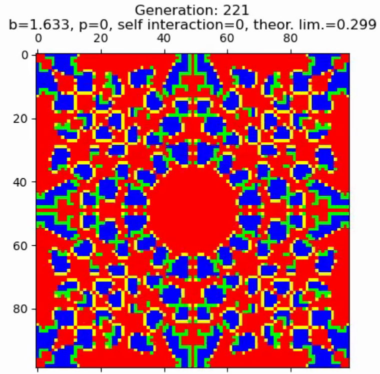
Figure: f_C plot for D invasion with $1.8 < b < 1.2$ and self-interaction

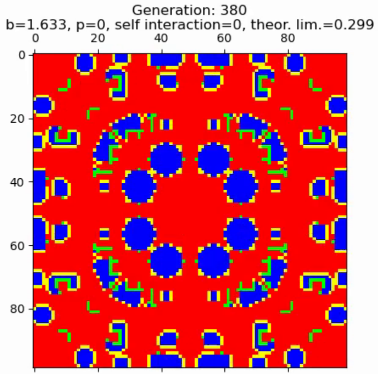
Defector Invasion: No Self-Interaction





Defector Invasion: No Self-Interaction (cont.)





Evolution gif

Defector Invasion: No Self-Interaction

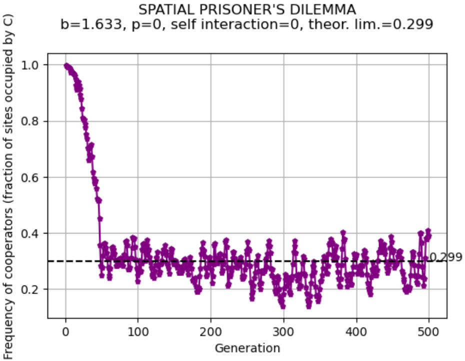


Figure: f_C plot for D invasion with $1.6 < b < 1.667$ and no self-interaction

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4 Conclusions

Concluding Remarks

- **Random Initial Configuration**:
  - For $1.75 < b < 1.8$: Cooperators dominated, forming stable clusters; defectors diminished.
  - For $1.8 < b < 2$: Observed spatial chaos with dynamic coexistence of cooperators and defectors, asymptotic limit $f_C = 0.318$ (?).
- **Defector Invasion**:
  - Emergence of fractal-like patterns indicating continuous strategy shifts, consistent with Nowak's findings.
  - Asymptotic limits: $f_C = 0.318$ (self-interaction) and $f_C = 0.299$ (no self-interaction), aligning with theoretical expectations.
- **Overall Insight**: The system's dynamics are highly sensitive to the temptation parameter b and initial conditions, highlighting the complex interplay between individual strategies and collective behavior.

Q&A

Thank you for your attention