graph theory o *network science*

introduction to network analysis in Python (NetPy)

Lovro Šubelj University of Ljubljana 19th Sep 2019

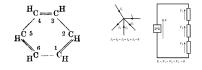
history graph theory

1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler) 1800s *travelling salesman* problem (William Hamilton)



1845 electrical circuit laws (Gustav Kirchhoff)

1857 *chemical structure* theory (August Kekulé)



history operations research

```
1956 shortest paths (Edsger Dijkstra)
1956 minimum spanning tree (Joseph Kruskal)
1956 maximum flow/minimum cut (Ford & Fulkerson)
1956 signed graph theory [CH56] (Cartwright & Harary)
1959 random graph theory [ER59] (Erdős & Rényi)
```

history sociometry

1934 children sociograms [Mor34] (Jacob Moreno)



1941 Southern women [DGG41] (Allison Davis)

1970 university karate club [Zac77] (Wayne Zachary)





1967 small-world experiment [Mil67] (Stanley Milgram)

1973 strength of weak ties [Gra73] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

history bibliometrics

1965 scientific paper citations [Pri65] (Derek de Solla Price)



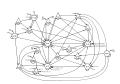
SCIENCE CITATION INDEX

1980s political scandals [HL03] (Mark Lombardi)

1986 neural wirings [WSTB86] (White et al.)

1999 transportation [Pel99] (Jon Pelletier)







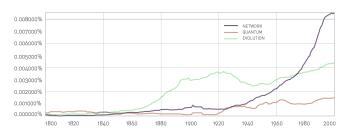
networks boom

< 2000 small graphs 10^2 - 10^3 nodes

 \approx 2000 communication networks 10^5 - 10^8 nodes

 \approx 2005 online social networks 10⁸ nodes

today Facebook graph $> 10^9$ users today Web graph $> 10^{12}$ pages

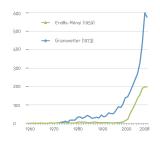


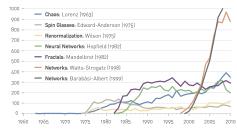
network *models*

1959 random graph theory [ER59] 1973 valued graphs theory [Gra73]

1998 *small-world network* structure [WS98]

1999 scale-free network structure [BA99]





networks language

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the next century will be the century of complexity."

Stephen Hawking

network *science*

problem understanding real networks

means

study of network properties design of mathematical models implementation of efficient algorithms

goals

network structure and evolution nodes, fragments, clusters, layers, network network dynamics and processes spreading, diffusion, epidemics

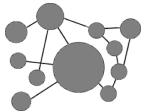
network analysis











network analysis

history references



A.-L. Barabási and R. Albert.

Emergence of scaling in random networks. Science, 286(5439):509–512, 1999.



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Dorwin Cartwright and Frank Harary.

Structural balance: A generalization of Heider's theory.

Psychological Review, 63(5):277-293, 1956.



A. Davis, B. B. Gardner, and M. R. Gardner.

Deep South.

Chicago University Press, Chicago, 1941.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World. Cambridge University Press. Cambridge. 2010.



P. Erdős and A. Rénvi.

On random graphs I.

Publ. Math. Debrecen, 6:290-297, 1959.



Leonhard Euler.

Solutio problematis ad geometriam situs pertinentis. Comment. Academiae Sci. I. Petropolitanae, 8:128–140, 1736.



L. Freeman.

A set of measures of centrality based on betweenness. *Sociometry*, 40(1):35–41, 1977.

11/13 @ NetPy

history references



Mark S. Granovetter.

The strength of weak ties.

Am. J. Sociol., 78(6):1360-1380, 1973.



César A. Hidalgo.

Disconnected, fragmented, or united? A trans-disciplinary review of network science. Appl. Netw. Sci., 1:6, 2016.



Robert Hobbs and Mark Lombardi

Mark Lombardi: Global Networks.

Independent Curators International, New York, 2003.



Stanley Milgram.

The small world problem. Psychol. Today, 1(1):60–67, 1967.



J. L. Moreno.

Who Shall Survive?

Beacon House, Beacon, 1934.



Mark E. J. Newman.

Networks: An Introduction.

Oxford University Press, Oxford, 2010.



Jon D. Pelletier.

Self-organization and scaling relationships of evolving river networks. Journal of Geophysical Research, 104(4):7359–7375, 1999.



D. J. de Solla Price.

Networks of scientific papers.

Science, 149:510-515, 1965.

history references



D. J. Watts and S. H. Strogatz.

Collective dynamics of 'small-world' networks. *Nature*, 393(6684):440–442, 1998.



J. G. White, E. Southgate, J. N. Thomson, and S. Brenner.

The structure of the nervous system of the nematode Caenorhabditis elegans. *Phil. Trans. R. Soc. Lond. B*, 314(1165):1–340, 1986.



Wayne W. Zachary.

An information flow model for conflict and fission in small groups.

J. Anthropol. Res., 33(4):452-473, 1977.