core-periphery structure

introduction to *network analysis in Python* (*NetPy*)

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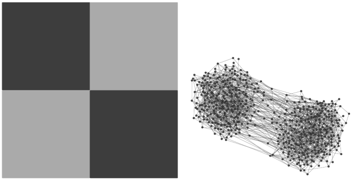
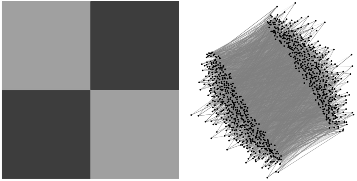
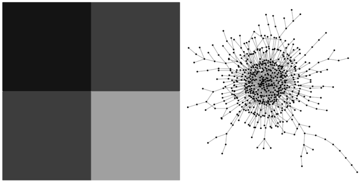
core-periphery *block model*



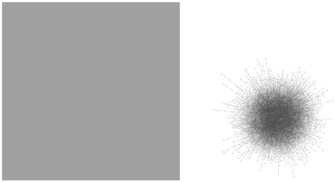
community block model [GN02]



disassortative block model [NL07]



core-periphery block model [Sei83]



random graph [ER59]

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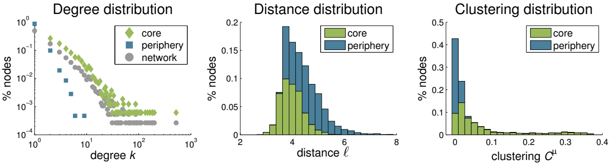
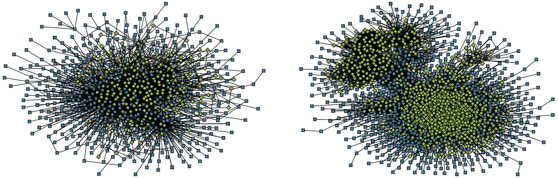
* origin of core-periphery structure in international relations

core-periphery *structure*

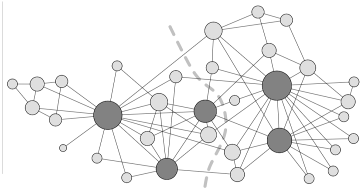
- *core/periphery nodes* have *higher/lower degrees k*
- *core/periphery nodes* are on *shorter/longer distances ℓ*
- *core/periphery nodes* have *higher/lower clustering C*



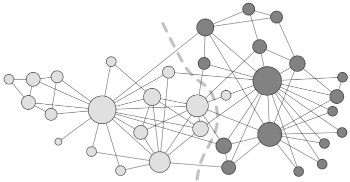


core-periphery *stochastic*

- $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$ *stochastic block model* [HLL83]
  - n_i is *size* of *cluster* C_i & p_{ij} is *link density* between C_i and C_j
- *density-based core-periphery* structure for $p_{11} \gg p_{12} \gg p_{22}$
- *lookalike core-periphery* for $n_1 p_{11} \gg 1$, $n_1 p_{12} \ll 1$, $n_2 p_{22} \approx 1$



non-corrected block model $p_{11} > p_{12} > p_{22}$



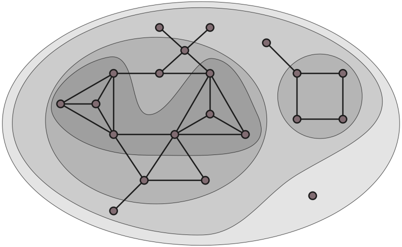
degree-corrected block model $p_{11} \approx p_{22} > p_{12}$

core-periphery *k-cores*

- *k-cores* are *subgraphs of nodes* with $\geq k$ *neighbors* [Sei83]

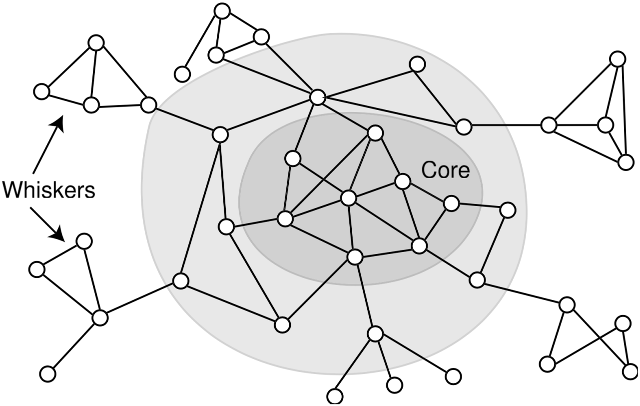
  remove nodes with degree $< k$ until no such node remains [BZ11]

- *k-shells* are *nodes of k-cores* that are *not in $k+1$-cores*
- *k-cores* are *nested* while *k-shells* form *decomposition*



0-cores are connected components & k-cores can be disconnected

core-periphery *nestedness*



nested cores & whiskers communities [LLDM09, YL13]

core-periphery *references*

V. Batagelj and M. Zaveršnik.
An O(m) algorithm for cores decomposition of networks.
Adv. Data Anal. Classif., 5(2):129–145, 2011.

P. Erdős and A. Rényi.
On random graphs I.
Publ. Math. Debrecen, 6:290–297, 1959.

M. Girvan and M. E. J Newman.
Community structure in social and biological networks.
P. Natl. Acad. Sci. USA, 99(12):7821–7826, 2002.

Paul W. Holland, Kathryn Blackmond Laskey, and Samuel Leinhardt.
Stochastic blockmodels: First steps.
Soc. Networks, 5(2):109–137, 1983.

Jure Leskovec, Kevin J Lang, Anirban Dasgupta, and Michael W Mahoney.
Community structure in large networks: Natural cluster sizes and the absence of large well-defined clusters.
Internet Math., 6(1):29–123, 2009.

Tilen Marc and Lovro Šubelj.
Convexity in complex networks.
Netw. Sci., 6(2):176–203, 2018.

M. E. J Newman and E. A Leicht.
Mixture models and exploratory analysis in networks.
P. Natl. Acad. Sci. USA, 104(23):9564–9569, 2007.

Stephen B. Seidman.
Network structure and minimum degree.
Soc. Networks, 5(3):269–287, 1983.

core-periphery *references*

J. Yang and Jure Leskovec.
Overlapping community detection at scale: A nonnegative matrix factorization approach.
In *Proceedings of the ACM International Conference on Web Search and Data Mining*, pages 587–596, Rome, Italy, 2013.