

link *bridging*

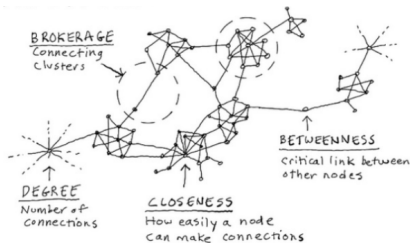
introduction to *network analysis in Python* (*NetPy*)

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bridging *measures*

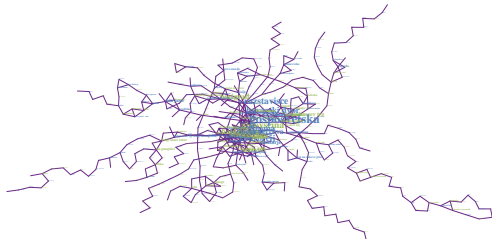
which *links* are most *important*?

- *link bridging measures* for (*un*)*directed* networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (*un*)*directed* networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “*scale-free*” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *shortest paths* *between* s and t
 - g_{st}^{ij} is number of *such shortest paths* *through* $\{i, j\}$

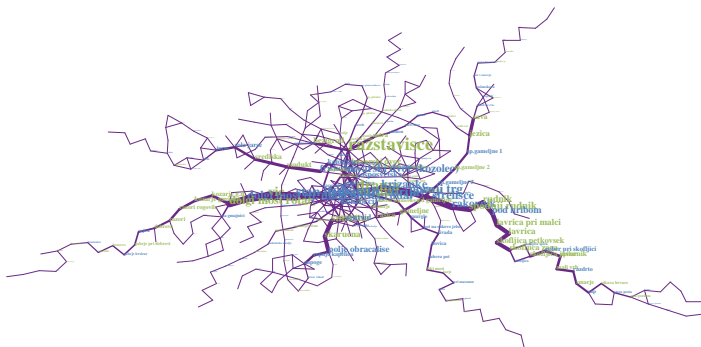
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is *{Vič, Stan in dom}*



[†] reduced to largest connected component of simple undirected graph

bridging *embeddedness*

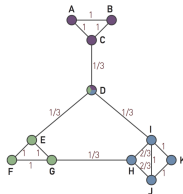
important *links* are *embedded between nodes*

- for *undirected* G *link embeddedness*[‡] θ [OSH⁺07] of $\{i, j\}$ is
 - Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

- μ -*corrected link embeddedness* $\tilde{\theta}$ [dNMB11] of $\{i, j\}$ is
 - μ is *maximum* number of *triangles* over *links*

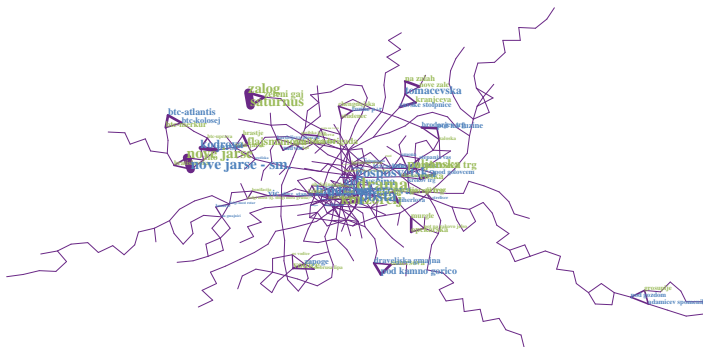
$$\tilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$



[‡] θ & $\tilde{\theta}$ better known as topological overlap indices/weights

networkology *embeddedness*

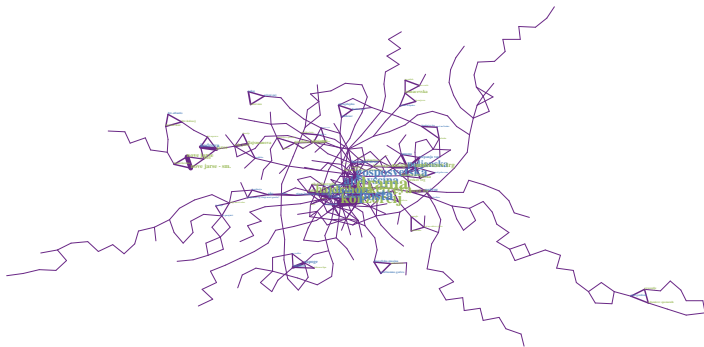
- *link embeddedness* θ in partial LPP network[§]
- *highest* $\theta_{ij} = 1.0$ links are {*Zalog*, *Saturnus*} *etc.*



[§] reduced to largest connected component of simple undirected graph

networkology μ -embeddedness

- μ -corrected embeddedness $\tilde{\theta}$ in partial LPP network¶
- highest $\tilde{\theta}_{ij} = 0.4$ links are {*Pošta*, *Konzorcij*} etc.



reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?

1 IA																		18 VIIIA																	
1	DC	34																5	9	6	13	14	15	16	17	18	19	20	21	22	23	24	EC		
	Degree Centrality																																Eigenvector Centrality		
2	BC	12	4	13															13	14	15	16	17	18	19	20	21	22	23	24	25	26	PR		
	Betweenness Centrality			Closeness Centrality															Subgraph Centrality	Clustering Coefficient	inverse Coeff	max. neigh. comp.	edge clustering coefficient	PageRank											
3	RL	2	12	8															91	1	14	2	15	3	16	4	17	2	18	1	19	20	LR		
	Range/Linked Betweenness			Information Centrality															odd Subgraph Centrality	SC _o	LAC	DMNC	SEC _o	LeaderRank											
4	BN	4	29	2															60	1	31	2	33	1	34	1	35	2	36	1	37	2	KS		
	BrandWalk Centrality			Radiality Centrality															even Subgraph Centrality	SC _e	KL	COC _o	cooper. weight Coeff	PECC _o	KatzRank										
5	RWBC	30	1	39	1	93	1	95	1	94	1	90	1	97	1	96	1	46	1	47	1	48	1	49	1	50	1	51	1	52	1	53	1	EC ₃	
	RandomWalk Betweenness			RandomWalk Closeness		CC _{3,3,4}		ECC _o		PR _o		KS _o		COC _o		RC _o		IG _o		DC _o		BCC _o		CCK _o		KS _{PR}		DC _{PR}		β		SC ₃		NC	
6	σ	2	36	2																													EC ₃		
	Stress Centrality			Eccentricity		WDC		DCEC		CCEC		BCEC		KSECC		PRECC		IGEC		DBCC		BCKS		CCPR		KS _{SG}		DCIG		DCC _o		SC ₃		LI	
7	BC _{3,3,4}	80	1	90	1	90	1	104	1	105	1	106	1	107	1	108	1	110	1	111	1	112	1	113	1	114	1	115	1	116	1	117	1	EC ₃	
	2,3,4 localized-BC			ECC ⁻¹		SDC		DCRC		CORC		BCRC		KSRC		PRRC		IGRC		DCKS		BCPR		CCIG		DCPR		BCIG		ECCRC		BCC _o		SC ₄	
				inverse Eccentricity		Sphere Degree Centrality																											4-localized-SC		
Z		mass																28	2	29	2	30	2	31	2	32	2	33	2	34	2	35	2	FC	
C		Name																	FD	US	DIS	ASS	DAM	UC											
		(Hybrid)																	Functional Closely	UniScore	Pairwise Dis-connectivity	Asymptotic Mixing	Damage	United compl. Centrality											
																			36	1	37	1	38	1	39	1	40	1	41	1	42	1	43	1	CM
																				Normalized α Centrality	Modular Centrality	HGI	HYP	HC											
																				Essentiality Index	Complexity Measure	Harary Graph Information	Hypocyclic Index	Homocyclic Centrality											

- Betweenness-based
- Distance-based
- Linear Combinations
- Subgraph-based
- Clustering Coefficient-based
- Edge Clustering Coefficient-based
- Spectral-based
- Miscellaneous

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bridging *references*



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bridging *references*



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