

link *bridging*

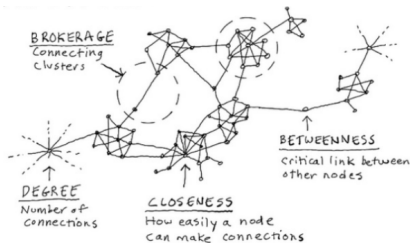
introduction to *network analysis in Python* (*NetPy*)

Lovro Šubelj  
University of Ljubljana  
10th Dec 2019

# bridging *measures*

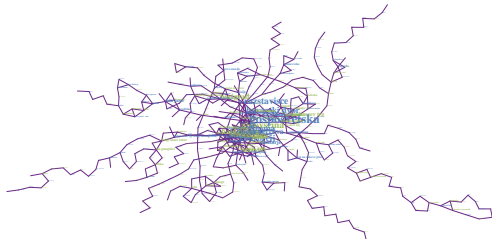
which *links* are most *important*?

- *link bridging measures* for (*un*)*directed* networks
  - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (*un*)*directed* networks
  - *topological overlap* measures [RSM<sup>+</sup>02, OSH<sup>+</sup>07, dNMB11]



# networkology *LPP*

- partial *LPP public bus transport network*\*
- $n = 416$  bus stops with  $\langle k \rangle = 2.72$  connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with  $\langle C \rangle = 0.09$  and  $\langle d \rangle = 14.26$
- “*scale-free*” with  $\gamma = 2.43$  for cutoff  $k_{min} = 2$



---

\* reduced to largest connected component of simple undirected graph

# bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed*  $G$  *link betweenness*  $\sigma$  [Fre77] of  $\{i, j\}$  is
  - $g_{st}$  is number of *shortest paths* *between*  $s$  and  $t$
  - $g_{st}^{ij}$  is number of *such shortest paths* *through*  $\{i, j\}$

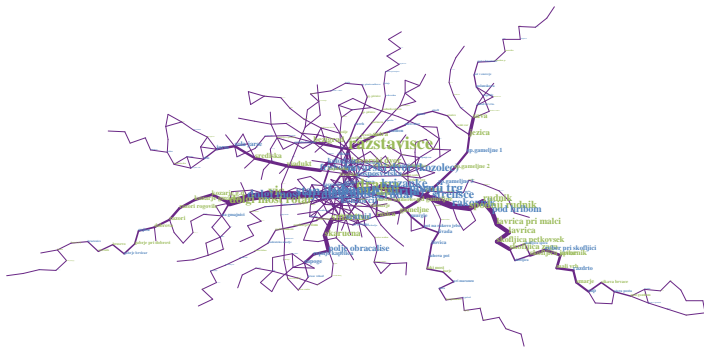
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- $\sigma$  considers *only shortest paths* [FBW91, New05]



# networkology *betweenness*

- *link betweenness*  $\sigma$  in partial LPP network<sup>†</sup>
- *highest*  $\sigma_{ij} = 0.176n^2$  link is *{Vič, Stan in dom}*



<sup>†</sup> reduced to largest connected component of simple undirected graph

# bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected G link embeddedness*<sup>‡</sup>  $\theta$  [OSH<sup>+</sup>07] of  $\{i, j\}$  is

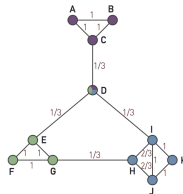
—  $\Gamma_i$  is set of *neighbors* or *neighborhood* of  $i$

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i + k_j - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

—  $\mu$ -corrected *link embeddedness*  $\tilde{\theta}$  [dNMB11] of  $\{i, j\}$  is

—  $\mu$  is *maximum* number of *triangles* over *links*

$$\tilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$

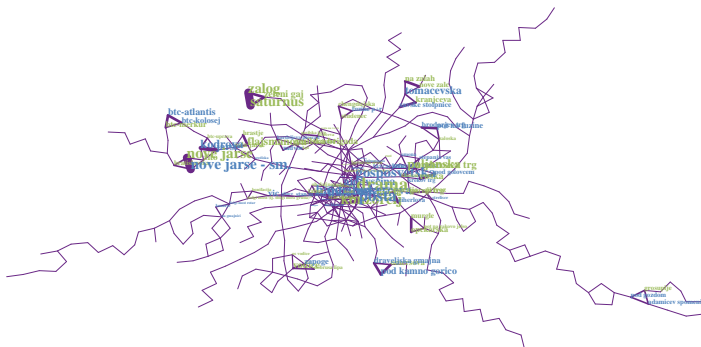


---

<sup>‡</sup> $\theta$  &  $\tilde{\theta}$  better known as topological overlap indices/weights

# networkology *embeddedness*

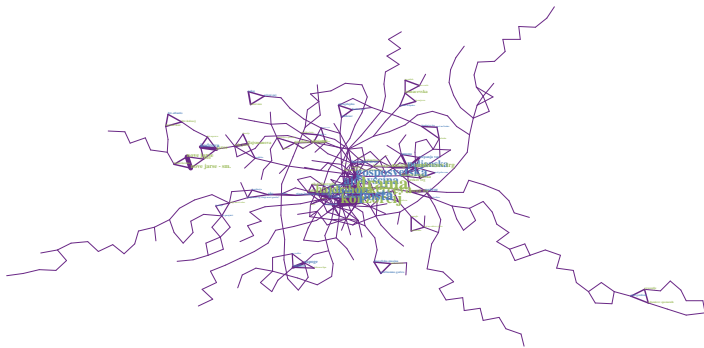
- *link embeddedness*  $\theta$  in partial LPP network<sup>§</sup>
- *highest*  $\theta_{ij} = 1.0$  links are {*Zalog*, *Saturnus*} *etc.*



<sup>§</sup> reduced to largest connected component of simple undirected graph

# networkology $\mu$ -embeddedness

- $\mu$ -corrected embeddedness  $\tilde{\theta}$  in partial LPP network¶
- highest  $\tilde{\theta}_{ij} = 0.4$  links are {*Pošta*, *Konzorcij*} etc.



reduced to largest connected component of simple undirected graph



# bridging *overview*

which *links* are most *important*?

1 IA																		18 VIIIA																			
1	DC	34																5	9	6	13	14	15	16	17	18	19	20	21	22	23	24	EC				
	Degree Centrality																																	Eigenvector Centrality			
2	BC	12	4	13															13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	PR			
	Betweenness Centrality			Closeness Centrality															Subgraph Centrality	Clustering Coefficient	inverse COEF	max. neigh. comp.	edge clustering coefficient	PageRank													
3	RL	2	12	8															11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	LR			
	Range/Linked Betweenness			Information Centrality															11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	LeaderRank			
4	BN	4	29	2	21	1	57	1	90	1	58	1	61	1	62	1	63	1	64	1	65	1	66	1	67	1	68	1	69	1	70	1	71	72	73		
	BrandWalk Centrality			Radiality Centrality	IG	Integration	DCox	BCox	CCox	ECCox	KSxox	PRox	IGox	RCox	DCox	BCox	SCx	KL	COCox	coex. weight COEF	CLique Level	3-localized SC	SCx	LI	ECx	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC			
5	RWBC	30	1	30	39	1	93	1	90	1	94	1	90	1	97	1	96	1	46	1	47	1	48	1	49	1	50	1	51	1	52	1	53	1	54	1	
	RandomWalk Betweenness			RandomWalk Closeness	CC <sub>2,3,4</sub>	2,3,4-localized CC	ECCox	PRox	KSxox	COCox	RCox	IGox	DCox	BCox	CCox	KSxox	DCPR	β	3-localized SC	SCx	LI	ECx	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC		
6	σ	2	26	2	67	71	1	72	1	73	1	74	1	75	1	76	1	77	1	78	1	79	1	80	1	81	1	82	1	83	1	84	1	85	1		
	Stress Centrality			Eccentricity	WDC	Weighted Degree	DCECC	COECC	BCECC	KSECC	PRECC	IGECC	DCC	BCC	CCPR	KSG	DCIG	DCCox	3-localized SC	SCx	LI	ECx	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC		
7	BC <sub>2,3,4</sub>	1	81	1	80	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	1	117	1	118	1	
	2,3,4-localized BC			ECC <sup>-1</sup>	SDC	Sphere Degree Centrality	DCRC	CORC	BCRC	KSRC	PRRC	IGRC	DCKS	BCPR	CCIG	DCPR	BCIG	ECCRC	BCCox	4-localized SC	SCx	LI	ECx	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC	3-localized EC		
Z		mass																28		29	30	31	32	33	34	35	36	37	38	39	40	41	42	43			
C		Name																FC		FD	US	DIS	ASS	DAM	UC												
		(Hybrid)																Functional Centrality		Functional Closeness	UniScore	Pairwise Dis-connectivity	Assortative Mixing	Damage	United comp. Centrality												

# bridging *references*



A.-L. Barabási.

*Network Science.*

Cambridge University Press, Cambridge, 2016.



Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.

*Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.*

Cambridge University Press, Cambridge, 2011.



Ernesto Estrada and Philip A. Knight.

*A First Course in Network Theory.*

Oxford University Press, 2015.



Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.

Centrality in valued graphs: A measure of betweenness based on network flow.

*Soc. Networks*, 13(2):141–154, 1991.



L. Freeman.

A set of measures of centrality based on betweenness.

*Sociometry*, 40(1):35–41, 1977.



M. E. J. Newman.

A measure of betweenness centrality based on random walks.

*Soc. Networks*, 27(1):39–54, 2005.



Mark E. J. Newman.

*Networks: An Introduction.*

Oxford University Press, Oxford, 2010.



J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási.

Structure and tie strengths in mobile communication networks.

*P. Natl. Acad. Sci. USA*, 104(18):7332–7336, 2007.

# bridging *references*



E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and Albert-László Barabási.  
Hierarchical organization of modularity in metabolic networks.  
*Science*, 297(5586):1551–1555, 2002.