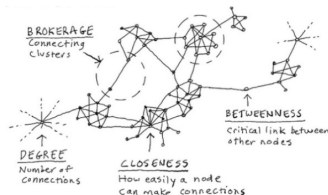# node *centrality*

introduction to *network analysis in Python* (*NetPy*)

Lovro Šubelj
University of Ljubljana
10th Dec 2019

centrality *measures*

which *nodes* are most *important*?

— *node centrality measures* for (*un*)*directed* networks
  – *clustering coefficients* [WS98, SV05, dNMB05]
  – *distance-based* centrality [Fre77, FBW91, New05]
  – *spectral analysis* centrality [Kat53, Bon87, BP98]

— *link analysis algorithms* for *directed* networks

networkology *LPP*

- — partial *LPP public bus transport network**
- — $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- — *giant component* 95.4% nodes (6 components)
- — "*small-world*" with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- — "*scale-free*" with $\gamma = 2.62$ for cutoff $k_{min} = 5$

---
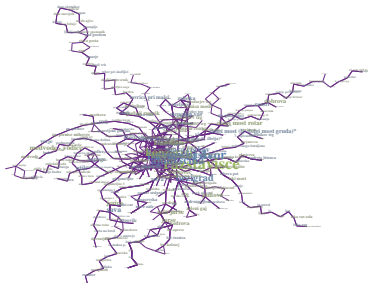
*reduced to largest connected component

centrality *clustering*

important *nodes* are *strongly* *embedded*

— for *undirected* G *clustering coefficient* C [WS98] of i is
  – t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \qquad C_i = 0 \text{ for } k_i \leq 1$$

— μ-*corrected clustering coefficient* C^μ [dNMB05] of i is
  – μ is *maximum* number of *triangles* over *links*

$$C_i^\mu = \frac{2t_i}{k_i\mu} \qquad C_i^\mu = 0 \text{ for } k_i = 0$$

— *clustering coefficient C* in partial LPP network[†]

— *highest $C_i = 1.0$* nodes are *Na Žalah etc.* with $k_i = 2$

[†] reduced to simple undirected graph

networkology μ-clustering

— μ-corrected clustering C^{μ} in partial LPP network[‡]

— highest $C_i^{\mu} = 0.44$ node is *Drama* with $k_i = 10$

[‡] reduced to simple undirected graph

centrality *closeness*

important *nodes* are *close to other* nodes

— for (*un*)*directed G closeness centrality* ℓ^{-1} [New10] of i is

  – d_{ij} is (*un*)*directed distance* between i and j
  – $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

— ℓ^{-1} spans *small range* in *small-world* networks

networkology *closeness*

— *closeness centrality* ℓ^{-1} in partial LPP network[§]

— *highest* $\ell_i^{-1} = 0.208$ node is *Gosposvetska* with $k_i = 14$
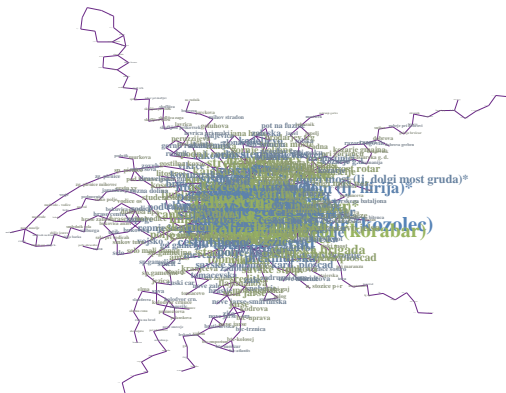
[§] reduced to simple undirected graph

centrality *betweenness*

important *nodes* are *bridges for other* nodes

— for (*un*)*directed G betweenness centrality* σ [Fre77] of *i* is
  – g_{st} is number of *shortest paths between s* and *t*
  – g_{st}^{i} is number of *such shortest paths through i*

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^{i}}{g_{st}}$$

— σ considers *only shortest paths* [FBW91, New05]

networkology *betweenness*

— *betweenness centrality σ* in partial LPP network[¶]

— *highest $\sigma_i = 0.235$* node is *Razstavišče* with $k_i = 11$
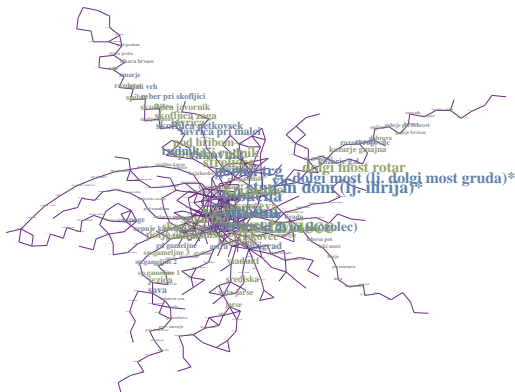
[¶] reduced to simple undirected graph

centrality *degrees*

important *nodes* are *linked by* *many* nodes

— for *undirected* *G* *degree centrality* *d* of *i* is
$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

— in *directed* *G* *in*-degree centrality d^{in} of *i* is
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

— in *directed* *G* *out-degree centrality* d^{out} of *i* is
$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *degrees*

— *degree centrality d* in partial LPP network
— *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
— *highest* d_i' node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$

centrality *eigenvector*

important *nodes* are *linked by important* nodes

— for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
  – e is *leading eigenvector* v_1 of A with *eigenvalue* λ_1^{-1}

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

— in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *eigenvector*

— *eigenvector centrality e* in partial LPP network
— *highest $e_i = 0.082$ node* is *Konzorcij* with $k_i = 30$
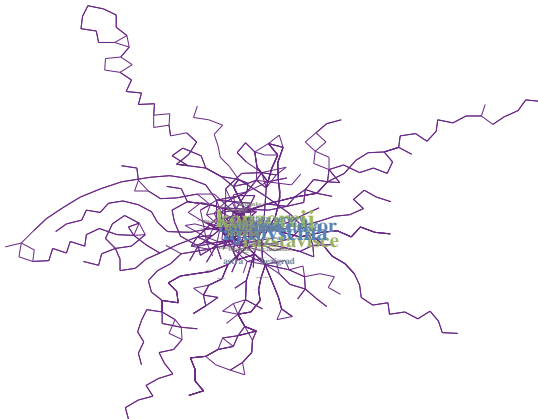
centrality *Katz*

nodes get small amount of *importance for free*

— for (*un*)*directed* G *Katz centrality* z [Kat53] of *i* is
 – α and β are *appropriate positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta$$

— for *convenience* $\beta = 1$ whereas $\alpha < \lambda_1^{-1}$
 – λ_1 is *leading eigenvalue* of A for *eigenvector* v_1

centrality *PageRank*

nodes distribute equal amount of *importance*

— for (*un*)*directed* **G** *PageRank centrality* p [BP98] of i is

    – α and β are *appropriate positive constants*

$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \beta$$

— for *convenience* $\beta = \frac{1-\alpha}{n}$ whereas $\alpha = 0.85$

see PageRank algorithm NetLogo demo

— *PageRank centrality p* in partial LPP network
— *highest* $p_i = 0.011$ node is *Razstavišče* with $k_i = 41$
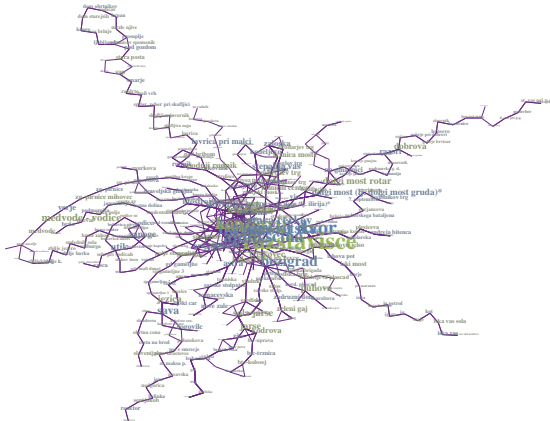
centrality *overview*

which *nodes* are most *important*?

©David Schoch (University of Konstanz)

- Betweenness-based
- Distance-based
- Linear Combinations
- Subgraph-based
- Clustering Coefficient-based
- Edge Clustering Coefficient-based
- Spectral-based
- Miscellaneous

centrality *references*

Phillip Bonacich.
Power and centrality: A family of measures.
American Journal of Sociology, 92(5):1170–1182, 1987.

S. Brin and L. Page.
The anatomy of a large-scale hypertextual Web search engine.
Comput. Networks ISDN, 30(1-7):107–117, 1998.

Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.
Exploratory Social Network Analysis with Pajek.
Cambridge University Press, Cambridge, 2005.

David Easley and Jon Kleinberg.
Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
Cambridge University Press, Cambridge, 2010.

Ernesto Estrada and Philip A. Knight.
A First Course in Network Theory.
Oxford University Press, 2015.

Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.
Centrality in valued graphs: A measure of betweenness based on network flow.
Soc. Networks, 13(2):141–154, 1991.

L. Freeman.
A set of measures of centrality based on betweenness.
Sociometry, 40(1):35–41, 1977.

Leo Katz.
A new status index derived from sociometric analysis.
Psychometrika, 18(1):39–43, 1953.

M. E. J. Newman.
A measure of betweenness centrality based on random walks.
Soc. Networks, 27(1):39–54, 2005.

Mark E. J. Newman.
Networks: An Introduction.
Oxford University Press, Oxford, 2010.

Sara Nadiv Soffer and Alexei Vázquez.
Network clustering coefficient without degree-correlation biases.
Phys. Rev. E, 71(5):057101, 2005.

D. J. Watts and S. H. Strogatz.
Collective dynamics of 'small-world' networks.
Nature, 393(6684):440–442, 1998.