link *bridging*

introduction to *network analysis in Python* (*NetPy*)

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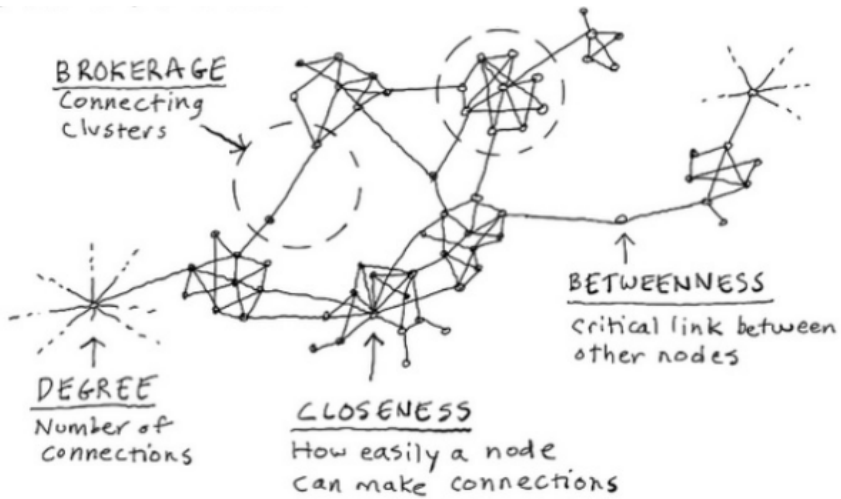
10th Dec 2019

bridging *measures*

which *links* are most *important*?

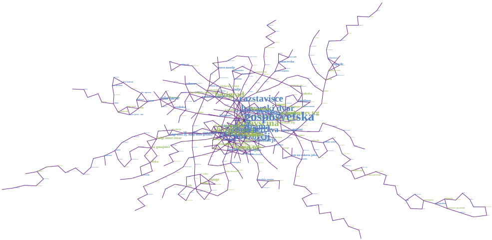
— *link bridging measures* for (*un*)*directed* networks
  – *betweenness-based* centrality [Fre77, FBW91, New05]

— *link embeddedness measures* for (*un*)*directed* networks
  – *topological overlap* measures [RSM+02, OSH+07, dNMB11]



networkology *LPP*

- partial *LPP* *public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- "*small-world*" with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- "*scale-free*" with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

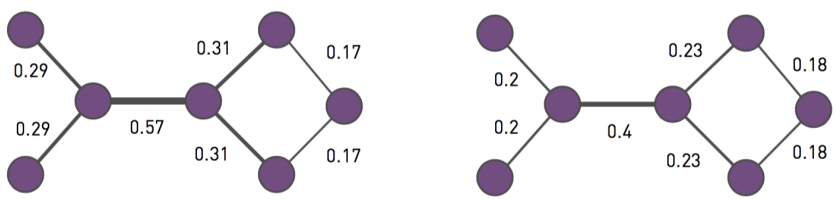
bridging *betweenness*

important *links* are *between other nodes*

— for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i, j\}$ is
  - g_{st} is number of *shortest paths between* s and t
  - g_{st}^{ij} is number of *such shortest paths through* $\{i, j\}$

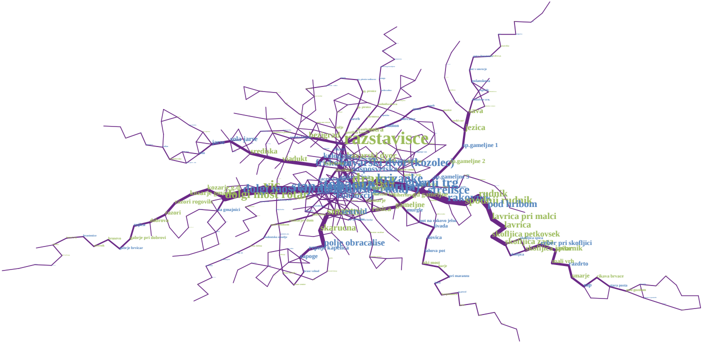
$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

— σ considers *only shortest paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network†
- *highest* $\sigma_{ij} = 0.176n^2$ link is $\{$*Vič, Stan in dom*$\}$



† reduced to largest connected component of simple undirected graph

bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected* G *link embeddedness*[‡] θ [OSH+07] of $\{i, j\}$ is

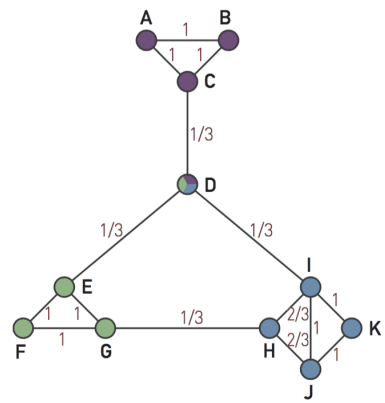
- Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \qquad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

— μ*-corrected link embeddedness* $\widetilde{\theta}$ [dNMB11] of $\{i, j\}$ is

- μ is *maximum* number of *triangles* over *links*

$$\widetilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$

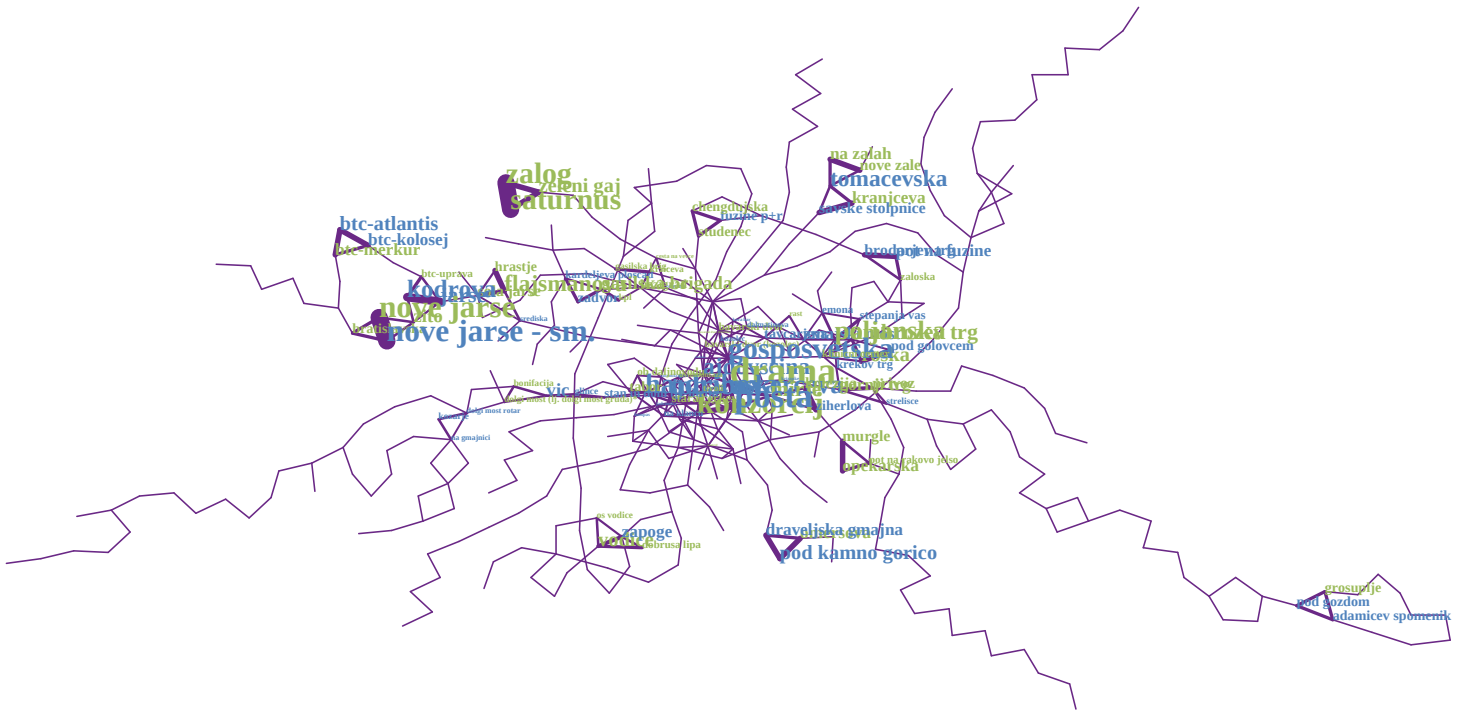


[‡] θ & $\widetilde{\theta}$ better known as topological overlap indices/weights

networkology *embeddedness*

— *link embeddedness* θ in partial LPP network[§]

— *highest* $\theta_{ij} = 1.0$ links are $\{$*Zalog, Saturnus*$\}$ *etc.*

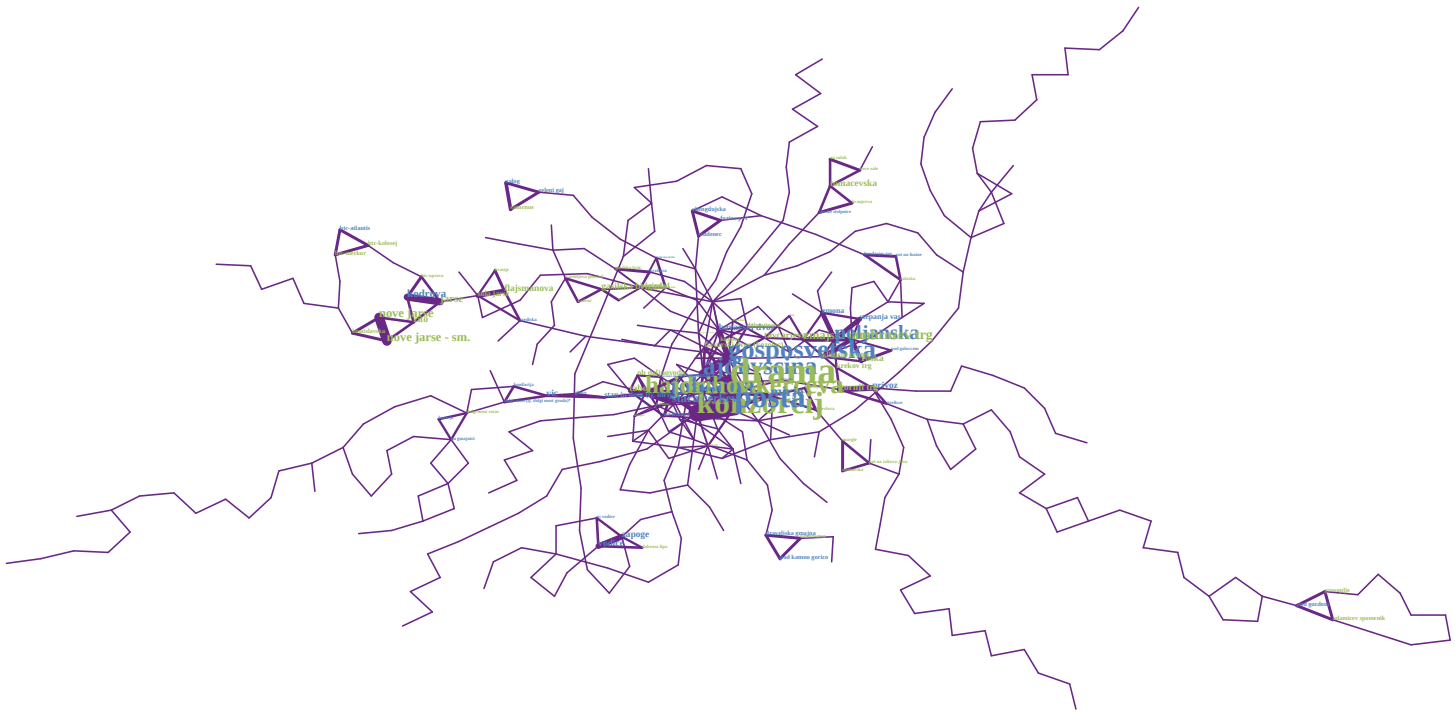


[§] reduced to largest connected component of simple undirected graph

networkology *μ-embeddedness*

— *μ-corrected embeddedness* $\widetilde{\theta}$ in partial LPP network[¶]

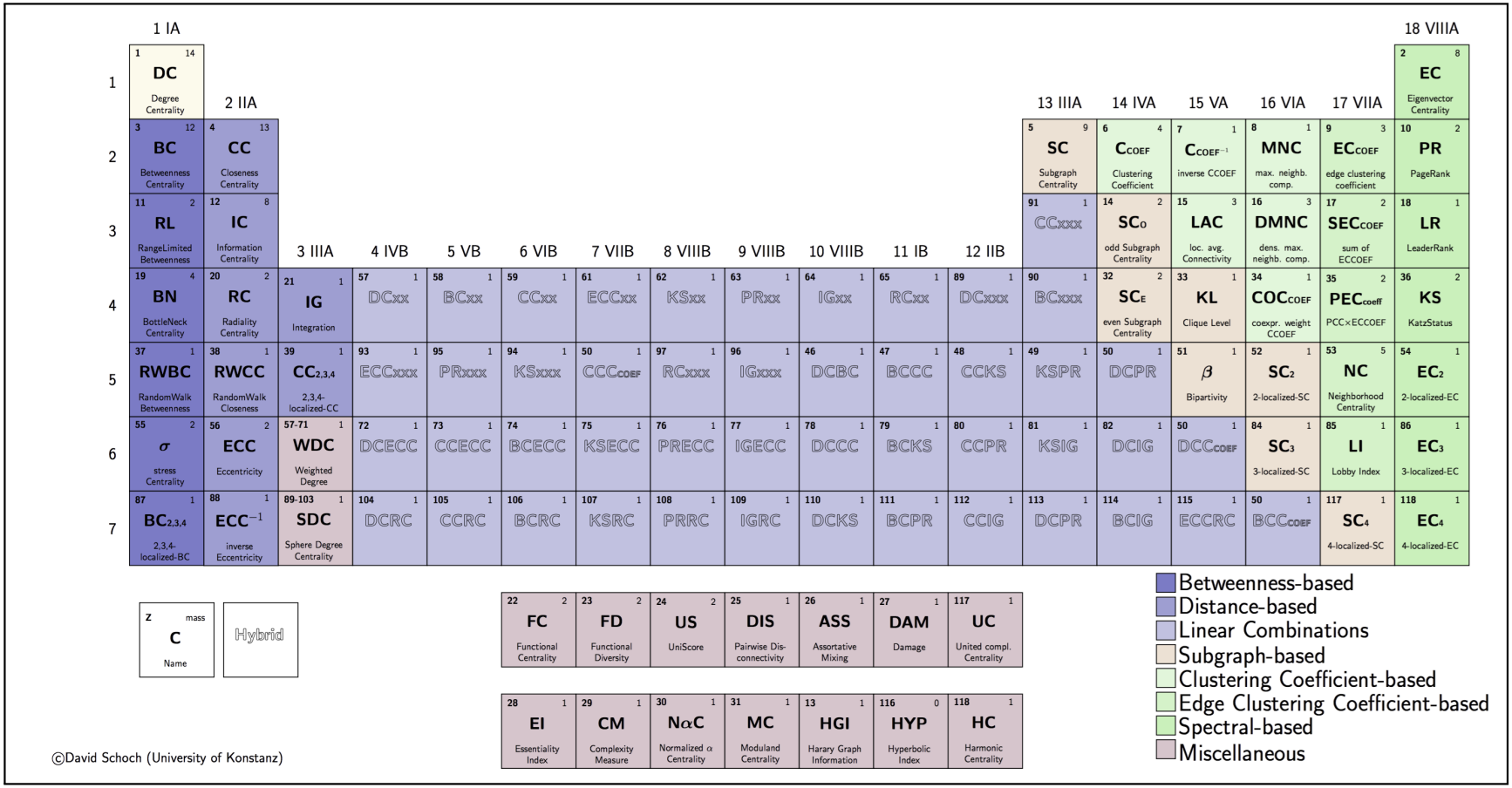
— *highest* $\widetilde{\theta}_{ij} = 0.4$ links are {*Pošta, Konzorcij*} *etc.*



[¶] reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?



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bridging *references*

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