## Large network structure and random graph models

You are given a selection of real networks of various size and origin.

- Zachary's karate club network (34 nodes)
- Davis's southern women network (32 nodes)
- Lusseau's bottlenose dolphins network (62 nodes)
- <u>Ingredients network by common compounds</u> (1,525 nodes)
- Map of Darknet from Tor network (7,178 nodes)
- Human protein-protein interaction network (19,634 nodes)
- Internet map of autonomous systems (75,885 nodes)
- Amazon product copurchase network (262,111 nodes)
- Paper citation network of APS (438,943 nodes)
- Small part of Google web graph (875,713 nodes)
- Road/highway network of Texas (1,379,917 nodes)

All networks are available in Pajek, edge list and LNA formats.



## I. Toy network construction and Pajek format

- 1. Using your programming library, **construct a small toy network** with a few nodes and edges. **Print out** its name, and the **number of nodes and edges**.
- 2. Using the methods provided by your programming library, read in real networks in Pajek format

above and **print out their basic statistics**. What is the size of the largest network you are able to construct in say a minute?

## II. Basic network statistics, connectivity, distances etc.

- 1. Compute and print out **basic statistics of real networks** above. Namely, the number of nodes n, the number of isolated nodes  $n_0$ , the number of edges m, the number of self-edges or loops  $m_0$ , the average node degree  $\langle k \rangle = 2m/n$  and the undirected density  $\rho = \langle k \rangle / (n-1)$ . Are the results **expected or surprising**? (Computational complexity of these computations is either constant  $\mathcal{O}(1)$  or linear  $\mathcal{O}(m)$  and should be applicable to any network you are able to fit in your memory.)
- 2. Using **depth-first search** methods provided by your programming library, compute **connected components of real networks** above. Print out the fraction of nodes in the largest connected component S and the number of all connected components S. Are the results **expected or surprising**? (Computational complexity of these computations is linear O(m) and should be applicable to any network you are able to fit in your memory.)
- 3. Using **breadth-first search** methods provided by your programming library, compute **distances between the nodes of real networks** above. Print out the average distance between the nodes  $\langle d \rangle$  and the maximum distance or diameter  $d_{max}$ . Are the results **expected or surprising**? (Computational complexity of these computations is inevitably quadratic  $\mathcal{O}(nm)$  and you should use approximation implemented in the started script.)
- 4. Using **triad counting** methods provided by your programming library, compute **node clustering coefficient of real networks** above. Print out the average node clustering coefficient  $\langle C \rangle$ . Are the results **expected or surprising**? (Computational complexity of these computations is superlinear  $\mathcal{O}(m\langle k \rangle)$  and should be applicable to all but the largest networks.)
- 5. Using plotting functionality provided by your programming library, compute degree distribution of real networks above. Plot degree distribution  $p_k$  on a doubly logarithmic plot. Are the results expected or surprising? (Computational complexity of these computations is linear  $\mathcal{O}(n)$  and should be applicable to any network you are able to fit in your memory.)
- 6. What is the size of the largest network you are able to analyze in say a minute?

## III. Random graphs, scale-free and small-world network models

- 1. Construct **Erdös-Rényi random graphs** G(n, m) with the same number of nodes n and edges m as **real networks** above. Print out their standard statistics. Are the results **expected or surprising**?
- 2. Next, construct **Barabási-Albert scale-free networks**  $G(n, \langle k \rangle/2)$  with the same number of nodes n and the average degree  $\langle k \rangle$  as **real networks** above. Print out their standard statistics. Are the results **expected or surprising**?

3. Finally, construct also **Watts-Strogatz small-world networks**  $G(n, \langle k \rangle, p)$  with the same number of nodes n and edges m as **real networks** above. Print out their standard statistics. Are the results **expected or surprising**?

